Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.7-Miscellaneous/137-4.7.3-c+d-x-^m-trig^n-trig^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [397]. This is test number [137].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (397)	0.00 (0)
Mathematica	99.75 (396)	0.25 (1)
Fricas	91.94 (365)	8.06 (32)
Maple	90.43 (359)	9.57 (38)
Maxima	85.89 (341)	14.11 (56)
Giac	61.96 (246)	38.04 (151)
Mupad	39.04 (155)	60.96 (242)
Sympy	31.74 (126)	68.26 (271)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

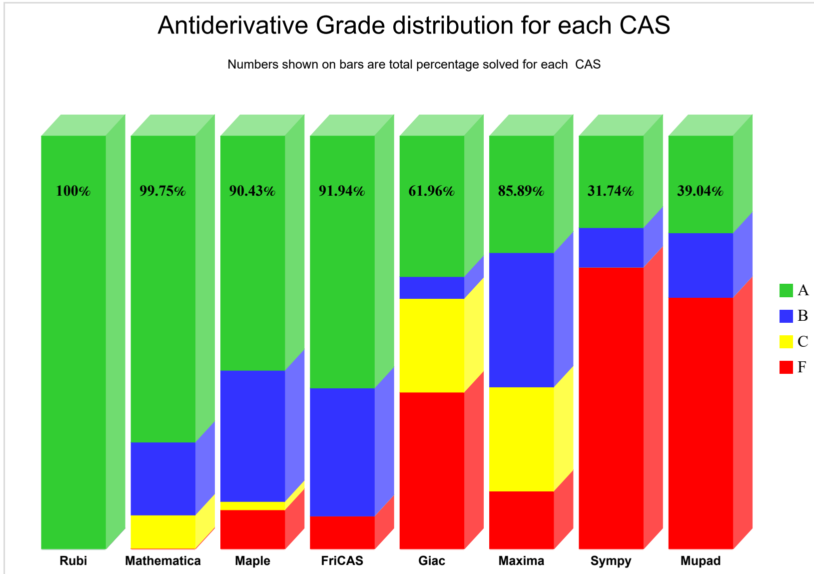
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

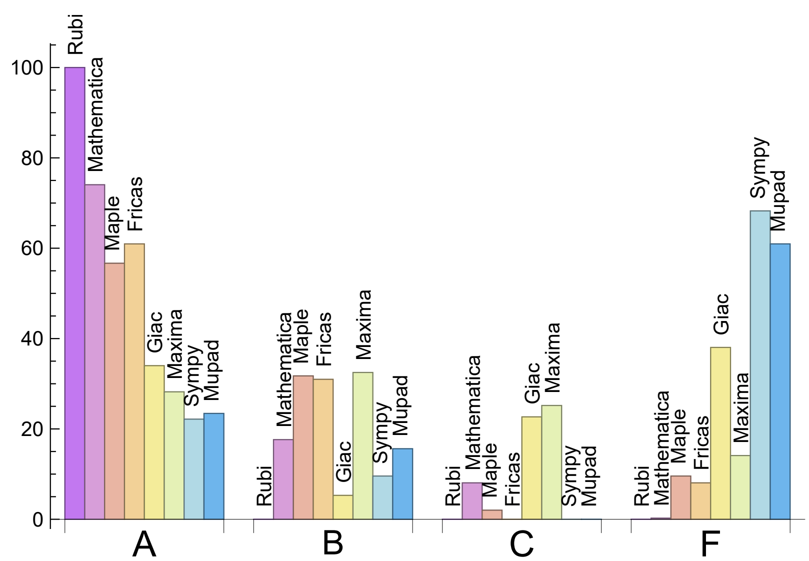
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	74.06	17.63	8.06	0.25
Fricas	60.96	30.98	0.00	8.06
Maple	56.68	31.74	2.02	9.57
Giac	34.01	5.29	22.67	38.04
Maxima	28.21	32.49	25.19	14.11
Mupad	N/A	15.62	0.00	60.96
Sympy	22.17	9.57	0.00	68.26

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	0.00 %	100.00 %	0.00 %
Maple	38	100.00 %	0.00 %	0.00 %
Fricas	32	0.00 %	0.00 %	100.00 %
Giac	151	96.03 %	3.97 %	0.00 %
Maxima	56	94.64 %	0.00 %	5.36 %
Sympy	271	67.90 %	18.08 %	14.02 %
Mupad	242	85.54 %	14.46 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

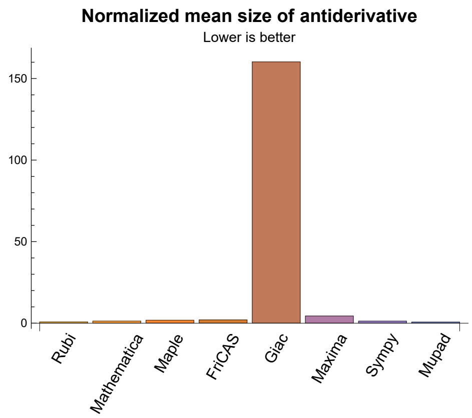
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

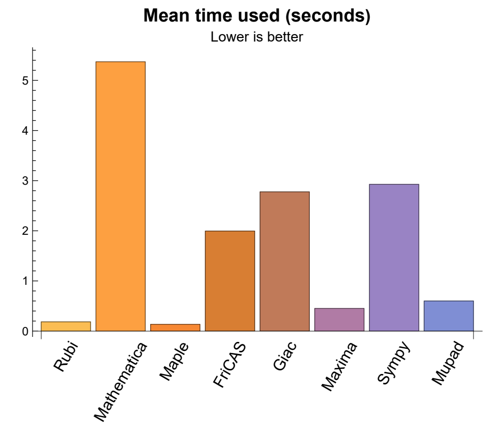
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.18	149.21	0.77	115.00	1.00
Mathematica	5.37	275.06	1.28	132.00	0.92
Maple	0.13	345.30	1.81	245.00	1.42
Maxima	0.45	858.40	4.40	308.00	1.47
Fricas	2.00	406.30	2.02	235.00	1.21
Sympy	2.93	175.54	1.26	0.00	0.00
Giac	2.78	50405.91	160.25	216.00	1.41
Mupad	0.60	84.90	0.67	-1.00	-0.01

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
150
100
50
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
5
4
3
2
1
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{31, 36, 37, 38, 43, 44, 45, 50, 51, 97, 102, 103, 104, 109, 110, 111, 116, 117, 163, 168, 169, 170, 175, 176, 177, 182, 183, 208, 213, 214, 215, 219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 245, 246, 251, 252, 253, 257, 258, 259, 263, 264, 265, 270, 271, 272, 276, 277, 278, 282, 283, 284, 288, 289, 290, 295, 296, 297, 301, 302, 303, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 379, 380, 381, 386, 387, 388, 392, 393, 394\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {41, 46, 164, 173, 222, 236, 237, 241, 242, 249, 254, 255, 261, 280, 286, 291, 292, 300, 304, 305, 306, 310, 311, 312, 318, 319, 331, 382, 383, 385, 390}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

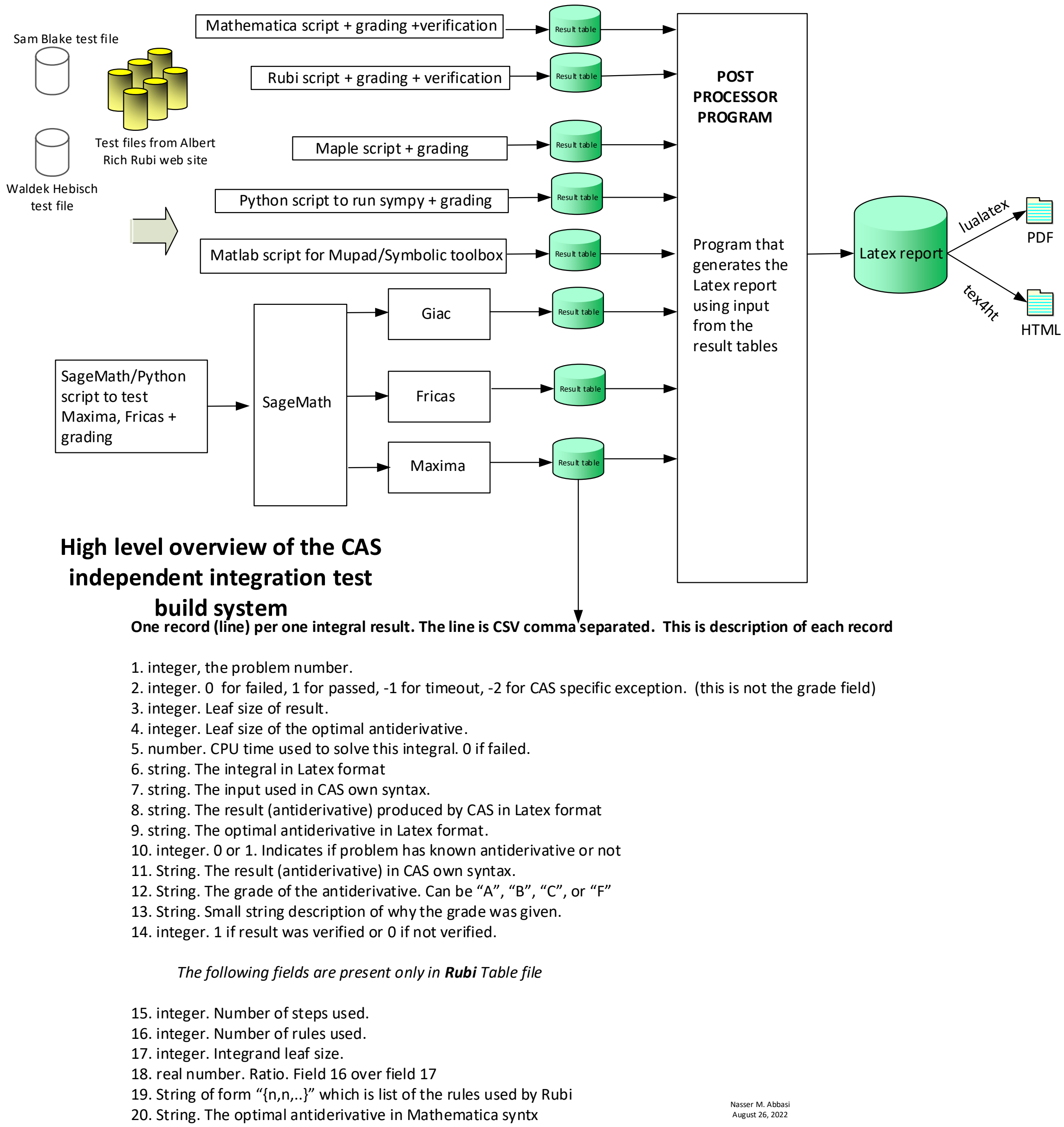
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 37, 38, 40, 43, 44, 45, 49, 50, 51, 52, 53, 54, 55, 56, 57, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 109, 110, 111, 112, 113, 116, 117, 124, 125, 126, 127, 128, 129, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 170, 174, 175, 176, 177, 182, 183, 184, 185, 186, 187, 188, 189, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 219, 220, 221, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 238, 239, 240, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 256, 257, 258, 259, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 282, 283, 284, 285, 288, 289, 290, 293, 294, 295, 296, 298, 301, 302, 303, 307, 308, 309, 313, 314, 315, 316, 317, 320, 321, 322, 323, 326, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 341, 343, 344, 346, 348, 349, 350, 351, 352, 353, 354, 355, 357, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 377, 378, 379, 380, 381, 386, 387, 388, 391, 392, 393, 394, 395, 396 }

B grade: { 32, 33, 34, 35, 39, 41, 42, 46, 47, 98, 99, 105, 106, 107, 114, 115, 145, 164, 165, 166, 171, 172, 173, 178, 179, 180, 181, 216, 218, 222, 223, 228, 235, 236, 241, 242, 250, 254, 255, 260, 261, 266, 280, 281, 286, 287, 291, 292, 299, 300, 304, 305, 306, 310, 311, 312, 318, 324, 325, 331, 340, 342, 376, 382, 383, 384, 385, 389, 390, 397 }

C grade: { 48, 58, 59, 60, 61, 62, 63, 108, 118, 119, 120, 121, 122, 123, 130, 131, 132, 133, 134, 135, 190, 191, 192, 193, 194, 195, 237, 319, 345, 347, 356, 358 }

F grade: { 297 }

2.1.3 Maple

A grade: { 5, 6, 7, 8, 9, 10, 11, 12, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 36, 37, 38, 43, 44, 45, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 102, 103, 104, 108, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 149, 150, 151, 152, 153, 159, 160, 161, 162, 163, 168, 169, 170, 175, 176, 177, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 213, 214, 215, 218, 219, 220, 221, 224, 225, 226, 227, 232, 233, 234, 237, 238, 239, 240, 244, 245, 246, 251, 252, 253, 256, 257, 258, 259, 263, 264, 265, 269, 270, 271, 272, 276, 277, 278, 281, 282, 283, 284, 287, 288, 289, 290, 294, 295, 296, 297, 301, 302, 303, 306, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 360, 362, 363, 364, 365, 366, 367, 371, 372, 373, 374, 375, 379, 380, 381, 385, 386, 387, 388, 391, 392, 393, 394, 395, 396 }

B grade: { 2, 3, 4, 14, 15, 16, 23, 24, 25, 32, 33, 34, 35, 39, 40, 41, 42, 46, 47, 48, 71, 72, 73, 80, 81, 82, 83, 89, 90, 91, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 137, 138, 139, 146, 147, 148, 155, 156, 157, 158, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 209, 210, 211, 212, 216, 217, 222, 223, 228, 229, 230, 231, 235, 236, 241, 242, 243, 247, 248, 249, 250, 254, 255, 260, 261, 266, 267, 268, 273, 274, 279, 280, 286, 291, 292, 293, 298, 299, 300, 304, 305, 310, 311, 312, 313, 317, 318, 319, 323, 324, 325, 361, 368, 369, 370, 376, 377, 378, 382, 383, 384, 389, 390, 397 }

C grade: { 174, 204, 262, 275, 331, 340, 347, 356 }

F grade: { 1, 13, 22, 70, 79, 88, 136, 145, 154, 285, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 357, 358, 359 }

2.1.4 Maxima

A grade: { 5, 17, 26, 31, 36, 37, 38, 43, 44, 45, 50, 51, 74, 92, 97, 102, 103, 104, 109, 110, 111, 116, 117, 140, 149, 158, 163, 168, 169, 170, 175, 176, 177, 182, 183, 208, 213, 214, 215, 219, 220, 221, 224, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 245, 246, 251, 252, 253, 257, 258, 259, 263, 264, 265, 270, 271, 272, 276, 277, 278, 282, 283, 284, 288, 289, 290, 295, 296, 297, 301, 302, 303, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 360, 361, 362, 363, 364, 368, 369, 370, 371, 379, 380, 381, 384, 386, 387, 388, 392, 393, 394 }

B grade: { 2, 3, 4, 14, 15, 16, 23, 24, 25, 32, 33, 34, 35, 39, 40, 41, 42, 46, 47, 48, 49, 71, 72, 73, 80, 81, 82, 83, 89, 90, 91, 98, 99, 100, 101, 105, 106, 107, 108, 112, 113, 114, 115, 137, 138, 139, 146, 147, 148, 155, 156, 157, 164, 165, 166, 167, 171, 172, 173, 174, 178, 179, 180, 181, 205, 206, 207, 209, 210, 211, 212, 216, 217, 222, 223, 228, 229, 230, 231, 235, 236, 241, 242, 243, 247, 248, 250, 254, 255, 256, 260, 262, 266, 267, 268, 269, 273, 274, 275, 279, 280, 281, 285, 286, 287, 291, 292, 293, 294, 298, 299, 304, 305, 306, 310, 311, 312, 313, 317, 318, 323, 324, 325, 376, 377, 382, 383, 391, 396 }

C grade: { 6, 7, 8, 9, 10, 11, 12, 18, 19, 20, 21, 27, 28, 29, 30, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 75, 76, 77, 78, 84, 85, 86, 87, 93, 94, 95, 96, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 365, 366, 367, 372, 373, 374, 375 }

F grade: { 1, 13, 22, 70, 79, 88, 136, 145, 154, 202, 203, 204, 218, 237, 249, 261, 300, 319, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 378, 385, 389, 390, 395, 397 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 31, 36, 37, 38, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 97, 102, 103, 104, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 154, 157, 158, 159, 160, 161, 163, 168, 169, 170, 174, 175, 176, 177, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 208, 213, 214, 215, 219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 245, 246, 251, 252, 253, 256, 257, 258, 259, 262, 263, 264, 265, 270, 271, 272, 276, 277, 278, 282, 283, 284, 288, 289, 290, 293, 294, 295, 296, 297, 301, 302, 303, 306, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 386, 387, 388, 391, 392, 393, 394, 396 }

B grade: { 8, 9, 21, 30, 32, 33, 34, 35, 39, 40, 41, 42, 46, 47, 78, 80, 81, 82, 86, 87, 96, 98, 99, 100, 101, 105, 106, 107, 108, 112, 113, 114, 115, 144, 153, 155, 156, 162, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 202, 203, 205, 206, 207, 209, 210, 211, 212, 216, 217, 218, 222, 223, 224, 228, 229, 230, 231, 235, 236, 237, 241, 242, 243, 247, 248, 249, 250, 254, 255, 260, 261, 266, 267, 268, 269, 273, 274, 275, 279, 280, 281, 285, 286, 287, 291, 292, 298, 299, 300, 304, 305, 310, 311, 312, 313, 317, 318, 319, 323, 324, 325, 376, 377, 378, 382, 383, 384, 385, 389, 390, 395, 397 }

C grade: { }

F grade: { 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359 }

2.1.6 Sympy

A grade: { 5, 10, 11, 12, 17, 26, 31, 36, 37, 38, 43, 44, 50, 51, 52, 57, 74, 92, 97, 102, 103, 104, 109, 110, 111, 116, 117, 140, 149, 163, 168, 169, 170, 175, 176, 177, 182, 183, 208, 213, 214, 215, 219, 220, 227, 232, 233, 234, 238, 239, 244, 245, 246, 251, 252, 253, 256, 257, 258, 259, 263, 264, 265, 270, 271, 276, 277, 282, 283, 288, 289, 290, 295, 296, 297, 301, 302, 303, 307, 308, 314, 315, 320, 321, 326, 327, 360, 364 }

B grade: { 2, 3, 4, 14, 15, 16, 23, 24, 25, 53, 54, 55, 56, 71, 72, 73, 80, 81, 82, 83, 89, 90, 91, 108, 137, 138, 139, 146, 147, 148, 155, 156, 157, 158, 361, 362, 363, 396 }

C grade: { }

F grade: { 1, 6, 7, 8, 9, 13, 18, 19, 20, 21, 22, 27, 28, 29, 30, 32, 33, 34, 35, 39, 40, 41, 42, 45, 46, 47, 48, 49, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 75, 76, 77, 78, 79, 84, 85, 86, 87, 88, 93, 94, 95, 96, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 142, 143, 144, 145, 150, 151, 152, 153, 154, 159, 160, 161, 162, 164, 165, 166, 167, 171, 172, 173, 174, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 216, 217, 218, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 235, 236, 237, 240, 241, 242, 243, 247, 248, 249, 250, 254, 255, 260, 261, 262, 266, 267, 268, 269, 272, 273, 274, 275, 278, 279, 280, 281, 284, 285, 286, 287, 291, 292, 293, 294, 298, 299, 300, 304, 305, 306, 309, 310, 311, 312, 313, 316, 317, 318, 319, 322, 323, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397 }

2.1.7 Giac

A grade: { 2, 3, 4, 5, 10, 11, 12, 14, 15, 16, 17, 23, 24, 25, 26, 31, 36, 37, 38, 43, 44, 45, 50, 71, 72, 73, 74, 80, 81, 82, 83, 89, 90, 91, 92, 97, 102, 103, 104, 109, 110, 111, 116, 117, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 163, 168, 169, 170, 175, 176, 177, 182, 183, 208, 213, 214, 215, 219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 246, 251, 252, 253, 257, 258, 259, 263, 264, 265, 270, 271, 272, 276, 277, 278, 282, 284, 288, 289, 290, 295, 296, 297, 301, 302, 303, 307, 308, 309, 314, 316, 320, 322, 326, 360, 361, 362, 363, 364, 365, 366, 371, 372, 379, 380, 381, 386, 387, 388, 392, 393, 394 }

B grade: { 42, 48, 49, 108, 174, 204, 250, 256, 262, 275, 293, 294, 367, 368, 369, 370, 373, 374, 375, 391, 396 }

C grade: { 6, 7, 8, 9, 18, 19, 20, 21, 27, 28, 29, 30, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 75, 76, 77, 78, 84, 85, 86, 87, 93, 94, 95, 96, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201 }

F grade: { 1, 13, 22, 32, 33, 34, 35, 39, 40, 41, 46, 47, 51, 70, 79, 88, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 136, 145, 154, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 202, 203, 205, 206, 207, 209, 210, 211, 212, 216, 217, 218, 222, 223, 224, 228, 229, 230, 231, 235, 236, 237, 241, 242, 243, 245, 247, 248, 249, 254, 255, 260, 261, 266, 267, 268, 269, 273, 274, 279, 280, 281, 283, 285, 286, 287, 291, 292, 298, 299, 300, 304, 305, 306, 310, 311, 312, 313, 315, 317, 318, 319, 321, 323, 324, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 376, 377, 378, 382, 383, 384, 385, 389, 390, 395, 397 }

2.1.8 Mupad

A grade: { 31, 36, 37, 38, 43, 44, 45, 50, 51, 97, 102, 103, 104, 109, 110, 111, 116, 117, 163, 168, 169, 170, 175, 176, 177, 182, 183, 208, 213, 214, 215, 219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 245, 246, 251, 252, 253, 257, 258, 259, 263, 264, 265, 270, 271, 272, 276, 277, 278, 282, 283, 284, 288, 289, 290, 295, 296, 297, 301, 302, 303, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 379, 380, 381, 386, 387, 388, 392, 393, 394 }

B grade: { 2, 3, 4, 5, 14, 15, 16, 17, 23, 24, 25, 26, 42, 48, 49, 71, 72, 73, 74, 80, 81, 82, 83, 89, 90, 91, 92, 108, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 174, 204, 212, 250, 256, 262, 275, 293, 294, 360, 361, 362, 363, 364, 368, 369, 370, 371, 391, 395, 396, 397 }

C grade: { }

F grade: { 1, 6, 7, 8, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 27, 28, 29, 30, 32, 33, 34, 35, 39, 40, 41, 46, 47, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 75, 76, 77, 78, 79, 84, 85, 86, 87, 88, 93, 94, 95, 96, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 142, 143, 144, 145, 150, 151, 152, 153, 154, 159, 160, 161, 162, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 209, 210, 211, 216, 217, 218, 222, 223, 224, 228, 229, 230, 231, 235, 236, 237, 241, 242, 243, 247, 248, 249, 254, 255, 260, 261, 266, 267, 268, 269, 273, 274, 279, 280, 281, 285, 286, 287, 291, 292, 298, 299, 300, 304, 305, 306, 310, 311, 312, 313, 317, 318, 319, 323, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 365, 366, 367, 372, 373, 374, 375, 376, 377, 378, 382, 383, 384, 385, 389, 390 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	162	0	0	96	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.325	0.024	0.000	0.331	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	86	853	586	255	502	181	245
N.S.	1	1.00	0.55	5.47	3.76	1.63	3.22	1.16	1.57
time (sec)	N/A	0.080	0.413	0.253	0.298	2.008	0.494	0.412	0.491

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	71	466	342	166	342	121	165
N.S.	1	1.00	0.59	3.88	2.85	1.38	2.85	1.01	1.38
time (sec)	N/A	0.059	0.255	0.105	0.281	2.460	0.333	0.439	0.854

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	50	215	171	92	175	73	100
N.S.	1	1.00	0.56	2.42	1.92	1.03	1.97	0.82	1.12
time (sec)	N/A	0.037	0.197	0.083	0.281	3.187	0.200	0.414	0.159

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	74	65	42	80	38	47
N.S.	1	1.00	0.68	1.48	1.30	0.84	1.60	0.76	0.94
time (sec)	N/A	0.019	0.103	0.052	0.260	3.096	0.121	0.444	0.699

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	60	84	143	80	0	569	-1
N.S.	1	1.00	0.92	1.29	2.20	1.23	0.00	8.75	-0.02
time (sec)	N/A	0.093	0.095	0.077	0.305	2.576	0.000	0.480	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	124	166	132	0	2870	-1
N.S.	1	1.00	0.94	1.46	1.95	1.55	0.00	33.76	-0.01
time (sec)	N/A	0.106	0.310	0.121	0.332	1.315	0.000	0.627	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	102	162	201	230	0	5398	-1
N.S.	1	1.00	0.89	1.42	1.76	2.02	0.00	47.35	-0.01
time (sec)	N/A	0.125	1.058	0.203	0.370	2.232	0.000	0.585	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	164	200	251	320	0	7592	-1
N.S.	1	1.00	1.14	1.39	1.74	2.22	0.00	52.72	-0.01
time (sec)	N/A	0.140	0.554	0.290	0.428	2.306	0.000	0.599	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	13	6	5	6	-1
N.S.	1	1.00	1.00	0.88	1.62	0.75	0.62	0.75	-0.12
time (sec)	N/A	0.019	0.007	0.039	0.292	2.129	0.510	0.434	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	15	24	22	19	-1
N.S.	1	1.00	1.00	0.94	0.94	1.50	1.38	1.19	-0.06
time (sec)	N/A	0.030	0.006	0.069	0.298	2.017	0.898	0.420	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	15	30	24	26	-1
N.S.	1	1.00	1.00	0.90	0.52	1.03	0.83	0.90	-0.03
time (sec)	N/A	0.040	0.008	0.056	0.287	1.494	0.674	0.421	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	252	0	0	188	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.68	0.00	0.00	-0.00
time (sec)	N/A	0.227	2.199	0.230	0.000	0.535	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	385	835	880	352	646	350	448
N.S.	1	1.00	1.88	4.07	4.29	1.72	3.15	1.71	2.19
time (sec)	N/A	0.149	1.513	0.201	0.320	3.781	0.704	0.443	1.409

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	121	447	499	227	391	231	289
N.S.	1	1.00	0.80	2.96	3.30	1.50	2.59	1.53	1.91
time (sec)	N/A	0.097	1.028	0.180	0.286	3.036	0.497	0.459	1.149

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	93	204	240	130	216	137	161
N.S.	1	1.00	0.90	1.98	2.33	1.26	2.10	1.33	1.56
time (sec)	N/A	0.053	0.624	0.131	0.280	2.449	0.492	0.461	0.868

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	44	71	85	59	85	69	59
N.S.	1	1.00	0.86	1.39	1.67	1.16	1.67	1.35	1.16
time (sec)	N/A	0.024	0.181	0.085	0.313	2.691	0.289	0.446	0.152

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	102	171	276	153	0	6059	-1
N.S.	1	1.00	0.84	1.41	2.28	1.26	0.00	50.07	-0.01
time (sec)	N/A	0.203	0.287	0.108	0.342	2.694	0.000	0.567	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	139	247	305	236	0	67350	-1
N.S.	1	1.00	0.83	1.47	1.82	1.40	0.00	400.89	-0.01
time (sec)	N/A	0.214	1.347	0.174	0.374	2.418	0.000	2.054	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	183	316	340	399	0	114422	-1
N.S.	1	1.00	0.83	1.43	1.54	1.81	0.00	517.75	-0.00
time (sec)	N/A	0.254	2.081	0.318	0.469	2.518	0.000	2.696	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	298	389	390	564	0	168646	-1
N.S.	1	1.00	1.10	1.44	1.44	2.09	0.00	624.61	-0.00
time (sec)	N/A	0.287	1.474	0.214	0.586	2.359	0.000	3.684	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	361	0	0	190	0	0	-1
N.S.	1	1.00	1.33	0.00	0.00	0.70	0.00	0.00	-0.00
time (sec)	N/A	0.228	27.149	0.181	0.000	0.733	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	158	1137	967	434	935	361	576
N.S.	1	1.00	0.61	4.37	3.72	1.67	3.60	1.39	2.22
time (sec)	N/A	0.158	1.751	0.289	0.302	2.648	1.040	0.472	1.944

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	135	586	549	283	602	241	366
N.S.	1	1.00	0.69	2.99	2.80	1.44	3.07	1.23	1.87
time (sec)	N/A	0.126	0.874	0.178	0.295	2.432	0.697	0.462	1.714

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	91	256	263	159	320	145	202
N.S.	1	1.00	0.68	1.91	1.96	1.19	2.39	1.08	1.51
time (sec)	N/A	0.071	0.489	0.188	0.276	2.594	0.455	0.439	1.249

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	75	85	92	76	138	75	94
N.S.	1	1.00	1.04	1.18	1.28	1.06	1.92	1.04	1.31
time (sec)	N/A	0.031	0.103	0.120	0.282	2.498	0.292	0.441	0.248

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	110	178	281	156	0	6046	-1
N.S.	1	1.00	0.85	1.38	2.18	1.21	0.00	46.87	-0.01
time (sec)	N/A	0.165	0.337	0.119	0.355	2.778	0.000	0.586	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	151	256	308	245	0	63510	-1
N.S.	1	1.00	0.84	1.43	1.72	1.37	0.00	354.80	-0.01
time (sec)	N/A	0.195	1.256	0.201	0.392	3.305	0.000	2.275	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	199	329	343	423	0	111694	-1
N.S.	1	1.00	0.87	1.44	1.50	1.85	0.00	487.75	-0.00
time (sec)	N/A	0.252	2.734	0.142	0.486	2.651	0.000	2.651	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	316	404	393	588	0	157526	-1
N.S.	1	1.00	1.10	1.41	1.37	2.05	0.00	548.87	-0.00
time (sec)	N/A	0.272	2.024	0.175	0.631	3.186	0.000	3.676	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.013	2.577	0.033	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	527	1159	1281	1204	0	0	-1
N.S.	1	1.00	3.49	7.68	8.48	7.97	0.00	0.00	-0.01
time (sec)	N/A	0.145	3.287	0.170	0.408	3.198	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	410	792	759	818	0	0	-1
N.S.	1	1.00	3.23	6.24	5.98	6.44	0.00	0.00	-0.01
time (sec)	N/A	0.125	1.807	0.099	0.378	2.802	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	287	477	411	502	0	0	-1
N.S.	1	1.00	3.09	5.13	4.42	5.40	0.00	0.00	-0.01
time (sec)	N/A	0.107	1.222	0.084	0.345	3.687	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	188	215	190	250	0	0	-1
N.S.	1	1.00	2.89	3.31	2.92	3.85	0.00	0.00	-0.02
time (sec)	N/A	0.062	5.452	0.075	0.345	2.258	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.015	3.569	0.045	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.015	7.004	0.045	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.147	2.326	0.029	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	458	716	2948	1021	0	0	-1
N.S.	1	1.00	2.20	3.44	14.17	4.91	0.00	0.00	-0.00
time (sec)	N/A	0.113	1.954	0.129	0.556	3.113	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	284	433	1770	669	0	0	-1
N.S.	1	1.00	1.95	2.97	12.12	4.58	0.00	0.00	-0.01
time (sec)	N/A	0.090	1.243	0.102	0.396	2.650	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	234	231	553	375	0	0	-1
N.S.	1	1.00	2.60	2.57	6.14	4.17	0.00	0.00	-0.01
time (sec)	N/A	0.046	2.185	0.064	0.377	2.594	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	131	68	259	62	0	801	88
N.S.	1	1.00	4.37	2.27	8.63	2.07	0.00	26.70	2.93
time (sec)	N/A	0.013	0.069	0.057	0.272	2.621	0.000	0.704	2.281

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.089	16.795	0.049	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.114	20.655	0.053	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.149	4.659	0.033	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	412	716	4551	1075	0	0	-1
N.S.	1	1.00	3.01	5.23	33.22	7.85	0.00	0.00	-0.01
time (sec)	N/A	0.165	6.506	0.118	0.518	2.523	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	277	409	1044	591	0	0	-1
N.S.	1	1.00	2.41	3.56	9.08	5.14	0.00	0.00	-0.01
time (sec)	N/A	0.122	6.364	0.127	0.474	3.302	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	94	162	1130	102	0	3482	147
N.S.	1	1.00	1.74	3.00	20.93	1.89	0.00	64.48	2.72
time (sec)	N/A	0.040	0.924	0.092	0.292	2.344	0.000	1.287	2.559

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	48	61	287	44	0	526	53
N.S.	1	1.00	1.37	1.74	8.20	1.26	0.00	15.03	1.51
time (sec)	N/A	0.022	0.092	0.068	0.278	2.345	0.000	0.496	1.716

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.104	10.301	0.069	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.119	9.499	0.066	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	179	234	277	222	294	1212	-1
N.S.	1	1.00	0.91	1.19	1.41	1.13	1.50	6.18	-0.01
time (sec)	N/A	0.298	2.476	0.059	0.508	2.441	127.206	0.731	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	157	187	256	167	665	753	-1
N.S.	1	1.00	0.93	1.11	1.52	0.99	3.96	4.48	-0.01
time (sec)	N/A	0.206	0.911	0.040	0.496	2.722	22.784	0.587	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	134	142	209	125	389	408	-1
N.S.	1	1.00	0.94	1.00	1.47	0.88	2.74	2.87	-0.01
time (sec)	N/A	0.161	0.281	0.041	0.489	2.082	3.093	0.503	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	134	142	209	125	389	408	-1
N.S.	1	1.00	0.94	1.00	1.47	0.88	2.74	2.87	-0.01
time (sec)	N/A	0.149	0.027	0.000	0.513	2.454	3.115	0.545	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	157	187	256	167	665	753	-1
N.S.	1	1.00	0.93	1.11	1.52	0.99	3.96	4.48	-0.01
time (sec)	N/A	0.190	0.038	0.000	0.523	2.591	22.693	0.584	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	179	234	277	222	294	1212	-1
N.S.	1	1.00	0.91	1.19	1.41	1.13	1.50	6.18	-0.01
time (sec)	N/A	0.225	0.055	0.000	0.512	3.315	127.595	0.727	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	1171	474	547	370	0	2475	-1
N.S.	1	1.00	2.88	1.17	1.35	0.91	0.00	6.10	-0.00
time (sec)	N/A	0.815	15.533	0.078	0.532	2.447	0.000	1.063	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	677	386	497	298	0	1545	-1
N.S.	1	1.00	1.92	1.09	1.41	0.84	0.00	4.38	-0.00
time (sec)	N/A	0.482	8.872	0.067	0.528	2.053	0.000	0.914	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	264	294	424	245	0	848	-1
N.S.	1	1.00	0.87	0.97	1.39	0.81	0.00	2.79	-0.00
time (sec)	N/A	0.317	6.108	0.063	0.514	2.113	0.000	0.679	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	264	294	424	245	0	848	-1
N.S.	1	1.00	0.87	0.97	1.39	0.81	0.00	2.79	-0.00
time (sec)	N/A	0.298	1.117	0.000	0.524	2.460	0.000	0.693	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	677	386	497	298	0	1545	-1
N.S.	1	1.00	1.92	1.09	1.41	0.84	0.00	4.38	-0.00
time (sec)	N/A	0.360	8.314	0.000	0.531	2.810	0.000	0.936	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	1171	474	547	370	0	2475	-1
N.S.	1	1.00	2.88	1.17	1.35	0.91	0.00	6.10	-0.00
time (sec)	N/A	0.447	13.419	0.000	0.525	3.435	0.000	1.036	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-2)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	550	470	551	406	0	2446	-1
N.S.	1	1.00	1.35	1.15	1.35	1.00	0.00	6.01	-0.00
time (sec)	N/A	0.718	15.247	0.062	0.524	3.844	0.000	1.236	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	393	376	503	316	0	1523	-1
N.S.	1	1.00	1.12	1.07	1.43	0.90	0.00	4.34	-0.00
time (sec)	N/A	0.453	1.510	0.056	0.537	2.461	0.000	0.941	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	264	286	429	244	0	830	-1
N.S.	1	1.00	0.88	0.96	1.43	0.82	0.00	2.78	-0.00
time (sec)	N/A	0.338	0.613	0.056	0.523	1.564	0.000	0.754	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	264	286	429	244	0	830	-1
N.S.	1	1.00	0.88	0.96	1.43	0.82	0.00	2.78	-0.00
time (sec)	N/A	0.312	0.052	0.000	0.505	1.586	0.000	0.713	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	393	376	503	316	0	1523	-1
N.S.	1	1.00	1.12	1.07	1.43	0.90	0.00	4.34	-0.00
time (sec)	N/A	0.374	2.629	0.000	0.515	0.965	0.000	0.923	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-2)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	550	470	551	406	0	2446	-1
N.S.	1	1.00	1.35	1.15	1.35	1.00	0.00	6.01	-0.00
time (sec)	N/A	0.468	11.434	0.000	0.526	1.326	0.000	1.230	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	282	0	0	186	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.70	0.00	0.00	-0.00
time (sec)	N/A	0.193	12.215	0.113	0.000	0.798	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	150	835	889	294	646	350	448
N.S.	1	1.00	0.73	4.07	4.34	1.43	3.15	1.71	2.19
time (sec)	N/A	0.146	1.626	0.195	0.327	2.191	0.720	0.451	1.898

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	127	447	505	183	391	231	290
N.S.	1	1.00	0.84	2.96	3.34	1.21	2.59	1.53	1.92
time (sec)	N/A	0.115	0.900	0.191	0.302	1.936	0.468	0.456	1.338

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	86	204	243	100	216	137	145
N.S.	1	1.00	0.83	1.98	2.36	0.97	2.10	1.33	1.41
time (sec)	N/A	0.054	0.514	0.138	0.277	2.786	0.301	0.447	1.131

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	71	71	86	46	85	69	58
N.S.	1	1.00	1.39	1.39	1.69	0.90	1.67	1.35	1.14
time (sec)	N/A	0.023	0.157	0.087	0.278	2.555	0.174	0.447	0.949

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	100	172	275	152	0	6279	-1
N.S.	1	1.00	0.83	1.42	2.27	1.26	0.00	51.89	-0.01
time (sec)	N/A	0.166	0.272	0.103	0.346	2.409	0.000	0.614	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	139	245	302	233	0	66726	-1
N.S.	1	1.00	0.83	1.46	1.80	1.39	0.00	397.18	-0.01
time (sec)	N/A	0.195	1.161	0.133	0.389	2.443	0.000	1.999	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	181	318	337	393	0	118262	-1
N.S.	1	1.00	0.82	1.44	1.52	1.78	0.00	535.12	-0.00
time (sec)	N/A	0.234	2.606	0.241	0.462	2.635	0.000	2.625	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	300	386	387	558	0	166374	-1
N.S.	1	1.00	1.11	1.43	1.43	2.07	0.00	616.20	-0.00
time (sec)	N/A	0.270	1.755	0.175	0.615	2.909	0.000	3.628	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	213	0	0	136	0	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.145	1.093	0.099	0.000	0.270	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	132	1951	735	466	1231	224	349
N.S.	1	1.00	1.01	14.89	5.61	3.56	9.40	1.71	2.66
time (sec)	N/A	0.112	1.116	0.266	0.303	0.911	1.088	0.506	1.684

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	106	1098	442	308	835	153	329
N.S.	1	1.00	1.01	10.46	4.21	2.93	7.95	1.46	3.13
time (sec)	N/A	0.087	0.576	0.163	0.289	2.001	0.755	0.463	1.691

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	77	531	232	180	493	94	179
N.S.	1	1.00	0.97	6.72	2.94	2.28	6.24	1.19	2.27
time (sec)	N/A	0.085	0.366	0.190	0.286	2.142	0.477	0.440	1.306

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	200	96	85	238	48	57
N.S.	1	1.00	1.02	3.77	1.81	1.60	4.49	0.91	1.08
time (sec)	N/A	0.036	0.301	0.131	0.276	1.040	0.306	0.441	1.040

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	65	114	162	88	0	669	-1
N.S.	1	1.00	0.83	1.46	2.08	1.13	0.00	8.58	-0.01
time (sec)	N/A	0.100	0.144	0.129	0.310	5.827	0.000	0.453	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	81	156	171	138	0	3218	-1
N.S.	1	1.00	0.78	1.50	1.64	1.33	0.00	30.94	-0.01
time (sec)	N/A	0.124	0.457	0.174	0.346	2.539	0.000	0.850	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	105	193	206	255	0	5600	-1
N.S.	1	1.00	0.83	1.52	1.62	2.01	0.00	44.09	-0.01
time (sec)	N/A	0.141	0.873	0.141	0.405	2.096	0.000	0.572	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	123	230	258	406	0	8508	-1
N.S.	1	1.00	0.78	1.46	1.63	2.57	0.00	53.85	-0.01
time (sec)	N/A	0.164	1.744	0.092	0.471	2.695	0.000	0.630	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	413	0	0	280	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.69	0.00	0.00	-0.00
time (sec)	N/A	0.277	53.665	0.136	0.000	0.575	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	238	1812	1339	471	1098	531	816
N.S.	1	1.00	0.72	5.49	4.06	1.43	3.33	1.61	2.47
time (sec)	N/A	0.268	3.376	0.390	0.324	2.151	1.479	0.542	4.606

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	369	992	766	296	690	351	516
N.S.	1	1.00	1.42	3.83	2.96	1.14	2.66	1.36	1.99
time (sec)	N/A	0.205	1.534	0.244	0.302	1.723	1.001	0.502	2.573

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	127	466	375	166	382	209	249
N.S.	1	1.00	0.69	2.53	2.04	0.90	2.08	1.14	1.35
time (sec)	N/A	0.142	0.884	0.184	0.296	2.451	0.656	0.476	0.809

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	94	163	139	76	163	106	99
N.S.	1	1.00	0.86	1.50	1.28	0.70	1.50	0.97	0.91
time (sec)	N/A	0.069	0.334	0.170	0.271	3.549	0.414	0.448	1.240

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	154	258	414	228	0	46675	-1
N.S.	1	1.00	0.83	1.39	2.24	1.23	0.00	252.30	-0.01
time (sec)	N/A	0.246	0.456	0.165	0.371	2.721	0.000	1.432	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	213	370	445	347	0	1014406	-1
N.S.	1	1.00	0.83	1.44	1.73	1.35	0.00	3947.11	-0.00
time (sec)	N/A	0.284	1.544	0.113	0.457	2.929	0.000	37.938	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	279	480	480	585	0	1737414	-1
N.S.	1	1.00	0.83	1.42	1.42	1.73	0.00	5140.28	-0.00
time (sec)	N/A	0.339	4.136	0.161	0.559	2.645	0.000	78.211	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	457	585	530	824	0	2449286	-1
N.S.	1	1.00	1.11	1.42	1.28	2.00	0.00	5930.47	-0.00
time (sec)	N/A	0.394	2.677	0.210	0.780	2.534	0.000	148.793	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	6.764	0.046	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	812	1295	1548	1367	0	0	-1
N.S.	1	1.00	2.44	3.89	4.65	4.11	0.00	0.00	-0.00
time (sec)	N/A	0.203	1.636	0.223	0.453	2.647	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	512	847	929	925	0	0	-1
N.S.	1	1.00	2.02	3.33	3.66	3.64	0.00	0.00	-0.00
time (sec)	N/A	0.142	0.963	0.164	0.385	2.851	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	288	479	513	562	0	0	-1
N.S.	1	1.00	1.68	2.80	3.00	3.29	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.593	0.136	0.336	2.729	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	176	177	200	277	0	0	-1
N.S.	1	1.00	1.87	1.88	2.13	2.95	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.218	0.096	0.337	5.131	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	8.221	0.098	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	3.871	0.108	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	1.269	0.046	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	592	921	3242	856	0	0	-1
N.S.	1	1.00	3.82	5.94	20.92	5.52	0.00	0.00	-0.01
time (sec)	N/A	0.159	6.562	0.139	0.803	2.100	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	311	581	1953	599	0	0	-1
N.S.	1	1.00	2.45	4.57	15.38	4.72	0.00	0.00	-0.01
time (sec)	N/A	0.136	6.049	0.112	0.608	1.572	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	268	305	645	384	0	0	-1
N.S.	1	1.00	2.76	3.14	6.65	3.96	0.00	0.00	-0.01
time (sec)	N/A	0.089	6.356	0.100	0.591	1.513	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	82	64	292	97	104	1375	67
N.S.	1	1.00	2.00	1.56	7.12	2.37	2.54	33.54	1.63
time (sec)	N/A	0.020	0.559	0.076	0.486	2.411	0.234	0.700	1.573

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.025	4.291	0.068	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.027	2.405	0.069	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.053	35.420	0.039	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	800	1673	7004	2770	0	0	-1
N.S.	1	1.00	1.92	4.02	16.84	6.66	0.00	0.00	-0.00
time (sec)	N/A	0.370	6.967	0.204	3.923	3.090	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	478	1056	3887	1742	0	0	-1
N.S.	1	1.00	1.55	3.43	12.62	5.66	0.00	0.00	-0.00
time (sec)	N/A	0.263	4.741	0.151	1.383	2.682	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	423	546	1938	970	0	0	-1
N.S.	1	1.00	2.36	3.05	10.83	5.42	0.00	0.00	-0.01
time (sec)	N/A	0.172	6.915	0.125	0.552	2.712	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	260	246	762	454	0	0	-1
N.S.	1	1.00	2.41	2.28	7.06	4.20	0.00	0.00	-0.01
time (sec)	N/A	0.080	1.820	0.096	0.401	3.162	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.062	35.640	0.181	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.058	41.317	0.075	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	1168	476	547	341	0	2479	-1
N.S.	1	1.00	2.88	1.17	1.35	0.84	0.00	6.11	-0.00
time (sec)	N/A	0.577	16.414	0.066	0.528	2.925	0.000	1.060	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	676	384	499	280	0	1548	-1
N.S.	1	1.00	1.92	1.09	1.41	0.79	0.00	4.39	-0.00
time (sec)	N/A	0.413	7.546	0.056	0.530	2.808	0.000	0.964	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	278	296	424	235	0	848	-1
N.S.	1	1.00	0.91	0.97	1.39	0.77	0.00	2.79	-0.00
time (sec)	N/A	0.328	6.113	0.056	0.497	2.484	0.000	0.672	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	264	296	424	235	0	848	-1
N.S.	1	1.00	0.87	0.97	1.39	0.77	0.00	2.79	-0.00
time (sec)	N/A	0.324	1.996	0.000	0.511	6.878	0.000	0.680	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	676	384	499	280	0	1548	-1
N.S.	1	1.00	1.92	1.09	1.41	0.79	0.00	4.39	-0.00
time (sec)	N/A	0.360	7.070	0.000	0.519	6.118	0.000	0.942	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	1168	476	547	341	0	2479	-1
N.S.	1	1.00	2.88	1.17	1.35	0.84	0.00	6.11	-0.00
time (sec)	N/A	0.437	12.048	0.001	0.538	1.558	0.000	1.066	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-2)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	206	251	285	347	0	1372	-1
N.S.	1	1.00	0.90	1.10	1.25	1.52	0.00	6.02	-0.00
time (sec)	N/A	0.292	3.604	0.076	0.513	5.230	0.000	1.097	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	187	206	264	249	0	852	-1
N.S.	1	1.00	0.94	1.03	1.32	1.24	0.00	4.26	-0.00
time (sec)	N/A	0.232	3.063	0.072	0.497	3.256	0.000	0.874	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	161	159	219	175	0	458	-1
N.S.	1	1.00	0.93	0.91	1.26	1.01	0.00	2.63	-0.01
time (sec)	N/A	0.176	0.818	0.072	0.509	3.405	0.000	0.693	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	161	159	219	175	0	458	-1
N.S.	1	1.00	0.93	0.91	1.26	1.01	0.00	2.63	-0.01
time (sec)	N/A	0.175	0.053	0.000	0.492	3.105	0.000	0.694	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	187	206	264	249	0	852	-1
N.S.	1	1.00	0.94	1.03	1.32	1.24	0.00	4.26	-0.00
time (sec)	N/A	0.208	1.066	0.000	0.508	3.978	0.000	0.899	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-2)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	206	251	285	347	0	1372	-1
N.S.	1	1.00	0.90	1.10	1.25	1.52	0.00	6.02	-0.00
time (sec)	N/A	0.260	2.055	0.000	0.510	3.213	0.000	1.107	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-2)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	3348	719	826	521	0	3717	-1
N.S.	1	1.00	5.44	1.17	1.34	0.85	0.00	6.04	-0.00
time (sec)	N/A	0.861	23.190	0.070	0.551	2.886	0.000	1.746	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	1041	580	760	427	0	2320	-1
N.S.	1	1.00	1.95	1.09	1.42	0.80	0.00	4.34	-0.00
time (sec)	N/A	0.644	11.383	0.066	0.548	3.663	0.000	1.272	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	432	447	680	356	0	1270	-1
N.S.	1	1.00	0.94	0.97	1.48	0.78	0.00	2.77	-0.00
time (sec)	N/A	0.498	6.973	0.066	0.530	2.622	0.000	0.886	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	432	447	680	356	0	1270	-1
N.S.	1	1.00	0.94	0.97	1.48	0.78	0.00	2.77	-0.00
time (sec)	N/A	0.504	6.831	0.000	0.532	3.444	0.000	0.891	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	1041	580	760	427	0	2320	-1
N.S.	1	1.00	1.95	1.09	1.42	0.80	0.00	4.34	-0.00
time (sec)	N/A	0.580	10.145	0.000	0.537	3.217	0.000	1.233	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-2)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	3348	719	826	521	0	3717	-1
N.S.	1	1.00	5.44	1.17	1.34	0.85	0.00	6.04	-0.00
time (sec)	N/A	0.757	22.012	0.000	0.564	3.068	0.000	1.716	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	362	0	0	188	0	0	-1
N.S.	1	1.00	1.33	0.00	0.00	0.69	0.00	0.00	-0.00
time (sec)	N/A	0.224	1.189	0.152	0.000	0.631	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	158	1134	967	378	935	361	576
N.S.	1	1.00	0.61	4.36	3.72	1.45	3.60	1.39	2.22
time (sec)	N/A	0.166	1.904	0.277	0.314	4.328	1.132	0.473	1.335

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	135	586	549	238	602	241	366
N.S.	1	1.00	0.69	2.99	2.80	1.21	3.07	1.23	1.87
time (sec)	N/A	0.115	0.972	0.177	0.295	3.314	0.738	0.476	2.057

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	89	256	263	130	320	145	202
N.S.	1	1.00	0.66	1.91	1.96	0.97	2.39	1.08	1.51
time (sec)	N/A	0.065	0.474	0.192	0.276	2.653	0.481	0.464	1.634

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	75	85	92	58	138	75	94
N.S.	1	1.00	1.04	1.18	1.28	0.81	1.92	1.04	1.31
time (sec)	N/A	0.032	0.134	0.125	0.268	2.135	0.298	0.462	0.313

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	110	178	281	155	0	6046	-1
N.S.	1	1.00	0.85	1.38	2.18	1.20	0.00	46.87	-0.01
time (sec)	N/A	0.167	0.278	0.132	0.354	2.470	0.000	0.589	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	151	256	308	235	0	63510	-1
N.S.	1	1.00	0.84	1.43	1.72	1.31	0.00	354.80	-0.01
time (sec)	N/A	0.196	1.711	0.224	0.397	2.503	0.000	2.151	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	197	329	343	397	0	111694	-1
N.S.	1	1.00	0.85	1.42	1.48	1.72	0.00	483.52	-0.00
time (sec)	N/A	0.252	3.843	0.151	0.463	3.321	0.000	2.655	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	316	404	393	568	0	157526	-1
N.S.	1	1.00	1.10	1.41	1.37	1.98	0.00	548.87	-0.00
time (sec)	N/A	0.281	2.262	0.224	0.600	2.707	0.000	3.562	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	1622	0	0	280	0	0	-1
N.S.	1	1.00	3.87	0.00	0.00	0.67	0.00	0.00	-0.00
time (sec)	N/A	0.311	51.142	0.181	0.000	0.835	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	563	1842	1339	527	1098	531	816
N.S.	1	1.00	1.71	5.58	4.06	1.60	3.33	1.61	2.47
time (sec)	N/A	0.265	3.805	0.354	0.327	2.928	1.539	0.511	4.382

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	195	1016	766	342	690	351	516
N.S.	1	1.00	0.75	3.92	2.96	1.32	2.66	1.36	1.99
time (sec)	N/A	0.207	2.352	0.248	0.295	2.951	1.081	0.480	2.426

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	252	484	375	193	382	209	295
N.S.	1	1.00	1.37	2.63	2.04	1.05	2.08	1.14	1.60
time (sec)	N/A	0.139	1.035	0.185	0.289	2.372	0.698	0.503	0.845

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	110	175	139	91	163	106	119
N.S.	1	1.00	1.01	1.61	1.28	0.83	1.50	0.97	1.09
time (sec)	N/A	0.074	0.380	0.168	0.265	2.120	0.423	0.470	0.469

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	154	257	413	229	0	44961	-1
N.S.	1	1.00	0.83	1.39	2.23	1.24	0.00	243.03	-0.01
time (sec)	N/A	0.243	0.465	0.165	0.361	1.596	0.000	1.512	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	212	372	446	339	0	1022022	-1
N.S.	1	1.00	0.82	1.45	1.74	1.32	0.00	3976.74	-0.00
time (sec)	N/A	0.269	2.043	0.125	0.439	1.657	0.000	37.490	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	283	478	481	567	0	1708998	-1
N.S.	1	1.00	0.84	1.41	1.42	1.68	0.00	5056.21	-0.00
time (sec)	N/A	0.343	3.102	0.202	0.540	2.472	0.000	76.710	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	451	588	531	811	0	2478918	-1
N.S.	1	1.00	1.09	1.42	1.29	1.96	0.00	6002.22	-0.00
time (sec)	N/A	0.420	2.809	0.260	0.786	1.292	0.000	155.210	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	255	0	0	190	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.67	0.00	0.00	-0.00
time (sec)	N/A	0.235	56.925	0.217	0.000	0.309	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	153	2125	1033	546	1334	359	576
N.S.	1	1.00	0.66	9.12	4.43	2.34	5.73	1.54	2.47
time (sec)	N/A	0.203	1.338	0.497	0.321	2.745	2.225	0.575	2.585

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	132	1132	602	349	857	241	366
N.S.	1	1.00	0.73	6.25	3.33	1.93	4.73	1.33	2.02
time (sec)	N/A	0.158	2.337	0.312	0.281	6.090	1.514	0.533	1.227

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	91	514	303	194	461	145	202
N.S.	1	1.00	0.71	3.98	2.35	1.50	3.57	1.12	1.57
time (sec)	N/A	0.103	0.518	0.236	0.292	2.306	1.010	0.497	0.806

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	63	176	119	87	201	75	84
N.S.	1	1.00	0.82	2.29	1.55	1.13	2.61	0.97	1.09
time (sec)	N/A	0.053	0.233	0.231	0.261	1.230	0.646	0.456	0.709

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	110	178	281	156	0	6046	-1
N.S.	1	1.00	0.85	1.38	2.18	1.21	0.00	46.87	-0.01
time (sec)	N/A	0.189	0.290	0.175	0.337	1.236	0.000	0.604	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	189	256	308	248	0	63798	-1
N.S.	1	1.00	1.06	1.43	1.72	1.39	0.00	356.41	-0.01
time (sec)	N/A	0.216	0.975	0.102	0.381	0.996	0.000	2.943	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	239	329	343	434	0	111694	-1
N.S.	1	1.00	1.02	1.40	1.46	1.85	0.00	475.29	-0.00
time (sec)	N/A	0.263	1.072	0.151	0.463	3.089	0.000	2.844	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	554	404	393	638	0	157526	-1
N.S.	1	1.00	1.93	1.41	1.37	2.22	0.00	548.87	-0.00
time (sec)	N/A	0.291	5.135	0.181	0.621	1.657	0.000	3.804	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	7.309	0.090	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	2486	1335	1654	1453	0	0	-1
N.S.	1	1.00	8.10	4.35	5.39	4.73	0.00	0.00	-0.00
time (sec)	N/A	0.234	6.364	0.625	0.483	3.992	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	1712	908	979	984	0	0	-1
N.S.	1	1.00	6.96	3.69	3.98	4.00	0.00	0.00	-0.00
time (sec)	N/A	0.200	6.299	0.562	0.406	3.239	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	511	544	529	594	0	0	-1
N.S.	1	1.00	2.82	3.01	2.92	3.28	0.00	0.00	-0.01
time (sec)	N/A	0.159	6.315	0.603	0.368	3.028	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	131	249	223	292	0	0	-1
N.S.	1	1.00	1.15	2.18	1.96	2.56	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.402	0.561	0.352	2.236	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.896	0.177	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	2.706	0.130	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.195	8.196	0.040	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	833	1056	17589	1233	0	0	-1
N.S.	1	1.00	2.79	3.53	58.83	4.12	0.00	0.00	-0.00
time (sec)	N/A	0.207	2.416	0.175	4.497	3.566	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	506	649	10994	797	0	0	-1
N.S.	1	1.00	2.34	3.00	50.90	3.69	0.00	0.00	-0.00
time (sec)	N/A	0.150	1.404	0.143	1.326	4.230	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	310	332	3199	448	0	0	-1
N.S.	1	1.00	2.23	2.39	23.01	3.22	0.00	0.00	-0.01
time (sec)	N/A	0.091	4.125	0.142	1.149	3.301	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	104	124	2110	95	0	1967	162
N.S.	1	1.00	1.79	2.14	36.38	1.64	0.00	33.91	2.79
time (sec)	N/A	0.037	0.759	0.129	0.301	2.773	0.000	1.068	2.305

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	4.051	0.098	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	4.430	0.117	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.025	10.146	0.060	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	1101	1877	7158	1751	0	0	-1
N.S.	1	1.00	3.65	6.22	23.70	5.80	0.00	0.00	-0.00
time (sec)	N/A	0.323	6.987	0.215	4.282	2.908	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	788	1203	3958	1139	0	0	-1
N.S.	1	1.00	3.08	4.70	15.46	4.45	0.00	0.00	-0.00
time (sec)	N/A	0.242	6.837	0.147	1.377	2.836	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	446	644	1966	657	0	0	-1
N.S.	1	1.00	2.65	3.83	11.70	3.91	0.00	0.00	-0.01
time (sec)	N/A	0.153	6.669	0.119	0.542	3.431	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	240	281	830	339	0	0	-1
N.S.	1	1.00	2.20	2.58	7.61	3.11	0.00	0.00	-0.01
time (sec)	N/A	0.086	6.194	0.086	0.419	3.119	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	6.567	0.075	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	9.984	0.073	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-2)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	550	470	551	376	0	2446	-1
N.S.	1	1.00	1.35	1.15	1.35	0.92	0.00	6.01	-0.00
time (sec)	N/A	0.617	14.612	0.063	0.522	3.328	0.000	1.250	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	393	376	503	294	0	1523	-1
N.S.	1	1.00	1.12	1.07	1.43	0.84	0.00	4.34	-0.00
time (sec)	N/A	0.412	1.418	0.056	0.512	2.912	0.000	0.936	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	264	286	429	233	0	830	-1
N.S.	1	1.00	0.88	0.96	1.43	0.78	0.00	2.78	-0.00
time (sec)	N/A	0.319	0.602	0.053	0.531	3.098	0.000	0.708	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	264	286	429	233	0	830	-1
N.S.	1	1.00	0.88	0.96	1.43	0.78	0.00	2.78	-0.00
time (sec)	N/A	0.301	0.056	0.000	0.514	3.102	0.000	0.695	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	393	376	503	294	0	1523	-1
N.S.	1	1.00	1.12	1.07	1.43	0.84	0.00	4.34	-0.00
time (sec)	N/A	0.376	0.137	0.000	0.518	3.267	0.000	0.950	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-2)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	550	470	551	376	0	2446	-1
N.S.	1	1.00	1.35	1.15	1.35	0.92	0.00	6.01	-0.00
time (sec)	N/A	0.473	10.663	0.000	0.528	3.385	0.000	1.183	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-2)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	1795	716	826	548	0	3705	-1
N.S.	1	1.00	2.92	1.16	1.34	0.89	0.00	6.02	-0.00
time (sec)	N/A	0.885	23.850	0.092	0.542	3.724	0.000	1.699	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	1043	583	760	446	0	2313	-1
N.S.	1	1.00	1.95	1.09	1.42	0.84	0.00	4.33	-0.00
time (sec)	N/A	0.632	12.323	0.090	0.532	2.897	0.000	1.277	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	435	444	680	365	0	1270	-1
N.S.	1	1.00	0.95	0.97	1.48	0.80	0.00	2.77	-0.00
time (sec)	N/A	0.490	6.221	0.088	0.522	3.513	0.000	0.899	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	403	444	680	365	0	1270	-1
N.S.	1	1.00	0.88	0.97	1.48	0.80	0.00	2.77	-0.00
time (sec)	N/A	0.476	5.719	0.000	0.525	2.860	0.000	0.892	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	1043	583	760	446	0	2313	-1
N.S.	1	1.00	1.95	1.09	1.42	0.84	0.00	4.33	-0.00
time (sec)	N/A	0.600	11.139	0.000	0.540	2.795	0.000	1.256	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-2)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	1795	716	826	548	0	3705	-1
N.S.	1	1.00	2.92	1.16	1.34	0.89	0.00	6.02	-0.00
time (sec)	N/A	0.717	21.973	0.002	0.558	2.508	0.000	1.686	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-2)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	550	477	561	445	0	2445	-1
N.S.	1	1.00	1.35	1.17	1.38	1.09	0.00	6.01	-0.00
time (sec)	N/A	0.635	5.185	0.073	0.535	3.357	0.000	2.516	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	391	383	511	326	0	1522	-1
N.S.	1	1.00	1.11	1.09	1.46	0.93	0.00	4.34	-0.00
time (sec)	N/A	0.456	2.777	0.069	0.519	4.177	0.000	1.767	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	264	293	439	242	0	830	-1
N.S.	1	1.00	0.88	0.98	1.47	0.81	0.00	2.78	-0.00
time (sec)	N/A	0.328	1.190	0.069	0.502	2.928	0.000	1.179	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	264	293	439	242	0	830	-1
N.S.	1	1.00	0.88	0.98	1.47	0.81	0.00	2.78	-0.00
time (sec)	N/A	0.334	0.037	0.000	0.513	2.962	0.000	1.206	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	391	383	511	326	0	1522	-1
N.S.	1	1.00	1.11	1.09	1.46	0.93	0.00	4.34	-0.00
time (sec)	N/A	0.400	0.092	0.002	0.511	3.081	0.000	1.778	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-2)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	550	477	561	445	0	2445	-1
N.S.	1	1.00	1.35	1.17	1.38	1.09	0.00	6.01	-0.00
time (sec)	N/A	0.467	2.611	0.000	0.526	4.254	0.000	2.542	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	104	150	0	244	0	0	-1
N.S.	1	1.00	0.93	1.34	0.00	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.198	0.124	0.000	3.385	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	112	0	162	0	0	-1
N.S.	1	1.00	0.87	1.35	0.00	1.95	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.116	0.108	0.000	3.149	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	60	0	45	0	206	56
N.S.	1	1.00	1.00	1.82	0.00	1.36	0.00	6.24	1.70
time (sec)	N/A	0.037	0.028	0.106	0.000	2.733	0.000	0.448	1.234

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	159	240	3740	508	0	0	-1
N.S.	1	1.00	0.88	1.33	20.78	2.82	0.00	0.00	-0.01
time (sec)	N/A	0.260	0.454	0.159	0.674	2.896	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	108	170	2842	370	0	0	-1
N.S.	1	1.00	1.02	1.60	26.81	3.49	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.360	0.162	0.555	3.751	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	62	109	1718	203	0	0	-1
N.S.	1	1.00	0.85	1.49	23.53	2.78	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.126	0.145	0.436	2.936	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.014	2.660	0.032	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	157	625	803	1410	0	0	-1
N.S.	1	1.00	0.99	3.96	5.08	8.92	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.074	0.126	0.571	4.165	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	126	432	497	974	0	0	-1
N.S.	1	1.00	0.95	3.27	3.77	7.38	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.093	0.087	0.541	4.427	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	100	266	284	594	0	0	-1
N.S.	1	1.00	1.04	2.77	2.96	6.19	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.045	0.073	0.531	2.809	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	70	123	115	310	0	0	148
N.S.	1	1.00	1.06	1.86	1.74	4.70	0.00	0.00	2.24
time (sec)	N/A	0.063	0.019	0.072	0.509	2.577	0.000	0.000	1.572

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.014	3.698	0.037	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.014	2.610	0.036	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	6.472	0.140	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	557	901	934	1075	0	0	-1
N.S.	1	1.00	2.03	3.28	3.40	3.91	0.00	0.00	-0.00
time (sec)	N/A	0.166	1.346	0.426	0.608	5.483	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	315	512	516	656	0	0	-1
N.S.	1	1.00	1.69	2.75	2.77	3.53	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.790	0.313	0.577	1.627	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	213	180	0	331	0	0	-1
N.S.	1	1.00	2.07	1.75	0.00	3.21	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.464	0.111	0.000	1.944	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	2.985	0.172	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	7.263	0.220	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	7.697	0.232	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	1734	650	692	1134	0	0	-1
N.S.	1	1.00	6.91	2.59	2.76	4.52	0.00	0.00	-0.00
time (sec)	N/A	0.196	6.426	0.234	0.577	2.094	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	525	388	383	688	0	0	-1
N.S.	1	1.00	2.85	2.11	2.08	3.74	0.00	0.00	-0.01
time (sec)	N/A	0.152	6.405	0.322	0.536	1.952	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	134	179	146	346	0	0	-1
N.S.	1	1.00	1.17	1.56	1.27	3.01	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.328	0.411	0.533	1.316	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.870	0.389	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	2.688	0.552	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.141	5.697	0.035	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	578	1242	1794	2600	0	0	-1
N.S.	1	1.00	2.34	5.03	7.26	10.53	0.00	0.00	-0.00
time (sec)	N/A	0.161	0.840	0.182	0.661	2.035	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	350	816	1078	1786	0	0	-1
N.S.	1	1.00	1.78	4.14	5.47	9.07	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.643	0.125	0.581	2.163	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	213	469	599	1098	0	0	-1
N.S.	1	1.00	1.68	3.69	4.72	8.65	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.376	0.101	0.578	2.069	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	141	208	269	554	0	0	-1
N.S.	1	1.00	1.99	2.93	3.79	7.80	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.132	0.081	0.559	1.683	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.029	4.476	0.074	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.028	5.880	0.081	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.144	15.620	0.035	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	760	1158	3257	1753	0	0	-1
N.S.	1	1.00	2.17	3.31	9.31	5.01	0.00	0.00	-0.00
time (sec)	N/A	0.435	3.124	0.550	1.042	3.611	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	593	556	1638	1067	0	0	-1
N.S.	1	1.00	2.62	2.46	7.25	4.72	0.00	0.00	-0.00
time (sec)	N/A	0.257	6.221	0.302	0.635	2.622	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	508	235	0	434	0	0	-1
N.S.	1	1.00	3.88	1.79	0.00	3.31	0.00	0.00	-0.01
time (sec)	N/A	0.089	3.237	0.138	0.000	3.408	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.131	10.570	0.098	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.151	10.038	0.108	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.177	15.795	0.046	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	1285	1223	5165	3475	0	0	-1
N.S.	1	1.00	3.95	3.76	15.89	10.69	0.00	0.00	-0.00
time (sec)	N/A	0.552	6.833	0.203	1.799	3.325	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	785	632	2528	1995	0	0	-1
N.S.	1	1.00	3.91	3.14	12.58	9.93	0.00	0.00	-0.00
time (sec)	N/A	0.310	6.703	0.172	0.753	3.126	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	210	270	1028	942	0	0	-1
N.S.	1	1.00	1.49	1.91	7.29	6.68	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.989	0.129	0.611	2.700	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.121	10.791	0.118	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.142	12.944	0.124	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.102	2.596	0.025	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	428	767	2944	1190	0	0	-1
N.S.	1	1.00	1.89	3.38	12.97	5.24	0.00	0.00	-0.00
time (sec)	N/A	0.128	1.095	0.187	0.718	1.226	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	256	463	1770	783	0	0	-1
N.S.	1	1.00	1.61	2.91	11.13	4.92	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.779	0.133	0.584	2.040	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	174	234	0	446	0	0	-1
N.S.	1	1.00	1.79	2.41	0.00	4.60	0.00	0.00	-0.01
time (sec)	N/A	0.047	1.762	0.073	0.000	1.404	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	93	67	259	60	0	1537	78
N.S.	1	1.00	3.21	2.31	8.93	2.07	0.00	53.00	2.69
time (sec)	N/A	0.013	0.061	0.070	0.466	1.144	0.000	0.712	2.583

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.061	11.030	0.071	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.086	19.652	0.079	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	3.204	0.040	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	431	356	1363	373	0	0	-1
N.S.	1	1.00	3.37	2.78	10.65	2.91	0.00	0.00	-0.01
time (sec)	N/A	0.137	6.440	0.109	0.562	1.174	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	276	199	418	210	0	0	-1
N.S.	1	1.00	2.88	2.07	4.35	2.19	0.00	0.00	-0.01
time (sec)	N/A	0.091	6.343	0.098	0.571	1.109	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	76	63	237	53	65	223	52
N.S.	1	1.00	1.90	1.58	5.92	1.32	1.62	5.58	1.30
time (sec)	N/A	0.019	0.323	0.073	0.493	1.420	0.090	0.595	1.436

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.025	3.618	0.060	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.025	5.441	0.063	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	14.419	0.036	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	532	677	11010	896	0	0	-1
N.S.	1	1.00	2.33	2.97	48.29	3.93	0.00	0.00	-0.00
time (sec)	N/A	0.143	1.454	0.128	1.615	2.099	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	362	345	0	511	0	0	-1
N.S.	1	1.00	2.50	2.38	0.00	3.52	0.00	0.00	-0.01
time (sec)	N/A	0.092	3.232	0.125	0.000	1.160	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	107	123	2123	93	0	2762	151
N.S.	1	1.00	1.91	2.20	37.91	1.66	0.00	49.32	2.70
time (sec)	N/A	0.036	0.344	0.139	0.532	2.210	0.000	1.188	1.161

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	4.072	0.084	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	4.404	0.102	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.182	22.433	0.037	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	998	1866	5709	2519	0	0	-1
N.S.	1	1.00	2.13	3.98	12.17	5.37	0.00	0.00	-0.00
time (sec)	N/A	0.521	2.773	0.641	2.172	2.494	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	616	1152	3202	1705	0	0	-1
N.S.	1	1.00	1.80	3.36	9.34	4.97	0.00	0.00	-0.00
time (sec)	N/A	0.367	1.652	0.509	0.992	2.641	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	379	568	1590	1035	0	0	-1
N.S.	1	1.00	1.73	2.59	7.26	4.73	0.00	0.00	-0.00
time (sec)	N/A	0.248	4.075	0.285	0.639	1.149	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	122	212	160	800	366	0	0	-1
N.S.	1	1.08	1.88	1.42	7.08	3.24	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.550	0.109	0.627	1.909	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.110	9.199	0.089	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.137	9.423	0.095	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.151	1.876	0.040	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	154	687	2360	1635	0	0	-1
N.S.	1	1.00	1.31	5.82	20.00	13.86	0.00	0.00	-0.01
time (sec)	N/A	0.181	1.504	0.141	0.669	1.622	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	113	351	772	950	0	0	-1
N.S.	1	1.00	1.28	3.99	8.77	10.80	0.00	0.00	-0.01
time (sec)	N/A	0.123	1.984	0.122	0.642	1.679	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	72	308	75	0	13091	55
N.S.	1	1.00	0.91	2.06	8.80	2.14	0.00	374.03	1.57
time (sec)	N/A	0.039	0.214	0.115	0.504	1.762	0.000	2.295	1.656

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	6.780	0.101	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	6.898	0.121	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.153	21.064	0.046	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	601	601	857	1613	8032	3173	0	0	-1
N.S.	1	1.00	1.43	2.68	13.36	5.28	0.00	0.00	-0.00
time (sec)	N/A	1.612	7.288	0.586	4.174	2.402	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	840	802	3814	1801	0	0	-1
N.S.	1	1.00	2.75	2.63	12.50	5.90	0.00	0.00	-0.00
time (sec)	N/A	0.600	7.196	0.330	1.277	1.844	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	174	520	267	1481	621	0	0	-1
N.S.	1	1.13	3.38	1.73	9.62	4.03	0.00	0.00	-0.01
time (sec)	N/A	0.123	5.395	0.161	0.787	2.001	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.178	21.028	0.220	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.175	29.153	0.088	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.692	36.045	0.040	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	616	0	3940	1747	0	0	-1
N.S.	1	1.00	1.59	0.00	10.18	4.51	0.00	0.00	-0.00
time (sec)	N/A	0.634	7.121	0.070	0.987	4.431	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	557	429	2205	1237	0	0	-1
N.S.	1	1.00	2.37	1.83	9.38	5.26	0.00	0.00	-0.00
time (sec)	N/A	0.373	6.644	0.245	0.702	3.594	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	282	182	1173	531	0	0	-1
N.S.	1	1.00	2.24	1.44	9.31	4.21	0.00	0.00	-0.01
time (sec)	N/A	0.108	2.553	0.145	0.689	3.690	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.371	42.346	0.083	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.385	17.808	0.086	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.131	5.056	0.030	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	425	489	3446	892	0	0	-1
N.S.	1	1.00	3.06	3.52	24.79	6.42	0.00	0.00	-0.01
time (sec)	N/A	0.171	6.472	0.113	0.643	2.944	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	286	301	668	540	0	0	-1
N.S.	1	1.00	2.49	2.62	5.81	4.70	0.00	0.00	-0.01
time (sec)	N/A	0.113	6.352	0.105	0.634	3.175	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	66	165	988	86	0	4474	150
N.S.	1	1.00	1.20	3.00	17.96	1.56	0.00	81.35	2.73
time (sec)	N/A	0.041	0.528	0.069	0.490	2.487	0.000	1.098	3.063

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	48	61	283	36	0	571	53
N.S.	1	1.00	1.37	1.74	8.09	1.03	0.00	16.31	1.51
time (sec)	N/A	0.022	0.085	0.081	0.268	2.077	0.000	0.494	2.187

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.090	6.790	0.104	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.114	10.232	0.046	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.058	180.003	0.030	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	530	1127	3831	1315	0	0	-1
N.S.	1	1.00	1.57	3.34	11.37	3.90	0.00	0.00	-0.00
time (sec)	N/A	0.282	3.094	0.238	1.599	3.716	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	526	584	1891	795	0	0	-1
N.S.	1	1.00	2.73	3.03	9.80	4.12	0.00	0.00	-0.01
time (sec)	N/A	0.177	6.924	0.188	0.752	3.323	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	538	267	0	435	0	0	-1
N.S.	1	1.00	4.60	2.28	0.00	3.72	0.00	0.00	-0.01
time (sec)	N/A	0.084	6.629	0.115	0.000	2.723	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.060	26.394	0.104	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.057	28.933	0.238	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	9.122	0.052	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	817	729	2413	592	0	0	-1
N.S.	1	1.00	3.15	2.81	9.32	2.29	0.00	0.00	-0.00
time (sec)	N/A	0.233	6.813	0.138	1.132	3.258	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	461	409	1228	354	0	0	-1
N.S.	1	1.00	2.73	2.42	7.27	2.09	0.00	0.00	-0.01
time (sec)	N/A	0.154	6.627	0.109	0.675	3.490	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	240	183	517	168	0	0	-1
N.S.	1	1.00	2.22	1.69	4.79	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.082	6.183	0.078	0.580	3.046	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.025	6.684	0.055	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	7.373	0.062	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.198	12.231	0.047	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	1790	1729	8853	3324	0	0	-1
N.S.	1	1.00	4.49	4.33	22.19	8.33	0.00	0.00	-0.00
time (sec)	N/A	0.648	7.360	0.257	5.368	4.306	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	1294	1115	4954	2268	0	0	-1
N.S.	1	1.00	3.98	3.43	15.24	6.98	0.00	0.00	-0.00
time (sec)	N/A	0.445	6.809	0.167	1.766	5.325	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	788	614	2447	1404	0	0	-1
N.S.	1	1.00	3.92	3.05	12.17	6.99	0.00	0.00	-0.00
time (sec)	N/A	0.271	6.689	0.132	0.764	3.584	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	141	212	270	1028	760	0	0	-1
N.S.	1	1.01	1.53	1.94	7.40	5.47	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.593	0.108	0.628	3.091	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.133	8.794	0.109	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.141	6.443	0.126	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.176	27.775	0.046	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	486	486	881	1629	8046	2226	0	0	-1
N.S.	1	1.00	1.81	3.35	16.56	4.58	0.00	0.00	-0.00
time (sec)	N/A	0.862	7.363	0.606	4.712	3.464	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	889	770	3828	1366	0	0	-1
N.S.	1	1.00	2.61	2.26	11.23	4.01	0.00	0.00	-0.00
time (sec)	N/A	0.440	7.192	0.337	1.269	2.817	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	182	660	344	0	592	0	0	-1
N.S.	1	1.12	4.07	2.12	0.00	3.65	0.00	0.00	-0.01
time (sec)	N/A	0.122	6.682	0.171	0.000	2.560	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.143	20.260	0.140	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.160	25.568	0.079	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.168	29.490	0.055	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	582	1329	5646	4193	0	0	-1
N.S.	1	1.00	1.83	4.18	17.75	13.19	0.00	0.00	-0.00
time (sec)	N/A	0.215	7.241	0.208	2.679	4.507	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	381	716	2728	2387	0	0	-1
N.S.	1	1.00	2.01	3.77	14.36	12.56	0.00	0.00	-0.01
time (sec)	N/A	0.139	6.940	0.164	0.931	3.424	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	236	325	1070	1193	0	0	-1
N.S.	1	1.00	2.15	2.95	9.73	10.85	0.00	0.00	-0.01
time (sec)	N/A	0.072	2.230	0.173	0.660	2.586	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.064	25.272	0.163	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.058	29.656	0.096	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	73	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.356	0.024	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.411	0.020	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.152	0.018	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	181	310	0	0	0	0	-1
N.S.	1	1.00	5.48	9.39	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.018	1.924	0.149	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.019	0.174	0.017	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.203	0.017	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	53	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.252	0.019	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	65	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.277	0.018	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	65	0	0	0	0	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.298	0.036	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	61	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.249	0.032	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	54	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.196	0.030	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	42	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.136	0.032	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	132	310	0	0	0	0	-1
N.S.	1	1.00	2.49	5.85	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	2.423	0.083	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	63	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.265	0.030	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	212	0	0	0	0	0	-1
N.S.	1	1.00	2.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	8.245	0.029	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	89	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.355	0.027	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	67	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.595	0.023	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	108	0	0	0	0	0	-1
N.S.	1	1.00	1.66	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.920	0.020	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	56	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.197	0.023	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	86	308	0	0	0	0	-1
N.S.	1	1.00	2.26	8.11	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.017	0.609	0.113	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.017	0.187	0.017	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	56	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.197	0.017	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	57	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.249	0.020	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	73	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.314	0.019	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	73	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.285	0.045	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	65	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.318	0.043	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	56	0	0	0	0	0	-1
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.201	0.042	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	46	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.148	0.042	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	106	308	0	0	0	0	-1
N.S.	1	1.00	1.83	5.31	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.789	0.131	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	65	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.235	0.033	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	114	0	0	0	0	0	-1
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	1.033	0.031	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	93	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.420	0.030	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	26	18	17	37	18	18
N.S.	1	1.00	0.71	0.84	0.58	0.55	1.19	0.58	0.58
time (sec)	N/A	0.029	0.020	0.079	0.267	2.924	1.114	0.401	0.092

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	154	260	146	200	440	167	212
N.S.	1	1.00	1.18	1.98	1.11	1.53	3.36	1.27	1.62
time (sec)	N/A	0.143	0.193	0.120	0.273	2.517	15.493	0.393	2.261

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	109	179	101	127	289	112	136
N.S.	1	1.00	0.95	1.56	0.88	1.10	2.51	0.97	1.18
time (sec)	N/A	0.107	0.149	0.092	0.274	3.134	7.622	0.389	0.343

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	107	60	70	155	64	73
N.S.	1	1.00	0.82	1.47	0.82	0.96	2.12	0.88	1.00
time (sec)	N/A	0.078	0.106	0.088	0.269	2.538	4.071	0.386	1.815

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	34	52	27	27	56	27	30
N.S.	1	1.00	0.83	1.27	0.66	0.66	1.37	0.66	0.73
time (sec)	N/A	0.042	0.031	0.073	0.268	2.170	2.193	0.392	1.715

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	58	97	62	0	51	-1
N.S.	1	1.00	0.86	1.02	1.70	1.09	0.00	0.89	-0.02
time (sec)	N/A	0.195	0.073	0.096	0.301	2.495	0.000	0.394	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	61	82	330	95	0	111	-1
N.S.	1	1.00	0.78	1.05	4.23	1.22	0.00	1.42	-0.01
time (sec)	N/A	0.177	0.158	0.125	0.313	2.687	0.000	0.400	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	77	104	365	158	0	201	-1
N.S.	1	1.00	0.78	1.05	3.69	1.60	0.00	2.03	-0.01
time (sec)	N/A	0.233	0.273	0.132	0.326	2.074	0.000	0.403	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	128	1000	244	283	0	1255	344
N.S.	1	1.00	0.65	5.05	1.23	1.43	0.00	6.34	1.74
time (sec)	N/A	0.177	0.636	0.126	0.306	1.975	0.000	0.535	0.650

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	105	580	173	188	0	682	216
N.S.	1	1.00	0.61	3.39	1.01	1.10	0.00	3.99	1.26
time (sec)	N/A	0.133	0.398	0.088	0.293	1.652	0.000	0.484	2.182

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	73	294	108	111	0	313	121
N.S.	1	1.00	0.65	2.62	0.96	0.99	0.00	2.79	1.08
time (sec)	N/A	0.108	0.402	0.076	0.278	2.366	0.000	0.458	0.313

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	119	55	54	0	106	53
N.S.	1	1.00	0.70	1.80	0.83	0.82	0.00	1.61	0.80
time (sec)	N/A	0.050	0.158	0.058	0.290	1.764	0.000	0.413	0.205

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	63	116	119	85	0	102	-1
N.S.	1	1.00	0.89	1.63	1.68	1.20	0.00	1.44	-0.01
time (sec)	N/A	0.202	0.119	0.096	0.329	1.870	0.000	0.427	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	81	169	120	131	0	308	-1
N.S.	1	1.00	0.79	1.66	1.18	1.28	0.00	3.02	-0.01
time (sec)	N/A	0.205	0.502	0.128	0.350	1.953	0.000	0.436	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	104	207	132	225	0	704	-1
N.S.	1	1.00	0.76	1.52	0.97	1.65	0.00	5.18	-0.01
time (sec)	N/A	0.269	0.917	0.156	0.335	1.412	0.000	0.487	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	125	243	143	343	0	1305	-1
N.S.	1	1.00	0.61	1.19	0.70	1.67	0.00	6.37	-0.00
time (sec)	N/A	0.264	0.922	0.187	0.360	2.687	0.000	0.500	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	541	849	606	929	0	0	-1
N.S.	1	1.00	2.12	3.33	2.38	3.64	0.00	0.00	-0.00
time (sec)	N/A	0.241	1.044	0.125	0.416	1.849	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	317	481	413	566	0	0	-1
N.S.	1	1.00	1.84	2.80	2.40	3.29	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.687	0.256	0.386	1.332	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	171	204	0	281	0	0	-1
N.S.	1	1.00	1.80	2.15	0.00	2.96	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.332	0.118	0.000	1.869	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.167	6.006	0.132	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.210	6.602	0.138	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.245	6.998	0.161	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	2517	965	610	1652	0	0	-1
N.S.	1	1.00	8.42	3.23	2.04	5.53	0.00	0.00	-0.00
time (sec)	N/A	0.354	6.475	0.139	0.420	4.148	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	1733	648	444	1126	0	0	-1
N.S.	1	1.00	7.16	2.68	1.83	4.65	0.00	0.00	-0.00
time (sec)	N/A	0.297	6.408	0.111	0.380	4.665	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	523	386	303	681	0	0	-1
N.S.	1	1.00	3.02	2.23	1.75	3.94	0.00	0.00	-0.01
time (sec)	N/A	0.227	6.404	0.147	0.363	4.477	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	254	177	0	340	0	0	-1
N.S.	1	1.00	2.37	1.65	0.00	3.18	0.00	0.00	-0.01
time (sec)	N/A	0.126	6.131	0.321	0.000	3.439	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	3.251	0.201	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.233	4.424	0.279	0.000	0.000	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.268	6.130	0.198	0.000	0.000	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	532	677	0	896	0	0	-1
N.S.	1	1.00	2.31	2.94	0.00	3.90	0.00	0.00	-0.00
time (sec)	N/A	0.230	2.286	0.158	0.000	5.236	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-2)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	364	369	0	513	0	0	-1
N.S.	1	1.00	2.48	2.51	0.00	3.49	0.00	0.00	-0.01
time (sec)	N/A	0.153	3.817	0.119	0.000	2.993	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	105	96	3330	93	0	365	150
N.S.	1	1.00	1.84	1.68	58.42	1.63	0.00	6.40	2.63
time (sec)	N/A	0.067	0.450	0.106	0.557	3.538	0.000	0.504	1.305

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.236	13.417	0.201	0.000	0.000	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.273	16.833	0.233	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.319	19.799	0.355	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	77	66	0	106	0	0	46
N.S.	1	1.00	1.35	1.16	0.00	1.86	0.00	0.00	0.81
time (sec)	N/A	0.047	0.034	0.107	0.000	3.030	0.000	0.000	2.214

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	111	26	144	118	31
N.S.	1	1.00	1.00	1.07	7.93	1.86	10.29	8.43	2.21
time (sec)	N/A	0.023	0.032	0.080	0.491	3.147	4.081	0.430	2.347

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	146	102	0	144	0	0	63
N.S.	1	1.00	2.18	1.52	0.00	2.15	0.00	0.00	0.94
time (sec)	N/A	0.091	0.332	0.158	0.000	2.986	0.000	0.000	2.317

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [124] had the largest ratio of [26]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.00	20	0.200
2	A	5	4	1.00	20	0.200
3	A	5	5	1.00	20	0.250
4	A	3	2	1.00	20	0.100
5	A	3	3	1.00	18	0.167
6	A	5	5	1.00	20	0.250
7	A	6	6	1.00	20	0.300
8	A	7	6	1.00	20	0.300
9	A	8	6	1.00	20	0.300
10	A	3	3	1.00	8	0.375
11	A	4	4	1.00	8	0.500
12	A	5	4	1.00	8	0.500
13	A	8	3	1.00	22	0.136
14	A	9	5	1.00	22	0.227
15	A	7	5	1.00	22	0.227
16	A	4	4	1.00	22	0.182
17	A	3	2	1.00	20	0.100
18	A	8	4	1.00	22	0.182
19	A	10	5	1.00	22	0.227
20	A	12	5	1.00	22	0.227
21	A	14	5	1.00	22	0.227
22	A	8	3	1.00	22	0.136
23	A	9	4	1.00	22	0.182
24	A	9	5	1.00	22	0.227
25	A	4	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	4	3	1.00	20	0.150
27	A	8	4	1.00	22	0.182
28	A	10	5	1.00	22	0.227
29	A	12	5	1.00	22	0.227
30	A	14	5	1.00	22	0.227
31	A	0	0	0.00	0	0.000
32	A	7	6	1.00	14	0.429
33	A	6	6	1.00	14	0.429
34	A	5	5	1.00	14	0.357
35	A	4	4	1.00	12	0.333
36	A	0	0	0.00	0	0.000
37	A	0	0	0.00	0	0.000
38	A	0	0	0.00	0	0.000
39	A	10	6	1.00	20	0.300
40	A	8	5	1.00	20	0.250
41	A	6	4	1.00	20	0.200
42	A	2	2	1.00	18	0.111
43	A	0	0	0.00	0	0.000
44	A	0	0	0.00	0	0.000
45	A	0	0	0.00	0	0.000
46	A	7	7	1.00	22	0.318
47	A	6	6	1.00	22	0.273
48	A	3	3	1.00	22	0.136
49	A	3	3	1.00	20	0.150
50	A	0	0	0.00	0	0.000
51	A	0	0	0.00	0	0.000
52	A	10	8	1.00	22	0.364
53	A	9	8	1.00	22	0.364
54	A	8	8	1.00	22	0.364
55	A	8	8	1.00	22	0.364
56	A	9	8	1.00	22	0.364
57	A	10	8	1.00	22	0.364
58	A	18	7	1.00	24	0.292
59	A	16	7	1.00	24	0.292
60	A	14	7	1.00	24	0.292

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	14	7	1.00	24	0.292
62	A	16	7	1.00	24	0.292
63	A	18	7	1.00	24	0.292
64	A	18	7	1.00	24	0.292
65	A	16	7	1.00	24	0.292
66	A	14	7	1.00	24	0.292
67	A	14	7	1.00	24	0.292
68	A	16	7	1.00	24	0.292
69	A	18	7	1.00	24	0.292
70	A	8	3	1.00	22	0.136
71	A	9	5	1.00	22	0.227
72	A	7	5	1.00	22	0.227
73	A	4	4	1.00	22	0.182
74	A	3	2	1.00	20	0.100
75	A	8	4	1.00	22	0.182
76	A	10	5	1.00	22	0.227
77	A	12	5	1.00	22	0.227
78	A	14	5	1.00	22	0.227
79	A	5	3	1.00	24	0.125
80	A	7	3	1.00	24	0.125
81	A	6	3	1.00	24	0.125
82	A	5	3	1.00	24	0.125
83	A	4	3	1.00	22	0.136
84	A	5	4	1.00	24	0.167
85	A	6	5	1.00	24	0.208
86	A	7	5	1.00	24	0.208
87	A	8	5	1.00	24	0.208
88	A	11	3	1.00	24	0.125
89	A	17	3	1.00	24	0.125
90	A	14	3	1.00	24	0.125
91	A	11	3	1.00	24	0.125
92	A	8	3	1.00	22	0.136
93	A	11	4	1.00	24	0.167
94	A	14	5	1.00	24	0.208
95	A	17	5	1.00	24	0.208

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	20	5	1.00	24	0.208
97	A	0	0	0.00	0	0.000
98	A	17	8	1.00	20	0.400
99	A	14	8	1.00	20	0.400
100	A	11	7	1.00	20	0.350
101	A	8	6	1.00	18	0.333
102	A	0	0	0.00	0	0.000
103	A	0	0	0.00	0	0.000
104	A	0	0	0.00	0	0.000
105	A	8	8	1.00	16	0.500
106	A	7	7	1.00	16	0.438
107	A	6	6	1.00	16	0.375
108	A	3	2	1.00	14	0.143
109	A	0	0	0.00	0	0.000
110	A	0	0	0.00	0	0.000
111	A	0	0	0.00	0	0.000
112	A	31	7	1.00	22	0.318
113	A	25	9	1.00	22	0.409
114	A	17	7	1.00	22	0.318
115	A	12	5	1.00	20	0.250
116	A	0	0	0.00	0	0.000
117	A	0	0	0.00	0	0.000
118	A	18	7	1.00	24	0.292
119	A	16	7	1.00	24	0.292
120	A	14	7	1.00	24	0.292
121	A	14	7	1.00	24	0.292
122	A	16	7	1.00	24	0.292
123	A	18	7	1.00	24	0.292
124	A	10	7	1.00	26	0.269
125	A	9	7	1.00	26	0.269
126	A	8	7	1.00	26	0.269
127	A	8	7	1.00	26	0.269
128	A	9	7	1.00	26	0.269
129	A	10	7	1.00	26	0.269
130	A	26	7	1.00	26	0.269

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	23	7	1.00	26	0.269
132	A	20	7	1.00	26	0.269
133	A	20	7	1.00	26	0.269
134	A	23	7	1.00	26	0.269
135	A	26	7	1.00	26	0.269
136	A	8	3	1.00	22	0.136
137	A	9	4	1.00	22	0.182
138	A	9	5	1.00	22	0.227
139	A	4	2	1.00	22	0.091
140	A	4	3	1.00	20	0.150
141	A	8	4	1.00	22	0.182
142	A	10	5	1.00	22	0.227
143	A	12	5	1.00	22	0.227
144	A	14	5	1.00	22	0.227
145	A	11	3	1.00	24	0.125
146	A	17	3	1.00	24	0.125
147	A	14	3	1.00	24	0.125
148	A	11	3	1.00	24	0.125
149	A	8	3	1.00	22	0.136
150	A	11	4	1.00	24	0.167
151	A	14	5	1.00	24	0.208
152	A	17	5	1.00	24	0.208
153	A	20	5	1.00	24	0.208
154	A	8	3	1.00	24	0.125
155	A	12	3	1.00	24	0.125
156	A	10	3	1.00	24	0.125
157	A	8	3	1.00	24	0.125
158	A	6	3	1.00	22	0.136
159	A	8	4	1.00	24	0.167
160	A	10	5	1.00	24	0.208
161	A	12	5	1.00	24	0.208
162	A	14	5	1.00	24	0.208
163	A	0	0	0.00	0	0.000
164	A	13	11	1.00	22	0.500
165	A	12	12	1.00	22	0.546

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	9	8	1.00	22	0.364
167	A	8	8	1.00	20	0.400
168	A	0	0	0.00	0	0.000
169	A	0	0	0.00	0	0.000
170	A	0	0	0.00	0	0.000
171	A	16	9	1.00	22	0.409
172	A	13	8	1.00	22	0.364
173	A	10	7	1.00	22	0.318
174	A	5	5	1.00	20	0.250
175	A	0	0	0.00	0	0.000
176	A	0	0	0.00	0	0.000
177	A	0	0	0.00	0	0.000
178	A	15	8	1.00	16	0.500
179	A	13	10	1.00	16	0.625
180	A	9	7	1.00	16	0.438
181	A	7	7	1.00	14	0.500
182	A	0	0	0.00	0	0.000
183	A	0	0	0.00	0	0.000
184	A	18	7	1.00	24	0.292
185	A	16	7	1.00	24	0.292
186	A	14	7	1.00	24	0.292
187	A	14	7	1.00	24	0.292
188	A	16	7	1.00	24	0.292
189	A	18	7	1.00	24	0.292
190	A	26	7	1.00	26	0.269
191	A	23	7	1.00	26	0.269
192	A	20	7	1.00	26	0.269
193	A	20	7	1.00	26	0.269
194	A	23	7	1.00	26	0.269
195	A	26	7	1.00	26	0.269
196	A	18	7	1.00	26	0.269
197	A	16	7	1.00	26	0.269
198	A	14	7	1.00	26	0.269
199	A	14	7	1.00	26	0.269
200	A	16	7	1.00	26	0.269

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	18	7	1.00	26	0.269
202	A	12	10	1.00	12	0.833
203	A	11	10	1.00	12	0.833
204	A	6	5	1.00	10	0.500
205	A	26	15	1.00	12	1.250
206	A	19	11	1.00	12	0.917
207	A	16	10	1.00	10	1.000
208	A	0	0	0.00	0	0.000
209	A	7	6	1.00	14	0.429
210	A	6	6	1.00	14	0.429
211	A	5	5	1.00	14	0.357
212	A	4	4	1.00	12	0.333
213	A	0	0	0.00	0	0.000
214	A	0	0	0.00	0	0.000
215	A	0	0	0.00	0	0.000
216	A	14	8	1.00	20	0.400
217	A	11	7	1.00	20	0.350
218	A	8	6	1.00	18	0.333
219	A	0	0	0.00	0	0.000
220	A	0	0	0.00	0	0.000
221	A	0	0	0.00	0	0.000
222	A	12	12	1.00	22	0.546
223	A	9	8	1.00	22	0.364
224	A	8	8	1.00	20	0.400
225	A	0	0	0.00	0	0.000
226	A	0	0	0.00	0	0.000
227	A	0	0	0.00	0	0.000
228	A	12	6	1.00	20	0.300
229	A	10	6	1.00	20	0.300
230	A	8	5	1.00	20	0.250
231	A	6	4	1.00	18	0.222
232	A	0	0	0.00	0	0.000
233	A	0	0	0.00	0	0.000
234	A	0	0	0.00	0	0.000
235	A	23	14	1.00	22	0.636

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	19	15	1.00	22	0.682
237	A	10	10	1.00	20	0.500
238	A	0	0	0.00	0	0.000
239	A	0	0	0.00	0	0.000
240	A	0	0	0.00	0	0.000
241	A	22	18	1.00	22	0.818
242	A	17	13	1.00	22	0.591
243	A	11	10	1.00	20	0.500
244	A	0	0	0.00	0	0.000
245	A	0	0	0.00	0	0.000
246	A	0	0	0.00	0	0.000
247	A	10	6	1.00	20	0.300
248	A	8	5	1.00	20	0.250
249	A	6	4	1.00	20	0.200
250	A	2	2	1.00	18	0.111
251	A	0	0	0.00	0	0.000
252	A	0	0	0.00	0	0.000
253	A	0	0	0.00	0	0.000
254	A	7	7	1.00	16	0.438
255	A	6	6	1.00	16	0.375
256	A	3	2	1.00	14	0.143
257	A	0	0	0.00	0	0.000
258	A	0	0	0.00	0	0.000
259	A	0	0	0.00	0	0.000
260	A	13	8	1.00	22	0.364
261	A	10	7	1.00	22	0.318
262	A	5	5	1.00	20	0.250
263	A	0	0	0.00	0	0.000
264	A	0	0	0.00	0	0.000
265	A	0	0	0.00	0	0.000
266	A	27	14	1.00	22	0.636
267	A	23	14	1.00	22	0.636
268	A	19	15	1.00	22	0.682
269	A	10	10	1.08	20	0.500
270	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	0	0	0.00	0	0.000
272	A	0	0	0.00	0	0.000
273	A	7	7	1.00	24	0.292
274	A	6	6	1.00	24	0.250
275	A	3	3	1.00	22	0.136
276	A	0	0	0.00	0	0.000
277	A	0	0	0.00	0	0.000
278	A	0	0	0.00	0	0.000
279	A	64	24	1.00	24	1.000
280	A	36	22	1.00	24	0.917
281	A	13	12	1.13	22	0.546
282	A	0	0	0.00	0	0.000
283	A	0	0	0.00	0	0.000
284	A	0	0	0.00	0	0.000
285	A	40	18	1.00	20	0.900
286	A	29	19	1.00	20	0.950
287	A	13	12	1.00	18	0.667
288	A	0	0	0.00	0	0.000
289	A	0	0	0.00	0	0.000
290	A	0	0	0.00	0	0.000
291	A	7	7	1.00	22	0.318
292	A	6	6	1.00	22	0.273
293	A	3	3	1.00	22	0.136
294	A	3	3	1.00	20	0.150
295	A	0	0	0.00	0	0.000
296	A	0	0	0.00	0	0.000
297	A	0	0	0.00	0	0.000
298	A	25	9	1.00	22	0.409
299	A	17	7	1.00	22	0.318
300	A	12	5	1.00	20	0.250
301	A	0	0	0.00	0	0.000
302	A	0	0	0.00	0	0.000
303	A	0	0	0.00	0	0.000
304	A	13	10	1.00	16	0.625
305	A	9	7	1.00	16	0.438

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	7	7	1.00	14	0.500
307	A	0	0	0.00	0	0.000
308	A	0	0	0.00	0	0.000
309	A	0	0	0.00	0	0.000
310	A	25	16	1.00	22	0.727
311	A	22	18	1.00	22	0.818
312	A	17	13	1.00	22	0.591
313	A	11	10	1.01	20	0.500
314	A	0	0	0.00	0	0.000
315	A	0	0	0.00	0	0.000
316	A	0	0	0.00	0	0.000
317	A	44	19	1.00	24	0.792
318	A	31	19	1.00	24	0.792
319	A	13	12	1.12	22	0.546
320	A	0	0	0.00	0	0.000
321	A	0	0	0.00	0	0.000
322	A	0	0	0.00	0	0.000
323	A	16	9	1.00	24	0.375
324	A	10	7	1.00	24	0.292
325	A	7	5	1.00	22	0.227
326	A	0	0	0.00	0	0.000
327	A	0	0	0.00	0	0.000
328	A	4	3	1.00	18	0.167
329	A	3	3	1.00	18	0.167
330	A	3	3	1.00	18	0.167
331	A	2	2	1.00	18	0.111
332	A	2	2	1.00	18	0.111
333	A	3	3	1.00	18	0.167
334	A	3	3	1.00	18	0.167
335	A	4	3	1.00	18	0.167
336	A	5	4	1.00	18	0.222
337	A	4	4	1.00	18	0.222
338	A	4	4	1.00	18	0.222
339	A	3	3	1.00	18	0.167
340	A	3	3	1.00	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	4	4	1.00	18	0.222
342	A	4	4	1.00	18	0.222
343	A	5	4	1.00	18	0.222
344	A	4	3	1.00	18	0.167
345	A	3	3	1.00	18	0.167
346	A	3	3	1.00	18	0.167
347	A	2	2	1.00	18	0.111
348	A	2	2	1.00	18	0.111
349	A	3	3	1.00	18	0.167
350	A	3	3	1.00	18	0.167
351	A	4	3	1.00	18	0.167
352	A	5	4	1.00	18	0.222
353	A	4	4	1.00	18	0.222
354	A	4	4	1.00	18	0.222
355	A	3	3	1.00	18	0.167
356	A	3	3	1.00	18	0.167
357	A	4	4	1.00	18	0.222
358	A	4	4	1.00	18	0.222
359	A	5	4	1.00	18	0.222
360	A	6	3	1.00	8	0.375
361	A	14	5	1.00	14	0.357
362	A	10	4	1.00	14	0.286
363	A	10	5	1.00	14	0.357
364	A	6	2	1.00	12	0.167
365	A	12	5	1.00	14	0.357
366	A	12	6	1.00	14	0.429
367	A	16	7	1.00	14	0.500
368	A	14	5	1.00	23	0.217
369	A	10	4	1.00	23	0.174
370	A	10	5	1.00	23	0.217
371	A	6	2	1.00	21	0.095
372	A	12	5	1.00	23	0.217
373	A	12	6	1.00	23	0.261
374	A	16	7	1.00	23	0.304
375	A	16	8	1.00	23	0.348

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	20	9	1.00	25	0.360
377	A	16	8	1.00	25	0.320
378	A	12	7	1.00	23	0.304
379	A	0	0	0.00	0	0.000
380	A	0	0	0.00	0	0.000
381	A	0	0	0.00	0	0.000
382	A	20	12	1.00	23	0.522
383	A	19	13	1.00	23	0.565
384	A	14	9	1.00	23	0.391
385	A	13	9	1.00	21	0.429
386	A	0	0	0.00	0	0.000
387	A	0	0	0.00	0	0.000
388	A	0	0	0.00	0	0.000
389	A	19	9	1.00	25	0.360
390	A	15	8	1.00	25	0.320
391	A	9	6	1.00	23	0.261
392	A	0	0	0.00	0	0.000
393	A	0	0	0.00	0	0.000
394	A	0	0	0.00	0	0.000
395	A	12	7	1.00	8	0.875
396	A	5	4	1.00	10	0.400
397	A	19	6	1.00	10	0.600

Chapter 3

Listing of integrals

Local contents

3.1 $\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=137

$$-\frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-3-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right.}{b}$$

```
[Out] -2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*b
*(d*x+c)/d)^m)-2^(-3-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a-b
*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4491, 12, 3389, 2212}

$$-\frac{2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2ib(c+dx)}{d}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -((2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c
 + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-3 - m)*(c + d*x)^m*Gamma[
1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/
d)^m)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx &= \int \frac{1}{2}(c+dx)^m \sin(2a+2bx)\,dx \\
&= \frac{1}{2}\int (c+dx)^m \sin(2a+2bx)\,dx \\
&= \frac{1}{4}i\int e^{-i(2a+2bx)}(c+dx)^m\,dx - \frac{1}{4}i\int e^{i(2a+2bx)}(c+dx)^m\,dx \\
&= -\frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}
\end{aligned}
$$

Mathematica [A]
time = 0.32, size = 162, normalized size = 1.18

$$
-\frac{2^{-3-m}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(\left(-\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(1+m,\frac{2ib(c+dx)}{d}\right)\left(\cos\left(2a-\frac{2bc}{d}\right)-i\sin\left(2a-\frac{2bc}{d}\right)\right)+\left(\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(1+m,-\frac{2ib(c+dx)}{d}\right)\left(\cos\left(2a-\frac{2bc}{d}\right)+i\sin\left(2a-\frac{2bc}{d}\right)\right)\right)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x],x]
[Out] -((2^(-3 - m)*(c + d*x)^m*((((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(
c + d*x))/d]*(Cos[2*a - (2*b*c)/d] - I*Sin[2*a - (2*b*c)/d]) + ((I*b*(c + d
*x))/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d]*(Cos[2*a - (2*b*c)/d] + I*Si
n[2*a - (2*b*c)/d])))/(b*((b^2*(c + d*x)^2)/d^2)^m))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$
\int (dx+c)^m \cos(bx+a)\sin(bx+a)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x)
[Out] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a), x)
```

Fricas [A]
time = 0.33, size = 96, normalized size = 0.70

$$-\frac{e^{\left(-\frac{dm\log\left(-\frac{2ib}{d}\right)+2i\,bc-2i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{2\,(i\,bdx+i\,bc)}{d}\right)+e^{\left(-\frac{dm\log\left(\frac{2ib}{d}\right)-2i\,bc+2i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{2\,(-i\,bdx-i\,bc)}{d}\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m + 1, -2*(I*b*d
*x + I*b*c)/d) + e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d)*gamma(m + 1,
 -2*(-I*b*d*x - I*b*c)/d))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*sin(a + b*x)*cos(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+bx)\sin(a+bx)(c+dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^m,x)
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^m, x)
```

3.2 $\int (c+dx)^4 \cos(a+bx) \sin(a+bx) \, dx$

Optimal. Leaf size=156

$$\frac{3cd^3x}{2b^3}+\frac{3d^4x^2}{4b^3}-\frac{(c+dx)^4}{4b}-\frac{3d^3(c+dx)\cos(a+bx)\sin(a+bx)}{2b^4}+\frac{d(c+dx)^3\cos(a+bx)\sin(a+bx)}{b^2}+\frac{3d^4\sin^2}{4}$$

```
[Out] 3/2*c*d^3*x/b^3+3/4*d^4*x^2/b^3-1/4*(d*x+c)^4/b-3/2*d^3*(d*x+c)*cos(b*x+a)*
sin(b*x+a)/b^4+d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2+3/4*d^4*sin(b*x+a)^2/b
^5-3/2*d^2*(d*x+c)^2*sin(b*x+a)^2/b^3+1/2*(d*x+c)^4*sin(b*x+a)^2/b
```

Rubi [A]
time = 0.08, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4489, 3392, 32, 3391}

$$\frac{3d^4\sin^2(a+bx)}{4b^5}-\frac{3d^3(c+dx)\sin(a+bx)\cos(a+bx)}{2b^4}-\frac{3d^2(c+dx)^2\sin^2(a+bx)}{2b^3}+\frac{d(c+dx)^3\sin(a+bx)\cos(a+bx)}{b^2}+\frac{(c+dx)^4\sin^2(a+bx)}{2b}+\frac{3cd^3x}{2b^3}+\frac{3d^4x^2}{4b^3}-\frac{(c+dx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (3*c*d^3*x)/(2*b^3) + (3*d^4*x^2)/(4*b^3) - (c + d*x)^4/(4*b) - (3*d^3*(c +
 d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b^4) + (d*(c + d*x)^3*Cos[a + b*x]*Sin[
a + b*x])/b^2 + (3*d^4*Sin[a + b*x]^2)/(4*b^5) - (3*d^2*(c + d*x)^2*Sin[a +
 b*x]^2)/(2*b^3) + ((c + d*x)^4*Sin[a + b*x]^2)/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^4\cos(a+bx)\sin(a+bx)\,dx &= \frac{(c+dx)^4\sin^2(a+bx)}{2b} - \frac{(2d)\int (c+dx)^3\sin^2(a+bx)\,dx}{b}\\ &= \frac{d(c+dx)^3\cos(a+bx)\sin(a+bx)}{b^2} - \frac{3d^2(c+dx)^2\sin^2(a+bx)}{2b^3} + \\ &= -\frac{(c+dx)^4}{4b} - \frac{3d^3(c+dx)\cos(a+bx)\sin(a+bx)}{2b^4} + \frac{d(c+dx)^3\text{co}}{}\\ &= \frac{3cd^3x}{2b^3} + \frac{3d^4x^2}{4b^3} - \frac{(c+dx)^4}{4b} - \frac{3d^3(c+dx)\cos(a+bx)\sin(a+bx)}{2b^4}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 86, normalized size = 0.55

$$\frac{-2(3d^4-6b^2d^2(c+dx)^2+2b^4(c+dx)^4)\cos(2(a+bx))+4bd(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\sin(2(a+bx))}{16b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x],x]
[Out] (-2*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)] +
4*b*d*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(16*b^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 852 vs. $2(142) = 284$.
time = 0.25, size = 853, normalized size = 5.47

method	result
risch	$-\frac{(2d^4x^4b^4+8b^4c\,d^3x^3+12b^4c^2d^2x^2+8b^4c^3dx+2b^4c^4-6b^2d^4x^2-12b^2c\,d^3x-6b^2c^2d^2+3d^4)\cos(2bx+2a)}{8b^5}+\frac{d(2b^2d^3x}{}$
norman	$\frac{\frac{\left(2b^4c^4-6b^2c^2d^2+3d^4\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^5}+\frac{cd\left(2b^2c^2-3d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^4}-\frac{d^4x^4}{4b}-\frac{c\,d^3x^3}{b}-\frac{3d^2\left(2b^2c^2-d^2\right)x^2}{4b^3}+\frac{2d^4x^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^2}}{}$
derivativedivides	Expression too large to display

default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2/b^4*a^4*d^4*cos(b*x+a)^2+2/b^3*a^3*c*d^3*cos(b*x+a)^2-4/b^4*a^3*d
^4*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)-3/b^
2*a^2*c^2*d^2*cos(b*x+a)^2+12/b^3*a^2*c*d^3*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*
cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+6/b^4*a^2*d^4*(-1/2*(b*x+a)^2*cos(b*x+
a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*si
n(b*x+a)^2)+2/b*a*c^3*d*cos(b*x+a)^2-12/b^2*a*c^2*d^2*(-1/2*(b*x+a)*cos(b*x
+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)-12/b^3*a*c*d^3*(-1/2*(b*x+a)
^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+
a)^2-1/4*sin(b*x+a)^2)-4/b^4*a*d^4*(-1/2*(b*x+a)^3*cos(b*x+a)^2+3/2*(b*x+a)
^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)*cos(b*x+a)^2-3/8*c
os(b*x+a)*sin(b*x+a)-3/8*b*x-3/8*a-1/2*(b*x+a)^3)-1/2*c^4*cos(b*x+a)^2+4/b*
c^3*d*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+6
/b^2*c^2*d^2*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a
)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+4/b^3*c*d^3*(-1/2*(b*x+a)^
3*cos(b*x+a)^2+3/2*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/4*
(b*x+a)*cos(b*x+a)^2-3/8*cos(b*x+a)*sin(b*x+a)-3/8*b*x-3/8*a-1/2*(b*x+a)^3)
+1/b^4*d^4*(-1/2*(b*x+a)^4*cos(b*x+a)^2+2*(b*x+a)^3*(1/2*cos(b*x+a)*sin(b*x
+a)+1/2*b*x+1/2*a)+3/2*(b*x+a)^2*cos(b*x+a)^2-3*(b*x+a)*(1/2*cos(b*x+a)*sin
(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)^2+3/4*sin(b*x+a)^2-3/4*(b*x+a)^4))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(142) = 284$.
time = 0.30, size = 586, normalized size = 3.76

$$-\frac{4\,c^{4}\cos\left(bx+a\right)^{2}-\frac{16\,ac^{3}d\cos\left(bx+a\right)^{2}}{b}+\frac{24\,a^{2}c^{2}d^{2}\cos\left(bx+a\right)^{2}}{b^{2}}-\frac{16\,a^{3}cd^{3}\cos\left(bx+a\right)^{2}}{b^{3}}+\frac{4\,a^{4}d^{4}\cos\left(bx+a\right)^{2}}{b^{4}}+\frac{4\,\left(2\,\left(bx+a\right)\cos\left(2\,bx+2\,a\right)-\sin\left(2\,bx+2\,a\right)\right)c^{3}d}{b}-\frac{12\,\left(2\,\left(bx+a\right)\cos\left(2\,bx+2\,a\right)-\sin\left(2\,bx+2\,a\right)\right)ac^{2}d^{2}}{b^{2}}+\frac{12\,\left(2\,\left(bx+a\right)\cos\left(2\,bx+2\,a\right)-\sin\left(2\,bx+2\,a\right)\right)a^{2}cd^{3}}{b^{3}}-\frac{4\,\left(2\,\left(bx+a\right)\cos\left(2\,bx+2\,a\right)-\sin\left(2\,bx+2\,a\right)\right)a^{3}d^{4}}{b^{4}}+\frac{6\,\left(\left(2\,\left(bx+a\right)^{2}-1\right)\cos\left(2\,bx+2\,a\right)-2\,\left(bx+a\right)\sin\left(2\,bx+2\,a\right)\right)c^{2}d^{2}}{b^{2}}-\frac{12\,\left(\left(2\,\left(bx+a\right)^{2}-1\right)\cos\left(2\,bx+2\,a\right)-2\,\left(bx+a\right)\sin\left(2\,bx+2\,a\right)\right)acd^{3}}{b^{3}}+\frac{6\,\left(\left(2\,\left(bx+a\right)^{2}-1\right)\cos\left(2\,bx+2\,a\right)-2\,\left(bx+a\right)\sin\left(2\,bx+2\,a\right)\right)a^{2}d^{4}}{b^{4}}+\frac{2\,\left(2\,\left(2\,\left(bx+a\right)^{3}-3\,bx-3\,a\right)\cos\left(2\,bx+2\,a\right)-3\,\left(2\,\left(bx+a\right)^{2}-1\right)\sin\left(2\,bx+2\,a\right)\right)cd^{3}}{b^{3}}-\frac{2\,\left(2\,\left(2\,\left(bx+a\right)^{3}-3\,bx-3\,a\right)\cos\left(2\,bx+2\,a\right)-3\,\left(2\,\left(bx+a\right)^{2}-1\right)\sin\left(2\,bx+2\,a\right)\right)ad^{4}}{b^{4}}+\frac{\left(\left(2\,\left(bx+a\right)^{4}-6\,\left(bx+a\right)^{2}+3\right)\cos\left(2\,bx+2\,a\right)-2\,\left(2\,\left(bx+a\right)^{3}-3\,bx-3\,a\right)\sin\left(2\,bx+2\,a\right)\right)d^{4}}{b^{4}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*(4*c^4*cos(b*x + a)^2 - 16*a*c^3*d*cos(b*x + a)^2/b + 24*a^2*c^2*d^2*c
os(b*x + a)^2/b^2 - 16*a^3*c*d^3*cos(b*x + a)^2/b^3 + 4*a^4*d^4*cos(b*x + a
)^2/b^4 + 4*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*c^3*d/b - 12*
(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 12*(2*(b*
x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a^2*c*d^3/b^3 - 4*(2*(b*x + a)*
cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a^3*d^4/b^4 + 6*((2*(b*x + a)^2 - 1)*c
os(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 - 12*((2*(b*x +
 a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/b^3 + 6
*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*a^2*
d^4/b^4 + 2*(2*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(2*(b*x +
 a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 - 2*(2*(2*(b*x + a)^3 - 3*b*x - 3*a)
```

```
*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a*d^4/b^4 + ((2
*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*cos(2*b*x + 2*a) - 2*(2*(b*x + a)^3 - 3*b
*x - 3*a)*sin(2*b*x + 2*a))*d^4/b^4)/b
```

Fricas [**A**]
time = 2.01, size = 255, normalized size = 1.63

$$\frac{b^4d^4x^4 + 4\,b^4cd^3x^3 + 3\,(2\,b^4c^2d^2 - b^2d^4)x^2 - (2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 2\,b^4c^4 - 6\,b^2c^2d^2 + 3\,d^4 + 6\,(2\,b^4c^2d^2 - b^2d^4)x^2 + 4\,(2\,b^4c^3d - 3\,b^2cd^3)x)\cos(bx+a)^2 + 2\,(2\,b^3d^4x^3 + 6\,b^3cd^3x^2 + 2\,b^3c^3d - 3\,bcd^3 + 3\,(2\,b^3c^2d^2 - bd^4)x)\cos(bx+a)\sin(bx+a) + 2\,(2\,b^4c^3d - 3\,b^2cd^3)x}{4\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 3*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - (2*b
^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4
*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^2 +
 2*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^
2*d^2 - b*d^4)*x)*cos(b*x + a)*sin(b*x + a) + 2*(2*b^4*c^3*d - 3*b^2*c*d^3)
*x)/b^5
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(153) = 306$.
time = 0.49, size = 502, normalized size = 3.22

$$\begin{cases} \frac{c^4\sin^2(a+bx)}{2b} + \frac{c^3dx\sin^2(a+bx)}{b} - \frac{c^3dx\cos^2(a+bx)}{b} + \frac{3c^2d^2x^2\sin^2(a+bx)}{2b} - \frac{3c^2d^2x^2\cos^2(a+bx)}{2b} + \frac{cd^3x^3\sin^2(a+bx)}{b} - \frac{cd^3x^3\cos^2(a+bx)}{b} + \frac{d^4x^4\sin^2(a+bx)}{4b} - \frac{d^4x^4\cos^2(a+bx)}{4b} + \frac{c^3d\sin(a+bx)\cos(a+bx)}{b^2} + \frac{3c^2d^2x\sin(a+bx)\cos(a+bx)}{b^2} + \frac{3cd^3x^2\sin(a+bx)\cos(a+bx)}{b^2} + \frac{d^4x^3\sin(a+bx)\cos(a+bx)}{b^2} - \frac{3c^2d^2\sin^2(a+bx)}{2b^3} - \frac{3cd^3x\sin^2(a+bx)}{2b^3} + \frac{3cd^3x\cos^2(a+bx)}{2b^3} - \frac{3d^4x^2\sin^2(a+bx)}{4b^3} + \frac{3d^4x^2\cos^2(a+bx)}{4b^3} - \frac{3cd^3\sin(a+bx)\cos(a+bx)}{2b^4} - \frac{3d^4x\sin(a+bx)\cos(a+bx)}{2b^4} + \frac{3d^4\sin^2(a+bx)}{4b^5} & \text{for}\: b \neq 0 \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sin(a)\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Piecewise((c**4*sin(a + b*x)**2/(2*b) + c**3*d*x*sin(a + b*x)**2/b - c**3*d
*x*cos(a + b*x)**2/b + 3*c**2*d**2*x**2*sin(a + b*x)**2/(2*b) - 3*c**2*d**2
*x**2*cos(a + b*x)**2/(2*b) + c*d**3*x**3*sin(a + b*x)**2/b - c*d**3*x**3*c
os(a + b*x)**2/b + d**4*x**4*sin(a + b*x)**2/(4*b) - d**4*x**4*cos(a + b*x)
**2/(4*b) + c**3*d*sin(a + b*x)*cos(a + b*x)/b**2 + 3*c**2*d**2*x*sin(a + b
*x)*cos(a + b*x)/b**2 + 3*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)/b**2 + d**4
*x**3*sin(a + b*x)*cos(a + b*x)/b**2 - 3*c**2*d**2*sin(a + b*x)**2/(2*b**3)
 - 3*c*d**3*x*sin(a + b*x)**2/(2*b**3) + 3*c*d**3*x*cos(a + b*x)**2/(2*b**3
) - 3*d**4*x**2*sin(a + b*x)**2/(4*b**3) + 3*d**4*x**2*cos(a + b*x)**2/(4*b
**3) - 3*c*d**3*sin(a + b*x)*cos(a + b*x)/(2*b**4) - 3*d**4*x*sin(a + b*x)*
cos(a + b*x)/(2*b**4) + 3*d**4*sin(a + b*x)**2/(4*b**5), Ne(b, 0)), ((c**4*
x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*co
s(a), True))
```

Giac [**A**]
time = 0.41, size = 181, normalized size = 1.16

$$-\frac{(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 12\,b^4c^2d^2x^2 + 8\,b^4c^3dx + 2\,b^4c^4 - 6\,b^2d^4x^2 - 12\,b^2cd^3x - 6\,b^2c^2d^2 + 3\,d^4)\cos(2\,bx+2\,a)}{8\,b^5} + \frac{(2\,b^3d^4x^3 + 6\,b^3cd^3x^2 + 6\,b^3c^2d^2x + 2\,b^3c^3d - 3\,bd^4x - 3\,bcd^3)\sin(2\,bx+2\,a)}{4\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x
+ 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x - 6*b^2*c^2*d^2 + 3*d^4)*cos(2
*b*x + 2*a)/b^5 + 1/4*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x +
2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

Mupad [B]
time = 0.49, size = 245, normalized size = 1.57

$$\frac{3\,x^2\,\cos\left(2\,a+2\,b\,x\right)\,\left(d^4-2\,b^2\,c^2\,d^2\right)}{4\,b^3}-\frac{\cos\left(2\,a+2\,b\,x\right)\,\left(\frac{b^4\,c^4}{2}-\frac{3\,b^2\,c^2\,d^2}{2}+\frac{3\,d^4}{4}\right)}{2\,b^5}-\frac{3\,x\,\sin\left(2\,a+2\,b\,x\right)\,\left(d^4-2\,b^2\,c^2\,d^2\right)}{4\,b^4}-\frac{d^4\,x^4\,\cos\left(2\,a+2\,b\,x\right)}{4\,b}-\frac{\sin\left(2\,a+2\,b\,x\right)\,\left(3\,c\,d^3-2\,b^2\,c^3\,d\right)}{4\,b^4}+\frac{x\,\cos\left(2\,a+2\,b\,x\right)\,\left(3\,c\,d^3-2\,b^2\,c^3\,d\right)}{2\,b^3}+\frac{d^4\,x^3\,\sin\left(2\,a+2\,b\,x\right)}{2\,b^2}-\frac{c\,d^3\,x^3\,\cos\left(2\,a+2\,b\,x\right)}{b}+\frac{3\,c\,d^3\,x^2\,\sin\left(2\,a+2\,b\,x\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^4,x)
```

```
[Out] (3*x^2*cos(2*a + 2*b*x)*(d^4 - 2*b^2*c^2*d^2))/(4*b^3) - (cos(2*a + 2*b*x)*
((3*d^4)/4 + (b^4*c^4)/2 - (3*b^2*c^2*d^2)/2))/(2*b^5) - (3*x*sin(2*a + 2*b
*x)*(d^4 - 2*b^2*c^2*d^2))/(4*b^4) - (d^4*x^4*cos(2*a + 2*b*x))/(4*b) - (si
n(2*a + 2*b*x)*(3*c*d^3 - 2*b^2*c^3*d))/(4*b^4) + (x*cos(2*a + 2*b*x)*(3*c*
d^3 - 2*b^2*c^3*d))/(2*b^3) + (d^4*x^3*sin(2*a + 2*b*x))/(2*b^2) - (c*d^3*x
^3*cos(2*a + 2*b*x))/b + (3*c*d^3*x^2*sin(2*a + 2*b*x))/(2*b^2)
```

3.3 $\int (c+dx)^3 \cos(a+bx) \sin(a+bx)\, dx$

Optimal. Leaf size=120

$$\frac{3d^3x}{8b^3}-\frac{(c+dx)^3}{4b}-\frac{3d^3\cos(a+bx)\sin(a+bx)}{8b^4}+\frac{3d(c+dx)^2\cos(a+bx)\sin(a+bx)}{4b^2}-\frac{3d^2(c+dx)\sin^2(a+b}{4b^3}$$

```
[Out] 3/8*d^3*x/b^3-1/4*(d*x+c)^3/b-3/8*d^3*cos(b*x+a)*sin(b*x+a)/b^4+3/4*d*(d*x+
c)^2*cos(b*x+a)*sin(b*x+a)/b^2-3/4*d^2*(d*x+c)*sin(b*x+a)^2/b^3+1/2*(d*x+c)
^3*sin(b*x+a)^2/b
```

Rubi [A]

time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4489, 3392, 32, 2715, 8}

$$-\frac{3d^3\sin(a+bx)\cos(a+bx)}{8b^4}-\frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3}+\frac{3d(c+dx)^2\sin(a+bx)\cos(a+bx)}{4b^2}+\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{3d^3x}{8b^3}-\frac{(c+dx)^3}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (3*d^3*x)/(8*b^3) - (c + d*x)^3/(4*b) - (3*d^3*Cos[a + b*x]*Sin[a + b*x])/(
8*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(4*b^2) - (3*d^2*(c +
d*x)*Sin[a + b*x]^2)/(4*b^3) + ((c + d*x)^3*Sin[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
```

```
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx = \frac{(c+dx)^3\sin^2(a+bx)}{2b} - \frac{(3d)\int (c+dx)^2\sin^2(a+bx)\,dx}{2b}$$
$$= \frac{3d(c+dx)^2\cos(a+bx)\sin(a+bx)}{4b^2} - \frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3} + \frac{(}{}$$
$$= -\frac{(c+dx)^3}{4b} - \frac{3d^3\cos(a+bx)\sin(a+bx)}{8b^4} + \frac{3d(c+dx)^2\cos(a+bx}{4b^2}$$
$$= \frac{3d^3x}{8b^3} - \frac{(c+dx)^3}{4b} - \frac{3d^3\cos(a+bx)\sin(a+bx)}{8b^4} + \frac{3d(c+dx)^2\cos(}{}$$

Mathematica [A]
time = 0.26, size = 71, normalized size = 0.59

$$\frac{-2b(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\cos(2(a+bx))+3d\left(-d^2+2b^2(c+dx)^2\right)\sin(2(a+bx))}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 3*d*(-d^2 +
 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(16*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(108) = 216$.
time = 0.10, size = 466, normalized size = 3.88 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2/b^3*a^3*d^3*cos(b*x+a)^2-3/2/b^2*a^2*c*d^2*cos(b*x+a)^2+3/b^3*a^2*
d^3*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+3/2
/b*a*c^2*d*cos(b*x+a)^2-6/b^2*a*c*d^2*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*
```

```
x+a)*sin(b*x+a)+1/4*b*x+1/4*a)-3/b^3*a*d^3*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*
x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)
^2)-1/2*c^3*cos(b*x+a)^2+3/b*c^2*d*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a
)*sin(b*x+a)+1/4*b*x+1/4*a)+3/b^2*c*d^2*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x+a
)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)
+1/b^3*d^3*(-1/2*(b*x+a)^3*cos(b*x+a)^2+3/2*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b
*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)*cos(b*x+a)^2-3/8*cos(b*x+a)*sin(b*x+a)-3/8
*b*x-3/8*a-1/2*(b*x+a)^3))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(108) = 216$.
time = 0.28, size = 342, normalized size = 2.85

$$-\frac{8\,c^3\cos\left(bx+a\right)^2 - \frac{24\,ac^2d\cos(bx+a)^2}{b} + \frac{24\,a^2cd^2\cos(bx+a)^2}{b^2} - \frac{8\,a^3d^3\cos(bx+a)^2}{b^3} + \frac{6\,(2\,(bx+a)\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a))c^2d}{b} - \frac{12\,(2\,(bx+a)\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a))acd^2}{b^2} + \frac{6\,(2\,(bx+a)\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a))a^2d^3}{b^3} + \frac{6\left(\left(2\,(bx+a)^2-1\right)\cos(2\,bx+2\,a)-2\,(bx+a)\sin(2\,bx+2\,a)\right)cd^2}{b^2} - \frac{6\left(\left(2\,(bx+a)^2-1\right)\cos(2\,bx+2\,a)-2\,(bx+a)\sin(2\,bx+2\,a)\right)ad^3}{b^3} + \frac{\left(2\left(2\,(bx+a)^3-3\,bx-3\,a\right)\cos(2\,bx+2\,a)-3\left(2\,(bx+a)^2-1\right)\sin(2\,bx+2\,a)\right)d^3}{b^3}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/16*(8*c^3*cos(b*x + a)^2 - 24*a*c^2*d*cos(b*x + a)^2/b + 24*a^2*c*d^2*co
s(b*x + a)^2/b^2 - 8*a^3*d^3*cos(b*x + a)^2/b^3 + 6*(2*(b*x + a)*cos(2*b*x
+ 2*a) - sin(2*b*x + 2*a))*c^2*d/b - 12*(2*(b*x + a)*cos(2*b*x + 2*a) - sin
(2*b*x + 2*a))*a*c*d^2/b^2 + 6*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x +
2*a))*a^2*d^3/b^3 + 6*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*s
in(2*b*x + 2*a))*c*d^2/b^2 - 6*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b
*x + a)*sin(2*b*x + 2*a))*a*d^3/b^3 + (2*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(
2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*d^3/b^3)/b
```

Fricas [A]
time = 2.46, size = 166, normalized size = 1.38

$$\frac{2\,b^3d^3x^3 + 6\,b^3cd^2x^2 - 2\left(2\,b^3d^3x^3 + 6\,b^3cd^2x^2 + 2\,b^3c^3 - 3\,bcd^2 + 3\left(2\,b^3c^2d - bd^3\right)x\right)\cos\left(bx+a\right)^2 + 3\left(2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d - d^3\right)\cos\left(bx+a\right)\sin\left(bx+a\right) + 3\left(2\,b^3c^2d - bd^3\right)x}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 +
 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b
^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) +
 3*(2*b^3*c^2*d - b*d^3)*x)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(116) = 232$.
time = 0.33, size = 342, normalized size = 2.85

$$\begin{cases} \frac{c^3\sin^2(a+bx)}{2b} + \frac{3c^2dx\sin^2(a+bx)}{4b} - \frac{3c^2dx\cos^2(a+bx)}{4b} + \frac{3cd^2x^2\sin^2(a+bx)}{4b} - \frac{3cd^2x^2\cos^2(a+bx)}{4b} + \frac{d^3x^3\sin^2(a+bx)}{4b} - \frac{d^3x^3\cos^2(a+bx)}{4b} + \frac{3c^2d\sin(a+bx)\cos(a+bx)}{4b^2} + \frac{3cd^2x\sin(a+bx)\cos(a+bx)}{2b^2} + \frac{3d^3x^2\sin(a+bx)\cos(a+bx)}{4b^2} - \frac{3cd^2\sin^2(a+bx)}{4b^3} - \frac{3d^3x\sin^2(a+bx)}{8b^3} + \frac{3d^3x\cos^2(a+bx)}{8b^3} - \frac{3d^3\sin(a+bx)\cos(a+bx)}{8b^4} & \text{for}\ b \neq 0 \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin(a)\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Piecewise((c**3*sin(a + b*x)**2/(2*b) + 3*c**2*d*x*sin(a + b*x)**2/(4*b) -
3*c**2*d*x*cos(a + b*x)**2/(4*b) + 3*c*d**2*x**2*sin(a + b*x)**2/(4*b) - 3*
c*d**2*x**2*cos(a + b*x)**2/(4*b) + d**3*x**3*sin(a + b*x)**2/(4*b) - d**3*
x**3*cos(a + b*x)**2/(4*b) + 3*c**2*d*sin(a + b*x)*cos(a + b*x)/(4*b**2) +
3*c*d**2*x*sin(a + b*x)*cos(a + b*x)/(2*b**2) + 3*d**3*x**2*sin(a + b*x)*co
s(a + b*x)/(4*b**2) - 3*c*d**2*sin(a + b*x)**2/(4*b**3) - 3*d**3*x*sin(a +
b*x)**2/(8*b**3) + 3*d**3*x*cos(a + b*x)**2/(8*b**3) - 3*d**3*sin(a + b*x)*
cos(a + b*x)/(8*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3
+ d**3*x**4/4)*sin(a)*cos(a), True))
```

Giac [A]
time = 0.44, size = 121, normalized size = 1.01

$$-\frac{(2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2dx+2\,b^3c^3-3\,bd^3x-3\,bcd^2)\cos\left(2\,bx+2\,a\right)}{8\,b^4}+\frac{3\,(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3)\sin\left(2\,bx+2\,a\right)}{16\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3
*x - 3*b*c*d^2)*cos(2*b*x + 2*a)/b^4 + 3/16*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x
+ 2*b^2*c^2*d - d^3)*sin(2*b*x + 2*a)/b^4
```

Mupad [B]
time = 0.85, size = 165, normalized size = 1.38

$$\frac{\cos\left(2\,a+2\,b\,x\right)\,\left(\frac{3\,c\,d^2}{4}-\frac{b^2\,c^3}{2}\right)}{2\,b^3}-\frac{3\,\sin\left(2\,a+2\,b\,x\right)\,\left(d^3-2\,b^2\,c^2\,d\right)}{16\,b^4}-\frac{d^3\,x^3\,\cos\left(2\,a+2\,b\,x\right)}{4\,b}+\frac{3\,d^3\,x^2\,\sin\left(2\,a+2\,b\,x\right)}{8\,b^2}+\frac{3\,x\,\cos\left(2\,a+2\,b\,x\right)\,\left(d^3-2\,b^2\,c^2\,d\right)}{8\,b^3}+\frac{3\,c\,d^2\,x\,\sin\left(2\,a+2\,b\,x\right)}{4\,b^2}-\frac{3\,c\,d^2\,x^2\,\cos\left(2\,a+2\,b\,x\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^3,x)
```

```
[Out] (cos(2*a + 2*b*x)*((3*c*d^2)/4 - (b^2*c^3)/2))/(2*b^3) - (3*sin(2*a + 2*b*x
)*(d^3 - 2*b^2*c^2*d))/(16*b^4) - (d^3*x^3*cos(2*a + 2*b*x))/(4*b) + (3*d^3
*x^2*sin(2*a + 2*b*x))/(8*b^2) + (3*x*cos(2*a + 2*b*x)*(d^3 - 2*b^2*c^2*d))
/(8*b^3) + (3*c*d^2*x*sin(2*a + 2*b*x))/(4*b^2) - (3*c*d^2*x^2*cos(2*a + 2*
b*x))/(4*b)
```

3.4 $\int (c+dx)^2 \cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=89

$$-\frac{cdx}{2b}-\frac{d^2x^2}{4b}+\frac{d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2}-\frac{d^2\sin^2(a+bx)}{4b^3}+\frac{(c+dx)^2\sin^2(a+bx)}{2b}$$

```
[Out] -1/2*c*d*x/b-1/4*d^2*x^2/b+1/2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2-1/4*d^2*
sin(b*x+a)^2/b^3+1/2*(d*x+c)^2*sin(b*x+a)^2/b
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {4489, 3391}

$$-\frac{d^2\sin^2(a+bx)}{4b^3}+\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}+\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{cdx}{2b}-\frac{d^2x^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -1/2*(c*d*x)/b - (d^2*x^2)/(4*b) + (d*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/
(2*b^2) - (d^2*Sin[a + b*x]^2)/(4*b^3) + ((c + d*x)^2*Sin[a + b*x]^2)/(2*b)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx &= \frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{d\int (c+dx)\sin^2(a+bx)\,dx}{b}\\ &= \frac{d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2}-\frac{d^2\sin^2(a+bx)}{4b^3}+\frac{(c+dx)^2\sin^2(a+bx)}{2b}\\ &= -\frac{cdx}{2b}-\frac{d^2x^2}{4b}+\frac{d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2}-\frac{d^2\sin^2(a+bx)}{4b^3}+\frac{(c+dx)^2\sin^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 50, normalized size = 0.56

$$\frac{(d^2-2b^2(c+dx)^2)\cos(2(a+bx))+2bd(c+dx)\sin(2(a+bx))}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] ((d^2 - 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 2*b*d*(c + d*x)*Sin[2*(a + b*
x)])/(8*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(79) = 158.
time = 0.08, size = 215, normalized size = 2.42

method	result
risch	$-\frac{(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2)\cos(2bx+2a)}{8b^3}+\frac{d(dx+c)\sin(2bx+2a)}{4b^2}$
derivativedivides	$-\frac{a^2d^2\left(\cos^2(bx+a)\right)}{2b^2}+\frac{acd\left(\cos^2(bx+a)\right)}{b}-\frac{2a\,d^2\left(-\frac{(bx+a)\left(\cos^2(bx+a)\right)}{2}+\frac{\cos(bx+a)\sin(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b^2}-\frac{c^2\left(\cos^2(bx+a)\right)}{2}+\frac{2cd\left(\right.}{}$
default	$-\frac{a^2d^2\left(\cos^2(bx+a)\right)}{2b^2}+\frac{acd\left(\cos^2(bx+a)\right)}{b}-\frac{2a\,d^2\left(-\frac{(bx+a)\left(\cos^2(bx+a)\right)}{2}+\frac{\cos(bx+a)\sin(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b^2}-\frac{c^2\left(\cos^2(bx+a)\right)}{2}+\frac{2cd\left(\right.}{}$
norman	$\frac{\frac{(2b^2c^2-d^2)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^3}+\frac{cd\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^2}+\frac{d^2x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^2}-\frac{d^2x^2}{4b}-\frac{cd\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}-\frac{cdx}{2b}-\frac{d^2x\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}+\frac{3d^2x^2}{}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2/b^2*a^2*d^2*cos(b*x+a)^2+1/b*a*c*d*cos(b*x+a)^2-2/b^2*a*d^2*(-1/2
*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)-1/2*c^2*cos(
b*x+a)^2+2/b*c*d*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b
*x+1/4*a)+1/b^2*d^2*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*si
n(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(79) = 158.
time = 0.28, size = 171, normalized size = 1.92

$$-\frac{4\,c^2\cos(bx+a)^2-\frac{8\,acd\cos(bx+a)^2}{b}+\frac{4\,a^2d^2\cos(bx+a)^2}{b^2}+\frac{2\,(2\,(bx+a)\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a))cd}{b}-\frac{2\,(2\,(bx+a)\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a))ad^2}{b^2}+\frac{\left(\left(2\,(bx+a)^2-1\right)\cos(2\,bx+2\,a)-2\,(bx+a)\sin(2\,bx+2\,a)\right)d^2}{b^2}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*(4*c^2*cos(b*x + a)^2 - 8*a*c*d*cos(b*x + a)^2/b + 4*a^2*d^2*cos(b*x +
 a)^2/b^2 + 2*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*c*d/b - 2*(
2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a*d^2/b^2 + ((2*(b*x + a)^
2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*d^2/b^2)/b
```

Fricas [A]
time = 3.19, size = 92, normalized size = 1.03

$$\frac{b^2d^2x^2 + 2\,b^2cdx - \left(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2\right)\cos\left(bx + a\right)^2 + 2\left(bd^2x + bcd\right)\cos\left(bx + a\right)\sin\left(bx + a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 -
 d^2)*cos(b*x + a)^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(78) = 156$.
time = 0.20, size = 175, normalized size = 1.97

$$\begin{cases} \frac{c^2\sin^2\left(a+bx\right)}{2b} + \frac{cdx\sin^2\left(a+bx\right)}{2b} - \frac{cdx\cos^2\left(a+bx\right)}{2b} + \frac{d^2x^2\sin^2\left(a+bx\right)}{4b} - \frac{d^2x^2\cos^2\left(a+bx\right)}{4b} + \frac{cd\sin\left(a+bx\right)\cos\left(a+bx\right)}{2b^2} + \frac{d^2x\sin\left(a+bx\right)\cos\left(a+bx\right)}{2b^2} - \frac{d^2\sin^2\left(a+bx\right)}{4b^3} & \text{for}\: b \neq 0 \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin\left(a\right)\cos\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Piecewise((c**2*sin(a + b*x)**2/(2*b) + c*d*x*sin(a + b*x)**2/(2*b) - c*d*x
*cos(a + b*x)**2/(2*b) + d**2*x**2*sin(a + b*x)**2/(4*b) - d**2*x**2*cos(a
+ b*x)**2/(4*b) + c*d*sin(a + b*x)*cos(a + b*x)/(2*b**2) + d**2*x*sin(a + b
*x)*cos(a + b*x)/(2*b**2) - d**2*sin(a + b*x)**2/(4*b**3), Ne(b, 0)), ((c**
2*x + c*d*x**2 + d**2*x**3/3)*sin(a)*cos(a), True))
```

Giac [A]
time = 0.41, size = 73, normalized size = 0.82

$$-\frac{\left(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2\right)\cos\left(2\,bx + 2\,a\right)}{8\,b^3} + \frac{\left(bd^2x + bcd\right)\sin\left(2\,bx + 2\,a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a)/b^3 +
 1/4*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a)/b^3
```

Mupad [B]
time = 0.16, size = 100, normalized size = 1.12

$$\frac{\cos\left(2\,a+2\,b\,x\right)\,\left(\frac{d^2}{4}-\frac{b^2\,c^2}{2}\right)}{2\,b^3}+\frac{d^2\,x\,\sin\left(2\,a+2\,b\,x\right)}{4\,b^2}-\frac{d^2\,x^2\,\cos\left(2\,a+2\,b\,x\right)}{4\,b}+\frac{c\,d\,\sin\left(2\,a+2\,b\,x\right)}{4\,b^2}-\frac{c\,d\,x\,\cos\left(2\,a+2\,b\,x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^2,x)
```

```
[Out] (cos(2*a + 2*b*x)*(d^2/4 - (b^2*c^2)/2))/(2*b^3) + (d^2*x*sin(2*a + 2*b*x))
/(4*b^2) - (d^2*x^2*cos(2*a + 2*b*x))/(4*b) + (c*d*sin(2*a + 2*b*x))/(4*b^2
) - (c*d*x*cos(2*a + 2*b*x))/(2*b)
```

3.5 $\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=50

$$-\frac{dx}{4b}+\frac{d\cos(a+bx)\sin(a+bx)}{4b^2}+\frac{(c+dx)\sin^2(a+bx)}{2b}$$

```
[Out] -1/4*d*x/b+1/4*d*cos(b*x+a)*sin(b*x+a)/b^2+1/2*(d*x+c)*sin(b*x+a)^2/b
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4489, 2715, 8}

$$\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}+\frac{(c+dx)\sin^2(a+bx)}{2b}-\frac{dx}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -1/4*(d*x)/b + (d*Cos[a + b*x]*Sin[a + b*x])/(4*b^2) + ((c + d*x)*Sin[a + b
*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx = \frac{(c+dx)\sin^2(a+bx)}{2b} - \frac{d\int \sin^2(a+bx)\,dx}{2b}$$
$$= \frac{d\cos(a+bx)\sin(a+bx)}{4b^2} + \frac{(c+dx)\sin^2(a+bx)}{2b} - \frac{d\int 1\,dx}{4b}$$
$$= -\frac{dx}{4b} + \frac{d\cos(a+bx)\sin(a+bx)}{4b^2} + \frac{(c+dx)\sin^2(a+bx)}{2b}$$

Mathematica [A]
time = 0.10, size = 34, normalized size = 0.68

$$\frac{-2b(c+dx)\cos(2(a+bx)) + d\sin(2(a+bx))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x],x]
[Out] (-2*b*(c + d*x)*Cos[2*(a + b*x)] + d*Sin[2*(a + b*x)])/(8*b^2)
```

Maple [A]
time = 0.05, size = 74, normalized size = 1.48

method	result	size
risch	$-\frac{(dx+c)\cos(2bx+2a)}{4b}+\frac{d\sin(2bx+2a)}{8b^2}$	36
derivativedivides	$\frac{\frac{da\left(\cos^2(bx+a)\right)}{2b}-\frac{c\left(\cos^2(bx+a)\right)}{2}+\frac{d\left(-\frac{(bx+a)\left(\cos^2(bx+a)\right)}{2}+\frac{\cos(bx+a)\sin(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b}}{b}$	74
default	$\frac{\frac{da\left(\cos^2(bx+a)\right)}{2b}-\frac{c\left(\cos^2(bx+a)\right)}{2}+\frac{d\left(-\frac{(bx+a)\left(\cos^2(bx+a)\right)}{2}+\frac{\cos(bx+a)\sin(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b}}{b}$	74
norman	$\frac{\frac{d\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b^2}-\frac{d\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b^2}-\frac{dx}{4b}+\frac{2c\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{3dx\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}-\frac{dx\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2/b*d*a*cos(b*x+a)^2-1/2*c*cos(b*x+a)^2+1/b*d*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a))
```

Maxima [A]
time = 0.26, size = 65, normalized size = 1.30

$$-\frac{4\,c\cos\left(bx+a\right)^2 - \frac{4\,ad\cos(bx+a)^2}{b} + \frac{(2\,(bx+a)\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a))d}{b}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*(4*c*cos(b*x + a)^2 - 4*a*d*cos(b*x + a)^2/b + (2*(b*x + a)*cos(2*b*x
+ 2*a) - sin(2*b*x + 2*a))*d/b)/b
```

Fricas [A]
time = 3.10, size = 42, normalized size = 0.84

$$\frac{bdx - 2\,(bdx + bc)\cos{(bx + a)}^2 + d\cos{(bx + a)}\sin{(bx + a)}}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a))/
b^2
```

Sympy [A]
time = 0.12, size = 80, normalized size = 1.60

$$\begin{cases} \frac{c\sin^2{(a+bx)}}{2b} + \frac{dx\sin^2{(a+bx)}}{4b} - \frac{dx\cos^2{(a+bx)}}{4b} + \frac{d\sin{(a+bx)}\cos{(a+bx)}}{4b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\sin{(a)}\cos{(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Piecewise((c*sin(a + b*x)**2/(2*b) + d*x*sin(a + b*x)**2/(4*b) - d*x*cos(a
+ b*x)**2/(4*b) + d*sin(a + b*x)*cos(a + b*x)/(4*b**2), Ne(b, 0)), ((c*x +
d*x**2/2)*sin(a)*cos(a), True))
```

Giac [A]
time = 0.44, size = 38, normalized size = 0.76

$$-\frac{(bdx + bc)\cos{(2\,bx + 2\,a)}}{4\,b^2} + \frac{d\sin{(2\,bx + 2\,a)}}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(b*d*x + b*c)*cos(2*b*x + 2*a)/b^2 + 1/8*d*sin(2*b*x + 2*a)/b^2
```

Mupad [B]
time = 0.70, size = 47, normalized size = 0.94

$$\frac{d\,\sin{(2\,a+2\,b\,x)}}{8\,b^2} - \frac{c\,\cos{(2\,a+2\,b\,x)}}{4\,b} - \frac{d\,x\,\cos{(2\,a+2\,b\,x)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x),x)
```

```
[Out] (d*sin(2*a + 2*b*x))/(8*b^2) - (c*cos(2*a + 2*b*x))/(4*b) - (d*x*cos(2*a +
2*b*x))/(4*b)
```

3.6 $\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=65

$$\frac{\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

```
[Out] 1/2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d+1/2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*
c/d)/d
```

Rubi [A]
time = 0.09, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4491, 12, 3384, 3380, 3383}

$$\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d) + (Cos[2*a - (2
*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx &= \int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx\\ &= \frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx\\ &= \frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx+\frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2b\right.}{c+dx}\\ &= \frac{\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 60, normalized size = 0.92

$$\frac{\mathrm{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)+\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x),x]
[Out] (CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d] + Cos[2*a - (2*b*c)/d]
*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Maple [A]
time = 0.08, size = 84, normalized size = 1.29

method	result	size
derivativedivides	$-\frac{\mathrm{sinIntegral}\left(-2bx-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{2d}-\frac{\mathrm{cosineIntegral}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{2d}$	84
default	$-\frac{\mathrm{sinIntegral}\left(-2bx-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{2d}-\frac{\mathrm{cosineIntegral}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{2d}$	84
risch	$-\frac{i\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\mathrm{expIntegral}\left(1,2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{4d}+\frac{i\mathrm{e}^{\frac{2i(ad-cb)}{d}}\mathrm{expIntegral}\left(1,-2ibx-2ia-\frac{2(-iad+ibc)}{d}\right)}{4d}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-1/2*Ci(2*b*x+2*a+2
*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d
```

Maxima [C] Result contains complex when optimal does not.
time = 0.31, size = 143, normalized size = 2.20

$$-\frac{b\left(-i\,E_{1}\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_{1}\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b\left(E_{1}\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_{1}\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*(b*(-I*exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp
_integral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/d
) + b*(exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integr
al_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d))/(b*d
)
```

Fricas [A]
time = 2.58, size = 80, normalized size = 1.23

$$\frac{\left(\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+2\,\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*((cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d))*s
in(-2*(b*c - a*d)/d) + 2*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)
/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(a+bx\right)\cos\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 569, normalized size = 8.75

$$\frac{\Im\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)\tan\left(a\right)^{2}\tan\left(\frac{bc}{d}\right)^{2}-\Im\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)\tan\left(a\right)^{2}\tan\left(\frac{bc}{d}\right)^{2}+2\,\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)\tan\left(a\right)^{2}\tan\left(\frac{bc}{d}\right)^{2}+2\,\Re\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)\tan\left(a\right)^{2}\tan\left(\frac{bc}{d}\right)+2\,\Re\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)\tan\left(a\right)^{2}\tan\left(\frac{bc}{d}\right)-2\,\Re\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)\tan\left(a\right)\tan\left(\frac{bc}{d}\right)^{2}-2\,\Re\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)\tan\left(a\right)\tan\left(\frac{bc}{d}\right)^{2}-\Im\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)\tan\left(a\right)^{2}+\Im\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)\tan\left(a\right)^{2}-2\,\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)\tan\left(a\right)^{2}+4\,\Im\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)\tan\left(a\right)\tan\left(\frac{bc}{d}\right)-4\,\Im\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)\tan\left(a\right)\tan\left(\frac{bc}{d}\right)+8\,\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)\tan\left(a\right)\tan\left(\frac{bc}{d}\right)-\Im\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)\tan\left(\frac{bc}{d}\right)^{2}+\Im\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)\tan\left(\frac{bc}{d}\right)^{2}-2\,\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)\tan\left(\frac{bc}{d}\right)^{2}+2\,\Re\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)\tan\left(a\right)+2\,\Re\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)\tan\left(a\right)-2\,\Re\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)\tan\left(\frac{bc}{d}\right)-2\,\Re\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)\tan\left(\frac{bc}{d}\right)+\Im\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)-\Im\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)+2\,\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)}{4\,\left(d\tan\left(a\right)^{2}\tan\left(\frac{bc}{d}\right)^{2}+d\tan\left(a\right)^{2}+d\tan\left(\frac{bc}{d}\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - imag_
part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*sin_integral
(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(2*b*x
+ 2*b*c/d))*tan(a)^2*tan(b*c/d) + 2*real_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(a)^2*tan(b*c/d) - 2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*
tan(b*c/d)^2 - 2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d
)^2 - imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + imag_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(a)^2 - 2*sin_integral(2*(b*d*x + b*c)/d)*tan(a
)^2 + 4*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) - 4*imag
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 8*sin_integral(2*
(b*d*x + b*c)/d)*tan(a)*tan(b*c/d) - imag_part(cos_integral(2*b*x + 2*b*c/d
))*tan(b*c/d)^2 + imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 -
2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(2
*b*x + 2*b*c/d))*tan(a) + 2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a
) - 2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*real_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + imag_part(cos_integral(2*b*x + 2*
b*c/d)) - imag_part(cos_integral(-2*b*x - 2*b*c/d)) + 2*sin_integral(2*(b*d
*x + b*c)/d))/(d*tan(a)^2*tan(b*c/d)^2 + d*tan(a)^2 + d*tan(b*c/d)^2 + d)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos (a+b\,x)\,\sin (a+b\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x))/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x))/(c + d*x), x)
```

3.7 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=85

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\sin(2a+2bx)}{2d(c+dx)}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}$$

```
[Out] b*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^2-b*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c/
d)/d^2-1/2*sin(2*b*x+2*a)/d/(d*x+c)
```

Rubi [A]
time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4491, 12, 3378, 3384, 3380, 3383}

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\sin(2a+2bx)}{2d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2 - Sin[2*a + 2*b
*x]/(2*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x
])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx &= \int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx\\ &= \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx\\ &= -\frac{\sin(2a+2bx)}{2d(c+dx)}+\frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d}\\ &= -\frac{\sin(2a+2bx)}{2d(c+dx)}+\frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d}-\frac{\left(b\sin\left(2a-\frac{2bc}{d}\right)\right)\int}{d}\\ &= \frac{b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\sin(2a+2bx)}{2d(c+dx)}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2b\right.}{d^2}\end{aligned}$$

Mathematica [A]
time = 0.31, size = 80, normalized size = 0.94

$$\frac{2b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)-\frac{d\sin(2(a+bx))}{c+dx}-2b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2b(c+dx)}{d}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] (2*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - (d*Sin[2*(a + b*
x)])/(c + d*x) - 2*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(
2*d^2)
```

Maple [A]
time = 0.12, size = 124, normalized size = 1.46

method	result
derivativedivides	$\frac{b\left(-\frac{2\sin(2bx+2a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{4\,\mathrm{sinIntegral}\left(-2bx-2a-\frac{2(-ad+cb)}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}+\frac{4\,\mathrm{cosineIntegral}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\cos\left(\right.}{d}}{d}\right.}{4}$
default	$\frac{b\left(-\frac{2\sin(2bx+2a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{4\,\mathrm{sinIntegral}\left(-2bx-2a-\frac{2(-ad+cb)}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}+\frac{4\,\mathrm{cosineIntegral}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\cos\left(\right.}{d}}{d}\right.}{4}$
risch	$-\frac{b\,\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{2d^2}-\frac{b\,\mathrm{e}^{\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,-2ibx-2ia-\frac{2(-iad+ibc)}{d}\right)}{2d^2}-\frac{(-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b*(-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2*(-2*Si(-2*b*x-2*a-2*(-a*d
+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b
*c)/d)/d)/d)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.33, size = 166, normalized size = 1.95

$$-\frac{b^2\left(-i\,E_2\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_2\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^2\left(E_2\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_2\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)}{4\,(bcd+(bx+a)d^2-ad^2)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(-I*exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*e
xp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)
/d) + b^2*(exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_in
tegral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d))/
((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A]
time = 1.31, size = 132, normalized size = 1.55

$$-\frac{2\,d\cos(bx+a)\sin(bx+a)+2\,(bdx+bc)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)-\left((bdx+bc)\,\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)}{2\,(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*cos(b*x + a)*sin(b*x + a) + 2*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)
*sin_integral(2*(b*d*x + b*c)/d) - ((b*d*x + b*c)*cos_integral(2*(b*d*x + b
*c)/d) + (b*d*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)
/d))/(d^3*x + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**2, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.63, size = 2870, normalized size = 33.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan
(b*c/d)^2 + b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(
a)^2*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(a)^2*tan(b*c/d) + 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*
tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b
*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 4*b*d*x*sin_integral(2*(b*d
*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b*c*real_part(cos_integral(2*
b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + b*c*real_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b*d*x*real_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - b*d*x*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 4*b*d*x*real_part(cos_integra
l(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 4*b*d*x*real_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 2*b*c*imag_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b*c*imag_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b*c*s
in_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - b*d*x*real_
part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - b*d*x*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 2*b*c*imag_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b*c*ima
g_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 4*b
```

```
*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b*d*x*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b*d*x*real_
part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - 2*b*d*x*imag_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 2*b*d*x*imag_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 4*b*d*x*sin_integral(2*(b
*d*x + b*c)/d)*tan(b*x)^2*tan(a) - b*c*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(b*x)^2*tan(a)^2 - b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*x)^2*tan(a)^2 + 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(b*c/d) - 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x
)^2*tan(b*c/d) + 4*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c
/d) + 4*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(
b*c/d) + 4*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*
tan(b*c/d) - 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(
b*c/d) + 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c
/d) - 4*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d) - b*c*rea
l_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - b*c*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 2*b*d*x*imag_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b*d*x*imag_part(
cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*b*d*x*sin_integral(
2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + b*c*real_part(cos_integral(2*b*x +
 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b*c*real_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(a)^2*tan(b*c/d)^2 + b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(b*x)^2 + b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 -
 2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 2*b*c*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 4*b*c*sin_inte
gral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - b*d*x*real_part(cos_integral(2*
b*x + 2*b*c/d))*tan(a)^2 - b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(a)^2 + 2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*
c/d) - 2*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d
) + 4*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) + 4*b*d*x*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) + 4*b*d*x*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) - 2*b*c*imag_part(cos_
integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d) + 2*b*c*imag_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) - 4*b*c*sin_integral(2*(b*d*x + b
*c)/d)*tan(a)^2*tan(b*c/d) - b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))
*tan(b*c/d)^2 - b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^
2 + 2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*
b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*b*c*s
in_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 2*d*tan(b*x)^2*tan(a)*
tan(b*c/d)^2 + 2*d*tan(b*x)*tan(a)^2*tan(b*c/d)^2 + b*c*real_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(b*x)^2 + b*c*real_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(b*x)^2 - 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)
+ 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a) - 4*b*d*x*sin_in
tegral(2*(b*d*x + b*c)/d)*tan(a) - b*c*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(a)^2 - b*c*real_part(cos_integral(-2*b...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)}{{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^2,x)
[Out] int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^2, x)
```

3.8 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=114

$$-\frac{b\cos(2a+2bx)}{2d^2(c+dx)}-\frac{b^2\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^3}-\frac{\sin(2a+2bx)}{4d(c+dx)^2}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}$$

```
[Out] -1/2*b*cos(2*b*x+2*a)/d^2/(d*x+c)-b^2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^
3-b^2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^3-1/4*sin(2*b*x+2*a)/d/(d*x+c)^2
```

Rubi [A]

time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4491, 12, 3378, 3384, 3380, 3383}

$$-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b\cos(2a+2bx)}{2d^2(c+dx)}-\frac{\sin(2a+2bx)}{4d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -1/2*(b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)) - (b^2*CosIntegral[(2*b*c)/d + 2*
b*x]*Sin[2*a - (2*b*c)/d])/d^3 - Sin[2*a + 2*b*x]/(4*d*(c + d*x)^2) - (b^2*
Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin(2a+2bx)}{2(c+dx)^3}\,dx$$
$$= \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx$$
$$= -\frac{\sin(2a+2bx)}{4d(c+dx)^2} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{2d}$$
$$= -\frac{b\cos(2a+2bx)}{2d^2(c+dx)} - \frac{\sin(2a+2bx)}{4d(c+dx)^2} - \frac{b^2\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d^2}$$
$$= -\frac{b\cos(2a+2bx)}{2d^2(c+dx)} - \frac{\sin(2a+2bx)}{4d(c+dx)^2} - \frac{\left(b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d^2} -$$
$$= -\frac{b\cos(2a+2bx)}{2d^2(c+dx)} - \frac{b^2\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^3} - \frac{\sin(2a+2bx)}{4d(c+dx)^2} - \frac{b^2\cos}{}$$

Mathematica [A]
time = 1.06, size = 102, normalized size = 0.89

$$-\frac{4b^2\mathrm{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right)+\frac{d(2b(c+dx)\cos(2(a+bx))+d\sin(2(a+bx)))}{(c+dx)^2}+4b^2\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2b(c+dx)}{d}\right)}{4d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^3,x]
```

[Out] -1/4*(4*b^2*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*(2*b*(c + d*x)*Cos[2*(a + b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + 4*b^2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^3

Maple [A]
time = 0.20, size = 162, normalized size = 1.42

method	result
derivativedivides	$\frac{b^2\left(-\frac{\sin(2bx+2a)}{(-ad+cb+d(bx+a))^2 d}+\frac{-\frac{2\cos(2bx+2a)}{(-ad+cb+d(bx+a))d}-\frac{2\left(-\frac{2\,\mathrm{sinIntegral}\left(-2bx-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{cosineInteg}}{}\right.}{d}}{d}\right.}{4}$
default	$\frac{b^2\left(-\frac{\sin(2bx+2a)}{(-ad+cb+d(bx+a))^2 d}+\frac{-\frac{2\cos(2bx+2a)}{(-ad+cb+d(bx+a))d}-\frac{2\left(-\frac{2\,\mathrm{sinIntegral}\left(-2bx-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{cosineInteg}}{}\right.}{d}}{d}\right.}{4}$
risch	$\frac{ib^2 e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{2d^3}-\frac{ib^2 e^{\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,-2ibx-2ia-\frac{2(-iad+ibc)}{d}\right)}{2d^3}+\frac{i}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] 1/4*b^2*(-sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d+(-2*cos(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d-2*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d)

Maxima [C] Result contains complex when optimal does not.
time = 0.37, size = 201, normalized size = 1.76

$$-\frac{b^3\left(-i\,E_3\left(\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)+i\,E_3\left(-\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^3\left(E_3\left(\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)+E_3\left(-\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)}{4\left(b^2c^2d-2abcd^2+(bx+a)^2d^3+a^2d^3+2(bcd^2-ad^3)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")

[Out] -1/4*(b^3*(-I*exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/d) + b^3*(exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(110) = 220$.
time = 2.23, size = 230, normalized size = 2.02

$$\frac{bd^2x - d^2\cos(bx+a)\sin(bx+a) + bcd - 2\left(bd^2x + bcd\right)\cos(bx+a)^2 - 2\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2\,(bdx+bc)}{d}\right) - \left(\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\operatorname{Ci}\left(\frac{2\,(bdx+bc)}{d}\right) + \left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\operatorname{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)}{2\left(d^5x^2 + 2\,cd^4x + c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*d^2*x - d^2*cos(b*x + a)*sin(b*x + a) + b*c*d - 2*(b*d^2*x + b*c*d)*
cos(b*x + a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2*(b*c - a*d)
/d)*sin_integral(2*(b*d*x + b*c)/d) - ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos_integral(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*c
os_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*
x + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.58, size = 5398, normalized size = 47.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a
)^2*tan(b*c/d)^2 - b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)
/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*real_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x^2*real_part
(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^2*d^2
*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^
2 - 2*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(
a)*tan(b*c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(
b*x)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^2*c*d*x*imag_part(cos_integral(-2*b*x -
```

```
2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 4*b^2*c*d*x*sin_integral(2*(b*
d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^2*d^2*x^2*imag_part(cos_
integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 + b^2*d^2*x^2*imag_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b^2*d^2*x^2*sin_integra
l(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2 + 4*b^2*d^2*x^2*imag_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 4*b^2*d^2*x^2*imag_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 8*b^2*d^2
*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d) + 4*b^2*c
*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d
) + 4*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)
^2*tan(b*c/d) - b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(b*x)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(
b*x)^2*tan(b*c/d)^2 - 4*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*
tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 4*b^2*c*d*x*real_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - b^2*d^2*x^2*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*sin_integra
l(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + b^2*c^2*imag_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^2*c^2*imag_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c^2*s
in_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2
*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 2*b^2*d^2
*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 2*b^2*c*
d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 2*b^2*c*
d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 4*b^2*c
*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2 - 2*b^2*d^2*x^2*re
al_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 2*b^2*d^2*x^
2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 8*b^2*c
*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)
- 8*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*t
an(b*c/d) + 16*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*
tan(b*c/d) + 2*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^
2*tan(b*c/d) + 2*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
a)^2*tan(b*c/d) + 2*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^2*c*d*x*imag_part(cos_integral(2*b
*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 4*b^2*c*d*x*sin_integral(2*(b
*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_integr
al(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b^2*c^2*real_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b^2*c^2*real_part(
cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 2*b^2*c*d*
x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - 2*b^2*c*
```

```
d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 4*b^2
*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + b*d^2*x*tan(
b*x)^2*tan(a)^2*tan(b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2
*b*c/d))*tan(b*x)^2 - b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(b*x)^2 + 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2 + 4*
b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 4*b^
2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - b^2*d
^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + b^2*d^2*x^2*imag
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - 2*b^2*d^2*x^2*sin_integral
(2*(b*d*x + b*c)/d)*tan(a)^2 - b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c
/d))*tan(b*x)^2*tan(a)^2 + b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(a)^2 - 2*b^2*c^2*sin_integral(...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)}{{\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^3,x)
[Out] int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^3, x)
```

3.9 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=144

$$-\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2}-\frac{2b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{\sin(2a+2bx)}{6d(c+dx)^3}+\frac{b^2\sin(2a+2bx)}{3d^3(c+dx)}+\frac{2b^3\sin\left(2a\right.}{}$$

```
[Out] -2/3*b^3*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^4-1/6*b*cos(2*b*x+2*a)/d^2/(d
*x+c)^2+2/3*b^3*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^4-1/6*sin(2*b*x+2*a)/d
/(d*x+c)^3+1/3*b^2*sin(2*b*x+2*a)/d^3/(d*x+c)
```

Rubi [A]
time = 0.14, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4491, 12, 3378, 3384, 3380, 3383}

$$-\frac{2b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{2b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{b^2\sin(2a+2bx)}{3d^3(c+dx)}-\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{6d(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] -1/6*(b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)^2) - (2*b^3*Cos[2*a - (2*b*c)/d]*C
osIntegral[(2*b*c)/d + 2*b*x])/(3*d^4) - Sin[2*a + 2*b*x]/(6*d*(c + d*x)^3)
 + (b^2*Sin[2*a + 2*b*x])/(3*d^3*(c + d*x)) + (2*b^3*Sin[2*a - (2*b*c)/d]*S
inIntegral[(2*b*c)/d + 2*b*x])/(3*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx &= \int \frac{\sin(2a+2bx)}{2(c+dx)^4}\,dx\\ &= \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^4}\,dx\\ &= -\frac{\sin(2a+2bx)}{6d(c+dx)^3}+\frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^3}\,dx}{3d}\\ &= -\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{6d(c+dx)^3}-\frac{b^2\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx}{3d^2}\\ &= -\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{6d(c+dx)^3}+\frac{b^2\sin(2a+2bx)}{3d^3(c+dx)}-\frac{\left(2b^3\right)\int \frac{\cos(2a+2bx)}{c+dx}\,d\cdots}{3d^3}\\ &= -\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{6d(c+dx)^3}+\frac{b^2\sin(2a+2bx)}{3d^3(c+dx)}-\frac{\left(2b^3\cos\left(2a-\frac{2bc}{d}\right)\right)\cdots}{3d^{\cdots}}\\ &= -\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2}-\frac{2b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{\sin(2a+2bx)}{6d(c+dx)^3}+\frac{b^2\,\mathrm{s}\cdots}{3\cdots}\end{aligned}$$

Mathematica [A]
time = 0.55, size = 164, normalized size = 1.14

$$\frac{-d\cos(2bx)\left(bd(c+dx)\cos(2a)+\left(d^2-2b^2(c+dx)^2\right)\sin(2a)\right)+d\left(\left(-d^2+2b^2(c+dx)^2\right)\cos(2a)+bd(c+dx)\sin(2a)\right)\sin(2bx)-4b^3(c+dx)^3\left(\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)-\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)\right)}{6d^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] (-(d*Cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*Sin[2*a
])) + d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*Sin[2*a])*Sin[
2*b*x] - 4*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x)
)/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d]))/(6*d^4*(c + d*
x)^3)
```

Maple [A]
time = 0.29, size = 200, normalized size = 1.39

method	result
derivativedivides	$\frac{b^3\left(-\frac{2\sin(2bx+2a)}{3(-ad+cb+d(bx+a))^3 d}+\frac{-\frac{2\cos(2bx+2a)}{3(-ad+cb+d(bx+a))^2 d}-\frac{2\left(-\frac{2\sin(2bx+2a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{4\,\mathrm{sinIntegral}\left(-2bx-2a-\frac{2(-ad+cb)}{d}\right)\sin}{d}}{}\right.}{}}{d}\right.}{4}$
default	$\frac{b^3\left(-\frac{2\sin(2bx+2a)}{3(-ad+cb+d(bx+a))^3 d}+\frac{-\frac{2\cos(2bx+2a)}{3(-ad+cb+d(bx+a))^2 d}-\frac{2\left(-\frac{2\sin(2bx+2a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{4\,\mathrm{sinIntegral}\left(-2bx-2a-\frac{2(-ad+cb)}{d}\right)\sin}{d}}{}\right.}{}}{d}\right.}{4}$
risch	$\frac{b^3 e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1, 2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{3d^4}+\frac{b^3 e^{\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1, -2ibx-2ia-\frac{2(-iad+ibc)}{d}\right)}{3d^4}+\frac{i(2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b^3*(-2/3*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^3/d+2/3*(-cos(2*b*x+2*a)/
(-a*d+c*b+d*(b*x+a))^2/d-(-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2*(-2*Si
(-2*b*x-2*a-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*
c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d)/d)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.43, size = 251, normalized size = 1.74

$$-\frac{b^4\left(-i\,E_4\left(\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)+i\,E_4\left(-\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^4\left(E_4\left(\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)+E_4\left(-\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)}{4\left(b^3c^3d-3ab^2c^2d^2+3a^2bcd^3+(bx+a)^3d^4-a^3d^4+3(bcd^3-ad^4)(bx+a)^2+3(b^2c^2d^2-2abcd^3+a^2d^4)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/4*(b^4*(-I*exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*e
xp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)
/d) + b^4*(exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_in
tegral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d))/
((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 +
 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*
(b*x + a))*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(134) = 268$.
time = 2.31, size = 320, normalized size = 2.22

$$\frac{bd^3x + bcd^2 - 2\left(bd^3x + bcd^2\right)\cos\left(bx + a\right)^2 + 2\left(2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d - d^3\right)\cos\left(bx + a\right)\sin\left(bx + a\right) + 4\left(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3\right)\sin\left(-\frac{2\,(bc - ad)}{d}\right)\operatorname{Si}\left(\frac{2\,(bdx + bc)}{d}\right) - 2\left(\left(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3\right)\operatorname{Ci}\left(\frac{2\,(bdx + bc)}{d}\right) + \left(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3\right)\operatorname{Ci}\left(-\frac{2\,(bdx + bc)}{d}\right)\right)\cos\left(-\frac{2\,(bc - ad)}{d}\right)}{6\left(d^7x^3 + 3\,cd^6x^2 + 3\,c^2d^5x + c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(b*d^3*x + b*c*d^2 - 2*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 + 2*(2*b^2*d^
3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 4*(b
^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-2*(b*c - a*d)/
d)*sin_integral(2*(b*d*x + b*c)/d) - 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*
b^3*c^2*d*x + b^3*c^3)*cos_integral(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b
^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-2*(b*d*x + b*c)/d))*c
os(-2*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a + bx\right)\cos\left(a + bx\right)}{\left(c + dx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**4, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.60, size = 7592, normalized size = 52.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/6*(2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan
(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d)
```

)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(2
*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 4*b^3*d^3*x^3*imag_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 8*b^3*d^3*x
^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 4*b^3*d
^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d
)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*ta
n(a)*tan(b*c/d)^2 + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^
2*tan(a)*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*
c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3*re
al_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b^3*d^3*x^3*
real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 8*b^3*d^3*x
^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) +
8*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*t
an(b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b
*x)^2*tan(a)^2*tan(b*c/d) + 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 24*b^3*c*d^2*x^2*sin_integral(
2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^3*d^3*x^3*real_part
(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3*rea
l_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 12*b^3*c*d
^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d
)^2 - 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2
*tan(a)*tan(b*c/d)^2 + 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan
(b*x)^2*tan(a)*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(2*b*x +
2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c^2*d*x*real_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c^2*d*x*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 4*b^3
*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 4*b^3
*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 8*b^
3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - 6*b^3*c*d^2*x
^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 6*b^3*c*d
^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 4*b^
3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) -
4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/
d) + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) +
24*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)
*tan(b*c/d) + 24*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*x)^2*tan(a)*tan(b*c/d) - 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2
*b*c/d))*tan(a)^2*tan(b*c/d) + 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(a)^2*tan(b*c/d) - 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c
)/d)*tan(a)^2*tan(b*c/d) - 12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*
b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 12*b^3*c^2*d*x*imag_part(cos_integ
ral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 24*b^3*c^2*d*x*sin_
integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 6*b^3*c*d^2*x^

```
2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 6*b^3*
c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2
 + 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)
^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c
/d)^2 + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 +
 12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*
tan(b*c/d)^2 - 12*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(b*x)^2*tan(a)*tan(b*c/d)^2 + 24*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/d
)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(2
*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^3*c^3*real_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^3*c^3*real_p
art(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^
3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + 2*b^3*d^3*x
^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 12*b^3*c*d^2*x^2*
imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 12*b^3*c*d^2*x
^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 24*b^3*c*d
^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - 2*b^3*d^3*x^3*re
al_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 - 2*b^3*d^3*x^3*real_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - 6*b^3...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^4,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^4, x)
```

3.10 $\int \frac{\cos(x)\sin(x)}{x}\,dx$

Optimal. Leaf size=8

$$\frac{\text{Si}(2x)}{2}$$

[Out] 1/2*Si(2*x)

Rubi [A]
time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4491, 12, 3380}

$$\frac{\text{Si}(2x)}{2}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]*Sin[x])/x,x]

[Out] SinIntegral[2*x]/2

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)\sin(x)}{x}\,dx &= \int \frac{\sin(2x)}{2x}\,dx\\ &= \frac{1}{2}\int \frac{\sin(2x)}{x}\,dx\\ &= \frac{\text{Si}(2x)}{2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$\frac{\mathrm{Si}(2x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/x,x]
```

```
[Out] SinIntegral[2*x]/2
```

Maple [A]
time = 0.04, size = 7, normalized size = 0.88

method	result	size
default	$\frac{\mathrm{sinIntegral}(2x)}{2}$	7
meijerg	$\frac{\mathrm{sinIntegral}(2x)}{2}$	7
risch	$-\frac{\pi\,\mathrm{csgn}(x)}{4}+\frac{\mathrm{sinIntegral}(2x)}{2}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*Si(2*x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.29, size = 13, normalized size = 1.62

$$-\frac{1}{4}i\,\mathrm{Ei}(2i\,x)+\frac{1}{4}i\,\mathrm{Ei}(-2i\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x,x, algorithm="maxima")
```

```
[Out] -1/4*I*Ei(2*I*x) + 1/4*I*Ei(-2*I*x)
```

Fricas [A]
time = 2.13, size = 6, normalized size = 0.75

$$\frac{1}{2}\,\mathrm{Si}\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sin_integral(2*x)
```

Sympy [A]
time = 0.51, size = 5, normalized size = 0.62

$$\frac{\mathrm{Si}\,(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x,x)
```

```
[Out] Si(2*x)/2
```

Giac [A]
time = 0.43, size = 6, normalized size = 0.75

$$\frac{1}{2}\,\mathrm{Si}\,(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x,x, algorithm="giac")
```

```
[Out] 1/2*sin_integral(2*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.12

$$\int \frac{\cos\left(x\right)\,\sin\left(x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x))/x,x)
```

```
[Out] int((cos(x)*sin(x))/x, x)
```

3.11 $\int \frac{\cos(x)\sin(x)}{x^2} \, dx$

Optimal. Leaf size=16

$$\text{CosIntegral}(2x) - \frac{\sin(2x)}{2x}$$

```
[Out] Ci(2*x)-1/2*sin(2*x)/x
```

Rubi [A]
time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4491, 12, 3378, 3383}

$$\text{CosIntegral}(2x) - \frac{\sin(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/x^2,x]
```

```
[Out] CosIntegral[2*x] - Sin[2*x]/(2*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(x)\sin(x)}{x^2}\,dx &= \int \frac{\sin(2x)}{2x^2}\,dx \\
&= \frac{1}{2}\int \frac{\sin(2x)}{x^2}\,dx \\
&= -\frac{\sin(2x)}{2x} + \int \frac{\cos(2x)}{x}\,dx \\
&= \operatorname{Ci}(2x) - \frac{\sin(2x)}{2x}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\text{CosIntegral}(2x) - \frac{\sin(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/x^2,x]
[Out] CosIntegral[2*x] - Sin[2*x]/(2*x)
```

Maple [A]
time = 0.07, size = 15, normalized size = 0.94

method	result	size
default	$\text{cosineIntegral}\left(2x\right) - \frac{\sin(2x)}{2x}$	15
risch	$\text{cosineIntegral}\left(2x\right) - \frac{i\pi\,\text{csgn}(ix)\text{csgn}(x)}{2} + \frac{i\pi\,\text{csgn}(ix)}{2} - \frac{\sin(2x)}{2x}$	35
meijerg	$\frac{\sqrt{\pi}\left(\frac{4}{\sqrt{\pi}} - \frac{4\gamma}{\sqrt{\pi}} - \frac{4\ln(2)}{\sqrt{\pi}} - \frac{4\ln(x)}{\sqrt{\pi}} - \frac{2\sin(2x)}{\sqrt{\pi}\,x} + \frac{4\,\text{cosineIntegral}(2x)}{\sqrt{\pi}} + \frac{4\gamma-4+4\ln(2)+4\ln(x)}{\sqrt{\pi}}\right)}{4}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/x^2,x,method=_RETURNVERBOSE)
[Out] Ci(2*x)-1/2*sin(2*x)/x
```

Maxima [C] Result contains complex when optimal does not.
time = 0.30, size = 15, normalized size = 0.94

$$\frac{1}{2}\,\Gamma(-1, 2i\,x) + \frac{1}{2}\,\Gamma(-1, -2i\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*gamma(-1, 2*I*x) + 1/2*gamma(-1, -2*I*x)
```

Fricas [A]
time = 2.02, size = 24, normalized size = 1.50

$$\frac{x\,\mathrm{Ci}\left(2\,x\right)+x\,\mathrm{Ci}\left(-2\,x\right)-2\,\cos\left(x\right)\sin\left(x\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos_integral(2*x) + x*cos_integral(-2*x) - 2*cos(x)*sin(x))/x
```

Sympy [A]
time = 0.90, size = 22, normalized size = 1.38

$$-\log\left(x\right)+\frac{\log\left(x^{2}\right)}{2}+\mathrm{Ci}\left(2x\right)-\frac{\sin\left(2x\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x**2,x)
```

```
[Out] -log(x) + log(x**2)/2 + Ci(2*x) - sin(2*x)/(2*x)
```

Giac [A]
time = 0.42, size = 19, normalized size = 1.19

$$\frac{2\,x\,\mathrm{Ci}\left(2\,x\right)-\sin\left(2\,x\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*x*cos_integral(2*x) - sin(2*x))/x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int\frac{\cos\left(x\right)\,\sin\left(x\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x))/x^2,x)
```

```
[Out] int((cos(x)*sin(x))/x^2, x)
```

3.12 $\int \frac{\cos(x)\sin(x)}{x^3}\,dx$

Optimal. Leaf size=29

$$-\frac{\cos(2x)}{2x}-\frac{\sin(2x)}{4x^2}-\text{Si}(2x)$$

```
[Out] -1/2*cos(2*x)/x-Si(2*x)-1/4*sin(2*x)/x^2
```

Rubi [A]
time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4491, 12, 3378, 3380}

$$-\text{Si}(2x)-\frac{\sin(2x)}{4x^2}-\frac{\cos(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/x^3,x]
```

```
[Out] -1/2*Cos[2*x]/x - Sin[2*x]/(4*x^2) - SinIntegral[2*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)\sin(x)}{x^3}\,dx &= \int \frac{\sin(2x)}{2x^3}\,dx\\ &= \frac{1}{2}\int \frac{\sin(2x)}{x^3}\,dx\\ &= -\frac{\sin(2x)}{4x^2} + \frac{1}{2}\int \frac{\cos(2x)}{x^2}\,dx\\ &= -\frac{\cos(2x)}{2x} - \frac{\sin(2x)}{4x^2} - \int \frac{\sin(2x)}{x}\,dx\\ &= -\frac{\cos(2x)}{2x} - \frac{\sin(2x)}{4x^2} - \mathrm{Si}(2x)\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 29, normalized size = 1.00

$$-\frac{\cos(2x)}{2x} - \frac{\sin(2x)}{4x^2} - \mathrm{Si}(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/x^3,x]
```

```
[Out] -1/2*Cos[2*x]/x - Sin[2*x]/(4*x^2) - SinIntegral[2*x]
```

Maple [**A**]
time = 0.06, size = 26, normalized size = 0.90

method	result	size
default	$-\frac{\cos(2x)}{2x} - \mathrm{sinIntegral}\left(2x\right) - \frac{\sin(2x)}{4x^2}$	26
risch	$\frac{\pi\,\mathrm{csgn}(x)}{2} - \mathrm{sinIntegral}\left(2x\right) - \frac{\cos(2x)}{2x} - \frac{\sin(2x)}{4x^2}$	31
meijerg	$\frac{\sqrt{\pi}\,\left(-\frac{2\cos(2x)}{x\sqrt{\pi}} - \frac{\sin(2x)}{x^2\sqrt{\pi}} - \frac{4\,\mathrm{sinIntegral}(2x)}{\sqrt{\pi}}\right)}{4}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*cos(2*x)/x-Si(2*x)-1/4*sin(2*x)/x^2
```

Maxima [**C**] Result contains complex when optimal does not.
time = 0.29, size = 15, normalized size = 0.52

$$i\,\Gamma(-2, 2i\,x) - i\,\Gamma(-2, -2i\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^3,x, algorithm="maxima")
```

```
[Out] I*gamma(-2, 2*I*x) - I*gamma(-2, -2*I*x)
```

Fricas [A]
time = 1.49, size = 30, normalized size = 1.03

$$-\frac{2\,x\cos\left(x\right)^{2}+2\,x^{2}\,\mathrm{Si}\left(2\,x\right)+\cos\left(x\right)\sin\left(x\right)-x}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*cos(x)^2 + 2*x^2*sin_integral(2*x) + cos(x)*sin(x) - x)/x^2
```

Sympy [A]
time = 0.67, size = 24, normalized size = 0.83

$$-\mathrm{Si}\left(2x\right)-\frac{\cos\left(2x\right)}{2x}-\frac{\sin\left(2x\right)}{4x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x**3,x)
```

```
[Out] -Si(2*x) - cos(2*x)/(2*x) - sin(2*x)/(4*x**2)
```

Giac [A]
time = 0.42, size = 26, normalized size = 0.90

$$-\frac{4\,x^{2}\,\mathrm{Si}\left(2\,x\right)+2\,x\cos\left(2\,x\right)+\sin\left(2\,x\right)}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*x^2*sin_integral(2*x) + 2*x*cos(2*x) + sin(2*x))/x^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(x\right)\,\sin\left(x\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x))/x^3,x)
```

```
[Out] int((cos(x)*sin(x))/x^3, x)
```

3.13 $\int (c+dx)^m \cos(a+bx) \sin^2(a+bx)\, dx$

Optimal. Leaf size=275

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \operatorname{Gamma}\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b}+\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \operatorname{Gamma}\left(\right.}{8b}$$

```
[Out] -1/8*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c
)/d)^m)+1/8*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(
d*x+c)/d)^m)+1/8*I*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*b*(
d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/8*I*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I*b*(
d*x+c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A]

time = 0.23, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4491, 3388, 2212}

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \operatorname{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}+\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \operatorname{Gamma}\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{8b}+\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \operatorname{Gamma}\left(m+1,\frac{ib(c+dx)}{d}\right)}{8b}-\frac{i3^{-m-1}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \operatorname{Gamma}\left(m+1,\frac{3ib(c+dx)}{d}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/8*I)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d
])/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/8)*(c + d*x)^m*Gamma[1 + m, (I*b*(c +
 d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + ((I/8)*3^(-1 - m
)*E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])
/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/8)*3^(-1 - m)*(c + d*x)^m*Gamma[1 + m,
((3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m\cos(a+bx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{4}(c+dx)^m\cos(a+bx)-\frac{1}{4}(c+dx)^m\cos(3a+3bx)\right)\,dx\\ &=\frac{1}{4}\int (c+dx)^m\cos(a+bx)\,dx-\frac{1}{4}\int (c+dx)^m\cos(3a+3bx)\,dx\\ &=\frac{1}{8}\int e^{-i(a+bx)}(c+dx)^m\,dx+\frac{1}{8}\int e^{i(a+bx)}(c+dx)^m\,dx-\frac{1}{8}\int e^{-i}\\ &=-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b}+\frac{ie^{-i(}}{}\end{aligned}$$

Mathematica [A]
time = 2.20, size = 252, normalized size = 0.92

$$-\frac{ie^{-\frac{3i(bc+ad)}{d}}(c+dx)^m\left(3e^{2i\left(2a+\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{ib(c+dx)}{d}\right)+\left(\frac{ib(c+dx)}{d}\right)^{-m}\left(-3e^{2ia+\frac{4ibc}{d}}\text{Gamma}\left(1+m,\frac{ib(c+dx)}{d}\right)+3^{-m}\left(-e^{6ia}\left(-\frac{ib(c+dx)}{d}\right)^m\left(\frac{ib(c+dx)}{d}\right)^{3m}\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-2m}\text{Gamma}\left(1+m,-\frac{3ib(c+dx)}{d}\right)+e^{\frac{6ibc}{d}}\text{Gamma}\left(1+m,\frac{3ib(c+dx)}{d}\right)\right)\right)\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/24*I)*(c + d*x)^m*((3*E^((2*I)*(2*a + (b*c)/d))*Gamma[1 + m, ((-I)*b*(
c + d*x))/d])/(((-I)*b*(c + d*x))/d)^m + (-3*E^((2*I)*a + ((4*I)*b*c)/d)*Ga
mma[1 + m, (I*b*(c + d*x))/d] + (-((E^((6*I)*a)*(((-I)*b*(c + d*x))/d)^m*((
I*b*(c + d*x))/d)^(3*m)*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/((b^2*(c + d*
x)^2)/d^2)^(2*m)) + E^(((6*I)*b*c)/d)*Gamma[1 + m, ((3*I)*b*(c + d*x))/d])/
3^m)/((I*b*(c + d*x))/d)^m))/(b*E^(((3*I)*(b*c + a*d))/d))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int (dx+c)^m\cos(bx+a)\left(\sin^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^2, x)
```

Fricas [A]
time = 0.53, size = 188, normalized size = 0.68

$$\frac{3i\,e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-i\,bc+i\,ad}{d}\right)}\Gamma\left(m+1,\frac{i\,bdx+i\,bc}{d}\right)+i\,e^{\left(-\frac{dm\log\left(-\frac{3i\,b}{d}\right)+3i\,bc-3i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{3\,(i\,bdx+i\,bc)}{d}\right)-3i\,e^{\left(-\frac{dm\log\left(-\frac{ib}{d}\right)+i\,bc-i\,ad}{d}\right)}\Gamma\left(m+1,\frac{-i\,bdx-i\,bc}{d}\right)-i\,e^{\left(-\frac{dm\log\left(\frac{3i\,b}{d}\right)-3i\,bc+3i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{3\,(-i\,bdx-i\,bc)}{d}\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*I*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x + I
*b*c)/d) + I*e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m + 1, -3
*(I*b*d*x + I*b*c)/d) - 3*I*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(
m + 1, (-I*b*d*x - I*b*c)/d) - I*e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)
/d)*gamma(m + 1, -3*(-I*b*d*x - I*b*c)/d))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin^2(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*sin(a + b*x)**2*cos(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)\sin(a+bx)^2(c+dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^m,x)
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^m, x)
```

3.14 $\int (c+dx)^4 \cos(a+bx) \sin^2(a+bx)\, dx$

Optimal. Leaf size=205

$$-\frac{160d^3(c+dx)\cos(a+bx)}{27b^4}+\frac{8d(c+dx)^3\cos(a+bx)}{9b^2}+\frac{160d^4\sin(a+bx)}{27b^5}-\frac{8d^2(c+dx)^2\sin(a+bx)}{3b^3}-\frac{8d^3(c}{}$$

```
[Out] -160/27*d^3*(d*x+c)*cos(b*x+a)/b^4+8/9*d*(d*x+c)^3*cos(b*x+a)/b^2+160/27*d^
4*sin(b*x+a)/b^5-8/3*d^2*(d*x+c)^2*sin(b*x+a)/b^3-8/27*d^3*(d*x+c)*cos(b*x+
a)*sin(b*x+a)^2/b^4+4/9*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2/b^2+8/81*d^4*si
n(b*x+a)^3/b^5-4/9*d^2*(d*x+c)^2*sin(b*x+a)^3/b^3+1/3*(d*x+c)^4*sin(b*x+a)^
3/b
```

Rubi [A]
time = 0.15, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4489, 3392, 3377, 2717, 3391}

$$\frac{8d^4\sin^3(a+bx)}{81b^5}+\frac{160d^4\sin(a+bx)}{27b^5}-\frac{160d^3(c+dx)\cos(a+bx)}{27b^4}-\frac{8d^3(c+dx)\sin^2(a+bx)\cos(a+bx)}{27b^4}-\frac{4d^2(c+dx)^2\sin^3(a+bx)}{9b^3}-\frac{8d^2(c+dx)^2\sin(a+bx)}{3b^3}+\frac{8d(c+dx)^3\cos(a+bx)}{9b^2}+\frac{4d(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{9b^2}+\frac{(c+dx)^4\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (-160*d^3*(c + d*x)*Cos[a + b*x])/(27*b^4) + (8*d*(c + d*x)^3*Cos[a + b*x])
/(9*b^2) + (160*d^4*Sin[a + b*x])/(27*b^5) - (8*d^2*(c + d*x)^2*Sin[a + b*x
])/(3*b^3) - (8*d^3*(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/(27*b^4) + (4*d*
(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^2) + (8*d^4*Sin[a + b*x]^3)/(
81*b^5) - (4*d^2*(c + d*x)^2*Sin[a + b*x]^3)/(9*b^3) + ((c + d*x)^4*Sin[a +
 b*x]^3)/(3*b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

```
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^4 \cos(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^4\sin^3(a+bx)}{3b} - \frac{(4d)\int (c+dx)^3\sin^3(a+bx)\,dx}{3b}$$
$$= \frac{4d(c+dx)^3\cos(a+bx)\sin^2(a+bx)}{9b^2} - \frac{4d^2(c+dx)^2\sin^3(a+bx)}{9b^3}$$
$$= \frac{8d(c+dx)^3\cos(a+bx)}{9b^2} - \frac{8d^3(c+dx)\cos(a+bx)\sin^2(a+bx)}{27b^4} +$$
$$= -\frac{16d^3(c+dx)\cos(a+bx)}{27b^4} + \frac{8d(c+dx)^3\cos(a+bx)}{9b^2} - \frac{8d^2(c+}{}$$
$$= -\frac{160d^3(c+dx)\cos(a+bx)}{27b^4} + \frac{8d(c+dx)^3\cos(a+bx)}{9b^2} + \frac{16d^4\sin}{2}$$
$$= -\frac{160d^3(c+dx)\cos(a+bx)}{27b^4} + \frac{8d(c+dx)^3\cos(a+bx)}{9b^2} + \frac{160d^4\,\mathrm{s}}{}$$

Mathematica [A]
time = 1.51, size = 385, normalized size = 1.88

$$\frac{324bd(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\cos(a+bx)-12bd(c+dx)\left(-2d^2+3b^2(c+dx)^2\right)\cos(3(a+bx))+81b^4c^4\sin(a+bx)-972b^2c^2d^2\sin(a+bx)+1944d^4\sin(a+bx)+324b^4c^3dx\sin(a+bx)-1944b^2cd^3x\sin(a+bx)+486b^4c^2d^2x^2\sin(a+bx)-972b^2d^4x^2\sin(a+bx)+324b^4cd^3x^3\sin(a+bx)+81b^4d^4x^4\sin(a+bx)-27b^4c^4\sin(3(a+bx))+36b^2c^2d^2\sin(3(a+bx))-8d^4\sin(3(a+bx))-108b^4c^3dx\sin(3(a+bx))+72b^2cd^3x\sin(3(a+bx))-162b^4c^2d^2x^2\sin(3(a+bx))+36b^2d^4x^2\sin(3(a+bx))-108b^4cd^3x^3\sin(3(a+bx))-27b^4d^4x^4\sin(3(a+bx))}{324b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (324*b*d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 12*b*d*(c + d*
x)*(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 81*b^4*c^4*Sin[a + b*x]
```

```
- 972*b^2*c^2*d^2*Sin[a + b*x] + 1944*d^4*Sin[a + b*x] + 324*b^4*c^3*d*x*Si
n[a + b*x] - 1944*b^2*c*d^3*x*Sin[a + b*x] + 486*b^4*c^2*d^2*x^2*Sin[a + b*
x] - 972*b^2*d^4*x^2*Sin[a + b*x] + 324*b^4*c*d^3*x^3*Sin[a + b*x] + 81*b^4
*d^4*x^4*Sin[a + b*x] - 27*b^4*c^4*Sin[3*(a + b*x)] + 36*b^2*c^2*d^2*Sin[3*
(a + b*x)] - 8*d^4*Sin[3*(a + b*x)] - 108*b^4*c^3*d*x*Sin[3*(a + b*x)] + 72
*b^2*c*d^3*x*Sin[3*(a + b*x)] - 162*b^4*c^2*d^2*x^2*Sin[3*(a + b*x)] + 36*b
^2*d^4*x^2*Sin[3*(a + b*x)] - 108*b^4*c*d^3*x^3*Sin[3*(a + b*x)] - 27*b^4*d
^4*x^4*Sin[3*(a + b*x)])/(324*b^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 834 vs. $2(187) = 374$.
time = 0.20, size = 835, normalized size = 4.07

method	result
risch	$\frac{d\left(b^{2} d^{3} x^{3}+3 b^{2} c\, d^{2} x^{2}+3 b^{2} c^{2} d x+b^{2} c^{3}-6 d^{3} x-6 c\, d^{2}\right) \cos \left(b x+a\right)}{b^{4}}+\frac{\left(d^{4} x^{4} b^{4}+4 b^{4} c\, d^{3} x^{3}+6 b^{4} c^{2} d^{2} x^{2}+4 b^{4} c^{3} d x+b^{4} c^{4}-12 b^{2}\right.}{4 b^{5}}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/3/b^4*a^4*d^4*sin(b*x+a)^3-4/3/b^3*a^3*c*d^3*sin(b*x+a)^3-4/b^4*a^3*
d^4*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+2/b^2*a^2*c^
2*d^2*sin(b*x+a)^3+12/b^3*a^2*c*d^3*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*
x+a)^2)*cos(b*x+a))+6/b^4*a^2*d^4*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(
2+sin(b*x+a)^2)*cos(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))-4/3/b*a*c^3*d*
sin(b*x+a)^3-12/b^2*a*c^2*d^2*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2
)*cos(b*x+a))-12/b^3*a*c*d^3*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin
(b*x+a)^2)*cos(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))-4/b^4*a*d^4*(1/3*(b
*x+a)^3*sin(b*x+a)^3+1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)-4/3*cos(b*x+
a)-4/3*(b*x+a)*sin(b*x+a)-2/9*(b*x+a)*sin(b*x+a)^3-2/27*(2+sin(b*x+a)^2)*co
s(b*x+a))+1/3*c^4*sin(b*x+a)^3+4/b*c^3*d*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+s
in(b*x+a)^2)*cos(b*x+a))+6/b^2*c^2*d^2*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x
+a)*(2+sin(b*x+a)^2)*cos(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))+4/b^3*c*d
^3*(1/3*(b*x+a)^3*sin(b*x+a)^3+1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)-4/
3*cos(b*x+a)-4/3*(b*x+a)*sin(b*x+a)-2/9*(b*x+a)*sin(b*x+a)^3-2/27*(2+sin(b*
x+a)^2)*cos(b*x+a))+1/b^4*d^4*(1/3*(b*x+a)^4*sin(b*x+a)^3+4/9*(b*x+a)^3*(2+
sin(b*x+a)^2)*cos(b*x+a)-8/3*(b*x+a)^2*sin(b*x+a)+160/27*sin(b*x+a)-16/3*(b
*x+a)*cos(b*x+a)-4/9*(b*x+a)^2*sin(b*x+a)^3-8/27*(b*x+a)*(2+sin(b*x+a)^2)*c
os(b*x+a)+8/81*sin(b*x+a)^3))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 880 vs. $2(187) = 374$.
time = 0.32, size = 880, normalized size = 4.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/324*(108*c^4*sin(b*x + a)^3 - 432*a*c^3*d*sin(b*x + a)^3/b + 648*a^2*c^2*
d^2*sin(b*x + a)^3/b^2 - 432*a^3*c*d^3*sin(b*x + a)^3/b^3 + 108*a^4*d^4*sin
(b*x + a)^3/b^4 - 36*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x +
a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*c^3*d/b + 108*(3*(b*x + a)*sin(3*b*
x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a*
c^2*d^2/b^2 - 108*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a)
+ cos(3*b*x + 3*a) - 9*cos(b*x + a))*a^2*c*d^3/b^3 + 36*(3*(b*x + a)*sin(3*
b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*
a^3*d^4/b^4 - 18*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a)
+ (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))
*c^2*d^2/b^2 + 36*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a)
 + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a)
)*a*c*d^3/b^3 - 18*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a
) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a
))*a^2*d^4/b^4 - 12*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2
 - 2)*cos(b*x + a) + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) - 27*
((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*c*d^3/b^3 + 12*((9*(b*x + a)^2 -
2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) + 3*(3*(b*x + a)^3
- 2*b*x - 2*a)*sin(3*b*x + 3*a) - 27*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x +
a))*a*d^4/b^4 - (12*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) - 324*((
b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) + (27*(b*x + a)^4 - 36*(b*x + a)^2 +
 8)*sin(3*b*x + 3*a) - 81*((b*x + a)^4 - 12*(b*x + a)^2 + 24)*sin(b*x + a))
*d^4/b^4)/b
```

Fricas [A]
time = 3.78, size = 352, normalized size = 1.72

$$-\frac{12\,(3\,b^3d^4x^3+9\,b^3cd^3x^2+3\,b^3c^3d-2\,bcd^3+(9\,b^3c^2d^2-2\,bd^4)x)\cos(bx+a)^3-36\,(3\,b^3d^4x^3+9\,b^3cd^3x^2+3\,b^3c^3d-14\,bcd^3+(9\,b^3c^2d^2-14\,bd^4)x)\cos(bx+a)-\left(27\,b^4d^4x^4+108\,b^4cd^3x^3+27\,b^4c^4-252\,b^2c^2d^2+488\,d^4+18\,(9\,b^4c^2d^2-14\,b^2d^4)x^2-(27\,b^4d^4x^4+108\,b^4cd^3x^3+27\,b^4c^4-36\,b^2c^2d^2+8\,d^4+18\,(9\,b^4c^2d^2-2\,b^2d^4)x^2+36\,(3\,b^4c^3d-2\,b^2cd^3)x)\cos(bx+a)^2+36\,(3\,b^4c^3d-14\,b^2cd^3)x\right)\sin(bx+a)}{81\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/81*(12*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 2*b*c*d^3 + (9*b
^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^3 - 36*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x
^2 + 3*b^3*c^3*d - 14*b*c*d^3 + (9*b^3*c^2*d^2 - 14*b*d^4)*x)*cos(b*x + a)
- (27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 - 252*b^2*c^2*d^2 + 488*
d^4 + 18*(9*b^4*c^2*d^2 - 14*b^2*d^4)*x^2 - (27*b^4*d^4*x^4 + 108*b^4*c*d^3
*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^2*d^4)
*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^3)*x)*cos(b*x + a)^2 + 36*(3*b^4*c^3*d -
 14*b^2*c*d^3)*x)*sin(b*x + a))/b^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(207) = 414$.

time = 0.70, size = 646, normalized size = 3.15

$$\begin{cases} \frac{c^4\sin^3(a+bx)}{3b}+\frac{4c^3dx\sin^3(a+bx)}{3b}+\frac{2c^2d^2x^2\sin^3(a+bx)}{b}+\frac{4cd^3x^3\sin^3(a+bx)}{3b}+\frac{d^4x^4\sin^3(a+bx)}{3b}+\frac{4c^3d\sin^2(a+bx)\cos(a+bx)}{3b^2}+\frac{8c^3d\cos^3(a+bx)}{9b^2}+\frac{4c^2d^2x\sin^2(a+bx)\cos(a+bx)}{b^2}+\frac{8c^2d^2x\cos^3(a+bx)}{3b^2}+\frac{4cd^3x^2\sin^2(a+bx)\cos(a+bx)}{b^2}+\frac{8cd^3x^2\cos^3(a+bx)}{3b^2}+\frac{4d^4x^3\sin^2(a+bx)\cos(a+bx)}{3b^2}+\frac{8d^4x^3\cos^3(a+bx)}{9b^2}-\frac{28c^2d^2\sin^3(a+bx)}{9b^3}-\frac{8c^2d^2\sin(a+bx)\cos^2(a+bx)}{3b^3}-\frac{56cd^3x\sin^3(a+bx)}{9b^3}-\frac{16cd^3x\sin(a+bx)\cos^2(a+bx)}{3b^3}-\frac{28d^4x^2\sin^3(a+bx)}{9b^3}-\frac{8d^4x^2\sin(a+bx)\cos^2(a+bx)}{3b^3}-\frac{56cd^3\sin^2(a+bx)\cos(a+bx)}{9b^4}-\frac{160cd^3\cos^3(a+bx)}{27b^4}-\frac{56d^4x\sin^2(a+bx)\cos(a+bx)}{9b^4}-\frac{160d^4x\cos^3(a+bx)}{27b^4}+\frac{488d^4\sin^3(a+bx)}{81b^5}+\frac{160d^4\sin(a+bx)\cos^2(a+bx)}{27b^5} & \text{for } b\neq 0 \\ \left(c^4x+2c^3dx^2+2c^2d^2x^3+cd^3x^4+\frac{d^4x^5}{5}\right)\sin^2(a)\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**4*sin(a + b*x)**3/(3*b) + 4*c**3*d*x*sin(a + b*x)**3/(3*b) +
2*c**2*d**2*x**2*sin(a + b*x)**3/b + 4*c*d**3*x**3*sin(a + b*x)**3/(3*b) +
d**4*x**4*sin(a + b*x)**3/(3*b) + 4*c**3*d*sin(a + b*x)**2*cos(a + b*x)/(3*
b**2) + 8*c**3*d*cos(a + b*x)**3/(9*b**2) + 4*c**2*d**2*x*sin(a + b*x)**2*c
os(a + b*x)/b**2 + 8*c**2*d**2*x*cos(a + b*x)**3/(3*b**2) + 4*c*d**3*x**2*s
in(a + b*x)**2*cos(a + b*x)/b**2 + 8*c*d**3*x**2*cos(a + b*x)**3/(3*b**2) +
 4*d**4*x**3*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 8*d**4*x**3*cos(a + b*
x)**3/(9*b**2) - 28*c**2*d**2*sin(a + b*x)**3/(9*b**3) - 8*c**2*d**2*sin(a
+ b*x)*cos(a + b*x)**2/(3*b**3) - 56*c*d**3*x*sin(a + b*x)**3/(9*b**3) - 16
*c*d**3*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 28*d**4*x**2*sin(a + b*x)
**3/(9*b**3) - 8*d**4*x**2*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 56*c*d**
3*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 160*c*d**3*cos(a + b*x)**3/(27*b*
*4) - 56*d**4*x*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 160*d**4*x*cos(a +
b*x)**3/(27*b**4) + 488*d**4*sin(a + b*x)**3/(81*b**5) + 160*d**4*sin(a + b
*x)*cos(a + b*x)**2/(27*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2
*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**2*cos(a), True))
```

Giac [A]
time = 0.44, size = 350, normalized size = 1.71

$$-\frac{(3b^3d^4x^3+9b^3cd^3x^2+9b^3c^2d^2x+3b^3c^3d-2bd^4x-2bcd^3)\cos(3bx+3a)}{27b^5}+\frac{(b^3d^4x^3+3b^3cd^3x^2+3b^3c^2d^2x+b^3c^3d-6bd^4x-6bcd^3)\cos(bx+a)}{b^5}-\frac{(27b^4d^4x^4+108b^4cd^3x^3+162b^4c^2d^2x^2+108b^4c^3dx+27b^4c^4-36b^2d^4x^2-72b^2cd^3x-36b^2c^2d^2+8d^4)\sin(3bx+3a)}{324b^5}+\frac{(b^4d^4x^4+4b^4cd^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-12b^2d^4x^2-24b^2cd^3x-12b^2c^2d^2+24d^4)\sin(bx+a)}{4b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/27*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*
b*d^4*x - 2*b*c*d^3)*cos(3*b*x + 3*a)/b^5 + (b^3*d^4*x^3 + 3*b^3*c*d^3*x^2
+ 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*cos(b*x + a)/b^5 - 1
/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*c^
3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8*d
^4)*sin(3*b*x + 3*a)/b^5 + 1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d
^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^2
*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

Mupad [B]
time = 1.41, size = 448, normalized size = 2.19

$$\frac{\sin(a+bx)^3\,(27b^4c^4-252b^2c^2d^2+488d^4)}{81b^5}-\frac{8\cos(a+bx)^3\,(20cd^3-3b^2c^3d)}{27b^4}+\frac{8\cos(a+bx)^2\,\sin(a+bx)\,(20d^4-9b^2c^2d^2)}{27b^5}-\frac{4\cos(a+bx)\,\sin(a+bx)^2\,(14cd^3-3b^2c^3d)}{9b^4}+\frac{8d^4x^3\cos(a+bx)^3}{9b^2}-\frac{8x\cos(a+bx)^3\,(20d^4-9b^2c^2d^2)}{27b^4}+\frac{d^4x^4\sin(a+bx)^3}{3b}-\frac{4x\sin(a+bx)^3\,(14cd^3-3b^2c^3d)}{9b^3}-\frac{2x^2\sin(a+bx)^3\,(14d^4-9b^2c^2d^2)}{9b^3}+\frac{8cd^3x^2\cos(a+bx)^3}{3b^2}+\frac{4d^4x^3\cos(a+bx)\,\sin(a+bx)^2}{3b^2}-\frac{8d^4x^2\cos(a+bx)^2\,\sin(a+bx)}{3b^3}+\frac{4cd^3x^3\sin(a+bx)^3}{3b}-\frac{4x\cos(a+bx)\,\sin(a+bx)^2\,(14d^4-9b^2c^2d^2)}{9b^4}+\frac{4cd^3x^2\cos(a+bx)\,\sin(a+bx)^2}{b^2}-\frac{16cd^3x\cos(a+bx)^2\,\sin(a+bx)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^4,x)
```

```
[Out] (sin(a + b*x)^3*(488*d^4 + 27*b^4*c^4 - 252*b^2*c^2*d^2))/(81*b^5) - (8*cos
(a + b*x)^3*(20*c*d^3 - 3*b^2*c^3*d))/(27*b^4) + (8*cos(a + b*x)^2*sin(a +
b*x)*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^5) - (4*cos(a + b*x)*sin(a + b*x)^2*(1
4*c*d^3 - 3*b^2*c^3*d))/(9*b^4) + (8*d^4*x^3*cos(a + b*x)^3)/(9*b^2) - (8*x
*cos(a + b*x)^3*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^4) + (d^4*x^4*sin(a + b*x)^
3)/(3*b) - (4*x*sin(a + b*x)^3*(14*c*d^3 - 3*b^2*c^3*d))/(9*b^3) - (2*x^2*s
in(a + b*x)^3*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^3) + (8*c*d^3*x^2*cos(a + b*x)
^3)/(3*b^2) + (4*d^4*x^3*cos(a + b*x)*sin(a + b*x)^2)/(3*b^2) - (8*d^4*x^2*
cos(a + b*x)^2*sin(a + b*x))/(3*b^3) + (4*c*d^3*x^3*sin(a + b*x)^3)/(3*b) -
 (4*x*cos(a + b*x)*sin(a + b*x)^2*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^4) + (4*c*
d^3*x^2*cos(a + b*x)*sin(a + b*x)^2)/b^2 - (16*c*d^3*x*cos(a + b*x)^2*sin(a
 + b*x))/(3*b^3)
```

3.15 $\int (c+dx)^3 \cos(a+bx) \sin^2(a+bx)\,dx$

Optimal. Leaf size=151

$$-\frac{14d^3\cos(a+bx)}{9b^4}+\frac{2d(c+dx)^2\cos(a+bx)}{3b^2}+\frac{2d^3\cos^3(a+bx)}{27b^4}-\frac{4d^2(c+dx)\sin(a+bx)}{3b^3}+\frac{d(c+dx)^2\cos(a+}{3b}$$

```
[Out] -14/9*d^3*cos(b*x+a)/b^4+2/3*d*(d*x+c)^2*cos(b*x+a)/b^2+2/27*d^3*cos(b*x+a)
^3/b^4-4/3*d^2*(d*x+c)*sin(b*x+a)/b^3+1/3*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)
^2/b^2-2/9*d^2*(d*x+c)*sin(b*x+a)^3/b^3+1/3*(d*x+c)^3*sin(b*x+a)^3/b
```

Rubi [A]
time = 0.10, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4489, 3392, 3377, 2718, 2713}

$$\frac{2d^3\cos^3(a+bx)}{27b^4}-\frac{14d^3\cos(a+bx)}{9b^4}-\frac{2d^2(c+dx)\sin^3(a+bx)}{9b^3}-\frac{4d^2(c+dx)\sin(a+bx)}{3b^3}+\frac{2d(c+dx)^2\cos(a+bx)}{3b^2}+\frac{d(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b^2}+\frac{(c+dx)^3\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (-14*d^3*Cos[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*Cos[a + b*x])/(3*b^2) + (
2*d^3*Cos[a + b*x]^3)/(27*b^4) - (4*d^2*(c + d*x)*Sin[a + b*x])/(3*b^3) + (
d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b^2) - (2*d^2*(c + d*x)*Sin[a
 + b*x]^3)/(9*b^3) + ((c + d*x)^3*Sin[a + b*x]^3)/(3*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
```

```
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3\cos(a+bx)\sin^2(a+bx)\,dx &= \frac{(c+dx)^3\sin^3(a+bx)}{3b} - \frac{d\int (c+dx)^2\sin^3(a+bx)\,dx}{b}\\
&= \frac{d(c+dx)^2\cos(a+bx)\sin^2(a+bx)}{3b^2} - \frac{2d^2(c+dx)\sin^3(a+bx)}{9b^3} +\\
&= \frac{2d(c+dx)^2\cos(a+bx)}{3b^2} + \frac{d(c+dx)^2\cos(a+bx)\sin^2(a+bx)}{3b^2} -\\
&= -\frac{2d^3\cos(a+bx)}{9b^4} + \frac{2d(c+dx)^2\cos(a+bx)}{3b^2} + \frac{2d^3\cos^3(a+bx)}{27b^4}\\
&= -\frac{14d^3\cos(a+bx)}{9b^4} + \frac{2d(c+dx)^2\cos(a+bx)}{3b^2} + \frac{2d^3\cos^3(a+bx)}{27b^4}
\end{aligned}$$

Mathematica [A]
time = 1.03, size = 121, normalized size = 0.80

$$-\frac{-81d(-2d^2+b^2(c+dx)^2)\cos(a+bx)+d(-2d^2+9b^2(c+dx)^2)\cos(3(a+bx))+6b(c+dx)\left(26d^2-3b^2(c+dx)^2+(-2d^2+3b^2(c+dx)^2)\cos(2(a+bx))\right)\sin(a+bx)}{108b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] -1/108*(-81*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + d*(-2*d^2 + 9*b^2*(
c + d*x)^2)*Cos[3*(a + b*x)] + 6*b*(c + d*x)*(26*d^2 - 3*b^2*(c + d*x)^2 +
(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/b^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(137) = 274$.
time = 0.18, size = 447, normalized size = 2.96 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/3/b^3*a^3*d^3*sin(b*x+a)^3+1/b^2*a^2*c*d^2*sin(b*x+a)^3+3/b^3*a^2*d
^3*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))-1/b*a*c^2*d*s
in(b*x+a)^3-6/b^2*a*c*d^2*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*co
s(b*x+a))-3/b^3*a*d^3*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)
^2)*cos(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))+1/3*c^3*sin(b*x+a)^3+3/b*c
^2*d*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+3/b^2*c*d^2
*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)-2/27*s
in(b*x+a)^3-4/9*sin(b*x+a))+1/b^3*d^3*(1/3*(b*x+a)^3*sin(b*x+a)^3+1/3*(b*x+
a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)-4/3*cos(b*x+a)-4/3*(b*x+a)*sin(b*x+a)-2/9*
(b*x+a)*sin(b*x+a)^3-2/27*(2+sin(b*x+a)^2)*cos(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(137) = 274$.
time = 0.29, size = 499, normalized size = 3.30

$$\frac{36\,c^3\sin\left(bx+a\right)^3-\frac{108\,ac^2d\sin(bx+a)^3}{b}+\frac{108\,a^2cd^2\sin(bx+a)^3}{b^2}-\frac{36\,a^3d^3\sin(bx+a)^3}{b^3}-\frac{9\,(3\,(bx+a)\sin(3\,bx+3\,a)-9\,(bx+a)\sin(bx+a)+\cos(3\,bx+3\,a)-9\,\cos(bx+a))c^2d}{b}+\frac{18\,(3\,(bx+a)\sin(3\,bx+3\,a)-9\,(bx+a)\sin(bx+a)+\cos(3\,bx+3\,a)-9\,\cos(bx+a))acd^2}{b^2}-\frac{9\,(3\,(bx+a)\sin(3\,bx+3\,a)-9\,(bx+a)\sin(bx+a)+\cos(3\,bx+3\,a)-9\,\cos(bx+a))a^2d^3}{b^3}-\frac{3\left(6\,(bx+a)\cos(3\,bx+3\,a)-54\,(bx+a)\cos(bx+a)+\left(9\,(bx+a)^2-2\right)\sin(3\,bx+3\,a)-27\left((bx+a)^2-2\right)\sin(bx+a)\right)cd^2}{b^2}+\frac{3\left(6\,(bx+a)\cos(3\,bx+3\,a)-54\,(bx+a)\cos(bx+a)+\left(9\,(bx+a)^2-2\right)\sin(3\,bx+3\,a)-27\left((bx+a)^2-2\right)\sin(bx+a)\right)ad^3}{b^3}-\frac{\left(\left(9\,(bx+a)^2-2\right)\cos(3\,bx+3\,a)-81\left((bx+a)^2-2\right)\cos(bx+a)+3\left(3\,(bx+a)^3-2\,bx-2\,a\right)\sin(3\,bx+3\,a)-27\left((bx+a)^3-6\,bx-6\,a\right)\sin(bx+a)\right)d^3}{b^3}}{108\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/108*(36*c^3*sin(b*x + a)^3 - 108*a*c^2*d*sin(b*x + a)^3/b + 108*a^2*c*d^2
*sin(b*x + a)^3/b^2 - 36*a^3*d^3*sin(b*x + a)^3/b^3 - 9*(3*(b*x + a)*sin(3*
b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*
c^2*d/b + 18*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos
(3*b*x + 3*a) - 9*cos(b*x + a))*a*c*d^2/b^2 - 9*(3*(b*x + a)*sin(3*b*x + 3*
a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a^2*d^3/
b^3 - 3*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) + (9*(b*x
 + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*c*d^2/b^
2 + 3*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) + (9*(b*x +
 a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*a*d^3/b^3
- ((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a)
 + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) - 27*((b*x + a)^3 - 6*b
*x - 6*a)*sin(b*x + a))*d^3/b^3)/b
```

Fricas [A]
time = 3.04, size = 227, normalized size = 1.50

$$-\frac{\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^3-3\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-14\,d^3\right)\cos\left(bx+a\right)-3\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+3\,b^3c^3-14\,bcd^2-\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+3\,b^3c^3-2\,bcd^2+\left(9\,b^3c^2d-2\,bd^3\right)x\right)\cos\left(bx+a\right)^2+\left(9\,b^3c^2d-14\,bd^3\right)x\right)\sin\left(bx+a\right)}{27\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/27*((9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)^
3 - 3*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 14*d^3)*cos(b*x + a)
```

```
- 3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 14*b*c*d^2 - (3*b^3*d^3*
x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3*c^2*d - 2*b*d^3)*x)*
cos(b*x + a)^2 + (9*b^3*c^2*d - 14*b*d^3)*x)*sin(b*x + a))/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(150) = 300$.
time = 0.50, size = 391, normalized size = 2.59

$$\begin{cases} \frac{c^3\sin^3(a+bx)}{3b} + \frac{c^2dx\sin^3(a+bx)}{b} + \frac{cd^2x^2\sin^3(a+bx)}{b} + \frac{d^3x^3\sin^3(a+bx)}{3b} + \frac{c^2d\sin^2(a+bx)\cos(a+bx)}{b^2} + \frac{2c^2d\cos^3(a+bx)}{3b^2} + \frac{2cd^2x\sin^2(a+bx)\cos(a+bx)}{b^2} + \frac{4cd^2x\cos^3(a+bx)}{3b^2} + \frac{d^3x^2\sin^2(a+bx)\cos(a+bx)}{b^2} + \frac{2d^3x^2\cos^3(a+bx)}{3b^2} - \frac{14cd^2\sin^3(a+bx)}{9b^3} - \frac{4cd^2\sin(a+bx)\cos^2(a+bx)}{3b^3} - \frac{14d^3x\sin^3(a+bx)}{9b^3} - \frac{4d^3x\sin(a+bx)\cos^2(a+bx)}{3b^3} - \frac{14d^3\sin^2(a+bx)\cos(a+bx)}{9b^4} - \frac{40d^3\cos^3(a+bx)}{27b^4} & \text{for } b \neq 0 \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin^2(a)\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**3*sin(a + b*x)**3/(3*b) + c**2*d*x*sin(a + b*x)**3/b + c*d**2
*x**2*sin(a + b*x)**3/b + d**3*x**3*sin(a + b*x)**3/(3*b) + c**2*d*sin(a +
b*x)**2*cos(a + b*x)/b**2 + 2*c**2*d*cos(a + b*x)**3/(3*b**2) + 2*c*d**2*x*
sin(a + b*x)**2*cos(a + b*x)/b**2 + 4*c*d**2*x*cos(a + b*x)**3/(3*b**2) + d
**3*x**2*sin(a + b*x)**2*cos(a + b*x)/b**2 + 2*d**3*x**2*cos(a + b*x)**3/(3
*b**2) - 14*c*d**2*sin(a + b*x)**3/(9*b**3) - 4*c*d**2*sin(a + b*x)*cos(a +
 b*x)**2/(3*b**3) - 14*d**3*x*sin(a + b*x)**3/(9*b**3) - 4*d**3*x*sin(a + b
*x)*cos(a + b*x)**2/(3*b**3) - 14*d**3*sin(a + b*x)**2*cos(a + b*x)/(9*b**4
) - 40*d**3*cos(a + b*x)**3/(27*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/
2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**2*cos(a), True))
```

Giac [A]
time = 0.46, size = 231, normalized size = 1.53

$$-\frac{(9\,b^2d^3x^2 + 18\,b^2cd^2x + 9\,b^2c^2d - 2\,d^3)\cos(3\,bx + 3\,a)}{108\,b^4} + \frac{3\,(b^2d^3x^2 + 2\,b^2cd^2x + b^2c^2d - 2\,d^3)\cos(bx + a)}{4\,b^4} - \frac{(3\,b^3d^3x^3 + 9\,b^3cd^2x^2 + 9\,b^3c^2dx + 3\,b^3c^3 - 2\,bd^3x - 2\,bcd^2)\sin(3\,bx + 3\,a)}{36\,b^4} + \frac{(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3 - 6\,bd^3x - 6\,bcd^2)\sin(bx + a)}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/108*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(3*b*x + 3
*a)/b^4 + 3/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a
)/b^4 - 1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 -
 2*b*d^3*x - 2*b*c*d^2)*sin(3*b*x + 3*a)/b^4 + 1/4*(b^3*d^3*x^3 + 3*b^3*c*d
^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x + a)/b^4
```

Mupad [B]
time = 1.15, size = 289, normalized size = 1.91

$$\frac{2\,d^3\,x^2\cos(a+b\,x)^3}{3\,b^2} - \frac{\sin(a+b\,x)^3\,(14\,c\,d^2 - 3\,b^2\,c^3)}{9\,b^3} - \frac{\cos(a+b\,x)\,\sin(a+b\,x)^2\,(14\,d^3 - 9\,b^2\,c^2\,d)}{9\,b^4} - \frac{x\sin(a+b\,x)^3\,(14\,d^3 - 9\,b^2\,c^2\,d)}{9\,b^3} - \frac{2\cos(a+b\,x)^3\,(20\,d^3 - 9\,b^2\,c^2\,d)}{27\,b^4} + \frac{d^3\,x^3\sin(a+b\,x)^3}{3\,b} - \frac{4\,c\,d^2\cos(a+b\,x)^2\sin(a+b\,x)}{3\,b^3} + \frac{4\,c\,d^2\,x\cos(a+b\,x)^3}{3\,b^2} - \frac{4\,d^3\,x\cos(a+b\,x)^2\sin(a+b\,x)}{3\,b^3} + \frac{d^3\,x^2\cos(a+b\,x)\sin(a+b\,x)^2}{b^2} + \frac{c\,d^2\,x^2\sin(a+b\,x)^3}{b} + \frac{2\,c\,d^2\,x\cos(a+b\,x)\sin(a+b\,x)^2}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] (2*d^3*x^2*cos(a + b*x)^3)/(3*b^2) - (sin(a + b*x)^3*(14*c*d^2 - 3*b^2*c^3)
)/(9*b^3) - (cos(a + b*x)*sin(a + b*x)^2*(14*d^3 - 9*b^2*c^2*d))/(9*b^4) -
(x*sin(a + b*x)^3*(14*d^3 - 9*b^2*c^2*d))/(9*b^3) - (2*cos(a + b*x)^3*(20*d
^3 - 9*b^2*c^2*d))/(27*b^4) + (d^3*x^3*sin(a + b*x)^3)/(3*b) - (4*c*d^2*cos
(a + b*x)^2*sin(a + b*x))/(3*b^3) + (4*c*d^2*x*cos(a + b*x)^3)/(3*b^2) - (4
*d^3*x*cos(a + b*x)^2*sin(a + b*x))/(3*b^3) + (d^3*x^2*cos(a + b*x)*sin(a +
 b*x)^2)/b^2 + (c*d^2*x^2*sin(a + b*x)^3)/b + (2*c*d^2*x*cos(a + b*x)*sin(a
 + b*x)^2)/b^2
```

3.16 $\int (c+dx)^2 \cos(a+bx) \sin^2(a+bx)\, dx$

Optimal. Leaf size=103

$$\frac{4d(c+dx)\cos(a+bx)}{9b^2} - \frac{4d^2\sin(a+bx)}{9b^3} + \frac{2d(c+dx)\cos(a+bx)\sin^2(a+bx)}{9b^2} - \frac{2d^2\sin^3(a+bx)}{27b^3} + \frac{(c+dx)^2}{}$$

```
[Out] 4/9*d*(d*x+c)*cos(b*x+a)/b^2-4/9*d^2*sin(b*x+a)/b^3+2/9*d*(d*x+c)*cos(b*x+a
)*sin(b*x+a)^2/b^2-2/27*d^2*sin(b*x+a)^3/b^3+1/3*(d*x+c)^2*sin(b*x+a)^3/b
```

Rubi [A]
time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4489, 3391, 3377, 2717}

$$-\frac{2d^2\sin^3(a+bx)}{27b^3} - \frac{4d^2\sin(a+bx)}{9b^3} + \frac{4d(c+dx)\cos(a+bx)}{9b^2} + \frac{2d(c+dx)\sin^2(a+bx)\cos(a+bx)}{9b^2} + \frac{(c+dx)^2\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (4*d*(c + d*x)*Cos[a + b*x])/(9*b^2) - (4*d^2*Sin[a + b*x])/(9*b^3) + (2*d*
(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^2) - (2*d^2*Sin[a + b*x]^3)/(27
*b^3) + ((c + d*x)^2*Sin[a + b*x]^3)/(3*b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
```

```
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^2\cos(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^2\sin^3(a+bx)}{3b} - \frac{(2d)\int (c+dx)\sin^3(a+bx)\,dx}{3b}$$
$$= \frac{2d(c+dx)\cos(a+bx)\sin^2(a+bx)}{9b^2} - \frac{2d^2\sin^3(a+bx)}{27b^3} + \frac{(c+dx)}{}$$
$$= \frac{4d(c+dx)\cos(a+bx)}{9b^2} + \frac{2d(c+dx)\cos(a+bx)\sin^2(a+bx)}{9b^2} - 2c$$
$$= \frac{4d(c+dx)\cos(a+bx)}{9b^2} - \frac{4d^2\sin(a+bx)}{9b^3} + \frac{2d(c+dx)\cos(a+bx)}{9b^2}$$

Mathematica [A]
time = 0.62, size = 93, normalized size = 0.90

$$\frac{54bd(c+dx)\cos(a+bx) - 6bd(c+dx)\cos(3(a+bx)) - 2(26d^2-9b^2(c+dx)^2+(-2d^2+9b^2(c+dx)^2)\cos(2(a+bx)))\sin(a+bx)}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (54*b*d*(c + d*x)*Cos[a + b*x] - 6*b*d*(c + d*x)*Cos[3*(a + b*x)] - 2*(26*d
^2 - 9*b^2*(c + d*x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin
[a + b*x])/(108*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(93) = 186.
time = 0.13, size = 204, normalized size = 1.98

method	result
risch	$\frac{d(dx+c)\cos(bx+a)}{2b^2} + \frac{(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\sin(bx+a)}{4b^3} - \frac{d(dx+c)\cos(3bx+3a)}{18b^2} - \frac{(9x^2d^2b^2+18b^2cdx+9b^2c^2}{108b^3}$
derivativedivides	$\frac{\frac{a^2d^2\left(\sin^3(bx+a)\right)}{3b^2} - \frac{2acd\left(\sin^3(bx+a)\right)}{3b} - \frac{2a\,d^2\left(\frac{(bx+a)\left(\sin^3(bx+a)\right)}{3}+\frac{\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{9}\right)}{b^2} + \frac{c^2\left(\sin^3(bx+a)\right)}{3} + \frac{2cd\left(\frac{(bx+}{}\right.}{}}{}$
default	$\frac{\frac{a^2d^2\left(\sin^3(bx+a)\right)}{3b^2} - \frac{2acd\left(\sin^3(bx+a)\right)}{3b} - \frac{2a\,d^2\left(\frac{(bx+a)\left(\sin^3(bx+a)\right)}{3}+\frac{\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{9}\right)}{b^2} + \frac{c^2\left(\sin^3(bx+a)\right)}{3} + \frac{2cd\left(\frac{(bx+}{}\right.}{}}{}$
norman	$\frac{\frac{8cd}{9b^2} - \frac{8d^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{9b^3} - \frac{8d^2\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{9b^3} + \frac{4d^2x}{9b^2} + \frac{8\left(9b^2c^2-8d^2\right)\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{27b^3} + \frac{8cd\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b^2} + \frac{4d^2x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b^2}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/3/b^2*a^2*d^2*sin(b*x+a)^3-2/3/b*a*c*d*sin(b*x+a)^3-2/b^2*a*d^2*(1/3
*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+1/3*c^2*sin(b*x+a)^3
+2/b*c*d*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+1/b^2*d
^2*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)-2/27
*sin(b*x+a)^3-4/9*sin(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(93) = 186$.
time = 0.28, size = 240, normalized size = 2.33

$$\frac{36\,c^2\sin\left(bx+a\right)^3-\frac{72\,acd\sin(bx+a)^3}{b}+\frac{36\,a^2d^2\sin(bx+a)^3}{b^2}-\frac{6\,(3\,(bx+a)\sin(3\,bx+3\,a)-9\,(bx+a)\sin(bx+a)+\cos(3\,bx+3\,a)-9\,\cos(bx+a))cd}{b}+\frac{6\,(3\,(bx+a)\sin(3\,bx+3\,a)-9\,(bx+a)\sin(bx+a)+\cos(3\,bx+3\,a)-9\,\cos(bx+a))ad^2}{b^2}-\frac{\left(6\,(bx+a)\cos(3\,bx+3\,a)-54\,(bx+a)\cos(bx+a)+\left(9\,(bx+a)^2-2\right)\sin(3\,bx+3\,a)-27\,\left((bx+a)^2-2\right)\sin(bx+a)\right)d^2}{b^2}}{108\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/108*(36*c^2*sin(b*x + a)^3 - 72*a*c*d*sin(b*x + a)^3/b + 36*a^2*d^2*sin(b
*x + a)^3/b^2 - 6*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a)
+ cos(3*b*x + 3*a) - 9*cos(b*x + a))*c*d/b + 6*(3*(b*x + a)*sin(3*b*x + 3*a
) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a*d^2/b^2
 - (6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) + (9*(b*x + a)
^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*d^2/b^2)/b
```

Fricas [A]
time = 2.45, size = 130, normalized size = 1.26

$$-\frac{6\,(bd^2x+bcd)\cos\left(bx+a\right)^3-18\,(bd^2x+bcd)\cos\left(bx+a\right)-\left(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-\left(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2\right)\cos\left(bx+a\right)^2-14\,d^2\right)\sin\left(bx+a\right)}{27\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/27*(6*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 18*(b*d^2*x + b*c*d)*cos(b*x +
a) - (9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - (9*b^2*d^2*x^2 + 18*b^2*c*
d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - 14*d^2)*sin(b*x + a))/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(102) = 204$.
time = 0.49, size = 216, normalized size = 2.10

$$\begin{cases} \frac{c^2\sin^3\left(a+bx\right)}{3b}+\frac{2cdx\sin^3\left(a+bx\right)}{3b}+\frac{d^2x^2\sin^3\left(a+bx\right)}{3b}+\frac{2cd\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{3b^2}+\frac{4cd\cos^3\left(a+bx\right)}{9b^2}+\frac{2d^2x\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{3b^2}+\frac{4d^2x\cos^3\left(a+bx\right)}{9b^2}-\frac{14d^2\sin^3\left(a+bx\right)}{27b^3}-\frac{4d^2\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{9b^3} & \text{for}\ b\neq 0 \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin^2\left(a\right)\cos\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**2*sin(a + b*x)**3/(3*b) + 2*c*d*x*sin(a + b*x)**3/(3*b) + d**
2*x**2*sin(a + b*x)**3/(3*b) + 2*c*d*sin(a + b*x)**2*cos(a + b*x)/(3*b**2)
+ 4*c*d*cos(a + b*x)**3/(9*b**2) + 2*d**2*x*sin(a + b*x)**2*cos(a + b*x)/(3
*b**2) + 4*d**2*x*cos(a + b*x)**3/(9*b**2) - 14*d**2*sin(a + b*x)**3/(27*b*
*3) - 4*d**2*sin(a + b*x)*cos(a + b*x)**2/(9*b**3), Ne(b, 0)), ((c**2*x + c
*d*x**2 + d**2*x**3/3)*sin(a)**2*cos(a), True))
```

Giac [A]
time = 0.46, size = 137, normalized size = 1.33

$$-\frac{(bd^2x+bcd)\cos(3bx+3a)}{18b^3}+\frac{(bd^2x+bcd)\cos(bx+a)}{2b^3}-\frac{(9b^2d^2x^2+18b^2cdx+9b^2c^2-2d^2)\sin(3bx+3a)}{108b^3}+\frac{(b^2d^2x^2+2b^2cdx+b^2c^2-2d^2)\sin(bx+a)}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/18*(b*d^2*x + b*c*d)*cos(3*b*x + 3*a)/b^3 + 1/2*(b*d^2*x + b*c*d)*cos(b*
x + a)/b^3 - 1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*sin(3
*b*x + 3*a)/b^3 + 1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x
 + a)/b^3
```

Mupad [B]
time = 0.87, size = 161, normalized size = 1.56

$$\frac{4d^2x\cos(a+bx)^3}{9b^2}-\frac{4d^2\cos(a+bx)^2\sin(a+bx)}{9b^3}-\frac{\sin(a+bx)^3(14d^2-9b^2c^2)}{27b^3}+\frac{d^2x^2\sin(a+bx)^3}{3b}+\frac{4cd\cos(a+bx)^3}{9b^2}+\frac{2cd\cos(a+bx)\sin(a+bx)^2}{3b^2}+\frac{2cdx\sin(a+bx)^3}{3b}+\frac{2d^2x\cos(a+bx)\sin(a+bx)^2}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] (4*d^2*x*cos(a + b*x)^3)/(9*b^2) - (4*d^2*cos(a + b*x)^2*sin(a + b*x))/(9*b
^3) - (sin(a + b*x)^3*(14*d^2 - 9*b^2*c^2))/(27*b^3) + (d^2*x^2*sin(a + b*x
)^3)/(3*b) + (4*c*d*cos(a + b*x)^3)/(9*b^2) + (2*c*d*cos(a + b*x)*sin(a + b
*x)^2)/(3*b^2) + (2*c*d*x*sin(a + b*x)^3)/(3*b) + (2*d^2*x*cos(a + b*x)*sin
(a + b*x)^2)/(3*b^2)
```

3.17 $\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=51

$$\frac{d\cos(a+bx)}{3b^2}-\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin^3(a+bx)}{3b}$$

```
[Out] 1/3*d*cos(b*x+a)/b^2-1/9*d*cos(b*x+a)^3/b^2+1/3*(d*x+c)*sin(b*x+a)^3/b
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4489, 2713}

$$-\frac{d\cos^3(a+bx)}{9b^2}+\frac{d\cos(a+bx)}{3b^2}+\frac{(c+dx)\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (d*Cos[a + b*x])/(3*b^2) - (d*Cos[a + b*x]^3)/(9*b^2) + ((c + d*x)*Sin[a + b*x]^3)/(3*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx &= \frac{(c+dx)\sin^3(a+bx)}{3b}-\frac{d\int \sin^3(a+bx)\,dx}{3b}\\ &= \frac{(c+dx)\sin^3(a+bx)}{3b}+\frac{d\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{3b^2}\\ &= \frac{d\cos(a+bx)}{3b^2}-\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 44, normalized size = 0.86

$$\frac{9d\cos(a+bx)-d\cos(3(a+bx))+12b(c+dx)\sin^3(a+bx)}{36b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2,x]
[Out] (9*d*Cos[a + b*x] - d*Cos[3*(a + b*x)] + 12*b*(c + d*x)*Sin[a + b*x]^3)/(36
*b^2)
```

Maple [A]
time = 0.08, size = 71, normalized size = 1.39

method	result	size
risch	$\frac{d\cos(bx+a)}{4b^2}+\frac{(dx+c)\sin(bx+a)}{4b}-\frac{d\cos(3bx+3a)}{36b^2}-\frac{(dx+c)\sin(3bx+3a)}{12b}$	64
derivativedivides	$\frac{-\frac{da\left(\sin^3(bx+a)\right)}{3b}+\frac{c\left(\sin^3(bx+a)\right)}{3}+\frac{d\left(\frac{(bx+a)\left(\sin^3(bx+a)\right)}{3}+\frac{\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{9}\right)}{b}}{b}$	71
default	$\frac{-\frac{da\left(\sin^3(bx+a)\right)}{3b}+\frac{c\left(\sin^3(bx+a)\right)}{3}+\frac{d\left(\frac{(bx+a)\left(\sin^3(bx+a)\right)}{3}+\frac{\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{9}\right)}{b}}{b}$	71
norman	$\frac{\frac{4d}{9b^2}+\frac{8c\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{4d\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b^2}+\frac{8dx\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/3/b*d*a*sin(b*x+a)^3+1/3*c*sin(b*x+a)^3+1/b*d*(1/3*(b*x+a)*sin(b*x+
a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a)))
```

Maxima [A]
time = 0.31, size = 85, normalized size = 1.67

$$\frac{12\,c\sin\left(bx+a\right)^3-\frac{12\,ad\sin(bx+a)^3}{b}-\frac{(3\,(bx+a)\sin(3\,bx+3\,a)-9\,(bx+a)\sin(bx+a)+\cos(3\,bx+3\,a)-9\,\cos(bx+a))d}{b}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
[Out] 1/36*(12*c*sin(b*x + a)^3 - 12*a*d*sin(b*x + a)^3/b - (3*(b*x + a)*sin(3*b*
x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*d/
b)/b
```

Fricas [A]
time = 2.69, size = 59, normalized size = 1.16

$$-\frac{d\cos\left(bx+a\right)^{3}-3\,d\cos\left(bx+a\right)-3\,\left(bdx-\left(bdx+bc\right)\cos\left(bx+a\right)^{2}+bc\right)\sin\left(bx+a\right)}{9\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/9*(d*cos(b*x + a)^3 - 3*d*cos(b*x + a) - 3*(b*d*x - (b*d*x + b*c)*cos(b*
x + a)^2 + b*c)*sin(b*x + a))/b^2
```

Sympy [A]
time = 0.29, size = 85, normalized size = 1.67

$$\begin{cases} \frac{c\sin^{3}\left(a+bx\right)}{3b}+\frac{dx\sin^{3}\left(a+bx\right)}{3b}+\frac{d\sin^{2}\left(a+bx\right)\cos\left(a+bx\right)}{3b^{2}}+\frac{2d\cos^{3}\left(a+bx\right)}{9b^{2}} & \text{for}\ b\neq 0\\ \left(cx+\frac{dx^{2}}{2}\right)\sin^{2}\left(a\right)\cos\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c*sin(a + b*x)**3/(3*b) + d*x*sin(a + b*x)**3/(3*b) + d*sin(a +
b*x)**2*cos(a + b*x)/(3*b**2) + 2*d*cos(a + b*x)**3/(9*b**2), Ne(b, 0)), ((
c*x + d*x**2/2)*sin(a)**2*cos(a), True))
```

Giac [A]
time = 0.45, size = 69, normalized size = 1.35

$$-\frac{d\cos\left(3\,bx+3\,a\right)}{36\,b^{2}}+\frac{d\cos\left(bx+a\right)}{4\,b^{2}}-\frac{\left(bdx+bc\right)\sin\left(3\,bx+3\,a\right)}{12\,b^{2}}+\frac{\left(bdx+bc\right)\sin\left(bx+a\right)}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/36*d*cos(3*b*x + 3*a)/b^2 + 1/4*d*cos(b*x + a)/b^2 - 1/12*(b*d*x + b*c)*
sin(3*b*x + 3*a)/b^2 + 1/4*(b*d*x + b*c)*sin(b*x + a)/b^2
```

Mupad [B]
time = 0.15, size = 59, normalized size = 1.16

$$\frac{\frac{2\,d\cos(a+b\,x)^3}{9}+b\left(\frac{c\sin(a+b\,x)^3}{3}+\frac{d\,x\sin(a+b\,x)^3}{3}\right)+\frac{d\,\cos(a+b\,x)\sin(a+b\,x)^2}{3}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x),x)
```

```
[Out] ((2*d*cos(a + b*x)^3)/9 + b*((c*sin(a + b*x)^3)/3 + (d*x*sin(a + b*x)^3)/3)
 + (d*cos(a + b*x)*sin(a + b*x)^2)/3)/b^2
```

3.18 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=121

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\sin\left(3a\right.}{}$$

```
[Out] -1/4*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d+1/4*Ci(b*c/d+b*x)*cos(a-b*c/d)/d+
1/4*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d-1/4*Si(b*c/d+b*x)*sin(a-b*c/d)/d
```

Rubi [A]
time = 0.20, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4491, 3384, 3380, 3383}

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d) - (Cos[3*a - (3*b*c)/d]
*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d) - (Sin[a - (b*c)/d]*SinIntegral[(b*c
)/d + b*x])/(4*d) + (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(
4*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx &= \int \left(\frac{\cos(a+bx)}{4(c+dx)} - \frac{\cos(3a+3bx)}{4(c+dx)}\right) dx \\
&= \frac{1}{4}\int \frac{\cos(a+bx)}{c+dx}\,dx - \frac{1}{4}\int \frac{\cos(3a+3bx)}{c+dx}\,dx \\
&= -\left(\frac{1}{4}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx\right) + \frac{1}{4}\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}\right.}{c+} \\
&= \frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{4d} - \frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d} - \frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}(}{4d}
\end{aligned}$$

Mathematica [A]
time = 0.29, size = 102, normalized size = 0.84

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right) - \cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) - \sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right) + \sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - Cos[3*a - (3*b*c)/d]*CosIntegr
al[(3*b*(c + d*x))/d] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Sin[3*a
 - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d)
```

Maple [A]
time = 0.11, size = 171, normalized size = 1.41

method	result
derivativedivides	$\dfrac{b\left(-\frac{\text{sinIntegral}\left(-bx-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\text{cosineIntegral}\left(bx+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}\right)}{4} - \dfrac{b\left(-\frac{3\,\text{sinIntegral}\left(-3bx-3a\right.}{}\right.}{b}$
default	$\dfrac{b\left(-\frac{\text{sinIntegral}\left(-bx-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\text{cosineIntegral}\left(bx+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}\right)}{4} - \dfrac{b\left(-\frac{3\,\text{sinIntegral}\left(-3bx-3a\right.}{}\right.}{b}$
risch	$\dfrac{e^{-\frac{3i(ad-cb)}{d}}\,\text{expIntegral}\left(1,3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{8d} - \dfrac{e^{-\frac{i(ad-cb)}{d}}\,\text{expIntegral}\left(1,ibx+ia-\frac{i(ad-cb)}{d}\right)}{8d} - \dfrac{e^{\frac{i(ad-cb)}{d}}\,\text{exp}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*b*(-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c
)/d)*cos((-a*d+b*c)/d)/d)-1/12*b*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*sin(3*(-
a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.34, size = 276, normalized size = 2.28

$$-\frac{b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b\left(E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)+b\left(-i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+b\left(-i\,E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_1\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{8\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*(b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integra
l_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*(exp_in
tegral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -3*(-
I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b*(-I*exp_integr
al_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(1, -(I*b*c +
I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(-I*exp_integral_e(1, 3*
(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integral_e(1, -3*(-I*b*c - I*(b
*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

Fricas [A]
time = 2.69, size = 153, normalized size = 1.26

$$\frac{\left(\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)+\operatorname{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-\left(\operatorname{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\operatorname{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)+2\,\sin\left(-\frac{3\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-2\,\sin\left(-\frac{bc-ad}{d}\right)\operatorname{Si}\left(\frac{bdx+bc}{d}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/8*((cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*cos(-
(b*c - a*d)/d) - (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d*x
+ b*c)/d))*cos(-3*(b*c - a*d)/d) + 2*sin(-3*(b*c - a*d)/d)*sin_integral(3*(
b*d*x + b*c)/d) - 2*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\cos(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.57, size = 6059, normalized size = 50.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/8*(real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c/d))*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_int
egral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(b*x + b*c/d))
*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(co
s_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/
2*b*c/d) + 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_int
egral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b
*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3
/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2
*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integr
al(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
+ 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d)^2 + real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_integral(b*x + b*c/d))*tan(3/
2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_integral(-b*x - b*c/d)
)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_integral(-3*b*
x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 4*real_part(cos_
integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d) - 4*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d) - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3
/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c/d)
)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(-b*x
- b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_part(cos_integr
al(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*real_p
art(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*t
an(1/2*b*c/d)^2 + 4*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*ta
n(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + real_part(cos_integral(3*b*x +
 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + real_part(cos_i
```

```
ntegral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + real
_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/
d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c/d))
*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(-b
*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos
_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan
(3/2*b*c/d) + 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(
1/2*a)^2*tan(3/2*b*c/d) - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d) + 2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*
a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 2*imag_part(cos_integral(-b*x - b*c/d))*
tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 4*sin_integral((b*d*x + b*c)/d)*
tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x +
 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 2*imag_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 4*sin_int
egral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 2*imag_
part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) -
2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b
*c/d) + 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b
*c/d) - 2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d) + 2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*imag_part(cos_integral(b*x + b*c/d))*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(-b*x
 - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 4*sin_integral((b
*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(c
os_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 2*imag
_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x), x)
```

3.19 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=168

$$-\frac{\cos(a+bx)}{4d(c+dx)}+\frac{\cos(3a+3bx)}{4d(c+dx)}+\frac{3b\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d^2}-\frac{b\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right.}{4d^2}$$

```
[Out] -1/4*cos(b*x+a)/d/(d*x+c)+1/4*cos(3*b*x+3*a)/d/(d*x+c)-1/4*b*cos(a-b*c/d)*S
i(b*c/d+b*x)/d^2+3/4*b*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^2+3/4*b*Ci(3*b*
c/d+3*b*x)*sin(3*a-3*b*c/d)/d^2-1/4*b*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^2
```

Rubi [A]
time = 0.21, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{\cos(a+bx)}{4d(c+dx)}+\frac{\cos(3a+3bx)}{4d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -1/4*Cos[a + b*x]/(d*(c + d*x)) + Cos[3*a + 3*b*x]/(4*d*(c + d*x)) + (3*b*C
osIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d^2) - (b*CosIntegra
l[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d^2) - (b*Cos[a - (b*c)/d]*SinIntegra
l[(b*c)/d + b*x])/(4*d^2) + (3*b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d
 + 3*b*x])/(4*d^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{\cos(a+bx)}{4(c+dx)^2}-\frac{\cos(3a+3bx)}{4(c+dx)^2}\right)dx\\
&= \frac{1}{4}\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx-\frac{1}{4}\int \frac{\cos(3a+3bx)}{(c+dx)^2}\,dx\\
&= -\frac{\cos(a+bx)}{4d(c+dx)}+\frac{\cos(3a+3bx)}{4d(c+dx)}-\frac{b\int \frac{\sin(a+bx)}{c+dx}\,dx}{4d}+\frac{(3b)\int \frac{\sin(3a+3bx)}{c+dx}\,dx}{4d}\\
&= -\frac{\cos(a+bx)}{4d(c+dx)}+\frac{\cos(3a+3bx)}{4d(c+dx)}+\frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx}{4d}-\frac{(b
\end{aligned}$$

$$= -\frac{\cos(a+bx)}{4d(c+dx)}+\frac{\cos(3a+3bx)}{4d(c+dx)}+\frac{3b\text{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d^2}-\frac{b\text{Ci}\left(\frac{bc}{d}\right.}{}$$

Mathematica [A]
time = 1.35, size = 139, normalized size = 0.83

$$-\frac{\frac{d\cos(a+bx)}{c+dx}-\frac{d\cos(3(a+bx))}{c+dx}-3b\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)+b\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right)+b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)-3b\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -1/4*((d*Cos[a + b*x])/(c + d*x) - (d*Cos[3*(a + b*x)])/(c + d*x) - 3*b*Cos
Integral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + b*CosIntegral[b*(c/d + x
)]*Sin[a - (b*c)/d] + b*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 3*b*Cos
[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/d^2
```

Maple [A]
time = 0.17, size = 247, normalized size = 1.47

method	result
derivativedivides	$\frac{b^2\left(-\frac{\cos(bx+a)}{(-ad+cb+d(bx+a))d}-\frac{-\frac{\mathrm{sinIntegral}\left(-bx-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\frac{\mathrm{cosineIntegral}\left(bx+a+\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}}{d}\right)}{4}-$
default	$\frac{b^2\left(-\frac{\cos(bx+a)}{(-ad+cb+d(bx+a))d}-\frac{-\frac{\mathrm{sinIntegral}\left(-bx-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\frac{\mathrm{cosineIntegral}\left(bx+a+\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}}{d}\right)}{4}-$
risch	$-\frac{3ib\,\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{8d^2}+\frac{ib\,\mathrm{e}^{-\frac{i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,ibx+ia-\frac{i(ad-cb)}{d}\right)}{8d^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*b^2*(-cos(b*x+a)/(-a*d+c*b+d*(b*x+a))/d-(-Si(-b*x-a-(-a*d+b*c)/d)*
cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)-1/12*b^2
*(-3*cos(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d-3*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)
/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d
)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.37, size = 305, normalized size = 1.82

$$-\frac{b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^2\left(E_2\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_2\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)-b^2\left(i\,E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)-b^2\left(i\,E_2\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_2\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{8\,(bcd+(bx+a)d^2-ad^2)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integ
ral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^2*(ex
p_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(2, -
3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b^2*(I*exp_i
ntegral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(2, -(I*b
*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^2*(I*exp_integral_e
(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(2, -3*(-I*b*c
- I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2
 - a*d^2)*b)
```

Fricas [A]
time = 2.42, size = 236, normalized size = 1.40

$$\frac{8\,d\cos(bx+a)^3+6\,(bdx+bc)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-2\,(bdx+bc)\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)-8\,d\cos(bx+a)-\left((bdx+bc)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+3\left((bdx+bc)\,\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{8\,(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*d*cos(b*x + a)^3 + 6*(b*d*x + b*c)*cos(-3*(b*c - a*d)/d)*sin_integra
l(3*(b*d*x + b*c)/d) - 2*(b*d*x + b*c)*cos(-(b*c - a*d)/d)*sin_integral((b*
d*x + b*c)/d) - 8*d*cos(b*x + a) - ((b*d*x + b*c)*cos_integral((b*d*x + b*c
)/d) + (b*d*x + b*c)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d) +
3*((b*d*x + b*c)*cos_integral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integr
al(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\cos(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**2, x)
```

Giac [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.05, size = 67350, normalized size = 400.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*b*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/
2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - b*d*
x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/
2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + b*d*x*imag_part(cos
_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2
*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b*d*x*imag_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b*d*x*sin_integral(3*(b*d*x + b*c)/d)*t
an(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 - 2*b*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/
2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b*
d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*b*d*x*real_part(c
os_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b*d*x*real_part(cos_integral(3*
b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan
```

```
(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 6*b*d*x*real_part(cos_integral(-3*b*x - 3*b*
c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d
)*tan(1/2*b*c/d)^2 + 2*b*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b
*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 + 2*b*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b*d*x*
real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(
3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b*d*x*real_part(c
os_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan
(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b*c*imag_part(cos_integral(
3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - b*c*imag_part(cos_integral(b*x + b*c/d))
*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d)^2 + b*c*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*
tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
- 3*b*c*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*
x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b*c*si
n_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b*c*sin_integral((b*d*x +
b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c
/d)^2*tan(1/2*b*c/d)^2 + 3*b*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*t
an(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + b
*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan
(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - b*d*x*imag_part(cos_integral(-b*x
 - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*
b*c/d)^2 - 3*b*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2
*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 6*b*d*x*sin_in
tegral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/
2*a)^2*tan(3/2*b*c/d)^2 + 2*b*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 4*b*d*x*ima
g_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^
2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 4*b*d*x*imag_part(cos_integr
al(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d) - 8*b*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/
2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*
c/d) - 2*b*c*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*
x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*b*c*real
_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^
2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 3*b*d*x*imag_part(cos_inte
gral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a
)^2*tan(1/2*b*c/d)^2 - b*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b
*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b*d*x*ima
g_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)
^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b*d*x*imag_part(cos_integral(-3*b*x -
3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b
```

```
*c/d)^2 - 6*b*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*
x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*b*d*x*sin_integral((b*d
*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/
2*b*c/d)^2 + 12*b*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)
^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 -
 12*b*d*x*imag_part(cos_integral(-3*b*x - 3*b*c...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^2, x)
```

3.20 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=221

$$-\frac{\cos(a+bx)}{8d(c+dx)^2}+\frac{\cos(3a+3bx)}{8d(c+dx)^2}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3\right.}{8d^3}$$

```
[Out] 9/8*b^2*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d^3-1/8*b^2*Ci(b*c/d+b*x)*cos(a-
b*c/d)/d^3-1/8*cos(b*x+a)/d/(d*x+c)^2+1/8*cos(3*b*x+3*a)/d/(d*x+c)^2-9/8*b^
2*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^3+1/8*b^2*Si(b*c/d+b*x)*sin(a-b*c/d)
/d^3+1/8*b*sin(b*x+a)/d^2/(d*x+c)-3/8*b*sin(3*b*x+3*a)/d^2/(d*x+c)
```

Rubi [A]
time = 0.25, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{b\sin(a+bx)}{8d^2(c+dx)}-\frac{3b\sin(3a+3bx)}{8d^2(c+dx)}-\frac{\cos(a+bx)}{8d(c+dx)^2}+\frac{\cos(3a+3bx)}{8d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] -1/8*Cos[a + b*x]/(d*(c + d*x)^2) + Cos[3*a + 3*b*x]/(8*d*(c + d*x)^2) - (b
^2*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^2*Cos[3*a -
(3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(8*d^3) + (b*Sin[a + b*x])/(8*d^
2*(c + d*x)) - (3*b*Sin[3*a + 3*b*x])/(8*d^2*(c + d*x)) + (b^2*Sin[a - (b*c
)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) - (9*b^2*Sin[3*a - (3*b*c)/d]*SinI
ntegral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule steps

$$
\begin{aligned}
\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx &= \int \left(\frac{\cos(a+bx)}{4(c+dx)^3} - \frac{\cos(3a+3bx)}{4(c+dx)^3}\right) dx \\
&= \frac{1}{4}\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx - \frac{1}{4}\int \frac{\cos(3a+3bx)}{(c+dx)^3}\,dx \\
&= -\frac{\cos(a+bx)}{8d(c+dx)^2} + \frac{\cos(3a+3bx)}{8d(c+dx)^2} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx}{8d} + \frac{(3b)\int \frac{\sin(3a+3bx)}{(c+dx)^2}\,dx}{8d} \\
&= -\frac{\cos(a+bx)}{8d(c+dx)^2} + \frac{\cos(3a+3bx)}{8d(c+dx)^2} + \frac{b\sin(a+bx)}{8d^2(c+dx)} - \frac{3b\sin(3a+3bx)}{8d^2(c+dx)} - \frac{b^2\int \cdots}{\cdots} \\
&= -\frac{\cos(a+bx)}{8d(c+dx)^2} + \frac{\cos(3a+3bx)}{8d(c+dx)^2} + \frac{b\sin(a+bx)}{8d^2(c+dx)} - \frac{3b\sin(3a+3bx)}{8d^2(c+dx)} + \frac{(9b^2 \cdots}{\cdots} \\
&= -\frac{\cos(a+bx)}{8d(c+dx)^2} + \frac{\cos(3a+3bx)}{8d(c+dx)^2} - \frac{b^2\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{8d^3} + \frac{9b^2\cos\left(3a \cdots\right.}{\cdots}
\end{aligned}
$$

Mathematica [A]
time = 2.08, size = 183, normalized size = 0.83

$$
\frac{-b^2\cos\left(a-\frac{bc}{d}\right)\mathrm{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+9b^2\cos\left(3a-\frac{3bc}{d}\right)\mathrm{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)+\frac{d(-d\cos(a+bx)+b(c+dx)\sin(a+bx))}{(c+dx)^2}+\frac{d(d\cos(3(a+bx))-3b(c+dx)\sin(3(a+bx)))}{(c+dx)^2}+b^2\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}+x\right)\right)-9b^2\sin\left(3a-\frac{3bc}{d}\right)\mathrm{Si}\left(\frac{3b(c+dx)}{d}\right)}{8d^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] (-(b^2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)]) + 9*b^2*Cos[3*a - (3*b*c)
/d]*CosIntegral[(3*b*(c + d*x))/d] + (d*(-(d*Cos[a + b*x]) + b*(c + d*x)*Si
n[a + b*x]))/(c + d*x)^2 + (d*(d*Cos[3*(a + b*x)] - 3*b*(c + d*x)*Sin[3*(a
+ b*x)]))/(c + d*x)^2 + b^2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 9*b
^2*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(8*d^3)
```

Maple [A]
time = 0.32, size = 316, normalized size = 1.43

method	result
derivativedivides	$\frac{b^3\left(-\frac{\cos(bx+a)}{2(-ad+cb+d(bx+a))^2 d}-\frac{-\frac{\sin(bx+a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{\mathrm{sinIntegral}\left(-bx-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\mathrm{cosineIntegral}\left(bx+a+\frac{-\cdots}{}\right)}{d}}{d}}{2d}\cdots\right.}{4}\cdots$
default	$\frac{b^3\left(-\frac{\cos(bx+a)}{2(-ad+cb+d(bx+a))^2 d}-\frac{-\frac{\sin(bx+a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{\mathrm{sinIntegral}\left(-bx-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\mathrm{cosineIntegral}\left(bx+a+\frac{-\cdots}{}\right)}{d}}{d}}{2d}\cdots\right.}{4}\cdots$
risch	$-\frac{9b^2 \mathrm{e}^{-\frac{3i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{16d^3}+\frac{b^2 \mathrm{e}^{-\frac{i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,ibx+ia-\frac{i(ad-cb)}{d}\right)}{16d^3}+\frac{b^2 \mathrm{e}^{\frac{i(a\cdots}{}}}{}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*b^3*(-1/2*cos(b*x+a)/(-a*d+c*b+d*(b*x+a))^2/d-1/2*(-sin(b*x+a)/(-a
*d+c*b+d*(b*x+a))/d+(-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+
(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)-1/12*b^3*(-3/2*cos(3*b*x+3*a)/(-a*
d+c*b+d*(b*x+a))^2/d-3/2*(-3*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d+3*(-3*Si
(-3*b*x-3*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*
c)/d)*cos(3*(-a*d+b*c)/d)/d)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.47, size = 340, normalized size = 1.54

$$-\frac{b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^3\left(E_3\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_3\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)-b^3\left(i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)-b^3\left(i\,E_3\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_3\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{8\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\,(bcd^2-ad^3)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integ
ral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^3*(ex
```

```
p_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(3, -
3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b^3*(I*exp_i
ntegral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(3, -(I*b
*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^3*(I*exp_integral_e
(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(3, -3*(-I*b*c
- I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d
^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Fricas [**A**]
time = 2.52, size = 399, normalized size = 1.81

$$\frac{8\,d^2\cos\left(bx+a\right)^3-8\,d^2\cos\left(bx+a\right)-18\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{3\left(bc-ad\right)}{d}\right)\operatorname{Si}\left(\frac{3\left(bdx+bc\right)}{d}\right)+2\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{bc-ad}{d}\right)\operatorname{Si}\left(\frac{bdx+bc}{d}\right)-\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+9\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(\frac{3\left(bdx+bc\right)}{d}\right)+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(-\frac{3\left(bdx+bc\right)}{d}\right)\right)\cos\left(-\frac{3\left(bc-ad\right)}{d}\right)+8\left(bd^2x+bcd-3\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2\right)\sin\left(bx+a\right)}{16\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(8*d^2*cos(b*x + a)^3 - 8*d^2*cos(b*x + a) - 18*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) + 2*(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-(b*c - a*d)/d)*sin_integral((b*d*
x + b*c)/d) - ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*x +
b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-(b*d*x + b*c)
/d))*cos(-(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_int
egral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integr
al(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d) + 8*(b*d^2*x + b*c*d - 3*(b*d
^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**3, x)
```

Giac [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.70, size = 114422, normalized size = 517.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*(9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2
*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
```

```
 - b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - b^2*d^
2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*
real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan
(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*x^2*im
ag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)
^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*b^2*d^2*x^2*imag_part(c
os_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 4*b^2*d^2*x^2*sin_integral((b*d*x
 + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d) - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b
*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/
d)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/
d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*
tan(1/2*b*c/d)^2 - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b
*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d
)^2 - 2*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan
(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b
^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 4*b^2*d^2*x
^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2
*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1
/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_in
tegral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*
a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_integral(3*(b*d
*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b
*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/
d)^2*tan(1/2*b*c/d)^2 - 2*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*ta
n(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d)^2 - 2*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*
x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^
2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 + 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*t
an(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + b^2*d^2*x^2*real
_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2
*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + b^2*d^2*x^2*real_part(cos_integral(-b*x -
b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c
/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 4*b^2*d^2*x
^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3
/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*b^2*d^2*x^2*real_par
```

```
t(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*ta
n(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 4*b^2*c*d*x*imag_part(cos_integr
al(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*b^2*c*d*x*imag_part(cos_integral(-b*x - b
*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/
d)^2*tan(1/2*b*c/d) + 8*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
) - 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*t
an(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^2*d^2*x^2*real
_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2
*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^2*d^2*x^2*real_part(cos_integral(-b*x -
b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c
/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 36*b^2*d^2*
x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*
tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*re
al_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3
/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part
(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + ...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a+b\,x)\,\sin(a+b\,x)^2}{(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^3, x)
```

3.21 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=270

$$-\frac{\cos(a+bx)}{12d(c+dx)^3}+\frac{b^2\cos(a+bx)}{24d^3(c+dx)}+\frac{\cos(3a+3bx)}{12d(c+dx)^3}-\frac{3b^2\cos(3a+3bx)}{8d^3(c+dx)}-\frac{9b^3\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\right.}{8d^4}$$

```
[Out] -1/12*cos(b*x+a)/d/(d*x+c)^3+1/24*b^2*cos(b*x+a)/d^3/(d*x+c)+1/12*cos(3*b*x
+3*a)/d/(d*x+c)^3-3/8*b^2*cos(3*b*x+3*a)/d^3/(d*x+c)+1/24*b^3*cos(a-b*c/d)*
Si(b*c/d+b*x)/d^4-9/8*b^3*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^4-9/8*b^3*Ci
(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^4+1/24*b^3*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^
4+1/24*b*sin(b*x+a)/d^2/(d*x+c)^2-1/8*b*sin(3*b*x+3*a)/d^2/(d*x+c)^2
```

Rubi [A]
time = 0.29, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$-\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{24d^4}+\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^2\cos(a+bx)}{24d^3(c+dx)}-\frac{3b^2\cos(3a+3bx)}{8d^3(c+dx)}+\frac{b\sin(a+bx)}{24d^2(c+dx)^2}-\frac{b\sin(3a+3bx)}{8d^2(c+dx)^2}-\frac{\cos(a+bx)}{12d(c+dx)^3}+\frac{\cos(3a+3bx)}{12d(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -1/12*Cos[a + b*x]/(d*(c + d*x)^3) + (b^2*Cos[a + b*x])/(24*d^3*(c + d*x))
+ Cos[3*a + 3*b*x]/(12*d*(c + d*x)^3) - (3*b^2*Cos[3*a + 3*b*x])/(8*d^3*(c
+ d*x)) - (9*b^3*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(8*d^
4) + (b^3*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(24*d^4) + (b*Sin[a
+ b*x])/(24*d^2*(c + d*x)^2) - (b*Sin[3*a + 3*b*x])/(8*d^2*(c + d*x)^2) + (
b^3*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(24*d^4) - (9*b^3*Cos[3*a
- (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^4)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx &= \int \left(\frac{\cos(a+bx)}{4(c+dx)^4} - \frac{\cos(3a+3bx)}{4(c+dx)^4}\right) dx \\
&= \frac{1}{4}\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx - \frac{1}{4}\int \frac{\cos(3a+3bx)}{(c+dx)^4}\,dx \\
&= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx}{12d} + \frac{b\int \frac{\sin(3a+3bx)}{(c+dx)^3}\,dx}{4d} \\
&= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} + \frac{b\sin(a+bx)}{24d^2(c+dx)^2} - \frac{b\sin(3a+3bx)}{8d^2(c+dx)^2} - \frac{b^2\int \cdots}{\cdots} \\
&= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{b^2\cos(a+bx)}{24d^3(c+dx)} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{8d^3(c+dx)} + \frac{b\cdots}{24\cdots} \\
&= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{b^2\cos(a+bx)}{24d^3(c+dx)} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{8d^3(c+dx)} + \frac{b\cdots}{24\cdots} \\
&= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{b^2\cos(a+bx)}{24d^3(c+dx)} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{8d^3(c+dx)} - \frac{9b\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A]
time = 1.47, size = 298, normalized size = 1.10

$$
\frac{d\cos(bx)\left(\left(-2d^2+b^2(c+dx)^2\right)\cos(a)+bd(c+dx)\sin(a)\right)-d\cos(3bx)\left(\left(-2d^2+9b^2(c+dx)^2\right)\cos(3a)+3bd(c+dx)\sin(3a)\right)+d\left(bd(c+dx)\cos(a)-\left(-2d^2+b^2(c+dx)^2\right)\sin(a)\right)\sin(bx)-d\left(3bd(c+dx)\cos(3a)-\left(-2d^2+9b^2(c+dx)^2\right)\sin(3a)\right)\sin(3bx)+b^3(c+dx)^3\left(\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right)+\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)\right)-27b^3(c+dx)^3\left(\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)+\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right)\right)}{24d^4(c+dx)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] (d*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a]) - d*
Cos[3*b*x]*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*x)*Sin[3*a
]) + d*(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x]
- d*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)*Sin[3*a])*Sin[
3*b*x] + b^3*(c + d*x)^3*(CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d] + Cos[a
 - (b*c)/d]*SinIntegral[b*(c/d + x)]) - 27*b^3*(c + d*x)^3*(CosIntegral[(3*
b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + Cos[3*a - (3*b*c)/d]*SinIntegral[(3*
b*(c + d*x))/d]))/(24*d^4*(c + d*x)^3)
```

Maple [A]
time = 0.21, size = 389, normalized size = 1.44

method	result
derivativedivides	$\frac{b^4\left(-\frac{\cos(bx+a)}{3(-ad+cb+d(bx+a))^3 d}-\frac{-\frac{\sin(bx+a)}{2(-ad+cb+d(bx+a))^2 d}+\frac{-\frac{\cos(bx+a)}{(-ad+cb+d(bx+a))d}-\frac{-\frac{\text{sinIntegral}\left(-bx-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)\cdots}{d}}{2d}}{3d}\cdots\right)}{4}\cdots$
default	$\frac{b^4\left(-\frac{\cos(bx+a)}{3(-ad+cb+d(bx+a))^3 d}-\frac{-\frac{\sin(bx+a)}{2(-ad+cb+d(bx+a))^2 d}+\frac{-\frac{\cos(bx+a)}{(-ad+cb+d(bx+a))d}-\frac{-\frac{\text{sinIntegral}\left(-bx-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)\cdots}{d}}{2d}}{3d}\cdots\right)}{4}\cdots$
risch	$\frac{9ib^3 e^{-\frac{3i(ad-cb)}{d}}\text{expIntegral}\left(1,3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{16d^4}-\frac{ib^3 e^{-\frac{i(ad-cb)}{d}}\text{expIntegral}\left(1,ibx+ia-\frac{i(ad-cb)}{d}\right)}{48d^4}+ib^3 e^{\frac{i(a\cdots}{}}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*b^4*(-1/3*cos(b*x+a)/(-a*d+c*b+d*(b*x+a))^3/d-1/3*(-1/2*sin(b*x+a)
/(-a*d+c*b+d*(b*x+a))^2/d+1/2*(-cos(b*x+a)/(-a*d+c*b+d*(b*x+a))/d-(-Si(-b*x
-a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/
d)/d)/d)/d)/d)-1/12*b^4*(-cos(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))^3/d-(-3/2*sin
(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))^2/d+3/2*(-3*cos(3*b*x+3*a)/(-a*d+c*b+d*(b*
x+a))/d-3*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*
x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.

time = 0.59, size = 390, normalized size = 1.44

$$-\frac{b^4\left(E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^4\left(E_4\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_4\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)-b^4\left(i\,E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)-b^4\left(i\,E_4\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_4\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{8\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\,(bcd^3-ad^4)(bx+a)^2+3\,(b^2c^2d^2-2\,abcd^3+a^2d^4)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/8*(b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integ
ral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^4*(ex
p_integral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(4, -
3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b^4*(I*exp_i
ntegral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(4, -(I*b
*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^4*(I*exp_integral_e
(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(4, -3*(-I*b*c
- I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c
^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b
*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(250) = 500$.
time = 2.36, size = 564, normalized size = 2.09

$$-\frac{8\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^3+54\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-2\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)-8\left(7\,b^2d^3x^2+14\,b^2cd^2x+7\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)-8\left(bd^3x+bcd^2-3\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^2\right)\sin\left(bx+a\right)-\left(\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+27\left(\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\mathrm{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{48\left(d^7x^3+3\,cd^6x^2+3\,c^2d^5x+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/48*(8*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a
)^3 + 54*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-3*(
b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^
2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x +
b*c)/d) - 8*(7*b^2*d^3*x^2 + 14*b^2*c*d^2*x + 7*b^2*c^2*d - 2*d^3)*cos(b*x
+ a) - 8*(b*d^3*x + b*c*d^2 - 3*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*sin(b*x
 + a) - ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_inte
gral((b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^
3*c^3)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d) + 27*((b^3*d^3*x
^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(3*(b*d*x + b*c
)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integr
al(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c
^2*d^5*x + c^3*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\cos(a+bx)}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**4, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.68, size = 168646, normalized size = 624.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/48*(27*b^3*d^3*x^3*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)
^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
^2 - b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/
2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + b^3*
d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2
*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 27*b^3*d^3*x
^3*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*
tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 54*b^3*d^3*x^
3*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^
2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*sin_integr
al((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*real_part(cos_integral(b
*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d) - 2*b^3*d^3*x^3*real_part(cos_integral(-b*x - b*c
/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)
^2*tan(1/2*b*c/d) + 54*b^3*d^3*x^3*real_part(cos_integral(3*b*x + 3*b*c/d))
*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan
(1/2*b*c/d)^2 + 54*b^3*d^3*x^3*real_part(cos_integral(-3*b*x - 3*b*c/d))*ta
n(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/
2*b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 + 2*b^3*d^3*x^3*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(
1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 54*b
^3*d^3*x^3*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 54*b^3*d
^3*x^3*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 81*b^3*c*d^
2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^
2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^3*c*d^2
*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan
(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^3*c*d^2*x^2*
imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2
```

```
*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 81*b^3*c*d^2*x^2*ima
g_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/
2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 162*b^3*c*d^2*x^2*s
in_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^3*c*d^2*x^2*sin_integra
l((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 27*b^3*d^3*x^3*imag_part(cos_integral(3
*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2 + b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b
*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - b^3*d^3*x
^3*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 27*b^3*d^3*x^3*imag_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a
)^2*tan(3/2*b*c/d)^2 + 54*b^3*d^3*x^3*sin_integral(3*(b*d*x + b*c)/d)*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 2*b^3
*d^3*x^3*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/
2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral
(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d) + 4*b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c
/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d) - 8*b^3*d^3*x^3*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^
2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) -
6*b^3*c*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^3*c
*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^
2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 27*b^3*d^3*x^
3*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^3*d^3*x^3*imag_part(cos_integr
al(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(1/2*b*c/d)^2 + b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*
b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 27*b^3*d
^3*x^3*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 54*b^3*d^3*x^3*sin_integra
l(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^
2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*sin_integral...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (a+b\,x)\,\sin (a+b\,x)^2}{(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^4,x)
[Out] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^4, x)
```

3.22 $\int (c+dx)^m \cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=271

$$-\frac{2^{-4-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-4-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right.}{b}$$

```
[Out] -2^(-4-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*b
*(d*x+c)/d)^m)-2^(-4-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a-b
*c/d))/((I*b*(d*x+c)/d)^m)+exp(4*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-4*I*b*(d
*x+c)/d)/(2^(6+2*m))/b/((-I*b*(d*x+c)/d)^m)+(d*x+c)^m*GAMMA(1+m,4*I*b*(d*x+
c)/d)/(2^(6+2*m))/b/exp(4*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.23, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4491, 3389, 2212}

$$-\frac{2^{-m-4}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{4ib(c+dx)}{d}\right)}{b}-\frac{2^{-m-4}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-2(m+3)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{4ib(c+dx)}{d}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] -((2^(-4 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c
 + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-4 - m)*(c + d*x)^m*Gamma[
1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/
d)^m) + (E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-4*I)*b*(c + d*
x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/d)^m) + ((c + d*x)^m*Gamma[1 +
 m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*E^((4*I)*(a - (b*c)/d))*((I*b*
(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \cos(a+bx)\sin^3(a+bx)\,dx &= \int \left(\frac{1}{4}(c+dx)^m \sin(2a+2bx) - \frac{1}{8}(c+dx)^m \sin(4a+4bx)\right) dx\\ &= -\left(\frac{1}{8}\int (c+dx)^m \sin(4a+4bx)\,dx\right) + \frac{1}{4}\int (c+dx)^m \sin(2a+2b\\ &= -\left(\frac{1}{16}i\int e^{-i(4a+4bx)}(c+dx)^m\,dx\right) + \frac{1}{16}i\int e^{i(4a+4bx)}(c+dx)^m\,dx\\ &= -\frac{2^{-4-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} -\end{aligned}$$

Mathematica [A]
time = 27.15, size = 361, normalized size = 1.33

$$\frac{2^{-2(3+m)}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-2m}\left(-2^{2+m}\left(\frac{b^2(c+dx)^2}{d^2}\right)^m\left(\left(-\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(1+m,\frac{2ib(c+dx)}{d}\right)\left(\cos\left(2a-\frac{2bc}{d}\right)-i\sin\left(2a-\frac{2bc}{d}\right)\right)+\left(\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(1+m,-\frac{2ib(c+dx)}{d}\right)\left(\cos\left(2a-\frac{2bc}{d}\right)+i\sin\left(2a-\frac{2bc}{d}\right)\right)\right)+e^{-\frac{4ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\left(\left(\frac{b^2(c+dx)^2}{d^2}\right)^m\text{Gamma}\left(1+m,\frac{4ib(c+dx)}{d}\right)\left(\cos\left(4a-\frac{4bc}{d}\right)-i\sin\left(4a-\frac{4bc}{d}\right)\right)+\left(\frac{ib(c+dx)}{d}\right)^{2m}\text{Gamma}\left(1+m,-\frac{4ib(c+dx)}{d}\right)\left(\cos\left(4a-\frac{4bc}{d}\right)+i\sin\left(4a-\frac{4bc}{d}\right)\right)\right)\left(\cos\left(\frac{4bc}{d}\right)+i\sin\left(\frac{4bc}{d}\right)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] ((c + d*x)^m*(-(2^(2 + m)*((b^2*(c + d*x)^2)/d^2)^m*((((-I)*b*(c + d*x))/d)
^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d]*(Cos[2*a - (2*b*c)/d] - I*Sin[2*a -
(2*b*c)/d]) + ((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d]*(C
os[2*a - (2*b*c)/d] + I*Sin[2*a - (2*b*c)/d]))) + ((((-I)*b*(c + d*x))/d)^m
*(((b^2*(c + d*x)^2)/d^2)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d]*(Cos[4*a -
(4*b*c)/d] - I*Sin[4*a - (4*b*c)/d]) + ((I*b*(c + d*x))/d)^(2*m)*Gamma[1 +
m, ((-4*I)*b*(c + d*x))/d]*(Cos[4*a - (4*b*c)/d] + I*Sin[4*a - (4*b*c)/d]))
*(Cos[(4*b*c)/d] + I*Sin[(4*b*c)/d]))/E^(((4*I)*b*c)/d)))/(2^(2*(3 + m))*b*
((b^2*(c + d*x)^2)/d^2)^(2*m))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \left(dx+c\right)^{m}\cos\left(bx+a\right)\left(\sin^{3}\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^3, x)
```

Fricas [A]
time = 0.73, size = 190, normalized size = 0.70

$$-\frac{4\,e^{\left(-\frac{dm\log\left(-\frac{2i\,b}{d}\right)+2i\,bc-2i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{2\,(i\,bdx+i\,bc)}{d}\right)-e^{\left(-\frac{dm\log\left(-\frac{4i\,b}{d}\right)+4i\,bc-4i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{4\,(i\,bdx+i\,bc)}{d}\right)+4\,e^{\left(-\frac{dm\log\left(\frac{2i\,b}{d}\right)-2i\,bc+2i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{2\,(-i\,bdx-i\,bc)}{d}\right)-e^{\left(-\frac{dm\log\left(\frac{4i\,b}{d}\right)-4i\,bc+4i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{4\,(-i\,bdx-i\,bc)}{d}\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(4*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m + 1, -2*(I*
b*d*x + I*b*c)/d) - e^(-(d*m*log(-4*I*b/d) + 4*I*b*c - 4*I*a*d)/d)*gamma(m
+ 1, -4*(I*b*d*x + I*b*c)/d) + 4*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)
/d)*gamma(m + 1, -2*(-I*b*d*x - I*b*c)/d) - e^(-(d*m*log(4*I*b/d) - 4*I*b*c
 + 4*I*a*d)/d)*gamma(m + 1, -4*(-I*b*d*x - I*b*c)/d))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)\sin(a+bx)^3(c+dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^m,x)
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^m, x)
```

3.23 $\int (c+dx)^4 \cos(a+bx) \sin^3(a+bx) \, dx$

Optimal. Leaf size=260

$$\frac{45cd^3x}{64b^3}+\frac{45d^4x^2}{128b^3}-\frac{3(c+dx)^4}{32b}-\frac{45d^3(c+dx)\cos(a+bx)\sin(a+bx)}{64b^4}+\frac{3d(c+dx)^3\cos(a+bx)\sin(a+bx)}{8b^2}+$$

```
[Out] 45/64*c*d^3*x/b^3+45/128*d^4*x^2/b^3-3/32*(d*x+c)^4/b-45/64*d^3*(d*x+c)*cos
(b*x+a)*sin(b*x+a)/b^4+3/8*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2+45/128*d^4
*sin(b*x+a)^2/b^5-9/16*d^2*(d*x+c)^2*sin(b*x+a)^2/b^3-3/32*d^3*(d*x+c)*cos(
b*x+a)*sin(b*x+a)^3/b^4+1/4*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3/b^2+3/128*d
^4*sin(b*x+a)^4/b^5-3/16*d^2*(d*x+c)^2*sin(b*x+a)^4/b^3+1/4*(d*x+c)^4*sin(b
*x+a)^4/b
```

Rubi [A]
time = 0.16, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4489, 3392, 32, 3391}

$$\frac{3d^4\sin^4(a+bx)}{128b^5}+\frac{45d^4\sin^2(a+bx)}{128b^5}-\frac{3d^3(c+dx)\sin^3(a+bx)\cos(a+bx)}{32b^4}-\frac{45d^3(c+dx)\sin(a+bx)\cos(a+bx)}{64b^4}-\frac{3d^2(c+dx)^2\sin^4(a+bx)}{16b^3}-\frac{9d^2(c+dx)^2\sin^2(a+bx)}{16b^3}+\frac{d(c+dx)^3\sin^3(a+bx)\cos(a+bx)}{4b^2}+\frac{3d(c+dx)^3\sin(a+bx)\cos(a+bx)}{8b^2}+\frac{(c+dx)^4\sin^4(a+bx)}{4b}+\frac{45cd^3x}{64b^3}+\frac{45d^4x^2}{128b^3}-\frac{3(c+dx)^4}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (45*c*d^3*x)/(64*b^3) + (45*d^4*x^2)/(128*b^3) - (3*(c + d*x)^4)/(32*b) - (
45*d^3*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(64*b^4) + (3*d*(c + d*x)^3*Cos
[a + b*x]*Sin[a + b*x])/(8*b^2) + (45*d^4*Sin[a + b*x]^2)/(128*b^5) - (9*d^
2*(c + d*x)^2*Sin[a + b*x]^2)/(16*b^3) - (3*d^3*(c + d*x)*Cos[a + b*x]*Sin[
a + b*x]^3)/(32*b^4) + (d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3)/(4*b^2)
+ (3*d^4*Sin[a + b*x]^4)/(128*b^5) - (3*d^2*(c + d*x)^2*Sin[a + b*x]^4)/(16
*b^3) + ((c + d*x)^4*Sin[a + b*x]^4)/(4*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^4\cos(a+bx)\sin^3(a+bx)\,dx &= \frac{(c+dx)^4\sin^4(a+bx)}{4b}-\frac{d\int (c+dx)^3\sin^4(a+bx)\,dx}{b}\\
&= \frac{d(c+dx)^3\cos(a+bx)\sin^3(a+bx)}{4b^2}-\frac{3d^2(c+dx)^2\sin^4(a+bx)}{16b^3}+\\
&= \frac{3d(c+dx)^3\cos(a+bx)\sin(a+bx)}{8b^2}-\frac{9d^2(c+dx)^2\sin^2(a+bx)}{16b^3}-\\
&= -\frac{3(c+dx)^4}{32b}-\frac{45d^3(c+dx)\cos(a+bx)\sin(a+bx)}{64b^4}+\frac{3d(c+dx)}{}\\
&= \frac{45cd^3x}{64b^3}+\frac{45d^4x^2}{128b^3}-\frac{3(c+dx)^4}{32b}-\frac{45d^3(c+dx)\cos(a+bx)\sin(a+}{64b^4}
\end{aligned}
$$

Mathematica [A]
time = 1.75, size = 158, normalized size = 0.61

$$\frac{-64(3d^4-6b^2d^2(c+dx)^2+2b^4(c+dx)^4)\cos(2(a+bx))+(3d^4-24b^2d^2(c+dx)^2+32b^4(c+dx)^4)\cos(4(a+bx))-8bd(c+dx)\left(-16(-3d^2+2b^2(c+dx)^2)+(-3d^2+8b^2(c+dx)^2)\cos(2(a+bx))\right)\sin(2(a+bx))}{1024b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-64*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)] +
 (3*d^4 - 24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Cos[4*(a + b*x)] - 8
*b*d*(c + d*x)*(-16*(-3*d^2 + 2*b^2*(c + d*x)^2) + (-3*d^2 + 8*b^2*(c + d*x
)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)])/(1024*b^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1136 vs. $2(236) = 472$.
time = 0.29, size = 1137, normalized size = 4.37

method	result
risch	$\frac{(32d^4x^4b^4+128b^4c\,d^3x^3+192b^4c^2d^2x^2+128b^4c^3dx+32b^4c^4-24b^2d^4x^2-48b^2c\,d^3x-24b^2c^2d^2+3d^4)\cos(4bx+4a)}{1024b^5} - d$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4/b^4*a^4*d^4*sin(b*x+a)^4-1/b^3*a^3*c*d^3*sin(b*x+a)^4-4/b^4*a^3*d^
4*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3
/32*b*x-3/32*a)+3/2/b^2*a^2*c^2*d^2*sin(b*x+a)^4+12/b^3*a^2*c*d^3*(1/4*(b*x
+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/3
2*a)+6/b^4*a^2*d^4*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a
)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/128*(2*sin(b
*x+a)^2+3)^2)-1/b*a*c^3*d*sin(b*x+a)^4-12/b^2*a*c^2*d^2*(1/4*(b*x+a)*sin(b*
x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)-12/b^
3*a*c*d^3*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*s
in(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/128*(2*sin(b*x+a)^2+3
)^2)-4/b^4*a*d^4*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a
)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)*sin(b*x+a)^4-3/1
28*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-27/256*b*x-27/256*a+9/32*(b*x+a
)*cos(b*x+a)^2-9/64*cos(b*x+a)*sin(b*x+a)+3/16*(b*x+a)^3)+1/4*c^4*sin(b*x+a
)^4+4/b*c^3*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*
cos(b*x+a)-3/32*b*x-3/32*a)+6/b^2*c^2*d^2*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(
b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(
b*x+a)^2-1/128*(2*sin(b*x+a)^2+3)^2)+4/b^3*c*d^3*(1/4*(b*x+a)^3*sin(b*x+a)^
4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*
a)-3/32*(b*x+a)*sin(b*x+a)^4-3/128*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)
-27/256*b*x-27/256*a+9/32*(b*x+a)*cos(b*x+a)^2-9/64*cos(b*x+a)*sin(b*x+a)+3
/16*(b*x+a)^3)+1/b^4*d^4*(1/4*(b*x+a)^4*sin(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b
*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^2*sin(b*x+a)
^4+3/8*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a
)+27/128*(b*x+a)^2+3/512*(2*sin(b*x+a)^2+3)^2+9/16*(b*x+a)^2*cos(b*x+a)^2-9
/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+9/32*sin(b*x+a)^2+9/32
*(b*x+a)^4))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 967 vs. $2(236) = 472$.
time = 0.30, size = 967, normalized size = 3.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/1024*(256*c^4*sin(b*x + a)^4 - 1024*a*c^3*d*sin(b*x + a)^4/b + 1536*a^2*c
^2*d^2*sin(b*x + a)^4/b^2 - 1024*a^3*c*d^3*sin(b*x + a)^4/b^3 + 256*a^4*d^4
*sin(b*x + a)^4/b^4 + 32*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2
*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*c^3*d/b - 96*(4*(b*x +
 a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8
*sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 96*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b
*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a^2*c*d^3
/b^3 - 32*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - s
in(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a^3*d^4/b^4 + 24*((8*(b*x + a)^2 - 1)
*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*s
in(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 - 48*((8*(b*x
+ a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*
(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/b^3 + 2
4*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x
+ 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*a^2*
d^4/b^4 + 4*(4*(8*(b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 64*(2*(b*x
+ a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x +
4*a) + 96*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 - 4*(4*(8*(b*x +
a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos
(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) + 96*(2*(b*x + a)^2
- 1)*sin(2*b*x + 2*a))*a*d^4/b^4 + ((32*(b*x + a)^4 - 24*(b*x + a)^2 + 3)*c
os(4*b*x + 4*a) - 64*(2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*cos(2*b*x + 2*a) -
 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a) + 128*(2*(b*x + a)^3 - 3*
b*x - 3*a)*sin(2*b*x + 2*a))*d^4/b^4)/b
```

Fricas [A]
time = 2.65, size = 434, normalized size = 1.67

$$\frac{20\,b^4d^4x^4 + 80\,b^4cd^3x^3 + (32\,b^4d^4x^4 + 128\,b^4cd^3x^3 + 32\,b^4c^4 - 24\,b^2c^2d^2 + 3\,d^4 + 24\,(8\,b^4c^2d^2 - b^2d^4)x^2 + 16\,(8\,b^4c^3d - 3\,b^2cd^3)x)\cos(bx+a)^4 + 3\,(40\,b^4c^2d^2 - 17\,b^2d^4)x^2 - (64\,b^4d^4x^4 + 256\,b^4cd^3x^3 + 64\,b^4c^4 - 120\,b^2c^2d^2 + 51\,d^4 + 24\,(16\,b^4c^2d^2 - 5\,b^2d^4)x^2 + 16\,(16\,b^4c^3d - 15\,b^2cd^3)x)\cos(bx+a)^2 + 2\,(40\,b^4c^3d - 51\,b^2cd^3)x - 2\,(2\,(8\,b^3d^4x^3 + 24\,b^3cd^3x^2 + 8\,b^3c^3d - 3\,bcd^3 + 3\,(8\,b^3c^2d^2 - bd^4)x)\cos(bx+a)^3 - (40\,b^3d^4x^3 + 120\,b^3cd^3x^2 + 40\,b^3c^3d - 51\,bcd^3 + 3\,(40\,b^3c^2d^2 - 17\,bd^4)x)\cos(bx+a))\sin(bx+a)}{128\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(20*b^4*d^4*x^4 + 80*b^4*c*d^3*x^3 + (32*b^4*d^4*x^4 + 128*b^4*c*d^3*
x^3 + 32*b^4*c^4 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^
2 + 16*(8*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^4 + 3*(40*b^4*c^2*d^2 -
17*b^2*d^4)*x^2 - (64*b^4*d^4*x^4 + 256*b^4*c*d^3*x^3 + 64*b^4*c^4 - 120*b^
2*c^2*d^2 + 51*d^4 + 24*(16*b^4*c^2*d^2 - 5*b^2*d^4)*x^2 + 16*(16*b^4*c^3*d
 - 15*b^2*c*d^3)*x)*cos(b*x + a)^2 + 2*(40*b^4*c^3*d - 51*b^2*c*d^3)*x - 2*
(2*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^3*c
^2*d^2 - b*d^4)*x)*cos(b*x + a)^3 - (40*b^3*d^4*x^3 + 120*b^3*c*d^3*x^2 + 4
0*b^3*c^3*d - 51*b*c*d^3 + 3*(40*b^3*c^2*d^2 - 17*b*d^4)*x)*cos(b*x + a))*s
in(b*x + a))/b^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(262) = 524$.
time = 1.04, size = 935, normalized size = 3.60

$$\begin{cases} \frac{c^{4}\sin^{4}(a+bx)}{4b} + \frac{5c^{3}dx\sin^{4}(a+bx)}{8b} - \frac{3c^{3}dx\sin^{2}(a+bx)\cos^{2}(a+bx)}{4b} - \frac{3c^{3}dx\cos^{4}(a+bx)}{8b} + \frac{15c^{2}d^{2}x^{2}\sin^{4}(a+bx)}{16b} - \frac{9c^{2}d^{2}x^{2}\sin^{2}(a+bx)\cos^{2}(a+bx)}{8b} - \frac{9c^{2}d^{2}x^{2}\cos^{4}(a+bx)}{16b} + \frac{5cd^{3}x^{3}\sin^{4}(a+bx)}{8b} - \frac{3cd^{3}x^{3}\sin^{2}(a+bx)\cos^{2}(a+bx)}{4b} - \frac{3cd^{3}x^{3}\cos^{4}(a+bx)}{8b} + \frac{5d^{4}x^{4}\sin^{4}(a+bx)}{32b} - \frac{3d^{4}x^{4}\sin^{2}(a+bx)\cos^{2}(a+bx)}{16b} - \frac{3d^{4}x^{4}\cos^{4}(a+bx)}{32b} + \frac{5c^{3}d\sin^{3}(a+bx)\cos(a+bx)}{8b^{2}} + \frac{3c^{3}d\sin(a+bx)\cos^{3}(a+bx)}{8b^{2}} + \frac{15c^{2}d^{2}x\sin^{3}(a+bx)\cos(a+bx)}{8b^{2}} + \frac{9c^{2}d^{2}x\sin(a+bx)\cos^{3}(a+bx)}{8b^{2}} + \frac{15cd^{3}x^{2}\sin^{3}(a+bx)\cos(a+bx)}{8b^{2}} + \frac{9cd^{3}x^{2}\sin(a+bx)\cos^{3}(a+bx)}{8b^{2}} + \frac{5d^{4}x^{3}\sin^{3}(a+bx)\cos(a+bx)}{8b^{2}} + \frac{3d^{4}x^{3}\sin(a+bx)\cos^{3}(a+bx)}{8b^{2}} - \frac{15c^{2}d^{2}\sin^{4}(a+bx)}{32b^{3}} + \frac{9c^{2}d^{2}\cos^{4}(a+bx)}{32b^{3}} - \frac{51cd^{3}x\sin^{4}(a+bx)}{64b^{3}} + \frac{9cd^{3}x\sin^{2}(a+bx)\cos^{2}(a+bx)}{32b^{3}} + \frac{45cd^{3}x\cos^{4}(a+bx)}{64b^{3}} - \frac{51d^{4}x^{2}\sin^{4}(a+bx)}{128b^{3}} + \frac{9d^{4}x^{2}\sin^{2}(a+bx)\cos^{2}(a+bx)}{64b^{3}} + \frac{45d^{4}x^{2}\cos^{4}(a+bx)}{128b^{3}} - \frac{51cd^{3}\sin^{3}(a+bx)\cos(a+bx)}{64b^{4}} - \frac{45cd^{3}\sin(a+bx)\cos^{3}(a+bx)}{64b^{4}} - \frac{51d^{4}x\sin^{3}(a+bx)\cos(a+bx)}{64b^{4}} - \frac{45d^{4}x\sin(a+bx)\cos^{3}(a+bx)}{64b^{4}} + \frac{51d^{4}\sin^{4}(a+bx)}{256b^{5}} - \frac{45d^{4}\cos^{4}(a+bx)}{256b^{5}} & \text{for}\: b \neq 0 \\ \left(c^{4}x + 2c^{3}dx^{2} + 2c^{2}d^{2}x^{3} + cd^{3}x^{4} + \frac{d^{4}x^{5}}{5}\right)\sin^{3}(a)\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c**4*sin(a + b*x)**4/(4*b) + 5*c**3*d*x*sin(a + b*x)**4/(8*b) -
3*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(4*b) - 3*c**3*d*x*cos(a + b*x)*
*4/(8*b) + 15*c**2*d**2*x**2*sin(a + b*x)**4/(16*b) - 9*c**2*d**2*x**2*sin(
a + b*x)**2*cos(a + b*x)**2/(8*b) - 9*c**2*d**2*x**2*cos(a + b*x)**4/(16*b)
 + 5*c*d**3*x**3*sin(a + b*x)**4/(8*b) - 3*c*d**3*x**3*sin(a + b*x)**2*cos(
a + b*x)**2/(4*b) - 3*c*d**3*x**3*cos(a + b*x)**4/(8*b) + 5*d**4*x**4*sin(a
 + b*x)**4/(32*b) - 3*d**4*x**4*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 3*
d**4*x**4*cos(a + b*x)**4/(32*b) + 5*c**3*d*sin(a + b*x)**3*cos(a + b*x)/(8
*b**2) + 3*c**3*d*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 15*c**2*d**2*x*si
n(a + b*x)**3*cos(a + b*x)/(8*b**2) + 9*c**2*d**2*x*sin(a + b*x)*cos(a + b*
x)**3/(8*b**2) + 15*c*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 9*c
*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 5*d**4*x**3*sin(a + b*x)
**3*cos(a + b*x)/(8*b**2) + 3*d**4*x**3*sin(a + b*x)*cos(a + b*x)**3/(8*b**
2) - 15*c**2*d**2*sin(a + b*x)**4/(32*b**3) + 9*c**2*d**2*cos(a + b*x)**4/(
32*b**3) - 51*c*d**3*x*sin(a + b*x)**4/(64*b**3) + 9*c*d**3*x*sin(a + b*x)*
*2*cos(a + b*x)**2/(32*b**3) + 45*c*d**3*x*cos(a + b*x)**4/(64*b**3) - 51*d
**4*x**2*sin(a + b*x)**4/(128*b**3) + 9*d**4*x**2*sin(a + b*x)**2*cos(a + b
*x)**2/(64*b**3) + 45*d**4*x**2*cos(a + b*x)**4/(128*b**3) - 51*c*d**3*sin(
a + b*x)**3*cos(a + b*x)/(64*b**4) - 45*c*d**3*sin(a + b*x)*cos(a + b*x)**3
/(64*b**4) - 51*d**4*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**4) - 45*d**4*x*s
in(a + b*x)*cos(a + b*x)**3/(64*b**4) + 51*d**4*sin(a + b*x)**4/(256*b**5)
- 45*d**4*cos(a + b*x)**4/(256*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 +
 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**3*cos(a), True))
```

Giac [A]
time = 0.47, size = 361, normalized size = 1.39

$$\frac{(32\,b^4d^4x^4 + 128\,b^4cd^3x^3 + 192\,b^4c^2d^2x^2 + 128\,b^4c^3dx + 32\,b^4c^4 - 24\,b^2d^4x^2 - 48\,b^2cd^3x - 24\,b^2c^2d^2 + 3\,d^4)\cos(4\,bx + 4\,a)}{1024\,b^5} - \frac{(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 12\,b^4c^2d^2x^2 + 8\,b^4c^3dx + 2\,b^4c^4 - 6\,b^2d^4x^2 - 12\,b^2cd^3x - 6\,b^2c^2d^2 + 3\,d^4)\cos(2\,bx + 2\,a)}{16\,b^5} - \frac{(8\,b^3d^4x^3 + 24\,b^3cd^3x^2 + 24\,b^3c^2d^2x + 8\,b^3c^3d - 3\,bd^4x - 3\,bcd^3)\sin(4\,bx + 4\,a)}{256\,b^5} + \frac{(2\,b^3d^4x^3 + 6\,b^3cd^3x^2 + 6\,b^3c^2d^2x + 2\,b^3c^3d - 3\,bd^4x - 3\,bcd^3)\sin(2\,bx + 2\,a)}{8\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/1024*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^4*
c^3*d*x + 32*b^4*c^4 - 24*b^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2 + 3
*d^4)*cos(4*b*x + 4*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4
*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x -
```

```
 6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 - 1/256*(8*b^3*d^4*x^3 + 24*b^
3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(4
*b*x + 4*a)/b^5 + 1/8*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x +
2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

Mupad [B]
time = 1.94, size = 576, normalized size = 2.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^4,x)
```

```
[Out] -(192*d^4*cos(2*a + 2*b*x) - 3*d^4*cos(4*a + 4*b*x) + 128*b^4*c^4*cos(2*a +
 2*b*x) - 32*b^4*c^4*cos(4*a + 4*b*x) - 256*b^3*c^3*d*sin(2*a + 2*b*x) + 32
*b^3*c^3*d*sin(4*a + 4*b*x) - 384*b^2*c^2*d^2*cos(2*a + 2*b*x) + 24*b^2*c^2
*d^2*cos(4*a + 4*b*x) - 384*b^2*d^4*x^2*cos(2*a + 2*b*x) + 24*b^2*d^4*x^2*c
os(4*a + 4*b*x) + 128*b^4*d^4*x^4*cos(2*a + 2*b*x) - 32*b^4*d^4*x^4*cos(4*a
 + 4*b*x) - 256*b^3*d^4*x^3*sin(2*a + 2*b*x) + 32*b^3*d^4*x^3*sin(4*a + 4*b
*x) + 384*b*c*d^3*sin(2*a + 2*b*x) - 12*b*c*d^3*sin(4*a + 4*b*x) + 384*b*d^
4*x*sin(2*a + 2*b*x) - 12*b*d^4*x*sin(4*a + 4*b*x) + 768*b^4*c^2*d^2*x^2*co
s(2*a + 2*b*x) - 192*b^4*c^2*d^2*x^2*cos(4*a + 4*b*x) - 768*b^2*c*d^3*x*cos
(2*a + 2*b*x) + 512*b^4*c^3*d*x*cos(2*a + 2*b*x) + 48*b^2*c*d^3*x*cos(4*a +
 4*b*x) - 128*b^4*c^3*d*x*cos(4*a + 4*b*x) + 512*b^4*c*d^3*x^3*cos(2*a + 2*
b*x) - 128*b^4*c*d^3*x^3*cos(4*a + 4*b*x) - 768*b^3*c^2*d^2*x*sin(2*a + 2*b
*x) - 768*b^3*c*d^3*x^2*sin(2*a + 2*b*x) + 96*b^3*c^2*d^2*x*sin(4*a + 4*b*x
) + 96*b^3*c*d^3*x^2*sin(4*a + 4*b*x))/(1024*b^5)
```

3.24 $\int (c+dx)^3 \cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=196

$$\frac{45d^3x}{256b^3}-\frac{3(c+dx)^3}{32b}-\frac{45d^3\cos(a+bx)\sin(a+bx)}{256b^4}+\frac{9d(c+dx)^2\cos(a+bx)\sin(a+bx)}{32b^2}-\frac{9d^2(c+dx)\sin^2(a}{32b^3}$$

```
[Out] 45/256*d^3*x/b^3-3/32*(d*x+c)^3/b-45/256*d^3*cos(b*x+a)*sin(b*x+a)/b^4+9/32
*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^2-9/32*d^2*(d*x+c)*sin(b*x+a)^2/b^3-3/
128*d^3*cos(b*x+a)*sin(b*x+a)^3/b^4+3/16*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)^
3/b^2-3/32*d^2*(d*x+c)*sin(b*x+a)^4/b^3+1/4*(d*x+c)^3*sin(b*x+a)^4/b
```

Rubi [A]
time = 0.13, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4489, 3392, 32, 2715, 8}

$$-\frac{3d^3\sin^3(a+bx)\cos(a+bx)}{128b^4}-\frac{45d^3\sin(a+bx)\cos(a+bx)}{256b^4}-\frac{3d^2(c+dx)\sin^4(a+bx)}{32b^3}-\frac{9d^2(c+dx)\sin^2(a+bx)}{32b^3}+\frac{3d(c+dx)^2\sin^3(a+bx)\cos(a+bx)}{16b^2}+\frac{9d(c+dx)^2\sin(a+bx)\cos(a+bx)}{32b^2}+\frac{(c+dx)^3\sin^4(a+bx)}{4b}+\frac{45d^3x}{256b^3}-\frac{3(c+dx)^3}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (45*d^3*x)/(256*b^3) - (3*(c + d*x)^3)/(32*b) - (45*d^3*Cos[a + b*x]*Sin[a
+ b*x])/(256*b^4) + (9*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(32*b^2) -
(9*d^2*(c + d*x)*Sin[a + b*x]^2)/(32*b^3) - (3*d^3*Cos[a + b*x]*Sin[a + b*x
]^3)/(128*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^3)/(16*b^2) - (
3*d^2*(c + d*x)*Sin[a + b*x]^4)/(32*b^3) + ((c + d*x)^3*Sin[a + b*x]^4)/(4*
b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\cos(a+bx)\sin^3(a+bx)\,dx &= \frac{(c+dx)^3\sin^4(a+bx)}{4b}-\frac{(3d)\int (c+dx)^2\sin^4(a+bx)\,dx}{4b}\\ &= \frac{3d(c+dx)^2\cos(a+bx)\sin^3(a+bx)}{16b^2}-\frac{3d^2(c+dx)\sin^4(a+bx)}{32b^3}+\\ &= \frac{9d(c+dx)^2\cos(a+bx)\sin(a+bx)}{32b^2}-\frac{9d^2(c+dx)\sin^2(a+bx)}{32b^3}-\\ &= -\frac{3(c+dx)^3}{32b}-\frac{45d^3\cos(a+bx)\sin(a+bx)}{256b^4}+\frac{9d(c+dx)^2\cos(a+}{32b^2}\\ &= \frac{45d^3x}{256b^3}-\frac{3(c+dx)^3}{32b}-\frac{45d^3\cos(a+bx)\sin(a+bx)}{256b^4}+\frac{9d(c+dx)^2}{}\end{aligned}$$

Mathematica [A]
time = 0.87, size = 135, normalized size = 0.69

$$\frac{-64b(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\cos(2(a+bx))+4b(c+dx)\left(-3d^2+8b^2(c+dx)^2\right)\cos(4(a+bx))-6d(-16(-d^2+2b^2(c+dx)^2)+(-d^2+8b^2(c+dx)^2)\cos(2(a+bx)))\sin(2(a+bx))}{1024b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-64*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 4*b*(c + d
*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] - 6*d*(-16*(-d^2 + 2*b^2*
(c + d*x)^2) + (-d^2 + 8*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)
])/(1024*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(178) = 356$.
time = 0.18, size = 586, normalized size = 2.99 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4/b^3*a^3*d^3*sin(b*x+a)^4+3/4/b^2*a^2*c*d^2*sin(b*x+a)^4+3/b^3*a^2
*d^3*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a
)-3/32*b*x-3/32*a)-3/4/b*a*c^2*d*sin(b*x+a)^4-6/b^2*a*c*d^2*(1/4*(b*x+a)*si
n(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)-3
/b^3*a*d^3*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*
sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/128*(2*sin(b*x+a)^2+
3)^2)+1/4*c^3*sin(b*x+a)^4+3/b*c^2*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*
x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)+3/b^2*c*d^2*(1/4*(b*x+a)
^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+
3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/128*(2*sin(b*x+a)^2+3)^2)+1/b^3*d^3*(1/4*(b
*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(
b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)*sin(b*x+a)^4-3/128*(sin(b*x+a)^3+3/2*sin
(b*x+a))*cos(b*x+a)-27/256*b*x-27/256*a+9/32*(b*x+a)*cos(b*x+a)^2-9/64*cos(
b*x+a)*sin(b*x+a)+3/16*(b*x+a)^3))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(178) = 356$.
time = 0.29, size = 549, normalized size = 2.80

$$\frac{256\,c^3\sin\left(bx+a\right)^4-\frac{768\,ac^2d\sin(bx+a)^4}{b}+\frac{768\,a^2cd^2\sin(bx+a)^4}{b^2}-\frac{256\,a^3d^3\sin(bx+a)^4}{b^3}+\frac{24\,(4\,(bx+a)\cos(4\,bx+4\,a)-16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)+8\,\sin(2\,bx+2\,a))c^2d}{b}-\frac{48\,(4\,(bx+a)\cos(4\,bx+4\,a)-16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)+8\,\sin(2\,bx+2\,a))acd^2}{b^2}+\frac{24\,(4\,(bx+a)\cos(4\,bx+4\,a)-16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)+8\,\sin(2\,bx+2\,a))a^2d^3}{b^3}+\frac{12\,\left(\left(8\,(bx+a)^2-1\right)\cos(4\,bx+4\,a)-16\,\left(2\,(bx+a)^2-1\right)\cos(2\,bx+2\,a)-4\,(bx+a)\sin(4\,bx+4\,a)+32\,(bx+a)\sin(2\,bx+2\,a)\right)cd^2}{b^2}-\frac{12\,\left(\left(8\,(bx+a)^2-1\right)\cos(4\,bx+4\,a)-16\,\left(2\,(bx+a)^2-1\right)\cos(2\,bx+2\,a)-4\,(bx+a)\sin(4\,bx+4\,a)+32\,(bx+a)\sin(2\,bx+2\,a)\right)ad^3}{b^3}+\frac{\left(4\,\left(8\,(bx+a)^3-3\,bx-3\,a\right)\cos(4\,bx+4\,a)-64\,\left(2\,(bx+a)^3-3\,bx-3\,a\right)\cos(2\,bx+2\,a)-3\,\left(8\,(bx+a)^2-1\right)\sin(4\,bx+4\,a)+96\,\left(2\,(bx+a)^2-1\right)\sin(2\,bx+2\,a)\right)d^3}{b^3}}{1024\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/1024*(256*c^3*sin(b*x + a)^4 - 768*a*c^2*d*sin(b*x + a)^4/b + 768*a^2*c*d
^2*sin(b*x + a)^4/b^2 - 256*a^3*d^3*sin(b*x + a)^4/b^3 + 24*(4*(b*x + a)*co
s(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2
*b*x + 2*a))*c^2*d/b - 48*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(
2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a*c*d^2/b^2 + 24*(4*(
b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a
) + 8*sin(2*b*x + 2*a))*a^2*d^3/b^3 + 12*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4
*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a
) + 32*(b*x + a)*sin(2*b*x + 2*a))*c*d^2/b^2 - 12*((8*(b*x + a)^2 - 1)*cos(
4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*
b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*a*d^3/b^3 + (4*(8*(b*x + a)^3 -
 3*b*x - 3*a)*cos(4*b*x + 4*a) - 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x
 + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) + 96*(2*(b*x + a)^2 - 1)*s
in(2*b*x + 2*a))*d^3/b^3)/b
```

Fricas [A]
time = 2.43, size = 283, normalized size = 1.44

$$\frac{40\,b^3d^3x^3+120\,b^3cd^2x^2+8\,(8\,b^3d^3x^3+24\,b^3cd^2x^2+8\,b^3c^3-3\,bcd^2+3\,(8\,b^3c^2d-bd^3)x)\cos\left(bx+a\right)^4-8\,(16\,b^3d^3x^3+48\,b^3cd^2x^2+16\,b^3c^3-15\,bcd^2+3\,(16\,b^3c^2d-5\,bd^3)x)\cos\left(bx+a\right)^2+3\,(40\,b^3c^2d-17\,bd^3)x-3\,\left(2\,(8\,b^2d^3x^2+16\,b^2cd^2x+8\,b^2c^2d-d^3)\cos\left(bx+a\right)^3-(40\,b^2d^3x^2+80\,b^2cd^2x+40\,b^2c^2d-17\,d^3)\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{256\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(40*b^3*d^3*x^3 + 120*b^3*c*d^2*x^2 + 8*(8*b^3*d^3*x^3 + 24*b^3*c*d^2
*x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^4 -
8*(16*b^3*d^3*x^3 + 48*b^3*c*d^2*x^2 + 16*b^3*c^3 - 15*b*c*d^2 + 3*(16*b^3*
c^2*d - 5*b*d^3)*x)*cos(b*x + a)^2 + 3*(40*b^3*c^2*d - 17*b*d^3)*x - 3*(2*(
8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3 - (40*b^
2*d^3*x^2 + 80*b^2*c*d^2*x + 40*b^2*c^2*d - 17*d^3)*cos(b*x + a))*sin(b*x +
 a))/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(197) = 394$.
time = 0.70, size = 602, normalized size = 3.07

$$\begin{cases} \frac{c^3\sin^4(a+bx)}{4b} + \frac{15c^2dx\sin^4(a+bx)}{32b} - \frac{9c^2dx\sin^2(a+bx)\cos^2(a+bx)}{16b} - \frac{9c^2dx\cos^4(a+bx)}{32b} + \frac{15cd^2x^2\sin^4(a+bx)}{32b} - \frac{9cd^2x^2\sin^2(a+bx)\cos^2(a+bx)}{16b} - \frac{9cd^2x^2\cos^4(a+bx)}{32b} + \frac{5d^3x^3\sin^4(a+bx)}{32b} - \frac{3d^3x^3\sin^2(a+bx)\cos^2(a+bx)}{16b} - \frac{3d^3x^3\cos^4(a+bx)}{32b} + \frac{15c^2d\sin^3(a+bx)\cos(a+bx)}{32b^2} + \frac{9c^2d\sin(a+bx)\cos^3(a+bx)}{32b^2} + \frac{15cd^2x\sin^3(a+bx)\cos(a+bx)}{16b^2} + \frac{9cd^2x\sin(a+bx)\cos^3(a+bx)}{16b^2} + \frac{15d^3x^2\sin^3(a+bx)\cos(a+bx)}{32b^2} + \frac{9d^3x^2\sin(a+bx)\cos^3(a+bx)}{32b^2} - \frac{15cd^2\sin^4(a+bx)}{64b^3} + \frac{9cd^2\cos^4(a+bx)}{64b^3} - \frac{51d^3x\sin^4(a+bx)}{256b^3} + \frac{9d^3x\sin^2(a+bx)\cos^2(a+bx)}{128b^3} + \frac{45d^3x\cos^4(a+bx)}{256b^3} - \frac{51d^3\sin^3(a+bx)\cos(a+bx)}{256b^4} - \frac{45d^3\sin(a+bx)\cos^3(a+bx)}{256b^4} & \text{for}\: b \neq 0 \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin^3(a)\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c**3*sin(a + b*x)**4/(4*b) + 15*c**2*d*x*sin(a + b*x)**4/(32*b)
- 9*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 9*c**2*d*x*cos(a + b*
x)**4/(32*b) + 15*c*d**2*x**2*sin(a + b*x)**4/(32*b) - 9*c*d**2*x**2*sin(a
+ b*x)**2*cos(a + b*x)**2/(16*b) - 9*c*d**2*x**2*cos(a + b*x)**4/(32*b) + 5
*d**3*x**3*sin(a + b*x)**4/(32*b) - 3*d**3*x**3*sin(a + b*x)**2*cos(a + b*x
)**2/(16*b) - 3*d**3*x**3*cos(a + b*x)**4/(32*b) + 15*c**2*d*sin(a + b*x)**
3*cos(a + b*x)/(32*b**2) + 9*c**2*d*sin(a + b*x)*cos(a + b*x)**3/(32*b**2)
+ 15*c*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 9*c*d**2*x*sin(a + b
*x)*cos(a + b*x)**3/(16*b**2) + 15*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(
32*b**2) + 9*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(32*b**2) - 15*c*d**2*s
in(a + b*x)**4/(64*b**3) + 9*c*d**2*cos(a + b*x)**4/(64*b**3) - 51*d**3*x*s
in(a + b*x)**4/(256*b**3) + 9*d**3*x*sin(a + b*x)**2*cos(a + b*x)**2/(128*b
**3) + 45*d**3*x*cos(a + b*x)**4/(256*b**3) - 51*d**3*sin(a + b*x)**3*cos(a
 + b*x)/(256*b**4) - 45*d**3*sin(a + b*x)*cos(a + b*x)**3/(256*b**4), Ne(b,
 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**3*cos
(a), True))
```

Giac [A]
time = 0.46, size = 241, normalized size = 1.23

$$\frac{(8b^3d^3x^3 + 24b^3cd^2x^2 + 24b^3c^2dx + 8b^3c^3 - 3bd^3x - 3bcd^2)\cos(4bx + 4a)}{256b^4} - \frac{(2b^3d^3x^3 + 6b^3cd^2x^2 + 6b^3c^2dx + 2b^3c^3 - 3bd^3x - 3bcd^2)\cos(2bx + 2a)}{16b^4} - \frac{3(8b^2d^3x^2 + 16b^2cd^2x + 8b^2c^2d - d^3)\sin(4bx + 4a)}{1024b^4} + \frac{3(2b^2d^3x^2 + 4b^2cd^2x + 2b^2c^2d - d^3)\sin(2bx + 2a)}{32b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

[Out] 1/256*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(4*b*x + 4*a)/b^4 - 1/16*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)/b^4 - 3/1024*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*sin(4*b*x + 4*a)/b^4 + 3/32*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin(2*b*x + 2*a)/b^4

Mupad [B]
time = 1.71, size = 366, normalized size = 1.87

$$-\frac{24\,d^3\,\sin\left(2\,a+2\,b\,x\right)-\frac{3\,d^3\,\sin\left(4\,a+4\,b\,x\right)}{4}+32\,b^3\,c^3\,\cos\left(2\,a+2\,b\,x\right)-8\,b^3\,c^3\,\cos\left(4\,a+4\,b\,x\right)-48\,b^2\,c^2\,d\,\sin\left(2\,a+2\,b\,x\right)+6\,b^2\,c^2\,d\,\sin\left(4\,a+4\,b\,x\right)+32\,b^3\,d^3\,x^3\,\cos\left(2\,a+2\,b\,x\right)-8\,b^3\,d^3\,x^3\,\cos\left(4\,a+4\,b\,x\right)-48\,b^2\,d^3\,x^2\,\sin\left(2\,a+2\,b\,x\right)+6\,b^2\,d^3\,x^2\,\sin\left(4\,a+4\,b\,x\right)-48\,b\,c\,d^2\,\cos\left(2\,a+2\,b\,x\right)+3\,b\,c\,d^2\,\cos\left(4\,a+4\,b\,x\right)-48\,b\,d^3\,x\,\cos\left(2\,a+2\,b\,x\right)+3\,b\,d^3\,x\,\cos\left(4\,a+4\,b\,x\right)+96\,b^3\,c^2\,d\,x\,\cos\left(2\,a+2\,b\,x\right)-24\,b^3\,c^2\,d\,x\,\cos\left(4\,a+4\,b\,x\right)-96\,b^2\,c\,d^2\,x\,\sin\left(2\,a+2\,b\,x\right)+12\,b^2\,c\,d^2\,x\,\sin\left(4\,a+4\,b\,x\right)+96\,b^3\,c\,d^2\,x^2\,\cos\left(2\,a+2\,b\,x\right)-24\,b^3\,c\,d^2\,x^2\,\cos\left(4\,a+4\,b\,x\right)}{256\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^3,x)

[Out] -(24*d^3*sin(2*a + 2*b*x) - (3*d^3*sin(4*a + 4*b*x))/4 + 32*b^3*c^3*cos(2*a + 2*b*x) - 8*b^3*c^3*cos(4*a + 4*b*x) - 48*b^2*c^2*d*sin(2*a + 2*b*x) + 6*b^2*c^2*d*sin(4*a + 4*b*x) + 32*b^3*d^3*x^3*cos(2*a + 2*b*x) - 8*b^3*d^3*x^3*cos(4*a + 4*b*x) - 48*b^2*d^3*x^2*sin(2*a + 2*b*x) + 6*b^2*d^3*x^2*sin(4*a + 4*b*x) - 48*b*c*d^2*cos(2*a + 2*b*x) + 3*b*c*d^2*cos(4*a + 4*b*x) - 48*b*d^3*x*cos(2*a + 2*b*x) + 3*b*d^3*x*cos(4*a + 4*b*x) + 96*b^3*c^2*d*x*cos(2*a + 2*b*x) - 24*b^3*c^2*d*x*cos(4*a + 4*b*x) - 96*b^2*c*d^2*x*sin(2*a + 2*b*x) + 12*b^2*c*d^2*x*sin(4*a + 4*b*x) + 96*b^3*c*d^2*x^2*cos(2*a + 2*b*x) - 24*b^3*c*d^2*x^2*cos(4*a + 4*b*x))/(256*b^4)

3.25 $\int (c+dx)^2\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=134

$$-\frac{3cdx}{16b}-\frac{3d^2x^2}{32b}+\frac{3d(c+dx)\cos(a+bx)\sin(a+bx)}{16b^2}-\frac{3d^2\sin^2(a+bx)}{32b^3}+\frac{d(c+dx)\cos(a+bx)\sin^3(a+bx)}{8b^2}-$$

```
[Out] -3/16*c*d*x/b-3/32*d^2*x^2/b+3/16*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2-3/32*
d^2*sin(b*x+a)^2/b^3+1/8*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)^3/b^2-1/32*d^2*sin
(b*x+a)^4/b^3+1/4*(d*x+c)^2*sin(b*x+a)^4/b
```

Rubi [A]
time = 0.07, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4489, 3391}

$$-\frac{d^2\sin^4(a+bx)}{32b^3}-\frac{3d^2\sin^2(a+bx)}{32b^3}+\frac{d(c+dx)\sin^3(a+bx)\cos(a+bx)}{8b^2}+\frac{3d(c+dx)\sin(a+bx)\cos(a+bx)}{16b^2}+\frac{(c+dx)^2\sin^4(a+bx)}{4b}-\frac{3cdx}{16b}-\frac{3d^2x^2}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-3*c*d*x)/(16*b) - (3*d^2*x^2)/(32*b) + (3*d*(c + d*x)*Cos[a + b*x]*Sin[a
+ b*x])/(16*b^2) - (3*d^2*Sin[a + b*x]^2)/(32*b^3) + (d*(c + d*x)*Cos[a + b
*x]*Sin[a + b*x]^3)/(8*b^2) - (d^2*Sin[a + b*x]^4)/(32*b^3) + ((c + d*x)^2*
Sin[a + b*x]^4)/(4*b)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^2\cos(a+bx)\sin^3(a+bx)\,dx = \frac{(c+dx)^2\sin^4(a+bx)}{4b} - \frac{d\int (c+dx)\sin^4(a+bx)\,dx}{2b}$$
$$= \frac{d(c+dx)\cos(a+bx)\sin^3(a+bx)}{8b^2} - \frac{d^2\sin^4(a+bx)}{32b^3} + \frac{(c+dx)^2}{}$$
$$= \frac{3d(c+dx)\cos(a+bx)\sin(a+bx)}{16b^2} - \frac{3d^2\sin^2(a+bx)}{32b^3} + \frac{d(c+dx}{}$$
$$= -\frac{3cdx}{16b} - \frac{3d^2x^2}{32b} + \frac{3d(c+dx)\cos(a+bx)\sin(a+bx)}{16b^2} - \frac{3d^2\sin^2(}{32}$$

Mathematica [A]
time = 0.49, size = 91, normalized size = 0.68

$$\frac{-16(-d^2+2b^2(c+dx)^2)\cos(2(a+bx))+(-d^2+8b^2(c+dx)^2)\cos(4(a+bx))-4bd(c+dx)(-8\sin(2(a+bx))+\sin(4(a+bx)))}{256b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-16*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (-d^2 + 8*b^2*(c + d*x)^
2)*Cos[4*(a + b*x)] - 4*b*d*(c + d*x)*(-8*Sin[2*(a + b*x)] + Sin[4*(a + b*x
)]))/(256*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(120) = 240.
time = 0.19, size = 256, normalized size = 1.91

method	result
risch	$\frac{(8x^2d^2b^2+16b^2cdx+8b^2c^2-d^2)\cos(4bx+4a)}{256b^3} - \frac{d(dx+c)\sin(4bx+4a)}{64b^2} - \frac{(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2)\cos(2bx+2a)}{16b^3}$
derivativedivides	$\frac{a^2d^2(\sin^4(bx+a))}{4b^2} - \frac{acd(\sin^4(bx+a))}{2b} - \frac{2a\,d^2\left(\frac{(bx+a)(\sin^4(bx+a))}{4} + \frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{16} - \frac{3bx}{32} - \frac{3a}{32}\right)}{b^2} + \frac{c^2(\mathrm{si}}{}$
default	$\frac{a^2d^2(\sin^4(bx+a))}{4b^2} - \frac{acd(\sin^4(bx+a))}{2b} - \frac{2a\,d^2\left(\frac{(bx+a)(\sin^4(bx+a))}{4} + \frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{16} - \frac{3bx}{32} - \frac{3a}{32}\right)}{b^2} + \frac{c^2(\mathrm{si}}{}$
norman	$-\frac{3d^2x^2}{32b} - \frac{3d^2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b^3} - \frac{3d^2\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b^3} + \frac{(16b^2c^2-5d^2)\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b^3} + \frac{3cd\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b^2} + \frac{11cd\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4/b^2*a^2*d^2*sin(b*x+a)^4-1/2/b*a*c*d*sin(b*x+a)^4-2/b^2*a*d^2*(1/4
*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*
x-3/32*a)+1/4*c^2*sin(b*x+a)^4+2/b*c*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(
b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)+1/b^2*d^2*(1/4*(b*x+a)
^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+
3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/128*(2*sin(b*x+a)^2+3)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(120) = 240$.
time = 0.28, size = 263, normalized size = 1.96

$$\frac{64\,c^2\sin\left(bx+a\right)^4-\frac{128\,acd\sin(bx+a)^4}{b}+\frac{64\,a^2d^2\sin(bx+a)^4}{b^2}+\frac{4\,(4\,(bx+a)\cos(4\,bx+4\,a)-16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)+8\,\sin(2\,bx+2\,a))cd}{b}-\frac{4\,(4\,(bx+a)\cos(4\,bx+4\,a)-16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)+8\,\sin(2\,bx+2\,a))ad^2}{b^2}+\frac{\left(\left(8\,(bx+a)^2-1\right)\cos(4\,bx+4\,a)-16\,\left(2\,(bx+a)^2-1\right)\cos(2\,bx+2\,a)-4\,(bx+a)\sin(4\,bx+4\,a)+32\,(bx+a)\sin(2\,bx+2\,a)\right)d^2}{b^2}}{256\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/256*(64*c^2*sin(b*x + a)^4 - 128*a*c*d*sin(b*x + a)^4/b + 64*a^2*d^2*sin(
b*x + a)^4/b^2 + 4*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x +
 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*c*d/b - 4*(4*(b*x + a)*cos(4
*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*
x + 2*a))*a*d^2/b^2 + ((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x +
a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*si
n(2*b*x + 2*a))*d^2/b^2)/b
```

Fricas [A]
time = 2.59, size = 159, normalized size = 1.19

$$\frac{5\,b^2d^2x^2+10\,b^2cdx+\left(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2\right)\cos\left(bx+a\right)^4-\left(16\,b^2d^2x^2+32\,b^2cdx+16\,b^2c^2-5\,d^2\right)\cos\left(bx+a\right)^2-2\,\left(2\,\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3-5\,\left(bd^2x+bcd\right)\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{32\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(5*b^2*d^2*x^2 + 10*b^2*c*d*x + (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*
c^2 - d^2)*cos(b*x + a)^4 - (16*b^2*d^2*x^2 + 32*b^2*c*d*x + 16*b^2*c^2 - 5
*d^2)*cos(b*x + a)^2 - 2*(2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 5*(b*d^2*x +
 b*c*d)*cos(b*x + a))*sin(b*x + a))/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(129) = 258$.
time = 0.45, size = 320, normalized size = 2.39

$$\begin{cases}\frac{c^2\sin^4\left(a+bx\right)}{4b}+\frac{5cdx\sin^4\left(a+bx\right)}{16b}-\frac{3cdx\sin^2\left(a+bx\right)\cos^2\left(a+bx\right)}{8b}-\frac{3cdx\cos^4\left(a+bx\right)}{16b}+\frac{5d^2x^2\sin^4\left(a+bx\right)}{32b}-\frac{3d^2x^2\sin^2\left(a+bx\right)\cos^2\left(a+bx\right)}{16b}-\frac{3d^2x^2\cos^4\left(a+bx\right)}{32b}+\frac{5cd\sin^3\left(a+bx\right)\cos\left(a+bx\right)}{16b^2}+\frac{3cd\sin\left(a+bx\right)\cos^3\left(a+bx\right)}{16b^2}+\frac{5d^2x\sin^3\left(a+bx\right)\cos\left(a+bx\right)}{16b^2}+\frac{3d^2x\sin\left(a+bx\right)\cos^3\left(a+bx\right)}{16b^2}-\frac{5d^2\sin^4\left(a+bx\right)}{64b^3}+\frac{3d^2\cos^4\left(a+bx\right)}{64b^3} & \text{for}\: b\neq 0\\\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin^3\left(a\right)\cos\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c**2*sin(a + b*x)**4/(4*b) + 5*c*d*x*sin(a + b*x)**4/(16*b) - 3*
c*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(8*b) - 3*c*d*x*cos(a + b*x)**4/(16*b
) + 5*d**2*x**2*sin(a + b*x)**4/(32*b) - 3*d**2*x**2*sin(a + b*x)**2*cos(a
+ b*x)**2/(16*b) - 3*d**2*x**2*cos(a + b*x)**4/(32*b) + 5*c*d*sin(a + b*x)*
*3*cos(a + b*x)/(16*b**2) + 3*c*d*sin(a + b*x)*cos(a + b*x)**3/(16*b**2) +
5*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 3*d**2*x*sin(a + b*x)*cos
(a + b*x)**3/(16*b**2) - 5*d**2*sin(a + b*x)**4/(64*b**3) + 3*d**2*cos(a +
b*x)**4/(64*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**3*
cos(a), True))
```

Giac [A]
time = 0.44, size = 145, normalized size = 1.08

$$\frac{(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2)\cos(4\,bx+4\,a)}{256\,b^3}-\frac{(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\cos(2\,bx+2\,a)}{16\,b^3}-\frac{(bd^2x+bcd)\sin(4\,bx+4\,a)}{64\,b^3}+\frac{(bd^2x+bcd)\sin(2\,bx+2\,a)}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/256*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(4*b*x + 4*a)/b^3
 - 1/16*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a)/b^
3 - 1/64*(b*d^2*x + b*c*d)*sin(4*b*x + 4*a)/b^3 + 1/8*(b*d^2*x + b*c*d)*sin
(2*b*x + 2*a)/b^3
```

Mupad [B]
time = 1.25, size = 202, normalized size = 1.51

$$\frac{8\,d^2\cos(2\,a+2\,b\,x)-\frac{d^2\cos(4\,a+4\,b\,x)}{2}-16\,b^2\,c^2\cos(2\,a+2\,b\,x)+4\,b^2\,c^2\cos(4\,a+4\,b\,x)+16\,b\,c\,d\sin(2\,a+2\,b\,x)-2\,b\,c\,d\sin(4\,a+4\,b\,x)-16\,b^2\,d^2\,x^2\cos(2\,a+2\,b\,x)+4\,b^2\,d^2\,x^2\cos(4\,a+4\,b\,x)+16\,b\,d^2\,x\sin(2\,a+2\,b\,x)-2\,b\,d^2\,x\sin(4\,a+4\,b\,x)-32\,b^2\,c\,d\,x\cos(2\,a+2\,b\,x)+8\,b^2\,c\,d\,x\cos(4\,a+4\,b\,x)}{128\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] (8*d^2*cos(2*a + 2*b*x) - (d^2*cos(4*a + 4*b*x))/2 - 16*b^2*c^2*cos(2*a + 2
*b*x) + 4*b^2*c^2*cos(4*a + 4*b*x) + 16*b*c*d*sin(2*a + 2*b*x) - 2*b*c*d*si
n(4*a + 4*b*x) - 16*b^2*d^2*x^2*cos(2*a + 2*b*x) + 4*b^2*d^2*x^2*cos(4*a +
4*b*x) + 16*b*d^2*x*sin(2*a + 2*b*x) - 2*b*d^2*x*sin(4*a + 4*b*x) - 32*b^2*
c*d*x*cos(2*a + 2*b*x) + 8*b^2*c*d*x*cos(4*a + 4*b*x))/(128*b^3)
```

3.26 $\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=72

$$-\frac{3dx}{32b}+\frac{3d\cos(a+bx)\sin(a+bx)}{32b^2}+\frac{d\cos(a+bx)\sin^3(a+bx)}{16b^2}+\frac{(c+dx)\sin^4(a+bx)}{4b}$$

```
[Out] -3/32*d*x/b+3/32*d*cos(b*x+a)*sin(b*x+a)/b^2+1/16*d*cos(b*x+a)*sin(b*x+a)^3
/b^2+1/4*(d*x+c)*sin(b*x+a)^4/b
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4489, 2715, 8}

$$\frac{d\sin^3(a+bx)\cos(a+bx)}{16b^2}+\frac{3d\sin(a+bx)\cos(a+bx)}{32b^2}+\frac{(c+dx)\sin^4(a+bx)}{4b}-\frac{3dx}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-3*d*x)/(32*b) + (3*d*Cos[a + b*x]*Sin[a + b*x])/(32*b^2) + (d*Cos[a + b*x
]*Sin[a + b*x]^3)/(16*b^2) + ((c + d*x)*Sin[a + b*x]^4)/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx = \frac{(c+dx)\sin^4(a+bx)}{4b} - \frac{d\int \sin^4(a+bx)\,dx}{4b}$$
$$= \frac{d\cos(a+bx)\sin^3(a+bx)}{16b^2} + \frac{(c+dx)\sin^4(a+bx)}{4b} - \frac{(3d)\int \sin^2(a}{16b}$$
$$= \frac{3d\cos(a+bx)\sin(a+bx)}{32b^2} + \frac{d\cos(a+bx)\sin^3(a+bx)}{16b^2} + \frac{(c+dx}{}$$
$$= -\frac{3dx}{32b} + \frac{3d\cos(a+bx)\sin(a+bx)}{32b^2} + \frac{d\cos(a+bx)\sin^3(a+bx)}{16b^2} +$$

Mathematica [A]
time = 0.10, size = 75, normalized size = 1.04

$$\frac{c\sin^4(a+bx)}{4b} + \frac{d(-2bx\cos(2(a+bx))+\sin(2(a+bx)))}{16b^2} - \frac{d(-4bx\cos(4(a+bx))+\sin(4(a+bx)))}{128b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (c*Sin[a + b*x]^4)/(4*b) + (d*(-2*b*x*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]))
/(16*b^2) - (d*(-4*b*x*Cos[4*(a + b*x)] + Sin[4*(a + b*x)]))/(128*b^2)
```

Maple [A]
time = 0.12, size = 85, normalized size = 1.18

method	result
risch	$\frac{(dx+c)\cos(4bx+4a)}{32b} - \frac{d\sin(4bx+4a)}{128b^2} - \frac{(dx+c)\cos(2bx+2a)}{8b} + \frac{d\sin(2bx+2a)}{16b^2}$
derivativedivides	$\frac{-\frac{da\left(\sin^4(bx+a)\right)}{4b} + \frac{c\left(\sin^4(bx+a)\right)}{4} + \frac{d\left(\frac{(bx+a)\left(\sin^4(bx+a)\right)}{4} + \frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{16} - \frac{3bx}{32} - \frac{3a}{32}\right)}{b}}{b}$
default	$\frac{-\frac{da\left(\sin^4(bx+a)\right)}{4b} + \frac{c\left(\sin^4(bx+a)\right)}{4} + \frac{d\left(\frac{(bx+a)\left(\sin^4(bx+a)\right)}{4} + \frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{16} - \frac{3bx}{32} - \frac{3a}{32}\right)}{b}}{b}$
norman	$\frac{\frac{3d\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{16b^2} + \frac{11d\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{16b^2} - \frac{11d\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{16b^2} - \frac{3d\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{16b^2} - \frac{3dx}{32b} + \frac{4c\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b} - \frac{3dx\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4/b*d*a*sin(b*x+a)^4+1/4*c*sin(b*x+a)^4+1/b*d*(1/4*(b*x+a)*sin(b*x+
a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a))
```

Maxima [A]
time = 0.28, size = 92, normalized size = 1.28

$$\frac{32\,c\sin\left(bx+a\right)^4-\frac{32\,ad\sin\left(bx+a\right)^4}{b}+\frac{\left(4\,(bx+a)\cos\left(4\,bx+4\,a\right)-16\,(bx+a)\cos\left(2\,bx+2\,a\right)-\sin\left(4\,bx+4\,a\right)+8\,\sin\left(2\,bx+2\,a\right)\right)d}{b}}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/128*(32*c*sin(b*x + a)^4 - 32*a*d*sin(b*x + a)^4/b + (4*(b*x + a)*cos(4*b
*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x
+ 2*a))*d/b)/b
```

Fricas [A]
time = 2.50, size = 76, normalized size = 1.06

$$\frac{8\,(bdx+bc)\cos\left(bx+a\right)^4+5\,bdx-16\,(bdx+bc)\cos\left(bx+a\right)^2-\left(2\,d\cos\left(bx+a\right)^3-5\,d\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(8*(b*d*x + b*c)*cos(b*x + a)^4 + 5*b*d*x - 16*(b*d*x + b*c)*cos(b*x +
 a)^2 - (2*d*cos(b*x + a)^3 - 5*d*cos(b*x + a))*sin(b*x + a))/b^2
```

Sympy [A]
time = 0.29, size = 138, normalized size = 1.92

$$\begin{cases} \frac{c\sin^4\left(a+bx\right)}{4b}+\frac{5dx\sin^4\left(a+bx\right)}{32b}-\frac{3dx\sin^2\left(a+bx\right)\cos^2\left(a+bx\right)}{16b}-\frac{3dx\cos^4\left(a+bx\right)}{32b}+\frac{5d\sin^3\left(a+bx\right)\cos\left(a+bx\right)}{32b^2}+\frac{3d\sin\left(a+bx\right)\cos^3\left(a+bx\right)}{32b^2} & \text{for}\ b\neq 0 \\ \left(cx+\frac{dx^2}{2}\right)\sin^3\left(a\right)\cos\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c*sin(a + b*x)**4/(4*b) + 5*d*x*sin(a + b*x)**4/(32*b) - 3*d*x*s
in(a + b*x)**2*cos(a + b*x)**2/(16*b) - 3*d*x*cos(a + b*x)**4/(32*b) + 5*d*
sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 3*d*sin(a + b*x)*cos(a + b*x)**3/(
32*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(a)**3*cos(a), True))
```

Giac [A]
time = 0.44, size = 75, normalized size = 1.04

$$\frac{(bdx+bc)\cos\left(4\,bx+4\,a\right)}{32\,b^2}-\frac{(bdx+bc)\cos\left(2\,bx+2\,a\right)}{8\,b^2}-\frac{d\sin\left(4\,bx+4\,a\right)}{128\,b^2}+\frac{d\sin\left(2\,bx+2\,a\right)}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(b*d*x + b*c)*cos(4*b*x + 4*a)/b^2 - 1/8*(b*d*x + b*c)*cos(2*b*x + 2*a
)/b^2 - 1/128*d*sin(4*b*x + 4*a)/b^2 + 1/16*d*sin(2*b*x + 2*a)/b^2
```

Mupad [B]
time = 0.25, size = 94, normalized size = 1.31

$$\frac{2\,d\,\sin\left(2\,a+2\,b\,x\right)-\frac{d\,\sin\left(4\,a+4\,b\,x\right)}{4}-2\,b\,c\,\sin\left(2\,a+2\,b\,x\right)^2+8\,b\,c\,\sin\left(a+b\,x\right)^2+4\,b\,d\,x\,\left(2\,\sin\left(a+b\,x\right)^2-1\right)-b\,d\,x\,\left(2\,\sin\left(2\,a+2\,b\,x\right)^2-1\right)}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x),x)
```

```
[Out] (2*d*sin(2*a + 2*b*x) - (d*sin(4*a + 4*b*x))/4 - 2*b*c*sin(2*a + 2*b*x)^2 +
 8*b*c*sin(a + b*x)^2 + 4*b*d*x*(2*sin(a + b*x)^2 - 1) - b*d*x*(2*sin(2*a +
 2*b*x)^2 - 1))/(32*b^2)
```

3.27 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=129

$$-\frac{\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{8d}+\frac{\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}$$

```
[Out] 1/4*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d-1/8*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*
b*x)/d-1/8*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d+1/4*Ci(2*b*c/d+2*b*x)*sin(2
*a-2*b*c/d)/d
```

Rubi [A]
time = 0.17, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4491, 3384, 3380, 3383}

$$-\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}-\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] -1/8*(CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/d + (CosIntegral
[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(4*d) + (Cos[2*a - (2*b*c)/d]*Sin
Integral[(2*b*c)/d + 2*b*x])/(4*d) - (Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b
*c)/d + 4*b*x])/(8*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)}-\frac{\sin(4a+4bx)}{8(c+dx)}\right)dx\\ &= -\left(\frac{1}{8}\int \frac{\sin(4a+4bx)}{c+dx}\,dx\right)+\frac{1}{4}\int \frac{\sin(2a+2bx)}{c+dx}\,dx\\ &= -\left(\frac{1}{8}\cos\left(4a-\frac{4bc}{d}\right)\int \frac{\sin\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx\right)+\frac{1}{4}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin(\\ &= -\frac{\operatorname{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{8d}+\frac{\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{4d}+\frac{\cos(2a-\end{aligned}$$

Mathematica [A]
time = 0.34, size = 110, normalized size = 0.85

$$-\frac{\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)\sin\left(4a-\frac{4bc}{d}\right)-2\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right)-2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)+\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] -1/8*(CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] - 2*CosIntegral[(
2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 2*Cos[2*a - (2*b*c)/d]*SinIntegral
[(2*b*(c + d*x))/d] + Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/
d
```

Maple [A]
time = 0.12, size = 178, normalized size = 1.38

method	result
derivativedivides	$\frac{-\frac{b\left(-\frac{4\,\text{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}-\frac{4\,\text{cosineIntegral}\left(4bx+4a+\frac{-4ad+4cb}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}\right)}{32}+\frac{b\left(-\frac{2\,\text{si}}{}\right.}{}}{b}$
default	$\frac{-\frac{b\left(-\frac{4\,\text{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}-\frac{4\,\text{cosineIntegral}\left(4bx+4a+\frac{-4ad+4cb}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}\right)}{32}+\frac{b\left(-\frac{2\,\text{si}}{}\right.}{}}{b}$

risch	$-\frac{i e^{-\frac{2i(ad-cb)}{d}} \operatorname{expIntegral}\left(1,2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{8d}+\frac{i e^{-\frac{4i(ad-cb)}{d}} \operatorname{expIntegral}\left(1,4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{16d}+\frac{i e^{\frac{2i(ad-}{d}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/32*b*(-4*Si(-4*b*x-4*a-4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4
*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)+1/8*b*(-2*Si(-2*b*x-2*a-2*(
-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a
*d+b*c)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.36, size = 281, normalized size = 2.18

$$-\frac{2 b\left(-i E_1\left(\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)+i E_1\left(-\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)-b\left(-i E_1\left(\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)+i E_1\left(-\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{4(bc-ad)}{d}\right)+2 b\left(E_1\left(\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)+E_1\left(-\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-b\left(E_1\left(\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)+E_1\left(-\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)\right)\sin\left(-\frac{4(bc-ad)}{d}\right)}{16 bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/16*(2*b*(-I*exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*
exp_integral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d
)/d) - b*(-I*exp_integral_e(1, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*ex
p_integral_e(1, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a*d)/
d) + 2*b*(exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_int
egral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d) -
b*(exp_integral_e(1, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e
(1, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d))/(b*d)
```

Fricas [A]
time = 2.78, size = 156, normalized size = 1.21

$$\frac{2\left(\operatorname{Ci}\left(\frac{2(bdx+bc)}{d}\right)+\operatorname{Ci}\left(-\frac{2(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-\left(\operatorname{Ci}\left(\frac{4(bdx+bc)}{d}\right)+\operatorname{Ci}\left(-\frac{4(bdx+bc)}{d}\right)\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-2\cos\left(-\frac{4(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{4(bdx+bc)}{d}\right)+4\cos\left(-\frac{2(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2(bdx+bc)}{d}\right)}{16 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/16*(2*(cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d)
)*sin(-2*(b*c - a*d)/d) - (cos_integral(4*(b*d*x + b*c)/d) + cos_integral(-
4*(b*d*x + b*c)/d))*sin(-4*(b*c - a*d)/d) - 2*cos(-4*(b*c - a*d)/d)*sin_int
egral(4*(b*d*x + b*c)/d) + 4*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x +
b*c)/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c),x)

[Out] Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x), x)

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.59, size = 6046, normalized size = 46.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")

```
[Out] -1/16*(imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b
*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^
2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x -
2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_i
ntegral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2
+ 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(
b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/
d)^2*tan(b*c/d)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*t
an(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(cos_integr
al(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 2*real
_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(
b*c/d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan
(2*b*c/d)^2*tan(b*c/d)^2 + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2
+ imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)
^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b
*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*t
an(2*b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)
^2*tan(2*b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*t
an(2*b*c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2
*b*c/d)^2 - 8*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*ta
n(2*b*c/d)^2*tan(b*c/d) + 8*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2
*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - 16*sin_integral(2*(b*d*x + b*c)/d)
*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(4*b*x
 + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*
b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + imag_part(cos_integr
al(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*sin_integral(4*(
b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 4*sin_integral(2*(b*d*x
```

```
+ b*c)/d)*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*imag_part(cos_integral(4*b*x
 + 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 4*imag_part(cos_
integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 8
*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^
2 + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2*tan(
b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/
d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*
tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan
(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan
(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/
d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*b*x +
 2*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + imag_part(cos_int
egral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*sin_integ
ral(4*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*sin_integra
l(2*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*real_part(cos
_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d) + 2*real_part(
cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d) - 4*real_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2 - 4*rea
l_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2 - 2
*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2
 - 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d
)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*
c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(
b*c/d) + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)
^2*tan(b*c/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(
2*b*c/d)^2*tan(b*c/d) - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2
*tan(2*b*c/d)^2*tan(b*c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(2*b*x + 2*b*c/d
))*tan(2*a)^2*tan(a)*tan(b*c/d)^2 + 4*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(2*a)^2*tan(a)*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4*b*
c/d))*tan(2*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4
*b*c/d))*tan(2*a)^2*tan(2*b*c/d)*tan(b*c/d)^2 +...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)\,\sin(a+b\,x)^3}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x),x)
[Out] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x), x)
```

3.28 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=179

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2}-\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{\sin(2a+2bx)}{4d(c+dx)}+\frac{\sin(4a+}{8d(c+}$$

```
[Out] -1/2*b*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^2+1/2*b*Ci(2*b*c/d+2*b*x)*cos(2
*a-2*b*c/d)/d^2+1/2*b*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^2-1/2*b*Si(2*b*c
/d+2*b*x)*sin(2*a-2*b*c/d)/d^2-1/4*sin(2*b*x+2*a)/d/(d*x+c)+1/8*sin(4*b*x+4
*a)/d/(d*x+c)
```

Rubi [A]
time = 0.19, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2}-\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^2}+\frac{b\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{\sin(2a+2bx)}{4d(c+dx)}+\frac{\sin(4a+4bx)}{8d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) - (b*Cos[4*
a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(2*d^2) - Sin[2*a + 2*b*x]/(
4*d*(c + d*x)) + Sin[4*a + 4*b*x]/(8*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]
*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) + (b*Sin[4*a - (4*b*c)/d]*SinInteg
ral[(4*b*c)/d + 4*b*x])/(2*d^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^2}-\frac{\sin(4a+4bx)}{8(c+dx)^2}\right)dx\\
&= -\left(\frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^2}\,dx\right)+\frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx\\
&= -\frac{\sin(2a+2bx)}{4d(c+dx)}+\frac{\sin(4a+4bx)}{8d(c+dx)}+\frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{2d}-\frac{b\int \frac{\cos(4a+4bx)}{c+dx}\,dx}{2d}\\
&= -\frac{\sin(2a+2bx)}{4d(c+dx)}+\frac{\sin(4a+4bx)}{8d(c+dx)}-\frac{\left(b\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\cos\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx}{2d}+\frac{(b
\\
&= \frac{b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d^2}-\frac{b\cos\left(4a-\frac{4bc}{d}\right)\operatorname{Ci}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{\sin(2a+}{4d(c+
\end{aligned}
$$

Mathematica [A]
time = 1.26, size = 151, normalized size = 0.84

$$
\frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)-4b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)-\frac{2d\sin(2(a+bx))}{c+dx}+\frac{d\sin(4(a+bx))}{c+dx}-4b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)+4b\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right)}{8d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (4*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 4*b*Cos[4*a - (4
*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] - (2*d*Sin[2*(a + b*x)])/(c + d*x)
+ (d*Sin[4*(a + b*x)])/(c + d*x) - 4*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*
b*(c + d*x))/d] + 4*b*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/
(8*d^2)
```

Maple [A]
time = 0.20, size = 256, normalized size = 1.43

method	result
derivativedivides	$-\frac{b^{2}\left(-\frac{4 \sin \left(4 b x +4 a \right)}{\left(-a d +c b +d \left(b x +a \right)\right) d}+\frac{-\frac{16\, \mathrm{sinIntegral}\left(-4 b x -4 a -\frac{4 \left(-a d +c b \right)}{d}\right) \sin \left(\frac{-4 a d +4 c b}{d}\right)}{d}+\frac{16\, \mathrm{cosineIntegral}\left(4 b x +4 a +\frac{-4 a d +4 c b}{d}\right) \cdots}{d}}{d} \cdots\right)}{32} \cdots$
default	$-\frac{b^{2}\left(-\frac{4 \sin \left(4 b x +4 a \right)}{\left(-a d +c b +d \left(b x +a \right)\right) d}+\frac{-\frac{16\, \mathrm{sinIntegral}\left(-4 b x -4 a -\frac{4 \left(-a d +c b \right)}{d}\right) \sin \left(\frac{-4 a d +4 c b}{d}\right)}{d}+\frac{16\, \mathrm{cosineIntegral}\left(4 b x +4 a +\frac{-4 a d +4 c b}{d}\right) \cdots}{d}}{d} \cdots\right)}{32} \cdots$
risch	$-\frac{b\,{\mathrm e}^{-\frac{2 i \left(a d -c b \right)}{d}} \mathrm{expIntegral}\left(1, 2 i b x +2 i a -\frac{2 i \left(a d -c b \right)}{d}\right)}{4 d^{2}}+\frac{b\,{\mathrm e}^{-\frac{4 i \left(a d -c b \right)}{d}} \mathrm{expIntegral}\left(1, 4 i b x +4 i a -\frac{4 i \left(a d -c b \right)}{d}\right)}{4 d^{2}}-\frac{b\,{\mathrm e}^{\frac{2 i \cdots}{}}}{} \cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/32*b^2*(-4*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))/d+4*(-4*Si(-4*b*x-4*
a-4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(
4*(-a*d+b*c)/d)/d)/d)+1/8*b^2*(-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2*(
-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a
*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.39, size = 308, normalized size = 1.72

$$-\frac{2\, b^{2} {\left(-i\, E_{2}\left(\frac{2\, {\left(-i\, b c - i\, {\left(b x + a\right)} d + i\, a d\right)}}{d}\right) + i\, E_{2}\left(-\frac{2\, {\left(-i\, b c - i\, {\left(b x + a\right)} d + i\, a d\right)}}{d}\right)\right)} \cos\left(-\frac{2\, {\left(b c - a d\right)}}{d}\right) - b^{2} {\left(-i\, E_{2}\left(\frac{4\, {\left(-i\, b c - i\, {\left(b x + a\right)} d + i\, a d\right)}}{d}\right) + i\, E_{2}\left(-\frac{4\, {\left(-i\, b c - i\, {\left(b x + a\right)} d + i\, a d\right)}}{d}\right)\right)} \cos\left(-\frac{4\, {\left(b c - a d\right)}}{d}\right) + 2\, b^{2} {\left(E_{2}\left(\frac{2\, {\left(-i\, b c - i\, {\left(b x + a\right)} d + i\, a d\right)}}{d}\right) + E_{2}\left(-\frac{2\, {\left(-i\, b c - i\, {\left(b x + a\right)} d + i\, a d\right)}}{d}\right)\right)} \sin\left(-\frac{2\, {\left(b c - a d\right)}}{d}\right) - b^{2} {\left(E_{2}\left(\frac{4\, {\left(-i\, b c - i\, {\left(b x + a\right)} d + i\, a d\right)}}{d}\right) + E_{2}\left(-\frac{4\, {\left(-i\, b c - i\, {\left(b x + a\right)} d + i\, a d\right)}}{d}\right)\right)} \sin\left(-\frac{4\, {\left(b c - a d\right)}}{d}\right)}{16\, {\left(b c d + {\left(b x + a\right)} d^{2} - a d^{2}\right)} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(2*b^2*(-I*exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
I*exp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a
*d)/d) - b^2*(-I*exp_integral_e(2, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
I*exp_integral_e(2, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a
*d)/d) + 2*b^2*(exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + e
xp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)
/d) - b^2*(exp_integral_e(2, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_in
tegral_e(2, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d))/
((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A]
time = 3.31, size = 245, normalized size = 1.37

$$\frac{2\, {\left(b d x + b c\right)} \sin\left(-\frac{4\, {\left(b c - a d\right)}}{d}\right) \operatorname{Si}\left(\frac{4\, {\left(b d x + b c\right)}}{d}\right) - 2\, {\left(b d x + b c\right)} \sin\left(-\frac{2\, {\left(b c - a d\right)}}{d}\right) \operatorname{Si}\left(\frac{2\, {\left(b d x + b c\right)}}{d}\right) + {\left({\left(b d x + b c\right)} \operatorname{Ci}\left(\frac{2\, {\left(b d x + b c\right)}}{d}\right) + {\left(b d x + b c\right)} \operatorname{Ci}\left(-\frac{2\, {\left(b d x + b c\right)}}{d}\right)\right)} \cos\left(-\frac{2\, {\left(b c - a d\right)}}{d}\right) - {\left({\left(b d x + b c\right)} \operatorname{Ci}\left(\frac{4\, {\left(b d x + b c\right)}}{d}\right) + {\left(b d x + b c\right)} \operatorname{Ci}\left(-\frac{4\, {\left(b d x + b c\right)}}{d}\right)\right)} \cos\left(-\frac{4\, {\left(b c - a d\right)}}{d}\right) + 4\, {\left(d \cos\left(b x + a\right)^{3} - d \cos\left(b x + a\right)\right)} \sin\left(b x + a\right)}{4\, {\left(d^{3} x + c d^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")

[Out] 1/4*(2*(b*d*x + b*c)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) - 2*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) + ((b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d) - ((b*d*x + b*c)*cos_integral(4*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c - a*d)/d) + 4*(d*cos(b*x + a)^3 - d*cos(b*x + a))*sin(b*x + a))/(d^3*x + c*d^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**2,x)

[Out] Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x)**2, x)

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.27, size = 63510, normalized size = 354.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")

[Out] -1/4*(b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 4*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 2*b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 2*b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 4*b*d*x*sin_integral(4*(b*d*x + b*c

```
)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2
- 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*
tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b*d*x*imag_part(cos_integ
ral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/
d)^2*tan(b*c/d)^2 - 4*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*ta
n(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b*d*x*imag_part(
cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*ta
n(2*b*c/d)^2*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d
))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 +
4*b*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*ta
n(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + b*c*real_part(cos_integral(4*b*x + 4*b
*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d
)^2 - b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*
tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - b*c*real_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/
d)^2*tan(b*c/d)^2 + b*c*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x
)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + b*d*x*real
_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan
(a)^2*tan(2*b*c/d)^2 + b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2
*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + b*d*x*real_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*ta
n(2*b*c/d)^2 + b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2
*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 - 4*b*d*x*real_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/
d)^2*tan(b*c/d) - 4*b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b
*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) + 2*b*c*imag_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a
)^2*tan(2*b*c/d)^2*tan(b*c/d) - 2*b*c*imag_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)
+ 4*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*
tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - b*d*x*real_part(cos_integral(4*b*x + 4
*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - b*d*x*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*
tan(a)^2*tan(b*c/d)^2 - b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - b*d*x*real_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*ta
n(b*c/d)^2 + 4*b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*
tan(b*x)^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 4*b*d*x*real_part(
cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*t
an(2*b*c/d)*tan(b*c/d)^2 - 2*b*c*imag_part(cos_integral(4*b*x + 4*b*c/d))*t
an(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 2*b*
c*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a
)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 4*b*c*sin_integral(4*(b*d*x + b*c)
/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 +
 b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan
```

```
(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + b*d*x*real_part(cos_integral(2*b*x +
2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 +
b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan
(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + b*d*x*real_part(cos_integral(-4*b*x -
 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 -
 2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan
(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^2,x)
[Out] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^2, x)
```

3.29 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=229

$$-\frac{b\cos(2a+2bx)}{4d^2(c+dx)}+\frac{b\cos(4a+4bx)}{4d^2(c+dx)}+\frac{b^2\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{d^3}-\frac{b^2\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin}{2d^3}$$

```
[Out] -1/4*b*cos(2*b*x+2*a)/d^2/(d*x+c)+1/4*b*cos(4*b*x+4*a)/d^2/(d*x+c)-1/2*b^2*
cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^3+b^2*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*b*
x)/d^3+b^2*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^3-1/2*b^2*Ci(2*b*c/d+2*b*x)
*sin(2*a-2*b*c/d)/d^3-1/8*sin(2*b*x+2*a)/d/(d*x+c)^2+1/16*sin(4*b*x+4*a)/d/
(d*x+c)^2
```

Rubi [A]
time = 0.25, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^3}+\frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b\cos(2a+2bx)}{4d^2(c+dx)}+\frac{b\cos(4a+4bx)}{4d^2(c+dx)}-\frac{\sin(2a+2bx)}{8d(c+dx)^2}+\frac{\sin(4a+4bx)}{16d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] -1/4*(b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)) + (b*Cos[4*a + 4*b*x])/(4*d^2*(c
+ d*x)) + (b^2*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/d^3 - (
b^2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d^3) - Sin[2*a
+ 2*b*x]/(8*d*(c + d*x)^2) + Sin[4*a + 4*b*x]/(16*d*(c + d*x)^2) - (b^2*Cos
[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^3) + (b^2*Cos[4*a -
(4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/d^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^3} - \frac{\sin(4a+4bx)}{8(c+dx)^3}\right) dx$$

$$= -\left(\frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^3}\,dx\right) + \frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx$$

$$= -\frac{\sin(2a+2bx)}{8d(c+dx)^2} + \frac{\sin(4a+4bx)}{16d(c+dx)^2} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{4d} - \frac{b\int \frac{\cos(4a+4bx)}{(c+dx)^2}\,dx}{4d}$$

$$= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)} + \frac{b\cos(4a+4bx)}{4d^2(c+dx)} - \frac{\sin(2a+2bx)}{8d(c+dx)^2} + \frac{\sin(4a+4bx)}{16d(c+dx)^2} - \frac{b^2}{}$$

$$= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)} + \frac{b\cos(4a+4bx)}{4d^2(c+dx)} - \frac{\sin(2a+2bx)}{8d(c+dx)^2} + \frac{\sin(4a+4bx)}{16d(c+dx)^2} + \frac{(b}{}$$

$$= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)} + \frac{b\cos(4a+4bx)}{4d^2(c+dx)} + \frac{b^2\operatorname{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{d^3} - \frac{b^2\mathrm{C}}{}$$

Mathematica [A]
time = 2.73, size = 199, normalized size = 0.87

$$\frac{16b^2\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)\sin\left(4a-\frac{4bc}{d}\right)+\frac{d(4b(c+dx)\cos(4(a+bx))+d\sin(4(a+bx)))}{(c+dx)^2}-2\left(4b^2\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right)+\frac{d(2b(c+dx)\cos(2(a+bx))+d\sin(2(a+bx)))}{(c+dx)^2}+4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)\right)+16b^2\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] (16*b^2*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + (d*(4*b*(c +
d*x)*Cos[4*(a + b*x)] + d*Sin[4*(a + b*x)]))/(c + d*x)^2 - 2*(4*b^2*CosInte
gral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*(2*b*(c + d*x)*Cos[2*(a +
 b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + 4*b^2*Cos[2*a - (2*b*c)/d]*SinI
ntegral[(2*b*(c + d*x))/d]) + 16*b^2*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*
(c + d*x))/d])/(16*d^3)
```

Maple [A]
time = 0.14, size = 329, normalized size = 1.44

method	result
derivativedivides	$-\frac{b^3\left(-\frac{2\sin(4bx+4a)}{(-ad+cb+d(bx+a))^2 d}+\frac{-\frac{8\cos(4bx+4a)}{(-ad+cb+d(bx+a))d}-\frac{8\left(-\frac{4\,\mathrm{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}-\frac{4\,\mathrm{cosineInte}}{}\right)}{d}}{d}\right)}{32}$
default	$-\frac{b^3\left(-\frac{2\sin(4bx+4a)}{(-ad+cb+d(bx+a))^2 d}+\frac{-\frac{8\cos(4bx+4a)}{(-ad+cb+d(bx+a))d}-\frac{8\left(-\frac{4\,\mathrm{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}-\frac{4\,\mathrm{cosineInte}}{}\right)}{d}}{d}\right)}{32}$
risch	$\frac{ib^2 \mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{4d^3}-\frac{ib^2 \mathrm{e}^{-\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{2d^3}-\frac{ib^2 \mathrm{e}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/32*b^3*(-2*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))^2/d+2*(-4*cos(4*b*x+
4*a)/(-a*d+c*b+d*(b*x+a))/d-4*(-4*Si(-4*b*x-4*a-4*(-a*d+b*c)/d)*cos(4*(-a*d
+b*c)/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)/d)/d)+1/8*
b^3*(-sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d+(-2*cos(2*b*x+2*a)/(-a*d+c*b+
d*(b*x+a))/d-2*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci
(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.49, size = 343, normalized size = 1.50

$$-\frac{2\,b^3\left(-i\,E_3\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_3\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^3\left(-i\,E_3\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_3\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)+2\,b^3\left(E_3\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_3\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-b^3\left(E_3\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_3\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)}{16\,\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\,(bcd^2-ad^3)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*b^3*(-I*exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
I*exp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a
*d)/d) - b^3*(-I*exp_integral_e(3, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
I*exp_integral_e(3, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a
*d)/d) + 2*b^3*(exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + e
xp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)
/d) - b^3*(exp_integral_e(3, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_in
tegral_e(3, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d))/
((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)
*(b*x + a))*b)
```

Fricas [A]
time = 2.65, size = 423, normalized size = 1.85

$$\frac{8\,(bd^2x+bcd)\cos(bx+a)^4+2\,bd^2x+2\,bcd-10\,(bd^2x+bcd)\cos(bx+a)^2+4\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\cos\left(-\frac{4\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{4\,(bdx+bc)}{d}\right)-2\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+2\,(d^2\cos(bx+a)^3-d^2\cos(bx+a))\sin(bx+a)-\left((b^2d^2x^2+2\,b^2cdx+b^2c^2)\operatorname{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+(b^2d^2x^2+2\,b^2cdx+b^2c^2)\operatorname{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+2\left((b^2d^2x^2+2\,b^2cdx+b^2c^2)\operatorname{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)+(b^2d^2x^2+2\,b^2cdx+b^2c^2)\operatorname{Ci}\left(-\frac{4\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)}{4\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*(b*d^2*x + b*c*d)*cos(b*x + a)^4 + 2*b*d^2*x + 2*b*c*d - 10*(b*d^2*x
 + b*c*d)*cos(b*x + a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-4*(
b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) + 2*(d^2
*cos(b*x + a)^3 - d^2*cos(b*x + a))*sin(b*x + a) - ((b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2)*cos_integral(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d) + 2*((b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(4*(b*d*x + b*c)/d) + (b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-4*(b*d*x + b*c)/d))*sin(-4*(
b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x)**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.65, size = 111694, normalized size = 487.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")

[Out] 1/8*(4*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*ta
n(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*im
ag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*t
an(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)
^2*tan(b*c/d)^2 - 4*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*t
an(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 8*
b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a
)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*b^2*d^2*x^2*sin_integral(2*(b*
d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*ta
n(b*c/d)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b
*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 4*b^2*d^2*
x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2
*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 8*b^2*d^2*x^2*real_part(cos_inte
gral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*
c/d)*tan(b*c/d)^2 + 8*b^2*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))
*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 4*
b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^
2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*b^2*d^2*x^2*real_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*ta
n(2*b*c/d)^2*tan(b*c/d)^2 - 8*b^2*d^2*x^2*real_part(cos_integral(4*b*x + 4*
b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)
^2 - 8*b^2*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*t
an(b*x)^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 8*b^2*c*d*x*imag_
part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(
a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*b^2*c*d*x*imag_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan
(b*c/d)^2 + 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x
)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 8*b^2*c*d*
x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a
)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 16*b^2*c*d*x*sin_integral(4*(b*d
*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan
(b*c/d)^2 - 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*
x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*b^2*d^2*x^2*imag_p
art(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a
)^2*tan(2*b*c/d)^2 + 2*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))
*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 - 2*b^2*d^2*x^2
*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)
^2*tan(a)^2*tan(2*b*c/d)^2 - 4*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x -
4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + 8*b^
2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^
2*tan(a)^2*tan(2*b*c/d)^2 + 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*t
an(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 - 8*b^2*d^2*x^2*i
```

```
mag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*
tan(a)*tan(2*b*c/d)^2*tan(b*c/d) + 8*b^2*d^2*x^2*imag_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*ta
n(b*c/d) - 16*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(
b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - 8*b^2*c*d*x*real_part(
cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*
tan(2*b*c/d)^2*tan(b*c/d) - 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b
*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d
) - 4*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan
(b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c/
d)^2 + 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2
*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*b^2*d^2*x^2*imag_part(cos_
integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan
(b*c/d)^2 - 8*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(
b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 4*b^2*d^2*x^2*sin_integral(2*(b*d
*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 16*
b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^
2*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 16*b^2*d^2*x^2*imag_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*tan
(2*b*c/d)*tan(b*c/d)^2 + 32*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan
(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 16*b^2*c
*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2
*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 16*b...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a+bx)\,\sin(a+bx)^3}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^3, x)
```

3.30 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=287

$$-\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}+\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegr}}{3d^4}$$

```
[Out] 4/3*b^3*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^4-1/3*b^3*Ci(2*b*c/d+2*b*x)*co
s(2*a-2*b*c/d)/d^4-1/12*b*cos(2*b*x+2*a)/d^2/(d*x+c)^2+1/12*b*cos(4*b*x+4*a
)/d^2/(d*x+c)^2-4/3*b^3*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^4+1/3*b^3*Si(2
*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^4-1/12*sin(2*b*x+2*a)/d/(d*x+c)^3+1/6*b^2*
sin(2*b*x+2*a)/d^3/(d*x+c)+1/24*sin(4*b*x+4*a)/d/(d*x+c)^3-1/3*b^2*sin(4*b*
x+4*a)/d^3/(d*x+c)
```

Rubi [A]
time = 0.27, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{4b^3\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^2\sin(2a+2bx)}{6d^3(c+dx)}-\frac{b^2\sin(4a+4bx)}{3d^3(c+dx)}-\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}+\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{12d(c+dx)^3}+\frac{\sin(4a+4bx)}{24d(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] -1/12*(b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)^2) + (b*Cos[4*a + 4*b*x])/(12*d^2
*(c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(
3*d^4) + (4*b^3*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(3*d^4
) - Sin[2*a + 2*b*x]/(12*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(6*d^3*(c
+ d*x)) + Sin[4*a + 4*b*x]/(24*d*(c + d*x)^3) - (b^2*Sin[4*a + 4*b*x])/(3*d
^3*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(
3*d^4) - (4*b^3*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(3*d^4
)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^4} - \frac{\sin(4a+4bx)}{8(c+dx)^4}\right)dx$$

$$= -\left(\frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^4}\,dx\right) + \frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^4}\,dx$$

$$= -\frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{\sin(4a+4bx)}{24d(c+dx)^3} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^3}\,dx}{6d} - \frac{b\int \frac{\cos(4a+4bx)}{(c+dx)^3}\,dx}{6d}$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{\sin(4a+4bx)}{24d(c+dx)^3} - b^2$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{6d^3(c+dx)} +$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{6d^3(c+dx)} +$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4} + 4b$$

Mathematica [A]
time = 2.02, size = 316, normalized size = 1.10

$$\frac{-2d\cos(2bx)\left(bd(c+dx)\cos(2a)+(d^2-2b^2(c+dx)^2)\sin(2a)\right)+d\cos(4bx)\left(2bd(c+dx)\cos(4a)+(d^2-8b^2(c+dx)^2)\sin(4a)\right)+2d\left((-d^2+2b^2(c+dx)^2)\cos(2a)+bd(c+dx)\sin(2a)\right)\sin(2bx)-d\left((-d^2+8b^2(c+dx)^2)\cos(4a)+2bd(c+dx)\sin(4a)\right)\sin(4bx)-8b^3(c+dx)^3\left(\cos\left(2a-\frac{2bc}{d}\right)\mathrm{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)-\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2b(c+dx)}{d}\right)\right)+32b^3(c+dx)^3\left(\cos\left(4a-\frac{4bc}{d}\right)\mathrm{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)-\sin\left(4a-\frac{4bc}{d}\right)\mathrm{Si}\left(\frac{4b(c+dx)}{d}\right)\right)}{24d^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] (-2*d*Cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*Sin[2*
a]) + d*Cos[4*b*x]*(2*b*d*(c + d*x)*Cos[4*a] + (d^2 - 8*b^2*(c + d*x)^2)*Si
n[4*a]) + 2*d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*Sin[2*a]
)*Sin[2*b*x] - d*((-d^2 + 8*b^2*(c + d*x)^2)*Cos[4*a] + 2*b*d*(c + d*x)*Sin
[4*a])*Sin[4*b*x] - 8*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*
b*(c + d*x))/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d]) + 32
*b^3*(c + d*x)^3*(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] - Sin
[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d]))/(24*d^4*(c + d*x)^3)
```

Maple [A]

time = 0.18, size = 404, normalized size = 1.41

method	result
derivativedivides	$-\frac{b^4\left(-\frac{4\sin(4bx+4a)}{3(-ad+cb+d(bx+a))^3d}+\frac{-\frac{8\cos(4bx+4a)}{3(-ad+cb+d(bx+a))^2d}-\frac{8\left(-\frac{4\sin(4bx+4a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{16\,\mathrm{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)}{d}}{}\right)}{}}{d}\right)}{32}$
default	$-\frac{b^4\left(-\frac{4\sin(4bx+4a)}{3(-ad+cb+d(bx+a))^3d}+\frac{-\frac{8\cos(4bx+4a)}{3(-ad+cb+d(bx+a))^2d}-\frac{8\left(-\frac{4\sin(4bx+4a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{16\,\mathrm{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)}{d}}{}\right)}{}}{d}\right)}{32}$
risch	$\frac{b^3 e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{6d^4}-\frac{2b^3 e^{-\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{3d^4}+\frac{b^3 e^{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/32*b^4*(-4/3*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))^3/d+4/3*(-2*cos(4*
b*x+4*a)/(-a*d+c*b+d*(b*x+a))^2/d-2*(-4*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))
/d+4*(-4*Si(-4*b*x-4*a-4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4*a
+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)/d)/d)/d)+1/8*b^4*(-2/3*sin(2*b*x+2*
a)/(-a*d+c*b+d*(b*x+a))^3/d+2/3*(-cos(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d-(
-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d
)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/
d)/d)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.63, size = 393, normalized size = 1.37

$$-\frac{2\,b^4\left(-i\,E_4\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_4\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^4\left(-i\,E_4\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_4\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)+2\,b^4\left(E_4\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_4\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-b^4\left(E_4\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_4\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)}{16\,\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\,(bcd^3-ad^4)(bx+a)^2+3\,(b^2c^2d^2-2\,abcd^3+a^2d^4)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/16*(2*b^4*(-I*exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
I*exp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a
*d)/d) - b^4*(-I*exp_integral_e(4, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
I*exp_integral_e(4, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a
*d)/d) + 2*b^4*(exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + e
xp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)
/d) - b^4*(exp_integral_e(4, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_in
tegral_e(4, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d))/
((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 +
 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*
(b*x + a))*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 588 vs. 2(267) = 534.
time = 3.19, size = 588, normalized size = 2.05

$$\frac{bd^3x+4\,(bd^3x+bcd^2)\cos(bx+a)^4+bcd^2-5\,(bd^3x+bcd^2)\cos(bx+a)^2-8\,(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\sin\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{4\,(bdx+bc)}{d}\right)+2\,(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)-\left((b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\,\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\,\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+4\left((b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\,\mathrm{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)+(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\,\mathrm{Ci}\left(-\frac{4\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)-2\left((8\,b^2d^3x^2+16\,b^2cd^2x+8\,b^2c^2d-d^3)\cos(bx+a)^3-(5\,b^2d^3x^2+10\,b^2cd^2x+5\,b^2c^2d-d^3)\cos(bx+a)\right)\sin(bx+a)}{6\,(d^7x^3+3\,cd^6x^2+3\,c^2d^5x+c^3d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(b*d^3*x + 4*(b*d^3*x + b*c*d^2)*cos(b*x + a)^4 + b*c*d^2 - 5*(b*d^3*x
+ b*c*d^2)*cos(b*x + a)^2 - 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + b^3*c^3)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + 2*(b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-2*(b*c - a*d)/d)
*sin_integral(2*(b*d*x + b*c)/d) - ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*
c^2*d*x + b^3*c^3)*cos_integral(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c
*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-
2*(b*c - a*d)/d) + 4*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*
c^3)*cos_integral(4*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b
^3*c^2*d*x + b^3*c^3)*cos_integral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c - a*d)/
d) - 2*((8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3
 - (5*b^2*d^3*x^2 + 10*b^2*c*d^2*x + 5*b^2*c^2*d - d^3)*cos(b*x + a))*sin(b
*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos(a+bx)}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x)**4, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.68, size = 157526, normalized size = 548.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/12*(8*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*t
an(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*
tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3*real_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d
)^2*tan(b*c/d)^2 + 8*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*
tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4
*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)
^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 4*b^3*d^3*x^3*imag_part(
cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2
*tan(2*b*c/d)^2*tan(b*c/d) + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*
tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 16*
b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^
2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 16*b^3*d^3*x^3*imag_part(
cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2
*tan(2*b*c/d)*tan(b*c/d)^2 - 32*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)
*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 4*
b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^
2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*b^3*d^3*x^3*imag_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*ta
n(2*b*c/d)^2*tan(b*c/d)^2 - 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*t
an(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 16*b
^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2
*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 16*b^3*d^3*x^3*imag_part(c
os_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*ta
n(2*b*c/d)^2*tan(b*c/d)^2 + 32*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*
tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 24*
b^3*c*d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x
)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 6*b^3*c*d^2*x^2*real_
part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(
a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 6*b^3*c*d^2*x^2*real_part(cos_integral(-
2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^
```

```
2*tan(b*c/d)^2 + 24*b^3*c*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))
*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 +
8*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x
)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integr
al(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/
d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2
*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + 8*b^3*d^3*x^3*real_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*t
an(2*b*c/d)^2 - 8*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(
2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - 8*b^3*d^3
*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(
2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) + 12*b^3*c*d^2*x^2*imag_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*
b*c/d)^2*tan(b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)
 + 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2
*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 8*b^3*d^3*x^3*real_part(co
s_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*ta
n(b*c/d)^2 - 2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b
*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3*real_part
(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^
2*tan(b*c/d)^2 - 8*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*ta
n(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 32*b^3*d^3*x^3*rea
l_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(
a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 32*b^3*d^3*x^3*real_part(cos_integral(-4*b
*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b
*c/d)^2 - 48*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b
*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 48*b^3*c*d
^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*ta
n(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 96*b^3*c*d^2*x^2*sin_integral
(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d
)*tan(b*c/d)^2 + 8*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan
(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3
*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^
2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*r...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a+b\,x)\,\sin(a+b\,x)^3}{(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^4,x)
[Out] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^4, x)
```

3.31 $\int (c+dx)^m \cot(a+bx)\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}((c+dx)^m\cot(a+bx),x)$$

```
[Out] Unintegrable((d*x+c)^m*cot(b*x+a),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m\cot(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m\cot(a+bx)\,dx = \int (c+dx)^m\cot(a+bx)\,dx$$

Mathematica [A]
time = 2.58, size = 0, normalized size = 0.00

$$\int (c+dx)^m\cot(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Cot[a + b*x], x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int (dx+c)^m\cos(bx+a)\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*cos(b*x + a)*csc(b*x + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cos(a+bx)\csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*csc(b*x+a),x)
[Out] Integral((c + d*x)**m*cos(a + b*x)*csc(a + b*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\cos(a+b\,x)\,{(c+d\,x)}^m}{\sin(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x),x)
[Out] int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x), x)
```

3.32 $\int (c+dx)^4 \cot(a+bx)\,dx$

Optimal. Leaf size=151

$$-\frac{i(c+dx)^5}{5d}+\frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{2id(c+dx)^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}+\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,e^{2i(\right.}}{b^3}$$

```
[Out] -1/5*I*(d*x+c)^5/d+(d*x+c)^4*ln(1-exp(2*I*(b*x+a)))/b-2*I*d*(d*x+c)^3*polyl
og(2,exp(2*I*(b*x+a)))/b^2+3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(b*x+a)))/b^3+
3*I*d^3*(d*x+c)*polylog(4,exp(2*I*(b*x+a)))/b^4-3/2*d^4*polylog(5,exp(2*I*(
b*x+a)))/b^5
```

Rubi [A]
time = 0.14, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3798, 2221, 2611, 6744, 2320, 6724}

$$-\frac{3d^4\text{Li}_5\left(e^{2i(a+bx)}\right)}{2b^5}+\frac{3id^3(c+dx)\text{Li}_4\left(e^{2i(a+bx)}\right)}{b^4}+\frac{3d^2(c+dx)^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{b^3}-\frac{2id(c+dx)^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{i(c+dx)^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x],x]
```

```
[Out] ((-1/5*I)*(c + d*x)^5)/d + ((c + d*x)^4*Log[1 - E^((2*I)*(a + b*x))])/b - (
(2*I)*d*(c + d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)
^2*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 + ((3*I)*d^3*(c + d*x)*PolyLog[4, E
^((2*I)*(a + b*x))])/b^4 - (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4\cot(a+bx)\,dx = -\frac{i(c+dx)^5}{5d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^4}{1-e^{2i(a+bx)}}\,dx$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{(4d)\int (c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{(6ia}{}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{3d^2(}{}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{3d^2(}{}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{3d^2(}{}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{3d^2(}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 527 vs. 2(151) = 302.
time = 3.29, size = 527, normalized size = 3.49

$$\frac{2ic^3d\pi x}{b} - 2ic^2d^2x^3 - icd^3x^4 - \frac{1}{5}id^4x^5 - \frac{4ic^3dx\text{ArcTan}(\tan(a))}{b} + 2c^3dx^2\cot(a) + \frac{2c^3d\pi\log\left(1+e^{-2ibx}\right)}{b^2} + \frac{6c^2d^2x^2\log\left(1-e^{2i(a+bx)}\right)}{b} + \frac{4cd^3x^3\log\left(1-e^{2i(a+bx)}\right)}{b} + \frac{d^4x^4\log\left(1-e^{2i(a+bx)}\right)}{b} + \frac{4c^3dx\log\left(1-e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)}{b} + \frac{4c^3d\text{ArcTan}(\tan(a))\log\left(1-e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)}{b^2} - \frac{2c^3d\pi\log(\cos(bx))}{b^2} + \frac{c^4\log(\sin(a+bx))}{b} - \frac{4c^3d\text{ArcTan}(\tan(a))\log(\sin(bx+\text{ArcTan}(\tan(a))))}{b^2} - \frac{2id^2x(3c^2+3cdx+d^2x^2)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} - \frac{2ic^3d\text{PolyLog}\left(2,e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)}{b^2} + \frac{3c^2d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3} + \frac{6cd^3x\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3} + \frac{3d^4x^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3} + \frac{3icd^3\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4} + \frac{3id^4x\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4} - \frac{3d^4\text{PolyLog}\left(5,e^{2i(a+bx)}\right)}{2b^5} - 2c^3de^{i\text{ArcTan}(\tan(a))}x^2\cot(a)\sqrt{\sec^2(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x],x]
```

```
[Out] ((2*I)*c^3*d*Pi*x)/b - (2*I)*c^2*d^2*x^3 - I*c*d^3*x^4 - (I/5)*d^4*x^5 - ((
4*I)*c^3*d*x*ArcTan[Tan[a]])/b + 2*c^3*d*x^2*Cot[a] + (2*c^3*d*Pi*Log[1 + E
^((-2*I)*b*x)])/b^2 + (6*c^2*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))])/b + (4*c
*d^3*x^3*Log[1 - E^((2*I)*(a + b*x))])/b + (d^4*x^4*Log[1 - E^((2*I)*(a + b
*x))])/b + (4*c^3*d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))])/b + (4*c^3
*d*ArcTan[Tan[a]]*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))])/b^2 - (2*c^3*d
*Pi*Log[Cos[b*x]])/b^2 + (c^4*Log[Sin[a + b*x]])/b - (4*c^3*d*ArcTan[Tan[a]
]*Log[Sin[b*x + ArcTan[Tan[a]]]])/b^2 - ((2*I)*d^2*x*(3*c^2 + 3*c*d*x + d^2
*x^2)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - ((2*I)*c^3*d*PolyLog[2, E^((2*
I)*(b*x + ArcTan[Tan[a]]))])/b^2 + (3*c^2*d^2*PolyLog[3, E^((2*I)*(a + b*x)
)])/b^3 + (6*c*d^3*x*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 + (3*d^4*x^2*Poly
Log[3, E^((2*I)*(a + b*x))])/b^3 + ((3*I)*c*d^3*PolyLog[4, E^((2*I)*(a + b*
x))])/b^4 + ((3*I)*d^4*x*PolyLog[4, E^((2*I)*(a + b*x))])/b^4 - (3*d^4*Poly
Log[5, E^((2*I)*(a + b*x))])/(2*b^5) - 2*c^3*d*E^(I*ArcTan[Tan[a]])*x^2*Cot
[a]*Sqrt[Sec[a]^2]
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1158 vs. 2(134) = 268.
time = 0.17, size = 1159, normalized size = 7.68

method	result	size
risch	Expression too large to display	1159

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3+4/b
*c*d^3*ln(exp(I*(b*x+a))+1)*x^3-12*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*
x-12*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-12*I/b^2*c*d^3*polylog(2,-e
xp(I*(b*x+a)))*x^2-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2-8*I/b^3*c*d
^3*a^3*x+12*I/b^2*d^2*c^2*a^2*x-8*I/b*c^3*d*a*x-12/b^3*c^2*d^2*a^2*ln(exp(I
*(b*x+a)))-4/b^2*c^3*d*a*ln(exp(I*(b*x+a))-1)+8/b^2*c^3*d*a*ln(exp(I*(b*x+a
)))-4/b^4*c*d^3*a^3*ln(exp(I*(b*x+a))-1)-4*I/b^2*d^4*polylog(2,exp(I*(b*x+a
)))*x^3-4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*polylog(4,-
exp(I*(b*x+a)))*x+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x+2*I/b^4*d^4*a^4*
x+24*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))-4*I/b^2*c^3*d*polylog(2,exp(I*(
```

```
b*x+a)))+24*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))-4*I/b^2*c^3*d*polylog(2,-
exp(I*(b*x+a)))-4*I/b^2*c^3*d*a^2-6*I/b^4*c*d^3*a^4+8*I/b^3*d^2*c^2*a^3+I*c
^4*x+1/5*I/d*c^5-24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*polylog(5,exp
(I*(b*x+a)))/b^5+1/b*c^4*ln(exp(I*(b*x+a))-1)+1/b*c^4*ln(exp(I*(b*x+a))+1)-
2/b*c^4*ln(exp(I*(b*x+a)))-1/5*I*d^4*x^5-2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))+1
2/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))+12/b^3*c^2*d^2*polylog(3,-exp(I*(b*
x+a)))+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)-1/b^5*d^4*a^4*ln(1-exp(I*(b*x+a))
)+12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2+12/b^3*d^4*polylog(3,exp(I*(b*x
+a)))*x^2+8/5*I/b^5*a^5*d^4-I*d^3*c*x^4-2*I*d^2*c^2*x^3-2*I*d*c^3*x^2+4/b*c
^3*d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*(b*x+a)))*a+4/b*c^3*d*ln
(exp(I*(b*x+a))+1)*x+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+1/b*d^4*ln(exp(I*(b*x
+a))+1)*x^4+8/b^4*c*d^3*a^3*ln(exp(I*(b*x+a)))+6/b^3*c^2*d^2*a^2*ln(exp(I*(
b*x+a))-1)+6/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2+6/b*c^2*d^2*ln(1-exp(I*(b*x
+a)))*x^2-6/b^3*c^2*d^2*ln(1-exp(I*(b*x+a)))*a^2+24/b^3*c*d^3*polylog(3,-ex
p(I*(b*x+a)))*x+24/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1281 vs. 2(130) = 260.
time = 0.41, size = 1281, normalized size = 8.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/10*(10*c^4*log(sin(b*x + a)) - 40*a*c^3*d*log(sin(b*x + a))/b + 60*a^2*c^
2*d^2*log(sin(b*x + a))/b^2 - 40*a^3*c*d^3*log(sin(b*x + a))/b^3 + 10*a^4*d
^4*log(sin(b*x + a))/b^4 + (-2*I*(b*x + a)^5*d^4 - 10*(I*b*c*d^3 - I*a*d^4)
*(b*x + a)^4 - 240*d^4*polylog(5, -e^(I*b*x + I*a)) - 240*d^4*polylog(5, e^
(I*b*x + I*a)) - 20*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a)^3
 - 20*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 - I*a^3*d^4)*(b*x
+ a)^2 - 10*(-I*(b*x + a)^4*d^4 + 4*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 6*
(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a)^2 + 4*(-I*b^3*c^3*d
+ 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b*c*d^3 + I*a^3*d^4)*(b*x + a))*arctan2(sin(b
*x + a), cos(b*x + a) + 1) - 10*(I*(b*x + a)^4*d^4 + 4*(I*b*c*d^3 - I*a*d^4
)*(b*x + a)^3 + 6*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a)^2 +
 4*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 - I*a^3*d^4)*(b*x + a
))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 40*(I*b^3*c^3*d - 3*I*a*b^2*c
^2*d^2 + 3*I*a^2*b*c*d^3 + I*(b*x + a)^3*d^4 - I*a^3*d^4 + 3*(I*b*c*d^3 - I
*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x +
a))*dilog(-e^(I*b*x + I*a)) - 40*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2
*b*c*d^3 + I*(b*x + a)^3*d^4 - I*a^3*d^4 + 3*(I*b*c*d^3 - I*a*d^4)*(b*x + a
)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a))*dilog(e^(I*b
*x + I*a)) + 5*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*
c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d
```

```
^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^
2 + 2*cos(b*x + a) + 1) + 5*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a
)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d -
3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(b*x + a)^2 +
sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 240*(-I*b*c*d^3 - I*(b*x + a)*d^4 +
I*a*d^4)*polylog(4, -e^(I*b*x + I*a)) - 240*(-I*b*c*d^3 - I*(b*x + a)*d^4 +
 I*a*d^4)*polylog(4, e^(I*b*x + I*a)) + 120*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b
*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, -e^(I*b
*x + I*a)) + 120*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2
*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, e^(I*b*x + I*a)))/b^4)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1204 vs. 2(130) = 260.
time = 3.20, size = 1204, normalized size = 7.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(24*d^4*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*polylog(5,
cos(b*x + a) - I*sin(b*x + a)) + 24*d^4*polylog(5, -cos(b*x + a) + I*sin(b*
x + a)) + 24*d^4*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) + 4*(I*b^3*d^4*
x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(cos(b*x +
a) + I*sin(b*x + a)) + 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*
d^2*x - I*b^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + 4*(-I*b^3*d^4*x
^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog(-cos(b*x +
a) + I*sin(b*x + a)) + 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d
^2*x + I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^4*d^4*x^4 +
4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x
+ a) + I*sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2
*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b
^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-
1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^4*c^4 - 4*a*b^3*c^3*d + 6
*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/2*cos(b*x + a) - 1/2*I*s
in(b*x + a) + 1/2) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4
*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)
*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 +
 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*
a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 24*(-I*b*d
^4*x - I*b*c*d^3)*polylog(4, cos(b*x + a) + I*sin(b*x + a)) + 24*(I*b*d^4*x
 + I*b*c*d^3)*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 24*(I*b*d^4*x + I
*b*c*d^3)*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) + 24*(-I*b*d^4*x - I*b
*c*d^3)*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^
2*c*d^3*x + b^2*c^2*d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 12*(b^
```

```
2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, cos(b*x + a) - I*sin(b*
x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*
x + a) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*p
olylog(3, -cos(b*x + a) - I*sin(b*x + a)))/b^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \cos(a+bx) \csc(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*csc(b*x+a),x)
[Out] Integral((c + d*x)**4*cos(a + b*x)*csc(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^4*cos(b*x + a)*csc(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)\,{(c+d\,x)}^4}{\sin(a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x),x)
[Out] int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x), x)
```

3.33 $\int (c+dx)^3 \cot(a+bx)\,dx$

Optimal. Leaf size=127

$$-\frac{i(c+dx)^4}{4d}+\frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a$$

```
[Out] -1/4*I*(d*x+c)^4/d+(d*x+c)^3*ln(1-exp(2*I*(b*x+a)))/b-3/2*I*d*(d*x+c)^2*pol
ylog(2,exp(2*I*(b*x+a)))/b^2+3/2*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^
3+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4
```

Rubi [A]
time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3798, 2221, 2611, 6744, 2320, 6724}

$$\frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^2(c+dx)\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}-\frac{3id(c+dx)^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}+\frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{i(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x],x]
```

```
[Out] ((-1/4*I)*(c + d*x)^4)/d + ((c + d*x)^3*Log[1 - E^((2*I)*(a + b*x))])/b - (
((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c +
d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) + (((3*I)/4)*d^3*PolyLog[4, E
^((2*I)*(a + b*x))])/b^4
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \cot(a+bx)\,dx = -\frac{i(c+dx)^4}{4d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1-e^{2i(a+bx)}}\,dx$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{(3d)\int (c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{(3id}{}$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{3d^2}{}$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{3d^2}{}$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{3d^2}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 410 vs. $2(127) = 254$.

time = 1.81, size = 410, normalized size = 3.23

$$\frac{6ib^3c^2d\pi x-4ib^4cd^2x^3-ib^4d^3x^4-12ib^3c^2dx\text{ArcTan}(\tan(a))+6b^4c^2dx^2\cot(a)+6b^2c^2d\pi\log\left(1+e^{-2ibx}\right)+12b^3cd^2x^2\log\left(1-e^{2i(a+bx)}\right)+4b^3d^3x^3\log\left(1-e^{2i(a+bx)}\right)+12b^3c^2dx\log\left(1-e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)+12b^2c^2d\text{ArcTan}(\tan(a))\log\left(1-e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)-6b^2c^2d\pi\log(\cos(bx))+4b^3c^3\log(\sin(a+bx))-12b^2c^2d\text{ArcTan}(\tan(a))\log(\sin(bx+\text{ArcTan}(\tan(a))))-6ib^2d^2x(2c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)-6ib^2c^2d\text{PolyLog}\left(2,e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)+6bcd^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)+6bd^3x\text{PolyLog}\left(3,e^{2i(a+bx)}\right)+3id^3\text{PolyLog}\left(4,e^{2i(a+bx)}\right)-6b^4c^2de^{i\text{ArcTan}(\tan(a))}x^2\cot(a)\sqrt{\sec^2(a)}}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x],x]
```

```
[Out] ((6*I)*b^3*c^2*d*Pi*x - (4*I)*b^4*c*d^2*x^3 - I*b^4*d^3*x^4 - (12*I)*b^3*c^
2*d*x*ArcTan[Tan[a]] + 6*b^4*c^2*d*x^2*Cot[a] + 6*b^2*c^2*d*Pi*Log[1 + E^((
-2*I)*b*x)] + 12*b^3*c*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] + 4*b^3*d^3*x^3
*Log[1 - E^((2*I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 - E^((2*I)*(b*x + ArcT
an[Tan[a]]))] + 12*b^2*c^2*d*ArcTan[Tan[a]]*Log[1 - E^((2*I)*(b*x + ArcTan[
Tan[a]]))] - 6*b^2*c^2*d*Pi*Log[Cos[b*x]] + 4*b^3*c^3*Log[Sin[a + b*x]] - 1
2*b^2*c^2*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] - (6*I)*b^2*d^2*x
*(2*c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))] - (6*I)*b^2*c^2*d*PolyLog[2, E
^((2*I)*(b*x + ArcTan[Tan[a]]))] + 6*b*c*d^2*PolyLog[3, E^((2*I)*(a + b*x))
] + 6*b*d^3*x*PolyLog[3, E^((2*I)*(a + b*x))] + (3*I)*d^3*PolyLog[4, E^((2*
I)*(a + b*x))] - 6*b^4*c^2*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^2]
)/(4*b^4)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 791 vs. 2(108) = 216.
time = 0.10, size = 792, normalized size = 6.24

method	result
risch	$-\frac{id^3x^4}{4}+\frac{c^3\ln\left(e^{i(bx+a)}+1\right)}{b}-\frac{2c^3\ln\left(e^{i(bx+a)}\right)}{b}+\frac{c^3\ln\left(e^{i(bx+a)}-1\right)}{b}+ic^3x+\frac{ic^4}{4d}-\frac{6icd^2\,\text{polylog}\left(2,e^{i(bx+a)}\right)x}{b^2}-\frac{6ic}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*I/b^4*d^3*a^4+6*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)))-I*d^2*c*x^3-3/2*I
*d*c^2*x^2-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+2/b^4*d^3*a^3*ln(exp(I*(b*x+a
)))+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,-exp(I*(b*x
+a)))+6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,exp(I*(b*x
+a)))*x-1/4*I*d^3*x^4+1/b*c^3*ln(exp(I*(b*x+a))+1)-2/b*c^3*ln(exp(I*(b*x+a)
))+1/b*c^3*ln(exp(I*(b*x+a))-1)+I*c^3*x+1/4*I/d*c^4-6*I/b^2*c*d^2*polylog(2
,exp(I*(b*x+a)))*x+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+1/b*d^3*ln(exp(I*(
b*x+a))+1)*x^3+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1-exp(I*(b*x+a
)))*a^3+6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-
1)+3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*
c^2*d*ln(1-exp(I*(b*x+a)))*a+3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b*c*d^2*l
n(1-exp(I*(b*x+a)))*x^2-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-6/b^3*c*d^2*a^
2*ln(exp(I*(b*x+a)))-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)-3*I/b^2*c^2*d*polyl
og(2,exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-3*I/b^2*d^3*p
```

```
olylog(2,-exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2+4*I
/b^3*c*d^2*a^3-3*I/b^2*c^2*d*a^2-2*I/b^3*d^3*a^3*x+6*I/b^2*c*d^2*a^2*x-6*I/
b*c^2*d*a*x-6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 759 vs. $2(104) = 208$.
time = 0.38, size = 759, normalized size = 5.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(4*c^3*log(sin(b*x + a)) - 12*a*c^2*d*log(sin(b*x + a))/b + 12*a^2*c*d^
2*log(sin(b*x + a))/b^2 - 4*a^3*d^3*log(sin(b*x + a))/b^3 + (-I*(b*x + a)^4
*d^3 - 4*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^3 + 24*I*d^3*polylog(4, -e^(I*b*x
+ I*a)) + 24*I*d^3*polylog(4, e^(I*b*x + I*a)) - 6*(I*b^2*c^2*d - 2*I*a*b*c
*d^2 + I*a^2*d^3)*(b*x + a)^2 - 4*(-I*(b*x + a)^3*d^3 + 3*(-I*b*c*d^2 + I*a
*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a))
*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 4*(I*(b*x + a)^3*d^3 + 3*(I*b*c*
d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b
*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 12*(I*b^2*c^2*d - 2*I*a
*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a
))*dilog(-e^(I*b*x + I*a)) - 12*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^
2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*dilog(e^(I*b*x + I*a
)) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d -
2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*c
os(b*x + a) + 1) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3
*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 - 2*cos(b*x + a) + 1) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polyl
og(3, -e^(I*b*x + I*a)) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, e
^(I*b*x + I*a)))/b^3)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(104) = 208$.
time = 2.80, size = 818, normalized size = 6.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4,
 cos(b*x + a) - I*sin(b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) + I*sin(
b*x + a)) + 6*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) - 3*(I*b^2*d
^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(cos(b*x + a) + I*sin(b*x + a)
```

```
) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(cos(b*x + a) -
 I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog
(-cos(b*x + a) + I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b
^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b^3*d^3*x^3 + 3*b^3*c*d^
2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + (
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) -
 I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*
d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1
/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^
2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^3*d^3*x^3
 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^
3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 6*(b*d^3*x + b*c*d^2)*polylog(
3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*
x + a) - I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) +
 I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) - I*sin(b
*x + a)))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cos (a+bx) \csc (a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*csc(b*x+a),x)
[Out] Integral((c + d*x)**3*cos(a + b*x)*csc(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^3*cos(b*x + a)*csc(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (a+b\,x)\,\left(c+d\,x\right)^3}{\sin (a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x),x)
[Out] int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x), x)
```

3.34 $\int (c+dx)^2 \cot(a+bx)\,dx$

Optimal. Leaf size=93

$$-\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}$$

```
[Out] -1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1-exp(2*I*(b*x+a)))/b-I*d*(d*x+c)*polylog(2
,exp(2*I*(b*x+a)))/b^2+1/2*d^2*polylog(3,exp(2*I*(b*x+a)))/b^3
```

Rubi [A]
time = 0.11, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3798, 2221, 2611, 2320, 6724}

$$\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}-\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{i(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x],x]
```

```
[Out] ((-1/3*I)*(c + d*x)^3)/d + ((c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))])/b - (
I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (d^2*PolyLog[3, E^((2*
I)*(a + b*x))])/(2*b^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^2 \cot(a+bx)\,dx &= -\frac{i(c+dx)^3}{3d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1-e^{2i(a+bx)}}\,dx \\
&= -\frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{(2d)\int (c+dx)\log\left(1-e^{2i(a+bx)}\right.}{b} \\
&= -\frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{(id^2)}{} \\
&= -\frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{d^2\text{Su}}{} \\
&= -\frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{d^2\text{Li}}{}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 287 vs. $2(93) = 186$.
time = 1.22, size = 287, normalized size = 3.09

$$
\frac{6ib^2cd\pi x - 2ib^3d^2x^3 - 12ib^2cdx\text{ArcTan}(\tan(a)) + 6b^3cdx^2\cot(a) + 6bcd\pi\log\left(1+e^{-2ibx}\right) + 6b^2d^2x^2\log\left(1-e^{2i(a+bx)}\right) + 12b^2cdx\log\left(1-e^{2i(bx+\text{ArcTan}(\tan(a)))}\right) + 12bcd\text{ArcTan}(\tan(a))\log\left(1-e^{2i(bx+\text{ArcTan}(\tan(a)))}\right) - 6bcd\pi\log(\cos(bx)) + 6b^2c^2\log(\sin(a+bx)) - 12bcd\text{ArcTan}(\tan(a))\log(\sin(bx+\text{ArcTan}(\tan(a)))) - 6ibd^2x\text{PolyLog}\left(2,e^{2i(a+bx)}\right) - 6ibcd\text{PolyLog}\left(2,e^{2i(bx+\text{ArcTan}(\tan(a)))}\right) + 3d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right) - 6b^3cde^{i\text{ArcTan}(\tan(a))}x^2\cot(a)\sqrt{\sec^2(a)}}{6b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x],x]
```

```
[Out] ((6*I)*b^2*c*d*Pi*x - (2*I)*b^3*d^2*x^3 - (12*I)*b^2*c*d*x*ArcTan[Tan[a]] +
 6*b^3*c*d*x^2*Cot[a] + 6*b*c*d*Pi*Log[1 + E^((-2*I)*b*x)] + 6*b^2*d^2*x^2*
Log[1 - E^((2*I)*(a + b*x))] + 12*b^2*c*d*x*Log[1 - E^((2*I)*(b*x + ArcTan[
Tan[a]]))] + 12*b*c*d*ArcTan[Tan[a]]*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]
```

```
))] - 6*b*c*d*Pi*Log[Cos[b*x]] + 6*b^2*c^2*Log[Sin[a + b*x]] - 12*b*c*d*Arc
Tan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] - (6*I)*b*d^2*x*PolyLog[2, E^((2
*I)*(a + b*x))] - (6*I)*b*c*d*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))]
+ 3*d^2*PolyLog[3, E^((2*I)*(a + b*x))] - 6*b^3*c*d*E^(I*ArcTan[Tan[a]])*x^
2*Cot[a]*Sqrt[Sec[a]^2])/(6*b^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(81) = 162$.
time = 0.08, size = 477, normalized size = 5.13

method	result
risch	$-\frac{4icdax}{b}-\frac{2id^2\,\mathrm{polylog}(2,e^{i(bx+a)})x}{b^2}+\frac{2cd\ln(1-e^{i(bx+a)})a}{b^2}+\frac{2cd\ln(e^{i(bx+a)}+1)x}{b}+\frac{2cd\ln(1-e^{i(bx+a)})x}{b}+\frac{2d^2\,\mathrm{polylog}(3}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -4*I/b*c*d*a*x-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))+1/b^3*d^2*a^2*ln(exp(I*(b*x
+a))-1)+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2
+1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+2/b*c*d*
ln(exp(I*(b*x+a))+1)*x+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c*d*a*ln(exp(I*
(b*x+a)))-2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)+2*I/b^2*d^2*a^2*x-2*I/b^2*c*d*a^
2-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*d^2*polylog(2,-exp(I*(b*x
+a)))*x-2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-2*I/b^2*c*d*polylog(2,exp(I*
(b*x+a)))-1/3*I*d^2*x^3+1/b*c^2*ln(exp(I*(b*x+a))-1)+1/b*c^2*ln(exp(I*(b*x+
a))+1)-2/b*c^2*ln(exp(I*(b*x+a)))+4/3*I/b^3*d^2*a^3-I*d*c*x^2+I*c^2*x+1/3*I
/d*c^3+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))
/b^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(78) = 156$.
time = 0.34, size = 411, normalized size = 4.42

$$\frac{6\,c^2\log\left(\sin\left(bx+a\right)\right)-\frac{12\,acd\log(\sin(bx+a))}{b}+\frac{6\,a^2d^2\log(\sin(bx+a))}{b^2}+\frac{-2i\,(bx+a)^3d^2-6\,(i\,bcd-i\,ad^2)(bx+a)^2+12\,d^2\mathrm{Li}_3(-e^{(i\,bx+i\,a)})+12\,d^2\mathrm{Li}_3(e^{(i\,bx+i\,a)})-6\left(-i\,(bx+a)^2d^2+2\,(-i\,bcd+i\,ad^2)(bx+a)\right)\arctan(\sin(bx+a),\cos(bx+a)+1)-6\left(i\,(bx+a)^2d^2+2\,(i\,bcd-i\,ad^2)(bx+a)\right)\arctan(\sin(bx+a),-\cos(bx+a)+1)-12\,(i\,bcd+i\,(bx+a)d^2-i\,ad^2)\mathrm{Li}_2(-e^{(i\,bx+i\,a)})-12\,(i\,bcd+i\,(bx+a)d^2-i\,ad^2)\mathrm{Li}_2(e^{(i\,bx+i\,a)})+3\left((bx+a)^2d^2+2\,(bcd-ad^2)(bx+a)\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2+2\,\cos(bx+a)+1\right)+3\left((bx+a)^2d^2+2\,(bcd-ad^2)(bx+a)\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2-2\,\cos(bx+a)+1\right)}{b^2}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(6*c^2*log(sin(b*x + a)) - 12*a*c*d*log(sin(b*x + a))/b + 6*a^2*d^2*log
(sin(b*x + a))/b^2 + (-2*I*(b*x + a)^3*d^2 - 6*(I*b*c*d - I*a*d^2)*(b*x + a
)^2 + 12*d^2*polylog(3, -e^(I*b*x + I*a)) + 12*d^2*polylog(3, e^(I*b*x + I*
a)) - 6*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2(sin
(b*x + a), cos(b*x + a) + 1) - 6*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)
*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 12*(I*b*c*d + I*(b*x
 + a)*d^2 - I*a*d^2)*dilog(-e^(I*b*x + I*a)) - 12*(I*b*c*d + I*(b*x + a)*d^
2 - I*a*d^2)*dilog(e^(I*b*x + I*a)) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2
```

```
)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 3*
((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 - 2*cos(b*x + a) + 1))/b^2)/b
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(78) = 156$.
time = 3.69, size = 502, normalized size = 5.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, cos
(b*x + a) - I*sin(b*x + a)) + 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x +
a)) + 2*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b
*c*d)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog
(cos(b*x + a) - I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(-cos(b*x +
 a) + I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(-cos(b*x + a) - I*sin
(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*sin
(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) - I
*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a)
+ 1/2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(
b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c
*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^
2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1))/b^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \cos (a + bx) \csc (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)*csc(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)*csc(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2}{\sin\left(a+b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x),x)
[Out] int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x), x)
```

3.35 $\int (c+dx)\cot(a+bx)\,dx$

Optimal. Leaf size=65

$$-\frac{i(c+dx)^2}{2d}+\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}$$

```
[Out] -1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b-1/2*I*d*polylog(2,exp(2
*I*(b*x+a)))/b^2
```

Rubi [A]
time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {3798, 2221, 2317, 2438}

$$-\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}+\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{i(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x],x]
```

```
[Out] ((-1/2*I)*(c + d*x)^2)/d + ((c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b - ((I
/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
```

```
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cot(a+bx)\,dx &= -\frac{i(c+dx)^2}{2d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1-e^{2i(a+bx)}}\,dx\\ &= -\frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{d\int \log\left(1-e^{2i(a+bx)}\right)\,dx}{b}\\ &= -\frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} + \frac{(id)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2i(a+bx)}\right)}{2b^2}\\ &= -\frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 188 vs. 2(65) = 130.
time = 5.45, size = 188, normalized size = 2.89

$$\frac{1}{2}dx^2\cot(a) + \frac{c(\log(\cos(a+bx)) + \log(\tan(a+bx)))}{b} - \frac{d\csc(a)\sec(a)\left(b^2e^{i\text{ArcTan}(\tan(a))}x^2 + \frac{\left(ibx(-\pi+2\text{ArcTan}(\tan(a))) - \pi\log\left(1+e^{-2ibx}\right) - 2(bx+\text{ArcTan}(\tan(a)))\log\left(1-e^{2i(bx+\text{ArcTan}(\tan(a)))}\right) + \pi\log(\cos(bx)) + 2\text{ArcTan}(\tan(a))\log(\sin(bx+\text{ArcTan}(\tan(a)))) + i\text{PolyLog}\left(2, e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)\right)\tan(a)}{\sqrt{1+\tan^2(a)}}\right)}{2b^2\sqrt{\sec^2(a)\left(\cos^2(a)+\sin^2(a)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cot[a + b*x],x]
```

```
[Out] (d*x^2*Cot[a])/2 + (c*(Log[Cos[a + b*x]] + Log[Tan[a + b*x]]))/b - (d*Csc[a
]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) -
 Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*
x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + A
rcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/S
qrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(55) = 110.
time = 0.08, size = 215, normalized size = 3.31

method	result
risch	$-\frac{idx^2}{2} + icx + \frac{c\ln\left(e^{i(bx+a)}-1\right)}{b} + \frac{c\ln\left(e^{i(bx+a)}+1\right)}{b} - \frac{2c\ln\left(e^{i(bx+a)}\right)}{b} - \frac{2idax}{b} - \frac{ida^2}{b^2} + \frac{d\ln\left(1-e^{i(bx+a)}\right)x}{b} + \frac{d\ln(1-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

[Out] -1/2*I*d*x^2+I*c*x+1/b*c*ln(exp(I*(b*x+a))-1)+1/b*c*ln(exp(I*(b*x+a))+1)-2/b*c*ln(exp(I*(b*x+a)))-2*I/b*d*a*x-I/b^2*d*a^2+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b*x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*polylog(2,-exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1)+2/b^2*d*a*ln(exp(I*(b*x+a)))

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(52) = 104$.
time = 0.35, size = 190, normalized size = 2.92

$$\frac{-i\,b^2dx^2-2i\,b^2cx-2i\,bdx\arctan\left(\sin\left(bx+a\right),-\cos\left(bx+a\right)+1\right)+2i\,bc\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)-1\right)-2\left(-i\,bdx-i\,bc\right)\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)+1\right)-2i\,d\mathrm{Li}_2\left(-e^{(i\,bx+i\,a)}\right)-2i\,d\mathrm{Li}_2\left(e^{(i\,bx+i\,a)}\right)+\left(bdx+bc\right)\log\left(\cos\left(bx+a\right)^2+\sin\left(bx+a\right)^2+2\,\cos\left(bx+a\right)+1\right)+\left(bdx+bc\right)\log\left(\cos\left(bx+a\right)^2+\sin\left(bx+a\right)^2-2\,\cos\left(bx+a\right)+1\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")

[Out] 1/2*(-I*b^2*d*x^2 - 2*I*b^2*c*x - 2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 2*I*b*c*arctan2(sin(b*x + a), cos(b*x + a) - 1) - 2*(-I*b*d*x - I*b*c)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*I*d*dilog(-e^(I*b*x + I*a)) - 2*I*d*dilog(e^(I*b*x + I*a)) + (b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1))/b^2

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(52) = 104$.
time = 2.26, size = 250, normalized size = 3.85

$$\frac{-i\,d\mathrm{Li}_2(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right))+i\,d\mathrm{Li}_2(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right))+i\,d\mathrm{Li}_2(-\cos\left(bx+a\right)+i\,\sin\left(bx+a\right))-i\,d\mathrm{Li}_2(-\cos\left(bx+a\right)-i\,\sin\left(bx+a\right))+\left(bdx+bc\right)\log\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)+1\right)+\left(bdx+bc\right)\log\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)+1\right)+\left(bc-ad\right)\log\left(-\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}i\,\sin\left(bx+a\right)+\frac{1}{2}\right)+\left(bc-ad\right)\log\left(-\frac{1}{2}\,\cos\left(bx+a\right)-\frac{1}{2}i\,\sin\left(bx+a\right)+\frac{1}{2}\right)+\left(bdx+ad\right)\log\left(-\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)+1\right)+\left(bdx+ad\right)\log\left(-\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)+1\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")

[Out] 1/2*(-I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*dilog(cos(b*x + a) - I*sin(b*x + a)) + I*d*dilog(-cos(b*x + a) + I*sin(b*x + a)) - I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b*d*x + b*c)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + (b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b*c - a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b*c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b*d*x + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + 1))/b^2

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+dx\right)\cos\left(a+bx\right)\csc\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)*csc(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*cos(b*x + a)*csc(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x))/sin(a + b*x),x)
```

```
[Out] int((cos(a + b*x)*(c + d*x))/sin(a + b*x), x)
```

3.36 $\int \frac{\cot(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{\cot(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)/(d*x+c),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[a + b*x]/(c + d*x),x]
[Out] Defer[Int][Cot[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\cot(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 3.57, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[a + b*x]/(c + d*x),x]
[Out] Integrate[Cot[a + b*x]/(c + d*x), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\csc(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)/(d*x+c),x)
[Out] int(cos(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(cos(b*x + a)*csc(b*x + a)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x)
[Out] Integral(cos(a + b*x)*csc(a + b*x)/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\cos(a+b\,x)}{\sin(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(sin(a + b*x)*(c + d*x)),x)
[Out] int(cos(a + b*x)/(sin(a + b*x)*(c + d*x)), x)
```

3.37 $\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=17

$$\operatorname{Int}\left(\frac{\cot(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)/(d*x+c)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[a + b*x]/(c + d*x)^2,x]
[Out] Defer[Int][Cot[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 7.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[a + b*x]/(c + d*x)^2,x]
[Out] Integrate[Cot[a + b*x]/(c + d*x)^2, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)\csc\left(bx+a\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)
[Out] int(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(cos(b*x + a)*csc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos{\left(a + bx \right)} \csc{\left(a + bx \right)}}{\left(c + dx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)**2,x)
[Out] Integral(cos(a + b*x)*csc(a + b*x)/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\cos\left(a+b\,x\right)}{\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(sin(a + b*x)*(c + d*x)^2),x)
[Out] int(cos(a + b*x)/(sin(a + b*x)*(c + d*x)^2), x)
```

3.38 $\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=23

$$\text{Int}((c+dx)^m \cot(a+bx)\csc(a+bx),x)$$

```
[Out] CannotIntegrate((d*x+c)^m*cot(b*x+a)*csc(b*x+a),x)
```

Rubi [A]
time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx = \int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx$$

Mathematica [A]
time = 2.33, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x], x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\left(\csc^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x)
[Out] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \cos(a + bx) \csc^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*csc(b*x+a)**2,x)
[Out] Integral((c + d*x)**m*cos(a + b*x)*csc(a + b*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos(a + b\,x)\,(c + d\,x)^m}{\sin(a + b\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x)^2,x)
[Out] int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x)^2, x)
```

3.39 $\int (c+dx)^4 \cot(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=208

$$-\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^4\csc(a+bx)}{b}+\frac{12id^2(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{12id^2(c+dx)}{}$$

```
[Out] -8*d*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b^2-(d*x+c)^4*csc(b*x+a)/b+12*I*d^2*
(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^3-12*I*d^2*(d*x+c)^2*polylog(2,exp(I
*(b*x+a)))/b^3-24*d^3*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^4+24*d^3*(d*x+c)
*polylog(3,exp(I*(b*x+a)))/b^4-24*I*d^4*polylog(4,-exp(I*(b*x+a)))/b^5+24*I
*d^4*polylog(4,exp(I*(b*x+a)))/b^5
```

Rubi [A]
time = 0.11, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4495, 4268, 2611, 6744, 2320, 6724}

$$-\frac{24id^4\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^5}+\frac{24id^4\text{Li}_4\left(e^{i(a+bx)}\right)}{b^5}-\frac{24d^3(c+dx)\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^4}+\frac{24d^3(c+dx)\text{Li}_3\left(e^{i(a+bx)}\right)}{b^4}+\frac{12id^2(c+dx)^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{12id^2(c+dx)^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}-\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^4\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] (-8*d*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^4*Csc[a + b*x]
)/b + ((12*I)*d^2*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((12*I)*d
^2*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (24*d^3*(c + d*x)*PolyLog
[3, -E^(I*(a + b*x))])/b^4 + (24*d^3*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])
/b^4 - ((24*I)*d^4*PolyLog[4, -E^(I*(a + b*x))])/b^5 + ((24*I)*d^4*PolyLog[
4, E^(I*(a + b*x))])/b^5
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \cot(a+bx)\csc(a+bx)\,dx = -\frac{(c+dx)^4\csc(a+bx)}{b}+\frac{(4d)\int (c+dx)^3\csc(a+bx)\,dx}{b}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^4\csc(a+bx)}{b}-\frac{(12d^2)\ldots}{}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^4\csc(a+bx)}{b}+\frac{12id^2(c\ldots}{}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^4\csc(a+bx)}{b}+\frac{12id^2(c\ldots}{}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^4\csc(a+bx)}{b}+\frac{12id^2(c\ldots}{}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^4\csc(a+bx)}{b}+\frac{12id^2(c\ldots}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 458 vs. $2(208) = 416$.
time = 1.95, size = 458, normalized size = 2.20

$$-\frac{8b^3c^3d\tanh^{-1}\left(e^{i(a+bx)}\right)+b^4c^4\csc(a+bx)+4b^4c^3dx\csc(a+bx)+6b^4c^2d^2x^2\csc(a+bx)+4b^4cd^3x^3\csc(a+bx)+b^4d^4x^4\csc(a+bx)-12b^3c^2d^2x\log\left(1-e^{i(a+bx)}\right)-12b^3cd^3x^2\log\left(1-e^{i(a+bx)}\right)-4b^3d^4x^3\log\left(1-e^{i(a+bx)}\right)+12b^3c^2d^2x\log\left(1+e^{i(a+bx)}\right)+12b^3cd^3x^2\log\left(1+e^{i(a+bx)}\right)+4b^3d^4x^3\log\left(1+e^{i(a+bx)}\right)-12ib^2d^2(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)+12ib^2d^2(c+dx)^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)+24bcd^3\text{PolyLog}\left(3,-e^{i(a+bx)}\right)+24bd^4x\text{PolyLog}\left(3,-e^{i(a+bx)}\right)-24bcd^3\text{PolyLog}\left(3,e^{i(a+bx)}\right)-24bd^4x\text{PolyLog}\left(3,e^{i(a+bx)}\right)+24id^4\text{PolyLog}\left(4,-e^{i(a+bx)}\right)-24id^4\text{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] -((8*b^3*c^3*d*ArcTanh[E^(I*(a + b*x))] + b^4*c^4*Csc[a + b*x] + 4*b^4*c^3*
d*x*Csc[a + b*x] + 6*b^4*c^2*d^2*x^2*Csc[a + b*x] + 4*b^4*c*d^3*x^3*Csc[a +
 b*x] + b^4*d^4*x^4*Csc[a + b*x] - 12*b^3*c^2*d^2*x*Log[1 - E^(I*(a + b*x))
] - 12*b^3*c*d^3*x^2*Log[1 - E^(I*(a + b*x))] - 4*b^3*d^4*x^3*Log[1 - E^(I*
(a + b*x))] + 12*b^3*c^2*d^2*x*Log[1 + E^(I*(a + b*x))] + 12*b^3*c*d^3*x^2*
Log[1 + E^(I*(a + b*x))] + 4*b^3*d^4*x^3*Log[1 + E^(I*(a + b*x))] - (12*I)*
b^2*d^2*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))] + (12*I)*b^2*d^2*(c + d*x)
^2*PolyLog[2, E^(I*(a + b*x))] + 24*b*c*d^3*PolyLog[3, -E^(I*(a + b*x))] +
24*b*d^4*x*PolyLog[3, -E^(I*(a + b*x))] - 24*b*c*d^3*PolyLog[3, E^(I*(a + b
*x))] - 24*b*d^4*x*PolyLog[3, E^(I*(a + b*x))] + (24*I)*d^4*PolyLog[4, -E^(
I*(a + b*x))] - (24*I)*d^4*PolyLog[4, E^(I*(a + b*x))])/b^5)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(190) = 380$.
time = 0.13, size = 716, normalized size = 3.44

method	result
risch	$\frac{24id^4\,\text{polylog}(4,e^{i(bx+a)})}{b^5}-\frac{8d\,c^3\,\text{arctanh}(e^{i(bx+a)})}{b^2}+\frac{8d^4a^3\,\text{arctanh}(e^{i(bx+a)})}{b^5}+\frac{24d^3c\,\text{polylog}(3,e^{i(bx+a)})}{b^4}-\frac{24d^3c\,\text{polylog}(3}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 24*I*d^4*polylog(4,exp(I*(b*x+a)))/b^5-2*I*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x
^2+4*c^3*d*x+c^4)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)-8*d/b^2*c^3*arctanh
(exp(I*(b*x+a)))+8*d^4/b^5*a^3*arctanh(exp(I*(b*x+a)))+24*d^3/b^4*c*polylog
(3,exp(I*(b*x+a)))-24*d^3/b^4*c*polylog(3,-exp(I*(b*x+a)))+24*d^4/b^4*polyl
og(3,exp(I*(b*x+a)))*x-24*d^4/b^4*polylog(3,-exp(I*(b*x+a)))*x+24*I*d^3/b^3
*c*polylog(2,-exp(I*(b*x+a)))*x-24*I*d^3/b^3*c*polylog(2,exp(I*(b*x+a)))*x-
12*d^2/b^2*c^2*ln(exp(I*(b*x+a))+1)*x-12*d^2/b^3*c^2*ln(exp(I*(b*x+a))+1)*a
+12*d^2/b^2*c^2*ln(1-exp(I*(b*x+a)))*x+12*d^2/b^3*c^2*ln(1-exp(I*(b*x+a)))*
a-4*d^4/b^2*ln(exp(I*(b*x+a))+1)*x^3-4*d^4/b^5*ln(exp(I*(b*x+a))+1)*a^3+4*d
^4/b^2*ln(1-exp(I*(b*x+a)))*x^3+4*d^4/b^5*ln(1-exp(I*(b*x+a)))*a^3+12*I*d^2
/b^3*c^2*polylog(2,-exp(I*(b*x+a)))-12*I*d^2/b^3*c^2*polylog(2,exp(I*(b*x+a
)))+12*I*d^4/b^3*polylog(2,-exp(I*(b*x+a)))*x^2-12*I*d^4/b^3*polylog(2,exp(
I*(b*x+a)))*x^2+24*d^2/b^3*c^2*a*arctanh(exp(I*(b*x+a)))-24*d^3/b^4*c*a^2*a
```

```
rctanh(exp(I*(b*x+a)))-12*d^3/b^2*c*ln(exp(I*(b*x+a))+1)*x^2+12*d^3/b^4*c*l
n(exp(I*(b*x+a))+1)*a^2+12*d^3/b^2*c*ln(1-exp(I*(b*x+a)))*x^2-12*d^3/b^4*c*
ln(1-exp(I*(b*x+a)))*a^2-24*I*d^4*polylog(4,-exp(I*(b*x+a)))/b^5
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2948 vs. $2(184) = 368$.
time = 0.56, size = 2948, normalized size = 14.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x + 2*
a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x +
2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(
2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x +
 a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a))*c^
3*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b)
- 6*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x + 2*
a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x +
2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(
2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x +
 a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a))*a*
c^2*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
*b^2) + 6*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*
x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*
b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) -
 (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos
(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x +
a))*a^2*c*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a
) + 1)*b^3) - 2*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*co
s(2*b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*
cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a)
+ 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*l
og(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(
b*x + a))*a^3*d^4/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x +
 2*a) + 1)*b^4) + c^4/sin(b*x + a) - 4*a*c^3*d/(b*sin(b*x + a)) + 6*a^2*c^2
*d^2/(b^2*sin(b*x + a)) - 4*a^3*c*d^3/(b^3*sin(b*x + a)) + a^4*d^4/(b^4*sin
(b*x + a)) - 2*(2*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b
^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) - ((b*x + a)^3*d^4 + 3*(b*c*d
^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)
)*cos(2*b*x + 2*a) + (-I*(b*x + a)^3*d^4 + 3*(-I*b*c*d^3 + I*a*d^4)*(b*x +
a)^2 + 3*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a))*sin(2*b*x
+ 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 2*((b*x + a)^3*d^4 + 3*(b
```

```
*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x
+ a) - ((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2
- 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (-I*(b*x + a)^3*d^4
+ 3*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^2 + 3*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3
- I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x +
a) + 1) - ((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d
^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2)*cos(b*x + a) - 6*(b^2*c^2*d^2 - 2*
a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a) - (b^
2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(
b*x + a))*cos(2*b*x + 2*a) - (I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*(b*x + a)^2
*d^4 + I*a^2*d^4 + 2*(I*b*c*d^3 - I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dil
og(-e^(I*b*x + I*a)) + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2
*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a) - (b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x +
a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (-I*
b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*(b*x + a)^2*d^4 - I*a^2*d^4 + 2*(-I*b*c*d^3
 + I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b*x
+ a)^3*d^4 + 3*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^2 + 3*(-I*b^2*c^2*d^2 + 2*I
*a*b*c*d^3 - I*a^2*d^4)*(b*x + a) + (I*(b*x + a)^3*d^4 + 3*(I*b*c*d^3 - I*a
*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a)
)*cos(2*b*x + 2*a) - ((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3
*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos
(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*(b*x + a)^3*d^4 + 3
*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a
^2*d^4)*(b*x + a) + (-I*(b*x + a)^3*d^4 + 3*(-I*b*c*d^3 + I*a*d^4)*(b*x + a
)^2 + 3*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a))*cos(2*b*x +
 2*a) + ((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2
 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 +
 sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 12*(d^4*cos(2*b*x + 2*a) + I*d^4*si
n(2*b*x + 2*a) - d^4)*polylog(4, -e^(I*b*x + I*a)) + 12*(d^4*cos(2*b*x + 2*
a) + I*d^4*sin(2*b*x + 2*a) - d^4)*polylog(4, e^(I*b*x + I*a)) + 12*(-I*b*c
*d^3 - I*(b*x + a)*d^4 + I*a*d^4 + (I*b*c*d^3 + I*(b*x + a)*d^4 - I*a*d^4)*
cos(2*b*x + 2*a) - (b*c*d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2*a))*poly
log(3, -e^(I*b*x + I*a)) + 12*(I*b*c*d^3 + I*(b*x + a)*d^4 - I*a*d^4 + (-I*
b*c*d^3 - I*(b*x + a)*d^4 + I*a*d^4)*cos(2*b*x + 2*a) + (b*c*d^3 + (b*x + a
)*d^4 - a*d^4)*sin(2*b*x + 2*a))*polylog(3, e^(...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1021 vs. $2(184) = 368$.
time = 3.11, size = 1021, normalized size = 4.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c
^4 - 12*I*d^4*polylog(4, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I
*d^4*polylog(4, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 12*I*d^4*poly
log(4, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(4, -
cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*(I*b^2*d^4*x^2 + 2*I*b^2*c*
d^3*x + I*b^2*c^2*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) +
6*(-I*b^2*d^4*x^2 - 2*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*dilog(cos(b*x + a) - I
*sin(b*x + a))*sin(b*x + a) + 6*(I*b^2*d^4*x^2 + 2*I*b^2*c*d^3*x + I*b^2*c^
2*d^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(-I*b^2*d^4*x
^2 - 2*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*dilog(-cos(b*x + a) - I*sin(b*x + a))
*sin(b*x + a) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^
3*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 2*(b^3*d^4*x^3 +
 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*log(cos(b*x + a) - I*sin(b*
x + a) + 1)*sin(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 -
 a^3*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) -
2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*log(-1/2*cos(b*x
+ a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 2*(b^3*d^4*x^3 + 3*b^3*c*d^
3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*log(-c
os(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*d^4*x^3 + 3*b^3*c*d
^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*log(-
cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 12*(b*d^4*x + b*c*d^3)*po
lylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 12*(b*d^4*x + b*c*d^
3)*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 12*(b*d^4*x + b
*c*d^3)*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*(b*d^4
*x + b*c*d^3)*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a))/(b^5
*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \cos(a+bx) \csc^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*csc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**4*cos(a + b*x)*csc(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*csc(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (a+b x) (c+d x)^4}{\sin (a+b x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x)^2,x)
[Out] int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x)^2, x)
```

3.40 $\int (c+dx)^3 \cot(a+bx) \csc(a+bx) \, dx$

Optimal. Leaf size=146

$$-\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{6id^2(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{P}}{}$$

```
[Out] -6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^2-(d*x+c)^3*csc(b*x+a)/b+6*I*d^2*(
d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^3-6*I*d^2*(d*x+c)*polylog(2,exp(I*(b*x+
a)))/b^3-6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)
))/b^4
```

Rubi [A]
time = 0.09, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4495, 4268, 2611, 2320, 6724}

$$-\frac{6d^3\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{Li}_3\left(e^{i(a+bx)}\right)}{b^4}+\frac{6id^2(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}-\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] (-6*d*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^3*Csc[a + b*x]
)/b + ((6*I)*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((6*I)*d^2*(
c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog[3, -E^(I*(a + b*
x))])/b^4 + (6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
```

```
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3\cot(a+bx)\csc(a+bx)\,dx &= -\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{(3d)\int (c+dx)^2\csc(a+bx)\,dx}{b}\\
&= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}-\frac{(6d^2)\int (c
\\
&= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{6id^2(c+
\\
&= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{6id^2(c+
\\
&= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{6id^2(c+
\end{aligned}$$

Mathematica [A]
time = 1.24, size = 284, normalized size = 1.95

$$-\frac{6b^2c^2d\tanh^{-1}\left(e^{i(a+bx)}\right)+b^3c^3\csc(a+bx)+3b^3c^2dx\csc(a+bx)+3b^3cd^2x^2\csc(a+bx)+b^3d^3x^3\csc(a+bx)-6b^2cd^2x\log\left(1-e^{i(a+bx)}\right)-3b^2d^3x^2\log\left(1-e^{i(a+bx)}\right)+6b^2cd^2x\log\left(1+e^{i(a+bx)}\right)+3b^2d^3x^2\log\left(1+e^{i(a+bx)}\right)-6ibd^2(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)+6ibd^2(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)+6d^3\text{PolyLog}\left(3,-e^{i(a+bx)}\right)-6d^3\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] -((6*b^2*c^2*d*ArcTanh[E^(I*(a + b*x))] + b^3*c^3*Csc[a + b*x] + 3*b^3*c^2*
d*x*Csc[a + b*x] + 3*b^3*c*d^2*x^2*Csc[a + b*x] + b^3*d^3*x^3*Csc[a + b*x]
- 6*b^2*c*d^2*x*Log[1 - E^(I*(a + b*x))] - 3*b^2*d^3*x^2*Log[1 - E^(I*(a +
```

b*x))] + 6*b^2*c*d^2*x*Log[1 + E^(I*(a + b*x))] + 3*b^2*d^3*x^2*Log[1 + E^(I*(a + b*x))] - (6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + (6*I)*b*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] + 6*d^3*PolyLog[3, -E^(I*(a + b*x))] - 6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4)

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(134) = 268$.
time = 0.10, size = 433, normalized size = 2.97

method	result
risch	$-\frac{6 i d^3 \operatorname{polylog}(2, e^{i(bx+a)}) x}{b^3} + \frac{12 d^2 c a \operatorname{arctanh}(e^{i(bx+a)})}{b^3} - \frac{6 i d^2 c \operatorname{polylog}(2, e^{i(bx+a)})}{b^3} - \frac{2 i (d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3) e^{i(bx+a)}}{b(e^{2i(bx+a)} - 1)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] -6*I*d^3/b^3*polylog(2,exp(I*(b*x+a)))*x+12*d^2/b^3*c*a*arctanh(exp(I*(b*x+a)))+6*I*d^2/b^3*c*polylog(2,-exp(I*(b*x+a)))-6*I*d^2/b^3*c*polylog(2,exp(I*(b*x+a)))-6*d/b^2*c^2*arctanh(exp(I*(b*x+a)))+3*d^3/b^4*ln(exp(I*(b*x+a))+1)*a^2-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)+3*d^3/b^2*ln(1-exp(I*(b*x+a)))*x^2-3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2+6*I*d^3/b^3*polylog(2,-exp(I*(b*x+a)))*x-3*d^3/b^2*ln(exp(I*(b*x+a))+1)*x^2+6*d^2/b^2*c*ln(1-exp(I*(b*x+a)))*x+6*d^2/b^3*c*ln(1-exp(I*(b*x+a)))*a+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4-6*d^3/b^4*a^2*arctanh(exp(I*(b*x+a)))-6*d^2/b^2*c*ln(exp(I*(b*x+a))+1)*x-6*d^2/b^3*c*ln(exp(I*(b*x+a))+1)*a

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1770 vs. $2(130) = 260$.
time = 0.40, size = 1770, normalized size = 12.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")

[Out] -1/2*(3*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a))*c^2*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b) - 6*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b

```
*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a)
)*a*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) +
1)*b^2) + 3*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*
b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(
2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(c
os(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x
+ a))*a^2*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a
) + 1)*b^3) + 2*c^3/sin(b*x + a) - 6*a*c^2*d/(b*sin(b*x + a)) + 6*a^2*c*d^2
/(b^2*sin(b*x + a)) - 2*a^3*d^3/(b^3*sin(b*x + a)) - 2*(6*((b*x + a)^2*d^3
+ 2*(b*c*d^2 - a*d^3)*(b*x + a) - ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b
*x + a))*cos(2*b*x + 2*a) + (-I*(b*x + a)^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*
(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 6*((
b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) - ((b*x + a)^2*d^3 + 2*(b*c*
d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-I*(b*x + a)^2*d^3 + 2*(-I*b*c*
d^2 + I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x
+ a) + 1) - 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*cos(b*x +
 a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3 - (b*c*d^2 + (b*x + a)*d^3 - a*d^
3)*cos(2*b*x + 2*a) - (I*b*c*d^2 + I*(b*x + a)*d^3 - I*a*d^3)*sin(2*b*x + 2
*a))*dilog(-e^(I*b*x + I*a)) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3 - (b*c*d
^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (-I*b*c*d^2 - I*(b*x + a)*d^
3 + I*a*d^3)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + 3*(-I*(b*x + a)^2*d
^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a) + (I*(b*x + a)^2*d^3 + 2*(I*b*c*d^2
 - I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a
*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*
cos(b*x + a) + 1) + 3*(I*(b*x + a)^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a
) + (-I*(b*x + a)^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*cos(2*b*x + 2
*a) + ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*l
og(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 12*(I*d^3*cos(2*
b*x + 2*a) - d^3*sin(2*b*x + 2*a) - I*d^3)*polylog(3, -e^(I*b*x + I*a)) + 1
2*(-I*d^3*cos(2*b*x + 2*a) + d^3*sin(2*b*x + 2*a) + I*d^3)*polylog(3, e^(I*
b*x + I*a)) + 4*(-I*(b*x + a)^3*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2)
*sin(b*x + a))/(-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x + 2*a) + 2*I*b^
3))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(130) = 260$.
time = 2.65, size = 669, normalized size = 4.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 6*d^3*p
olylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 6*d^3*polylog(3, co
```

```
s(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a)
+ I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) - I*sin(b*x
 + a))*sin(b*x + a) + 6*(I*b*d^3*x + I*b*c*d^2)*dilog(cos(b*x + a) + I*sin(
b*x + a))*sin(b*x + a) + 6*(-I*b*d^3*x - I*b*c*d^2)*dilog(cos(b*x + a) - I*
sin(b*x + a))*sin(b*x + a) + 6*(I*b*d^3*x + I*b*c*d^2)*dilog(-cos(b*x + a)
+ I*sin(b*x + a))*sin(b*x + a) + 6*(-I*b*d^3*x - I*b*c*d^2)*dilog(-cos(b*x
+ a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*
c^2*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2
 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*
x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a) + 1/2*
I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*
log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3*(b^2*d^3
*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) + I*sin(b*x
 + a) + 1)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^
2*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a))/(b^4*sin(b*x +
 a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cos(a+bx) \csc^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*csc(b*x+a)**2,x)
[Out] Integral((c + d*x)**3*cos(a + b*x)*csc(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^3*cos(b*x + a)*csc(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)\,\left(c+d\,x\right)^3}{\sin\left(a+b\,x\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x)^2,x)
[Out] int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x)^2, x)
```

3.41 $\int (c+dx)^2 \cot(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=90

$$-\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{2id^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}$$

```
[Out] -4*d*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^2-(d*x+c)^2*csc(b*x+a)/b+2*I*d^2*pol
ylog(2,-exp(I*(b*x+a)))/b^3-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3
```

Rubi [A]
time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4495, 4268, 2317, 2438}

$$\frac{2id^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}-\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] (-4*d*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^2*Csc[a + b*x])/
b + ((2*I)*d^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((2*I)*d^2*PolyLog[2, E^
(I*(a + b*x))])/b^3
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
```

```
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2\cot(a+bx)\csc(a+bx)\,dx = -\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{(2d)\int (c+dx)\csc(a+bx)\,dx}{b}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}-\frac{(2d^2)\int \mathrm{l}}{}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{(2id^2)\,\mathrm{Su}}{}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{2id^2\mathrm{Li}_2(}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 234 vs. 2(90) = 180.
time = 2.18, size = 234, normalized size = 2.60

$$\frac{-8bcd\tanh^{-1}\left(\cos(a)-\sin(a)\tan\left(\frac{bx}{2}\right)\right)-2b^2(c+dx)^2\csc(a)+4d^2\left(2\mathrm{ArcTan}(\tan(a))\tanh^{-1}\left(\cos(a)-\sin(a)\tan\left(\frac{bx}{2}\right)\right)+\frac{\left((bx+\mathrm{ArcTan}(\tan(a)))\left(\log\left(1-e^{i(bx+\mathrm{ArcTan}(\tan(a)))}\right)-\log\left(1+e^{i(bx+\mathrm{ArcTan}(\tan(a)))}\right)\right)+i\,\mathrm{PolyLog}\left(2,-e^{i(bx+\mathrm{ArcTan}(\tan(a)))}\right)-i\,\mathrm{PolyLog}\left(2,e^{i(bx+\mathrm{ArcTan}(\tan(a)))}\right)\right)\sec(a)}{\sqrt{\sec^2(a)}}\right)+b^2(c+dx)^2\csc\left(\frac{a}{2}\right)\csc\left(\frac{1}{2}(a+bx)\right)\sin\left(\frac{bx}{2}\right)-b^2(c+dx)^2\sec\left(\frac{a}{2}\right)\sec\left(\frac{1}{2}(a+bx)\right)\sin\left(\frac{bx}{2}\right)}{2b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] (-8*b*c*d*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] - 2*b^2*(c + d*x)^2*Csc[a]
+ 4*d^2*(2*ArcTan[Tan[a]]*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + (((b*x +
ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(I*(b*x
+ ArcTan[Tan[a]]))]) + I*PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))] - I*Poly
Log[2, E^(I*(b*x + ArcTan[Tan[a]]))])*Sec[a])/Sqrt[Sec[a]^2]) + b^2*(c + d*
x)^2*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2] - b^2*(c + d*x)^2*Sec[a/2]*Sec[
(a + b*x)/2]*Sin[(b*x)/2])/(2*b^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(82) = 164.
time = 0.06, size = 231, normalized size = 2.57

method	result
risch	$-\frac{2i\left(x^2d^2+2cdx+c^2\right)e^{i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)}-\frac{4dc\,\mathrm{arctanh}\left(e^{i(bx+a)}\right)}{b^2}+\frac{2d^2\ln\left(1-e^{i(bx+a)}\right)x}{b^2}+\frac{2d^2\ln\left(1-e^{i(bx+a)}\right)a}{b^3}-\frac{2id^2\,\mathrm{p}}{}$
derivativedivides	$\frac{-\frac{a^2d^2}{b^2\sin(bx+a)}+\frac{2acd}{b\sin(bx+a)}-\frac{2a\,d^2\left(-\frac{bx+a}{\sin(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))\right)}{b^2}-\frac{c^2}{\sin(bx+a)}+\frac{2cd\left(-\frac{bx+a}{\sin(bx+a)}+\ln(\csc(bx+a)-\cot(\right.}{b}}{b}$

default	$\frac{-\frac{a^2 d^2}{b^2 \sin(bx+a)}+\frac{2acd}{b\sin(bx+a)}-\frac{2a\,d^2\left(-\frac{bx+a}{\sin(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))\right)}{b^2}-\frac{c^2}{\sin(bx+a)}+\frac{2cd\left(-\frac{bx+a}{\sin(bx+a)}+\ln(\csc(bx+a)-\cot(bx}{b}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b^2*a^2*d^2/sin(b*x+a)+2/b*a*c*d/sin(b*x+a)-2/b^2*a*d^2*(-(b*x+a)/s
in(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)))-c^2/sin(b*x+a)+2/b*c*d*(-(b*x+a)/sin(b
*x+a)+ln(csc(b*x+a)-cot(b*x+a)))+1/b^2*d^2*(-(b*x+a)^2/sin(b*x+a)+2*(b*x+a)
*ln(1-exp(I*(b*x+a)))-2*(b*x+a)*ln(exp(I*(b*x+a))+1)+2*I*dilog(exp(I*(b*x+a
))+1)-2*I*dilog(1-exp(I*(b*x+a)))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(78) = 156$.
time = 0.38, size = 553, normalized size = 6.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (2*(b*d^2*x + b*c*d - (b*d^2*x + b*c*d)*cos(2*b*x + 2*a) + (-I*b*d^2*x - I*
b*c*d)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 2*(b*c*d
*cos(2*b*x + 2*a) + I*b*c*d*sin(2*b*x + 2*a) - b*c*d)*arctan2(sin(b*x + a),
 cos(b*x + a) - 1) - 2*(b*d^2*x*cos(2*b*x + 2*a) + I*b*d^2*x*sin(2*b*x + 2*
a) - b*d^2*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*cos(b*x + a) + 2*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*
b*x + 2*a) - d^2)*dilog(-e^(I*b*x + I*a)) - 2*(d^2*cos(2*b*x + 2*a) + I*d^2
*sin(2*b*x + 2*a) - d^2)*dilog(e^(I*b*x + I*a)) - (I*b*d^2*x + I*b*c*d + (-
I*b*d^2*x - I*b*c*d)*cos(2*b*x + 2*a) + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a))
*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-I*b*d^2*x -
I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(2*b*x + 2*a) - (b*d^2*x + b*c*d)*sin(2*
b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 2*(
-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x - I*b^2*c^2)*sin(b*x + a))/(-I*b^3*cos(2*b*x
 + 2*a) + b^3*sin(2*b*x + 2*a) + I*b^3)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(78) = 156$.
time = 2.59, size = 375, normalized size = 4.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*dilog(cos(b*x + a) + I*sin(b*
x + a))*sin(b*x + a) - I*d^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x +
 a) + I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - I*d^2*dilo
g(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + (b*d^2*x + b*c*d)*log(cos(
b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x
 + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x
 + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - (b*c*d - a*d^2)*log(-1/2*c
os(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - (b*d^2*x + a*d^2)*lo
g(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(
-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a))/(b^3*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \cos (a+bx) \csc^2 (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*csc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)*csc(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)*csc(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos \left(a+b\,x\right)\,{\left(c+d\,x\right)}^2}{{\sin \left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x)^2,x)
```

```
[Out] int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x)^2, x)
```

3.42 $\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=30

$$-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}$$

```
[Out] -d*arctanh(cos(b*x+a))/b^2-(d*x+c)*csc(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4495, 3855}

$$-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] -((d*ArcTanh[Cos[a + b*x]])/b^2) - ((c + d*x)*Csc[a + b*x])/b
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx &= -\frac{(c+dx)\csc(a+bx)}{b}+\frac{d\int \csc(a+bx)\,dx}{b}\\ &= -\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 131 vs. 2(30) = 60.
time = 0.07, size = 131, normalized size = 4.37

$$-\frac{dx\csc(a)}{b}-\frac{c\csc(a+bx)}{b}-\frac{d\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}+\frac{d\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}+\frac{dx\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{bx}{2}\right)}{2b}-\frac{dx\sec\left(\frac{a}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{bx}{2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] -((d*x*Csc[a])/b) - (c*Csc[a + b*x])/b - (d*Log[Cos[a/2 + (b*x)/2]])/b^2 +
(d*Log[Sin[a/2 + (b*x)/2]])/b^2 + (d*x*Csc[a/2]*Csc[a/2 + (b*x)/2]*Sin[(b*x
)/2])/(2*b) - (d*x*Sec[a/2]*Sec[a/2 + (b*x)/2]*Sin[(b*x)/2])/(2*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(30) = 60$.
time = 0.06, size = 68, normalized size = 2.27

method	result	size
derivativedivides	$\frac{\frac{da}{b\sin(bx+a)}-\frac{c}{\sin(bx+a)}+\frac{d\left(-\frac{bx+a}{\sin(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))\right)}{b}}{b}$	68
default	$\frac{\frac{da}{b\sin(bx+a)}-\frac{c}{\sin(bx+a)}+\frac{d\left(-\frac{bx+a}{\sin(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))\right)}{b}}{b}$	68
risch	$-\frac{2i(dx+c)e^{i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)}-\frac{d\ln\left(e^{i(bx+a)}+1\right)}{b^2}+\frac{d\ln\left(e^{i(bx+a)}-1\right)}{b^2}$	70
norman	$\frac{-\frac{c}{2b}-\frac{c\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}-\frac{dx}{2b}-\frac{dx\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}+\frac{d\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/b*d*a/sin(b*x+a)-c/sin(b*x+a)+1/b*d*(-(b*x+a)/sin(b*x+a)+ln(csc(b*x+
a)-cot(b*x+a))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(30) = 60$.
time = 0.27, size = 259, normalized size = 8.63

$$-\frac{\frac{\left(4\,(bx+a)\cos(bx+a)\sin(2\,bx+2\,a)-4\,(bx+a)\cos(2\,bx+2\,a)\sin(bx+a)+\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2-2\cos(2\,bx+2\,a)+1\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2+2\cos(bx+a)+1\right)-\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2-2\cos(2\,bx+2\,a)+1\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2-2\cos(bx+a)+1\right)+4\,(bx+a)\sin(bx+a)\right)d}{\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2-2\cos(2\,bx+2\,a)+1\right)b}+\frac{2\,c}{\sin(bx+a)}-\frac{2\,ad}{b\sin(bx+a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x +
2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x
+ 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (co
s(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x
 + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a))*
d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b) +
2*c/sin(b*x + a) - 2*a*d/(b*sin(b*x + a)))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(30) = 60$.
time = 2.62, size = 62, normalized size = 2.07

$$-\frac{2\,bdx+d\log\left(\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)\sin\left(bx+a\right)-d\log\left(-\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)\sin\left(bx+a\right)+2\,bc}{2\,b^{2}\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + d*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - d*log(-1/2*cos
(b*x + a) + 1/2)*sin(b*x + a) + 2*b*c)/(b^2*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c+dx\right)\cos\left(a+bx\right)\csc^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)*csc(a + b*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 801 vs. $2(30) = 60$.
time = 0.70, size = 801, normalized size = 26.70

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 +
b*d*x*tan(1/2*b*x)^2 - d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x
)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b
*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a) + d*log(4
*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^
2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^
2 + 1))*tan(1/2*b*x)^2*tan(1/2*a) + b*d*x*tan(1/2*a)^2 - d*log(4*(tan(1/2*b
*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a
)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*t
an(1/2*b*x)*tan(1/2*a)^2 + d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1
/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1
/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^2 + b*c*t
an(1/2*b*x)^2 + b*c*tan(1/2*a)^2 + b*d*x + d*log(4*(tan(1/2*b*x)^4*tan(1/2*
```

```
a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*
b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x) -
d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(
1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(
1/2*a)^2 + 1))*tan(1/2*b*x) + d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(
1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*ta
n(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*a) - d*log(4*(tan(1/
2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(
1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 + 1))*
tan(1/2*a) + b*c)/(b^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*tan(1/2*b*x)*tan(1/2
*a)^2 - b^2*tan(1/2*b*x) - b^2*tan(1/2*a))
```

Mupad [B]
time = 2.28, size = 88, normalized size = 2.93

$$-\frac{d\,\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{b^2}+\frac{d\,\ln\left(d\,2\mathrm{i}-d\,\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}\,2\mathrm{i}\right)}{b^2}-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(c+d\,x\right)\,2\mathrm{i}}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x))/sin(a + b*x)^2,x)
```

```
[Out] (d*log(d*2i - d*exp(a*1i)*exp(b*x*1i)*2i))/b^2 - (d*log(exp(a*1i + b*x*1i)*
1i + 1i))/b^2 - (exp(a*1i + b*x*1i)*(c + d*x)*2i)/(b*(exp(a*2i + b*x*2i) -
1))
```

3.43 $\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(cot(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x),x]
```

```
[Out] Defer[Int][(Cot[a + b*x]*Csc[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 16.80, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)\left(\csc^{2}\left(bx+a\right)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -((b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*
d)*sin(2*b*x + 2*a)^2 - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))*integrate(sin
(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*
cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 + 2*(b*d^2*
x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) + (b*d^2*x + b*c*d + (b*d^2*x +
b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^2
*x + b*c*d)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x
 + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*
b*c*d*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x
 + a)), x) + 2*cos(b*x + a)*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a)*sin(b*x +
 a) + 2*sin(b*x + a))/(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x +
b*c)*sin(2*b*x + 2*a)^2 + b*c - 2*(b*d*x + b*c)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*csc(b*x + a)^2/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*csc(a + b*x)**2/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(cos(b*x + a)*csc(b*x + a)^2/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos\left(a+b\,x\right)}{{\sin\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(sin(a + b*x)^2*(c + d*x)),x)
[Out] int(cos(a + b*x)/(sin(a + b*x)^2*(c + d*x)), x)
```

3.44 $\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(cot(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2,x]
[Out] Defer[Int][(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 20.65, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2,x]
[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\left(\csc^2(bx+a)\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -2*((b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d
)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(2*b*x + 2*a)
^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a))*integrate(sin(
b*x + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*
b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*
x^2 + 3*b*c^2*d*x + b*c^3)*sin(b*x + a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 +
3*b*c^2*d*x + b*c^3)*cos(b*x + a)), x) + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d
 + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*
b*c*d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^
2*d)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 +
3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos
(b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(b*x + a
)^2 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)), x)
 + cos(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x
+ a))/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(
2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)^2 - 2*(b*
d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*csc(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a+bx\right)\csc^{2}\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)**2/(d*x+c)**2,x)
[Out] Integral(cos(a + b*x)*csc(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(cos(b*x + a)*csc(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos\left(a+b\,x\right)}{{\sin\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(sin(a + b*x)^2*(c + d*x)^2),x)
[Out] int(cos(a + b*x)/(sin(a + b*x)^2*(c + d*x)^2), x)
```

3.45 $\int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left((c+dx)^m\cot(a+bx)\csc^2(a+bx),x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*cot(b*x+a)*csc(b*x+a)^2,x)
```

Rubi [A]
time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m\cot(a+bx)\csc^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2,x]
[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m\cot(a+bx)\csc^2(a+bx)\,dx = \int (c+dx)^m\cot(a+bx)\csc^2(a+bx)\,dx$$

Mathematica [A]
time = 4.66, size = 0, normalized size = 0.00

$$\int (c+dx)^m\cot(a+bx)\csc^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int (dx+c)^m\cos(bx+a)\left(\csc^3(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x)
[Out] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
[Out] integral((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*csc(b*x+a)**3,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos\left(a+b\,x\right)\,{\left(c+d\,x\right)}^m}{{\sin\left(a+b\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x)^3,x)
[Out] int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x)^3, x)
```

3.46 $\int (c+dx)^4 \cot(a+bx) \csc^2(a+bx)\,dx$

Optimal. Leaf size=137

$$-\frac{2id(c+dx)^3}{b^2}-\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4\csc^2(a+bx)}{2b}+\frac{6d^2(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\frac{6id^3(c+$$

```
[Out] -2*I*d*(d*x+c)^3/b^2-2*d*(d*x+c)^3*cot(b*x+a)/b^2-1/2*(d*x+c)^4*csc(b*x+a)^
2/b+6*d^2*(d*x+c)^2*ln(1-exp(2*I*(b*x+a)))/b^3-6*I*d^3*(d*x+c)*polylog(2,ex
p(2*I*(b*x+a)))/b^4+3*d^4*polylog(3,exp(2*I*(b*x+a)))/b^5
```

Rubi [A]

time = 0.17, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {4495, 4269, 3798, 2221, 2611, 2320, 6724}

$$\frac{3d^4\text{Li}_3\left(e^{2i(a+bx)}\right)}{b^5}-\frac{6id^3(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^4}+\frac{6d^2(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4\csc^2(a+bx)}{2b}-\frac{2id(c+dx)^3}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] ((-2*I)*d*(c + d*x)^3)/b^2 - (2*d*(c + d*x)^3*Cot[a + b*x])/b^2 - ((c + d*x
)^4*Csc[a + b*x]^2)/(2*b) + (6*d^2*(c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))]
)/b^3 - ((6*I)*d^3*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^4 + (3*d^4*
PolyLog[3, E^((2*I)*(a + b*x))])/b^5
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^4 \cot(a+bx) \csc^2(a+bx)\,dx = -\frac{(c+dx)^4 \csc^2(a+bx)}{2b} + \frac{(2d)\int (c+dx)^3 \csc^2(a+bx)\,dx}{b}$$
$$= -\frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+dx)^4 \csc^2(a+bx)}{2b} + \frac{(6d^2)\int (c+$$
$$= -\frac{2id(c+dx)^3}{b^2} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+dx)^4 \csc^2(a+bx}{2b}$$
$$= -\frac{2id(c+dx)^3}{b^2} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+dx)^4 \csc^2(a+bx}{2b}$$
$$= -\frac{2id(c+dx)^3}{b^2} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+dx)^4 \csc^2(a+bx}{2b}$$
$$= -\frac{2id(c+dx)^3}{b^2} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+dx)^4 \csc^2(a+bx}{2b}$$
$$= -\frac{2id(c+dx)^3}{b^2} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+dx)^4 \csc^2(a+bx}{2b}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 412 vs. $2(137) = 274$.
time = 6.51, size = 412, normalized size = 3.01

$$-\frac{(c+dx)^4\csc^2(a+bx)}{2b}-\frac{d^4e^{-ia}\csc(a)\left(2b^2x^2\left(2be^{2ia}x+3i(-1+e^{2ia})\log\left(1-e^{2i(a+bx)}\right)\right)+6b(-1+e^{2ia})x\text{PolyLog}(2,e^{2i(a+bx)})+3i(-1+e^{2ia})\text{PolyLog}(3,e^{2i(a+bx)})\right)}{2b^5}+\frac{6c^2d^2\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b^3(\cos^2(a)+\sin^2(a))}+\frac{2\csc(a)\csc(a+bx)(c^3d\sin(bx)+3c^2d^2x\sin(bx)+3cd^3x^2\sin(bx)+d^4x^3\sin(bx))}{b^2}-\frac{6cd^3\csc(a)\sec(a)\left(b^2e^{i\text{ArcTan}(\tan(a))}x^2+\frac{\left(ibx(-\pi+2\text{ArcTan}(\tan(a)))-\pi\log(1+e^{-2ibx})-2(bx+\text{ArcTan}(\tan(a)))\log\left(1-e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)+\pi\log(\cos(bx))+2\text{ArcTan}(\tan(a))\log(\sin(bx+\text{ArcTan}(\tan(a))))+i\text{PolyLog}\left(2,e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)\right)\tan(a)}{\sqrt{1+\tan^2(a)}}\right)}{b^4\sqrt{\sec^2(a)(\cos^2(a)+\sin^2(a))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -1/2*((c + d*x)^4*Csc[a + b*x]^2)/b - (d^4*Csc[a]*(2*b^2*x^2*(2*b*E^((2*I)*
a)*x + (3*I)*(-1 + E^((2*I)*a))*Log[1 - E^((2*I)*(a + b*x))]) + 6*b*(-1 + E
^((2*I)*a))*x*PolyLog[2, E^((2*I)*(a + b*x))] + (3*I)*(-1 + E^((2*I)*a))*Po
lyLog[3, E^((2*I)*(a + b*x))]))/(2*b^5*E^(I*a)) + (6*c^2*d^2*Csc[a]*(-(b*x*
Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 +
Sin[a]^2)) + (2*Csc[a]*Csc[a + b*x]*(c^3*d*Sin[b*x] + 3*c^2*d^2*x*Sin[b*x]
+ 3*c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin[b*x]))/b^2 - (6*c*d^3*Csc[a]*Sec[a]*(b
^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 +
 E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[
Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a
]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan
[a]^2]))/(b^4*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(127) = 254$.
time = 0.12, size = 716, normalized size = 5.23

method	result
risch	$\frac{12d^4 \operatorname{polylog}(3,-e^{i(bx+a)})}{b^5}+\frac{12d^4 \operatorname{polylog}(3,e^{i(bx+a)})}{b^5}+\frac{6d^2c^2 \ln(e^{i(bx+a)}-1)}{b^3}+\frac{6d^2c^2 \ln(e^{i(bx+a)}+1)}{b^3}-\frac{12d^2c^2 \ln(e^{i(bx+a)}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 12*d^4*polylog(3,-exp(I*(b*x+a)))/b^5+12*d^4*polylog(3,exp(I*(b*x+a)))/b^5+
2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x^
2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2*I*d^4*x^3*exp(2*I*(b*x+a)
)+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b*x+a))-6*I*c^2*d^2*x*exp(2
*I*(b*x+a))+2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a))+6*I*c*d^3*x^2+6*I*c^2*d^
2*x+2*I*c^3*d)/b^2/(exp(2*I*(b*x+a))-1)^2+6*d^2/b^3*c^2*ln(exp(I*(b*x+a))-1
)+6*d^2/b^3*c^2*ln(exp(I*(b*x+a))+1)-12*d^2/b^3*c^2*ln(exp(I*(b*x+a)))+6*d^
4/b^5*a^2*ln(exp(I*(b*x+a))-1)-12*d^4/b^5*a^2*ln(exp(I*(b*x+a)))+6*d^4/b^3*
ln(1-exp(I*(b*x+a)))*x^2-6*d^4/b^5*ln(1-exp(I*(b*x+a)))*a^2+6*d^4/b^3*ln(ex
p(I*(b*x+a))+1)*x^2-4*I*d^4/b^2*x^3+8*I*d^4/b^5*a^3+24*d^3/b^4*c*a*ln(exp(I
*(b*x+a)))-12*d^3/b^4*c*a*ln(exp(I*(b*x+a))-1)+12*d^3/b^3*c*ln(exp(I*(b*x+a
))+1)*x+12*d^3/b^3*c*ln(1-exp(I*(b*x+a)))*x+12*d^3/b^4*c*ln(1-exp(I*(b*x+a)
))*a-12*I*d^4/b^4*polylog(2,-exp(I*(b*x+a)))*x+12*I*d^4/b^4*a^2*x-12*I*d^3/
b^4*c*polylog(2,-exp(I*(b*x+a)))-12*I*d^3/b^4*c*polylog(2,exp(I*(b*x+a)))-1
2*I*d^3/b^2*c*x^2-12*I*d^3/b^4*c*a^2-12*I*d^4/b^4*polylog(2,exp(I*(b*x+a)))
*x-24*I*d^3/b^3*c*a*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4551 vs. $2(124) = 248$.
time = 0.52, size = 4551, normalized size = 33.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (
2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x
+ a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) +
1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*c^3*d/((2*(2*cos(2*b*x + 2*a) - 1)*
cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x +
4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos
(2*b*x + 2*a) - 1)*b) - 24*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*si
n(2*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*
b*x + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) -
 cos(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*a*c^2*d^2/((2*(
2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x
```

```
 + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*si
n(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^2) + 24*(4*(b*x + a)*cos(2*b*x
 + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a)
+ sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b
*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b
*x + 2*a))*a^2*c*d^3/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*
b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*
a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^3) -
 8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (2*(b
*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x + a)
*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*s
in(4*b*x + 4*a) + sin(2*b*x + 2*a))*a^3*d^4/((2*(2*cos(2*b*x + 2*a) - 1)*co
s(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*
a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2
*b*x + 2*a) - 1)*b^4) + 6*(8*(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2
*sin(2*b*x + 2*a)^2 - 4*(b*x + a)^2*cos(2*b*x + 2*a) - 4*((b*x + a)^2*cos(2
*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*(2*cos(2*b*
x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2
- sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x +
2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*co
s(b*x + a) + 1) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x
+ 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*s
in(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*
x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*((b*x + a)^2*sin(2*b*x
+ 2*a) + b*x - (b*x + a)*cos(2*b*x + 2*a) + a)*sin(4*b*x + 4*a) + 4*(b*x +
a)*sin(2*b*x + 2*a))*c^2*d^2/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a)
- cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*
b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1
)*b^2) - 12*(8*(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2
*a)^2 - 4*(b*x + a)^2*cos(2*b*x + 2*a) - 4*((b*x + a)^2*cos(2*b*x + 2*a) +
(b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*(2*cos(2*b*x + 2*a) - 1)*
cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x +
4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos
(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1
) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*c
os(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a
) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin
(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*((b*x + a)^2*sin(2*b*x + 2*a) + b*x -
 (b*x + a)*cos(2*b*x + 2*a) + a)*sin(4*b*x + 4*a) + 4*(b*x + a)*sin(2*b*x +
 2*a))*a*c*d^3/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x +
4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^3) + 6*(8*
(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 - 4*(b*x
+ a)^2*cos(2*b*x + 2*a) - 4*((b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2
*b*x + 2*a))*cos(4*b*x + 4*a) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a
```

```
) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(
4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) -
 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (2*(2*cos(2
*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)
^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x
 + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2
*cos(b*x + a) + 1) - 4*((b*x + a)^2*sin(2*b*x + 2*a) + b*x - (b*x + a)*cos(
2*b*x + 2*a) + a)*sin(4*b*x + 4*a) + 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^4/
((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(
2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1075 vs. $2(124) = 248$.
time = 2.52, size = 1075, normalized size = 7.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^
4*c^4 + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*cos
(b*x + a)*sin(b*x + a) - 12*(-I*b*d^4*x - I*b*c*d^3 + (I*b*d^4*x + I*b*c*d^
3)*cos(b*x + a)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 12*(I*b*d^4*x + I
*b*c*d^3 + (-I*b*d^4*x - I*b*c*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*
sin(b*x + a)) - 12*(I*b*d^4*x + I*b*c*d^3 + (-I*b*d^4*x - I*b*c*d^3)*cos(b*
x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 12*(-I*b*d^4*x - I*b*c*d^
3 + (I*b*d^4*x + I*b*c*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*sin(b*x
 + a)) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - (b^2*d^4*x^2 + 2*b^
2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a)
+ 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - (b^2*d^4*x^2 + 2*b^2*
c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin(b*x + a) +
1) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4 - (b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)
- 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4 - (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2
*d^4)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 6
*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4 - (b^2*d^4*x^2 + 2*b^
2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*si
n(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4 -
(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^2)*log(-
cos(b*x + a) - I*sin(b*x + a) + 1) + 12*(d^4*cos(b*x + a)^2 - d^4)*polylog(
3, cos(b*x + a) + I*sin(b*x + a)) + 12*(d^4*cos(b*x + a)^2 - d^4)*polylog(3
, cos(b*x + a) - I*sin(b*x + a)) + 12*(d^4*cos(b*x + a)^2 - d^4)*polylog(3,
 -cos(b*x + a) + I*sin(b*x + a)) + 12*(d^4*cos(b*x + a)^2 - d^4)*polylog(3,
 -cos(b*x + a) - I*sin(b*x + a)))/(b^5*cos(b*x + a)^2 - b^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*csc(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,{\left(c+d\,x\right)}^4}{{\sin\left(a+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x)^3,x)
```

```
[Out] int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x)^3, x)
```

3.47 $\int (c+dx)^3 \cot(a+bx) \csc^2(a+bx)\, dx$

Optimal. Leaf size=115

$$-\frac{3id(c+dx)^2}{2b^2}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}+\frac{3d^2(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\frac{3id^3\text{Poly}}{}$$

```
[Out] -3/2*I*d*(d*x+c)^2/b^2-3/2*d*(d*x+c)^2*cot(b*x+a)/b^2-1/2*(d*x+c)^3*csc(b*x
+a)^2/b+3*d^2*(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b^3-3/2*I*d^3*polylog(2,exp(2*
I*(b*x+a)))/b^4
```

Rubi [A]
time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4495, 4269, 3798, 2221, 2317, 2438}

$$-\frac{3id^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^4}+\frac{3d^2(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}-\frac{3id(c+dx)^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] (((-3*I)/2)*d*(c + d*x)^2)/b^2 - (3*d*(c + d*x)^2*Cot[a + b*x])/(2*b^2) - (
(c + d*x)^3*Csc[a + b*x]^2)/(2*b) + (3*d^2*(c + d*x)*Log[1 - E^((2*I)*(a +
b*x))])/b^3 - (((3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^4
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{(c+dx)^3\csc^2(a+bx)}{2b} + \frac{(3d)\int (c+dx)^2\csc^2(a+bx)\,dx}{2b}$$

$$= -\frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\csc^2(a+bx)}{2b} + \frac{(3d^2)\int (c+ \ldots$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\csc^2(a+bx \ldots}{2b}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\csc^2(a+bx \ldots}{2b}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\csc^2(a+bx \ldots}{2b}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\csc^2(a+bx \ldots}{2b}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 277 vs. $2(115) = 230$.
time = 6.36, size = 277, normalized size = 2.41

$$-\frac{(c+dx)^3\csc^2(a+bx)}{2b} + \frac{3cd^2\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b^3(\cos^2(a)+\sin^2(a))} + \frac{3\csc(a)\csc(a+bx)\left(c^2d\sin(bx)+2cd^2x\sin(bx)+d^3x^2\sin(bx)\right)}{2b^2} - \frac{3d^3\csc(a)\sec(a)\left(b^2e^{i\mathrm{ArcTan}(\tan(a))}x^2+\frac{\left(ibx(-\pi+2\mathrm{ArcTan}(\tan(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx+\mathrm{ArcTan}(\tan(a)))\log\left(1-e^{2i(bx+\mathrm{ArcTan}(\tan(a)))}\right)+\pi\log(\cos(bx))+2\mathrm{ArcTan}(\tan(a))\log(\sin(bx+\mathrm{ArcTan}(\tan(a))))+i\mathrm{PolyLog}\left(2,e^{2i(bx+\mathrm{ArcTan}(\tan(a)))}\right)\right)\tan(a)}{\sqrt{1+\tan^2(a)}}\right)}{2b^4\sqrt{\sec^2(a)\left(\cos^2(a)+\sin^2(a)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

[Out] -1/2*((c + d*x)^3*Csc[a + b*x]^2)/b + (3*c*d^2*Csc[a]*(-(b*x*Cos[a]) + Log[
Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (
3*Csc[a]*Csc[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[b*
x]))/(2*b^2) - (3*d^3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x
*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Ta
n[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*Arc
Tan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + A
rcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^4*Sqrt[Sec[a]^2*(Cos[a]
^2 + Sin[a]^2)])

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(101) = 202$.
time = 0.13, size = 409, normalized size = 3.56

method	result
risch	$\frac{2b\,d^3x^3e^{2i(bx+a)}-3id^3x^2e^{2i(bx+a)}+6bc\,d^2x^2e^{2i(bx+a)}-6ic\,d^2x\,e^{2i(bx+a)}+6b\,c^2dx\,e^{2i(bx+a)}-3ic^2d\,e^{2i(bx+a)}+3id^3x^2+2b\,c^3e^{2i(bx+a}}{b^2\left(e^{2i(bx+a)}-1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] (2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*ex
p(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*
I*c^2*d*exp(2*I*(b*x+a))+3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))+6*I*c*d^2*x+3
*I*c^2*d)/b^2/(exp(2*I*(b*x+a))-1)^2+3*d^2/b^3*c*ln(exp(I*(b*x+a))-1)+3*d^2
/b^3*c*ln(exp(I*(b*x+a))+1)-6*d^2/b^3*c*ln(exp(I*(b*x+a)))-3*I*d^3/b^2*x^2-
6*I*d^3/b^3*a*x-3*I*d^3/b^4*a^2+3*d^3/b^3*ln(1-exp(I*(b*x+a)))*x+3*d^3/b^4*
ln(1-exp(I*(b*x+a)))*a-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4+3*d^3/b^3*ln(e
xp(I*(b*x+a))+1)*x-3*I*d^3/b^4*polylog(2,-exp(I*(b*x+a)))-3*d^3/b^4*a*ln(ex
p(I*(b*x+a))-1)+6*d^3/b^4*a*ln(exp(I*(b*x+a)))

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1044 vs. $2(98) = 196$.
time = 0.47, size = 1044, normalized size = 9.08

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")

[Out] (6*b^2*c^2*d + 6*(b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cos(4*b*x + 4*a)
- 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a) - (-I*b*d^3*x - I*b*c*d^2)*sin(4*b
*x + 4*a) - 2*(I*b*d^3*x + I*b*c*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a
), cos(b*x + a) + 1) + 6*(b*c*d^2*cos(4*b*x + 4*a) - 2*b*c*d^2*cos(2*b*x +
2*a) + I*b*c*d^2*sin(4*b*x + 4*a) - 2*I*b*c*d^2*sin(2*b*x + 2*a) + b*c*d^2)

```
*arctan2(sin(b*x + a), cos(b*x + a) - 1) - 6*(b*d^3*x*cos(4*b*x + 4*a) - 2*
b*d^3*x*cos(2*b*x + 2*a) + I*b*d^3*x*sin(4*b*x + 4*a) - 2*I*b*d^3*x*sin(2*b
*x + 2*a) + b*d^3*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 6*(b^2*d^3*
x^2 + 2*b^2*c*d^2*x)*cos(4*b*x + 4*a) - 2*(2*I*b^3*d^3*x^3 + 2*I*b^3*c^3 +
3*b^2*c^2*d + 3*(2*I*b^3*c*d^2 - b^2*d^3)*x^2 + 6*(I*b^3*c^2*d - b^2*c*d^2)
*x)*cos(2*b*x + 2*a) - 6*(d^3*cos(4*b*x + 4*a) - 2*d^3*cos(2*b*x + 2*a) + I
*d^3*sin(4*b*x + 4*a) - 2*I*d^3*sin(2*b*x + 2*a) + d^3)*dilog(-e^(I*b*x + I
*a)) - 6*(d^3*cos(4*b*x + 4*a) - 2*d^3*cos(2*b*x + 2*a) + I*d^3*sin(4*b*x +
 4*a) - 2*I*d^3*sin(2*b*x + 2*a) + d^3)*dilog(e^(I*b*x + I*a)) - 3*(I*b*d^3
*x + I*b*c*d^2 + (I*b*d^3*x + I*b*c*d^2)*cos(4*b*x + 4*a) + 2*(-I*b*d^3*x -
 I*b*c*d^2)*cos(2*b*x + 2*a) - (b*d^3*x + b*c*d^2)*sin(4*b*x + 4*a) + 2*(b*
d^3*x + b*c*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*
cos(b*x + a) + 1) - 3*(I*b*d^3*x + I*b*c*d^2 + (I*b*d^3*x + I*b*c*d^2)*cos(
4*b*x + 4*a) + 2*(-I*b*d^3*x - I*b*c*d^2)*cos(2*b*x + 2*a) - (b*d^3*x + b*c
*d^2)*sin(4*b*x + 4*a) + 2*(b*d^3*x + b*c*d^2)*sin(2*b*x + 2*a))*log(cos(b*
x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 6*(I*b^2*d^3*x^2 + 2*I*b^
2*c*d^2*x)*sin(4*b*x + 4*a) + 2*(2*b^3*d^3*x^3 + 2*b^3*c^3 - 3*I*b^2*c^2*d
+ 3*(2*b^3*c*d^2 + I*b^2*d^3)*x^2 + 6*(b^3*c^2*d + I*b^2*c*d^2)*x)*sin(2*b*
x + 2*a))/(-2*I*b^4*cos(4*b*x + 4*a) + 4*I*b^4*cos(2*b*x + 2*a) + 2*b^4*sin
(4*b*x + 4*a) - 4*b^4*sin(2*b*x + 2*a) - 2*I*b^4)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(98) = 196$.
time = 3.30, size = 591, normalized size = 5.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 + 3*(b^2*d^3*x
^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) - 3*(I*d^3*cos(b*
x + a)^2 - I*d^3)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*d^3*cos(b*x
+ a)^2 + I*d^3)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*(-I*d^3*cos(b*x +
a)^2 + I*d^3)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 3*(I*d^3*cos(b*x + a)
^2 - I*d^3)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b*d^3*x + b*c*d^2 -
(b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1)
- 3*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*log(cos(b*x +
a) - I*sin(b*x + a) + 1) - 3*(b*c*d^2 - a*d^3 - (b*c*d^2 - a*d^3)*cos(b*x +
 a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 3*(b*c*d^2 - a*d
^3 - (b*c*d^2 - a*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*
x + a) + 1/2) - 3*(b*d^3*x + a*d^3 - (b*d^3*x + a*d^3)*cos(b*x + a)^2)*log(
-cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b*d^3*x + a*d^3 - (b*d^3*x + a*d^3
)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1))/(b^4*cos(b*x + a
)^2 - b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cos(a+bx) \csc^3(a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**3*cos(a + b*x)*csc(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)*csc(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)\,(c+d\,x)^3}{\sin(a+b\,x)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x)^3,x)
```

```
[Out] int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x)^3, x)
```

3.48 $\int (c+dx)^2 \cot(a+bx)\csc^2(a+bx)\,dx$

Optimal. Leaf size=54

$$-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+dx)^2\csc^2(a+bx)}{2b}+\frac{d^2\log(\sin(a+bx))}{b^3}$$

```
[Out] -d*(d*x+c)*cot(b*x+a)/b^2-1/2*(d*x+c)^2*csc(b*x+a)^2/b+d^2*ln(sin(b*x+a))/b
^3
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4495, 4269, 3556}

$$\frac{d^2\log(\sin(a+bx))}{b^3}-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+dx)^2\csc^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -((d*(c + d*x)*Cot[a + b*x])/b^2) - ((c + d*x)^2*Csc[a + b*x]^2)/(2*b) + (d
^2*Log[Sin[a + b*x]])/b^3
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2 \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{(c+dx)^2\csc^2(a+bx)}{2b} + \frac{d\int (c+dx)\csc^2(a+bx)\,dx}{b}$$
$$= -\frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\csc^2(a+bx)}{2b} + \frac{d^2\int \cot(a+ \ldots}{b^2}$$
$$= -\frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\csc^2(a+bx)}{2b} + \frac{d^2\log(\sin(a \ldots}{b^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.92, size = 94, normalized size = 1.74

$$\frac{2ibd^2x - 2id^2\text{ArcTan}(\tan(a+bx)) - 2bd^2x\cot(a) - b^2(c+dx)^2\csc^2(a+bx) + d^2\log\left(\sin^2(a+bx)\right) + 2bd(c+dx)\csc(a)\csc(a+bx)\sin(bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] ((2*I)*b*d^2*x - (2*I)*d^2*ArcTan[Tan[a + b*x]] - 2*b*d^2*x*Cot[a] - b^2*(c
 + d*x)^2*Csc[a + b*x]^2 + d^2*Log[Sin[a + b*x]^2] + 2*b*d*(c + d*x)*Csc[a]
*Csc[a + b*x]*Sin[b*x])/(2*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(52) = 104.
time = 0.09, size = 162, normalized size = 3.00

method	result
risch	$-\frac{2id^2x}{b^2} - \frac{2id^2a}{b^3} + \frac{2b\,d^2x^2\mathrm{e}^{2i(bx+a)} + 4bcdx\,\mathrm{e}^{2i(bx+a)} + 2b\,c^2\mathrm{e}^{2i(bx+a)} - 2id^2x\,\mathrm{e}^{2i(bx+a)} - 2icd\,\mathrm{e}^{2i(bx+a)} + 2id^2x + 2id \ldots}{b^2\left(\mathrm{e}^{2i(bx+a)}-1\right)^2}$
derivativedivides	$\frac{-\frac{a^2d^2}{2b^2\sin(bx+a)^2} + \frac{acd}{b\sin(bx+a)^2} - \frac{2a\,d^2\left(-\frac{bx+a}{2\sin(bx+a)^2} - \frac{\cot(bx+a)}{2}\right)}{b^2} - \frac{c^2}{2\sin(bx+a)^2} + \frac{2cd\left(-\frac{bx+a}{2\sin(bx+a)^2} - \frac{\cot(bx+a)}{2}\right)}{b} + \frac{d^2\left(- \ldots\right.}{\ldots}}{b}$
default	$\frac{-\frac{a^2d^2}{2b^2\sin(bx+a)^2} + \frac{acd}{b\sin(bx+a)^2} - \frac{2a\,d^2\left(-\frac{bx+a}{2\sin(bx+a)^2} - \frac{\cot(bx+a)}{2}\right)}{b^2} - \frac{c^2}{2\sin(bx+a)^2} + \frac{2cd\left(-\frac{bx+a}{2\sin(bx+a)^2} - \frac{\cot(bx+a)}{2}\right)}{b} + \frac{d^2\left(- \ldots\right.}{\ldots}}{b}$
norman	$\frac{-\frac{c^2}{8b} - \frac{c^2\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b} - \frac{d^2x^2}{8b} - \frac{cd\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b^2} + \frac{cd\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b^2} - \frac{cdx}{4b} - \frac{d^2x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b^2} + \frac{d^2x\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b^2} - \frac{d^2x^2\left(t \ldots\right.}{\ldots}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2/b^2*a^2*d^2/sin(b*x+a)^2+1/b*a*c*d/sin(b*x+a)^2-2/b^2*a*d^2*(-1/2
*(b*x+a)/sin(b*x+a)^2-1/2*cot(b*x+a))-1/2*c^2/sin(b*x+a)^2+2/b*c*d*(-1/2*(b
*x+a)/sin(b*x+a)^2-1/2*cot(b*x+a))+1/b^2*d^2*(-1/2*(b*x+a)^2/sin(b*x+a)^2-(
b*x+a)*cot(b*x+a)+ln(sin(b*x+a))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1130 vs. $2(52) = 104$.
time = 0.29, size = 1130, normalized size = 20.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (
2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x
+ a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) +
1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*c*d/((2*(2*cos(2*b*x + 2*a) - 1)*co
s(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*
a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2
*b*x + 2*a) - 1)*b) - 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2
*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x
 + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - co
s(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*a*d^2/((2*(2*cos(2
*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)
^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x
 + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^2) + (8*(b*x + a)^2*cos(2*b*x + 2*a)^
2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 - 4*(b*x + a)^2*cos(2*b*x + 2*a) - 4*(
(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
 + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*co
s(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a)
 - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin(
b*x + a)^2 + 2*cos(b*x + a) + 1) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x +
4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*s
in(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a
) - 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*((b*x
+ a)^2*sin(2*b*x + 2*a) + b*x - (b*x + a)*cos(2*b*x + 2*a) + a)*sin(4*b*x +
 4*a) + 4*(b*x + a)*sin(2*b*x + 2*a))*d^2/((2*(2*cos(2*b*x + 2*a) - 1)*cos(
4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)
^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b
*x + 2*a) - 1)*b^2) - c^2/sin(b*x + a)^2 + 2*a*c*d/(b*sin(b*x + a)^2) - a^2
*d^2/(b^2*sin(b*x + a)^2))/b
```

Fricas [A]
time = 2.34, size = 102, normalized size = 1.89

$$\frac{b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + 2\,(bd^2x + bcd)\cos(bx+a)\sin(bx+a) + 2\left(d^2\cos(bx+a)^2 - d^2\right)\log\left(\frac{1}{2}\,\sin(bx+a)\right)}{2\left(b^3\cos(bx+a)^2 - b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)
*sin(b*x + a) + 2*(d^2*cos(b*x + a)^2 - d^2)*log(1/2*sin(b*x + a)))/(b^3*co
s(b*x + a)^2 - b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \cos(a+bx) \csc^3(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)*csc(a + b*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3482 vs. $2(52) = 104$.
time = 1.29, size = 3482, normalized size = 64.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4*
tan(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*d^2*x^2*ta
n(1/2*b*x)^2*tan(1/2*a)^4 + b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^2*c*d
*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 4*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*
b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - 4*b*d^2*x*tan(1/2*b*x)^3*tan(1/2*a)
^4 + b^2*d^2*x^2*tan(1/2*b*x)^4 + 4*b^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2
 + 2*b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 4*b*c*d*tan(1/2*b*x)^4*tan(1/2*a
)^3 + b^2*d^2*x^2*tan(1/2*a)^4 + 2*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 4*
b*c*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4 + 4*b*d^2*x*
tan(1/2*b*x)^4*tan(1/2*a) + 8*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 24*b*
d^2*x*tan(1/2*b*x)^3*tan(1/2*a)^2 - 4*d^2*log(16*(tan(1/2*b*x)^8*tan(1/2*a)
^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/
2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(
1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^
5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^
3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1
/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1
/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan
(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^2 + 24*b*d^2*x*tan(1/2*b*x)^2*tan
(1/2*a)^3 - 8*d^2*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*ta
```

```
n(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*
tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x
)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2
*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1
/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2
*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*
x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*
b*x)^3*tan(1/2*a)^3 + 2*b^2*c*d*x*tan(1/2*a)^4 + 4*b*d^2*x*tan(1/2*b*x)*tan
(1/2*a)^4 - 4*d^2*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*ta
n(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*
tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x
)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2
*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1
/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2
*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*
x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*
b*x)^2*tan(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^2 + b^2*c^2*tan(1/2*b*x)^4
 + 4*b*c*d*tan(1/2*b*x)^4*tan(1/2*a) + 2*b^2*d^2*x^2*tan(1/2*a)^2 + 4*b^2*c
^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 24*b*c*d*tan(1/2*b*x)^3*tan(1/2*a)^2 + 24*
b*c*d*tan(1/2*b*x)^2*tan(1/2*a)^3 + b^2*c^2*tan(1/2*a)^4 + 4*b*c*d*tan(1/2*
b*x)*tan(1/2*a)^4 + 4*b^2*c*d*x*tan(1/2*b*x)^2 - 4*b*d^2*x*tan(1/2*b*x)^3 -
 24*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a) + 8*d^2*log(16*(tan(1/2*b*x)^8*tan(1/
2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*ta
n(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*
tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b
*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2
*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*t
an(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + t
an(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2
*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a) + 4*b^2*c*d*x*tan(1/2*a)^2 -
24*b*d^2*x*tan(1/2*b*x)*tan(1/2*a)^2 + 16*d^2*log(16*(tan(1/2*b*x)^8*tan(1/
2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*ta
n(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*
tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b
*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2
*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*t
an(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + t
an(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2
*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 4*b*d^2*x*tan(1/2*a)^3 +
8*d^2*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 +
 tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)
^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*
a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(
1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*
tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^
```

```
2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a
) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^...
```

Mupad [B]
time = 2.56, size = 147, normalized size = 2.72

$$\frac{\frac{(c+d\,x)^2}{b}+\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,(c+d\,x)^2}{b}}{1+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}-2\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}}-\frac{d^2\,x\,2\mathrm{i}}{b^2}+\frac{b\,c^2+2\,b\,c\,d\,x-c\,d\,2\mathrm{i}+b\,d^2\,x^2-d^2\,x\,2\mathrm{i}}{b^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}+\frac{d^2\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x)^3,x)
```

```
[Out] ((c + d*x)^2/b + (exp(a*2i + b*x*2i)*(c + d*x)^2)/b)/(exp(a*4i + b*x*4i) -
2*exp(a*2i + b*x*2i) + 1) - (d^2*x*2i)/b^2 + (b*c^2 - c*d*2i - d^2*x*2i + b
*d^2*x^2 + 2*b*c*d*x)/(b^2*(exp(a*2i + b*x*2i) - 1)) + (d^2*log(exp(a*2i)*e
xp(b*x*2i) - 1))/b^3
```

3.49 $\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx$

Optimal. Leaf size=35

$$-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\csc^2(a+bx)}{2b}$$

```
[Out] -1/2*d*cot(b*x+a)/b^2-1/2*(d*x+c)*csc(b*x+a)^2/b
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4495, 3852, 8}

$$-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\csc^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -1/2*(d*Cot[a + b*x])/b^2 - ((c + d*x)*Csc[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx &= -\frac{(c+dx)\csc^2(a+bx)}{2b}+\frac{d\int \csc^2(a+bx)\,dx}{2b}\\ &= -\frac{(c+dx)\csc^2(a+bx)}{2b}-\frac{d\text{Subst}(\int 1\,dx,x,\cot(a+bx))}{2b^2}\\ &= -\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\csc^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 48, normalized size = 1.37

$$-\frac{d\cot(a+bx)}{2b^2}-\frac{c\csc^2(a+bx)}{2b}-\frac{dx\csc^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -1/2*(d*Cot[a + b*x])/b^2 - (c*Csc[a + b*x]^2)/(2*b) - (d*x*Csc[a + b*x]^2)
/(2*b)
```

Maple [A]
time = 0.07, size = 61, normalized size = 1.74

method	result	size
derivativedivides	$\frac{\frac{da}{2b\sin(bx+a)^2}-\frac{c}{2\sin(bx+a)^2}+\frac{d\left(-\frac{bx+a}{2\sin(bx+a)^2}-\frac{\cot(bx+a)}{2}\right)}{b}}{b}$	61
default	$\frac{\frac{da}{2b\sin(bx+a)^2}-\frac{c}{2\sin(bx+a)^2}+\frac{d\left(-\frac{bx+a}{2\sin(bx+a)^2}-\frac{\cot(bx+a)}{2}\right)}{b}}{b}$	61
risch	$\frac{2bdx\,\mathrm{e}^{2i(bx+a)}-id\,\mathrm{e}^{2i(bx+a)}+2bc\,\mathrm{e}^{2i(bx+a)}+id}{b^2\left(\mathrm{e}^{2i(bx+a)}-1\right)^2}$	63
norman	$\frac{-\frac{c}{8b}-\frac{c\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}-\frac{d\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b^2}+\frac{d\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b^2}-\frac{dx}{8b}-\frac{dx\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}-\frac{dx\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2/b*d*a/sin(b*x+a)^2-1/2*c/sin(b*x+a)^2+1/b*d*(-1/2*(b*x+a)/sin(b*x+
a)^2-1/2*cot(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(31) = 62.
time = 0.28, size = 287, normalized size = 8.20

$$\frac{\frac{2\left(4\,(bx+a)\cos(2\,bx+2\,a)^2+4\,(bx+a)\sin(2\,bx+2\,a)^2-(2\,(bx+a)\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a))\cos(4\,bx+4\,a)-2\,(bx+a)\cos(2\,bx+2\,a)-(2\,(bx+a)\sin(2\,bx+2\,a)-\cos(2\,bx+2\,a)+1)\sin(4\,bx+4\,a)+\sin(2\,bx+2\,a)\right)d}{\left(2\,(2\,\cos(2\,bx+2\,a)-1)\cos(4\,bx+4\,a)-\cos(4\,bx+4\,a)^2-4\,\cos(2\,bx+2\,a)^2-\sin(4\,bx+4\,a)^2+4\,\sin(4\,bx+4\,a)\sin(2\,bx+2\,a)-4\,\sin(2\,bx+2\,a)^2+4\,\cos(2\,bx+2\,a)-1\right)b}-\frac{c}{\sin(bx+a)^2}+\frac{ad}{b\sin(bx+a)^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (
2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x
+ a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) +
```

```
1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*d/((2*(2*cos(2*b*x + 2*a) - 1)*cos(
4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)
^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b
*x + 2*a) - 1)*b) - c/sin(b*x + a)^2 + a*d/(b*sin(b*x + a)^2))/b
```

Fricas [A]
time = 2.35, size = 44, normalized size = 1.26

$$\frac{bdx + d\cos\left(bx + a\right)\sin\left(bx + a\right) + bc}{2\left(b^2\cos\left(bx + a\right)^2 - b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x + d*cos(b*x + a)*sin(b*x + a) + b*c)/(b^2*cos(b*x + a)^2 - b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + dx\right)\cos\left(a + bx\right)\csc^3\left(a + bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)*csc(a + b*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. 2(31) = 62.
time = 0.50, size = 526, normalized size = 15.03

$$-\frac{bdx\tan\left(\frac{1}{2}\,bx\right)^4\tan\left(\frac{1}{2}\,a\right)^4 + bc\tan\left(\frac{1}{2}\,bx\right)^4\tan\left(\frac{1}{2}\,a\right)^4 + 2\,bdx\tan\left(\frac{1}{2}\,bx\right)^4\tan\left(\frac{1}{2}\,a\right)^2 + 2\,bdx\tan\left(\frac{1}{2}\,bx\right)^2\tan\left(\frac{1}{2}\,a\right)^4 + 2\,bc\tan\left(\frac{1}{2}\,bx\right)^4\tan\left(\frac{1}{2}\,a\right)^2 - 2\,d\tan\left(\frac{1}{2}\,bx\right)^4\tan\left(\frac{1}{2}\,a\right)^3 + 2\,bc\tan\left(\frac{1}{2}\,bx\right)^2\tan\left(\frac{1}{2}\,a\right)^4 - 2\,d\tan\left(\frac{1}{2}\,bx\right)^3\tan\left(\frac{1}{2}\,a\right)^4 + bdx\tan\left(\frac{1}{2}\,bx\right)^4 + 4\,bdx\tan\left(\frac{1}{2}\,bx\right)^2\tan\left(\frac{1}{2}\,a\right)^2 + bdx\tan\left(\frac{1}{2}\,a\right)^4 + bc\tan\left(\frac{1}{2}\,bx\right)^4 + 2\,d\tan\left(\frac{1}{2}\,bx\right)^4\tan\left(\frac{1}{2}\,a\right) + 4\,bc\tan\left(\frac{1}{2}\,bx\right)^2\tan\left(\frac{1}{2}\,a\right)^2 + 12\,d\tan\left(\frac{1}{2}\,bx\right)^3\tan\left(\frac{1}{2}\,a\right)^2 + 12\,d\tan\left(\frac{1}{2}\,bx\right)^2\tan\left(\frac{1}{2}\,a\right)^3 + bc\tan\left(\frac{1}{2}\,a\right)^4 + 2\,d\tan\left(\frac{1}{2}\,bx\right)\tan\left(\frac{1}{2}\,a\right)^4 + 2\,bdx\tan\left(\frac{1}{2}\,bx\right)^2 + 2\,bdx\tan\left(\frac{1}{2}\,a\right)^2 + 2\,bc\tan\left(\frac{1}{2}\,bx\right)^2 - 2\,d\tan\left(\frac{1}{2}\,bx\right)^3 - 12\,d\tan\left(\frac{1}{2}\,bx\right)^2\tan\left(\frac{1}{2}\,a\right) + 2\,bc\tan\left(\frac{1}{2}\,a\right)^2 - 12\,d\tan\left(\frac{1}{2}\,bx\right)\tan\left(\frac{1}{2}\,a\right)^2 - 2\,d\tan\left(\frac{1}{2}\,a\right)^3 + bdx + bc + 2\,d\tan\left(\frac{1}{2}\,bx\right) + 2\,d\tan\left(\frac{1}{2}\,a\right)}{8\left(b^2\tan\left(\frac{1}{2}\,bx\right)^4\tan\left(\frac{1}{2}\,a\right)^2 + 2\,b^2\tan\left(\frac{1}{2}\,bx\right)^3\tan\left(\frac{1}{2}\,a\right)^3 + b^2\tan\left(\frac{1}{2}\,bx\right)^2\tan\left(\frac{1}{2}\,a\right)^4 - 2\,b^2\tan\left(\frac{1}{2}\,bx\right)^3\tan\left(\frac{1}{2}\,a\right) - 4\,b^2\tan\left(\frac{1}{2}\,bx\right)^2\tan\left(\frac{1}{2}\,a\right)^2 - 2\,b^2\tan\left(\frac{1}{2}\,bx\right)\tan\left(\frac{1}{2}\,a\right)^3 + b^2\tan\left(\frac{1}{2}\,bx\right)^2 + 2\,b^2\tan\left(\frac{1}{2}\,bx\right)\tan\left(\frac{1}{2}\,a\right) + b^2\tan\left(\frac{1}{2}\,a\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 +
 2*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4
+ 2*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 2*b
*c*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + b*d*x*ta
n(1/2*b*x)^4 + 4*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*d*x*tan(1/2*a)^4 + b
*c*tan(1/2*b*x)^4 + 2*d*tan(1/2*b*x)^4*tan(1/2*a) + 4*b*c*tan(1/2*b*x)^2*ta
n(1/2*a)^2 + 12*d*tan(1/2*b*x)^3*tan(1/2*a)^2 + 12*d*tan(1/2*b*x)^2*tan(1/2
*a)^3 + b*c*tan(1/2*a)^4 + 2*d*tan(1/2*b*x)*tan(1/2*a)^4 + 2*b*d*x*tan(1/2*
b*x)^2 + 2*b*d*x*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2 - 2*d*tan(1/2*b*x)^3 -
 12*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b*c*tan(1/2*a)^2 - 12*d*tan(1/2*b*x)*ta
```

```
n(1/2*a)^2 - 2*d*tan(1/2*a)^3 + b*d*x + b*c + 2*d*tan(1/2*b*x) + 2*d*tan(1/
2*a))/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*tan(1/2*b*x)^3*tan(1/2*a)^3
+ b^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*b^2*tan(1/2*b*x)^3*tan(1/2*a) - 4*b^2
*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*tan(1/2*b*x)*tan(1/2*a)^3 + b^2*tan(1/
2*b*x)^2 + 2*b^2*tan(1/2*b*x)*tan(1/2*a) + b^2*tan(1/2*a)^2)
```

Mupad [B]
time = 1.72, size = 53, normalized size = 1.51

$$\frac{d\,1\mathrm{i}-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,\left(-b\,\left(2\,c+2\,d\,x\right)+d\,1\mathrm{i}\right)}{b^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x))/sin(a + b*x)^3,x)
```

```
[Out] (d*1i - exp(a*2i + b*x*2i)*(d*1i - b*(2*c + 2*d*x)))/(b^2*(exp(a*2i + b*x*2
i) - 1)^2)
```

3.50 $\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\cot(a+bx)\csc^2(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(cot(b*x+a)*csc(b*x+a)^2/(d*x+c),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x),x]
[Out] Defer[Int][(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x), x]
```

Rubi steps

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 10.30, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x),x]
[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x), x]
```

Maple [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\left(\csc^3(bx+a)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] -(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 -
 (2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) -
 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*
d^2 + (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(4*b*x + 4*a)^2 + 4*(b
^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^2
 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^4*x^2 + 2*b^2
*c*d^3*x + b^2*c^2*d^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^4*x^2
+ 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4*x^2 + 2*b^2*
c*d^3*x + b^2*c^2*d^2 - 2*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(2
*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^
2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
+ 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*cos(b*x + a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*sin(b*x + a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*
x + b^2*c^3)*cos(b*x + a)), x) - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2
 + (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*
d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^2 +
2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^4*x^2 + 2*b^2*c*
d^3*x + b^2*c^2*d^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^4*x^2 + 2
*b^2*c*d^3*x + b^2*c^2*d^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4*x^2 + 2*b^2*c*d
^3*x + b^2*c^2*d^2 - 2*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(2*b*
x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*
cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*cos(b*x + a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*sin(b*x + a)^2 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3)*cos(b*x + a)), x) - (d*cos(2*b*x + 2*a) + 2*(b*d*x + b*c)*sin(2*b
*x + 2*a) - d)*sin(4*b*x + 4*a) - d*sin(2*b*x + 2*a))/(b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2
+ 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 -
2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a)
- 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x, algorithm="fricas")
[Out] integral(cos(b*x + a)*csc(b*x + a)^3/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\csc^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)**3/(d*x+c),x)
[Out] Integral(cos(a + b*x)*csc(a + b*x)**3/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x, algorithm="giac")
[Out] integrate(cos(b*x + a)*csc(b*x + a)^3/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos(a+b\,x)}{{\sin(a+b\,x)}^3\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(sin(a + b*x)^3*(c + d*x)),x)
[Out] int(cos(a + b*x)/(sin(a + b*x)^3*(c + d*x)), x)
```

3.51 $\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2},x\right)$$

[Out] CannotIntegrate(cot(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x)

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2,x]

[Out] Defer[Int][(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 9.50, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2,x]

[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2, x]

Maple [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\left(\csc^3(bx+a)\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 -
 2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) -
 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + 3*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^
2*c^2*d^3*x + b^2*c^3*d^2 + (b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*
x + b^2*c^3*d^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*
b^2*c^2*d^3*x + b^2*c^3*d^2)*cos(2*b*x + 2*a)^2 + (b^2*d^5*x^3 + 3*b^2*c*d^
4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^5*x^3
+ 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*sin(4*b*x + 4*a)*sin(2*b
*x + 2*a) + 4*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^
2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x
+ b^2*c^3*d^2 - 2*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^
3*d^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^
2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)
/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c
^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b
^2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2
 + 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x
^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) - 3*(b^
2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2 + (b^2*d^5*x^3
+ 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*cos(4*b*x + 4*a)^2 + 4*(
b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*cos(2*b*x +
2*a)^2 + (b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*si
n(4*b*x + 4*a)^2 - 4*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2
*c^3*d^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^5*x^3 + 3*b^2*c*d^4*
x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^5*x^3 +
3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2 - 2*(b^2*d^5*x^3 + 3*b^2*c*
d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a)
 - 4*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*cos(2*
b*x + 2*a))*integrate(sin(b*x + a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c
^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b
^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4
*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^
2 - 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x +
b^2*c^4)*cos(b*x + a)), x) - 2*(d*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(2*b*
```

```
x + 2*a) - d)*sin(4*b*x + 4*a) - 2*d*sin(2*b*x + 2*a))/(b^2*d^3*x^3 + 3*b^2
*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2
*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2
*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4
*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)
^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^
3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*
b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*co
s(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(cos(b*x + a)*csc(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\csc^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)**3/(d*x+c)**2,x)
[Out] Integral(cos(a + b*x)*csc(a + b*x)**3/(c + d*x)**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos(a+b\,x)}{\sin(a+b\,x)^3\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(sin(a + b*x)^3*(c + d*x)^2),x)
[Out] int(cos(a + b*x)/(sin(a + b*x)^3*(c + d*x)^2), x)
```

3.52 $\int (c+dx)^{5/2} \cos(a+bx) \sin(a+bx) \, dx$

Optimal. Leaf size=196

$$\frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b}-\frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}$$

```
[Out] -1/4*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+5/16*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2
-15/128*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/P
i^(1/2))*Pi^(1/2)/b^(7/2)+15/128*d^(5/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d
^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/64*d^2*cos(2*b*x+2*a)
*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.30, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4491, 12, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{7/2}}+\frac{15\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3}+\frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{16b^2}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b^3) - ((c + d*x)^(5/2)*Cos[2*a
 + 2*b*x])/(4*b) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(128*b^(7/2)) + (15*d^(5/2)*Sqrt[
Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c
)/d])/(128*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(16*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx &= \int \frac{1}{2}(c+dx)^{5/2}\sin(2a+2bx)\,dx \\
&= \frac{1}{2}\int (c+dx)^{5/2}\sin(2a+2bx)\,dx \\
&= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{(5d)\int (c+dx)^{3/2}\cos(2a+2bx)\,dx}{8b} \\
&= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{16b^2} - (15 \\
&= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + 5d(\\
&= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + 5d(\\
&= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + 5d(\\
&= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} - 15d
\end{aligned}
$$

Mathematica [A]
time = 2.48, size = 179, normalized size = 0.91

$$
\frac{-15d^2\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+15d^2\sqrt{\pi}\,S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)+2\sqrt{\frac{b}{d}}\sqrt{c+dx}\,(-((-15d^2+16b^2(c+dx)^2)\cos(2(a+bx)))+20bd(c+dx)\sin(2(a+bx)))}{128b^3\sqrt{\frac{b}{d}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-15*d^2*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])
/Sqrt[Pi]] + 15*d^2*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
*Sin[2*a - (2*b*c)/d] + 2*Sqrt[b/d]*Sqrt[c + d*x]*(-((-15*d^2 + 16*b^2*(c +
 d*x)^2)*Cos[2*(a + b*x)]) + 20*b*d*(c + d*x)*Sin[2*(a + b*x)]))/(128*b^3*S
qrt[b/d])
```

Maple [A]
time = 0.06, size = 234, normalized size = 1.19

method	result

derivativedivides	$$\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2c}{d}\right.}{4b}\right.}{}\right.}{}}{d}$$
default	$$\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2c}{d}\right.}{4b}\right.}{}\right.}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+5/8/b*d*(1/4/b
*d*(d*x+c)^(3/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+c)
^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2
*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*(a*d
-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 277, normalized size = 1.41

$$\frac{\sqrt{2}\left(160\sqrt{2}(dx+c)^{\frac{3}{2}}b^2d\sin\left(\frac{2((dx+c)b-bc+ad)}{d}\right)-8\left(16\sqrt{2}(dx+c)^{\frac{5}{2}}b^3-15\sqrt{2}\sqrt{dx+c}\,bd^2\right)\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)-15\left(-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)-(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{2ib}{d}}\right)-15\left((i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{2ib}{d}}\right)\right)}{1024b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/1024*sqrt(2)*(160*sqrt(2)*(d*x + c)^(3/2)*b^2*d*sin(2*((d*x + c)*b - b*c
+ a*d)/d) - 8*(16*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c)*b*
d^2)*cos(2*((d*x + c)*b - b*c + a*d)/d) - 15*(-(I - 1)*4^(1/4)*sqrt(pi)*d^3
*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/
d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 15*((I
 + 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*
4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x +
c)*sqrt(-2*I*b/d)))/b^4
```

Fricas [A]
time = 2.44, size = 222, normalized size = 1.13

$$-\frac{15\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\,(16\,b^3d^2x^2+32\,b^3cdx+16\,b^3c^2-15\,bd^2-2\,(16\,b^3d^2x^2+32\,b^3cdx+16\,b^3c^2-15\,bd^2)\cos(bx+a)^2+40\,(b^2d^2x+b^2cd)\cos(bx+a)\sin(bx+a))\sqrt{dx+c}}{128\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/128*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d
*x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*
x + 16*b^3*c^2 - 15*b*d^2 - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 -
 15*b*d^2)*cos(b*x + a)^2 + 40*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)*sin(b*x +
 a))*sqrt(d*x + c))/b^4
```

Sympy [A]
time = 127.21, size = 294, normalized size = 1.50

$$-\frac{3b^{\frac{3}{2}}\sqrt{\frac{d}{b}}\,(c+dx)^{\frac{9}{2}}\cos\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{9}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{9}{4}\\\frac{3}{2},\frac{7}{4},\frac{13}{4}\end{matrix}\middle|-\frac{b^2(c+dx)^2}{d^2}\right)}{4d^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{13}{4}\right)}-\frac{3\sqrt{b}\sqrt{\frac{d}{b}}\,(c+dx)^{\frac{7}{2}}\sin\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{7}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\\frac{1}{2},\frac{5}{4},\frac{11}{4}\end{matrix}\middle|-\frac{b^2(c+dx)^2}{d^2}\right)}{8d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{\pi}\sqrt{\frac{d}{b}}\,(c+dx)^3\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}\,d\sqrt{\frac{b}{d}}}\right)}{2d}+\frac{\sqrt{\pi}\sqrt{\frac{d}{b}}\,(c+dx)^3\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}\,d\sqrt{\frac{b}{d}}}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -3*b**(3/2)*sqrt(d/b)*(c + d*x)**(9/2)*cos(2*a - 2*b*c/d)*gamma(3/4)*gamma(
9/4)*hyper((3/4, 9/4), (3/2, 7/4, 13/4), -b**2*(c + d*x)**2/d**2)/(4*d**(5/
2)*gamma(7/4)*gamma(13/4)) - 3*sqrt(b)*sqrt(d/b)*(c + d*x)**(7/2)*sin(2*a -
 2*b*c/d)*gamma(1/4)*gamma(7/4)*hyper((1/4, 7/4), (1/2, 5/4, 11/4), -b**2*(
c + d*x)**2/d**2)/(8*d**(3/2)*gamma(5/4)*gamma(11/4)) + sqrt(pi)*sqrt(d/b)*
(c + d*x)**3*sin(2*a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt
(b/d)))/(2*d) + sqrt(pi)*sqrt(d/b)*(c + d*x)**3*cos(2*a - 2*b*c/d)*fresnels
(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d)
```

Giac [C] Result contains complex when optimal does not.
time = 0.73, size = 1212, normalized size = 6.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")

[Out] -1/256*(64*(-I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 12*c*d^2*((-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) + d^3*((I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + (-I*sqrt(pi)*(64*b^3*c^3 + 48*I*b^2*c^2*d - 36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3) + 48*(I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 2*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c^2)/d

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\sin(a+b\,x)\,\left(c+d\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2),x)

[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2), x)

3.53 $\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=168

$$-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b}-\frac{3d^{3/2}\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}-\frac{3d^{3/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+}}{\sqrt{d}\,\sqrt{\pi}}\right.}{32b^{5/2}}$$

```
[Out] -1/4*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-3/32*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(
2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*Fre
snelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b
^(5/2)+3/16*d*sin(2*b*x+2*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.21, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4491, 12, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{32b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -1/4*((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/b - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(32*b^(5/
2)) - (3*d^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
Pi])]*Sin[2*a - (2*b*c)/d])/(32*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b
*x])/(16*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}(c+dx)^{3/2}\sin(2a+2bx)\,dx$$
$$= \frac{1}{2}\int (c+dx)^{3/2}\sin(2a+2bx)\,dx$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{(3d)\int \sqrt{c+dx}\,\cos(2a+2bx)\,dx}{8b}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2} - \frac{(3d^2}{}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2} - \frac{(3d^2}{}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2} - \frac{(3d\,c}{}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} - \frac{3d^{3/2}\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{}}{\sqrt{d}}\right.}{32b^{5/2}}$$

Mathematica [A]
time = 0.91, size = 157, normalized size = 0.93

$$\frac{-3d\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\,\sqrt{c+dx}}{\sqrt{\pi}}\right)-3d\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\,\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)-2\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\,(4b(c+dx)\cos(2(a+bx))-3d\sin(2(a+bx)))}{32\left(\frac{b}{d}\right)^{5/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-3*d*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sq
rt[Pi]] - 3*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2
*a - (2*b*c)/d] - 2*Sqrt[b/d]*Sqrt[c + d*x]*(4*b*(c + d*x)*Cos[2*(a + b*x)]
 - 3*d*Sin[2*(a + b*x)]))/(32*(b/d)^(5/2)*d^2)
```

Maple [A]
time = 0.04, size = 187, normalized size = 1.11

method	result

derivativedivides	$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+}}{\sqrt{\pi}\sqrt{\frac{b}{d}}}\right.\right.}{}\right.}{4b}}{d}$
default	$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+}}{\sqrt{\pi}\sqrt{\frac{b}{d}}}\right.\right.}{}\right.}{4b}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+3/8/b*d*(1/4/b
*d*(d*x+c)^(1/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^(1
/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+
sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.50, size = 256, normalized size = 1.52

$$-\frac{\sqrt{2}\left(32\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^{2}\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)-24\sqrt{2}\,\sqrt{dx+c}\,bd\sin\left(\frac{2((dx+c)b-bc+ad)}{d}\right)+3\left((i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^{2}\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^{2}\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{2i\,b}{d}}\right)+3\left(-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^{2}\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^{2}\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{2i\,b}{d}}\right)\right)}{256\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/256*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b^2*cos(2*((d*x + c)*b - b*c + a
*d)/d) - 24*sqrt(2)*sqrt(d*x + c)*b*d*sin(2*((d*x + c)*b - b*c + a*d)/d) +
3*((I + 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I
- 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d
*x + c)*sqrt(2*I*b/d)) + 3*(-(I - 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*c
os(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-2*
(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^3
```

Fricas [A]
time = 2.72, size = 167, normalized size = 0.99

$$-\frac{3\,\pi d^{2}\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+3\,\pi d^{2}\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-4\left(2\,b^{2}dx+3\,bd\cos(bx+a)\sin(bx+a)+2\,b^{2}c-4\left(b^{2}dx+b^{2}c\right)\cos(bx+a)^{2}\right)\sqrt{dx+c}}{32\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/32*(3*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 3*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(2*b^2*d*x + 3*b*d*cos(b*x + a)*
sin(b*x + a) + 2*b^2*c - 4*(b^2*d*x + b^2*c)*cos(b*x + a)^2)*sqrt(d*x + c))
/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(163) = 326$.
time = 22.78, size = 665, normalized size = 3.96

$$-\frac{5\sqrt{\pi}\sqrt{\frac{d}{b}}(c+dx)^2\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)\Gamma\left(\frac{1}{4}\right)}{32d\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{\pi}\sqrt{\frac{d}{b}}(c+dx)^2\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}d\sqrt{\frac{b}{d}}}\right)}{2d}-\frac{21\sqrt{\pi}\sqrt{\frac{d}{b}}(c+dx)^2\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)\Gamma\left(\frac{3}{4}\right)}{32d\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{\pi}\sqrt{\frac{d}{b}}(c+dx)^2\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}d\sqrt{\frac{b}{d}}}\right)}{2d}-\frac{15\sqrt{\pi}d\sqrt{\frac{d}{b}}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)\Gamma\left(\frac{1}{4}\right)}{512b^2\Gamma\left(\frac{9}{4}\right)}-\frac{63\sqrt{\pi}d\sqrt{\frac{d}{b}}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)\Gamma\left(\frac{3}{4}\right)}{512b^2\Gamma\left(\frac{11}{4}\right)}+\frac{5\sqrt{\frac{d}{b}}(c+dx)^{\frac{3}{2}}\sin\left(2a-\frac{2bc}{d}\right)\sin\left(\frac{2bc}{d}+2bx\right)\Gamma\left(\frac{1}{4}\right)}{64\sqrt{b}\sqrt{d}\Gamma\left(\frac{9}{4}\right)}-\frac{21\sqrt{\frac{d}{b}}(c+dx)^{\frac{3}{2}}\cos\left(2a-\frac{2bc}{d}\right)\cos\left(\frac{2bc}{d}+2bx\right)\Gamma\left(\frac{3}{4}\right)}{64\sqrt{b}\sqrt{d}\Gamma\left(\frac{11}{4}\right)}+\frac{15\sqrt{d}\sqrt{\frac{d}{b}}\sqrt{c+dx}\sin\left(2a-\frac{2bc}{d}\right)\cos\left(\frac{2bc}{d}+2bx\right)\Gamma\left(\frac{1}{4}\right)}{256b^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}+\frac{63\sqrt{d}\sqrt{\frac{d}{b}}\sqrt{c+dx}\sin\left(\frac{2bc}{d}+2bx\right)\cos\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{3}{4}\right)}{256b^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -5*sqrt(pi)*sqrt(d/b)*(c + d*x)**2*sin(2*a - 2*b*c/d)*fresnelc(2*sqrt(b)*sq
rt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(1/4)/(32*d*gamma(9/4)) + sqrt(pi)*sqr
t(d/b)*(c + d*x)**2*sin(2*a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)
*d*sqrt(b/d)))/(2*d) - 21*sqrt(pi)*sqrt(d/b)*(c + d*x)**2*cos(2*a - 2*b*c/d
)*fresnels(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(3/4)/(32*d*gam
ma(11/4)) + sqrt(pi)*sqrt(d/b)*(c + d*x)**2*cos(2*a - 2*b*c/d)*fresnels(2*b
*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) - 15*sqrt(pi)*d*sqrt(d/b)*sin(
2*a - 2*b*c/d)*fresnelc(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(1
/4)/(512*b**2*gamma(9/4)) - 63*sqrt(pi)*d*sqrt(d/b)*cos(2*a - 2*b*c/d)*fres
nels(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(3/4)/(512*b**2*gamma
(11/4)) + 5*sqrt(d/b)*(c + d*x)**(3/2)*sin(2*a - 2*b*c/d)*sin(2*b*c/d + 2*b
*x)*gamma(1/4)/(64*sqrt(b)*sqrt(d)*gamma(9/4)) - 21*sqrt(d/b)*(c + d*x)**(3
/2)*cos(2*a - 2*b*c/d)*cos(2*b*c/d + 2*b*x)*gamma(3/4)/(64*sqrt(b)*sqrt(d)*
gamma(11/4)) + 15*sqrt(d)*sqrt(d/b)*sqrt(c + d*x)*sin(2*a - 2*b*c/d)*cos(2*
b*c/d + 2*b*x)*gamma(1/4)/(256*b**(3/2)*gamma(9/4)) + 63*sqrt(d)*sqrt(d/b)*
sqrt(c + d*x)*sin(2*b*c/d + 2*b*x)*cos(2*a - 2*b*c/d)*gamma(3/4)/(256*b**(3
/2)*gamma(11/4))
```

Giac [C] Result contains complex when optimal does not.
time = 0.59, size = 753, normalized size = 4.48

$$-\frac{16\left(-\frac{i\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}+\frac{i\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}\right)c^2+d^2\left(\frac{-\frac{i\sqrt{\pi}(16b^2c^2-8ibcd-3d^2)d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b^2}-\frac{2i\left(4i(dx+c)^{\frac{3}{2}}bd-8i\sqrt{dx+c}bcd-3\sqrt{dx+c}d^2\right)e^{\left(-\frac{2(-i(dx+c)b+ibc-iad)}{d}\right)}}{b^2}}{d^2}+\frac{\frac{i\sqrt{\pi}(16b^2c^2+8ibcd-3d^2)d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b^2}-\frac{2i\left(4i(dx+c)^{\frac{3}{2}}bd-8i\sqrt{dx+c}bcd+3\sqrt{dx+c}d^2\right)e^{\left(-\frac{2(i(dx+c)b-ibc+iad)}{d}\right)}}{b^2}}{d^2}\right)+8\left(\frac{i\sqrt{\pi}(4bc-id)d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-\frac{i\sqrt{\pi}(4bc+id)d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{2\sqrt{dx+c}de^{\left(-\frac{2(i(dx+c)b-ibc+iad)}{d}\right)}}{b}+\frac{2\sqrt{dx+c}de^{\left(-\frac{2(-i(dx+c)b+ibc-iad)}{d}\right)}}{b}\right)c}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/64*(16*(-I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
 + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) +
I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*((-I
*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*
c*d - 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d
^2 + (I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x +
 c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b
^2)/d^2) + 8*(I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b
^2*d^2) + 1)*b) - I*sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d
)/d)/b + 2*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c)/
d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+bx)\sin(a+bx)\,(c+dx)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2), x)
```

3.54 $\int \sqrt{c+dx} \cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=142

$$-\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b}+\frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{d}\sqrt{\pi}S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}$$

```
[Out] 1/8*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*d^(
1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2)
)*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/4*cos(2*b*x+2*a)*(d*x+c)^(1/2
)/b
```

Rubi [A]
time = 0.16, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4491, 12, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -1/4*(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/b + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*
c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) -
 (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*S
in[2*a - (2*b*c)/d])/(8*b^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}\sqrt{c+dx}\sin(2a+2bx)\,dx$$
$$= \frac{1}{2}\int \sqrt{c+dx}\sin(2a+2bx)\,dx$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{d\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}dx}{8b}$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{\left(d\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx}{8b} -$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Subst}\left(\int\cos\left(\frac{2bx^2}{d}\right)d\right.}{4b}$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c-}}{\sqrt{d}\sqrt{}}\right.}{8b^{3/2}}$$

Mathematica [A]
time = 0.28, size = 134, normalized size = 0.94

$$\frac{-2\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(2(a+bx))+\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-\sqrt{\pi}\,S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{8b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-2*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + Sqrt[Pi]*Cos[2*a - (2*b*c)/d
]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[Pi]*FresnelS[(2*Sqr
t[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(8*b*Sqrt[b/d])
```

Maple [A]
time = 0.04, size = 142, normalized size = 1.00

method	result
derivativedivides	$\frac{-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\text{S}\left(\frac{2b\sqrt{dx}}{\sqrt{\pi}\sqrt{}}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{d}$

default	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx-}}{\sqrt{\pi}\sqrt{}}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+1/16/b*d*Pi^(1
/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+
c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.49, size = 209, normalized size = 1.47

$$-\frac{\sqrt{2}\left(8\sqrt{2}\sqrt{dx+c}\,b\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)+\left((i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{2ib}{d}}\right)+\left(-(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{2ib}{d}}\right)\right)}{64\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/64*sqrt(2)*(8*sqrt(2)*sqrt(d*x + c)*b*cos(2*((d*x + c)*b - b*c + a*d)/d)
 + ((I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I +
 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x
+ c)*sqrt(2*I*b/d)) + (-(I + 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(
b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*
d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^2
```

Fricas [A]
time = 2.08, size = 125, normalized size = 0.88

$$\frac{\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-2\left(2b\cos\left(bx+a\right)^2-b\right)\sqrt{dx+c}}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*
sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-2*(b*c - a*d)/d) - 2*(2*b*cos(b*x + a)^2 - b)*sqrt(d*x + c))/b^
2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(133) = 266$.
time = 3.09, size = 389, normalized size = 2.74

$$-\frac{b^{\frac{3}{2}}\sqrt{\frac{d}{b}}\,(c+dx)^{\frac{5}{2}}\cos\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{b^2(c+dx)^2}{d^2}\right)}{4d^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}-\frac{\sqrt{b}\sqrt{\frac{d}{b}}\,(c+dx)^{\frac{3}{2}}\sin\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle|-\frac{b^2(c+dx)^2}{d^2}\right)}{8d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{\pi}\,c\sqrt{\frac{d}{b}}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}\,d\sqrt{\frac{b}{d}}}\right)}{2d}+\frac{\sqrt{\pi}\,c\sqrt{\frac{d}{b}}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}\,d\sqrt{\frac{b}{d}}}\right)}{2d}+\frac{\sqrt{\pi}\,x\sqrt{\frac{d}{b}}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}\,d\sqrt{\frac{b}{d}}}\right)}{2}+\frac{\sqrt{\pi}\,x\sqrt{\frac{d}{b}}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}\,d\sqrt{\frac{b}{d}}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -b**(3/2)*sqrt(d/b)*(c + d*x)**(5/2)*cos(2*a - 2*b*c/d)*gamma(3/4)*gamma(5/
4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4), -b**2*(c + d*x)**2/d**2)/(4*d**(5/2)*
gamma(7/4)*gamma(9/4)) - sqrt(b)*sqrt(d/b)*(c + d*x)**(3/2)*sin(2*a - 2*b*c
/d)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*(c + d*x
)**2/d**2)/(8*d**(3/2)*gamma(5/4)*gamma(7/4)) + sqrt(pi)*c*sqrt(d/b)*sin(2*
a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) + sqr
t(pi)*c*sqrt(d/b)*cos(2*a - 2*b*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d
*sqrt(b/d)))/(2*d) + sqrt(pi)*x*sqrt(d/b)*sin(2*a - 2*b*c/d)*fresnelc(2*b*s
qrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2 + sqrt(pi)*x*sqrt(d/b)*cos(2*a - 2*b
*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2
```

Giac [C] Result contains complex when optimal does not.
time = 0.50, size = 408, normalized size = 2.87

$$-\frac{4\left(-\frac{i\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}+\frac{i\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}\right)c+\frac{i\sqrt{\pi}\,(4bc-id)d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-\frac{i\sqrt{\pi}\,(4bc+id)d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{2\sqrt{dx+c}\,de^{\left(-\frac{2(i(dx+c)b-ibc+iad)}{d}\right)}}{b}+\frac{2\sqrt{dx+c}\,de^{\left(-\frac{2(-i(dx+c)b+ibc-iad)}{d}\right)}}{b}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(4*(-I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + I
*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2
*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c + I*sqrt(pi)*
(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(
pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sq
rt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 2*sqrt(d*x + c)*
d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2),x)
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2), x)
```

3.55 $\int \sqrt{c+dx} \cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=142

$$-\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b}+\frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{d}\sqrt{\pi}S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\dots}{8b^{3/2}}$$

```
[Out] 1/8*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*d^(
1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2)
)*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/4*cos(2*b*x+2*a)*(d*x+c)^(1/2
)/b
```

Rubi [A]

time = 0.15, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4491, 12, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -1/4*(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/b + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*
c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) -
 (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*S
in[2*a - (2*b*c)/d])/(8*b^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}\sqrt{c+dx}\sin(2a+2bx)\,dx$$
$$= \frac{1}{2}\int \sqrt{c+dx}\sin(2a+2bx)\,dx$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{d\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}dx}{8b}$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{\left(d\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx}{8b} -$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Subst}\left(\int \cos\left(\frac{2bx^2}{d}\right) dx\right.}{4b}$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c}}{\sqrt{d}\sqrt{}}\right.}{8b^{3/2}}$$

Mathematica [A]
time = 0.03, size = 134, normalized size = 0.94

$$\frac{-2\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(2(a+bx))+\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-\sqrt{\pi}\,S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{8b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
[Out] (-2*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + Sqrt[Pi]*Cos[2*a - (2*b*c)/d
]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[Pi]*FresnelS[(2*Sqr
t[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(8*b*Sqrt[b/d])
```

Maple [A]
time = 0.00, size = 142, normalized size = 1.00

method	result
derivativedivides	$\frac{-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\text{S}\left(\frac{2b\sqrt{dx}}{\sqrt{\pi}}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{d}$

default	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx-}}{\sqrt{\pi}\,\sqrt{}}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+1/16/b*d*Pi^(1
/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+
c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 209, normalized size = 1.47

$$-\frac{\sqrt{2}\left(8\sqrt{2}\sqrt{dx+c}\,b\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)+\left((i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{2ib}{d}}\right)+\left(-(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{2ib}{d}}\right)\right)}{64\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/64*sqrt(2)*(8*sqrt(2)*sqrt(d*x + c)*b*cos(2*((d*x + c)*b - b*c + a*d)/d)
 + ((I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I +
 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x
+ c)*sqrt(2*I*b/d)) + (-(I + 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(
b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*
d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^2
```

Fricas [A]
time = 2.45, size = 125, normalized size = 0.88

$$\frac{\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-2\left(2b\cos(bx+a)^2-b\right)\sqrt{dx+c}}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*
sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-2*(b*c - a*d)/d) - 2*(2*b*cos(b*x + a)^2 - b)*sqrt(d*x + c))/b^
2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(133) = 266$.
time = 3.12, size = 389, normalized size = 2.74

$$-\frac{b^{\frac{3}{2}}\sqrt{\frac{d}{b}}\,(c+dx)^{\frac{5}{2}}\cos\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{b^2(c+dx)^2}{d^2}\right)}{4d^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}-\frac{\sqrt{b}\sqrt{\frac{d}{b}}\,(c+dx)^{\frac{3}{2}}\sin\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle|-\frac{b^2(c+dx)^2}{d^2}\right)}{8d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{\pi}\,c\sqrt{\frac{d}{b}}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}\,d\sqrt{\frac{b}{d}}}\right)}{2d}+\frac{\sqrt{\pi}\,c\sqrt{\frac{d}{b}}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}\,d\sqrt{\frac{b}{d}}}\right)}{2d}+\frac{\sqrt{\pi}\,x\sqrt{\frac{d}{b}}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}\,d\sqrt{\frac{b}{d}}}\right)}{2}+\frac{\sqrt{\pi}\,x\sqrt{\frac{d}{b}}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}\,d\sqrt{\frac{b}{d}}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -b**(3/2)*sqrt(d/b)*(c + d*x)**(5/2)*cos(2*a - 2*b*c/d)*gamma(3/4)*gamma(5/
4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4), -b**2*(c + d*x)**2/d**2)/(4*d**(5/2)*
gamma(7/4)*gamma(9/4)) - sqrt(b)*sqrt(d/b)*(c + d*x)**(3/2)*sin(2*a - 2*b*c
/d)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*(c + d*x
)**2/d**2)/(8*d**(3/2)*gamma(5/4)*gamma(7/4)) + sqrt(pi)*c*sqrt(d/b)*sin(2*
a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) + sqr
t(pi)*c*sqrt(d/b)*cos(2*a - 2*b*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d
*sqrt(b/d)))/(2*d) + sqrt(pi)*x*sqrt(d/b)*sin(2*a - 2*b*c/d)*fresnelc(2*b*s
qrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2 + sqrt(pi)*x*sqrt(d/b)*cos(2*a - 2*b
*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2
```

Giac [C] Result contains complex when optimal does not.
time = 0.55, size = 408, normalized size = 2.87

$$-\frac{4\left(-\frac{i\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+\frac{i\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}\right)c+\frac{i\sqrt{\pi}\,(4\,bc-i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{i\sqrt{\pi}\,(4\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{2\sqrt{dx+c}\,de^{\left(-\frac{2(i\,(dx+c)b-i\,bc+i\,ad)}{d}\right)}}{b}+\frac{2\sqrt{dx+c}\,de^{\left(-\frac{2(-i\,(dx+c)b+i\,bc-i\,ad)}{d}\right)}}{b}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(4*(-I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + I
*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2
*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c + I*sqrt(pi)*
(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(
pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sq
rt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 2*sqrt(d*x + c)*
d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2),x)
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2), x)
```

3.56 $\int (c+dx)^{3/2} \cos(a+bx) \sin(a+bx) \, dx$

Optimal. Leaf size=168

$$-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b}-\frac{3d^{3/2}\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}-\frac{3d^{3/2}\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+}}{\sqrt{d}\,\sqrt{\pi}}\right.}{32b^{5/2}}$$

```
[Out] -1/4*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-3/32*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(
2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*Fre
snelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b
^(5/2)+3/16*d*sin(2*b*x+2*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.19, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4491, 12, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{32b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -1/4*((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/b - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(32*b^(5/
2)) - (3*d^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
Pi])]*Sin[2*a - (2*b*c)/d])/(32*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b
*x])/(16*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}(c+dx)^{3/2}\sin(2a+2bx)\,dx$$
$$= \frac{1}{2}\int (c+dx)^{3/2}\sin(2a+2bx)\,dx$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{(3d)\int \sqrt{c+dx}\,\cos(2a+2bx)\,dx}{8b}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2} - \frac{(3d^2}{}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2} - \frac{(3d^2}{}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2} - \frac{(3d\,c}{}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} - \frac{3d^{3/2}\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{}}{\sqrt{c}}\right.}{32b^{5/2}}$$

Mathematica [A]
time = 0.04, size = 157, normalized size = 0.93

$$\frac{-3d\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\,\sqrt{c+dx}}{\sqrt{\pi}}\right)-3d\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\,\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)-2\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\,(4b(c+dx)\cos(2(a+bx))-3d\sin(2(a+bx)))}{32\left(\frac{b}{d}\right)^{5/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-3*d*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sq
rt[Pi]] - 3*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2
*a - (2*b*c)/d] - 2*Sqrt[b/d]*Sqrt[c + d*x]*(4*b*(c + d*x)*Cos[2*(a + b*x)]
 - 3*d*Sin[2*(a + b*x)]))/(32*(b/d)^(5/2)*d^2)
```

Maple [A]
time = 0.00, size = 187, normalized size = 1.11

method	result

derivativedivides	$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)S\left(\frac{2b\sqrt{dx+}}{\sqrt{\pi}\sqrt{\frac{b}{d}}}\right.\right.}{}\right.}{4b}}{d}$
default	$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)S\left(\frac{2b\sqrt{dx+}}{\sqrt{\pi}\sqrt{\frac{b}{d}}}\right.\right.}{}\right.}{4b}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+3/8/b*d*(1/4/b
*d*(d*x+c)^(1/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^(1
/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+
sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 256, normalized size = 1.52

$$-\frac{\sqrt{2}\left(32\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^{2}\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-24\sqrt{2}\sqrt{dx+c}\,bd\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)+3\left((i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^{2}\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^{2}\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\sin\left(-\frac{2\,(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{2ib}{d}}\right)+3\left(-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^{2}\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)+(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^{2}\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\sin\left(-\frac{2\,(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{2ib}{d}}\right)\right)}{256\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/256*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b^2*cos(2*((d*x + c)*b - b*c + a
*d)/d) - 24*sqrt(2)*sqrt(d*x + c)*b*d*sin(2*((d*x + c)*b - b*c + a*d)/d) +
3*((I + 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I
- 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d
*x + c)*sqrt(2*I*b/d)) + 3*(-(I - 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*c
os(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-2*
(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^3
```

Fricas [A]
time = 2.59, size = 167, normalized size = 0.99

$$-\frac{3\,\pi d^{2}\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)S\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\,\pi d^{2}\sqrt{\frac{b}{\pi d}}\,C\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\left(2\,b^{2}dx+3\,bd\cos\left(bx+a\right)\sin\left(bx+a\right)+2\,b^{2}c-4\left(b^{2}dx+b^{2}c\right)\cos\left(bx+a\right)^{2}\right)\sqrt{dx+c}}{32\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/32*(3*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 3*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(2*b^2*d*x + 3*b*d*cos(b*x + a)*
sin(b*x + a) + 2*b^2*c - 4*(b^2*d*x + b^2*c)*cos(b*x + a)^2)*sqrt(d*x + c))
/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(163) = 326$.
time = 22.69, size = 665, normalized size = 3.96

$$-\frac{5\sqrt{\pi}\sqrt{\frac{d}{b}}(c+dx)^2\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)\Gamma\left(\frac{1}{4}\right)}{32d\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{\pi}\sqrt{\frac{d}{b}}(c+dx)^2\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}d\sqrt{\frac{b}{d}}}\right)}{2d}-\frac{21\sqrt{\pi}\sqrt{\frac{d}{b}}(c+dx)^2\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)\Gamma\left(\frac{3}{4}\right)}{32d\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{\pi}\sqrt{\frac{d}{b}}(c+dx)^2\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}d\sqrt{\frac{b}{d}}}\right)}{2d}-\frac{15\sqrt{\pi}d\sqrt{\frac{d}{b}}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)\Gamma\left(\frac{1}{4}\right)}{512b^2\Gamma\left(\frac{9}{4}\right)}-\frac{63\sqrt{\pi}d\sqrt{\frac{d}{b}}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)\Gamma\left(\frac{3}{4}\right)}{512b^2\Gamma\left(\frac{11}{4}\right)}+\frac{5\sqrt{\frac{d}{b}}(c+dx)^{\frac{3}{2}}\sin\left(2a-\frac{2bc}{d}\right)\sin\left(\frac{2bc}{d}+2bx\right)\Gamma\left(\frac{1}{4}\right)}{64\sqrt{b}\sqrt{d}\Gamma\left(\frac{9}{4}\right)}-\frac{21\sqrt{\frac{d}{b}}(c+dx)^{\frac{3}{2}}\cos\left(2a-\frac{2bc}{d}\right)\cos\left(\frac{2bc}{d}+2bx\right)\Gamma\left(\frac{3}{4}\right)}{64\sqrt{b}\sqrt{d}\Gamma\left(\frac{11}{4}\right)}+\frac{15\sqrt{d}\sqrt{\frac{d}{b}}\sqrt{c+dx}\sin\left(2a-\frac{2bc}{d}\right)\cos\left(\frac{2bc}{d}+2bx\right)\Gamma\left(\frac{1}{4}\right)}{256b^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}+\frac{63\sqrt{d}\sqrt{\frac{d}{b}}\sqrt{c+dx}\sin\left(\frac{2bc}{d}+2bx\right)\cos\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{3}{4}\right)}{256b^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -5*sqrt(pi)*sqrt(d/b)*(c + d*x)**2*sin(2*a - 2*b*c/d)*fresnelc(2*sqrt(b)*sq
rt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(1/4)/(32*d*gamma(9/4)) + sqrt(pi)*sqr
t(d/b)*(c + d*x)**2*sin(2*a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)
*d*sqrt(b/d)))/(2*d) - 21*sqrt(pi)*sqrt(d/b)*(c + d*x)**2*cos(2*a - 2*b*c/d
)*fresnels(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(3/4)/(32*d*gam
ma(11/4)) + sqrt(pi)*sqrt(d/b)*(c + d*x)**2*cos(2*a - 2*b*c/d)*fresnels(2*b
*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) - 15*sqrt(pi)*d*sqrt(d/b)*sin(
2*a - 2*b*c/d)*fresnelc(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(1
/4)/(512*b**2*gamma(9/4)) - 63*sqrt(pi)*d*sqrt(d/b)*cos(2*a - 2*b*c/d)*fres
nels(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(3/4)/(512*b**2*gamma
(11/4)) + 5*sqrt(d/b)*(c + d*x)**(3/2)*sin(2*a - 2*b*c/d)*sin(2*b*c/d + 2*b
*x)*gamma(1/4)/(64*sqrt(b)*sqrt(d)*gamma(9/4)) - 21*sqrt(d/b)*(c + d*x)**(3
/2)*cos(2*a - 2*b*c/d)*cos(2*b*c/d + 2*b*x)*gamma(3/4)/(64*sqrt(b)*sqrt(d)*
gamma(11/4)) + 15*sqrt(d)*sqrt(d/b)*sqrt(c + d*x)*sin(2*a - 2*b*c/d)*cos(2*
b*c/d + 2*b*x)*gamma(1/4)/(256*b**(3/2)*gamma(9/4)) + 63*sqrt(d)*sqrt(d/b)*
sqrt(c + d*x)*sin(2*b*c/d + 2*b*x)*cos(2*a - 2*b*c/d)*gamma(3/4)/(256*b**(3
/2)*gamma(11/4))
```

Giac [C] Result contains complex when optimal does not.
time = 0.58, size = 753, normalized size = 4.48

$$-\frac{16\left(-\frac{i\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(-\frac{2(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}+\frac{i\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(-\frac{2(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}\right)c^2+d^2\left(-\frac{\frac{i\sqrt{\pi}(16b^2c^2-8ibcd-3d^2)d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(-\frac{2(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b^2}-\frac{2i\left(4i(dx+c)^{\frac{3}{2}}bd-8i\sqrt{dx+c}bcd-3\sqrt{dx+c}d^2\right)e^{\left(-\frac{2(-i(dx+c)b+ibc-iad)}{d}\right)}}{b^2}}{d^2}+\frac{\frac{i\sqrt{\pi}(16b^2c^2+8ibcd-3d^2)d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(-\frac{2(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b^2}-\frac{2i\left(4i(dx+c)^{\frac{3}{2}}bd-8i\sqrt{dx+c}bcd+3\sqrt{dx+c}d^2\right)e^{\left(-\frac{2(i(dx+c)b-ibc+iad)}{d}\right)}}{b^2}}{d^2}\right)+8\left(\frac{i\sqrt{\pi}(4bc-id)d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(-\frac{2(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-\frac{i\sqrt{\pi}(4bc+id)d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(-\frac{2(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{2\sqrt{dx+c}de^{\left(-\frac{2(i(dx+c)b-ibc+iad)}{d}\right)}}{b}+\frac{2\sqrt{dx+c}de^{\left(-\frac{2(-i(dx+c)b+ibc-iad)}{d}\right)}}{b}\right)c}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/64*(16*(-I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
 + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) +
I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*((-I
*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*
c*d - 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d
^2 + (I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x +
 c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b
^2)/d^2) + 8*(I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b
^2*d^2) + 1)*b) - I*sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d
)/d)/b + 2*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c)/
d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\sin(a+b\,x)\,\left(c+d\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2), x)
```

3.57 $\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=196

$$\frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b}-\frac{15d^{5/2}\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{128b^{7/2}}$$

```
[Out] -1/4*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+5/16*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2
-15/128*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/P
i^(1/2))*Pi^(1/2)/b^(7/2)+15/128*d^(5/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d
^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/64*d^2*cos(2*b*x+2*a)
*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.22, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4491, 12, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{128b^{7/2}}+\frac{15\sqrt{\pi}\,d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3}+\frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{16b^2}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b^3) - ((c + d*x)^(5/2)*Cos[2*a
 + 2*b*x])/(4*b) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(128*b^(7/2)) + (15*d^(5/2)*Sqrt[
Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c
)/d])/(128*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(16*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}(c+dx)^{5/2}\sin(2a+2bx)\,dx$$

$$= \frac{1}{2}\int (c+dx)^{5/2}\sin(2a+2bx)\,dx$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{(5d)\int (c+dx)^{3/2}\cos(2a+2bx)\,d}{8b}$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{16b^2} - (15$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + 5d($$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + 5d($$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + 5d($$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} - 15d$$

Mathematica [A]
time = 0.05, size = 179, normalized size = 0.91

$$\frac{-15d^2\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+15d^2\sqrt{\pi}\,S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)+2\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(-\left(\left(-15d^2+16b^2(c+dx)^2\right)\cos(2(a+bx))\right)+20bd(c+dx)\sin(2(a+bx))\right)}{128b^3\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-15*d^2*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])
/Sqrt[Pi]] + 15*d^2*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
*Sin[2*a - (2*b*c)/d] + 2*Sqrt[b/d]*Sqrt[c + d*x]*(-((-15*d^2 + 16*b^2*(c +
 d*x)^2)*Cos[2*(a + b*x)]) + 20*b*d*(c + d*x)*Sin[2*(a + b*x)]))/(128*b^3*S
qrt[b/d])
```

Maple [A]
time = 0.00, size = 234, normalized size = 1.19

method	result

derivativedivides	$\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2c\cdots}{d}\cdots\right)}{4b}\cdots\right)}{\cdots}\right)}{\cdots}}{d}$
default	$\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2c\cdots}{d}\cdots\right)}{4b}\cdots\right)}{\cdots}\right)}{\cdots}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+5/8/b*d*(1/4/b
*d*(d*x+c)^(3/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+c)
^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2
*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*(a*d
-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 277, normalized size = 1.41

$$\frac{\sqrt{2}\left(160\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2d\sin\left(\frac{2\left((dx+c)b-bc+ad\right)}{d}\right)-8\left(16\sqrt{2}\,(dx+c)^{\frac{5}{2}}b^3-15\sqrt{2}\,\sqrt{dx+c}\,bd^2\right)\cos\left(\frac{2\left((dx+c)b-bc+ad\right)}{d}\right)-15\left(-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)-(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{2ib}{d}}\right)-15\left((i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{2ib}{d}}\right)\right)}{1024\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/1024*sqrt(2)*(160*sqrt(2)*(d*x + c)^(3/2)*b^2*d*sin(2*((d*x + c)*b - b*c
+ a*d)/d) - 8*(16*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c)*b*
d^2)*cos(2*((d*x + c)*b - b*c + a*d)/d) - 15*(-(I - 1)*4^(1/4)*sqrt(pi)*d^3
*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/
d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 15*((I
 + 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*
4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x +
c)*sqrt(-2*I*b/d)))/b^4
```

Fricas [A]
time = 3.31, size = 222, normalized size = 1.13

$$-\frac{15\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-15\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\,\left(16\,b^3d^2x^2+32\,b^3cdx+16\,b^3c^2-15\,bd^2-2\,\left(16\,b^3d^2x^2+32\,b^3cdx+16\,b^3c^2-15\,bd^2\right)\cos\left(bx+a\right)^2+40\,\left(b^2d^2x+b^2cd\right)\cos\left(bx+a\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{128\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/128*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d
*x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*
x + 16*b^3*c^2 - 15*b*d^2 - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 -
 15*b*d^2)*cos(b*x + a)^2 + 40*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)*sin(b*x +
 a))*sqrt(d*x + c))/b^4
```

Sympy [A]
time = 127.59, size = 294, normalized size = 1.50

$$-\frac{3b^{\frac{3}{2}}\sqrt{\frac{d}{b}}\,(c+dx)^{\frac{9}{2}}\cos\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{9}{4}\right){}_{2}F_{3}\left(\begin{matrix}\frac{3}{4},\frac{9}{4}\\\frac{3}{2},\frac{7}{4},\frac{13}{4}\end{matrix}\middle|-\frac{b^2(c+dx)^2}{d^2}\right)}{4d^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{13}{4}\right)}-\frac{3\sqrt{b}\sqrt{\frac{d}{b}}\,(c+dx)^{\frac{7}{2}}\sin\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{7}{4}\right){}_{2}F_{3}\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\\frac{1}{2},\frac{5}{4},\frac{11}{4}\end{matrix}\middle|-\frac{b^2(c+dx)^2}{d^2}\right)}{8d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{\pi}\sqrt{\frac{d}{b}}\,(c+dx)^3\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}\,d\sqrt{\frac{b}{d}}}\right)}{2d}+\frac{\sqrt{\pi}\sqrt{\frac{d}{b}}\,(c+dx)^3\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}\,d\sqrt{\frac{b}{d}}}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -3*b**(3/2)*sqrt(d/b)*(c + d*x)**(9/2)*cos(2*a - 2*b*c/d)*gamma(3/4)*gamma(
9/4)*hyper((3/4, 9/4), (3/2, 7/4, 13/4), -b**2*(c + d*x)**2/d**2)/(4*d**(5/
2)*gamma(7/4)*gamma(13/4)) - 3*sqrt(b)*sqrt(d/b)*(c + d*x)**(7/2)*sin(2*a -
 2*b*c/d)*gamma(1/4)*gamma(7/4)*hyper((1/4, 7/4), (1/2, 5/4, 11/4), -b**2*(
c + d*x)**2/d**2)/(8*d**(3/2)*gamma(5/4)*gamma(11/4)) + sqrt(pi)*sqrt(d/b)*
(c + d*x)**3*sin(2*a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt
(b/d)))/(2*d) + sqrt(pi)*sqrt(d/b)*(c + d*x)**3*cos(2*a - 2*b*c/d)*fresnels
(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d)
```

Giac [C] Result contains complex when optimal does not.
time = 0.73, size = 1212, normalized size = 6.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(64*(-I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) +
 I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 12*c*d^
2*((-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x +
 c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/
b^2)/d^2 + (I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt
(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d
)/d)/b^2)/d^2) + d^3*((I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2
 + 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(1
6*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x +
c)*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*s
qrt(d*x + c)*d^3)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + (-I*
sqrt(pi)*(64*b^3*c^3 + 48*I*b^2*c^2*d - 36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(
sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d
- 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c
)^(3/2)*b*d^2 - 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-2*(I
*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3) + 48*(I*sqrt(pi)*(4*b*c - I*d)*d
*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c -
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(pi)*(4*b*c + I*
d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*
c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e
^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 2*sqrt(d*x + c)*d*e^(-2*(-I*(d*
x + c)*b + I*b*c - I*a*d)/d)/b)*c^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\sin(a+b\,x)\,\left(c+d\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2), x)
```

3.58 $\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=406

$$\frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}+\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}$$

```
[Out] 5/8*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/72*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b^2+
1/4*(d*x+c)^(5/2)*sin(b*x+a)/b-1/12*(d*x+c)^(5/2)*sin(3*b*x+3*a)/b-5/864*d^
(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*6^(1/2)*Pi^(1/2)/b^(7/2)-5/864*d^(5/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d
^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d^(5/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*
(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)-15/16*d^2*sin(
b*x+a)*(d*x+c)^(1/2)/b^3+5/144*d^2*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.81, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}-\frac{15d^2\sqrt{c+dx}\,\sin(a+bx)}{16b^3}+\frac{5d^2\sqrt{c+dx}\,\sin(3a+3bx)}{144b^3}+\frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)}{4b}-\frac{(c+dx)^{5/2}\sin(3a+3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(8*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a +
 3*b*x])/(72*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]
*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(144*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/
2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(
16*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + ((c + d*x)^(5/
2)*Sin[a + b*x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(144*b^3) -
 ((c + d*x)^(5/2)*Sin[3*a + 3*b*x])/(12*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\cos(a+bx) - \frac{1}{4}(c+dx)^{5/2}\cos(3a+3bx)\right)\,dx$$

$$= \frac{1}{4}\int (c+dx)^{5/2}\cos(a+bx)\,dx - \frac{1}{4}\int (c+dx)^{5/2}\cos(3a+3bx)$$

$$= \frac{(c+dx)^{5/2}\sin(a+bx)}{4b} - \frac{(c+dx)^{5/2}\sin(3a+3bx)}{12b} + \frac{(5d)\int(c}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} + \frac{(c+}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} - \frac{15d}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} - \frac{15d}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} - \frac{15d}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} + \frac{15d}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 15.53, size = 1171, normalized size = 2.88

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/8*I)*c^2*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])
/Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))
/d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (c*d*(Sqrt[b/d]*
Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)
/d] + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqr
t[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*
b*Sqrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(8*b^3) + ((b/d)^(3
/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*
c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d])) - Sqrt[2*Pi]*F
resnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (
4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2*b*(
```

```
c - 5*d*x)*Cos[a + b*x] + d*(-15 + 4*b^2*x^2)*Sin[a + b*x])))/(32*b^5) - (c
^2*(-(Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c
 + d*x]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a
 - (3*b*c)/d] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(24*Sqr
t[3]*b*Sqrt[b/d]) - (c*d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi
]*Sqrt[c + d*x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) +
 Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*C
os[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(
Cos[3*(a + b*x)] + 2*b*x*Sin[3*(a + b*x)])))/(24*Sqrt[3]*b^3) - ((b/d)^(3/2
)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c
^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d])) - Sqrt[2
*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b
*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d
*Sqrt[c + d*x]*(-2*b*(c - 5*d*x)*Cos[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*Sin
[3*(a + b*x)])))/(288*Sqrt[3]*b^5)
```

Maple [A]

time = 0.08, size = 474, normalized size = 1.17

method	result
derivativedivides	$\frac{\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\cdots}$

default	$\frac{\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(5/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-5/8/b*d*(-1/2/b*d
*(d*x+c)^(3/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2
)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((
a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/2
4/b*d*(d*x+c)^(5/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+5/24/b*d*(-1/6/b*d*(d*
x+c)^(3/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2)*
sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1
/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*
x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 547, normalized size = 1.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3456*(240*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d) - 2160*
(d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d) - 5*(-(I + 1)*9^(1/4)*
sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1
/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(
d*x + c)*sqrt(3*I*b/d)) - 405*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/
4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin
(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 405*(-(I - 1)*sqrt(2)*sq
```

```
rt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)
*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d))
 - 5*((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c -
a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*
c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 24*(12*(d*x + c)^(5/2)*b^4
/d - 5*sqrt(d*x + c)*b^2*d)*sin(3*((d*x + c)*b - b*c + a*d)/d) - 216*(4*(d*
x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*sin(((d*x + c)*b - b*c + a*d)/
d))*d/b^5
```

Fricas [**A**]
time = 2.45, size = 370, normalized size = 0.91

$$-\frac{5\,\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-405\,\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-405\,\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+5\,\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)+24\,\left(10\,(b^2d^2x+b^2cd)\cos(bx+a)^3-30\,(b^2d^2x+b^2cd)\cos(bx+a)-\left(12\,b^3d^2x^2+24\,b^3cdx+12\,b^3c^2-35\,bd^2-(12\,b^3d^2x^2+24\,b^3cdx+12\,b^3c^2-5\,bd^2)\cos(bx+a)^2\right)\sin(bx+a)\right)\sqrt{dx+c}}{864\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(s
qrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*s
qrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) + 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(
6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(10*(b^2*d^2*x
+ b^2*c*d)*cos(b*x + a)^3 - 30*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (12*b^3
*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 35*b*d^2 - (12*b^3*d^2*x^2 + 24*b^3*
c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/
b^4
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [**C**] Result contains complex when optimal does not.
time = 1.06, size = 2475, normalized size = 6.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/1728*(72*(3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)) - sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)) - sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*s
qrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b
*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*(9*(sqrt(2)*sqrt(pi)*(4*b^2*
c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2)
 + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt
(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 - (sqrt(6)*s
qrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x
 + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b
^2)/d^2 + 9*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*
a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)
*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I
*b*c + I*a*d)/d)/b^2)/d^2 - (sqrt(6)*sqrt(pi)*(12*b^2*c^2 + 4*I*b*c*d - d^2
)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(2*I
*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3*(
I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) - d^3*(27*(sqrt(2)*sqrt(pi)*(8*
b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(
b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(-4*I*(d*x + c)^(5/2)*b^2*d + 12*I*
(d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)
*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c
)*b + I*b*c - I*a*d)/d)/b^3)/d^3 - (sqrt(6)*sqrt(pi)*(72*b^3*c^3 - 36*I*b^2
*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b^3) + 6*(-12*I*(d*x + c)^(5/2)*b^2*d + 36*I*(d*x + c)^(3/
2)*b^2*c*d - 36*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*s
qrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^(-3*(-I*(d*x + c)*b + I*b*c
 - I*a*d)/d)/b^3)/d^3 + 27*(sqrt(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d -
18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)*b^3) - 2*(4*I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d +
 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)
*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^
3)/d^3 - (sqrt(6)*sqrt(pi)*(72*b^3*c^3 + 36*I*b^2*c^2*d - 18*b*c*d^2 - 5*I*
d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
```

```
)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 6*(
12*I*(d*x + c)^(5/2)*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x +
 c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 5*I*s
qrt(d*x + c)*d^3)*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3) - 36*(
9*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) - sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d
*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - sqrt(6)*sqrt
(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
 1)*b) - 18*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*I
*sqrt(d*x + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*I*sqrt(d*x
 + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^(-
3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)\sin(a+bx)^2(c+dx)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2),x)
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

3.59 $\int (c+dx)^{3/2} \cos(a+bx) \sin^2(a+bx)\,dx$

Optimal. Leaf size=353

$$\frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2}-\frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}$$

```
[Out] 1/4*(d*x+c)^(3/2)*sin(b*x+a)/b-1/12*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b+1/144*d^
(3/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/144*d^(3/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d^
(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))
*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d^(3/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d
*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/8*d*cos(b*x+a)
*(d*x+c)^(1/2)/b^2-1/24*d*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]

time = 0.48, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\,\sqrt{\frac{6}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2}-\frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)}{4b}-\frac{(c+dx)^{3/2}\sin(3a+3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(8*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*x
])/(24*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt
[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a -
 (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/
2)) - (d^(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + ((
c + d*x)^(3/2)*Sin[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(12
*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\cos(a+bx) - \frac{1}{4}(c+dx)^{3/2}\cos(3a+3bx)\right) dx$$

$$= \frac{1}{4}\int (c+dx)^{3/2}\cos(a+bx)\,dx - \frac{1}{4}\int (c+dx)^{3/2}\cos(3a+3bx)$$

$$= \frac{(c+dx)^{3/2}\sin(a+bx)}{4b} - \frac{(c+dx)^{3/2}\sin(3a+3bx)}{12b} + \frac{d\int \sqrt{c+}}{}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2} + \frac{(c+dx}{}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2} + \frac{(c+dx}{}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2} + \frac{(c+dx}{}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2} - \frac{3d^{3/2}\sqrt{}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.87, size = 677, normalized size = 1.92

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/8*I)*c*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/S
qrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d
])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (d*(Sqrt[b/d]*Sqrt
[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/d]
+ 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/
Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b*Sq
rt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(16*b^3) - (c*(-(Sqrt[2
*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - S
qrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]
 + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(24*Sqrt[3]*b*Sqrt[
b/d]) - (d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x
]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[b/d]*Sqr
t[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*
```

```
c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(Cos[3*(a + b*x
)] + 2*b*x*Sin[3*(a + b*x)])))/(48*Sqrt[3]*b^3)
```

Maple [A]
time = 0.07, size = 386, normalized size = 1.09

method	result
derivativedivides	$\frac{\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b}{\sqrt{\pi}}\right.\right.}{}\right)}{4b}}{}$
default	$\frac{\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b}{\sqrt{\pi}}\right.\right.}{}\right)}{4b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(3/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-3/8/b*d*(-1/2/b*d
*(d*x+c)^(1/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)))-1/24/b*d*(d*x+c)^(3/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/8/b*d*(-
1/6/b*d*(d*x+c)^(1/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^
(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(
1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 497, normalized size = 1.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/576*(48*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 144*(
d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d + 24*sqrt(d*x + c)*b^
```

```
2*cos(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*cos(((d*x + c)
*b - b*c + a*d)/d) + ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*
cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4
)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 27*((I - 1)*sqr
t(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(2)*sqr
t(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d
)) - 27*(-(I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)
- (I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)
^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^
2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*d/b^4
```

Fricas [A]
time = 2.05, size = 298, normalized size = 0.84

$$\frac{\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-27\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)-24\left(bd\cos\left(bx+a\right)^3-3\,bd\cos\left(bx+a\right)-2\left(b^2dx+b^2c-\left(b^2dx+b^2c\right)\cos\left(bx+a\right)^2\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{144\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt
(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(
b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2
)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-(b*c - a*d)/d) - sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt
(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*d*cos(b*x + a)^3 -
3*b*d*cos(b*x + a) - 2*(b^2*d*x + b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2)
*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.91, size = 1545, normalized size = 4.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
[Out] -1/288*(12*(3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)) - sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/s
qrt(b^2*d^2) + 1)) + 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I
*b*d/sqrt(b^2*d^2) + 1)) - sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(9*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 +
4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*
b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x +
 c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 - (sqrt(6)*sqrt(pi
)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b^2) - 6*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b
*c*d + sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^
2 + 9*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*d -
 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c +
 I*a*d)/d)/b^2)/d^2 - (sqrt(6)*sqrt(pi)*(12*b^2*c^2 + 4*I*b*c*d - d^2)*d*er
f(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(
-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(2*I*(d*x
+ c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3*(I*(d*x
 + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) - 4*(9*sqrt(2)*sqrt(pi)*(2*b*c + I*d)
*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - sqrt(6)*sqrt(
pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b) + 9*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)*b) - sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sq
rt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c +
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 18*I*sqrt(d*x + c)*d*e^
((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*I*sqrt(d*x + c)*d*e^(-3*(I*(d*x +
 c)*b - I*b*c + I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b
*c - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*
d)/d)/b)*c)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

3.60 $\int \sqrt{c+dx}\, \cos(a+bx)\sin^2(a+bx)\, dx$

Optimal. Leaf size=304

$$-\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}+\frac{\sqrt{d}\ \sqrt{}}{}$$

```
[Out] 1/72*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1/72*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1
/8*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^
(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/4*sin(b*x+
a)*(d*x+c)^(1/2)/b-1/12*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.32, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}+\frac{\sqrt{c+dx}\sin(a+bx)}{4b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] -1/4*(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/b^(3/2) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (
3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2)) + (Sqrt[c + d*x]*Sin[
a + b*x])/(4*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(12*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\cos(a+bx)-\frac{1}{4}\sqrt{c+dx}\cos(3a+3bx)\right)dx$$
$$= \frac{1}{4}\int \sqrt{c+dx}\cos(a+bx)\,dx-\frac{1}{4}\int \sqrt{c+dx}\cos(3a+3bx)\,dx$$
$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}+\frac{d\int \frac{\sin(3a+3bx)}{\sqrt{c+dx}}}{24b}$$
$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}+\frac{\left(d\cos\left(3a-\right.\right.}{}$$
$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}+\frac{\cos\left(3a-\frac{3bc}{d}\right)}{}$$
$$= -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.11, size = 264, normalized size = 0.87

$$\frac{-9ie^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)+\frac{\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+\sqrt{6\pi}\,\text{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\sin\left(3a-\frac{3bc}{d}\right)-6\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin(3(a+bx))}{\sqrt{\frac{b}{d}}}}{72b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (((-9*I)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt
[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/
Sqrt[(I*b*(c + d*x))/d]))/E^((I*(b*c + a*d))/d) + (Sqrt[6*Pi]*Cos[3*a - (3*
b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[6*Pi]*FresnelC[
Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] - 6*Sqrt[b/d]*Sqrt
[c + d*x]*Sin[3*(a + b*x)])/Sqrt[b/d])/(72*b)
```

Maple [A]
time = 0.06, size = 294, normalized size = 0.97

method	result

derivativedivides	$\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b}{\sqrt{\pi}}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{}$
default	$\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b}{\sqrt{\pi}}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(1/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-1/16/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)
*b*(d*x+c)^(1/2)/d))-1/24/b*d*(d*x+c)^(1/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d
)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresne
lS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/
d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 424, normalized size = 1.39

$$-\frac{\left(\frac{24\sqrt{dx+c}\,b^2\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{72\sqrt{dx+c}\,b^2\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+\left(-(i+1)\cdot 9^{\frac14}\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\cos\left(-\frac{3(bc-ad)}{d}\right)+(i-1)\cdot 9^{\frac14}\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{3ib}{d}}\right)-9\left(-(i+1)\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\cos\left(-\frac{bc-ad}{d}\right)+(i-1)\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{ib}{d}}\right)-9\left((i-1)\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\cos\left(-\frac{bc-ad}{d}\right)-(i+1)\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{ib}{d}}\right)+\left((i-1)\cdot 9^{\frac14}\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\cos\left(-\frac{3(bc-ad)}{d}\right)-(i+1)\cdot 9^{\frac14}\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{3ib}{d}}\right)\right)d}{288\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/288*(24*sqrt(d*x + c)*b^2*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 72*sqrt
(d*x + c)*b^2*sin(((d*x + c)*b - b*c + a*d)/d)/d + (-(I + 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(
3*I*b/d)) - 9*(-(I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)
/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(I*b/d)) - 9*((I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*c
os(-(b*c - a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c -
 a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((I - 1)*9^(1/4)*sqrt(2)*sqrt(p
i)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(p
i)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d
)))*d/b^3
```

Fricas [A]
time = 2.11, size = 245, normalized size = 0.81

$$\frac{\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\,C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)-24\left(b\cos(bx+a)^2-b\right)\sqrt{dx+c}\,\sin(bx+a)}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c -
a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*
sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c
- a*d)/d) + sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^2 - b)*sqrt(d*x +
 c)*sin(b*x + a))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^2(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.68, size = 848, normalized size = 2.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/144*(9*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)*b) - sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*(2*b*c -
 I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - sqrt(
6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b) - 6*(3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
```

```
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)) - sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(
sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c - 18*I*sqrt(d*x + c)*d*e^((I*(d*x +
 c)*b - I*b*c + I*a*d)/d)/b - 6*I*sqrt(d*x + c)*d*e^(-3*(I*(d*x + c)*b - I*
b*c + I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d
)/d)/b + 6*I*sqrt(d*x + c)*d*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^2\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2),x)
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

3.61 $\int \sqrt{c+dx} \cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=304

$$-\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}+\frac{\sqrt{d}\ \sqrt{}}{}$$

```
[Out] 1/72*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1/72*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1
/8*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^
(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/4*sin(b*x+
a)*(d*x+c)^(1/2)/b-1/12*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.30, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}+\frac{\sqrt{c+dx}\sin(a+bx)}{4b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] -1/4*(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/b^(3/2) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (
3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2)) + (Sqrt[c + d*x]*Sin[
a + b*x])/(4*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(12*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{c+dx}\,\cos(a+bx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{4}\sqrt{c+dx}\,\cos(a+bx)-\frac{1}{4}\sqrt{c+dx}\,\cos(3a+3bx)\right)\,dx \\
&= \frac{1}{4}\int \sqrt{c+dx}\,\cos(a+bx)\,dx-\frac{1}{4}\int \sqrt{c+dx}\,\cos(3a+3bx)\,dx \\
&= \frac{\sqrt{c+dx}\,\sin(a+bx)}{4b}-\frac{\sqrt{c+dx}\,\sin(3a+3bx)}{12b}+\frac{d\int \frac{\sin(3a+3bx)}{\sqrt{c+dx}}\,\cdots}{24b} \\
&= \frac{\sqrt{c+dx}\,\sin(a+bx)}{4b}-\frac{\sqrt{c+dx}\,\sin(3a+3bx)}{12b}+\frac{\left(d\cos\left(3a-\frac{3b\cdots}{\cdots}\right.\right.\cdots}{\cdots} \\
&= \frac{\sqrt{c+dx}\,\sin(a+bx)}{4b}-\frac{\sqrt{c+dx}\,\sin(3a+3bx)}{12b}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\cdots}{\cdots} \\
&= -\frac{\sqrt{d}\,\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{d}\,\sqrt{\frac{\pi}{6}}\,\cos\cdots}{\cdots}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.12, size = 264, normalized size = 0.87

$$\frac{-9ie^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\mathrm{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\mathrm{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)+\frac{\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+\sqrt{6\pi}\,\mathrm{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\sin\left(3a-\frac{3bc}{d}\right)-6\sqrt{\frac{b}{d}}\sqrt{c+dx}\,\sin(3(a+bx))}{\sqrt{\frac{b}{d}}}}{72b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (((-9*I)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt
[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/
Sqrt[(I*b*(c + d*x))/d]))/E^((I*(b*c + a*d))/d) + (Sqrt[6*Pi]*Cos[3*a - (3*
b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[6*Pi]*FresnelC[
Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] - 6*Sqrt[b/d]*Sqrt
[c + d*x]*Sin[3*(a + b*x)])/Sqrt[b/d])/(72*b)
```

Maple [A]
time = 0.00, size = 294, normalized size = 0.97

method	result

derivativedivides	$\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b}{\sqrt{\pi}}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{}$
default	$\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b}{\sqrt{\pi}}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(1/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-1/16/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)
*b*(d*x+c)^(1/2)/d))-1/24/b*d*(d*x+c)^(1/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d
)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresne
lS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/
d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 424, normalized size = 1.39

$$-\frac{\left(\frac{24\sqrt{dx+c}\,b^2\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{72\sqrt{dx+c}\,b^2\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+\left(-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)+(i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{3ib}{d}}\right)-9\left(-(i+1)\,\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i-1)\,\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{ib}{d}}\right)-9\left((i-1)\,\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)-(i+1)\,\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{ib}{d}}\right)+\left((i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{3ib}{d}}\right)\right)d}{288\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/288*(24*sqrt(d*x + c)*b^2*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 72*sqrt
(d*x + c)*b^2*sin(((d*x + c)*b - b*c + a*d)/d)/d + (-(I + 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(
3*I*b/d)) - 9*(-(I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)
/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(I*b/d)) - 9*((I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*c
os(-(b*c - a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c -
 a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((I - 1)*9^(1/4)*sqrt(2)*sqrt(p
i)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(p
i)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d
)))*d/b^3
```

Fricas [A]
time = 2.46, size = 245, normalized size = 0.81

$$\frac{\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)-24\left(b\cos(bx+a)^2-b\right)\sqrt{dx+c}\,\sin(bx+a)}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c -
a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*
sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c
- a*d)/d) + sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^2 - b)*sqrt(d*x +
 c)*sin(b*x + a))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c+dx}\,\sin^2(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.69, size = 848, normalized size = 2.79

$$\frac{\frac{9\sqrt{2}\sqrt{\pi}(2bc+id)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-\frac{\sqrt{6}\sqrt{\pi}(6bc-id)d\,\mathrm{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(-\frac{3(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{9\sqrt{2}\sqrt{\pi}(2bc-id)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{-ibc+iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-\frac{\sqrt{6}\sqrt{\pi}(6bc+id)d\,\mathrm{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(-\frac{3(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-6\left(\cdots\right)\cdots}{144\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/144*(9*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)*b) - sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*(2*b*c -
 I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - sqrt(
6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b) - 6*(3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
```

```
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)) - sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(
sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c - 18*I*sqrt(d*x + c)*d*e^((I*(d*x +
 c)*b - I*b*c + I*a*d)/d)/b - 6*I*sqrt(d*x + c)*d*e^(-3*(I*(d*x + c)*b - I*
b*c + I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d
)/d)/b + 6*I*sqrt(d*x + c)*d*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^2\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2),x)
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

3.62 $\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=353

$$\frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2}-\frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}$$

```
[Out] 1/4*(d*x+c)^(3/2)*sin(b*x+a)/b-1/12*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b+1/144*d^
(3/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/144*d^(3/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d^
(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))
*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d^(3/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d
*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/8*d*cos(b*x+a)
*(d*x+c)^(1/2)/b^2-1/24*d*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.36, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2}-\frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)}{4b}-\frac{(c+dx)^{3/2}\sin(3a+3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(8*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*x
])/(24*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt
[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a -
 (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/
2)) - (d^(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + ((
c + d*x)^(3/2)*Sin[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(12
*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\cos(a+bx) - \frac{1}{4}(c+dx)^{3/2}\cos(3a+3bx)\right)\,dx$$

$$= \frac{1}{4}\int (c+dx)^{3/2}\cos(a+bx)\,dx - \frac{1}{4}\int (c+dx)^{3/2}\cos(3a+3bx)\,d$$

$$= \frac{(c+dx)^{3/2}\sin(a+bx)}{4b} - \frac{(c+dx)^{3/2}\sin(3a+3bx)}{12b} + \frac{d\int\sqrt{c+}}{}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2} + \frac{(c+dx)^3}{}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2} + \frac{(c+dx)^3}{}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2} + \frac{(c+dx)^3}{}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}}{}$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.
time = 8.31, size = 677, normalized size = 1.92

$$-\frac{ice^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{8b}+\frac{d\left(\sqrt{\frac{b}{d}}\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(-3d\cos\left(a-\frac{bc}{d}\right)+2bc\sin\left(a-\frac{bc}{d}\right)\right)+\sqrt{\frac{b}{d}}\sqrt{2\pi}\,S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(2bc\cos\left(a-\frac{bc}{d}\right)+3d\sin\left(a-\frac{bc}{d}\right)\right)+2b\sqrt{c+dx}\,(3\cos(a+bx)+2bx\sin(a+bx))\right)}{16b^3}-\frac{c\left(-\sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)-\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\sin\left(3a-\frac{3bc}{d}\right)+2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\,\sin(3(a+bx))\right)}{24\sqrt{3}\,b\sqrt{\frac{b}{d}}}-\frac{d\left(\sqrt{\frac{b}{d}}\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\left(-d\cos\left(3a-\frac{3bc}{d}\right)+2bc\sin\left(3a-\frac{3bc}{d}\right)\right)+\sqrt{\frac{b}{d}}\sqrt{2\pi}\,S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\left(2bc\cos\left(3a-\frac{3bc}{d}\right)+d\sin\left(3a-\frac{3bc}{d}\right)\right)+2\sqrt{3}\,b\sqrt{c+dx}\,(\cos(3(a+bx))+2bx\sin(3(a+bx)))\right)}{48\sqrt{3}\,b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/8*I)*c*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/S
qrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d
])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (d*(Sqrt[b/d]*Sqrt
[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/d]
+ 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/
Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b*Sq
rt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(16*b^3) - (c*(-(Sqrt[2
*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - S
qrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]
 + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(24*Sqrt[3]*b*Sqrt[
b/d]) - (d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x
]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[b/d]*Sqr
t[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*
```

```
c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(Cos[3*(a + b*x
)] + 2*b*x*Sin[3*(a + b*x)])))/(48*Sqrt[3]*b^3)
```

Maple [A]
time = 0.00, size = 386, normalized size = 1.09

method	result
derivativedivides	$\frac{\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{}}\right.\right.}{}\right.}{4b}}{}$
default	$\frac{\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{}}\right.\right.}{}\right.}{4b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(3/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-3/8/b*d*(-1/2/b*d
*(d*x+c)^(1/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)))-1/24/b*d*(d*x+c)^(3/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/8/b*d*(-
1/6/b*d*(d*x+c)^(1/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^
(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(
1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 497, normalized size = 1.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/576*(48*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 144*(
d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d + 24*sqrt(d*x + c)*b^
```

```
2*cos(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*cos(((d*x + c)
*b - b*c + a*d)/d) + ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*
cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4
)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 27*((I - 1)*sqr
t(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(2)*sqr
t(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d
)) - 27*(-(I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)
- (I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)
^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^
2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*d/b^4
```

Fricas [A]
time = 2.81, size = 298, normalized size = 0.84

$$\frac{\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-27\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)-24\left(bd\cos\left(bx+a\right)^3-3\,bd\cos\left(bx+a\right)-2\left(b^2dx+b^2c-\left(b^2dx+b^2c\right)\cos\left(bx+a\right)^2\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{144\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt
(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(
b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2
)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-(b*c - a*d)/d) - sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt
(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*d*cos(b*x + a)^3 -
3*b*d*cos(b*x + a) - 2*(b^2*d*x + b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2)
*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin^2\left(a+bx\right)\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.94, size = 1545, normalized size = 4.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/288*(12*(3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)) - sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/s
qrt(b^2*d^2) + 1)) + 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I
*b*d/sqrt(b^2*d^2) + 1)) - sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(9*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 +
4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*
b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x +
 c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 - (sqrt(6)*sqrt(pi
)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b^2) - 6*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b
*c*d + sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^
2 + 9*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*d -
 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c +
 I*a*d)/d)/b^2)/d^2 - (sqrt(6)*sqrt(pi)*(12*b^2*c^2 + 4*I*b*c*d - d^2)*d*er
f(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(
-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(2*I*(d*x
+ c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3*(I*(d*x
 + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) - 4*(9*sqrt(2)*sqrt(pi)*(2*b*c + I*d)
*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - sqrt(6)*sqrt(
pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b) + 9*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)*b) - sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sq
rt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c +
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 18*I*sqrt(d*x + c)*d*e^
((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*I*sqrt(d*x + c)*d*e^(-3*(I*(d*x +
 c)*b - I*b*c + I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b
*c - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*
d)/d)/b)*c)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

3.63 $\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=406

$$\frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}+\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}$$

```
[Out] 5/8*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/72*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b^2+
1/4*(d*x+c)^(5/2)*sin(b*x+a)/b-1/12*(d*x+c)^(5/2)*sin(3*b*x+3*a)/b-5/864*d^
(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*6^(1/2)*Pi^(1/2)/b^(7/2)-5/864*d^(5/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d
^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d^(5/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*
(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)-15/16*d^2*sin(
b*x+a)*(d*x+c)^(1/2)/b^3+5/144*d^2*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.45, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}-\frac{15d^2\sqrt{c+dx}\sin(a+bx)}{16b^3}+\frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{144b^3}+\frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)}{4b}-\frac{(c+dx)^{5/2}\sin(3a+3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(8*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a +
 3*b*x])/(72*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]
*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(144*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/
2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(
16*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + ((c + d*x)^(5/
2)*Sin[a + b*x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(144*b^3) -
 ((c + d*x)^(5/2)*Sin[3*a + 3*b*x])/(12*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3385

Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3386

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3387

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]

Rule 3432

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]

Rule 3433

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]

Rule 4491

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\cos(a+bx) - \frac{1}{4}(c+dx)^{5/2}\cos(3a+3bx)\right)\,dx$$

$$= \frac{1}{4}\int (c+dx)^{5/2}\cos(a+bx)\,dx - \frac{1}{4}\int (c+dx)^{5/2}\cos(3a+3bx)$$

$$= \frac{(c+dx)^{5/2}\sin(a+bx)}{4b} - \frac{(c+dx)^{5/2}\sin(3a+3bx)}{12b} + \frac{(5d)\int(c}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} + \frac{(c+}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} - \frac{15d}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} - \frac{15d}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} - \frac{15d}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} + \frac{15d}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.42, size = 1171, normalized size = 2.88

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/8*I)*c^2*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])
/Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))
/d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (c*d*(Sqrt[b/d]*
Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)
/d] + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqr
t[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*
b*Sqrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(8*b^3) + ((b/d)^(3
/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*
c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d])) - Sqrt[2*Pi]*F
resnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (
4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2*b*(
```

```
c - 5*d*x)*Cos[a + b*x] + d*(-15 + 4*b^2*x^2)*Sin[a + b*x])))/(32*b^5) - (c
^2*(-(Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c
+ d*x]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a
- (3*b*c)/d] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(24*Sqr
t[3]*b*Sqrt[b/d]) - (c*d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi
]*Sqrt[c + d*x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) +
 Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*C
os[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(
Cos[3*(a + b*x)] + 2*b*x*Sin[3*(a + b*x)])))/(24*Sqrt[3]*b^3) - ((b/d)^(3/2
)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c
^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d])) - Sqrt[2
*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b
*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d
*Sqrt[c + d*x]*(-2*b*(c - 5*d*x)*Cos[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*Sin
[3*(a + b*x)])))/(288*Sqrt[3]*b^5)
```

Maple [A]
time = 0.00, size = 474, normalized size = 1.17

method	result
derivativedivides	$\frac{\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}}{\cdots}$

default	$\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b} - \frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b} + \frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b} - \frac{d\sqrt{2}}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(5/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-5/8/b*d*(-1/2/b*d
*(d*x+c)^(3/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2
)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((
a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/2
4/b*d*(d*x+c)^(5/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+5/24/b*d*(-1/6/b*d*(d*
x+c)^(3/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2)*
sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1
/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*
x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 547, normalized size = 1.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3456*(240*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d) - 2160*
(d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d) - 5*(-(I + 1)*9^(1/4)*
sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1
/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(
d*x + c)*sqrt(3*I*b/d)) - 405*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/
4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin
(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 405*(-(I - 1)*sqrt(2)*sq
```

```
rt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)
*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d))
 - 5*((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c -
a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*
c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 24*(12*(d*x + c)^(5/2)*b^4
/d - 5*sqrt(d*x + c)*b^2*d)*sin(3*((d*x + c)*b - b*c + a*d)/d) - 216*(4*(d*
x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*sin(((d*x + c)*b - b*c + a*d)/
d))*d/b^5
```

Fricas [A]
time = 3.43, size = 370, normalized size = 0.91

$$-\frac{5\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+5\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)+24\left(10\left(b^2d^2x+b^2cd\right)\cos(bx+a)^3-30\left(b^2d^2x+b^2cd\right)\cos(bx+a)-\left(12\,b^3d^2x^2+24\,b^3cdx+12\,b^3c^2-35\,bd^2-\left(12\,b^3d^2x^2+24\,b^3cdx+12\,b^3c^2-5\,bd^2\right)\cos(bx+a)^2\right)\sin(bx+a)\right)\sqrt{dx+c}}{864\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(s
qrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*s
qrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) + 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(
6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(10*(b^2*d^2*x
+ b^2*c*d)*cos(b*x + a)^3 - 30*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (12*b^3
*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 35*b*d^2 - (12*b^3*d^2*x^2 + 24*b^3*
c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/
b^4
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [C] Result contains complex when optimal does not.
time = 1.04, size = 2475, normalized size = 6.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/1728*(72*(3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)) - sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)) - sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*s
qrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b
*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*(9*(sqrt(2)*sqrt(pi)*(4*b^2*
c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2)
 + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt
(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 - (sqrt(6)*s
qrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x
+ c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b
^2)/d^2 + 9*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*
a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)
*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I
*b*c + I*a*d)/d)/b^2)/d^2 - (sqrt(6)*sqrt(pi)*(12*b^2*c^2 + 4*I*b*c*d - d^2
)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(2*I
*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3*(
I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) - d^3*(27*(sqrt(2)*sqrt(pi)*(8*
b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(
b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(-4*I*(d*x + c)^(5/2)*b^2*d + 12*I*
(d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)
*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c
)*b + I*b*c - I*a*d)/d)/b^3)/d^3 - (sqrt(6)*sqrt(pi)*(72*b^3*c^3 - 36*I*b^2
*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b^3) + 6*(-12*I*(d*x + c)^(5/2)*b^2*d + 36*I*(d*x + c)^(3/
2)*b^2*c*d - 36*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*s
qrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^(-3*(-I*(d*x + c)*b + I*b*c
 - I*a*d)/d)/b^3)/d^3 + 27*(sqrt(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d -
18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)*b^3) - 2*(4*I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d +
 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)
*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^
3)/d^3 - (sqrt(6)*sqrt(pi)*(72*b^3*c^3 + 36*I*b^2*c^2*d - 18*b*c*d^2 - 5*I*
d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
```

```
)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 6*(
12*I*(d*x + c)^(5/2)*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x +
 c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 5*I*s
qrt(d*x + c)*d^3)*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3) - 36*(
9*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) - sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d
*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - sqrt(6)*sqrt
(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
 1)*b) - 18*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*I
*sqrt(d*x + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*I*sqrt(d*x
 + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^(-
3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c^2)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)\sin(a+bx)^2(c+dx)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

3.64 $\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=407

$$\frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{15d^2\sqrt{c+dx}\,\cos(4a+4bx)}{2048b^3}+\frac{(c+dx)^{5/2}\cos(4a}{32b}$$

```
[Out] -1/8*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(5/2)*cos(4*b*x+4*a)/b+5/3
2*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/256*d*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b
^2+15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*
x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)-15/8192*d^(5/2)*FresnelS(2*b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1
/2)/b^(7/2)-15/256*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+15/256*d^(5/2)*FresnelS(2*b^(1/2)*(d*x
+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/128*d^2*co
s(2*b*x+2*a)*(d*x+c)^(1/2)/b^3-15/2048*d^2*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.72, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15\sqrt{\pi}\,d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{15d^2\sqrt{c+dx}\,\cos(4a+4bx)}{2048b^3}+\frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2}-\frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*
a + 2*b*x])/(8*b) - (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) + ((
c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a -
(4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^
(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4
096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*
Sin[2*a + 2*b*x])/(32*b^2) - (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^
2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx) - \frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right)$$

$$= -\left(\frac{1}{8}\int (c+dx)^{5/2}\sin(4a+4bx)\,dx\right) + \frac{1}{4}\int (c+dx)^{5/2}\sin(2a$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b} - \frac{(5d)}{}$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b} + \frac{5d(c}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d}{}$$

Mathematica [A]
time = 15.25, size = 550, normalized size = 1.35

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos
[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^
2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a +
 b*x)] - 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 512*b^3*c*d*x*Sqrt[c + d
*x]*Cos[4*(a + b*x)] + 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 15*
Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresnel
C[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Fresn
elS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b
/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (
```

```
2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sq
rt[c + d*x]*Sin[2*(a + b*x)] - 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] -
 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(8192*b^4)
```

Maple [A]
time = 0.06, size = 470, normalized size = 1.15

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right.}{4b}\right.}{}\right.}{}$
default	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right.}{4b}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+5/16/b*d*(1/4
/b*d*(d*x+c)^(3/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos
(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*(a
*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/64/b*d*(
d*x+c)^(5/2)*cos(4/d*b*(d*x+c)+4*(a*d-b*c)/d)-5/64/b*d*(1/8/b*d*(d*x+c)^(3/
```

```
2)*sin(4/d*b*(d*x+c)+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d
*b*(d*x+c)+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d
-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(
a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 551, normalized size = 1.35

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/32768*(640*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) - 5120
*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 16*(64*(d*x + c)^
(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d) +
256*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)*b
- b*c + a*d)/d) - 240*((I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/
4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 15*(-(I -
1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I + 1)*s
qrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x
 + c)*sqrt(I*b/d)) - 15*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos
(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*
(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 240*(-(I + 1)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/
4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d
*x + c)*sqrt(-2*I*b/d)))*d/b^5
```

Fricas [A]
time = 3.84, size = 406, normalized size = 1.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) -
 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c
)*sqrt(b/(pi*d))) + 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(320*b^3*d^2*x^2 + 640*b^3*c*d*x +
 320*b^3*c^2 + 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*x + 64*b^3*c^2 - 15*b*d^2)*c
os(b*x + a)^4 - 255*b*d^2 - 8*(128*b^3*d^2*x^2 + 256*b^3*c*d*x + 128*b^3*c^
```

```
2 - 75*b*d^2)*cos(b*x + a)^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3
- 5*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [C] Result contains complex when optimal does not.
time = 1.24, size = 2446, normalized size = 6.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/16384*(512*(I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d
^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*
c^3 + 24*c*d^2*((I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf
(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*
c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(-8*I*(d*x +
 c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-4*(-I*(
d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2
+ 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
1)*b^2) - 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt
(d*x + c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 16*(-I*s
qrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt
(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*
d - 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2
 + 16*(I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x
+ c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/
```

```
b^2)/d^2) + d^3*((-I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*I*b^2*c^2*d - 72*b
*c*d^2 + 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
*b^3) - 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x + c)^(3/2)*b^2*c*d -
192*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)
*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d
)/b^3)/d^3 + (I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 + 192*I*b^2*c^2*d - 72*b*c*d^
2 - 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3)
- 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x + c)^(3/2)*b^2*c*d - 192*I*
sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(3/2)*b*d^2 + 72*sqrt(d*x + c)*b*c*d
^2 + 15*I*sqrt(d*x + c)*d^3)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/
d^3 + 32*(I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d^3)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)
^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d
- 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*
d^3)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 32*(-I*sqrt(pi)*(
64*b^3*c^3 + 48*I*b^2*c^2*d - 36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*
(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*
x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*
d^2 - 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-2*(I*(d*x + c)
*b - I*b*c + I*a*d)/d)/b^3)/d^3) + 192*(-I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d
*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(
I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(2)*sqrt
(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*
b) + 8*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt
(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b) - 8*I*sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b) + 16*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)
/b - 4*sqrt(d*x + c)*d*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 16*sqrt
(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b - 4*sqrt(d*x + c)*d
*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c^2)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^3\,\left(c+d\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2),x)
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

3.65 $\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=351

$$-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-$$

```
[Out] -1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b+3/1
024*d^(3/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/1024*d^(3/2)*FresnelC(2*b^(1/2)*2^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5
/2)-3/64*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/
Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/64*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^
(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+3/32*d*sin(2*b*x+2*a)*(d*
x+c)^(1/2)/b^2-3/256*d*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.45, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2}-\frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] -1/8*((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/b + ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) + (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(64*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) - (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx) - \frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right)\,dx$$

$$= -\left(\frac{1}{8}\int (c+dx)^{3/2}\sin(4a+4bx)\,dx\right) + \frac{1}{4}\int (c+dx)^{3/2}\sin(2a+$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} - \frac{(3d)\int}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d^{3/2}}{}$$

Mathematica [A]
time = 1.51, size = 393, normalized size = 1.12

$$\frac{-128bc\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(2(a+bx)) - 128b\sqrt{\frac{b}{d}}dx\sqrt{c+dx}\cos(2(a+bx)) + 32bc\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(4(a+bx)) + 32b\sqrt{\frac{b}{d}}dx\sqrt{c+dx}\cos(4(a+bx)) + 3d\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right) - 48d\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + 3d\sqrt{2\pi}\,\mathrm{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(4a-\frac{4bc}{d}\right) - 48d\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right) + 96\sqrt{\frac{b}{d}}d\sqrt{c+dx}\sin(2(a+bx)) - 12\sqrt{\frac{b}{d}}d\sqrt{c+dx}\sin(4(a+bx))}{1024b^2\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] + 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*
x)] + 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 3*d*Sqrt[2*Pi]*Co
s[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sq
rt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
+ 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (
4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Si
n[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 12*Sqr
t[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(1024*b^2*Sqrt[b/d])
```

Maple [A]
time = 0.06, size = 376, normalized size = 1.07

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx}}{\sqrt{\pi}\sqrt{}}\right.\right.}{}\right.}{8b}$
default	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx}}{\sqrt{\pi}\sqrt{}}\right.\right.}{}\right.}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+3/16/b*d*(1/4
/b*d*(d*x+c)^(1/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d
)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))+1
/64/b*d*(d*x+c)^(3/2)*cos(4/d*b*(d*x+c)+4*(a*d-b*c)/d)-3/64/b*d*(1/8/b*d*(d
*x+c)^(1/2)*sin(4/d*b*(d*x+c)+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+
c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.54, size = 503, normalized size = 1.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4096*(128*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 512*
(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 48*sqrt(d*x + c)
*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) + 384*sqrt(d*x + c)*b^2*sin(2*((d*x
 + c)*b - b*c + a*d)/d) + 24*(-(I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^
2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/
d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + 3*((I
```

```
+ 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I - 1)*s
qrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x +
 c)*sqrt(I*b/d)) + 3*(-(I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*
(b*c - a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c -
 a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + 24*((I - 1)*4^(1/4)*sqrt(2)*s
qrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(2)
*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt
(-2*I*b/d)))*d/b^4
```

Fricas [A]
time = 2.46, size = 316, normalized size = 0.90

$$\frac{3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-48\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-48\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+16\left(16\left(b^2dx+b^2c\right)\cos\left(bx+a\right)^4+10\,b^2dx+10\,b^2c-32\left(b^2dx+b^2c\right)\cos\left(bx+a\right)^2-3\left(2\,bd\cos\left(bx+a\right)^3-5\,bd\cos\left(bx+a\right)\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{1024\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2
*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fr
esnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 4
8*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d))) - 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(
b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4
+ 10*b^2*d*x + 10*b^2*c - 32*(b^2*d*x + b^2*c)*cos(b*x + a)^2 - 3*(2*b*d*co
s(b*x + a)^3 - 5*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin^3(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**3*cos(a + b*x), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.94, size = 1523, normalized size = 4.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2048*(64*(I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
```

```
b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^
2 + d^2*((I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(-8*I*(d*x + c)^(3/
2)*b*d + 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-4*(-I*(d*x + c
)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*
b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2)
 - 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x +
c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 16*(-I*sqrt(pi)
*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^
2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d - 3*s
qrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 16*(
I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*
c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^
2) + 16*(-I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a
*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*d)
*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c
- I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sqrt(pi)*(4*b*c
+ I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-
I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 16*sqrt(d*x + c
)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 4*sqrt(d*x + c)*d*e^(-4*(I
*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)
*b + I*b*c - I*a*d)/d)/b - 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b + I*b*c
- I*a*d)/d)/b)*c)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

3.66 $\int \sqrt{c+dx} \cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=299

$$-\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{8b}+\frac{\sqrt{c+dx}\,\cos(4a+4bx)}{32b}-\frac{\sqrt{d}\,\sqrt{\frac{\pi}{2}}\,\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}$$

```
[Out] -1/128*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/128*FresnelS(2*b^(1/2)*2^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^
(3/2)+1/16*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/
2))*d^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/
Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/8*cos(2*b*x+2*a)*(d*x
+c)^(1/2)/b+1/32*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.34, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$,
Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{d}\,\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\,\sqrt{d}\,\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{d}\,\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{\pi}\,\sqrt{d}\,\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{8b}+\frac{\sqrt{c+dx}\,\cos(4a+4bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] -1/8*(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/b + (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) - (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\,\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\,\sin(2a+2bx) - \frac{1}{8}\sqrt{c+dx}\,\sin(4a+4bx)\right) dx$$

$$= -\left(\frac{1}{8}\int \sqrt{c+dx}\,\sin(4a+4bx)\,dx\right) + \frac{1}{4}\int \sqrt{c+dx}\,\sin(2a+2b$$

$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\,\cos(4a+4bx)}{32b} - \frac{d\int \frac{\cos(4a+}{\sqrt{c+}}}{64b}$$

$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\,\cos(4a+4bx)}{32b} - \frac{(d\cos(4a}{}$$

$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\,\cos(4a+4bx)}{32b} - \frac{\cos(4a-}{}$$

$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\,\cos(4a+4bx)}{32b} - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}}{}$$

Mathematica [**A**]
time = 0.61, size = 264, normalized size = 0.88

$$\frac{-16\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(2(a+bx))+4\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(4(a+bx))-\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+8\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+\sqrt{2\pi}\,S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(4a-\frac{4bc}{d}\right)-8\sqrt{\pi}\,S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{128b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 4*Sqrt[b/d]*Sqrt[c + d*x]*C
os[4*(a + b*x)] - Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/
d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sq
rt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 8*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
 + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(128*b*Sqrt[b/d])
```

Maple [**A**]
time = 0.06, size = 286, normalized size = 0.96

method	result

derivativedivides	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx}}{\sqrt{\pi}\ \sqrt{}}\right.\right.}{16b\sqrt{\frac{b}{d}}}}{}$
default	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx}}{\sqrt{\pi}\ \sqrt{}}\right.\right.}{16b\sqrt{\frac{b}{d}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+1/32/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x
+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d))+1/64/b*d*(d*x+c)^(1/2)*cos(4/d*b*(d*x+c)+4*(a*d-b*c)/d)-1/256/b*d*
2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(
1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 429, normalized size = 1.43

$$\frac{\left(\frac{16\sqrt{dx+c}\,b^2\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{64\sqrt{dx+c}\,b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d}-4\left((i-1)\cdot4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(i+1)\cdot4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{2ib}{d}}\right)-\left(-(i-1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)-(i+1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4(bc-ad)}{d}\right)\right)\mathrm{erf}\left(2\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)-\left((i+1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)+(i-1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4(bc-ad)}{d}\right)\right)\mathrm{erf}\left(2\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)-4\left(-(i+1)\cdot4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)-(i-1)\cdot4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{2ib}{d}}\right)\right)d}{512\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/512*(16*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 64*sqrt(
d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 4*((I - 1)*4^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqr
t(2*I*b/d)) - (-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*
d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*e
rf(2*sqrt(d*x + c)*sqrt(I*b/d)) - ((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/
4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-
4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 4*(-(I + 1)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*
sqrt(-2*I*b/d)))*d/b^3
```

Fricas [**A**]
time = 1.56, size = 244, normalized size = 0.82

$$-\frac{\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)-8\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+8\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\left(8\,b\cos\left(bx+a\right)^4-16\,b\cos\left(bx+a\right)^2+5\,b\right)\sqrt{dx+c}}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_si
n(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 8*pi*d*sq
rt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d
))) + 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin
(-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 16*b*cos(b*x + a)^2 + 5*b)*sqr
t(d*x + c))/b^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c+dx}\,\sin^3\left(a+bx\right)\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x), x)
```

Giac [**C**] Result contains complex when optimal does not.
time = 0.75, size = 830, normalized size = 2.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/256*(-I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I
*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*(I*sqrt(2)*sqrt(pi)*d*erf
(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*
c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d
*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-
I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*er
f(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a
```

```
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c + 8*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sqrt(pi)*(4*b*c + I*d)*d*
erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 16*sqrt(d*x + c)*d*e^(-2
*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 4*sqrt(d*x + c)*d*e^(-4*(I*(d*x + c
)*b - I*b*c + I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*
c - I*a*d)/d)/b - 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/
d)/b)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)\,\sin(a+bx)^3\,\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

3.67 $\int \sqrt{c+dx} \cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=299

$$-\frac{\sqrt{c+dx}\ \cos(2a+2bx)}{8b}+\frac{\sqrt{c+dx}\ \cos(4a+4bx)}{32b}-\frac{\sqrt{d}\ \sqrt{\frac{\pi}{2}}\ \cos\left(4a-\frac{4bc}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{b}\ \sqrt{\frac{2}{\pi}}\ \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}$$

```
[Out] -1/128*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/128*FresnelS(2*b^(1/2)*2^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^
(3/2)+1/16*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/
2))*d^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/
Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/8*cos(2*b*x+2*a)*(d*x
+c)^(1/2)/b+1/32*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.31, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{\sqrt{\frac{\pi}{2}}\ \sqrt{d}\ \cos\left(4a-\frac{4bc}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\ \sqrt{b}\ \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\ \sqrt{d}\ \cos\left(2a-\frac{2bc}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{b}\ \sqrt{c+dx}}{\sqrt{\pi}\ \sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\ \sqrt{d}\ \sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\ \sqrt{\frac{2}{\pi}}\ \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{\pi}\ \sqrt{d}\ \sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\ \sqrt{c+dx}}{\sqrt{d}\ \sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{c+dx}\ \cos(2a+2bx)}{8b}+\frac{\sqrt{c+dx}\ \cos(4a+4bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] -1/8*(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/b + (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) - (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\, \cos(a+bx)\sin^3(a+bx)\, dx = \int \left(\frac{1}{4}\sqrt{c+dx}\, \sin(2a+2bx) - \frac{1}{8}\sqrt{c+dx}\, \sin(4a+4bx)\right)\, dx$$

$$= -\left(\frac{1}{8}\int \sqrt{c+dx}\, \sin(4a+4bx)\, dx\right) + \frac{1}{4}\int \sqrt{c+dx}\, \sin(2a+2b$$

$$= -\frac{\sqrt{c+dx}\, \cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\, \cos(4a+4bx)}{32b} - \frac{d\int \frac{\cos(4a+}{\sqrt{c+}}}{64b}$$

$$= -\frac{\sqrt{c+dx}\, \cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\, \cos(4a+4bx)}{32b} - \frac{(d\cos(4a}{}$$

$$= -\frac{\sqrt{c+dx}\, \cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\, \cos(4a+4bx)}{32b} - \frac{\cos(4a-}{}$$

$$= -\frac{\sqrt{c+dx}\, \cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\, \cos(4a+4bx)}{32b} - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}}{}$$

Mathematica [A]
time = 0.05, size = 264, normalized size = 0.88

$$\frac{-16\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(2(a+bx)) + 4\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(4(a+bx)) - \sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right) + 8\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + \sqrt{2\pi}\,S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(4a-\frac{4bc}{d}\right) - 8\sqrt{\pi}\,S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{128b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 4*Sqrt[b/d]*Sqrt[c + d*x]*C
os[4*(a + b*x)] - Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/
d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sq
rt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 8*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
 + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(128*b*Sqrt[b/d])
```

Maple [A]
time = 0.00, size = 286, normalized size = 0.96

method	result

derivativedivides	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx}}{\sqrt{\pi}\,\cdots}\right.\right.}{16b\sqrt{\frac{b}{d}}}}{\cdots}$
default	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx}}{\sqrt{\pi}\,\cdots}\right.\right.}{16b\sqrt{\frac{b}{d}}}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+1/32/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x
+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d))+1/64/b*d*(d*x+c)^(1/2)*cos(4/d*b*(d*x+c)+4*(a*d-b*c)/d)-1/256/b*d*
2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(
1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.50, size = 429, normalized size = 1.43

$$\frac{\left(\frac{16\sqrt{dx+c}\,b^2\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{64\sqrt{dx+c}\,b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d}-4\left((i-1)\cdot4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(i+1)\cdot4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{2ib}{d}}\right)-\left(-(i-1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)-(i+1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4(bc-ad)}{d}\right)\right)\mathrm{erf}\left(2\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)-\left((i+1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)+(i-1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4(bc-ad)}{d}\right)\right)\mathrm{erf}\left(2\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)-4\left(-(i+1)\cdot4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)-(i-1)\cdot4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{2ib}{d}}\right)\right)d}{512\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/512*(16*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 64*sqrt(
d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 4*((I - 1)*4^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqr
t(2*I*b/d)) - (-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*
d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*e
rf(2*sqrt(d*x + c)*sqrt(I*b/d)) - ((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/
4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-
4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 4*(-(I + 1)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*
sqrt(-2*I*b/d)))*d/b^3
```

Fricas [A]
time = 1.59, size = 244, normalized size = 0.82

$$-\frac{\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-8\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+8\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-4\left(8\,b\cos\left(bx+a\right)^{4}-16\,b\cos\left(bx+a\right)^{2}+5\,b\right)\sqrt{dx+c}}{128\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_si
n(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 8*pi*d*sq
rt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d
))) + 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin
(-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 16*b*cos(b*x + a)^2 + 5*b)*sqr
t(d*x + c))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^{3}\left(a+bx\right)\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.71, size = 830, normalized size = 2.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/256*(-I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I
*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*(I*sqrt(2)*sqrt(pi)*d*erf
(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*
c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d
*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-
I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*er
f(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a
```

```
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c + 8*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sqrt(pi)*(4*b*c + I*d)*d*
erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 16*sqrt(d*x + c)*d*e^(-2
*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 4*sqrt(d*x + c)*d*e^(-4*(I*(d*x + c
)*b - I*b*c + I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*
c - I*a*d)/d)/b - 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/
d)/b)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)\,\sin(a+bx)^3\,\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

3.68 $\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=351

$$-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-$$

```
[Out] -1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b+3/1
024*d^(3/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/1024*d^(3/2)*FresnelC(2*b^(1/2)*2^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5
/2)-3/64*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/
Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/64*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^
(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+3/32*d*sin(2*b*x+2*a)*(d*
x+c)^(1/2)/b^2-3/256*d*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.37, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2}-\frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] -1/8*((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/b + ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) + (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(64*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) - (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx) - \frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right) d\!\!$$

$$= -\left(\frac{1}{8}\int (c+dx)^{3/2}\sin(4a+4bx)\,dx\right) + \frac{1}{4}\int (c+dx)^{3/2}\sin(2a+$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} - \frac{(3d)\int}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d^{3/2}}{}$$

Mathematica [A]
time = 2.63, size = 393, normalized size = 1.12

$$\frac{-128bc\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(2(a+bx)) - 128b\sqrt{\frac{b}{d}}dx\sqrt{c+dx}\cos(2(a+bx)) + 32bc\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(4(a+bx)) + 32b\sqrt{\frac{b}{d}}dx\sqrt{c+dx}\cos(4(a+bx)) + 3d\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right) - 48d\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + 3d\sqrt{2\pi}\,\mathrm{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(4a-\frac{4bc}{d}\right) - 48d\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right) + 96\sqrt{\frac{b}{d}}d\sqrt{c+dx}\sin(2(a+bx)) - 12\sqrt{\frac{b}{d}}d\sqrt{c+dx}\sin(4(a+bx))}{1024b^2\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] + 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*
x)] + 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 3*d*Sqrt[2*Pi]*Co
s[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sq
rt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
+ 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (
4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Si
n[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 12*Sqr
t[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(1024*b^2*Sqrt[b/d])
```

Maple [A]
time = 0.00, size = 376, normalized size = 1.07

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx}}{\sqrt{\pi}\,\sqrt{}}\right.\right.}{}\right.}{8b}$
default	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx}}{\sqrt{\pi}\,\sqrt{}}\right.\right.}{}\right.}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+3/16/b*d*(1/4
/b*d*(d*x+c)^(1/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d
)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))+1
/64/b*d*(d*x+c)^(3/2)*cos(4/d*b*(d*x+c)+4*(a*d-b*c)/d)-3/64/b*d*(1/8/b*d*(d
*x+c)^(1/2)*sin(4/d*b*(d*x+c)+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+
c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 503, normalized size = 1.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4096*(128*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 512*
(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 48*sqrt(d*x + c)
*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) + 384*sqrt(d*x + c)*b^2*sin(2*((d*x
 + c)*b - b*c + a*d)/d) + 24*(-(I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^
2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/
d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + 3*((I
```

```
+ 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I - 1)*s
qrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x +
 c)*sqrt(I*b/d)) + 3*(-(I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*
(b*c - a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c -
 a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + 24*((I - 1)*4^(1/4)*sqrt(2)*s
qrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(2)
*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt
(-2*I*b/d)))*d/b^4
```

Fricas [A]
time = 0.96, size = 316, normalized size = 0.90

$$\frac{3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-48\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-48\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+16\left(16\left(b^2dx+b^2c\right)\cos\left(bx+a\right)^4+10\,b^2dx+10\,b^2c-32\left(b^2dx+b^2c\right)\cos\left(bx+a\right)^2-3\left(2\,bd\cos\left(bx+a\right)^3-5\,bd\cos\left(bx+a\right)\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{1024\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2
*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fr
esnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 4
8*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d))) - 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(
b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4
+ 10*b^2*d*x + 10*b^2*c - 32*(b^2*d*x + b^2*c)*cos(b*x + a)^2 - 3*(2*b*d*co
s(b*x + a)^3 - 5*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin^3\left(a+bx\right)\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**3*cos(a + b*x), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.92, size = 1523, normalized size = 4.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2048*(64*(I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
```

```
b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^
2 + d^2*((I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(-8*I*(d*x + c)^(3/
2)*b*d + 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-4*(-I*(d*x + c
)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*
b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2)
 - 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x +
c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 16*(-I*sqrt(pi)
*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^
2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d - 3*s
qrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 16*(
I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*
c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^
2) + 16*(-I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a
*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*d)
*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c
- I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sqrt(pi)*(4*b*c
+ I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-
I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 16*sqrt(d*x + c
)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 4*sqrt(d*x + c)*d*e^(-4*(I
*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)
*b + I*b*c - I*a*d)/d)/b - 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b + I*b*c
- I*a*d)/d)/b)*c)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

3.69 $\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=407

$$\frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{15d^2\sqrt{c+dx}\,\cos(4a+4bx)}{2048b^3}+\frac{(c+dx)^{5/2}\cos(4a+}{32b}$$

```
[Out] -1/8*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(5/2)*cos(4*b*x+4*a)/b+5/3
2*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/256*d*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b
^2+15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*
x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)-15/8192*d^(5/2)*FresnelS(2*b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1
/2)/b^(7/2)-15/256*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+15/256*d^(5/2)*FresnelS(2*b^(1/2)*(d*x
+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/128*d^2*co
s(2*b*x+2*a)*(d*x+c)^(1/2)/b^3-15/2048*d^2*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A]

time = 0.47, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15\sqrt{\pi}\,d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{15d^2\sqrt{c+dx}\,\cos(4a+4bx)}{2048b^3}+\frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2}-\frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*
a + 2*b*x])/(8*b) - (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) + ((
c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a -
(4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^
(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4
096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*
Sin[2*a + 2*b*x])/(32*b^2) - (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^
2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx) - \frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right) d$$

$$= -\left(\frac{1}{8}\int (c+dx)^{5/2}\sin(4a+4bx)\,dx\right) + \frac{1}{4}\int (c+dx)^{5/2}\sin(2a+$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b} - \frac{(5d)\int}{}$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b} + \frac{5d(c}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2}{}$$

Mathematica [A]
time = 11.43, size = 550, normalized size = 1.35

$$\frac{-1024b^3c^2\sqrt{c+dx}\cos(2(a+bx)) + 960bd^2\sqrt{c+dx}\cos(2(a+bx)) - 2048b^3cdx\sqrt{c+dx}\cos(2(a+bx)) - 1024b^3d^2x^2\sqrt{c+dx}\cos(2(a+bx)) + 256b^3c^2\sqrt{c+dx}\cos(4(a+bx)) - 60bd^2\sqrt{c+dx}\cos(4(a+bx)) + 512b^3cdx\sqrt{c+dx}\cos(4(a+bx)) + 256b^3d^2x^2\sqrt{c+dx}\cos(4(a+bx)) + 15\sqrt{\frac{b}{d}}d^3\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right) - 480\sqrt{\frac{b}{d}}d^3\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - 15\sqrt{\frac{b}{d}}d^3\sqrt{2\pi}\,S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(4a-\frac{4bc}{d}\right) + 480\sqrt{\frac{b}{d}}d^3\sqrt{\pi}\,S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right) + 1280b^2cd\sqrt{c+dx}\sin(2(a+bx)) + 1280b^2d^2x\sqrt{c+dx}\sin(2(a+bx)) - 160b^2cd\sqrt{c+dx}\sin(4(a+bx)) - 160b^2d^2x\sqrt{c+dx}\sin(4(a+bx))}{8192b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos
[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^
2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a +
 b*x)] - 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 512*b^3*c*d*x*Sqrt[c + d
*x]*Cos[4*(a + b*x)] + 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 15*
Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresnel
C[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Fresn
elS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b
/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (
```

2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] - 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(8192*b^4)

Maple [A]

time = 0.00, size = 470, normalized size = 1.15

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2}{d}\right.}{4b}\right.}{}\right.}{}$
default	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2}{d}\right.}{4b}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+5/16/b*d*(1/4
/b*d*(d*x+c)^(3/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos
(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*(a
*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/64/b*d*(
d*x+c)^(5/2)*cos(4/d*b*(d*x+c)+4*(a*d-b*c)/d)-5/64/b*d*(1/8/b*d*(d*x+c)^(3/
```

```
2)*sin(4/d*b*(d*x+c)+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d
*b*(d*x+c)+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d
-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(
a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 551, normalized size = 1.35

$$-\frac{\left(640\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)-5120\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-16\left(\frac{64\,(dx+c)^{\frac{5}{2}}b^4}{d}-15\sqrt{dx+c}\,b^2d\right)\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)+256\left(\frac{16\,(dx+c)^{\frac{5}{2}}b^4}{d}-15\sqrt{dx+c}\,b^2d\right)\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-240\left((i-1)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)+(i+1)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2\,(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{2i\,b}{d}}\right)-15\left(-(i-1)\sqrt{2}\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)-(i+1)\sqrt{2}\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4\,(bc-ad)}{d}\right)\right)\operatorname{erf}\left(2\sqrt{dx+c}\sqrt{\frac{i\,b}{d}}\right)-15\left((i+1)\sqrt{2}\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)+(i-1)\sqrt{2}\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4\,(bc-ad)}{d}\right)\right)\operatorname{erf}\left(2\sqrt{dx+c}\sqrt{-\frac{i\,b}{d}}\right)-240\left(-(i+1)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2\,(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{2i\,b}{d}}\right)\right)d}{32768\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/32768*(640*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) - 5120
*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 16*(64*(d*x + c)^
(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d) +
256*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)*b
- b*c + a*d)/d) - 240*((I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/
4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 15*(-(I -
1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I + 1)*s
qrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x
 + c)*sqrt(I*b/d)) - 15*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos
(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*
(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 240*(-(I + 1)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/
4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d
*x + c)*sqrt(-2*I*b/d)))*d/b^5
```

Fricas [A]
time = 1.33, size = 406, normalized size = 1.00

$$\frac{15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)-480\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+480\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+4\left(320\,b^3d^2x^2+640\,b^3cdx+320\,b^3c^2+8\left(64\,b^3d^2x^2+128\,b^3cdx+64\,b^3c^2-15\,bd^2\right)\cos\left(bx+a\right)^4-255\,bd^2-8\left(128\,b^3d^2x^2+256\,b^3cdx+128\,b^3c^2-75\,bd^2\right)\cos\left(bx+a\right)^2-160\left(2\left(b^2d^2x+b^2cd\right)\cos\left(bx+a\right)^3-5\left(b^2d^2x+b^2cd\right)\cos\left(bx+a\right)\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{8192\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) -
 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c
)*sqrt(b/(pi*d))) + 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(320*b^3*d^2*x^2 + 640*b^3*c*d*x +
 320*b^3*c^2 + 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*x + 64*b^3*c^2 - 15*b*d^2)*c
os(b*x + a)^4 - 255*b*d^2 - 8*(128*b^3*d^2*x^2 + 256*b^3*c*d*x + 128*b^3*c^
```

```
2 - 75*b*d^2)*cos(b*x + a)^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3
- 5*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [**C**] Result contains complex when optimal does not.
time = 1.23, size = 2446, normalized size = 6.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/16384*(512*(I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d
^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*
c^3 + 24*c*d^2*((I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf
(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*
c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(-8*I*(d*x +
 c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-4*(-I*(
d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2
+ 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
1)*b^2) - 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt
(d*x + c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 16*(-I*s
qrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt
(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*
d - 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2
 + 16*(I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x
+ c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/
```

```
b^2)/d^2) + d^3*((-I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*I*b^2*c^2*d - 72*b
*c*d^2 + 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
*b^3) - 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x + c)^(3/2)*b^2*c*d -
192*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)
*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d
)/b^3)/d^3 + (I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 + 192*I*b^2*c^2*d - 72*b*c*d^
2 - 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3)
- 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x + c)^(3/2)*b^2*c*d - 192*I*
sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(3/2)*b*d^2 + 72*sqrt(d*x + c)*b*c*d
^2 + 15*I*sqrt(d*x + c)*d^3)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/
d^3 + 32*(I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d^3)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)
^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d
- 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*
d^3)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 32*(-I*sqrt(pi)*(
64*b^3*c^3 + 48*I*b^2*c^2*d - 36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*
(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*
x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*
d^2 - 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-2*(I*(d*x + c)
*b - I*b*c + I*a*d)/d)/b^3)/d^3) + 192*(-I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d
*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(
I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(2)*sqrt
(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*
b) + 8*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt
(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b) - 8*I*sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b) + 16*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)
/b - 4*sqrt(d*x + c)*d*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 16*sqrt
(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b - 4*sqrt(d*x + c)*d
*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c^2)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2),x)
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

3.70 $\int (c+dx)^m \cos^2(a+bx) \sin(a+bx) \, dx$

Optimal. Leaf size=267

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b}-\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(\right.}{8b}$$

```
[Out] -1/8*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/
d)^m)-1/8*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*x+
c)/d)^m)-1/8*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*b*(d*x+c)
/d)/b/((-I*b*(d*x+c)/d)^m)-1/8*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I*b*(d*x+c)/d
)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.19, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4491, 3389, 2212}

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}-\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{8b}-\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{ib(c+dx)}{d}\right)}{8b}-\frac{3^{-m-1}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{3ib(c+dx)}{d}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -1/8*(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(
b*(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])
/(8*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - (3^(-1 - m)*E^((3*I)*(a
- (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(8*b*(((-I)*b
*(c + d*x))/d)^m) - (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(c + d*x)
)/d])/(8*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx &= \int \left(\frac{1}{4}(c+dx)^m \sin(a+bx)+\frac{1}{4}(c+dx)^m \sin(3a+3bx)\right)\,dx \\
&= \frac{1}{4}\int (c+dx)^m \sin(a+bx)\,dx+\frac{1}{4}\int (c+dx)^m \sin(3a+3bx)\,dx \\
&= \frac{1}{8}i\int e^{-i(a+bx)}(c+dx)^m\,dx-\frac{1}{8}i\int e^{i(a+bx)}(c+dx)^m\,dx+\frac{1}{8}i\int e^{-} \\
&= -\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b}-\frac{e^{-i\left(a-\right.}}{}
\end{aligned}$$

Mathematica [A]
time = 12.22, size = 282, normalized size = 1.06

$$\frac{e^{-\frac{i(bc+ad)}{d}}(c+dx)^m\left(-e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,\frac{ib(c+dx)}{d}\right)\right)}{8b}-\frac{3^{-1-m}e^{-\frac{3i(bc+ad)}{d}}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(e^{6ia}\left(\frac{ib(c+dx)}{d}\right)^{2m}\text{Gamma}\left(1+m,-\frac{3ib(c+dx)}{d}\right)+e^{\frac{6ibc}{d}}\left(\frac{b^2(c+dx)^2}{d^2}\right)^m\text{Gamma}\left(1+m,\frac{3ib(c+dx)}{d}\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] ((c + d*x)^m*(-((E^((2*I)*a)*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(((-I)*b*(
c + d*x))/d)^m) - (E^(((2*I)*b*c)/d)*Gamma[1 + m, (I*b*(c + d*x))/d])/((I*b
*(c + d*x))/d)^m))/(8*b*E^((I*(b*c + a*d))/d)) - (3^(-1 - m)*(c + d*x)^m*(E
^((6*I)*a)*((I*b*(c + d*x))/d)^(2*m)*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d] +
 E^(((6*I)*b*c)/d)*((b^2*(c + d*x)^2)/d^2)^m*Gamma[1 + m, ((3*I)*b*(c + d*x
))/d]))/(8*b*E^(((3*I)*(b*c + a*d))/d)*((I*b*(c + d*x))/d)^m*((b^2*(c + d*x
)^2)/d^2)^m)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cos^2(bx+a)\right)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a), x)
```

Fricas [A]
time = 0.80, size = 186, normalized size = 0.70

$$-\frac{3 e^{\left(-\frac{d m \log\left(\frac{i b}{d}\right) - i b c + i a d}{d}\right)} \Gamma\left(m+1, \frac{i b d x + i b c}{d}\right) + e^{\left(-\frac{d m \log\left(-\frac{3 i b}{d}\right) + 3 i b c - 3 i a d}{d}\right)} \Gamma\left(m+1, -\frac{3 (i b d x + i b c)}{d}\right) + 3 e^{\left(-\frac{d m \log\left(-\frac{i b}{d}\right) + i b c - i a d}{d}\right)} \Gamma\left(m+1, \frac{-i b d x - i b c}{d}\right) + e^{\left(-\frac{d m \log\left(\frac{3 i b}{d}\right) - 3 i b c + 3 i a d}{d}\right)} \Gamma\left(m+1, -\frac{3 (-i b d x - i b c)}{d}\right)}{24 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
[Out] -1/24*(3*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x + I*
b*c)/d) + e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m + 1, -3*(I
*b*d*x + I*b*c)/d) + 3*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(m + 1
, (-I*b*d*x - I*b*c)/d) + e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)*gam
ma(m + 1, -3*(-I*b*d*x - I*b*c)/d))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^{m} \sin(a + bx) \cos^{2}(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a),x)
[Out] Integral((c + d*x)**m*sin(a + b*x)*cos(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^m,x)
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^m, x)
```

3.71 $\int (c+dx)^4 \cos^2(a+bx) \sin(a+bx)\,dx$

Optimal. Leaf size=205

$$-\frac{160d^4\cos(a+bx)}{27b^5}+\frac{8d^2(c+dx)^2\cos(a+bx)}{3b^3}-\frac{8d^4\cos^3(a+bx)}{81b^5}+\frac{4d^2(c+dx)^2\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^4\cos}{3b}$$

```
[Out] -160/27*d^4*cos(b*x+a)/b^5+8/3*d^2*(d*x+c)^2*cos(b*x+a)/b^3-8/81*d^4*cos(b*
x+a)^3/b^5+4/9*d^2*(d*x+c)^2*cos(b*x+a)^3/b^3-1/3*(d*x+c)^4*cos(b*x+a)^3/b-
160/27*d^3*(d*x+c)*sin(b*x+a)/b^4+8/9*d*(d*x+c)^3*sin(b*x+a)/b^2-8/27*d^3*(
d*x+c)*cos(b*x+a)^2*sin(b*x+a)/b^4+4/9*d*(d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)/
b^2
```

Rubi [A]
time = 0.15, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4490, 3392, 3377, 2718, 3391}

$$-\frac{8d^4\cos^3(a+bx)}{81b^5}-\frac{160d^4\cos(a+bx)}{27b^5}-\frac{160d^3(c+dx)\sin(a+bx)}{27b^4}-\frac{8d^3(c+dx)\sin(a+bx)\cos^2(a+bx)}{27b^4}+\frac{4d^2(c+dx)^2\cos^3(a+bx)}{9b^3}+\frac{8d^2(c+dx)^2\cos(a+bx)}{3b^3}+\frac{8d(c+dx)^3\sin(a+bx)}{9b^2}+\frac{4d(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{9b^2}-\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (-160*d^4*Cos[a + b*x])/(27*b^5) + (8*d^2*(c + d*x)^2*Cos[a + b*x])/(3*b^3)
 - (8*d^4*Cos[a + b*x]^3)/(81*b^5) + (4*d^2*(c + d*x)^2*Cos[a + b*x]^3)/(9*
b^3) - ((c + d*x)^4*Cos[a + b*x]^3)/(3*b) - (160*d^3*(c + d*x)*Sin[a + b*x]
)/(27*b^4) + (8*d*(c + d*x)^3*Sin[a + b*x])/(9*b^2) - (8*d^3*(c + d*x)*Cos[
a + b*x]^2*Sin[a + b*x])/(27*b^4) + (4*d*(c + d*x)^3*Cos[a + b*x]^2*Sin[a +
 b*x])/(9*b^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

```
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^4 \cos^2(a+bx)\sin(a+bx)\,dx &= -\frac{(c+dx)^4\cos^3(a+bx)}{3b} + \frac{(4d)\int (c+dx)^3\cos^3(a+bx)\,dx}{3b} \\
&= \frac{4d^2(c+dx)^2\cos^3(a+bx)}{9b^3} - \frac{(c+dx)^4\cos^3(a+bx)}{3b} + \frac{4d(c+dx)^3}{} \\
&= -\frac{8d^4\cos^3(a+bx)}{81b^5} + \frac{4d^2(c+dx)^2\cos^3(a+bx)}{9b^3} - \frac{(c+dx)^4\cos^3(a}{3b} \\
&= \frac{8d^2(c+dx)^2\cos(a+bx)}{3b^3} - \frac{8d^4\cos^3(a+bx)}{81b^5} + \frac{4d^2(c+dx)^2\cos^3(a}{9b^3} \\
&= -\frac{16d^4\cos(a+bx)}{27b^5} + \frac{8d^2(c+dx)^2\cos(a+bx)}{3b^3} - \frac{8d^4\cos^3(a+bx)}{81b^5} \\
&= -\frac{160d^4\cos(a+bx)}{27b^5} + \frac{8d^2(c+dx)^2\cos(a+bx)}{3b^3} - \frac{8d^4\cos^3(a+bx}{81b^5}
\end{aligned}$$

Mathematica [A]

time = 1.63, size = 150, normalized size = 0.73

$$-\frac{81(24d^4-12b^2d^2(c+dx)^2+b^4(c+dx)^4)\cos(a+bx)+(8d^4-36b^2d^2(c+dx)^2+27b^4(c+dx)^4)\cos(3(a+bx))-24bd(c+dx)\left(-82d^2+15b^2(c+dx)^2+(-2d^2+3b^2(c+dx)^2)\cos(2(a+bx))\right)\sin(a+bx)}{324b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -1/324*(81*(24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x]
 + (8*d^4 - 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cos[3*(a + b*x)] -
```

```
24*b*d*(c + d*x)*(-82*d^2 + 15*b^2*(c + d*x)^2 + (-2*d^2 + 3*b^2*(c + d*x)
^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/b^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 834 vs. $2(187) = 374$.
time = 0.20, size = 835, normalized size = 4.07

method	result
risch	$-\frac{(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-12b^2d^4x^2-24b^2c\,d^3x-12b^2c^2d^2+24d^4)\cos(bx+a)}{4b^5}+\frac{d(b^2d^3x^3+3$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/3/b^4*a^4*d^4*cos(b*x+a)^3+4/3/b^3*a^3*c*d^3*cos(b*x+a)^3-4/b^4*a^3
*d^4*(-1/3*(b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))-2/b^2*a^2*
c^2*d^2*cos(b*x+a)^3+12/b^3*a^2*c*d^3*(-1/3*(b*x+a)*cos(b*x+a)^3+1/9*(2+cos
(b*x+a)^2)*sin(b*x+a))+6/b^4*a^2*d^4*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+
a)*(2+cos(b*x+a)^2)*sin(b*x+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))+4/3/b*a*c^
3*d*cos(b*x+a)^3-12/b^2*a*c^2*d^2*(-1/3*(b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x
+a)^2)*sin(b*x+a))-12/b^3*a*c*d^3*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+a)*
(2+cos(b*x+a)^2)*sin(b*x+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))-4/b^4*a*d^4*(
-1/3*(b*x+a)^3*cos(b*x+a)^3+1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/3*s
in(b*x+a)+4/3*(b*x+a)*cos(b*x+a)+2/9*(b*x+a)*cos(b*x+a)^3-2/27*(2+cos(b*x+a
)^2)*sin(b*x+a))-1/3*c^4*cos(b*x+a)^3+4/b*c^3*d*(-1/3*(b*x+a)*cos(b*x+a)^3+
1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+6/b^2*c^2*d^2*(-1/3*(b*x+a)^2*cos(b*x+a)^3
+2/9*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))+
4/b^3*c*d^3*(-1/3*(b*x+a)^3*cos(b*x+a)^3+1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin
(b*x+a)-4/3*sin(b*x+a)+4/3*(b*x+a)*cos(b*x+a)+2/9*(b*x+a)*cos(b*x+a)^3-2/27
*(2+cos(b*x+a)^2)*sin(b*x+a))+1/b^4*d^4*(-1/3*(b*x+a)^4*cos(b*x+a)^3+4/9*(b
*x+a)^3*(2+cos(b*x+a)^2)*sin(b*x+a)+8/3*(b*x+a)^2*cos(b*x+a)-160/27*cos(b*x
+a)-16/3*(b*x+a)*sin(b*x+a)+4/9*(b*x+a)^2*cos(b*x+a)^3-8/27*(b*x+a)*(2+cos(
b*x+a)^2)*sin(b*x+a)-8/81*cos(b*x+a)^3))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(187) = 374$.
time = 0.33, size = 889, normalized size = 4.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/324*(108*c^4*cos(b*x + a)^3 - 432*a*c^3*d*cos(b*x + a)^3/b + 648*a^2*c^2
*d^2*cos(b*x + a)^3/b^2 - 432*a^3*c*d^3*cos(b*x + a)^3/b^3 + 108*a^4*d^4*co
s(b*x + a)^3/b^4 + 36*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x +
 a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*c^3*d/b - 108*(3*(b*x + a)*cos(3*b
*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a
*c^2*d^2/b^2 + 108*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a)
 - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a^2*c*d^3/b^3 - 36*(3*(b*x + a)*cos(3
*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))
*a^3*d^4/b^4 + 18*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 -
 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a)
)*c^2*d^2/b^2 - 36*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2
- 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a
))*a*c*d^3/b^3 + 18*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2
 - 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x +
a))*a^2*d^4/b^4 + 12*(3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) + 27
*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2 - 2)*sin(3*b*x +
 3*a) - 81*((b*x + a)^2 - 2)*sin(b*x + a))*c*d^3/b^3 - 12*(3*(3*(b*x + a)^3
 - 2*b*x - 2*a)*cos(3*b*x + 3*a) + 27*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x +
 a) - (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*sin(b*x +
 a))*a*d^4/b^4 + ((27*(b*x + a)^4 - 36*(b*x + a)^2 + 8)*cos(3*b*x + 3*a) +
81*((b*x + a)^4 - 12*(b*x + a)^2 + 24)*cos(b*x + a) - 12*(3*(b*x + a)^3 - 2
*b*x - 2*a)*sin(3*b*x + 3*a) - 324*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a)
)*d^4/b^4)/b
```

Fricas [A]
time = 2.19, size = 294, normalized size = 1.43

$$-\frac{\left(27 b^4 d^4 x^4 + 108 b^4 c d^3 x^3 + 27 b^4 c^4 - 36 b^2 c^2 d^2 + 8 d^4 + 18 \left(9 b^4 c^2 d^2 - 2 b^2 d^4\right) x^2 + 36 \left(3 b^4 c^3 d - 2 b^2 c d^3\right) x\right) \cos\left(bx+a\right)^3 - 24 \left(9 b^2 d^4 x^2 + 18 b^2 c d^3 x + 9 b^2 c^2 d^2 - 20 d^4\right) \cos\left(bx+a\right) - 12 \left(6 b^3 d^4 x^3 + 18 b^3 c d^3 x^2 + 6 b^3 c^3 d - 40 b c d^3 + \left(3 b^3 d^4 x^3 + 9 b^3 c d^3 x^2 + 3 b^3 c^3 d - 2 b c d^3 + \left(9 b^3 c^2 d^2 - 2 b d^4\right) x\right) \cos\left(bx+a\right)^2 + 2 \left(9 b^3 c^2 d^2 - 20 b d^4\right) x\right) \sin\left(bx+a\right)}{81 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/81*((27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 +
8*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^3)
*x)*cos(b*x + a)^3 - 24*(9*b^2*d^4*x^2 + 18*b^2*c*d^3*x + 9*b^2*c^2*d^2 - 2
0*d^4)*cos(b*x + a) - 12*(6*b^3*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*c^3*d -
40*b*c*d^3 + (3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 2*b*c*d^3 + (
9*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 + 2*(9*b^3*c^2*d^2 - 20*b*d^4)*x
)*sin(b*x + a))/b^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(207) = 414$.
time = 0.72, size = 646, normalized size = 3.15

$$\begin{cases} - \frac{c^{4} \cos^{3}{\left(a + b x \right)}}{3 b} - \frac{4 c^{3} d x \cos^{3}{\left(a + b x \right)}}{3 b} - \frac{2 c^{2} d^{2} x^{2} \cos^{3}{\left(a + b x \right)}}{b} - \frac{4 c d^{3} x^{3} \cos^{3}{\left(a + b x \right)}}{3 b} - \frac{d^{4} x^{4} \cos^{3}{\left(a + b x \right)}}{3 b} + \frac{8 c^{3} d \sin^{3}{\left(a + b x \right)}}{9 b^{2}} + \frac{4 c^{3} d \sin{\left(a + b x \right)} \cos^{2}{\left(a + b x \right)}}{3 b^{2}} + \frac{8 c^{2} d^{2} x \sin^{3}{\left(a + b x \right)}}{3 b^{2}} + \frac{4 c^{2} d^{2} x \sin{\left(a + b x \right)} \cos^{2}{\left(a + b x \right)}}{b^{2}} + \frac{8 c d^{3} x^{2} \sin^{3}{\left(a + b x \right)}}{3 b^{2}} + \frac{4 c d^{3} x^{2} \sin{\left(a + b x \right)} \cos^{2}{\left(a + b x \right)}}{b^{2}} + \frac{8 d^{4} x^{3} \sin^{3}{\left(a + b x \right)}}{9 b^{2}} + \frac{4 d^{4} x^{3} \sin{\left(a + b x \right)} \cos^{2}{\left(a + b x \right)}}{3 b^{2}} + \frac{8 c^{2} d^{2} \sin^{2}{\left(a + b x \right)} \cos{\left(a + b x \right)}}{3 b^{3}} + \frac{28 c^{2} d^{2} \cos^{3}{\left(a + b x \right)}}{9 b^{3}} + \frac{16 c d^{3} x \sin^{2}{\left(a + b x \right)} \cos{\left(a + b x \right)}}{3 b^{3}} + \frac{56 c d^{3} x \cos^{3}{\left(a + b x \right)}}{9 b^{3}} + \frac{8 d^{4} x^{2} \sin^{2}{\left(a + b x \right)} \cos{\left(a + b x \right)}}{3 b^{3}} + \frac{28 d^{4} x^{2} \cos^{3}{\left(a + b x \right)}}{9 b^{3}} - \frac{160 c d^{3} \sin^{3}{\left(a + b x \right)}}{27 b^{4}} - \frac{56 c d^{3} \sin{\left(a + b x \right)} \cos^{2}{\left(a + b x \right)}}{9 b^{4}} - \frac{160 d^{4} x \sin^{3}{\left(a + b x \right)}}{27 b^{4}} - \frac{56 d^{4} x \sin{\left(a + b x \right)} \cos^{2}{\left(a + b x \right)}}{9 b^{4}} - \frac{160 d^{4} \sin^{2}{\left(a + b x \right)} \cos{\left(a + b x \right)}}{27 b^{5}} - \frac{488 d^{4} \cos^{3}{\left(a + b x \right)}}{81 b^{5}} & \text{for}\: b \neq 0 \\\left(c^{4} x + 2 c^{3} d x^{2} + 2 c^{2} d^{2} x^{3} + c d^{3} x^{4} + \frac{d^{4} x^{5}}{5}\right) \sin{\left(a \right)} \cos^{2}{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**4*cos(a + b*x)**3/(3*b) - 4*c**3*d*x*cos(a + b*x)**3/(3*b) -
 2*c**2*d**2*x**2*cos(a + b*x)**3/b - 4*c*d**3*x**3*cos(a + b*x)**3/(3*b) -
 d**4*x**4*cos(a + b*x)**3/(3*b) + 8*c**3*d*sin(a + b*x)**3/(9*b**2) + 4*c*
*3*d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2) + 8*c**2*d**2*x*sin(a + b*x)**3/
(3*b**2) + 4*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**2/b**2 + 8*c*d**3*x**2*
sin(a + b*x)**3/(3*b**2) + 4*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**2/b**2
+ 8*d**4*x**3*sin(a + b*x)**3/(9*b**2) + 4*d**4*x**3*sin(a + b*x)*cos(a + b
*x)**2/(3*b**2) + 8*c**2*d**2*sin(a + b*x)**2*cos(a + b*x)/(3*b**3) + 28*c*
*2*d**2*cos(a + b*x)**3/(9*b**3) + 16*c*d**3*x*sin(a + b*x)**2*cos(a + b*x)
/(3*b**3) + 56*c*d**3*x*cos(a + b*x)**3/(9*b**3) + 8*d**4*x**2*sin(a + b*x)
**2*cos(a + b*x)/(3*b**3) + 28*d**4*x**2*cos(a + b*x)**3/(9*b**3) - 160*c*d
**3*sin(a + b*x)**3/(27*b**4) - 56*c*d**3*sin(a + b*x)*cos(a + b*x)**2/(9*b
**4) - 160*d**4*x*sin(a + b*x)**3/(27*b**4) - 56*d**4*x*sin(a + b*x)*cos(a
+ b*x)**2/(9*b**4) - 160*d**4*sin(a + b*x)**2*cos(a + b*x)/(27*b**5) - 488*
d**4*cos(a + b*x)**3/(81*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**
2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*cos(a)**2, True))
```

Giac [**A**]
time = 0.45, size = 350, normalized size = 1.71

$$-\frac{\left(27\,b^4d^4x^4+108\,b^4cd^3x^3+162\,b^4c^2d^2x^2+108\,b^4c^3dx+27\,b^4c^4-36\,b^2d^4x^2-72\,b^2cd^3x-36\,b^2c^2d^2+8\,d^4\right)\cos\left(3\,bx+3\,a\right)}{324\,b^5}-\frac{\left(b^4d^4x^4+4\,b^4cd^3x^3+6\,b^4c^2d^2x^2+4\,b^4c^3dx+b^4c^4-12\,b^2d^4x^2-24\,b^2cd^3x-12\,b^2c^2d^2+24\,d^4\right)\cos\left(bx+a\right)}{4\,b^5}+\frac{\left(3\,b^3d^4x^3+9\,b^3cd^3x^2+9\,b^3c^2d^2x+3\,b^3c^3d-2\,bd^4x-2\,bcd^3\right)\sin\left(3\,bx+3\,a\right)}{27\,b^5}+\frac{\left(b^3d^4x^3+3\,b^3cd^3x^2+3\,b^3c^2d^2x+b^3c^3d-6\,bd^4x-6\,bcd^3\right)\sin\left(bx+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*
c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8
*d^4)*cos(3*b*x + 3*a)/b^5 - 1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2
*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b
^2*c^2*d^2 + 24*d^4)*cos(b*x + a)/b^5 + 1/27*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x
^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*sin(3*b*x + 3*a
)/b^5 + (b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*
d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

Mupad [**B**]
time = 1.90, size = 448, normalized size = 2.19

$$\frac{4\,x\,\cos\left(a+b\,x\right)^3\,\left(14\,c\,d^3-3\,b^2\,c^3\,d\right)}{9\,b^3}-\frac{\cos\left(a+b\,x\right)^3\,\left(27\,b^4\,c^4-252\,b^2\,c^2\,d^2+488\,d^4\right)}{81\,b^5}-\frac{8\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\,\left(20\,d^4-9\,b^2\,c^2\,d^2\right)}{27\,b^5}-\frac{4\,\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(14\,c\,d^3-3\,b^2\,c^3\,d\right)}{9\,b^4}-\frac{d^4\,x^4\,\cos\left(a+b\,x\right)^3}{3\,b}-\frac{8\,\sin\left(a+b\,x\right)^3\,\left(20\,c\,d^3-3\,b^2\,c^3\,d\right)}{27\,b^4}+\frac{8\,d^4\,x^3\,\sin\left(a+b\,x\right)^3}{9\,b^2}-\frac{8\,x\,\sin\left(a+b\,x\right)^3\,\left(20\,d^4-9\,b^2\,c^2\,d^2\right)}{27\,b^4}+\frac{2\,x^2\,\cos\left(a+b\,x\right)^3\,\left(14\,d^4-9\,b^2\,c^2\,d^2\right)}{9\,b^3}-\frac{4\,c\,d^3\,x^3\,\cos\left(a+b\,x\right)^3}{3\,b}+\frac{4\,d^4\,x^3\,\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{3\,b^2}+\frac{8\,d^4\,x^2\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{3\,b^3}+\frac{8\,c\,d^3\,x^2\,\sin\left(a+b\,x\right)^3}{3\,b^2}-\frac{4\,x\,\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(14\,d^4-9\,b^2\,c^2\,d^2\right)}{9\,b^4}+\frac{4\,c\,d^3\,x^2\,\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{b^2}+\frac{16\,c\,d^3\,x\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^4,x)
```

```
[Out] (4*x*cos(a + b*x)^3*(14*c*d^3 - 3*b^2*c^3*d))/(9*b^3) - (cos(a + b*x)^3*(48
8*d^4 + 27*b^4*c^4 - 252*b^2*c^2*d^2))/(81*b^5) - (8*cos(a + b*x)*sin(a + b
```

```
*x)^2*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^5) - (4*cos(a + b*x)^2*sin(a + b*x)*(
14*c*d^3 - 3*b^2*c^3*d))/(9*b^4) - (d^4*x^4*cos(a + b*x)^3)/(3*b) - (8*sin(
a + b*x)^3*(20*c*d^3 - 3*b^2*c^3*d))/(27*b^4) + (8*d^4*x^3*sin(a + b*x)^3)/
(9*b^2) - (8*x*sin(a + b*x)^3*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^4) + (2*x^2*c
os(a + b*x)^3*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^3) - (4*c*d^3*x^3*cos(a + b*x)
^3)/(3*b) + (4*d^4*x^3*cos(a + b*x)^2*sin(a + b*x))/(3*b^2) + (8*d^4*x^2*co
s(a + b*x)*sin(a + b*x)^2)/(3*b^3) + (8*c*d^3*x^2*sin(a + b*x)^3)/(3*b^2) -
 (4*x*cos(a + b*x)^2*sin(a + b*x)*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^4) + (4*c*
d^3*x^2*cos(a + b*x)^2*sin(a + b*x))/b^2 + (16*c*d^3*x*cos(a + b*x)*sin(a +
 b*x)^2)/(3*b^3)
```

3.72 $\int (c+dx)^3 \cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=151

$$\frac{4d^2(c+dx)\cos(a+bx)}{3b^3}+\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}-\frac{14d^3\sin(a+bx)}{9b^4}+\frac{2d(c+dx)^2}{3\ldots}$$

```
[Out] 4/3*d^2*(d*x+c)*cos(b*x+a)/b^3+2/9*d^2*(d*x+c)*cos(b*x+a)^3/b^3-1/3*(d*x+c)
^3*cos(b*x+a)^3/b-14/9*d^3*sin(b*x+a)/b^4+2/3*d*(d*x+c)^2*sin(b*x+a)/b^2+1/
3*d*(d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)/b^2+2/27*d^3*sin(b*x+a)^3/b^4
```

Rubi [A]

time = 0.12, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4490, 3392, 3377, 2717, 2713}

$$\frac{2d^3\sin^3(a+bx)}{27b^4}-\frac{14d^3\sin(a+bx)}{9b^4}+\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}+\frac{4d^2(c+dx)\cos(a+bx)}{3b^3}+\frac{2d(c+dx)^2\sin(a+bx)}{3b^2}+\frac{d(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b^2}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (4*d^2*(c + d*x)*Cos[a + b*x])/(3*b^3) + (2*d^2*(c + d*x)*Cos[a + b*x]^3)/(
9*b^3) - ((c + d*x)^3*Cos[a + b*x]^3)/(3*b) - (14*d^3*Sin[a + b*x])/(9*b^4)
 + (2*d*(c + d*x)^2*Sin[a + b*x])/(3*b^2) + (d*(c + d*x)^2*Cos[a + b*x]^2*S
in[a + b*x])/(3*b^2) + (2*d^3*Sin[a + b*x]^3)/(27*b^4)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
```

```
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^3\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^3\cos^3(a+bx)}{3b}+\frac{d\int (c+dx)^2\cos^3(a+bx)\,dx}{b}$$
$$=\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}+\frac{d(c+dx)^2\,\text{co}\ldots}{\ldots}$$
$$=\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}+\frac{2d(c+dx)^2\,\text{s}\ldots}{3b^2\ldots}$$
$$=\frac{4d^2(c+dx)\cos(a+bx)}{3b^3}+\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^3\,\text{co}\ldots}{3\ldots}$$
$$=\frac{4d^2(c+dx)\cos(a+bx)}{3b^3}+\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^3\,\text{co}\ldots}{3\ldots}$$

Mathematica [**A**]
time = 0.90, size = 127, normalized size = 0.84

$$\frac{-27b(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\cos(a+bx)-3b(c+dx)\left(-2d^2+3b^2(c+dx)^2\right)\cos(3(a+bx))+2d\left(-82d^2+45b^2(c+dx)^2+\left(-2d^2+9b^2(c+dx)^2\right)\cos(2(a+bx))\right)\sin(a+bx)}{108b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (-27*b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 3*b*(c + d*x)*(-
2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 2*d*(-82*d^2 + 45*b^2*(c + d*
x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/(108*b^
4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(137) = 274$.
time = 0.19, size = 447, normalized size = 2.96 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/3/b^3*a^3*d^3*cos(b*x+a)^3-1/b^2*a^2*c*d^2*cos(b*x+a)^3+3/b^3*a^2*d^
3*(-1/3*(b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+1/b*a*c^2*d*c
os(b*x+a)^3-6/b^2*a*c*d^2*(-1/3*(b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*s
in(b*x+a))-3/b^3*a*d^3*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x+
a)^2)*sin(b*x+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))-1/3*c^3*cos(b*x+a)^3+3/b
*c^2*d*(-1/3*(b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+3/b^2*c*
d^2*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+2/
27*cos(b*x+a)^3+4/9*cos(b*x+a))+1/b^3*d^3*(-1/3*(b*x+a)^3*cos(b*x+a)^3+1/3*
(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/3*sin(b*x+a)+4/3*(b*x+a)*cos(b*x+a)
+2/9*(b*x+a)*cos(b*x+a)^3-2/27*(2+cos(b*x+a)^2)*sin(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(137) = 274$.
time = 0.30, size = 505, normalized size = 3.34

$$-\frac{36\,c^3\cos\left(bx+a\right)^3-\frac{108\,ac^2d\cos\left(bx+a\right)^3}{b}+\frac{108\,a^2cd^2\cos\left(bx+a\right)^3}{b^2}-\frac{36\,a^3d^3\cos\left(bx+a\right)^3}{b^3}+\frac{9\,\left(3\,\left(bx+a\right)\cos\left(3\,bx+3\,a\right)+9\,\left(bx+a\right)\cos\left(bx+a\right)-\sin\left(3\,bx+3\,a\right)-9\,\sin\left(bx+a\right)\right)c^2d}{b}-\frac{18\,\left(3\,\left(bx+a\right)\cos\left(3\,bx+3\,a\right)+9\,\left(bx+a\right)\cos\left(bx+a\right)-\sin\left(3\,bx+3\,a\right)-9\,\sin\left(bx+a\right)\right)acd^2}{b^2}+\frac{9\,\left(3\,\left(bx+a\right)\cos\left(3\,bx+3\,a\right)+9\,\left(bx+a\right)\cos\left(bx+a\right)-\sin\left(3\,bx+3\,a\right)-9\,\sin\left(bx+a\right)\right)a^2d^3}{b^3}+\frac{3\,\left(\left(9\,\left(bx+a\right)^2-2\right)\cos\left(3\,bx+3\,a\right)+27\,\left(\left(bx+a\right)^2-2\right)\cos\left(bx+a\right)-6\,\left(bx+a\right)\sin\left(3\,bx+3\,a\right)-54\,\left(bx+a\right)\sin\left(bx+a\right)\right)cd^2}{b^2}-\frac{3\,\left(\left(9\,\left(bx+a\right)^2-2\right)\cos\left(3\,bx+3\,a\right)+27\,\left(\left(bx+a\right)^2-2\right)\cos\left(bx+a\right)-6\,\left(bx+a\right)\sin\left(3\,bx+3\,a\right)-54\,\left(bx+a\right)\sin\left(bx+a\right)\right)ad^3}{b^3}+\frac{\left(3\,\left(3\,\left(bx+a\right)^3-2\,bx-2\,a\right)\cos\left(3\,bx+3\,a\right)+27\,\left(\left(bx+a\right)^3-6\,bx-6\,a\right)\cos\left(bx+a\right)-\left(9\,\left(bx+a\right)^2-2\right)\sin\left(3\,bx+3\,a\right)-81\,\left(\left(bx+a\right)^2-2\right)\sin\left(bx+a\right)\right)d^3}{b^3}}{108\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/108*(36*c^3*cos(b*x + a)^3 - 108*a*c^2*d*cos(b*x + a)^3/b + 108*a^2*c*d^
2*cos(b*x + a)^3/b^2 - 36*a^3*d^3*cos(b*x + a)^3/b^3 + 9*(3*(b*x + a)*cos(3
*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))
*c^2*d/b - 18*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - si
n(3*b*x + 3*a) - 9*sin(b*x + a))*a*c*d^2/b^2 + 9*(3*(b*x + a)*cos(3*b*x + 3
*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a^2*d^3
/b^3 + 3*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2)*cos(b
*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a))*c*d^2/b
^2 - 3*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2)*cos(b*x
 + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a))*a*d^3/b^3
 + (3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) + 27*((b*x + a)^3 - 6*
b*x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 81*((b*x +
 a)^2 - 2)*sin(b*x + a))*d^3/b^3)/b
```

Fricas [A]
time = 1.94, size = 183, normalized size = 1.21

$$-\frac{3\,\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+3\,b^3c^3-2\,bcd^2+\left(9\,b^3c^2d-2\,bd^3\right)x\right)\cos\left(bx+a\right)^3-36\,\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)-\left(18\,b^2d^3x^2+36\,b^2cd^2x+18\,b^2c^2d-40\,d^3+\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^2\right)\sin\left(bx+a\right)}{27\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/27*(3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3*
c^2*d - 2*b*d^3)*x)*cos(b*x + a)^3 - 36*(b*d^3*x + b*c*d^2)*cos(b*x + a) -
```

```
(18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - 40*d^3 + (9*b^2*d^3*x^2 +
 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)^2)*sin(b*x + a))/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(150) = 300$.
time = 0.47, size = 391, normalized size = 2.59

$$\begin{cases} -\frac{c^3\cos^3(a+bx)}{3b} - \frac{c^2dx\cos^3(a+bx)}{b} - \frac{cd^2x^2\cos^3(a+bx)}{b} - \frac{d^3x^3\cos^3(a+bx)}{3b} + \frac{2c^2d\sin^3(a+bx)}{3b^2} + \frac{c^2d\sin(a+bx)\cos^2(a+bx)}{b^2} + \frac{4cd^2x\sin^3(a+bx)}{3b^2} + \frac{2cd^2x\sin(a+bx)\cos^2(a+bx)}{b^2} + \frac{2d^3x^2\sin^3(a+bx)}{3b^2} + \frac{d^3x^2\sin(a+bx)\cos^2(a+bx)}{b^2} + \frac{4cd^2\sin^2(a+bx)\cos(a+bx)}{3b^3} + \frac{14cd^2\cos^3(a+bx)}{9b^3} + \frac{4d^3x\sin^2(a+bx)\cos(a+bx)}{3b^3} + \frac{14d^3x\cos^3(a+bx)}{9b^3} - \frac{40d^3\sin^3(a+bx)}{27b^4} - \frac{14d^3\sin(a+bx)\cos^2(a+bx)}{9b^4} & \text{for } b \neq 0 \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin(a)\cos^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**3*cos(a + b*x)**3/(3*b) - c**2*d*x*cos(a + b*x)**3/b - c*d**
2*x**2*cos(a + b*x)**3/b - d**3*x**3*cos(a + b*x)**3/(3*b) + 2*c**2*d*sin(a
 + b*x)**3/(3*b**2) + c**2*d*sin(a + b*x)*cos(a + b*x)**2/b**2 + 4*c*d**2*x
*sin(a + b*x)**3/(3*b**2) + 2*c*d**2*x*sin(a + b*x)*cos(a + b*x)**2/b**2 +
2*d**3*x**2*sin(a + b*x)**3/(3*b**2) + d**3*x**2*sin(a + b*x)*cos(a + b*x)*
*2/b**2 + 4*c*d**2*sin(a + b*x)**2*cos(a + b*x)/(3*b**3) + 14*c*d**2*cos(a
+ b*x)**3/(9*b**3) + 4*d**3*x*sin(a + b*x)**2*cos(a + b*x)/(3*b**3) + 14*d*
*3*x*cos(a + b*x)**3/(9*b**3) - 40*d**3*sin(a + b*x)**3/(27*b**4) - 14*d**3
*sin(a + b*x)*cos(a + b*x)**2/(9*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2
/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)*cos(a)**2, True))
```

Giac [A]
time = 0.46, size = 231, normalized size = 1.53

$$-\frac{(3b^3d^3x^3 + 9b^3cd^2x^2 + 9b^3c^2dx + 3b^3c^3 - 2bd^3x - 2bcd^2)\cos(3bx + 3a)}{36b^4} - \frac{(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + b^3c^3 - 6bd^3x - 6bcd^2)\cos(bx + a)}{4b^4} + \frac{(9b^2d^3x^2 + 18b^2cd^2x + 9b^2c^2d - 2d^3)\sin(3bx + 3a)}{108b^4} + \frac{3(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3)\sin(bx + a)}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^
3*x - 2*b*c*d^2)*cos(3*b*x + 3*a)/b^4 - 1/4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
+ 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*x + a)/b^4 + 1/108
*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*sin(3*b*x + 3*a)/b^
4 + 3/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a)/b^4
```

Mupad [B]
time = 1.34, size = 290, normalized size = 1.92

$$\frac{\cos(a+bx)^3(14cd^2 - 3b^2c^3)}{9b^3} - \frac{2\sin(a+bx)^3(20d^3 - 9b^2c^2d)}{27b^4} - \frac{\cos(a+bx)^2\sin(a+bx)(14d^3 - 9b^2c^2d)}{9b^4} + \frac{x\cos(a+bx)^3(14d^3 - 9b^2c^2d)}{9b^3} - \frac{d^3x^3\cos(a+bx)^3}{3b} + \frac{2d^3x^2\sin(a+bx)^3}{3b^2} + \frac{4cd^2\cos(a+bx)\sin(a+bx)^2}{3b^3} + \frac{4d^3x\cos(a+bx)\sin(a+bx)^2}{3b^3} + \frac{4cd^2x\sin(a+bx)^3}{3b^2} - \frac{cd^2x^2\cos(a+bx)^3}{b} + \frac{d^3x^2\cos(a+bx)^2\sin(a+bx)}{b^2} + \frac{2cd^2x\cos(a+bx)^2\sin(a+bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^3,x)
```

```
[Out] (cos(a + b*x)^3*(14*c*d^2 - 3*b^2*c^3))/(9*b^3) - (2*sin(a + b*x)^3*(20*d^3
 - 9*b^2*c^2*d))/(27*b^4) - (cos(a + b*x)^2*sin(a + b*x)*(14*d^3 - 9*b^2*c^
```

```
2*d))/(9*b^4) + (x*cos(a + b*x)^3*(14*d^3 - 9*b^2*c^2*d))/(9*b^3) - (d^3*x^
3*cos(a + b*x)^3)/(3*b) + (2*d^3*x^2*sin(a + b*x)^3)/(3*b^2) + (4*c*d^2*cos
(a + b*x)*sin(a + b*x)^2)/(3*b^3) + (4*d^3*x*cos(a + b*x)*sin(a + b*x)^2)/(
3*b^3) + (4*c*d^2*x*sin(a + b*x)^3)/(3*b^2) - (c*d^2*x^2*cos(a + b*x)^3)/b
+ (d^3*x^2*cos(a + b*x)^2*sin(a + b*x))/b^2 + (2*c*d^2*x*cos(a + b*x)^2*sin
(a + b*x))/b^2
```

3.73 $\int (c+dx)^2 \cos^2(a+bx) \sin(a+bx) \, dx$

Optimal. Leaf size=103

$$\frac{4d^2\cos(a+bx)}{9b^3}+\frac{2d^2\cos^3(a+bx)}{27b^3}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}+\frac{4d(c+dx)\sin(a+bx)}{9b^2}+\frac{2d(c+dx)\cos^2(a+bx)}{9b^2}$$

```
[Out] 4/9*d^2*cos(b*x+a)/b^3+2/27*d^2*cos(b*x+a)^3/b^3-1/3*(d*x+c)^2*cos(b*x+a)^3
/b+4/9*d*(d*x+c)*sin(b*x+a)/b^2+2/9*d*(d*x+c)*cos(b*x+a)^2*sin(b*x+a)/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4490, 3391, 3377, 2718}

$$\frac{2d^2\cos^3(a+bx)}{27b^3}+\frac{4d^2\cos(a+bx)}{9b^3}+\frac{4d(c+dx)\sin(a+bx)}{9b^2}+\frac{2d(c+dx)\sin(a+bx)\cos^2(a+bx)}{9b^2}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (4*d^2*Cos[a + b*x])/(9*b^3) + (2*d^2*Cos[a + b*x]^3)/(27*b^3) - ((c + d*x)
^2*Cos[a + b*x]^3)/(3*b) + (4*d*(c + d*x)*Sin[a + b*x])/(9*b^2) + (2*d*(c +
 d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(9*b^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
```

```
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^2\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^2\cos^3(a+bx)}{3b} + \frac{(2d)\int (c+dx)\cos^3(a+bx)\,dx}{3b}$$
$$= \frac{2d^2\cos^3(a+bx)}{27b^3} - \frac{(c+dx)^2\cos^3(a+bx)}{3b} + \frac{2d(c+dx)\cos^2(a+}{9b^2}$$
$$= \frac{2d^2\cos^3(a+bx)}{27b^3} - \frac{(c+dx)^2\cos^3(a+bx)}{3b} + \frac{4d(c+dx)\sin(a+b}{9b^2}$$
$$= \frac{4d^2\cos(a+bx)}{9b^3} + \frac{2d^2\cos^3(a+bx)}{27b^3} - \frac{(c+dx)^2\cos^3(a+bx)}{3b} + \frac{4}{}$$

Mathematica [A]
time = 0.51, size = 86, normalized size = 0.83

$$-\frac{27(-2d^2+b^2(c+dx)^2)\cos(a+bx)+(-2d^2+9b^2(c+dx)^2)\cos(3(a+bx))-6bd(c+dx)(9\sin(a+bx)+\sin(3(a+bx)))}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x],x]
[Out] -1/108*(27*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + (-2*d^2 + 9*b^2*(c + d
*x)^2)*Cos[3*(a + b*x)] - 6*b*d*(c + d*x)*(9*Sin[a + b*x] + Sin[3*(a + b*x)
]))/b^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(93) = 186.
time = 0.14, size = 204, normalized size = 1.98

method	result
risch	$-\frac{(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\cos(bx+a)}{4b^3}+\frac{d(dx+c)\sin(bx+a)}{2b^2}-\frac{(9x^2d^2b^2+18b^2cdx+9b^2c^2-2d^2)\cos(3bx+3a)}{108b^3}$
derivativedivides	$-\frac{a^2d^2(\cos^3(bx+a))}{3b^2}+\frac{2acd(\cos^3(bx+a))}{3b}-\frac{2a\,d^2\left(-\frac{(bx+a)(\cos^3(bx+a))}{3}+\frac{(2+\cos^2(bx+a))\sin(bx+a)}{9}\right)}{b^2}-\frac{c^2(\cos^3(bx+a))}{3}+$
default	$-\frac{a^2d^2(\cos^3(bx+a))}{3b^2}+\frac{2acd(\cos^3(bx+a))}{3b}-\frac{2a\,d^2\left(-\frac{(bx+a)(\cos^3(bx+a))}{3}+\frac{(2+\cos^2(bx+a))\sin(bx+a)}{9}\right)}{b^2}-\frac{c^2(\cos^3(bx+a))}{3}+$
norman	$\frac{\frac{d^2x^2(\tan^2(\frac{bx}{2}+\frac{a}{2}))}{b}+\frac{-18b^2c^2+28d^2}{27b^3}-\frac{d^2x^2}{3b}+\frac{16d^2(\tan^2(\frac{bx}{2}+\frac{a}{2}))}{9b^3}+\frac{(-6b^2c^2+4d^2)(\tan^4(\frac{bx}{2}+\frac{a}{2}))}{3b^3}+\frac{4cd\tan(\frac{bx}{2}+\frac{a}{2})}{3b^2}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/3/b^2*a^2*d^2*cos(b*x+a)^3+2/3/b*a*c*d*cos(b*x+a)^3-2/b^2*a*d^2*(-1
/3*(b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))-1/3*c^2*cos(b*x+a)
^3+2/b*c*d*(-1/3*(b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+1/b^
2*d^2*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+
2/27*cos(b*x+a)^3+4/9*cos(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(93) = 186$.
time = 0.28, size = 243, normalized size = 2.36

$$-\frac{36\,c^2\cos\left(bx+a\right)^3-\frac{72\,acd\cos(bx+a)^3}{b}+\frac{36\,a^2d^2\cos(bx+a)^3}{b^2}+\frac{6\,(3\,(bx+a)\cos(3\,bx+3\,a)+9\,(bx+a)\cos(bx+a)-\sin(3\,bx+3\,a)-9\,\sin(bx+a))cd}{b}-\frac{6\,(3\,(bx+a)\cos(3\,bx+3\,a)+9\,(bx+a)\cos(bx+a)-\sin(3\,bx+3\,a)-9\,\sin(bx+a))ad^2}{b^2}+\frac{\left(\left(9\,(bx+a)^2-2\right)\cos(3\,bx+3\,a)+27\left((bx+a)^2-2\right)\cos(bx+a)-6\,(bx+a)\sin(3\,bx+3\,a)-54\,(bx+a)\sin(bx+a)\right)d^2}{b^2}}{108\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/108*(36*c^2*cos(b*x + a)^3 - 72*a*c*d*cos(b*x + a)^3/b + 36*a^2*d^2*cos(
b*x + a)^3/b^2 + 6*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a)
 - sin(3*b*x + 3*a) - 9*sin(b*x + a))*c*d/b - 6*(3*(b*x + a)*cos(3*b*x + 3*
a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a*d^2/b^
2 + ((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2)*cos(b*x +
a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a))*d^2/b^2)/b
```

Fricas [A]
time = 2.79, size = 100, normalized size = 0.97

$$-\frac{\left(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2\right)\cos\left(bx+a\right)^3-12\,d^2\cos\left(bx+a\right)-6\left(2\,bd^2x+2\,bcd+\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2\right)\sin\left(bx+a\right)}{27\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/27*((9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^3 -
12*d^2*cos(b*x + a) - 6*(2*b*d^2*x + 2*b*c*d + (b*d^2*x + b*c*d)*cos(b*x +
a)^2)*sin(b*x + a))/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(102) = 204$.
time = 0.30, size = 216, normalized size = 2.10

$$\begin{cases} -\frac{c^2\cos^3\left(a+bx\right)}{3b}-\frac{2cdx\cos^3\left(a+bx\right)}{3b}-\frac{d^2x^2\cos^3\left(a+bx\right)}{3b}+\frac{4cd\sin^3\left(a+bx\right)}{9b^2}+\frac{2cd\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{3b^2}+\frac{4d^2x\sin^3\left(a+bx\right)}{9b^2}+\frac{2d^2x\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{3b^2}+\frac{4d^2\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{9b^3}+\frac{14d^2\cos^3\left(a+bx\right)}{27b^3} & \text{for}\ b\neq 0 \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin\left(a\right)\cos^2\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**2*cos(a + b*x)**3/(3*b) - 2*c*d*x*cos(a + b*x)**3/(3*b) - d*
*2*x**2*cos(a + b*x)**3/(3*b) + 4*c*d*sin(a + b*x)**3/(9*b**2) + 2*c*d*sin(
a + b*x)*cos(a + b*x)**2/(3*b**2) + 4*d**2*x*sin(a + b*x)**3/(9*b**2) + 2*d
**2*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**2) + 4*d**2*sin(a + b*x)**2*cos(a
+ b*x)/(9*b**3) + 14*d**2*cos(a + b*x)**3/(27*b**3), Ne(b, 0)), ((c**2*x +
c*d*x**2 + d**2*x**3/3)*sin(a)*cos(a)**2, True))
```

Giac [A]
time = 0.45, size = 137, normalized size = 1.33

$$-\frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2)\cos(3\,bx+3\,a)}{108\,b^3}-\frac{(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2)\cos(bx+a)}{4\,b^3}+\frac{(bd^2x+bcd)\sin(3\,bx+3\,a)}{18\,b^3}+\frac{(bd^2x+bcd)\sin(bx+a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(3*b*x + 3*a)/
b^3 - 1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)/b^3 +
1/18*(b*d^2*x + b*c*d)*sin(3*b*x + 3*a)/b^3 + 1/2*(b*d^2*x + b*c*d)*sin(b*x
 + a)/b^3
```

Mupad [B]
time = 1.13, size = 145, normalized size = 1.41

$$\frac{12\,d^2\cos(a+b\,x)+2\,d^2\cos(a+b\,x)^3-9\,b^2\,c^2\cos(a+b\,x)^3+12\,b\,d^2\,x\sin(a+b\,x)-9\,b^2\,d^2\,x^2\cos(a+b\,x)^3+12\,b\,c\,d\sin(a+b\,x)-18\,b^2\,c\,d\,x\cos(a+b\,x)^3+6\,b\,d^2\,x\cos(a+b\,x)^2\sin(a+b\,x)+6\,b\,c\,d\cos(a+b\,x)^2\sin(a+b\,x)}{27\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^2,x)
```

```
[Out] (12*d^2*cos(a + b*x) + 2*d^2*cos(a + b*x)^3 - 9*b^2*c^2*cos(a + b*x)^3 + 12
*b*d^2*x*sin(a + b*x) - 9*b^2*d^2*x^2*cos(a + b*x)^3 + 12*b*c*d*sin(a + b*x
) - 18*b^2*c*d*x*cos(a + b*x)^3 + 6*b*d^2*x*cos(a + b*x)^2*sin(a + b*x) + 6
*b*c*d*cos(a + b*x)^2*sin(a + b*x))/(27*b^3)
```

3.74 $\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=51

$$-\frac{(c+dx)\cos^3(a+bx)}{3b}+\frac{d\sin(a+bx)}{3b^2}-\frac{d\sin^3(a+bx)}{9b^2}$$

```
[Out] -1/3*(d*x+c)*cos(b*x+a)^3/b+1/3*d*sin(b*x+a)/b^2-1/9*d*sin(b*x+a)^3/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4490, 2713}

$$-\frac{d\sin^3(a+bx)}{9b^2}+\frac{d\sin(a+bx)}{3b^2}-\frac{(c+dx)\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -1/3*((c + d*x)*Cos[a + b*x]^3)/b + (d*Sin[a + b*x])/(3*b^2) - (d*Sin[a + b
*x]^3)/(9*b^2)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx &= -\frac{(c+dx)\cos^3(a+bx)}{3b}+\frac{d\int \cos^3(a+bx)\,dx}{3b}\\
&= -\frac{(c+dx)\cos^3(a+bx)}{3b}-\frac{d\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(a+bx)\right)}{3b^2}\\
&= -\frac{(c+dx)\cos^3(a+bx)}{3b}+\frac{d\sin(a+bx)}{3b^2}-\frac{d\sin^3(a+bx)}{9b^2}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 71, normalized size = 1.39

$$-\frac{c\cos^3(a+bx)}{3b}+\frac{d(-bx\cos(a+bx)+\sin(a+bx))}{4b^2}+\frac{d(-3bx\cos(3(a+bx))+\sin(3(a+bx)))}{36b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -1/3*(c*Cos[a + b*x]^3)/b + (d*(-(b*x*Cos[a + b*x]) + Sin[a + b*x]))/(4*b^2
) + (d*(-3*b*x*Cos[3*(a + b*x)] + Sin[3*(a + b*x)]))/(36*b^2)
```

Maple [A]
time = 0.09, size = 71, normalized size = 1.39

method	result
risch	$-\frac{(dx+c)\cos(bx+a)}{4b}+\frac{d\sin(bx+a)}{4b^2}-\frac{(dx+c)\cos(3bx+3a)}{12b}+\frac{d\sin(3bx+3a)}{36b^2}$
derivativedivides	$\frac{\frac{da\left(\cos^3(bx+a)\right)}{3b}-\frac{c\left(\cos^3(bx+a)\right)}{3}+\frac{d\left(-\frac{(bx+a)\left(\cos^3(bx+a)\right)}{3}+\frac{\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{9}\right)}{b}}{b}$
default	$\frac{\frac{da\left(\cos^3(bx+a)\right)}{3b}-\frac{c\left(\cos^3(bx+a)\right)}{3}+\frac{d\left(-\frac{(bx+a)\left(\cos^3(bx+a)\right)}{3}+\frac{\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{9}\right)}{b}}{b}$
norman	$\frac{-\frac{2c\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{dx\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{2c}{3b}+\frac{2d\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{3b^2}+\frac{4d\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{9b^2}+\frac{2d\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b^2}-\frac{dx}{3b}-\frac{dx\left(\tan^4\left(\frac{bx}{2}+\ldots\right.\right.}{b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/3/b*d*a*cos(b*x+a)^3-1/3*c*cos(b*x+a)^3+1/b*d*(-1/3*(b*x+a)*cos(b*x+
a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a)))
```

Maxima [A]
time = 0.28, size = 86, normalized size = 1.69

$$-\frac{12\,c\cos\left(bx+a\right)^3-\frac{12\,ad\cos(bx+a)^3}{b}+\frac{(3\,(bx+a)\cos(3\,bx+3\,a)+9\,(bx+a)\cos(bx+a)-\sin(3\,bx+3\,a)-9\,\sin(bx+a))d}{b}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/36*(12*c*cos(b*x + a)^3 - 12*a*d*cos(b*x + a)^3/b + (3*(b*x + a)*cos(3*b
*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*d
/b)/b
```

Fricas [A]
time = 2.55, size = 46, normalized size = 0.90

$$-\frac{3\left(bdx+bc\right)\cos\left(bx+a\right)^{3}-\left(d\cos\left(bx+a\right)^{2}+2\,d\right)\sin\left(bx+a\right)}{9\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/9*(3*(b*d*x + b*c)*cos(b*x + a)^3 - (d*cos(b*x + a)^2 + 2*d)*sin(b*x + a
))/b^2
```

Sympy [A]
time = 0.17, size = 85, normalized size = 1.67

$$\begin{cases} -\frac{c\cos^{3}\left(a+bx\right)}{3b}-\frac{dx\cos^{3}\left(a+bx\right)}{3b}+\frac{2d\sin^{3}\left(a+bx\right)}{9b^{2}}+\frac{d\sin\left(a+bx\right)\cos^{2}\left(a+bx\right)}{3b^{2}} & \text{for}\: b\neq 0 \\ \left(cx+\frac{dx^{2}}{2}\right)\sin\left(a\right)\cos^{2}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-c*cos(a + b*x)**3/(3*b) - d*x*cos(a + b*x)**3/(3*b) + 2*d*sin(a
 + b*x)**3/(9*b**2) + d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2), Ne(b, 0)), (
(c*x + d*x**2/2)*sin(a)*cos(a)**2, True))
```

Giac [A]
time = 0.45, size = 69, normalized size = 1.35

$$-\frac{\left(bdx+bc\right)\cos\left(3\,bx+3\,a\right)}{12\,b^{2}}-\frac{\left(bdx+bc\right)\cos\left(bx+a\right)}{4\,b^{2}}+\frac{d\sin\left(3\,bx+3\,a\right)}{36\,b^{2}}+\frac{d\sin\left(bx+a\right)}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/12*(b*d*x + b*c)*cos(3*b*x + 3*a)/b^2 - 1/4*(b*d*x + b*c)*cos(b*x + a)/b
^2 + 1/36*d*sin(3*b*x + 3*a)/b^2 + 1/4*d*sin(b*x + a)/b^2
```

Mupad [B]
time = 0.95, size = 58, normalized size = 1.14

$$\frac{\frac{2\,d\,\sin\left(a+b\,x\right)}{9}-b\,\left(\frac{c\,{\cos\left(a+b\,x\right)}^3}{3}+\frac{d\,x\,{\cos\left(a+b\,x\right)}^3}{3}\right)+\frac{d\,{\cos\left(a+b\,x\right)}^2\,\sin\left(a+b\,x\right)}{9}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x),x)
```

```
[Out] ((2*d*sin(a + b*x))/9 - b*((c*cos(a + b*x)^3)/3 + (d*x*cos(a + b*x)^3)/3) +
 (d*cos(a + b*x)^2*sin(a + b*x))/9)/b^2
```

3.75 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=121

$$\frac{\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d}+\frac{\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{4d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(3\right.}{}$$

```
[Out] 1/4*cos(a-b*c/d)*Si(b*c/d+b*x)/d+1/4*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d+1
/4*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d+1/4*Ci(b*c/d+b*x)*sin(a-b*c/d)/d
```

Rubi [A]
time = 0.17, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4491, 3384, 3380, 3383}

$$\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d) + (CosIntegral[
(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d) + (Cos[a - (b*c)/d]*SinIntegral[(b*c
)/d + b*x])/(4*d) + (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(
4*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx &= \int \left(\frac{\sin(a+bx)}{4(c+dx)}+\frac{\sin(3a+3bx)}{4(c+dx)}\right)dx\\ &= \frac{1}{4}\int \frac{\sin(a+bx)}{c+dx}\,dx+\frac{1}{4}\int \frac{\sin(3a+3bx)}{c+dx}\,dx\\ &= \frac{1}{4}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx+\frac{1}{4}\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\\ &= \frac{\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d}+\frac{\mathrm{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{4d}+\frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{b}{d}\right.}{4d}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 100, normalized size = 0.83

$$\frac{\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)+\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right)+\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)+\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + CosIntegral[b*(c/d +
 x)]*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Cos[3*a
 - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d)
```

Maple [A]
time = 0.10, size = 172, normalized size = 1.42

method	result
derivativedivides	$\frac{\frac{b\left(-\frac{3\,\text{sinIntegral}\left(-3bx-3a-\frac{3(-ad+cb)}{d}\right)\cos\left(\frac{-3ad+3cb}{d}\right)}{d}-\frac{3\,\text{cosineIntegral}\left(3bx+3a+\frac{-3ad+3cb}{d}\right)\sin\left(\frac{-3ad+3cb}{d}\right)}{d}\right)}{12}+\frac{b\left(-\frac{\text{sinInteg}}{}\right.}{}}{b}$
default	$\frac{\frac{b\left(-\frac{3\,\text{sinIntegral}\left(-3bx-3a-\frac{3(-ad+cb)}{d}\right)\cos\left(\frac{-3ad+3cb}{d}\right)}{d}-\frac{3\,\text{cosineIntegral}\left(3bx+3a+\frac{-3ad+3cb}{d}\right)\sin\left(\frac{-3ad+3cb}{d}\right)}{d}\right)}{12}+\frac{b\left(-\frac{\text{sinInte}}{}\right.}{}}{b}$
risch	$-\frac{i\,e^{-\frac{3i(ad-cb)}{d}}\,\text{expIntegral}\left(1,3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{8d}-\frac{i\,e^{-\frac{i(ad-cb)}{d}}\,\text{expIntegral}\left(1,ibx+ia-\frac{i(ad-cb)}{d}\right)}{8d}+\frac{i\,e^{\frac{i(ad-cb)}{d}}\,\text{e}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/12*b*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*
b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)+1/4*b*(-Si(-b*x-a-(-a*d+b*c)
/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.35, size = 275, normalized size = 2.27

$$-\frac{b\left(i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b\left(-i\,E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_1\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)+b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+b\left(E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{8\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*(b*(I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_int
egral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b*(-I
*exp_integral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integral_e
(1, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b*(exp_
integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(1, -(I*b*
c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(exp_integral_e(1, 3
*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -3*(-I*b*c - I*(b*
x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

Fricas [A]
time = 2.41, size = 152, normalized size = 1.26

$$\frac{\left(\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+\left(\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)+2\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right)+2\,\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/8*((cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*sin(-
(b*c - a*d)/d) + (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d*x
+ b*c)/d))*sin(-3*(b*c - a*d)/d) + 2*cos(-3*(b*c - a*d)/d)*sin_integral(3*(
b*d*x + b*c)/d) + 2*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\cos^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 6279, normalized size = 51.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/8*(imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(b*x + b*c/d))*tan(3
/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_inte
gral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3
/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral((b*
d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 +
 2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d) + 2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)
^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*real_part(cos_integral(
3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2
 + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(b*x + b*c/d))*tan(
3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_int
egral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d
))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - i
mag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d
)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3
/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/
2*a)^2*tan(3/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)^2 - 2*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 4*imag_part(cos_integral(b*x + b*c/d))*tan(
3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*imag_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
+ 8*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*
tan(1/2*b*c/d) - imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(
1/2*a)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)
^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(-b*x - b*c/d))*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(-3*b*x -
3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*sin_integral(3*(b*
d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*sin_integral((
b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*imag_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2
```

```
*b*c/d)^2 - 4*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*
a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 8*sin_integral(3*(b*d*x + b*c)/d)*ta
n(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + imag_part(cos_integ
ral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag
_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*
tan(1/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/
2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral((b*d*x + b*c)/d)*
tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(3*b
*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(c
os_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 -
imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d)^2 + imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral((b*d*x + b*c)/d)*tan(1/2
*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(3*b*x +
3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 2*real_part(cos_integr
al(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 2*real_par
t(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 2*r
eal_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)
^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan
(3/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(
1/2*a)^2*tan(3/2*b*c/d)^2 + 2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*
a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*real_part(cos_integral(-b*x - b*c/d))*
tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*real_part(cos_integral(b*x + b
*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*real_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*real_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
+ 2*real_part(cos_integral(-b*x - b*c/d))*tan(1...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (a+b\,x)^2\,\sin (a+b\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x),x)
[Out] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x), x)
```

3.76 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=168

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{\sin(a+bx)}{4d(c+dx)}-\frac{\sin(3a+3bx)}{4d(c+dx)}-$$

```
[Out] 3/4*b*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d^2+1/4*b*Ci(b*c/d+b*x)*cos(a-b*c/
d)/d^2-3/4*b*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^2-1/4*b*Si(b*c/d+b*x)*sin
(a-b*c/d)/d^2-1/4*sin(b*x+a)/d/(d*x+c)-1/4*sin(3*b*x+3*a)/d/(d*x+c)
```

Rubi [A]

time = 0.20, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{\sin(a+bx)}{4d(c+dx)}-\frac{\sin(3a+3bx)}{4d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] (b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*Cos[3*a - (3
*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d^2) - Sin[a + b*x]/(4*d*(c + d
*x)) - Sin[3*a + 3*b*x]/(4*d*(c + d*x)) - (b*Sin[a - (b*c)/d]*SinIntegral[(
b*c)/d + b*x])/(4*d^2) - (3*b*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d +
3*b*x])/(4*d^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{\sin(a+bx)}{4(c+dx)^2}+\frac{\sin(3a+3bx)}{4(c+dx)^2}\right)dx\\
&= \frac{1}{4}\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx+\frac{1}{4}\int \frac{\sin(3a+3bx)}{(c+dx)^2}\,dx\\
&= -\frac{\sin(a+bx)}{4d(c+dx)}-\frac{\sin(3a+3bx)}{4d(c+dx)}+\frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{4d}+\frac{(3b)\int \frac{\cos(3a+3bx)}{c+dx}\,dx}{4d}\\
&= -\frac{\sin(a+bx)}{4d(c+dx)}-\frac{\sin(3a+3bx)}{4d(c+dx)}+\frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx}{4d}+\frac{(\ldots}{\ldots}\\
&= \frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{\sin(a+bx)}{4d(c+dx)}\ldots
\end{aligned}$$

Mathematica [A]
time = 1.16, size = 139, normalized size = 0.83

$$-\frac{-b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)-3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)+\frac{d\sin(a+bx)}{c+dx}+\frac{d\sin(3(a+bx))}{c+dx}+b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)+3b\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -1/4*(-(b*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)]) - 3*b*Cos[3*a - (3*b*c
)/d]*CosIntegral[(3*b*(c + d*x))/d] + (d*Sin[a + b*x])/(c + d*x) + (d*Sin[3
*(a + b*x)])/(c + d*x) + b*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 3*b*
Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/d^2
```

Maple [A]
time = 0.13, size = 245, normalized size = 1.46

method	result
derivativedivides	$\frac{b^2\left(-\frac{3\sin(3bx+3a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{9\,\text{sinIntegral}\left(-3bx-3a-\frac{3(-ad+cb)}{d}\right)\sin\left(\frac{-3ad+3cb}{d}\right)}{d}+\frac{9\,\text{cosineIntegral}\left(3bx+3a+\frac{-3ad+3cb}{d}\right)\cos\left(\frac{-3}{}\right.}{d}}{d}\right.}{12}$
default	$\frac{b^2\left(-\frac{3\sin(3bx+3a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{9\,\text{sinIntegral}\left(-3bx-3a-\frac{3(-ad+cb)}{d}\right)\sin\left(\frac{-3ad+3cb}{d}\right)}{d}+\frac{9\,\text{cosineIntegral}\left(3bx+3a+\frac{-3ad+3cb}{d}\right)\cos\left(\frac{-3}{}\right.}{d}}{d}\right.}{12}$
risch	$-\frac{3b\,\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\,\text{expIntegral}\left(1,3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{8d^2}-\frac{b\,\mathrm{e}^{-\frac{i(ad-cb)}{d}}\,\text{expIntegral}\left(1,ibx+ia-\frac{i(ad-cb)}{d}\right)}{8d^2}-\frac{b\,\mathrm{e}^{\frac{i(ad-cb)}{d}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/12*b^2*(-3*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d+3*(-3*Si(-3*b*x-3*a
-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3
*(-a*d+b*c)/d)/d)/d)+1/4*b^2*(-sin(b*x+a)/(-a*d+c*b+d*(b*x+a))/d+(-Si(-b*x-
a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d
)/d)/d))
```

<u>Maxima</u> [C] Result contains complex when optimal does not.
time = 0.39, size = 302, normalized size = 1.80

$$-\frac{b^2\left(iE_2\left(\frac{ibc+i(bx+a)d-iad}{d}\right)-iE_2\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^2\left(-iE_2\left(\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)+iE_2\left(-\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{3(bc-ad)}{d}\right)+b^2\left(E_2\left(\frac{ibc+i(bx+a)d-iad}{d}\right)+E_2\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+b^2\left(E_2\left(\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)+E_2\left(-\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)\right)\sin\left(-\frac{3(bc-ad)}{d}\right)}{8(bcd+(bx+a)d^2-ad^2)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(b^2*(I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_i
ntegral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^2
*(-I*exp_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integr
al_e(2, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b^2
*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(2,
-(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^2*(exp_integra
l_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(2, -3*(-I*b*c
 - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^
2 - a*d^2)*b)
```

<u>Fricas</u> [A]
time = 2.44, size = 233, normalized size = 1.39

$$-\frac{8d\cos(bx+a)^2\sin(bx+a)+6(bdx+bc)\sin\left(-\frac{3(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3(bdx+bc)}{d}\right)+2(bdx+bc)\sin\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)-\left((bdx+bc)\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+(bdx+bc)\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-3\left((bdx+bc)\mathrm{Ci}\left(\frac{3(bdx+bc)}{d}\right)+(bdx+bc)\mathrm{Ci}\left(-\frac{3(bdx+bc)}{d}\right)\right)\cos\left(-\frac{3(bc-ad)}{d}\right)}{8(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(8*d*cos(b*x + a)^2*sin(b*x + a) + 6*(b*d*x + b*c)*sin(-3*(b*c - a*d)/
d)*sin_integral(3*(b*d*x + b*c)/d) + 2*(b*d*x + b*c)*sin(-(b*c - a*d)/d)*si
n_integral((b*d*x + b*c)/d) - ((b*d*x + b*c)*cos_integral((b*d*x + b*c)/d)
+ (b*d*x + b*c)*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d) - 3*((b
*d*x + b*c)*cos_integral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-3
*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\cos^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**2, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.00, size = 66726, normalized size = 397.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*b*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/
2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + b*d*
x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/
2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + b*d*x*real_part(cos
_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2
*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b*d*x*real_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(b*x + b*c/
d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d) + 2*b*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*
x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d) - 4*b*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b*d*x*imag_part
(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 6*b*d*x*imag_part(cos_integr
al(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)
^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 12*b*d*x*sin_integral(3*(b*d*x + b*c)/
d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*t
```

an(1/2*b*c/d)^2 + 2*b*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)
^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 2*b*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*b*d*x*sin
_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b*d*x*imag_part(cos_integral(3*
b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b*d*x*imag_part(cos_integral(-3*b*x - 3*b*
c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d)^2 + 12*b*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^
2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 + 3*b*c*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + b*c*real_
part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + b*c*real_part(cos_integral
(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b*c*real_part(cos_integral(-3*b*x - 3*b*
c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d
)^2*tan(1/2*b*c/d)^2 + 3*b*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan
(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - b*d
*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3
/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - b*d*x*real_part(cos_integral(-b*x -
 b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*
c/d)^2 + 3*b*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*t
an(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 4*b*d*x*real_par
t(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan
(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 4*b*d*x*real_part(cos_integral(-b
*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d) - 2*b*c*imag_part(cos_integral(b*x + b*c/d))*tan(3/
2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d) + 2*b*c*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*b*c*s
in_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan
(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 3*b*d*x*real_part(cos_integral(
3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*t
an(1/2*b*c/d)^2 + b*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2
*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b*d*x*real_par
t(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(1/2*b*c/d)^2 - 3*b*d*x*real_part(cos_integral(-3*b*x - 3*b*c
/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)
^2 + 12*b*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 12*b*d*
x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*t
an(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 6*b*c*imag_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan

```
(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 6*b*c*imag_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 12*b*c*sin_integral(3*(b*d*x + b*c)/d)*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (a+b\,x)^2\,\sin (a+b\,x)}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^2,x)
[Out] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^2, x)
```

3.77 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=221

$$-\frac{b\cos(a+bx)}{8d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{8d^2(c+dx)}-\frac{9b^2\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{8d^3}-\frac{b^2\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a\right.}{8d^3}$$

```
[Out] -1/8*b*cos(b*x+a)/d^2/(d*x+c)-3/8*b*cos(3*b*x+3*a)/d^2/(d*x+c)-1/8*b^2*cos(
a-b*c/d)*Si(b*c/d+b*x)/d^3-9/8*b^2*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^3-9
/8*b^2*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^3-1/8*b^2*Ci(b*c/d+b*x)*sin(a-b
*c/d)/d^3-1/8*sin(b*x+a)/d/(d*x+c)^2-1/8*sin(3*b*x+3*a)/d/(d*x+c)^2
```

Rubi [A]
time = 0.23, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{b\cos(a+bx)}{8d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{8d^2(c+dx)}-\frac{\sin(a+bx)}{8d(c+dx)^2}-\frac{\sin(3a+3bx)}{8d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -1/8*(b*Cos[a + b*x])/(d^2*(c + d*x)) - (3*b*Cos[3*a + 3*b*x])/(8*d^2*(c +
d*x)) - (9*b^2*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(8*d^3)
 - (b^2*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d^3) - Sin[a + b*x]
/(8*d*(c + d*x)^2) - Sin[3*a + 3*b*x]/(8*d*(c + d*x)^2) - (b^2*Cos[a - (b*c
)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) - (9*b^2*Cos[3*a - (3*b*c)/d]*SinI
ntegral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx &= \int \left(\frac{\sin(a+bx)}{4(c+dx)^3}+\frac{\sin(3a+3bx)}{4(c+dx)^3}\right)dx \\
&= \frac{1}{4}\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx+\frac{1}{4}\int \frac{\sin(3a+3bx)}{(c+dx)^3}\,dx \\
&= -\frac{\sin(a+bx)}{8d(c+dx)^2}-\frac{\sin(3a+3bx)}{8d(c+dx)^2}+\frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{8d}+\frac{(3b)\int \frac{\cos(3a+3bx)}{(c+dx)^2}\,dx}{8d} \\
&= -\frac{b\cos(a+bx)}{8d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{8d^2(c+dx)}-\frac{\sin(a+bx)}{8d(c+dx)^2}-\frac{\sin(3a+3bx)}{8d(c+dx)^2}-\frac{b^2\int \\
&= -\frac{b\cos(a+bx)}{8d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{8d^2(c+dx)}-\frac{\sin(a+bx)}{8d(c+dx)^2}-\frac{\sin(3a+3bx)}{8d(c+dx)^2}-\frac{(9b \\
&= -\frac{b\cos(a+bx)}{8d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{8d^2(c+dx)}-\frac{9b^2\text{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{8d^3}-\frac{b
\end{aligned}$$

Mathematica [A]
time = 2.61, size = 181, normalized size = 0.82

$$-\frac{9b^2\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)+b^2\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right)+\frac{d(b(c+dx)\cos(a+bx)+d\sin(a+bx))}{(c+dx)^2}+\frac{d(3b(c+dx)\cos(3(a+bx))+d\sin(3(a+bx)))}{(c+dx)^2}+b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)+9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -1/8*(9*b^2*CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + b^2*CosIn
tegral[b*(c/d + x)]*Sin[a - (b*c)/d] + (d*(b*(c + d*x)*Cos[a + b*x] + d*Sin
[a + b*x]))/(c + d*x)^2 + (d*(3*b*(c + d*x)*Cos[3*(a + b*x)] + d*Sin[3*(a +
 b*x)]))/(c + d*x)^2 + b^2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 9*b^
2*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/d^3
```

Maple [A]
time = 0.24, size = 318, normalized size = 1.44

method	result
derivativedivides	$\frac{b^3\left(-\frac{3\sin(3bx+3a)}{2(-ad+cb+d(bx+a))^2 d}+\frac{-\frac{9\cos(3bx+3a)}{2(-ad+cb+d(bx+a))d}-\frac{9\left(-\frac{3\,\text{sinIntegral}\left(-3bx-3a-\frac{3(-ad+cb)}{d}\right)\cos\left(\frac{-3ad+3cb}{d}\right)}{d}-\frac{3\,\text{cosineInteg}}{}\right.}{2d}}{d}\right.}{12}$
default	$\frac{b^3\left(-\frac{3\sin(3bx+3a)}{2(-ad+cb+d(bx+a))^2 d}+\frac{-\frac{9\cos(3bx+3a)}{2(-ad+cb+d(bx+a))d}-\frac{9\left(-\frac{3\,\text{sinIntegral}\left(-3bx-3a-\frac{3(-ad+cb)}{d}\right)\cos\left(\frac{-3ad+3cb}{d}\right)}{d}-\frac{3\,\text{cosineInteg}}{}\right.}{2d}}{d}\right.}{12}$
risch	$\frac{9ib^2\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\,\text{expIntegral}\left(1,3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{16d^3}+\frac{ib^2\mathrm{e}^{-\frac{i(ad-cb)}{d}}\,\text{expIntegral}\left(1,ibx+ia-\frac{i(ad-cb)}{d}\right)}{16d^3}-\frac{ib^2\mathrm{e}^{\frac{i(ad-}{d}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/12*b^3*(-3/2*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))^2/d+3/2*(-3*cos(3*b
*x+3*a)/(-a*d+c*b+d*(b*x+a))/d-3*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*cos(3*(-
a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)/d)+1
/4*b^3*(-1/2*sin(b*x+a)/(-a*d+c*b+d*(b*x+a))^2/d+1/2*(-cos(b*x+a)/(-a*d+c*b
+d*(b*x+a))/d-(-Si(-b*x-a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+
b*c)/d)*sin((-a*d+b*c)/d)/d)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.46, size = 337, normalized size = 1.52

$$-\frac{b^3\left(i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^3\left(-i\,E_3\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_3\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)+b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+b^3\left(E_3\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_3\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{8\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\,(bcd^2-ad^3)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(b^3*(I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_i
ntegral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^3
```

```
*(-I*exp_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integr
al_e(3, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b^3
*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(3,
-(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^3*(exp_integra
l_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(3, -3*(-I*b*c
 - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*
d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Fricas [A]
time = 2.64, size = 393, normalized size = 1.78

$$-\frac{8\,d^2\cos\left(bx+a\right)^2\sin\left(bx+a\right)+24\,\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3+18\,\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{3\,(bdx+bc)}{d}\right)+2\,\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left(-\frac{bc-ad}{d}\right)\operatorname{Si}\left(\frac{bdx+bc}{d}\right)-16\,\left(bd^2x+bcd\right)\cos\left(bx+a\right)+\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+9\,\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{16\,\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(8*d^2*cos(b*x + a)^2*sin(b*x + a) + 24*(b*d^2*x + b*c*d)*cos(b*x + a
)^3 + 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-3*(b*c - a*d)/d)*sin_in
tegral(3*(b*d*x + b*c)/d) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-(b
*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - 16*(b*d^2*x + b*c*d)*cos(b*x +
 a) + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*x + b*c)/d)
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-(b*d*x + b*c)/d))*sin
(-(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(3*
(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-3*(b
*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.63, size = 118262, normalized size = 535.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/16*(9*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^
2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
```

2 + b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 18*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 9*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 18*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 2*b^2*d^2*x^2*sin_integral((b*d*x

```
+ b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b
*c/d)^2 + 4*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2
*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4
*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b
*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 8*b^2*d^2*x
^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2
*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 4*b^2*c*d*x*real_part(cos_int
egral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 4*b^2*c*d*x*real_part(cos_integral(-b*x
- b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d) - 9*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c
/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)
^2 + b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/
2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^2*d^2*x^2*imag_part
(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan
(1/2*a)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x -
3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b
*c/d)^2 - 18*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan
(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*x^2*sin_
integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{{\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^3, x)
```

3.78 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=270

$$-\frac{b\cos(a+bx)}{24d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{8d^2(c+dx)^2}-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{24d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}\right.}{8d^4}$$

```
[Out] -9/8*b^3*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d^4-1/24*b^3*Ci(b*c/d+b*x)*cos(
a-b*c/d)/d^4-1/24*b*cos(b*x+a)/d^2/(d*x+c)^2-1/8*b*cos(3*b*x+3*a)/d^2/(d*x+
c)^2+9/8*b^3*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^4+1/24*b^3*Si(b*c/d+b*x)*
sin(a-b*c/d)/d^4-1/12*sin(b*x+a)/d/(d*x+c)^3+1/24*b^2*sin(b*x+a)/d^3/(d*x+c
)-1/12*sin(3*b*x+3*a)/d/(d*x+c)^3+3/8*b^2*sin(3*b*x+3*a)/d^3/(d*x+c)
```

Rubi [A]
time = 0.27, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{24d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}+\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^2\sin(a+bx)}{24d^3(c+dx)}+\frac{3b^2\sin(3a+3bx)}{8d^3(c+dx)}-\frac{b\cos(a+bx)}{24d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{8d^2(c+dx)^2}-\frac{\sin(a+bx)}{12d(c+dx)^3}-\frac{\sin(3a+3bx)}{12d(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] -1/24*(b*Cos[a + b*x])/(d^2*(c + d*x)^2) - (b*Cos[3*a + 3*b*x])/(8*d^2*(c +
 d*x)^2) - (b^3*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(24*d^4) - (9*
b^3*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(8*d^4) - Sin[a +
b*x]/(12*d*(c + d*x)^3) + (b^2*Sin[a + b*x])/(24*d^3*(c + d*x)) - Sin[3*a +
 3*b*x]/(12*d*(c + d*x)^3) + (3*b^2*Sin[3*a + 3*b*x])/(8*d^3*(c + d*x)) + (
b^3*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(24*d^4) + (9*b^3*Sin[3*a
- (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^4)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx &= \int \left(\frac{\sin(a+bx)}{4(c+dx)^4}+\frac{\sin(3a+3bx)}{4(c+dx)^4}\right)dx\\
&= \frac{1}{4}\int \frac{\sin(a+bx)}{(c+dx)^4}\,dx+\frac{1}{4}\int \frac{\sin(3a+3bx)}{(c+dx)^4}\,dx\\
&= -\frac{\sin(a+bx)}{12d(c+dx)^3}-\frac{\sin(3a+3bx)}{12d(c+dx)^3}+\frac{b\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx}{12d}+\frac{b\int \frac{\cos(3a+3bx)}{(c+dx)^3}\,dx}{4d}\\
&= -\frac{b\cos(a+bx)}{24d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{8d^2(c+dx)^2}-\frac{\sin(a+bx)}{12d(c+dx)^3}-\frac{\sin(3a+3bx)}{12d(c+dx)^3}-\frac{b^2 \cdots}{\cdots}\\
&= -\frac{b\cos(a+bx)}{24d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{8d^2(c+dx)^2}-\frac{\sin(a+bx)}{12d(c+dx)^3}+\frac{b^2\sin(a+bx)}{24d^3(c+dx)}-\frac{\sin\cdots}{12\cdots}\\
&= -\frac{b\cos(a+bx)}{24d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{8d^2(c+dx)^2}-\frac{\sin(a+bx)}{12d(c+dx)^3}+\frac{b^2\sin(a+bx)}{24d^3(c+dx)}-\frac{\sin\cdots}{12\cdots}\\
&= -\frac{b\cos(a+bx)}{24d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{8d^2(c+dx)^2}-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{24d^4}-\frac{9b^3\cos\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A]
time = 1.76, size = 300, normalized size = 1.11

$$
-\frac{d\cos(bx)\left(bd(c+dx)\cos(a)-\left(-2d^2+b^2(c+dx)^2\right)\sin(a)\right)+d\cos(3bx)\left(3bd(c+dx)\cos(3a)-\left(-2d^2+9b^2(c+dx)^2\right)\sin(3a)\right)-d\left(\left(-2d^2+b^2(c+dx)^2\right)\cos(a)+bd(c+dx)\sin(a)\right)\sin(bx)-d\left(\left(-2d^2+9b^2(c+dx)^2\right)\cos(3a)+3bd(c+dx)\sin(3a)\right)\sin(3bx)+b^3(c+dx)^3\left(\cos\left(a-\frac{bc}{d}\right)\mathrm{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)-\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}+x\right)\right)\right)+27b^3(c+dx)^3\left(\cos\left(3a-\frac{3bc}{d}\right)\mathrm{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)-\sin\left(3a-\frac{3bc}{d}\right)\mathrm{Si}\left(\frac{3b(c+dx)}{d}\right)\right)}{24d^4(c+dx)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] -1/24*(d*Cos[b*x]*(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^2*(c + d*x)^2)*Sin[a]
) + d*Cos[3*b*x]*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)*S
in[3*a]) - d*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a])*Sin
[b*x] - d*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*x)*Sin[3*a]
)*Sin[3*b*x] + b^3*(c + d*x)^3*(Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] -
 Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)]) + 27*b^3*(c + d*x)^3*(Cos[3*a -
 (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] - Sin[3*a - (3*b*c)/d]*SinIntegr
al[(3*b*(c + d*x))/d]))/(d^4*(c + d*x)^3)
```

Maple [A]
time = 0.18, size = 386, normalized size = 1.43

method	result
derivativedivides	$\frac{b^4\left(-\frac{\sin(3bx+3a)}{(-ad+cb+d(bx+a))^3 d}+\frac{-\frac{3\cos(3bx+3a)}{2(-ad+cb+d(bx+a))^2 d}-\frac{3\left(-\frac{3\sin(3bx+3a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{9\,\mathrm{sinIntegral}\left(-3bx-3a-\frac{3(-ad+cb)}{d}\right)\sin\left(\cdots\right.}{d}\cdots}{\cdots}\right)}{2d}}{d}\cdots\right)}{12}\cdots$
default	$\frac{b^4\left(-\frac{\sin(3bx+3a)}{(-ad+cb+d(bx+a))^3 d}+\frac{-\frac{3\cos(3bx+3a)}{2(-ad+cb+d(bx+a))^2 d}-\frac{3\left(-\frac{3\sin(3bx+3a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{9\,\mathrm{sinIntegral}\left(-3bx-3a-\frac{3(-ad+cb)}{d}\right)\sin\left(\cdots\right.}{d}\cdots}{\cdots}\right)}{2d}}{d}\cdots\right)}{12}\cdots$
risch	$\frac{9b^3 e^{-\frac{3i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{16d^4}+\frac{b^3 e^{-\frac{i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,ibx+ia-\frac{i(ad-cb)}{d}\right)}{48d^4}+\frac{b^3 e^{\frac{i(ad-cb}{d}}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/12*b^4*(-sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))^3/d+(-3/2*cos(3*b*x+3*a
)/(-a*d+c*b+d*(b*x+a))^2/d-3/2*(-3*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d+3*
(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-
a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d)/d)/d)/d)+1/4*b^4*(-1/3*sin(b*x+a)/(-a*d+
c*b+d*(b*x+a))^3/d+1/3*(-1/2*cos(b*x+a)/(-a*d+c*b+d*(b*x+a))^2/d-1/2*(-sin(
b*x+a)/(-a*d+c*b+d*(b*x+a))/d+(-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d
+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)/d))
```

Maple [C] Result contains complex when optimal does not.

time = 0.61, size = 387, normalized size = 1.43

$$-\frac{b^4\left(i\,E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^4\left(-i\,E_4\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_4\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)+b^4\left(E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+b^4\left(E_4\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_4\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{8\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\,(bcd^3-ad^4)(bx+a)^2+3\,(b^2c^2d^2-2\,abcd^3+a^2d^4)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/8*(b^4*(I*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_i
ntegral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^4
*(-I*exp_integral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integr
al_e(4, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b^4
*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(4,
-(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^4*(exp_integra
l_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(4, -3*(-I*b*c
 - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*
c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(
b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. 2(250) = 500.
time = 2.91, size = 558, normalized size = 2.07

$$-\frac{24\,(bd^3x+bcd^2)\cos(bx+a)^3-54\,(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\sin\left(-\frac{3\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-2\,(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\sin\left(-\frac{bc-ad}{d}\right)\operatorname{Si}\left(\frac{bdx+bc}{d}\right)-16\,(bd^3x+bcd^2)\cos(bx+a)+\left((b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)+(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\operatorname{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+27\left((b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\operatorname{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\operatorname{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)+8\,(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3)\cos(bx+a)^2)\sin(bx+a)}{48\,(d^7x^3+3\,cd^6x^2+3\,c^2d^5x+c^3d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/48*(24*(b*d^3*x + b*c*d^2)*cos(b*x + a)^3 - 54*(b^3*d^3*x^3 + 3*b^3*c*d^
2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*
x + b*c)/d) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*s
in(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - 16*(b*d^3*x + b*c*d^2)*c
os(b*x + a) + ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*co
s_integral((b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + b^3*c^3)*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d) + 27*((b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(3*(b*d*x
 + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_
integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d) + 8*(2*b^2*d^3*x^2 + 4*
b^2*c*d^2*x + 2*b^2*c^2*d - (9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d -
 2*d^3)*cos(b*x + a)^2)*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x
+ c^3*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\cos^2(a+bx)}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**4, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.63, size = 166374, normalized size = 616.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/48*(27*b^3*d^3*x^3*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)
^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
^2 + b^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/
2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + b^3*
d^3*x^3*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2
*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 27*b^3*d^3*x
^3*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*
tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3
*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2
*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*b^3*d^3*x^3*imag_par
t(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*b^3*d^3*x^3*sin_integral((b*
d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d) - 54*b^3*d^3*x^3*imag_part(cos_integral(3*b*x +
3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b
*c/d)*tan(1/2*b*c/d)^2 + 54*b^3*d^3*x^3*imag_part(cos_integral(-3*b*x - 3*b
*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/
d)*tan(1/2*b*c/d)^2 - 108*b^3*d^3*x^3*sin_integral(3*(b*d*x + b*c)/d)*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b
*c/d)^2 + 2*b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2
*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 -
 2*b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*b^3*d
^3*x^3*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*
a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 54*b^3*d^3*x^3*imag_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 54*b^3*d^3*x^3*imag_part(co
s_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(
1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 108*b^3*d^3*x^3*sin_integral(3
*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 81*b^3*c*d^2*x^2*real_part(cos_integral(3*
b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan
```

```
(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^3*c*d^2*x^2*real_part(cos_integral(b*x
 + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^3*c*d^2*x^2*real_part(cos_integral(-b*x - b
*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/
d)^2*tan(1/2*b*c/d)^2 + 81*b^3*c*d^2*x^2*real_part(cos_integral(-3*b*x - 3*
b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c
/d)^2*tan(1/2*b*c/d)^2 + 27*b^3*d^3*x^3*real_part(cos_integral(3*b*x + 3*b*
c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d
)^2 - b^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - b^3*d^3*x^3*real_par
t(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)^2 + 27*b^3*d^3*x^3*real_part(cos_integral(-3*b*x
- 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2
*b*c/d)^2 + 4*b^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)
^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) +
 4*b^3*d^3*x^3*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^3*c*d
^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^3*c*d^2*x^2*
imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2
*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^3*c*d^2*x^2*sin_i
ntegral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2
*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 27*b^3*d^3*x^3*real_part(cos_integr
al(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^
2*tan(1/2*b*c/d)^2 + b^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b^3*d
^3*x^3*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*
tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 27*b^3*d^3*x^3*real_part(cos_i
ntegral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(1/2*b*c/d)^2 + 108*b^3*d^3*x^3*real_part(cos_integral(3*b*x + 3
*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/
d)*tan(1/2*b*c/d)^2 + 108*b^3*d^3*x^3*real_part(cos_integral(-3*b*x - 3*b*c
/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*t
an(1/2*b*c/d)^2 - 162*b^3*c*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d)
)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*ta...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a+b\,x)^2\,\sin(a+b\,x)}{(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^4,x)
[Out] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^4, x)
```

3.79 $\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=162

$$\frac{(c+dx)^{1+m}}{8d(1+m)}+\frac{i2^{-2(3+m)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{4ib(c+dx)}{d}\right)}{b}-\frac{i2^{-2(3+m)}e^{-4i\left(a-\frac{bc}{d}\right)}(}{}$$

```
[Out] 1/8*(d*x+c)^(1+m)/d/(1+m)+I*exp(4*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-4*I*b*(
d*x+c)/d)/(2^(6+2*m))/b/((-I*b*(d*x+c)/d)^m)-I*(d*x+c)^m*GAMMA(1+m,4*I*b*(d
*x+c)/d)/(2^(6+2*m))/b/exp(4*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.14, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4491, 3388, 2212}

$$\frac{i2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{4ib(c+dx)}{d}\right)}{b}-\frac{i2^{-2(m+3)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{4ib(c+dx)}{d}\right)}{b}+\frac{(c+dx)^{m+1}}{8d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(1 + m)/(8*d*(1 + m)) + (I*E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Ga
mma[1 + m, ((-4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/d)
^m) - (I*(c + d*x)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*
E^((4*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)^m - \frac{1}{8}(c+dx)^m\cos(4a+4bx)\right)dx \\
&= \frac{(c+dx)^{1+m}}{8d(1+m)} - \frac{1}{8}\int (c+dx)^m\cos(4a+4bx)\,dx \\
&= \frac{(c+dx)^{1+m}}{8d(1+m)} - \frac{1}{16}\int e^{-i(4a+4bx)}(c+dx)^m\,dx - \frac{1}{16}\int e^{i(4a+4bx)}(\\
&= \frac{(c+dx)^{1+m}}{8d(1+m)} + \frac{i4^{-3-m}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+\right.}{b}
\end{aligned}
$$

Mathematica [A]
time = 1.09, size = 213, normalized size = 1.31

$$\frac{4^{-3-m}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(2^{3+2m}b(c+dx)\left(\frac{b^2(c+dx)^2}{d^2}\right)^m - id(1+m)\left(-\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(1+m,\frac{4ib(c+dx)}{d}\right)\left(\cos\left(4a-\frac{4bc}{d}\right)-i\sin\left(4a-\frac{4bc}{d}\right)\right)+id(1+m)\left(\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(1+m,-\frac{4ib(c+dx)}{d}\right)\left(\cos\left(4a-\frac{4bc}{d}\right)+i\sin\left(4a-\frac{4bc}{d}\right)\right)\right)}{bd(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (4^(-3 - m)*(c + d*x)^m*(2^(3 + 2*m)*b*(c + d*x)*((b^2*(c + d*x)^2)/d^2)^m
- I*d*(1 + m)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d]*
(Cos[4*a - (4*b*c)/d] - I*Sin[4*a - (4*b*c)/d]) + I*d*(1 + m)*((I*b*(c + d*
x))/d)^m*Gamma[1 + m, ((-4*I)*b*(c + d*x))/d]*(Cos[4*a - (4*b*c)/d] + I*Sin
[4*a - (4*b*c)/d])))/(b*d*(1 + m)*((b^2*(c + d*x)^2)/d^2)^m)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (dx+c)^m\left(\cos^2(bx+a)\right)\left(\sin^2(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*((d*m + d)*integrate((d*x + c)^m*cos(4*b*x + 4*a), x) - e^(m*log(d*x +
 c) + log(d*x + c)))/(d*m + d)
```

Fricas [A]
time = 0.27, size = 136, normalized size = 0.84

$$\frac{(i\,dm+i\,d)e^{\left(-\frac{dm\log\left(-\frac{4i\,b}{d}\right)+4i\,bc-4i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{4\,(i\,bdx+i\,bc)}{d}\right)+(-i\,dm-i\,d)e^{\left(-\frac{dm\log\left(\frac{4i\,b}{d}\right)-4i\,bc+4i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{4\,(-i\,bdx-i\,bc)}{d}\right)+8\,(bdx+bc)(dx+c)^m}{64\,(bdm+bd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/64*((I*d*m + I*d)*e^(-(d*m*log(-4*I*b/d) + 4*I*b*c - 4*I*a*d)/d)*gamma(m
+ 1, -4*(I*b*d*x + I*b*c)/d) + (-I*d*m - I*d)*e^(-(d*m*log(4*I*b/d) - 4*I*b
*c + 4*I*a*d)/d)*gamma(m + 1, -4*(-I*b*d*x - I*b*c)/d) + 8*(b*d*x + b*c)*(d
*x + c)^m)/(b*d*m + b*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^2\,{\left(c+d\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^m, x)
```

3.80 $\int (c+dx)^4 \cos^2(a+bx) \sin^2(a+bx)\, dx$

Optimal. Leaf size=131

$$\frac{(c+dx)^5}{40d}+\frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4}-\frac{d(c+dx)^3\cos(4a+4bx)}{32b^2}-\frac{3d^4\sin(4a+4bx)}{1024b^5}+\frac{3d^2(c+dx)^2\sin(4a-}{128b^3}$$

```
[Out] 1/40*(d*x+c)^5/d+3/256*d^3*(d*x+c)*cos(4*b*x+4*a)/b^4-1/32*d*(d*x+c)^3*cos(
4*b*x+4*a)/b^2-3/1024*d^4*sin(4*b*x+4*a)/b^5+3/128*d^2*(d*x+c)^2*sin(4*b*x+
4*a)/b^3-1/32*(d*x+c)^4*sin(4*b*x+4*a)/b
```

Rubi [A]
time = 0.11, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4491, 3377, 2717}

$$-\frac{3d^4\sin(4a+4bx)}{1024b^5}+\frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4}+\frac{3d^2(c+dx)^2\sin(4a+4bx)}{128b^3}-\frac{d(c+dx)^3\cos(4a+4bx)}{32b^2}-\frac{(c+dx)^4\sin(4a+4bx)}{32b}+\frac{(c+dx)^5}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^5/(40*d) + (3*d^3*(c + d*x)*Cos[4*a + 4*b*x])/(256*b^4) - (d*(c +
 d*x)^3*Cos[4*a + 4*b*x])/(32*b^2) - (3*d^4*Sin[4*a + 4*b*x])/(1024*b^5) +
(3*d^2*(c + d*x)^2*Sin[4*a + 4*b*x])/(128*b^3) - ((c + d*x)^4*Sin[4*a + 4*b
*x])/(32*b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^4 \cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^4 - \frac{1}{8}(c+dx)^4\cos(4a+4bx)\right)\,dx$$
$$= \frac{(c+dx)^5}{40d} - \frac{1}{8}\int (c+dx)^4\cos(4a+4bx)\,dx$$
$$= \frac{(c+dx)^5}{40d} - \frac{(c+dx)^4\sin(4a+4bx)}{32b} + \frac{d\int (c+dx)^3\sin(4a+4bx)}{8b}$$
$$= \frac{(c+dx)^5}{40d} - \frac{d(c+dx)^3\cos(4a+4bx)}{32b^2} - \frac{(c+dx)^4\sin(4a+4bx)}{32b}$$
$$= \frac{(c+dx)^5}{40d} - \frac{d(c+dx)^3\cos(4a+4bx)}{32b^2} + \frac{3d^2(c+dx)^2\sin(4a+4b}{128b^3}$$
$$= \frac{(c+dx)^5}{40d} + \frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4} - \frac{d(c+dx)^3\cos(4a+4bx}{32b^2}$$
$$= \frac{(c+dx)^5}{40d} + \frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4} - \frac{d(c+dx)^3\cos(4a+4bx}{32b^2}$$

Mathematica [A]
time = 1.12, size = 132, normalized size = 1.01

$$\frac{128b^5x(5c^4+10c^3dx+10c^2d^2x^2+5cd^3x^3+d^4x^4)+20bd(c+dx)\left(3d^2-8b^2(c+dx)^2\right)\cos(4(a+bx))-5(3d^4-24b^2d^2(c+dx)^2+32b^4(c+dx)^4)\sin(4(a+bx))}{5120b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (128*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) +
20*b*d*(c + d*x)*(3*d^2 - 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] - 5*(3*d^4 -
24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Sin[4*(a + b*x)])/(5120*b^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1950 vs. $2(119) = 238$.
time = 0.27, size = 1951, normalized size = 14.89

method	result
risch	$\frac{d^4x^5}{40}+\frac{d^3c\,x^4}{8}+\frac{d^2c^2x^3}{4}+\frac{d\,c^3x^2}{4}+\frac{c^4x}{8}+\frac{c^5}{40d}-\frac{d\left(8b^2d^3x^3+24b^2c\,d^2x^2+24b^2c^2dx+8b^2c^3-3d^3x-3c\,d^2\right)\cos(4}{256b^4}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/b^4*a^4*d^4*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+
1/8*b*x+1/8*a)-4/b^3*a^3*c*d^3*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)
```

*sin(b*x+a)+1/8*b*x+1/8*a)-4/b^4*a^3*d^4*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+
a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/4*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)
^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/64*(2*sin(b*x+a)^2+3)^2)+6/b
^2*a^2*c^2*d^2*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*
b*x+1/8*a)+12/b^3*a^2*c*d^3*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/
2*a)-1/16*(b*x+a)^2+1/4*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*
x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/64*(2*sin(b*x+a)^2+3)^2)+6/b^4*a^2*d^4*((
b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/8*(b*x+a)*cos(b*x+a)^
2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/12*(b*x+a)^3-(b*x+a)^2*(-1/4
*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/8*(b*x+a)*sin(b*
x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a))-4/b*a*c^3*d*(-1/4*sin
(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)-12/b^2*a*c^2*
d^2*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/4*
sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x
+3/8*a)-1/64*(2*sin(b*x+a)^2+3)^2)-12/b^3*a*c*d^3*((b*x+a)^2*(-1/2*cos(b*x+
a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b
*x+a)+7/64*b*x+7/64*a-1/12*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(
b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)
^3+3/2*sin(b*x+a))*cos(b*x+a))-4/b^4*a*d^4*((b*x+a)^3*(-1/2*cos(b*x+a)*sin(
b*x+a)+1/2*b*x+1/2*a)-3/16*(b*x+a)^2*cos(b*x+a)^2+3/8*(b*x+a)*(1/2*cos(b*x+
a)*sin(b*x+a)+1/2*b*x+1/2*a)-21/128*(b*x+a)^2-3/32*sin(b*x+a)^2-3/32*(b*x+a
)^4-(b*x+a)^3*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)
-3/16*(b*x+a)^2*sin(b*x+a)^4+3/8*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a)
)*cos(b*x+a)+3/8*b*x+3/8*a)+3/512*(2*sin(b*x+a)^2+3)^2)+c^4*(-1/4*sin(b*x+a
)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)+4/b*c^3*d*((b*x+a)*
(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/4*sin(b*x+a)^2-
(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/64*
(2*sin(b*x+a)^2+3)^2)+6/b^2*c^2*d^2*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+
1/2*b*x+1/2*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x
+7/64*a-1/12*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*
x+a)+3/8*b*x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x
+a))*cos(b*x+a))+4/b^3*c*d^3*((b*x+a)^3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x
+1/2*a)-3/16*(b*x+a)^2*cos(b*x+a)^2+3/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+
1/2*b*x+1/2*a)-21/128*(b*x+a)^2-3/32*sin(b*x+a)^2-3/32*(b*x+a)^4-(b*x+a)^3*
(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^
2*sin(b*x+a)^4+3/8*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3
/8*b*x+3/8*a)+3/512*(2*sin(b*x+a)^2+3)^2)+1/b^4*d^4*((b*x+a)^4*(-1/2*cos(b*
x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^3*cos(b*x+a)^2+3/4*(b*x+a)^2*(1/
2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/32*(b*x+a)*cos(b*x+a)^2-3/64*cos(b
*x+a)*sin(b*x+a)-21/256*b*x-21/256*a-7/16*(b*x+a)^3-1/10*(b*x+a)^5-(b*x+a)^
4*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/4*(b*x+a)
^3*sin(b*x+a)^4+3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a
)+3/8*b*x+3/8*a)+3/32*(b*x+a)*sin(b*x+a)^4+3/128*(sin(b*x+a)^3+3/2*sin(b*x+
a))*cos(b*x+a)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 735 vs. $2(119) = 238$.
time = 0.30, size = 735, normalized size = 5.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/5120*(160*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^4 - 640*(4*b*x + 4*a - sin(4
*b*x + 4*a))*a*c^3*d/b + 960*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*c^2*d^2/b
^2 - 640*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^3*c*d^3/b^3 + 160*(4*b*x + 4*a
- sin(4*b*x + 4*a))*a^4*d^4/b^4 + 160*(8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*
x + 4*a) - cos(4*b*x + 4*a))*c^3*d/b - 480*(8*(b*x + a)^2 - 4*(b*x + a)*sin
(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*c^2*d^2/b^2 + 480*(8*(b*x + a)^2 - 4*(b
*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^2*c*d^3/b^3 - 160*(8*(b*x +
a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^3*d^4/b^4 + 40*(3
2*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(4
*b*x + 4*a))*c^2*d^2/b^2 - 80*(32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*
a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a))*a*c*d^3/b^3 + 40*(32*(b*x + a)
^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a)
)*a^2*d^4/b^4 + 20*(32*(b*x + a)^4 - 3*(8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a)
 - 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a))*c*d^3/b^3 - 20*(32*(b*
x + a)^4 - 3*(8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 4*(8*(b*x + a)^3 - 3*b*
x - 3*a)*sin(4*b*x + 4*a))*a*d^4/b^4 + (128*(b*x + a)^5 - 20*(8*(b*x + a)^3
 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 5*(32*(b*x + a)^4 - 24*(b*x + a)^2 + 3)*
sin(4*b*x + 4*a))*d^4/b^4)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(119) = 238$.
time = 0.91, size = 466, normalized size = 3.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/1280*(32*b^5*d^4*x^5 + 160*b^5*c*d^3*x^4 - 40*(8*b^3*d^4*x^3 + 24*b^3*c*d
^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a
)^4 + 40*(8*b^5*c^2*d^2 - b^3*d^4)*x^3 + 40*(8*b^5*c^3*d - 3*b^3*c*d^3)*x^2
 + 40*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^
3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 + 5*(32*b^5*c^4 - 24*b^3*c^2*d^2 + 3*b
*d^4)*x - 5*(2*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 32*b^4*c^4 - 24*b^2*c^
2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^2 + 16*(8*b^4*c^3*d - 3*b^2*
c*d^3)*x)*cos(b*x + a)^3 - (32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 32*b^4*c^4
```

```
 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^2 + 16*(8*b^4*c^
3*d - 3*b^2*c*d^3)*x)*cos(b*x + a))*sin(b*x + a))/b^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1231 vs. $2(126) = 252$.
time = 1.09, size = 1231, normalized size = 9.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**4*x*sin(a + b*x)**4/8 + c**4*x*sin(a + b*x)**2*cos(a + b*x)**
2/4 + c**4*x*cos(a + b*x)**4/8 + c**3*d*x**2*sin(a + b*x)**4/4 + c**3*d*x**
2*sin(a + b*x)**2*cos(a + b*x)**2/2 + c**3*d*x**2*cos(a + b*x)**4/4 + c**2*
d**2*x**3*sin(a + b*x)**4/4 + c**2*d**2*x**3*sin(a + b*x)**2*cos(a + b*x)**
2/2 + c**2*d**2*x**3*cos(a + b*x)**4/4 + c*d**3*x**4*sin(a + b*x)**4/8 + c*
d**3*x**4*sin(a + b*x)**2*cos(a + b*x)**2/4 + c*d**3*x**4*cos(a + b*x)**4/8
 + d**4*x**5*sin(a + b*x)**4/40 + d**4*x**5*sin(a + b*x)**2*cos(a + b*x)**2
/20 + d**4*x**5*cos(a + b*x)**4/40 + c**4*sin(a + b*x)**3*cos(a + b*x)/(8*b
) - c**4*sin(a + b*x)*cos(a + b*x)**3/(8*b) + c**3*d*x*sin(a + b*x)**3*cos(
a + b*x)/(2*b) - c**3*d*x*sin(a + b*x)*cos(a + b*x)**3/(2*b) + 3*c**2*d**2*
x**2*sin(a + b*x)**3*cos(a + b*x)/(4*b) - 3*c**2*d**2*x**2*sin(a + b*x)*cos
(a + b*x)**3/(4*b) + c*d**3*x**3*sin(a + b*x)**3*cos(a + b*x)/(2*b) - c*d**
3*x**3*sin(a + b*x)*cos(a + b*x)**3/(2*b) + d**4*x**4*sin(a + b*x)**3*cos(a
 + b*x)/(8*b) - d**4*x**4*sin(a + b*x)*cos(a + b*x)**3/(8*b) - c**3*d*sin(a
 + b*x)**4/(8*b**2) - c**3*d*cos(a + b*x)**4/(8*b**2) - 3*c**2*d**2*x*sin(a
 + b*x)**4/(32*b**2) + 9*c**2*d**2*x*sin(a + b*x)**2*cos(a + b*x)**2/(16*b*
*2) - 3*c**2*d**2*x*cos(a + b*x)**4/(32*b**2) - 3*c*d**3*x**2*sin(a + b*x)*
*4/(32*b**2) + 9*c*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(16*b**2) - 3*
c*d**3*x**2*cos(a + b*x)**4/(32*b**2) - d**4*x**3*sin(a + b*x)**4/(32*b**2)
 + 3*d**4*x**3*sin(a + b*x)**2*cos(a + b*x)**2/(16*b**2) - d**4*x**3*cos(a
+ b*x)**4/(32*b**2) - 3*c**2*d**2*sin(a + b*x)**3*cos(a + b*x)/(32*b**3) +
3*c**2*d**2*sin(a + b*x)*cos(a + b*x)**3/(32*b**3) - 3*c*d**3*x*sin(a + b*x
)**3*cos(a + b*x)/(16*b**3) + 3*c*d**3*x*sin(a + b*x)*cos(a + b*x)**3/(16*b
**3) - 3*d**4*x**2*sin(a + b*x)**3*cos(a + b*x)/(32*b**3) + 3*d**4*x**2*sin
(a + b*x)*cos(a + b*x)**3/(32*b**3) + 3*c*d**3*sin(a + b*x)**4/(64*b**4) +
3*c*d**3*cos(a + b*x)**4/(64*b**4) + 3*d**4*x*sin(a + b*x)**4/(256*b**4) -
9*d**4*x*sin(a + b*x)**2*cos(a + b*x)**2/(128*b**4) + 3*d**4*x*cos(a + b*x)
**4/(256*b**4) + 3*d**4*sin(a + b*x)**3*cos(a + b*x)/(256*b**5) - 3*d**4*si
n(a + b*x)*cos(a + b*x)**3/(256*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2
+ 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**2*cos(a)**2, True))
```

Giac [A]
time = 0.51, size = 224, normalized size = 1.71

$$\frac{1}{40}\,d^{4}x^{5}+\frac{1}{8}\,cd^{3}x^{4}+\frac{1}{4}\,c^{2}d^{2}x^{3}+\frac{1}{4}\,c^{3}dx^{2}+\frac{1}{8}\,c^{4}x-\frac{(8\,b^{3}d^{4}x^{3}+24\,b^{3}cd^{3}x^{2}+24\,b^{3}c^{2}d^{2}x+8\,b^{3}c^{3}d-3\,bd^{4}x-3\,bcd^{3})\cos(4\,bx+4\,a)}{256\,b^{5}}-\frac{(32\,b^{4}d^{4}x^{4}+128\,b^{4}cd^{3}x^{3}+192\,b^{4}c^{2}d^{2}x^{2}+128\,b^{4}c^{3}dx+32\,b^{4}c^{4}-24\,b^{2}d^{4}x^{2}-48\,b^{2}cd^{3}x-24\,b^{2}c^{2}d^{2}+3\,d^{4})\sin(4\,bx+4\,a)}{1024\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/40*d^4*x^5 + 1/8*c*d^3*x^4 + 1/4*c^2*d^2*x^3 + 1/4*c^3*d*x^2 + 1/8*c^4*x
- 1/256*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*d
- 3*b*d^4*x - 3*b*c*d^3)*cos(4*b*x + 4*a)/b^5 - 1/1024*(32*b^4*d^4*x^4 + 12
8*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^4*c^3*d*x + 32*b^4*c^4 - 24*b
^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2 + 3*d^4)*sin(4*b*x + 4*a)/b^5
```

Mupad [B]
time = 1.68, size = 349, normalized size = 2.66

$$-\frac{15\,d^4\,\sin\left(4\,a+4\,b\,x\right)-640\,b^5\,c^4\,x+160\,b^4\,c^4\,\sin\left(4\,a+4\,b\,x\right)-128\,b^5\,d^4\,x^5+160\,b^3\,c^3\,d\,\cos\left(4\,a+4\,b\,x\right)-1280\,b^5\,c^3\,d\,x^2-640\,b^5\,c\,d^3\,x^4-120\,b^2\,c^2\,d^2\,\sin\left(4\,a+4\,b\,x\right)+160\,b^3\,d^4\,x^3\,\cos\left(4\,a+4\,b\,x\right)-1280\,b^5\,c^2\,d^2\,x^3-120\,b^2\,d^4\,x^2\,\sin\left(4\,a+4\,b\,x\right)+160\,b^4\,d^4\,x^4\,\sin\left(4\,a+4\,b\,x\right)-60\,b\,c\,d^3\,\cos\left(4\,a+4\,b\,x\right)-60\,b\,d^4\,x\,\cos\left(4\,a+4\,b\,x\right)+960\,b^4\,c^2\,d^2\,x^2\,\sin\left(4\,a+4\,b\,x\right)-240\,b^2\,c\,d^3\,x\,\sin\left(4\,a+4\,b\,x\right)+640\,b^4\,c^3\,d\,x\,\sin\left(4\,a+4\,b\,x\right)+480\,b^3\,c^2\,d^2\,x\,\cos\left(4\,a+4\,b\,x\right)+480\,b^3\,c\,d^3\,x^2\,\cos\left(4\,a+4\,b\,x\right)+640\,b^4\,c\,d^3\,x^3\,\sin\left(4\,a+4\,b\,x\right)}{5120\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^4,x)
```

```
[Out] -(15*d^4*sin(4*a + 4*b*x) - 640*b^5*c^4*x + 160*b^4*c^4*sin(4*a + 4*b*x) -
128*b^5*d^4*x^5 + 160*b^3*c^3*d*cos(4*a + 4*b*x) - 1280*b^5*c^3*d*x^2 - 640
*b^5*c*d^3*x^4 - 120*b^2*c^2*d^2*sin(4*a + 4*b*x) + 160*b^3*d^4*x^3*cos(4*a
 + 4*b*x) - 1280*b^5*c^2*d^2*x^3 - 120*b^2*d^4*x^2*sin(4*a + 4*b*x) + 160*b
^4*d^4*x^4*sin(4*a + 4*b*x) - 60*b*c*d^3*cos(4*a + 4*b*x) - 60*b*d^4*x*cos(
4*a + 4*b*x) + 960*b^4*c^2*d^2*x^2*sin(4*a + 4*b*x) - 240*b^2*c*d^3*x*sin(4
*a + 4*b*x) + 640*b^4*c^3*d*x*sin(4*a + 4*b*x) + 480*b^3*c^2*d^2*x*cos(4*a
+ 4*b*x) + 480*b^3*c*d^3*x^2*cos(4*a + 4*b*x) + 640*b^4*c*d^3*x^3*sin(4*a +
 4*b*x))/(5120*b^5)
```

3.81 $\int (c+dx)^3 \cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=105

$$\frac{(c+dx)^4}{32d}+\frac{3d^3\cos(4a+4bx)}{1024b^4}-\frac{3d(c+dx)^2\cos(4a+4bx)}{128b^2}+\frac{3d^2(c+dx)\sin(4a+4bx)}{256b^3}-\frac{(c+dx)^3\sin(4a+4}{32b}$$

```
[Out] 1/32*(d*x+c)^4/d+3/1024*d^3*cos(4*b*x+4*a)/b^4-3/128*d*(d*x+c)^2*cos(4*b*x+
4*a)/b^2+3/256*d^2*(d*x+c)*sin(4*b*x+4*a)/b^3-1/32*(d*x+c)^3*sin(4*b*x+4*a)
/b
```

Rubi [A]
time = 0.09, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4491, 3377, 2718}

$$\frac{3d^3\cos(4a+4bx)}{1024b^4}+\frac{3d^2(c+dx)\sin(4a+4bx)}{256b^3}-\frac{3d(c+dx)^2\cos(4a+4bx)}{128b^2}-\frac{(c+dx)^3\sin(4a+4bx)}{32b}+\frac{(c+dx)^4}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^4/(32*d) + (3*d^3*Cos[4*a + 4*b*x])/(1024*b^4) - (3*d*(c + d*x)^2
*Cos[4*a + 4*b*x])/(128*b^2) + (3*d^2*(c + d*x)*Sin[4*a + 4*b*x])/(256*b^3)
 - ((c + d*x)^3*Sin[4*a + 4*b*x])/(32*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^3 \cos^2(a+bx) \sin^2(a+bx)\, dx = \int \left(\frac{1}{8}(c+dx)^3 - \frac{1}{8}(c+dx)^3 \cos(4a+4bx)\right) dx$$
$$= \frac{(c+dx)^4}{32d} - \frac{1}{8}\int (c+dx)^3 \cos(4a+4bx)\, dx$$
$$= \frac{(c+dx)^4}{32d} - \frac{(c+dx)^3 \sin(4a+4bx)}{32b} + \frac{(3d)\int (c+dx)^2 \sin(4a+4$$
$$= \frac{(c+dx)^4}{32d} - \frac{3d(c+dx)^2 \cos(4a+4bx)}{128b^2} - \frac{(c+dx)^3 \sin(4a+4bx)}{32b}$$
$$= \frac{(c+dx)^4}{32d} - \frac{3d(c+dx)^2 \cos(4a+4bx)}{128b^2} + \frac{3d^2(c+dx) \sin(4a+4b}{256b^3}$$
$$= \frac{(c+dx)^4}{32d} + \frac{3d^3 \cos(4a+4bx)}{1024b^4} - \frac{3d(c+dx)^2 \cos(4a+4bx)}{128b^2} + \frac{3d}{}$$

Mathematica [A]
time = 0.58, size = 106, normalized size = 1.01

$$\frac{32b^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3) - 3d(-d^2+8b^2(c+dx)^2)\cos(4(a+bx)) - 4b(c+dx)\left(-3d^2+8b^2(c+dx)^2\right)\sin(4(a+bx))}{1024b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (32*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 3*d*(-d^2 + 8*b^2*(
c + d*x)^2)*Cos[4*(a + b*x)] - 4*b*(c + d*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)*S
in[4*(a + b*x)])/(1024*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1097 vs. $2(95) = 190$.
time = 0.16, size = 1098, normalized size = 10.46

method	result
risch	$\frac{d^3x^4}{32} + \frac{d^2cx^3}{8} + \frac{3dc^2x^2}{16} + \frac{c^3x}{8} + \frac{c^4}{32d} - \frac{3d(8x^2d^2b^2+16b^2cdx+8b^2c^2-d^2)\cos(4bx+4a)}{1024b^4} - \frac{(8b^2d^3x^3+24b^2cd^2}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b^3*a^3*d^3*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)
+1/8*b*x+1/8*a)+3/b^2*a^2*c*d^2*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a
)*sin(b*x+a)+1/8*b*x+1/8*a)+3/b^3*a^2*d^3*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x
```

```
+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/4*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a
)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/64*(2*sin(b*x+a)^2+3)^2)-3/
b*a*c^2*d*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1
/8*a)-6/b^2*a*c*d^2*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/1
6*(b*x+a)^2+1/4*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*co
s(b*x+a)+3/8*b*x+3/8*a)-1/64*(2*sin(b*x+a)^2+3)^2)-3/b^3*a*d^3*((b*x+a)^2*(
-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos
(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/12*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+
a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/3
2*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a))+c^3*(-1/4*sin(b*x+a)*cos(b*x+a)
^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)+3/b*c^2*d*((b*x+a)*(-1/2*cos(b*
x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/4*sin(b*x+a)^2-(b*x+a)*(-1/
4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/64*(2*sin(b*x+a
)^2+3)^2)+3/b^2*c*d^2*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/12*(
b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3
/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a
))+1/b^3*d^3*((b*x+a)^3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/16*(b*
x+a)^2*cos(b*x+a)^2+3/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-2
1/128*(b*x+a)^2-3/32*sin(b*x+a)^2-3/32*(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*x+a
)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^2*sin(b*x+a)^4+3
/8*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/
512*(2*sin(b*x+a)^2+3)^2))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(95) = 190$.
time = 0.29, size = 442, normalized size = 4.21

$$\frac{32\,(4\,bx+4\,a-\sin(4\,bx+4\,a))c^3-\frac{96\,(4\,bx+4\,a-\sin(4\,bx+4\,a))ac^2d}{b}+\frac{96\,(4\,bx+4\,a-\sin(4\,bx+4\,a))a^2cd^2}{b^2}-\frac{32\,(4\,bx+4\,a-\sin(4\,bx+4\,a))a^3d^3}{b^3}+\frac{24\left(8\,(bx+a)^2-4\,(bx+a)\sin(4\,bx+4\,a)-\cos(4\,bx+4\,a)\right)c^2d}{b}-\frac{48\left(8\,(bx+a)^2-4\,(bx+a)\sin(4\,bx+4\,a)-\cos(4\,bx+4\,a)\right)acd^2}{b^2}+\frac{24\left(8\,(bx+a)^2-4\,(bx+a)\sin(4\,bx+4\,a)-\cos(4\,bx+4\,a)\right)a^2d^3}{b^3}+\frac{4\left(32\,(bx+a)^3-12\,(bx+a)\cos(4\,bx+4\,a)-3\left(8\,(bx+a)^2-1\right)\sin(4\,bx+4\,a)\right)cd^2}{b^2}-\frac{4\left(32\,(bx+a)^3-12\,(bx+a)\cos(4\,bx+4\,a)-3\left(8\,(bx+a)^2-1\right)\sin(4\,bx+4\,a)\right)ad^3}{b^3}+\frac{\left(32\,(bx+a)^4-3\left(8\,(bx+a)^2-1\right)\cos(4\,bx+4\,a)-4\left(8\,(bx+a)^3-3\,bx-3\,a\right)\sin(4\,bx+4\,a)\right)d^3}{b^3}}{1024\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1024*(32*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^3 - 96*(4*b*x + 4*a - sin(4*b
*x + 4*a))*a*c^2*d/b + 96*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*c*d^2/b^2 -
32*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^3*d^3/b^3 + 24*(8*(b*x + a)^2 - 4*(b*
x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*c^2*d/b - 48*(8*(b*x + a)^2 - 4
*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*c*d^2/b^2 + 24*(8*(b*x +
a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^2*d^3/b^3 + 4*(32
*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*
b*x + 4*a))*c*d^2/b^2 - 4*(32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) -
 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a))*a*d^3/b^3 + (32*(b*x + a)^4 - 3*(8
*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*
b*x + 4*a))*d^3/b^3)/b
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(95) = 190$.

time = 2.00, size = 308, normalized size = 2.93

$$\frac{4\,b^{4}d^{3}x^{4}+16\,b^{4}cd^{2}x^{3}-3\,(8\,b^{2}d^{3}x^{2}+16\,b^{2}cd^{2}x+8\,b^{2}c^{2}d-d^{3})\cos\left(bx+a\right)^{4}+3\,(8\,b^{4}c^{2}d-b^{2}d^{3})x^{2}+3\,(8\,b^{2}d^{3}x^{2}+16\,b^{2}cd^{2}x+8\,b^{2}c^{2}d-d^{3})\cos\left(bx+a\right)^{2}+2\,(8\,b^{4}c^{3}-3\,b^{2}cd^{2})x-2\left(2\,(8\,b^{3}d^{3}x^{3}+24\,b^{3}cd^{2}x^{2}+8\,b^{3}c^{3}-3\,bcd^{2}+3\,(8\,b^{3}c^{2}d-bd^{3})x)\cos\left(bx+a\right)^{3}-(8\,b^{3}d^{3}x^{3}+24\,b^{3}cd^{2}x^{2}+8\,b^{3}c^{3}-3\,bcd^{2}+3\,(8\,b^{3}c^{2}d-bd^{3})x)\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{128\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/128*(4*b^4*d^3*x^4 + 16*b^4*c*d^2*x^3 - 3*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x
 + 8*b^2*c^2*d - d^3)*cos(b*x + a)^4 + 3*(8*b^4*c^2*d - b^2*d^3)*x^2 + 3*(8
*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 2*(8*b^
4*c^3 - 3*b^2*c*d^2)*x - 2*(2*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 8*b^3*c^3
 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^3 - (8*b^3*d^3*x^3 +
 24*b^3*c*d^2*x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(
b*x + a))*sin(b*x + a))/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(100) = 200$.
time = 0.76, size = 835, normalized size = 7.95

$$\begin{cases} \frac{c^{3}x\sin^{4}(a+bx)}{8}+\frac{c^{3}x\sin^{2}(a+bx)\cos^{2}(a+bx)}{4}+\frac{c^{3}x\cos^{4}(a+bx)}{8}+\frac{3c^{2}dx^{2}\sin^{4}(a+bx)}{16}+\frac{3c^{2}dx^{2}\sin^{2}(a+bx)\cos^{2}(a+bx)}{8}+\frac{3c^{2}dx^{2}\cos^{4}(a+bx)}{16}+\frac{cd^{2}x^{3}\sin^{4}(a+bx)}{8}+\frac{cd^{2}x^{3}\sin^{2}(a+bx)\cos^{2}(a+bx)}{4}+\frac{cd^{2}x^{3}\cos^{4}(a+bx)}{8}+\frac{d^{3}x^{4}\sin^{4}(a+bx)}{32}+\frac{d^{3}x^{4}\sin^{2}(a+bx)\cos^{2}(a+bx)}{16}+\frac{d^{3}x^{4}\cos^{4}(a+bx)}{32}+\frac{c^{3}\sin^{3}(a+bx)\cos(a+bx)}{8b}-\frac{c^{3}\sin(a+bx)\cos^{3}(a+bx)}{8b}+\frac{3c^{2}dx\sin^{3}(a+bx)\cos(a+bx)}{8b}-\frac{3c^{2}dx\sin(a+bx)\cos^{3}(a+bx)}{8b}+\frac{3cd^{2}x^{2}\sin^{3}(a+bx)\cos(a+bx)}{8b}-\frac{3cd^{2}x^{2}\sin(a+bx)\cos^{3}(a+bx)}{8b}+\frac{d^{3}x^{3}\sin^{3}(a+bx)\cos(a+bx)}{8b}-\frac{d^{3}x^{3}\sin(a+bx)\cos^{3}(a+bx)}{8b}-\frac{3c^{2}d\sin^{4}(a+bx)}{32b^{2}}-\frac{3c^{2}d\cos^{4}(a+bx)}{32b^{2}}-\frac{3cd^{2}x\sin^{4}(a+bx)}{64b^{2}}+\frac{9cd^{2}x\sin^{2}(a+bx)\cos^{2}(a+bx)}{32b^{2}}-\frac{3cd^{2}x\cos^{4}(a+bx)}{64b^{2}}-\frac{3d^{3}x^{2}\sin^{4}(a+bx)}{128b^{2}}+\frac{9d^{3}x^{2}\sin^{2}(a+bx)\cos^{2}(a+bx)}{64b^{2}}-\frac{3d^{3}x^{2}\cos^{4}(a+bx)}{128b^{2}}-\frac{3cd^{2}\sin^{3}(a+bx)\cos(a+bx)}{64b^{3}}+\frac{3cd^{2}\sin(a+bx)\cos^{3}(a+bx)}{64b^{3}}-\frac{3d^{3}x\sin^{3}(a+bx)\cos(a+bx)}{64b^{3}}+\frac{3d^{3}x\sin(a+bx)\cos^{3}(a+bx)}{64b^{3}}+\frac{3d^{3}\sin^{4}(a+bx)}{256b^{4}}+\frac{3d^{3}\cos^{4}(a+bx)}{256b^{4}} & \text{for}\: b \neq 0 \\ \left(c^{3}x+\frac{3c^{2}dx^{2}}{2}+cd^{2}x^{3}+\frac{d^{3}x^{4}}{4}\right)\sin^{2}(a)\cos^{2}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**3*x*sin(a + b*x)**4/8 + c**3*x*sin(a + b*x)**2*cos(a + b*x)**
2/4 + c**3*x*cos(a + b*x)**4/8 + 3*c**2*d*x**2*sin(a + b*x)**4/16 + 3*c**2*
d*x**2*sin(a + b*x)**2*cos(a + b*x)**2/8 + 3*c**2*d*x**2*cos(a + b*x)**4/16
 + c*d**2*x**3*sin(a + b*x)**4/8 + c*d**2*x**3*sin(a + b*x)**2*cos(a + b*x)
**2/4 + c*d**2*x**3*cos(a + b*x)**4/8 + d**3*x**4*sin(a + b*x)**4/32 + d**3
*x**4*sin(a + b*x)**2*cos(a + b*x)**2/16 + d**3*x**4*cos(a + b*x)**4/32 + c
**3*sin(a + b*x)**3*cos(a + b*x)/(8*b) - c**3*sin(a + b*x)*cos(a + b*x)**3/
(8*b) + 3*c**2*d*x*sin(a + b*x)**3*cos(a + b*x)/(8*b) - 3*c**2*d*x*sin(a +
b*x)*cos(a + b*x)**3/(8*b) + 3*c*d**2*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*
b) - 3*c*d**2*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b) + d**3*x**3*sin(a + b
*x)**3*cos(a + b*x)/(8*b) - d**3*x**3*sin(a + b*x)*cos(a + b*x)**3/(8*b) -
3*c**2*d*sin(a + b*x)**4/(32*b**2) - 3*c**2*d*cos(a + b*x)**4/(32*b**2) - 3
*c*d**2*x*sin(a + b*x)**4/(64*b**2) + 9*c*d**2*x*sin(a + b*x)**2*cos(a + b*
x)**2/(32*b**2) - 3*c*d**2*x*cos(a + b*x)**4/(64*b**2) - 3*d**3*x**2*sin(a
+ b*x)**4/(128*b**2) + 9*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(64*b**2
) - 3*d**3*x**2*cos(a + b*x)**4/(128*b**2) - 3*c*d**2*sin(a + b*x)**3*cos(a
 + b*x)/(64*b**3) + 3*c*d**2*sin(a + b*x)*cos(a + b*x)**3/(64*b**3) - 3*d**
3*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) + 3*d**3*x*sin(a + b*x)*cos(a +
b*x)**3/(64*b**3) + 3*d**3*sin(a + b*x)**4/(256*b**4) + 3*d**3*cos(a + b*x)
**4/(256*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*
x**4/4)*sin(a)**2*cos(a)**2, True))
```

Giac [A]
time = 0.46, size = 153, normalized size = 1.46

$$\frac{1}{32}\,d^3x^4+\frac{1}{8}\,cd^2x^3+\frac{3}{16}\,c^2dx^2+\frac{1}{8}\,c^3x-\frac{3\,(8\,b^2d^3x^2+16\,b^2cd^2x+8\,b^2c^2d-d^3)\cos\left(4\,bx+4\,a\right)}{1024\,b^4}-\frac{(8\,b^3d^3x^3+24\,b^3cd^2x^2+24\,b^3c^2dx+8\,b^3c^3-3\,bd^3x-3\,bcd^2)\sin\left(4\,bx+4\,a\right)}{256\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/32*d^3*x^4 + 1/8*c*d^2*x^3 + 3/16*c^2*d*x^2 + 1/8*c^3*x - 3/1024*(8*b^2*d
^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(4*b*x + 4*a)/b^4 - 1/256*(
8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*b*d^3*x -
 3*b*c*d^2)*sin(4*b*x + 4*a)/b^4
```

Mupad [B]
time = 1.69, size = 329, normalized size = 3.13

$$x^2\left(\frac{3\,c^2\,d}{64}+\frac{9\,d^3}{512\,b^2}\right)+x^2\left(\frac{9\,c^2\,d}{64}-\frac{9\,d^3}{512\,b^2}\right)+x\left(\frac{c^3}{32}+\frac{9\,c\,d^2}{256\,b^2}\right)+x\left(\frac{3\,c^3}{32}-\frac{9\,c\,d^2}{256\,b^2}\right)+\frac{d^3\,x^4}{32}-\frac{x\,\cos\left(4\,a+4\,b\,x\right)\,\left(\frac{c^3}{4}+\frac{9\,c\,d^2}{32\,b^2}\right)}{8}+\frac{x\,\cos\left(4\,a+4\,b\,x\right)\,\left(\frac{c^3}{8}-\frac{3\,c\,d^2}{64\,b^2}\right)}{4}+\frac{c\,d^2\,x^3}{8}+\frac{\cos\left(4\,a+4\,b\,x\right)\,\left(\frac{3\,d^3}{128}-\frac{3\,b^2\,c^2\,d}{16}\right)}{8\,b^4}+\frac{\sin\left(4\,a+4\,b\,x\right)\,\left(3\,c\,d^2-8\,b^2\,c^3\right)}{256\,b^3}-\frac{x^2\,\cos\left(4\,a+4\,b\,x\right)\,\left(\frac{3\,c^2\,d}{8}+\frac{9\,d^3}{64\,b^2}\right)}{8}+\frac{x^2\,\cos\left(4\,a+4\,b\,x\right)\,\left(\frac{3\,c^2\,d}{16}-\frac{3\,d^3}{128\,b^2}\right)}{4}-\frac{d^3\,x^3\,\sin\left(4\,a+4\,b\,x\right)}{32\,b}+\frac{3\,x\,\sin\left(4\,a+4\,b\,x\right)\,\left(d^3-8\,b^2\,c^2\,d\right)}{256\,b^3}-\frac{3\,c\,d^2\,x^2\,\sin\left(4\,a+4\,b\,x\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] x^2*((3*c^2*d)/64 + (9*d^3)/(512*b^2)) + x^2*((9*c^2*d)/64 - (9*d^3)/(512*b
^2)) + x*(c^3/32 + (9*c*d^2)/(256*b^2)) + x*((3*c^3)/32 - (9*c*d^2)/(256*b^
2)) + (d^3*x^4)/32 - (x*cos(4*a + 4*b*x)*(c^3/4 + (9*c*d^2)/(32*b^2)))/8 +
(x*cos(4*a + 4*b*x)*(c^3/8 - (3*c*d^2)/(64*b^2)))/4 + (c*d^2*x^3)/8 + (cos(
4*a + 4*b*x)*((3*d^3)/128 - (3*b^2*c^2*d)/16))/(8*b^4) + (sin(4*a + 4*b*x)*
(3*c*d^2 - 8*b^2*c^3))/(256*b^3) - (x^2*cos(4*a + 4*b*x)*((3*c^2*d)/8 + (9*
d^3)/(64*b^2)))/8 + (x^2*cos(4*a + 4*b*x)*((3*c^2*d)/16 - (3*d^3)/(128*b^2)
))/4 - (d^3*x^3*sin(4*a + 4*b*x))/(32*b) + (3*x*sin(4*a + 4*b*x)*(d^3 - 8*b
^2*c^2*d))/(256*b^3) - (3*c*d^2*x^2*sin(4*a + 4*b*x))/(32*b)
```

3.82 $\int (c+dx)^2 \cos^2(a+bx) \sin^2(a+bx)\, dx$

Optimal. Leaf size=79

$$\frac{(c+dx)^3}{24d}-\frac{d(c+dx)\cos(4a+4bx)}{64b^2}+\frac{d^2\sin(4a+4bx)}{256b^3}-\frac{(c+dx)^2\sin(4a+4bx)}{32b}$$

```
[Out] 1/24*(d*x+c)^3/d-1/64*d*(d*x+c)*cos(4*b*x+4*a)/b^2+1/256*d^2*sin(4*b*x+4*a)
/b^3-1/32*(d*x+c)^2*sin(4*b*x+4*a)/b
```

Rubi [A]
time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4491, 3377, 2717}

$$\frac{d^2\sin(4a+4bx)}{256b^3}-\frac{d(c+dx)\cos(4a+4bx)}{64b^2}-\frac{(c+dx)^2\sin(4a+4bx)}{32b}+\frac{(c+dx)^3}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^3/(24*d) - (d*(c + d*x)*Cos[4*a + 4*b*x])/(64*b^2) + (d^2*Sin[4*a
 + 4*b*x])/(256*b^3) - ((c + d*x)^2*Sin[4*a + 4*b*x])/(32*b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^2 \cos^2(a+bx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)^2 - \frac{1}{8}(c+dx)^2\cos(4a+4bx)\right)dx \\
&= \frac{(c+dx)^3}{24d} - \frac{1}{8}\int (c+dx)^2\cos(4a+4bx)\,dx \\
&= \frac{(c+dx)^3}{24d} - \frac{(c+dx)^2\sin(4a+4bx)}{32b} + \frac{d\int (c+dx)\sin(4a+4bx}{16b} \\
&= \frac{(c+dx)^3}{24d} - \frac{d(c+dx)\cos(4a+4bx)}{64b^2} - \frac{(c+dx)^2\sin(4a+4bx)}{32b} \\
&= \frac{(c+dx)^3}{24d} - \frac{d(c+dx)\cos(4a+4bx)}{64b^2} + \frac{d^2\sin(4a+4bx)}{256b^3} - \frac{(c+}{}
\end{aligned}
$$

Mathematica [A]
time = 0.37, size = 77, normalized size = 0.97

$$\frac{32b^3x(3c^2+3cdx+d^2x^2) - 12bd(c+dx)\cos(4(a+bx)) - 3(-d^2+8b^2(c+dx)^2)\sin(4(a+bx))}{768b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (32*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) - 12*b*d*(c + d*x)*Cos[4*(a + b*x)] -
 3*(-d^2 + 8*b^2*(c + d*x)^2)*Sin[4*(a + b*x)])/(768*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(71) = 142$.
time = 0.19, size = 531, normalized size = 6.72 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/b^2*a^2*d^2*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+
1/8*b*x+1/8*a)-2/b*a*c*d*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b
*x+a)+1/8*b*x+1/8*a)-2/b^2*a*d^2*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b
*x+1/2*a)-1/16*(b*x+a)^2+1/4*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*s
in(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/64*(2*sin(b*x+a)^2+3)^2)+c^2*(-1/4*s
in(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)+2/b*c*d*((b
*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/4*sin(b*x
+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)
-1/64*(2*sin(b*x+a)^2+3)^2)+1/b^2*d^2*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a
)+1/2*b*x+1/2*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b
*x+7/64*a-1/12*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(
b*x+a)+3/8*b*x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b
*x+a))*cos(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(71) = 142$.
time = 0.29, size = 232, normalized size = 2.94

$$\frac{24\,(4\,bx+4\,a-\sin\left(4\,bx+4\,a\right))c^2-\frac{48\,(4\,bx+4\,a-\sin(4\,bx+4\,a))acd}{b}+\frac{24\,(4\,bx+4\,a-\sin(4\,bx+4\,a))a^2d^2}{b^2}+\frac{12\left(8\,(bx+a)^2-4\,(bx+a)\sin(4\,bx+4\,a)-\cos(4\,bx+4\,a)\right)cd}{b}-\frac{12\left(8\,(bx+a)^2-4\,(bx+a)\sin(4\,bx+4\,a)-\cos(4\,bx+4\,a)\right)ad^2}{b^2}+\frac{\left(32\,(bx+a)^3-12\,(bx+a)\cos(4\,bx+4\,a)-3\left(8\,(bx+a)^2-1\right)\sin(4\,bx+4\,a)\right)d^2}{b^2}}{768\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/768*(24*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^2 - 48*(4*b*x + 4*a - sin(4*b*
x + 4*a))*a*c*d/b + 24*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*d^2/b^2 + 12*(8
*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*c*d/b - 12*
(8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*d^2/b^2
 + (32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*
sin(4*b*x + 4*a))*d^2/b^2)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(71) = 142$.
time = 2.14, size = 180, normalized size = 2.28

$$\frac{8\,b^3d^2x^3+24\,b^3cdx^2-24\,(bd^2x+bcd)\cos\left(bx+a\right)^4+24\,(bd^2x+bcd)\cos\left(bx+a\right)^2+3\,(8\,b^3c^2-bd^2)x-3\left(2\left(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2\right)\cos\left(bx+a\right)^3-\left(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2\right)\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{192\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/192*(8*b^3*d^2*x^3 + 24*b^3*c*d*x^2 - 24*(b*d^2*x + b*c*d)*cos(b*x + a)^4
 + 24*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 3*(8*b^3*c^2 - b*d^2)*x - 3*(2*(8*
b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(b*x + a)^3 - (8*b^2*d^2*x
^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(b*x + a))*sin(b*x + a))/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(70) = 140$.
time = 0.48, size = 493, normalized size = 6.24

$$\begin{cases} \frac{c^2x\sin^4(a+bx)}{8}+\frac{c^2x\sin^2(a+bx)\cos^2(a+bx)}{4}+\frac{c^2x\cos^4(a+bx)}{8}+\frac{cdx^2\sin^4(a+bx)}{8}+\frac{cdx^2\sin^2(a+bx)\cos^2(a+bx)}{4}+\frac{cdx^2\cos^4(a+bx)}{8}+\frac{d^2x^3\sin^4(a+bx)}{24}+\frac{d^2x^3\sin^2(a+bx)\cos^2(a+bx)}{12}+\frac{d^2x^3\cos^4(a+bx)}{24}+\frac{c^2\sin^3(a+bx)\cos(a+bx)}{8b}-\frac{c^2\sin(a+bx)\cos^3(a+bx)}{8b}+\frac{cdx\sin^3(a+bx)\cos(a+bx)}{4b}-\frac{cdx\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{d^2x^2\sin^3(a+bx)\cos(a+bx)}{8b}-\frac{d^2x^2\sin(a+bx)\cos^3(a+bx)}{8b}-\frac{cd\sin^4(a+bx)}{16b^2}-\frac{cd\cos^4(a+bx)}{16b^2}-\frac{d^2x\sin^4(a+bx)}{64b^2}+\frac{3d^2x\sin^2(a+bx)\cos^2(a+bx)}{32b^2}-\frac{d^2x\cos^4(a+bx)}{64b^2}-\frac{d^2\sin^3(a+bx)\cos(a+bx)}{64b^3}+\frac{d^2\sin(a+bx)\cos^3(a+bx)}{64b^3} & \text{for } b \neq 0 \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin^2(a)\cos^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**2*x*sin(a + b*x)**4/8 + c**2*x*sin(a + b*x)**2*cos(a + b*x)**
2/4 + c**2*x*cos(a + b*x)**4/8 + c*d*x**2*sin(a + b*x)**4/8 + c*d*x**2*sin(
a + b*x)**2*cos(a + b*x)**2/4 + c*d*x**2*cos(a + b*x)**4/8 + d**2*x**3*sin(
a + b*x)**4/24 + d**2*x**3*sin(a + b*x)**2*cos(a + b*x)**2/12 + d**2*x**3*c
os(a + b*x)**4/24 + c**2*sin(a + b*x)**3*cos(a + b*x)/(8*b) - c**2*sin(a +
b*x)*cos(a + b*x)**3/(8*b) + c*d*x*sin(a + b*x)**3*cos(a + b*x)/(4*b) - c*d
```

```
*x*sin(a + b*x)*cos(a + b*x)**3/(4*b) + d**2*x**2*sin(a + b*x)**3*cos(a + b
*x)/(8*b) - d**2*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b) - c*d*sin(a + b*x)
**4/(16*b**2) - c*d*cos(a + b*x)**4/(16*b**2) - d**2*x*sin(a + b*x)**4/(64*
b**2) + 3*d**2*x*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**2) - d**2*x*cos(a +
 b*x)**4/(64*b**2) - d**2*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) + d**2*sin
(a + b*x)*cos(a + b*x)**3/(64*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*
x**3/3)*sin(a)**2*cos(a)**2, True))
```

Giac [A]
time = 0.44, size = 94, normalized size = 1.19

$$\frac{1}{24}\,d^2x^3+\frac{1}{8}\,cdx^2+\frac{1}{8}\,c^2x-\frac{(bd^2x+bcd)\cos\left(4\,bx+4\,a\right)}{64\,b^3}-\frac{(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2)\sin\left(4\,bx+4\,a\right)}{256\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/24*d^2*x^3 + 1/8*c*d*x^2 + 1/8*c^2*x - 1/64*(b*d^2*x + b*c*d)*cos(4*b*x +
 4*a)/b^3 - 1/256*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*sin(4*b*
x + 4*a)/b^3
```

Mupad [B]
time = 1.31, size = 179, normalized size = 2.27

$$x\left(\frac{c^2}{32}+\frac{3\,d^2}{256\,b^2}\right)+x\left(\frac{3\,c^2}{32}-\frac{3\,d^2}{256\,b^2}\right)+\frac{d^2\,x^3}{24}+\frac{\sin\left(4\,a+4\,b\,x\right)\,\left(d^2-8\,b^2\,c^2\right)}{256\,b^3}-\frac{x\,\cos\left(4\,a+4\,b\,x\right)\,\left(\frac{c^2}{4}+\frac{3\,d^2}{32\,b^2}\right)}{8}+\frac{x\,\cos\left(4\,a+4\,b\,x\right)\,\left(\frac{c^2}{8}-\frac{d^2}{64\,b^2}\right)}{4}+\frac{c\,d\,x^2}{8}-\frac{d^2\,x^2\,\sin\left(4\,a+4\,b\,x\right)}{32\,b}-\frac{c\,d\,\cos\left(4\,a+4\,b\,x\right)}{64\,b^2}-\frac{c\,d\,x\,\sin\left(4\,a+4\,b\,x\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] x*(c^2/32 + (3*d^2)/(256*b^2)) + x*((3*c^2)/32 - (3*d^2)/(256*b^2)) + (d^2*
x^3)/24 + (sin(4*a + 4*b*x)*(d^2 - 8*b^2*c^2))/(256*b^3) - (x*cos(4*a + 4*b
*x)*(c^2/4 + (3*d^2)/(32*b^2)))/8 + (x*cos(4*a + 4*b*x)*(c^2/8 - d^2/(64*b^
2)))/4 + (c*d*x^2)/8 - (d^2*x^2*sin(4*a + 4*b*x))/(32*b) - (c*d*cos(4*a + 4
*b*x))/(64*b^2) - (c*d*x*sin(4*a + 4*b*x))/(16*b)
```

3.83 $\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=53

$$\frac{(c+dx)^2}{16d}-\frac{d\cos(4a+4bx)}{128b^2}-\frac{(c+dx)\sin(4a+4bx)}{32b}$$

```
[Out] 1/16*(d*x+c)^2/d-1/128*d*cos(4*b*x+4*a)/b^2-1/32*(d*x+c)*sin(4*b*x+4*a)/b
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {4491, 3377, 2718}

$$-\frac{d\cos(4a+4bx)}{128b^2}-\frac{(c+dx)\sin(4a+4bx)}{32b}+\frac{(c+dx)^2}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^2/(16*d) - (d*Cos[4*a + 4*b*x])/(128*b^2) - ((c + d*x)*Sin[4*a +
4*b*x])/(32*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)-\frac{1}{8}(c+dx)\cos(4a+4bx)\right)\,dx$$
$$= \frac{(c+dx)^2}{16d}-\frac{1}{8}\int (c+dx)\cos(4a+4bx)\,dx$$
$$= \frac{(c+dx)^2}{16d}-\frac{(c+dx)\sin(4a+4bx)}{32b}+\frac{d\int \sin(4a+4bx)\,dx}{32b}$$
$$= \frac{(c+dx)^2}{16d}-\frac{d\cos(4a+4bx)}{128b^2}-\frac{(c+dx)\sin(4a+4bx)}{32b}$$

Mathematica [A]
time = 0.30, size = 54, normalized size = 1.02

$$-\frac{8(a+bx)(-2bc+ad-bdx)+d\cos(4(a+bx))+4b(c+dx)\sin(4(a+bx))}{128b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] -1/128*(8*(a + b*x)*(-2*b*c + a*d - b*d*x) + d*Cos[4*(a + b*x)] + 4*b*(c + d*x)*Sin[4*(a + b*x)])/b^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(47) = 94$.
time = 0.13, size = 200, normalized size = 3.77

method	result
risch	$\frac{d\,x^2}{16}+\frac{cx}{8}-\frac{d\cos(4bx+4a)}{128b^2}-\frac{(dx+c)\sin(4bx+4a)}{32b}$
derivativedivides	$-\frac{da\left(-\frac{\sin(bx+a)\left(\cos^3(bx+a)\right)}{4}+\frac{\cos(bx+a)\sin(bx+a)}{8}+\frac{bx}{8}+\frac{a}{8}\right)}{b}+c\left(-\frac{\sin(bx+a)\left(\cos^3(bx+a)\right)}{4}+\frac{\cos(bx+a)\sin(bx+a)}{8}+\frac{bx}{8}+\frac{a}{8}\right)$
default	$-\frac{da\left(-\frac{\sin(bx+a)\left(\cos^3(bx+a)\right)}{4}+\frac{\cos(bx+a)\sin(bx+a)}{8}+\frac{bx}{8}+\frac{a}{8}\right)}{b}+c\left(-\frac{\sin(bx+a)\left(\cos^3(bx+a)\right)}{4}+\frac{\cos(bx+a)\sin(bx+a)}{8}+\frac{bx}{8}+\frac{a}{8}\right)$
norman	$\frac{cx}{8}+\frac{d\,x^2}{16}-\frac{c\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}+\frac{7c\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}-\frac{7c\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{c\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{cx\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}+\frac{3cx\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b*d*a*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b
*x+1/8*a)+c*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x
+1/8*a)+1/b*d*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x
+a)^2+1/4*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+
a)+3/8*b*x+3/8*a)-1/64*(2*sin(b*x+a)^2+3)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(47) = 94$.
time = 0.28, size = 96, normalized size = 1.81

$$\frac{4\,(4\,bx + 4\,a - \sin\left(4\,bx + 4\,a\right))c - \frac{4\,(4\,bx + 4\,a - \sin(4\,bx + 4\,a))ad}{b} + \frac{\left(8\,(bx + a)^2 - 4\,(bx + a)\sin(4\,bx + 4\,a) - \cos(4\,bx + 4\,a)\right)d}{b}}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/128*(4*(4*b*x + 4*a - sin(4*b*x + 4*a))*c - 4*(4*b*x + 4*a - sin(4*b*x +
4*a))*a*d/b + (8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4
*a))*d/b)/b
```

Fricas [A]
time = 1.04, size = 85, normalized size = 1.60

$$\frac{b^2dx^2 - d\cos\left(bx + a\right)^4 + 2\,b^2cx + d\cos\left(bx + a\right)^2 - 2\,\left(2\,(bdx + bc)\cos\left(bx + a\right)^3 - (bdx + bc)\cos\left(bx + a\right)\right)\sin\left(bx + a\right)}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(b^2*d*x^2 - d*cos(b*x + a)^4 + 2*b^2*c*x + d*cos(b*x + a)^2 - 2*(2*(b
*d*x + b*c)*cos(b*x + a)^3 - (b*d*x + b*c)*cos(b*x + a))*sin(b*x + a))/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(46) = 92$.
time = 0.31, size = 238, normalized size = 4.49

$$\begin{cases} \frac{cx\sin^4(a+bx)}{8} + \frac{cx\sin^2(a+bx)\cos^2(a+bx)}{4} + \frac{cx\cos^4(a+bx)}{8} + \frac{dx^2\sin^4(a+bx)}{16} + \frac{dx^2\sin^2(a+bx)\cos^2(a+bx)}{8} + \frac{dx^2\cos^4(a+bx)}{16} + \frac{c\sin^3(a+bx)\cos(a+bx)}{8b} - \frac{c\sin(a+bx)\cos^3(a+bx)}{8b} + \frac{dx\sin^3(a+bx)\cos(a+bx)}{8b} - \frac{dx\sin(a+bx)\cos^3(a+bx)}{8b} - \frac{d\sin^4(a+bx)}{32b^2} - \frac{d\cos^4(a+bx)}{32b^2} & \text{for}\ b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\sin^2(a)\cos^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c*x*sin(a + b*x)**4/8 + c*x*sin(a + b*x)**2*cos(a + b*x)**2/4 +
c*x*cos(a + b*x)**4/8 + d*x**2*sin(a + b*x)**4/16 + d*x**2*sin(a + b*x)**2*
cos(a + b*x)**2/8 + d*x**2*cos(a + b*x)**4/16 + c*sin(a + b*x)**3*cos(a + b
*x)/(8*b) - c*sin(a + b*x)*cos(a + b*x)**3/(8*b) + d*x*sin(a + b*x)**3*cos(
a + b*x)/(8*b) - d*x*sin(a + b*x)*cos(a + b*x)**3/(8*b) - d*sin(a + b*x)**4
```

```
/(32*b**2) - d*cos(a + b*x)**4/(32*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(
a)**2*cos(a)**2, True))
```

Giac [A]
time = 0.44, size = 48, normalized size = 0.91

$$\frac{1}{16}\,dx^2+\frac{1}{8}\,cx-\frac{d\cos\left(4\,bx+4\,a\right)}{128\,b^2}-\frac{(bdx+bc)\sin\left(4\,bx+4\,a\right)}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*d*x^2 + 1/8*c*x - 1/128*d*cos(4*b*x + 4*a)/b^2 - 1/32*(b*d*x + b*c)*si
n(4*b*x + 4*a)/b^2
```

Mupad [B]
time = 1.04, size = 57, normalized size = 1.08

$$\frac{c\,x}{8}+\frac{d\,x^2}{16}-\frac{d\,\cos\left(4\,a+4\,b\,x\right)}{128\,b^2}-\frac{c\,\sin\left(4\,a+4\,b\,x\right)}{32\,b}-\frac{d\,x\,\sin\left(4\,a+4\,b\,x\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x),x)
```

```
[Out] (c*x)/8 + (d*x^2)/16 - (d*cos(4*a + 4*b*x))/(128*b^2) - (c*sin(4*a + 4*b*x)
)/(32*b) - (d*x*sin(4*a + 4*b*x))/(32*b)
```

3.84 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=78

$$-\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\log(c+dx)}{8d}+\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}$$

```
[Out] -1/8*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d+1/8*ln(d*x+c)/d+1/8*Si(4*b*c/d+4*
b*x)*sin(4*a-4*b*c/d)/d
```

Rubi [A]
time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4491, 3384, 3380, 3383}

$$-\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\log(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] -1/8*(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/d + Log[c + d*x]
/(8*d) + (Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(8*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx &= \int \left(\frac{1}{8(c+dx)}-\frac{\cos(4a+4bx)}{8(c+dx)}\right)dx \\
&= \frac{\log(c+dx)}{8d}-\frac{1}{8}\int \frac{\cos(4a+4bx)}{c+dx}\,dx \\
&= \frac{\log(c+dx)}{8d}-\frac{1}{8}\cos\left(4a-\frac{4bc}{d}\right)\int \frac{\cos\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx+\frac{1}{8}\sin\left(4a-\frac{4bc}{d}\right. \\
&= -\frac{\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\log(c+dx)}{8d}+\frac{\sin\left(4a-\frac{4bc}{d}\right)\mathrm{Si}\left(\frac{4bc}{d}+4b\right.}{8d}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 65, normalized size = 0.83

$$\frac{-\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)+\log(c+dx)+\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x),x]
[Out] (-(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d]) + Log[c + d*x] + Si
n[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(8*d)
```

Maple [A]
time = 0.13, size = 114, normalized size = 1.46

method	result
risch	$\frac{\ln(dx+c)}{8d}+\frac{e^{-\frac{4i(ad-cb)}{d}}\text{expIntegral}\left(1,4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{16d}+\frac{e^{\frac{4i(ad-cb)}{d}}\text{expIntegral}\left(1,-4ibx-4ia-\frac{4(-iad+ibc)}{d}\right.}{16d}$
derivativedivides	$\frac{-\frac{b\left(-\frac{4\,\text{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}+\frac{4\,\text{cosineIntegral}\left(4bx+4a+\frac{-4ad+4cb}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}\right)}{32}+\frac{b\ln(-ad}{}}{b}$
default	$\frac{-\frac{b\left(-\frac{4\,\text{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}+\frac{4\,\text{cosineIntegral}\left(4bx+4a+\frac{-4ad+4cb}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}\right)}{32}+\frac{b\ln(-ad}{}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/32*b*(-4*Si(-4*b*x-4*a-4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4
*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)+1/8*b*ln(-a*d+c*b+d*(b*x+a)
)/d)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.31, size = 162, normalized size = 2.08

$$\frac{b\left(E_1\left(\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)+E_1\left(-\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{4(bc-ad)}{d}\right)+b\left(iE_1\left(\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)-iE_1\left(-\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+2b\log\left(bc+(bx+a)d-ad\right)}{16\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/16*(b*(exp_integral_e(1, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_inte
gral_e(1, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a*d)/d) + b
*(I*exp_integral_e(1, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integra
l_e(1, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d) + 2*b*
log(b*c + (b*x + a)*d - a*d))/(b*d)
```

Fricas [A]
time = 5.83, size = 88, normalized size = 1.13

$$-\frac{\left(\operatorname{Ci}\left(\frac{4(bdx+bc)}{d}\right)+\operatorname{Ci}\left(-\frac{4(bdx+bc)}{d}\right)\right)\cos\left(-\frac{4(bc-ad)}{d}\right)-2\sin\left(-\frac{4(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{4(bdx+bc)}{d}\right)-2\log\left(dx+c\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/16*((cos_integral(4*(b*d*x + b*c)/d) + cos_integral(-4*(b*d*x + b*c)/d))
*cos(-4*(b*c - a*d)/d) - 2*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*
c)/d) - 2*log(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(a+bx\right)\cos^2\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.45, size = 669, normalized size = 8.58

$$\frac{2\log\left(|dx+c|\right)\tan\left(2a\right)^2\tan\left(\frac{2bc}{d}\right)^2-\Re\left(\operatorname{Ci}\left(4bx+\frac{4bc}{d}\right)\right)\tan\left(2a\right)^2\tan\left(\frac{2bc}{d}\right)^2-\Re\left(\operatorname{Ci}\left(-4bx-\frac{4bc}{d}\right)\right)\tan\left(2a\right)^2\tan\left(\frac{2bc}{d}\right)^2+2\Im\left(\operatorname{Ci}\left(4bx+\frac{4bc}{d}\right)\right)\tan\left(2a\right)^2\tan\left(\frac{2bc}{d}\right)-2\Im\left(\operatorname{Ci}\left(-4bx-\frac{4bc}{d}\right)\right)\tan\left(2a\right)^2\tan\left(\frac{2bc}{d}\right)+4\operatorname{Si}\left(\frac{4(bdx+bc)}{d}\right)\tan\left(2a\right)^2\tan\left(\frac{2bc}{d}\right)-2\Im\left(\operatorname{Ci}\left(4bx+\frac{4bc}{d}\right)\right)\tan\left(2a\right)\tan\left(\frac{2bc}{d}\right)^2+2\Im\left(\operatorname{Ci}\left(-4bx-\frac{4bc}{d}\right)\right)\tan\left(2a\right)\tan\left(\frac{2bc}{d}\right)^2-4\operatorname{Si}\left(\frac{4(bdx+bc)}{d}\right)\tan\left(2a\right)\tan\left(\frac{2bc}{d}\right)^2+2\log\left(|dx+c|\right)\tan\left(2a\right)^2+\Re\left(\operatorname{Ci}\left(4bx+\frac{4bc}{d}\right)\right)\tan\left(2a\right)^2+\Re\left(\operatorname{Ci}\left(-4bx-\frac{4bc}{d}\right)\right)\tan\left(2a\right)^2-4\Re\left(\operatorname{Ci}\left(4bx+\frac{4bc}{d}\right)\right)\tan\left(2a\right)\tan\left(\frac{2bc}{d}\right)-4\Re\left(\operatorname{Ci}\left(-4bx-\frac{4bc}{d}\right)\right)\tan\left(2a\right)\tan\left(\frac{2bc}{d}\right)+2\log\left(|dx+c|\right)\tan\left(\frac{2bc}{d}\right)^2+\Re\left(\operatorname{Ci}\left(4bx+\frac{4bc}{d}\right)\right)\tan\left(\frac{2bc}{d}\right)^2+\Re\left(\operatorname{Ci}\left(-4bx-\frac{4bc}{d}\right)\right)\tan\left(\frac{2bc}{d}\right)^2+2\Im\left(\operatorname{Ci}\left(4bx+\frac{4bc}{d}\right)\right)\tan\left(2a\right)-2\Im\left(\operatorname{Ci}\left(-4bx-\frac{4bc}{d}\right)\right)\tan\left(2a\right)+4\operatorname{Si}\left(\frac{4(bdx+bc)}{d}\right)\tan\left(2a\right)-2\Im\left(\operatorname{Ci}\left(4bx+\frac{4bc}{d}\right)\right)\tan\left(\frac{2bc}{d}\right)+2\Im\left(\operatorname{Ci}\left(-4bx-\frac{4bc}{d}\right)\right)\tan\left(\frac{2bc}{d}\right)-4\operatorname{Si}\left(\frac{4(bdx+bc)}{d}\right)\tan\left(\frac{2bc}{d}\right)+2\log\left(|dx+c|\right)-\Re\left(\operatorname{Ci}\left(4bx+\frac{4bc}{d}\right)\right)-\Re\left(\operatorname{Ci}\left(-4bx-\frac{4bc}{d}\right)\right)}{16\left(d\tan\left(2a\right)^2\tan\left(\frac{2bc}{d}\right)^2+d\tan\left(2a\right)^2+d\tan\left(\frac{2bc}{d}\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/16*(2*log(abs(d*x + c))*tan(2*a)^2*tan(2*b*c/d)^2 - real_part(cos_integra
l(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - real_part(cos_integral(-4*b
*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(4*b*x +
 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - 2*imag_part(cos_integral(-4*b*x - 4*b*
c/d))*tan(2*a)^2*tan(2*b*c/d) + 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^
2*tan(2*b*c/d) - 2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*
b*c/d)^2 + 2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d
)^2 - 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)^2 + 2*log(abs
(d*x + c))*tan(2*a)^2 + real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2
 + real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 4*real_part(cos_i
ntegral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d) - 4*real_part(cos_integral(
-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d) + 2*log(abs(d*x + c))*tan(2*b*c/d)
^2 + real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2 + real_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(4*b
*x + 4*b*c/d))*tan(2*a) - 2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2
*a) + 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a) - 2*imag_part(cos_integral
(4*b*x + 4*b*c/d))*tan(2*b*c/d) + 2*imag_part(cos_integral(-4*b*x - 4*b*c/d
))*tan(2*b*c/d) - 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d) + 2*log(ab
s(d*x + c)) - real_part(cos_integral(4*b*x + 4*b*c/d)) - real_part(cos_inte
gral(-4*b*x - 4*b*c/d)))/(d*tan(2*a)^2*tan(2*b*c/d)^2 + d*tan(2*a)^2 + d*ta
n(2*b*c/d)^2 + d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x), x)
```

3.85 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=104

$$-\frac{1}{8d(c+dx)}+\frac{\cos(4a+4bx)}{8d(c+dx)}+\frac{b\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}$$

```
[Out] -1/8/d/(d*x+c)+1/8*cos(4*b*x+4*a)/d/(d*x+c)+1/2*b*cos(4*a-4*b*c/d)*Si(4*b*c
/d+4*b*x)/d^2+1/2*b*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^2
```

Rubi [A]
time = 0.12, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$\frac{b\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}+\frac{\cos(4a+4bx)}{8d(c+dx)}-\frac{1}{8d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -1/8*1/(d*(c + d*x)) + Cos[4*a + 4*b*x]/(8*d*(c + d*x)) + (b*CosIntegral[(4
*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(2*d^2) + (b*Cos[4*a - (4*b*c)/d]*Si
nIntegral[(4*b*c)/d + 4*b*x])/(2*d^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{1}{8(c+dx)^2}-\frac{\cos(4a+4bx)}{8(c+dx)^2}\right)dx \\
&= -\frac{1}{8d(c+dx)}-\frac{1}{8}\int \frac{\cos(4a+4bx)}{(c+dx)^2}\,dx \\
&= -\frac{1}{8d(c+dx)}+\frac{\cos(4a+4bx)}{8d(c+dx)}+\frac{b\int \frac{\sin(4a+4bx)}{c+dx}\,dx}{2d} \\
&= -\frac{1}{8d(c+dx)}+\frac{\cos(4a+4bx)}{8d(c+dx)}+\frac{\left(b\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\sin\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx}{2d}+\frac{(b}{} \\
&= -\frac{1}{8d(c+dx)}+\frac{\cos(4a+4bx)}{8d(c+dx)}+\frac{b\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{2d^2}+\frac{b\cos(4}{}
\end{aligned}$$

Mathematica [A]
time = 0.46, size = 81, normalized size = 0.78

$$\frac{\frac{d(-1+\cos(4(a+bx)))}{c+dx}+4b\mathrm{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)\sin\left(4a-\frac{4bc}{d}\right)+4b\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Si}\left(\frac{4b(c+dx)}{d}\right)}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] ((d*(-1 + Cos[4*(a + b*x)]))/(c + d*x) + 4*b*CosIntegral[(4*b*(c + d*x))/d]
*Sin[4*a - (4*b*c)/d] + 4*b*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x)
)/d])/(8*d^2)
```

Maple [A]
time = 0.17, size = 156, normalized size = 1.50

method	result

risch	$-\frac{1}{8d(dx+c)}-\frac{ib\,\mathrm{e}^{-\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{4d^2}+\frac{ib\,\mathrm{e}^{\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,-4ibx-4ia-\frac{4(-ia}{}\right.}{4d^2}$
derivativedivides	$-\frac{b^2\left(-\frac{4\cos(4bx+4a)}{(-ad+cb+d(bx+a))d}-\frac{4\left(-\frac{4\,\mathrm{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}-\frac{4\,\mathrm{cosineIntegral}\left(4bx+4a+\frac{-4ad+4cb}{d}\right)\mathrm{si}}{d}\right.}{d}\right.}{32}}{b}$
default	$-\frac{b^2\left(-\frac{4\cos(4bx+4a)}{(-ad+cb+d(bx+a))d}-\frac{4\left(-\frac{4\,\mathrm{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}-\frac{4\,\mathrm{cosineIntegral}\left(4bx+4a+\frac{-4ad+4cb}{d}\right)\mathrm{si}}{d}\right.}{d}\right.}{32}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/32*b^2*(-4*cos(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))/d-4*(-4*Si(-4*b*x-4*
a-4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(
4*(-a*d+b*c)/d)/d)/d)-1/8*b^2/(-a*d+c*b+d*(b*x+a))/d)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.35, size = 171, normalized size = 1.64

$$\frac{b^2\left(E_2\left(\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)+E_2\left(-\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{4(bc-ad)}{d}\right)+b^2\left(iE_2\left(\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)-iE_2\left(-\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-2b^2}{16(bcd+(bx+a)d^2-ad^2)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(b^2*(exp_integral_e(2, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_in
tegral_e(2, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a*d)/d) +
 b^2*(I*exp_integral_e(2, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_int
egral_e(2, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d) -
2*b^2)/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A]
time = 2.54, size = 138, normalized size = 1.33

$$\frac{4d\cos(bx+a)^4-4d\cos(bx+a)^2+2(bdx+bc)\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{4(bdx+bc)}{d}\right)+\left((bdx+bc)\,\mathrm{Ci}\left(\frac{4(bdx+bc)}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{4(bdx+bc)}{d}\right)\right)\sin\left(-\frac{4(bc-ad)}{d}\right)}{4(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*d*cos(b*x + a)^4 - 4*d*cos(b*x + a)^2 + 2*(b*d*x + b*c)*cos(-4*(b*c
- a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + ((b*d*x + b*c)*cos_integral(4*(
b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-4*(b*d*x + b*c)/d))*sin(-4*(b
*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\cos^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**2, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.85, size = 3218, normalized size = 30.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2
*tan(2*b*c/d)^2 - b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x
)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*b*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan
(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*b*d*x*real_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 2*b*d*x*real_part(cos_in
tegral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 2*b*d*x*re
al_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2
 - 2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*
tan(2*b*c/d)^2 + b*c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*
tan(2*a)^2*tan(2*b*c/d)^2 - b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d))*t
an(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*b*c*sin_integral(4*(b*d*x + b*c)/
d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - b*d*x*imag_part(cos_integral(4*
b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + b*d*x*imag_part(cos_integral(-4*b
*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 2*b*d*x*sin_integral(4*(b*d*x + b*
c)/d)*tan(2*b*x)^2*tan(2*a)^2 + 4*b*d*x*imag_part(cos_integral(4*b*x + 4*b*
c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) - 4*b*d*x*imag_part(cos_integral(-
4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 8*b*d*x*sin_integral
(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 2*b*c*real_part(co
s_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 2*b*c*r
eal_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/
d) - b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/
d)^2 + b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b
```

```
*c/d)^2 - 2*b*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d)
^2 - 2*b*c*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*t
an(2*b*c/d)^2 - 2*b*c*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^
2*tan(2*a)*tan(2*b*c/d)^2 + b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*
tan(2*a)^2*tan(2*b*c/d)^2 - b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))
*tan(2*a)^2*tan(2*b*c/d)^2 + 2*b*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*
a)^2*tan(2*b*c/d)^2 + 2*b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(
2*b*x)^2*tan(2*a) + 2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2
*b*x)^2*tan(2*a) - b*c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^
2*tan(2*a)^2 + b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*t
an(2*a)^2 - 2*b*c*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2 -
 2*b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)
 - 2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c
/d) + 4*b*c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*
tan(2*b*c/d) - 4*b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2
*tan(2*a)*tan(2*b*c/d) + 8*b*c*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2
*tan(2*a)*tan(2*b*c/d) + 2*b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*t
an(2*a)^2*tan(2*b*c/d) + 2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*
tan(2*a)^2*tan(2*b*c/d) - b*c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(
2*b*x)^2*tan(2*b*c/d)^2 + b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*b*x)^2*tan(2*b*c/d)^2 - 2*b*c*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)
^2*tan(2*b*c/d)^2 - 2*b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*
a)*tan(2*b*c/d)^2 - 2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2
*a)*tan(2*b*c/d)^2 + b*c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^
2*tan(2*b*c/d)^2 - b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2
*tan(2*b*c/d)^2 + 2*b*c*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*
c/d)^2 + b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2 - b*d*
x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 + 2*b*d*x*sin_inte
gral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2 + 2*b*c*real_part(cos_integral(4*b*x +
 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 2*b*c*real_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*b*x)^2*tan(2*a) - b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/
d))*tan(2*a)^2 + b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2
 - 2*b*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2 - 2*b*c*real_part(cos
_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 2*b*c*real_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) + 4*b*d*x*imag_part(
cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d) - 4*b*d*x*imag_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d) + 8*b*d*x*sin_integral(
4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d) + 2*b*c*real_part(cos_integral(4*b
*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 2*b*c*real_part(cos_integral(-4*b*
x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - b*d*x*imag_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*b*c/d)^2 + b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d)
)*tan(2*b*c/d)^2 - 2*b*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d)^2 -
 2*b*c*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 2
*b*c*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 + b*
c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^2,x)
[Out] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^2, x)
```

3.86 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=127

$$-\frac{1}{16d(c+dx)^2}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}+\frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b\sin(4a+4bx)}{4d^2(c+dx)}-\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)}{d^3}$$

```
[Out] -1/16/d/(d*x+c)^2+b^2*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^3+1/16*cos(4*b*x
+4*a)/d/(d*x+c)^2-b^2*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^3-1/4*b*sin(4*b*
x+4*a)/d^2/(d*x+c)
```

Rubi [A]
time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$\frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b\sin(4a+4bx)}{4d^2(c+dx)}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}-\frac{1}{16d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] -1/16*1/(d*(c + d*x)^2) + Cos[4*a + 4*b*x]/(16*d*(c + d*x)^2) + (b^2*Cos[4*
a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/d^3 - (b*Sin[4*a + 4*b*x])/(
4*d^2*(c + d*x)) - (b^2*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x]
)/d^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{1}{8(c+dx)^3}-\frac{\cos(4a+4bx)}{8(c+dx)^3}\right)dx$$
$$= -\frac{1}{16d(c+dx)^2}-\frac{1}{8}\int \frac{\cos(4a+4bx)}{(c+dx)^3}\,dx$$
$$= -\frac{1}{16d(c+dx)^2}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}+\frac{b\int \frac{\sin(4a+4bx)}{(c+dx)^2}\,dx}{4d}$$
$$= -\frac{1}{16d(c+dx)^2}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}-\frac{b\sin(4a+4bx)}{4d^2(c+dx)}+\frac{b^2\int \frac{\cos(4a+4bx)}{c+dx}\,dx}{d^2}$$
$$= -\frac{1}{16d(c+dx)^2}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}-\frac{b\sin(4a+4bx)}{4d^2(c+dx)}+\frac{\left(b^2\cos\left(4a-\frac{4bc}{d}\right)\right)}{d^2}$$
$$= -\frac{1}{16d(c+dx)^2}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}+\frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b\sin}{4d}$$

Mathematica [A]
time = 0.87, size = 105, normalized size = 0.83

$$\frac{16b^2\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)+\frac{d(-d+d\cos(4(a+bx))-4b(c+dx)\sin(4(a+bx)))}{(c+dx)^2}-16b^2\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^3,x]
[Out] (16*b^2*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] + (d*(-d + d*Co
s[4*(a + b*x)] - 4*b*(c + d*x)*Sin[4*(a + b*x)]))/(c + d*x)^2 - 16*b^2*Sin[
4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(16*d^3)
```

Maple [A]
time = 0.14, size = 193, normalized size = 1.52

method	result
derivativedivides	$-\frac{b^3\left(-\frac{2\cos(4bx+4a)}{(-ad+cb+d(bx+a))^2 d}-\frac{2\left(-\frac{4\sin(4bx+4a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{16\,\mathrm{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}+\frac{16\,\mathrm{cosineInte}}{}}{d}\right.}{d}\right.}{32}}{b}$
default	$-\frac{b^3\left(-\frac{2\cos(4bx+4a)}{(-ad+cb+d(bx+a))^2 d}-\frac{2\left(-\frac{4\sin(4bx+4a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{16\,\mathrm{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}+\frac{16\,\mathrm{cosineInte}}{}}{d}\right.}{d}\right.}{32}}{b}$
risch	$-\frac{1}{16d(dx+c)^2}-\frac{b^2 e^{-\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{2d^3}-\frac{b^2 e^{\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,-4ibx-4ia-\frac{4(-}{}\right.}{2d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/32*b^3*(-2*cos(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))^2/d-2*(-4*sin(4*b*x+
4*a)/(-a*d+c*b+d*(b*x+a))/d+4*(-4*Si(-4*b*x-4*a-4*(-a*d+b*c)/d)*sin(4*(-a*d
+b*c)/d)/d+4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)/d)/d)-1/16
*b^3/(-a*d+c*b+d*(b*x+a))^2/d)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.41, size = 206, normalized size = 1.62

$$\frac{b^3\left(E_3\left(\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)+E_3\left(-\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{4(bc-ad)}{d}\right)+b^3\left(iE_3\left(\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)-iE_3\left(-\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-b^3}{16\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\left(bcd^2-ad^3\right)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(b^3*(exp_integral_e(3, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_in
tegral_e(3, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a*d)/d) +
 b^3*(I*exp_integral_e(3, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_int
egral_e(3, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d) -
b^3)/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a
*d^3)*(b*x + a))*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(121) = 242$.
time = 2.10, size = 255, normalized size = 2.01

$$\frac{d^2 \cos(bx+a)^4 - d^2 \cos(bx+a)^2 - 2\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{4\,(bdx+bc)}{d}\right) + \left(\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\operatorname{Ci}\left(\frac{4\,(bdx+bc)}{d}\right) + \left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\operatorname{Ci}\left(-\frac{4\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right) - 2\left(2\,(bd^2x + bcd)\cos(bx+a)^3 - (bd^2x + bcd)\cos(bx+a)\right)\sin(bx+a)}{2\left(d^5x^2 + 2\,cd^4x + c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + ((b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(4*(b*d*x + b*c)/d) + (b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c
- a*d)/d) - 2*(2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - (b*d^2*x + b*c*d)*cos(b
*x + a))*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\cos^2(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.57, size = 5600, normalized size = 44.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^2*d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*ta
n(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b
*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(cos
_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 8*b^2*d^
2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan
(2*b*c/d) - 16*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan
(2*a)^2*tan(2*b*c/d) + 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d
))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(cos_integ
ral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 16*b^2*d^2*x^
2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 8*
```

b^2*c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*
tan(2*b*c/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(
2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(4
*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 4*b^2*d^2*x^2*real_part(cos_inte
gral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 16*b^2*d^2*x^2*real_part(
cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 16*b^2*
d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan
(2*b*c/d) - 16*b^2*c*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x
)^2*tan(2*a)^2*tan(2*b*c/d) + 16*b^2*c*d*x*imag_part(cos_integral(-4*b*x -
4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 32*b^2*c*d*x*sin_integral(
4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 4*b^2*d^2*x^2*rea
l_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 4*b^2*d
^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^
2 + 16*b^2*c*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(
2*a)*tan(2*b*c/d)^2 - 16*b^2*c*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d)
)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 32*b^2*c*d*x*sin_integral(4*(b*d*x
 + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 4*b^2*d^2*x^2*real_part(c
os_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*d^2*x^2*rea
l_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*c^
2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*
c/d)^2 + 4*b^2*c^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*t
an(2*a)^2*tan(2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b
*c/d))*tan(2*b*x)^2*tan(2*a) + 8*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x
- 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 16*b^2*d^2*x^2*sin_integral(4*(b*d*x +
b*c)/d)*tan(2*b*x)^2*tan(2*a) - 8*b^2*c*d*x*real_part(cos_integral(4*b*x +
4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 8*b^2*c*d*x*real_part(cos_integral(-4*b
*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 8*b^2*d^2*x^2*imag_part(cos_integr
al(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 8*b^2*d^2*x^2*imag_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) + 16*b^2*d^2*x^2*si
n_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d) + 32*b^2*c*d*x*real
_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 3
2*b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)
*tan(2*b*c/d) - 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(
2*a)^2*tan(2*b*c/d) + 8*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d
))*tan(2*a)^2*tan(2*b*c/d) - 16*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)
*tan(2*a)^2*tan(2*b*c/d) - 8*b^2*c^2*imag_part(cos_integral(4*b*x + 4*b*c/d
))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 8*b^2*c^2*imag_part(cos_integral(
-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 16*b^2*c^2*sin_in
tegral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 8*b^2*c*d*
x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 8*
b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/
d)^2 + 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(
2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*
a)*tan(2*b*c/d)^2 + 16*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)
*tan(2*b*c/d)^2 + 8*b^2*c^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*

```
b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 8*b^2*c^2*imag_part(cos_integral(-4*b*x -
4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 16*b^2*c^2*sin_integral(4*
(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 8*b^2*c*d*x*real_pa
rt(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 8*b^2*c*d*x*r
eal_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*
d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2 + 4*b^2*d^2*x
^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 - 16*b^2*c*d*x*im
ag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 16*b^2*c*d*x
*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 32*b^2*c
*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a) - 4*b^2*d^2*x^2*
real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 - 4*b^2*d^2*x^2*real_pa
rt(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 -...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^2}{{\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^3, x)
```

3.87 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=158

$$-\frac{1}{24d(c+dx)^3}+\frac{\cos(4a+4bx)}{24d(c+dx)^3}-\frac{b^2\cos(4a+4bx)}{3d^3(c+dx)}-\frac{4b^3\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{3d^4}-\frac{b\sin(4a+4bx)}{12d^2(c+dx)^2}$$

```
[Out] -1/24/d/(d*x+c)^3+1/24*cos(4*b*x+4*a)/d/(d*x+c)^3-1/3*b^2*cos(4*b*x+4*a)/d^
3/(d*x+c)-4/3*b^3*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*b*x)/d^4-4/3*b^3*Ci(4*b*c/d
+4*b*x)*sin(4*a-4*b*c/d)/d^4-1/12*b*sin(4*b*x+4*a)/d^2/(d*x+c)^2
```

Rubi [A]
time = 0.16, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$-\frac{4b^3\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}-\frac{b^2\cos(4a+4bx)}{3d^3(c+dx)}-\frac{b\sin(4a+4bx)}{12d^2(c+dx)^2}+\frac{\cos(4a+4bx)}{24d(c+dx)^3}-\frac{1}{24d(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -1/24*1/(d*(c + d*x)^3) + Cos[4*a + 4*b*x]/(24*d*(c + d*x)^3) - (b^2*Cos[4*
a + 4*b*x])/(3*d^3*(c + d*x)) - (4*b^3*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4
*a - (4*b*c)/d])/(3*d^4) - (b*Sin[4*a + 4*b*x])/(12*d^2*(c + d*x)^2) - (4*b
^3*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{1}{8(c+dx)^4}-\frac{\cos(4a+4bx)}{8(c+dx)^4}\right)dx$$
$$= -\frac{1}{24d(c+dx)^3}-\frac{1}{8}\int \frac{\cos(4a+4bx)}{(c+dx)^4}\,dx$$
$$= -\frac{1}{24d(c+dx)^3}+\frac{\cos(4a+4bx)}{24d(c+dx)^3}+\frac{b\int \frac{\sin(4a+4bx)}{(c+dx)^3}\,dx}{6d}$$
$$= -\frac{1}{24d(c+dx)^3}+\frac{\cos(4a+4bx)}{24d(c+dx)^3}-\frac{b\sin(4a+4bx)}{12d^2(c+dx)^2}+\frac{b^2\int \frac{\cos(4a+4bx)}{(c+dx)^2}\,dx}{3d^2}$$
$$= -\frac{1}{24d(c+dx)^3}+\frac{\cos(4a+4bx)}{24d(c+dx)^3}-\frac{b^2\cos(4a+4bx)}{3d^3(c+dx)}-\frac{b\sin(4a+4bx)}{12d^2(c+dx)^2}-$$
$$= -\frac{1}{24d(c+dx)^3}+\frac{\cos(4a+4bx)}{24d(c+dx)^3}-\frac{b^2\cos(4a+4bx)}{3d^3(c+dx)}-\frac{b\sin(4a+4bx)}{12d^2(c+dx)^2}-$$
$$= -\frac{1}{24d(c+dx)^3}+\frac{\cos(4a+4bx)}{24d(c+dx)^3}-\frac{b^2\cos(4a+4bx)}{3d^3(c+dx)}-\frac{4b^3\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{3d^4}$$

Mathematica [A]
time = 1.74, size = 123, normalized size = 0.78

$$-\frac{32b^3\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)\sin\left(4a-\frac{4bc}{d}\right)+\frac{d\left(\left(-d^2+8b^2(c+dx)^2\right)\cos(4(a+bx))+d(d+2b(c+dx)\sin(4(a+bx)))\right)}{(c+dx)^3}+32b^3\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right)}{24d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -1/24*(32*b^3*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + (d*((-d
^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] + d*(d + 2*b*(c + d*x)*Sin[4*(a +
```

```
b*x)]))))/(c + d*x)^3 + 32*b^3*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*
x))/d])/d^4
```

Maple [A]
time = 0.09, size = 230, normalized size = 1.46

method	result
derivativedivides	$-\frac{b^4\left(-\frac{4\cos(4bx+4a)}{3(-ad+cb+d(bx+a))^3 d}-\frac{4\left(-\frac{2\sin(4bx+4a)}{(-ad+cb+d(bx+a))^2 d}+\frac{-\frac{8\cos(4bx+4a)}{(-ad+cb+d(bx+a))d}-\frac{8\left(-\frac{4\,\mathrm{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)c\cdots}{d}\cdots\right)}{\cdots}}{\cdots}\right)}{3d}\cdots\right)}{32}\cdots}{b}$
default	$-\frac{b^4\left(-\frac{4\cos(4bx+4a)}{3(-ad+cb+d(bx+a))^3 d}-\frac{4\left(-\frac{2\sin(4bx+4a)}{(-ad+cb+d(bx+a))^2 d}+\frac{-\frac{8\cos(4bx+4a)}{(-ad+cb+d(bx+a))d}-\frac{8\left(-\frac{4\,\mathrm{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)c\cdots}{d}\cdots\right)}{\cdots}}{\cdots}\right)}{3d}\cdots\right)}{32}\cdots}{b}$
risch	$-\frac{1}{24d(dx+c)^3}+\frac{2ib^3 \mathrm{e}^{-\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{3d^4}-\frac{2ib^3 \mathrm{e}^{\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,-4ibx-4ia-\cdots\right)}{3d^4}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/32*b^4*(-4/3*cos(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))^3/d-4/3*(-2*sin(4*
b*x+4*a)/(-a*d+c*b+d*(b*x+a))^2/d+2*(-4*cos(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))
/d-4*(-4*Si(-4*b*x-4*a-4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*x+4*a
+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)/d)/d)/d)-1/24*b^4/(-a*d+c*b+d*(b*x+
a))^3/d)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.47, size = 258, normalized size = 1.63

$$\frac{3\,b^4\left(E_4\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_4\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)+3\,b^4\left(i\,E_4\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_4\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)-2\,b^4\cdots}{48\,\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\,(bcd^3-ad^4)(bx+a)^2+3\,(b^2c^2d^2-2\,abcd^3+a^2d^4)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*b^4*(exp_integral_e(4, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_
integral_e(4, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a*d)/d)
 + 3*b^4*(I*exp_integral_e(4, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp
_integral_e(4, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d
) - 2*b^4)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4
- a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3
+ a^2*d^4)*(b*x + a))*b)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(146) = 292$.
time = 2.69, size = 406, normalized size = 2.57

$$-\frac{b^2d^3x^2+2\,b^2cd^2x+b^2c^2d+\left(8\,b^2d^3x^2+16\,b^2cd^2x+8\,b^2c^2d-d^3\right)\cos\left(bx+a\right)^4-\left(8\,b^2d^3x^2+16\,b^2cd^2x+8\,b^2c^2d-d^3\right)\cos\left(bx+a\right)^2+4\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{4\,(bdx+bc)}{d}\right)+\left(2\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^3-\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)\right)\sin\left(bx+a\right)+2\left(\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\operatorname{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)+\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\operatorname{Ci}\left(-\frac{4\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)}{3\left(d^7x^3+3\,cd^6x^2+3\,c^2d^5x+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d + (8*b^2*d^3*x^2 + 16*b^2*c*d
^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^4 - (8*b^2*d^3*x^2 + 16*b^2*c*d^2*x
+ 8*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*
b^3*c^2*d*x + b^3*c^3)*cos(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d
) + (2*(b*d^3*x + b*c*d^2)*cos(b*x + a)^3 - (b*d^3*x + b*c*d^2)*cos(b*x + a
))*sin(b*x + a) + 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c
^3)*cos_integral(4*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + b^3*c^3)*cos_integral(-4*(b*d*x + b*c)/d))*sin(-4*(b*c - a*d)/d
))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(a+bx\right)\cos^2\left(a+bx\right)}{\left(c+dx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**4, x)
```

Giac [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.63, size = 8508, normalized size = 53.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")

[Out] -1/12*(8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 16*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 16*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 48*b^3*c*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 16*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2 + 32*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) - 32*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 64*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 48*b^3*c*d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 48*b^3*c*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 + 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 + 24*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 24*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 48*b^3*c^2*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 16*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 48*b^3*c*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2 - 16*b^3*d^3*x^3*real_part(cos_integral(4*b

```
*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 16*b^3*d^3*x^3*real_part(cos_int
egral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) + 96*b^3*c*d^2*x^2*imag_
part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) - 96
*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2
*a)*tan(2*b*c/d) + 192*b^3*c*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*
b*x)^2*tan(2*a)*tan(2*b*c/d) + 16*b^3*d^3*x^3*real_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 16*b^3*d^3*x^3*real_part(cos_integral
(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 48*b^3*c^2*d*x*real_part(cos_
integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 48*b^3*c^
2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan
(2*b*c/d) - 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2
*b*x)^2*tan(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4
*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*sin_integral(4*(b*d
*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*real_part(cos_int
egral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*real_part(
cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 48*b^3*c^2*d*x*re
al_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2
 - 48*b^3*c^2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*ta
n(2*a)*tan(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b
*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - 24*b^3*c*d^2*x^2*imag_part(cos_integral(
-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 48*b^3*c*d^2*x^2*sin_integra
l(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*d^3*x^2*tan(2*b*x)^2
*tan(2*a)^2*tan(2*b*c/d)^2 + 8*b^3*c^3*imag_part(cos_integral(4*b*x + 4*b*c
/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^3*c^3*imag_part(cos_integ
ral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*c^3*
sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*ta...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^2\,\sin(a+b\,x)^2}{(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^4,x)
[Out] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^4, x)
```

3.88 $\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=407

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{ib(c+dx)}{d}\right)}{16b}-\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1\right.}{16b}$$

```
[Out] -1/16*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)
/d)^m)-1/16*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*
x+c)/d)^m)-1/32*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*b*(d*x
+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/32*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I*b*(d*x+
c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)+1/32*5^(-1-m)*exp(5*I*(a-b*c
/d))*(d*x+c)^m*GAMMA(1+m,-5*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+1/32*5^(-
1-m)*(d*x+c)^m*GAMMA(1+m,5*I*b*(d*x+c)/d)/b/exp(5*I*(a-b*c/d))/((I*b*(d*x+c
)/d)^m)
```

Rubi [A]
time = 0.28, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4491, 3389, 2212}

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b}-\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{32b}+\frac{5^{-m-1}e^{5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{5ib(c+dx)}{d}\right)}{32b}-\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{ib(c+dx)}{d}\right)}{16b}-\frac{3^{-m-1}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{3ib(c+dx)}{d}\right)}{32b}+\frac{5^{-m-1}e^{-5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{5ib(c+dx)}{d}\right)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -1/16*(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/
(b*(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d]
)/(16*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - (3^(-1 - m)*E^((3*I)*(
a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(32*b*(((-I
)*b*(c + d*x))/d)^m) - (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(c + d
*x))/d])/(32*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + (5^(-1 - m)
*E^((5*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-5*I)*b*(c + d*x))/d])/
(32*b*(((-I)*b*(c + d*x))/d)^m) + (5^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((5*
I)*b*(c + d*x))/d])/(32*b*E^((5*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)^m \sin(a+bx) + \frac{1}{16}(c+dx)^m \sin(3a+3bx) - \frac{1}{16}\right.\\ &= \frac{1}{16}\int (c+dx)^m \sin(3a+3bx)\,dx - \frac{1}{16}\int (c+dx)^m \sin(5a+5bx\\ &= \frac{1}{32}i\int e^{-i(3a+3bx)}(c+dx)^m\,dx - \frac{1}{32}i\int e^{i(3a+3bx)}(c+dx)^m\,dx -\\ &= -\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{16b} - \frac{e^{-i(}}{}\end{aligned}$$

Mathematica [A]
time = 53.67, size = 413, normalized size = 1.01

$$\frac{15^{-1-m}e^{-\frac{5i(bc+ad)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-2m}\left(-2\,15^{1+m}e^{6ia+\frac{4ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^m\text{Gamma}\left(1+m,-\frac{ib(c+dx)}{d}\right)+\left(-\frac{ib(c+dx)}{d}\right)^m\left(-2\,15^{1+m}e^{4ia+\frac{6ibc}{d}}\left(\frac{b^2(c+dx)^2}{d^2}\right)^m\text{Gamma}\left(1+m,\frac{ib(c+dx)}{d}\right)-5^{1+m}e^{2i\left(4a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2m}\text{Gamma}\left(1+m,-\frac{3ib(c+dx)}{d}\right)-5^{1+m}e^{2ia+\frac{8ibc}{d}}\left(\frac{b^2(c+dx)^2}{d^2}\right)^m\text{Gamma}\left(1+m,\frac{3ib(c+dx)}{d}\right)+3^{1+m}e^{10ia}\left(\frac{ib(c+dx)}{d}\right)^{2m}\text{Gamma}\left(1+m,-\frac{5ib(c+dx)}{d}\right)+3^{1+m}e^{\frac{10ibc}{d}}\left(\frac{b^2(c+dx)^2}{d^2}\right)^m\text{Gamma}\left(1+m,\frac{5ib(c+dx)}{d}\right)\right)\right)}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (15^(-1 - m)*(c + d*x)^m*(-2*15^(1 + m)*E^((6*I)*a + ((4*I)*b*c)/d)*((I*b*(
c + d*x))/d)^m*((b^2*(c + d*x)^2)/d^2)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d]
 + (((-I)*b*(c + d*x))/d)^m*(-2*15^(1 + m)*E^((4*I)*a + ((6*I)*b*c)/d)*((b^
2*(c + d*x)^2)/d^2)^m*Gamma[1 + m, (I*b*(c + d*x))/d] - 5^(1 + m)*E^((2*I)*
(4*a + (b*c)/d))*((I*b*(c + d*x))/d)^(2*m)*Gamma[1 + m, ((-3*I)*b*(c + d*x)
)/d] - 5^(1 + m)*E^((2*I)*a + ((8*I)*b*c)/d)*((b^2*(c + d*x)^2)/d^2)^m*Gamm
a[1 + m, ((3*I)*b*(c + d*x))/d] + 3^(1 + m)*E^((10*I)*a)*((I*b*(c + d*x))/d
)^(2*m)*Gamma[1 + m, ((-5*I)*b*(c + d*x))/d] + 3^(1 + m)*E^(((10*I)*b*c)/d)
*((b^2*(c + d*x)^2)/d^2)^m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d])))/(32*b*E^(
((5*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^(2*m))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cos^2(bx+a)\right) \left(\sin^3(bx+a)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^3, x)
```

Fricas [A]
time = 0.58, size = 280, normalized size = 0.69

$$-\frac{30\,e^{\left(-\frac{dm\log\left(\frac{i\,b}{d}\right)-i\,bc+i\,ad}{d}\right)}\Gamma\left(m+1,\frac{i\,bdx+i\,bc}{d}\right)+5\,e^{\left(-\frac{dm\log\left(-\frac{3i\,b}{d}\right)+3i\,bc-3i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{3\,(i\,bdx+i\,bc)}{d}\right)-3\,e^{\left(-\frac{dm\log\left(-\frac{5i\,b}{d}\right)+5i\,bc-5i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{5\,(i\,bdx+i\,bc)}{d}\right)+30\,e^{\left(-\frac{dm\log\left(-\frac{i\,b}{d}\right)+i\,bc-i\,ad}{d}\right)}\Gamma\left(m+1,\frac{-i\,bdx-i\,bc}{d}\right)+5\,e^{\left(-\frac{dm\log\left(\frac{3i\,b}{d}\right)-3i\,bc+3i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{3\,(-i\,bdx-i\,bc)}{d}\right)-3\,e^{\left(-\frac{dm\log\left(\frac{5i\,b}{d}\right)-5i\,bc+5i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{5\,(-i\,bdx-i\,bc)}{d}\right)}{480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/480*(30*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x +
I*b*c)/d) + 5*e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m + 1, -
3*(I*b*d*x + I*b*c)/d) - 3*e^(-(d*m*log(-5*I*b/d) + 5*I*b*c - 5*I*a*d)/d)*g
amma(m + 1, -5*(I*b*d*x + I*b*c)/d) + 30*e^(-(d*m*log(-I*b/d) + I*b*c - I*a
*d)/d)*gamma(m + 1, (-I*b*d*x - I*b*c)/d) + 5*e^(-(d*m*log(3*I*b/d) - 3*I*b
*c + 3*I*a*d)/d)*gamma(m + 1, -3*(-I*b*d*x - I*b*c)/d) - 3*e^(-(d*m*log(5*I
*b/d) - 5*I*b*c + 5*I*a*d)/d)*gamma(m + 1, -5*(-I*b*d*x - I*b*c)/d))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^m,x)
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^m, x)
```

3.89 $\int (c+dx)^4 \cos^2(a+bx) \sin^3(a+bx) \, dx$

Optimal. Leaf size=330

$$-\frac{3d^4 \cos(a+bx)}{b^5}+\frac{3d^2(c+dx)^2 \cos(a+bx)}{2b^3}-\frac{(c+dx)^4 \cos(a+bx)}{8b}-\frac{d^4 \cos(3a+3bx)}{162b^5}+\frac{d^2(c+dx)^2 \cos(3a+}{36b^3}$$

```
[Out] -3*d^4*cos(b*x+a)/b^5+3/2*d^2*(d*x+c)^2*cos(b*x+a)/b^3-1/8*(d*x+c)^4*cos(b*
x+a)/b-1/162*d^4*cos(3*b*x+3*a)/b^5+1/36*d^2*(d*x+c)^2*cos(3*b*x+3*a)/b^3-1
/48*(d*x+c)^4*cos(3*b*x+3*a)/b+3/6250*d^4*cos(5*b*x+5*a)/b^5-3/500*d^2*(d*x
+c)^2*cos(5*b*x+5*a)/b^3+1/80*(d*x+c)^4*cos(5*b*x+5*a)/b-3*d^3*(d*x+c)*sin(
b*x+a)/b^4+1/2*d*(d*x+c)^3*sin(b*x+a)/b^2-1/54*d^3*(d*x+c)*sin(3*b*x+3*a)/b
^4+1/36*d*(d*x+c)^3*sin(3*b*x+3*a)/b^2+3/1250*d^3*(d*x+c)*sin(5*b*x+5*a)/b^
4-1/100*d*(d*x+c)^3*sin(5*b*x+5*a)/b^2
```

Rubi [A]

time = 0.27, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4491, 3377, 2718}

$$-\frac{3d^4 \cos(a+bx)}{b^5}-\frac{d^4 \cos(3a+3bx)}{162b^5}+\frac{3d^4 \cos(5a+5bx)}{6250b^5}-\frac{3d^3(c+dx)\sin(a+bx)}{b^4}-\frac{d^3(c+dx)\sin(3a+3bx)}{54b^4}+\frac{3d^3(c+dx)\sin(5a+5bx)}{1250b^4}+\frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}+\frac{d^2(c+dx)^2\cos(3a+3bx)}{36b^3}-\frac{3d^2(c+dx)^2\cos(5a+5bx)}{500b^3}+\frac{d(c+dx)^3\sin(a+bx)}{2b^2}+\frac{d(c+dx)^3\sin(3a+3bx)}{36b^2}-\frac{d(c+dx)^3\sin(5a+5bx)}{100b^2}-\frac{(c+dx)^4\cos(a+bx)}{8b}-\frac{(c+dx)^4\cos(3a+3bx)}{48b}+\frac{(c+dx)^4\cos(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-3*d^4*Cos[a + b*x])/b^5 + (3*d^2*(c + d*x)^2*Cos[a + b*x])/(2*b^3) - ((c
 + d*x)^4*Cos[a + b*x])/(8*b) - (d^4*Cos[3*a + 3*b*x])/(162*b^5) + (d^2*(c +
 d*x)^2*Cos[3*a + 3*b*x])/(36*b^3) - ((c + d*x)^4*Cos[3*a + 3*b*x])/(48*b)
 + (3*d^4*Cos[5*a + 5*b*x])/(6250*b^5) - (3*d^2*(c + d*x)^2*Cos[5*a + 5*b*x]
)/(500*b^3) + ((c + d*x)^4*Cos[5*a + 5*b*x])/(80*b) - (3*d^3*(c + d*x)*Sin[
a + b*x])/b^4 + (d*(c + d*x)^3*Sin[a + b*x])/(2*b^2) - (d^3*(c + d*x)*Sin[3
*a + 3*b*x])/(54*b^4) + (d*(c + d*x)^3*Sin[3*a + 3*b*x])/(36*b^2) + (3*d^3*
(c + d*x)*Sin[5*a + 5*b*x])/(1250*b^4) - (d*(c + d*x)^3*Sin[5*a + 5*b*x])/(
100*b^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^4\cos^2(a+bx)\sin^3(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)^4\sin(a+bx)+\frac{1}{16}(c+dx)^4\sin(3a+3bx)-\frac{1}{16}(c\right.\\ &= \frac{1}{16}\int (c+dx)^4\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^4\sin(5a+5bx)\\ &= -\frac{(c+dx)^4\cos(a+bx)}{8b}-\frac{(c+dx)^4\cos(3a+3bx)}{48b}+\frac{(c+dx)^4\,c}{8}\\ &= -\frac{(c+dx)^4\cos(a+bx)}{8b}-\frac{(c+dx)^4\cos(3a+3bx)}{48b}+\frac{(c+dx)^4\,c}{8}\\ &= \frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}-\frac{(c+dx)^4\cos(a+bx)}{8b}+\frac{d^2(c+dx)^2}{3}\\ &= \frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}-\frac{(c+dx)^4\cos(a+bx)}{8b}+\frac{d^2(c+dx)^2}{3}\\ &= -\frac{3d^4\cos(a+bx)}{b^5}+\frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}-\frac{(c+dx)^4\cos(a}{8b}\end{aligned}$$

Mathematica [A]
time = 3.38, size = 238, normalized size = 0.72

$$\frac{-506250(24d^4-12b^2d^2(c+dx)^2+b^4(c+dx)^4)\cos(a+bx)-3125(8d^4-36b^2d^2(c+dx)^2+27b^4(c+dx)^4)\cos(3(a+bx))+81(24d^4-300b^2d^2(c+dx)^2+625b^4(c+dx)^4)\cos(5(a+bx))+120bd(c+dx)\left(17475b^2c^2-101794d^2+34950b^2cdx+17475b^2d^2x^2+16(-68d^2+75b^2(c+dx)^2)\cos(2(a+bx))-27(-6d^2+25b^2(c+dx)^2)\cos(4(a+bx))\right)\sin(a+bx)}{4050000b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-506250*(24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x] -
 3125*(8*d^4 - 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cos[3*(a + b*x)
] + 81*(24*d^4 - 300*b^2*d^2*(c + d*x)^2 + 625*b^4*(c + d*x)^4)*Cos[5*(a +
b*x)] + 120*b*d*(c + d*x)*(17475*b^2*c^2 - 101794*d^2 + 34950*b^2*c*d*x + 1
7475*b^2*d^2*x^2 + 16*(-68*d^2 + 75*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] - 27*
(-6*d^2 + 25*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[a + b*x])/(4050000*b^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1811 vs. $2(304) = 608$.
time = 0.39, size = 1812, normalized size = 5.49

method	result
risch	$-\frac{(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-12b^2d^4x^2-24b^2c\,d^3x-12b^2c^2d^2+24d^4)\cos(bx+a)}{8b^5}+\frac{d(b^2d^3x^3+3b^2$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/b^4*a^4*d^4*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)-4/b^3
*a^3*c*d^3*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)-4/b^4*a^3*d^4
*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a
)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a
)^5)+6/b^2*a^2*c^2*d^2*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)+1
2/b^3*a^2*c*d^3*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3
+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)
-1/25*sin(b*x+a)^5)+6/b^4*a^2*d^4*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+
a)+4/15*cos(b*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+2/135*
(2+sin(b*x+a)^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^
2)*cos(b*x+a)-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*sin(b*x
+a)^2)*cos(b*x+a))-4/b*a*c^3*d*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x
+a)^3)-12/b^2*a*c^2*d^2*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(
b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*co
s(b*x+a)-1/25*sin(b*x+a)^5)-12/b^3*a*c*d^3*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)
*cos(b*x+a)+4/15*cos(b*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)
^3+2/135*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*si
n(b*x+a)^2)*cos(b*x+a)-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/
3*sin(b*x+a)^2)*cos(b*x+a))-4/b^4*a*d^4*(-1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*co
s(b*x+a)+2/5*(b*x+a)^2*sin(b*x+a)-856/1125*sin(b*x+a)+4/5*(b*x+a)*cos(b*x+a
)+1/15*(b*x+a)^2*sin(b*x+a)^3+2/45*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+22/3
375*sin(b*x+a)^3+1/5*(b*x+a)^3*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+
a)-3/25*(b*x+a)^2*sin(b*x+a)^5-6/125*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+
a)^2)*cos(b*x+a)+6/625*sin(b*x+a)^5)+c^4*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/
15*cos(b*x+a)^3)+4/b*c^3*d*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*s
in(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)
*cos(b*x+a)-1/25*sin(b*x+a)^5)+6/b^2*c^2*d^2*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^
2)*cos(b*x+a)+4/15*cos(b*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+
a)^3+2/135*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*
sin(b*x+a)^2)*cos(b*x+a)-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+
4/3*sin(b*x+a)^2)*cos(b*x+a))+4/b^3*c*d^3*(-1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*
cos(b*x+a)+2/5*(b*x+a)^2*sin(b*x+a)-856/1125*sin(b*x+a)+4/5*(b*x+a)*cos(b*x
+a)+1/15*(b*x+a)^2*sin(b*x+a)^3+2/45*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+22
/3375*sin(b*x+a)^3+1/5*(b*x+a)^3*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*
x+a)-3/25*(b*x+a)^2*sin(b*x+a)^5-6/125*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*
```

```
x+a)^2)*cos(b*x+a)+6/625*sin(b*x+a)^5)+1/b^4*d^4*(-1/3*(b*x+a)^4*(2+sin(b*x
+a)^2)*cos(b*x+a)+8/15*(b*x+a)^3*sin(b*x+a)+8/5*(b*x+a)^2*cos(b*x+a)-3424/1
125*cos(b*x+a)-3424/1125*(b*x+a)*sin(b*x+a)+4/45*(b*x+a)^3*sin(b*x+a)^3+4/4
5*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+88/3375*(b*x+a)*sin(b*x+a)^3+88/101
25*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5*(b*x+a)^4*(8/3+sin(b*x+a)^4+4/3*sin(b*x+
a)^2)*cos(b*x+a)-4/25*(b*x+a)^3*sin(b*x+a)^5-12/125*(b*x+a)^2*(8/3+sin(b*x+
a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)+24/625*(b*x+a)*sin(b*x+a)^5+24/3125*(8/3+
sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1339 vs. $2(304) = 608$.
time = 0.32, size = 1339, normalized size = 4.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4050000*(270000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c^4 - 1080000*(3*co
s(b*x + a)^5 - 5*cos(b*x + a)^3)*a*c^3*d/b + 1620000*(3*cos(b*x + a)^5 - 5*
cos(b*x + a)^3)*a^2*c^2*d^2/b^2 - 1080000*(3*cos(b*x + a)^5 - 5*cos(b*x + a
)^3)*a^3*c*d^3/b^3 + 270000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*a^4*d^4/b
^4 + 4500*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) -
450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450
*sin(b*x + a))*c^3*d/b - 13500*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a
)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*s
in(3*b*x + 3*a) + 450*sin(b*x + a))*a*c^2*d^2/b^2 + 13500*(45*(b*x + a)*cos
(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a)
- 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a^2*c*d^3/b^
3 - 4500*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) - 4
50*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450*
sin(b*x + a))*a^3*d^4/b^4 + 450*(27*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) -
 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*cos(b*x
+ a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3*b*x + 3*a) + 13
500*(b*x + a)*sin(b*x + a))*c^2*d^2/b^2 - 900*(27*(25*(b*x + a)^2 - 2)*cos(
5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 6750*((b*x + a)^2
 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3*b
*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*a*c*d^3/b^3 + 450*(27*(25*(b*x +
a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 675
0*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*
x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*a^2*d^4/b^4 + 60*(1
35*(25*(b*x + a)^3 - 6*b*x - 6*a)*cos(5*b*x + 5*a) - 1875*(3*(b*x + a)^3 -
2*b*x - 2*a)*cos(3*b*x + 3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x +
 a) - 81*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*si
n(3*b*x + 3*a) + 101250*((b*x + a)^2 - 2)*sin(b*x + a))*c*d^3/b^3 - 60*(135
```

```
*(25*(b*x + a)^3 - 6*b*x - 6*a)*cos(5*b*x + 5*a) - 1875*(3*(b*x + a)^3 - 2*
b*x - 2*a)*cos(3*b*x + 3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a
) - 81*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*sin(
3*b*x + 3*a) + 101250*((b*x + a)^2 - 2)*sin(b*x + a))*a*d^4/b^4 + (81*(625*
(b*x + a)^4 - 300*(b*x + a)^2 + 24)*cos(5*b*x + 5*a) - 3125*(27*(b*x + a)^4
 - 36*(b*x + a)^2 + 8)*cos(3*b*x + 3*a) - 506250*((b*x + a)^4 - 12*(b*x + a
)^2 + 24)*cos(b*x + a) - 1620*(25*(b*x + a)^3 - 6*b*x - 6*a)*sin(5*b*x + 5*
a) + 37500*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) + 2025000*((b*x +
 a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^4/b^4)/b
```

Fricas [A]
time = 2.15, size = 471, normalized size = 1.43

$$\frac{81\,{\left(625\,b^{4} d^{4} x^{4} + 2500\,b^{4} c d^{3} x^{3} + 625\,b^{4} c^{4} - 300\,b^{2} c^{2} d^{2} + 24\,d^{4} + 150\,{\left(25\,b^{4} c^{2} d^{2} - 2\,b^{2} d^{4}\right)} x^{2} + 100\,{\left(25\,b^{4} c^{3} d - 6\,b^{2} c d^{3}\right)} x\right)} \cos\left(b x + a\right)^{5} - 5\,{\left(16875\,b^{4} d^{4} x^{4} + 67500\,b^{4} c d^{3} x^{3} + 16875\,b^{4} c^{4} - 11700\,b^{2} c^{2} d^{2} + 1736\,d^{4} + 450\,{\left(225\,b^{4} c^{2} d^{2} - 26\,b^{2} d^{4}\right)} x^{2} + 900\,{\left(75\,b^{4} c^{3} d - 26\,b^{2} c d^{3}\right)} x\right)} \cos\left(b x + a\right)^{3} + 120\,{\left(2925\,b^{2} d^{4} x^{2} + 5850\,b^{2} c d^{3} x + 2925\,b^{2} c^{2} d^{2} - 6284\,d^{4}\right)} \cos\left(b x + a\right) + 60\,{\left(1950\,b^{3} d^{4} x^{3} + 5850\,b^{3} c d^{3} x^{2} + 1950\,b^{3} c^{3} d - 12568\,b c d^{3} - 27\,{\left(25\,b^{3} d^{4} x^{3} + 75\,b^{3} c d^{3} x^{2} + 25\,b^{3} c^{3} d - 6\,b c d^{3} + 3\,{\left(25\,b^{3} c^{2} d^{2} - 2\,b d^{4}\right)} x\right)} \cos\left(b x + a\right)^{4} + {\left(975\,b^{3} d^{4} x^{3} + 2925\,b^{3} c d^{3} x^{2} + 975\,b^{3} c^{3} d - 434\,b c d^{3} + {\left(2925\,b^{3} c^{2} d^{2} - 434\,b d^{4}\right)} x\right)} \cos\left(b x + a\right)^{2} + 2\,{\left(2925\,b^{3} c^{2} d^{2} - 6284\,b d^{4}\right)} x\right)} \sin\left(b x + a\right)}{253125\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/253125*(81*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 625*b^4*c^4 - 300*b^2*
c^2*d^2 + 24*d^4 + 150*(25*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 100*(25*b^4*c^3*d
 - 6*b^2*c*d^3)*x)*cos(b*x + a)^5 - 5*(16875*b^4*d^4*x^4 + 67500*b^4*c*d^3*
x^3 + 16875*b^4*c^4 - 11700*b^2*c^2*d^2 + 1736*d^4 + 450*(225*b^4*c^2*d^2 -
 26*b^2*d^4)*x^2 + 900*(75*b^4*c^3*d - 26*b^2*c*d^3)*x)*cos(b*x + a)^3 + 12
0*(2925*b^2*d^4*x^2 + 5850*b^2*c*d^3*x + 2925*b^2*c^2*d^2 - 6284*d^4)*cos(b
*x + a) + 60*(1950*b^3*d^4*x^3 + 5850*b^3*c*d^3*x^2 + 1950*b^3*c^3*d - 1256
8*b*c*d^3 - 27*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 25*b^3*c^3*d - 6*b*c*d^
3 + 3*(25*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^4 + (975*b^3*d^4*x^3 + 292
5*b^3*c*d^3*x^2 + 975*b^3*c^3*d - 434*b*c*d^3 + (2925*b^3*c^2*d^2 - 434*b*d
^4)*x)*cos(b*x + a)^2 + 2*(2925*b^3*c^2*d^2 - 6284*b*d^4)*x)*sin(b*x + a))/
b^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(325) = 650$.
time = 1.48, size = 1098, normalized size = 3.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**4*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**4*cos(a + b*x
)**5/(15*b) - 4*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 8*c**3*d*x
*cos(a + b*x)**5/(15*b) - 2*c**2*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**3/
b - 4*c**2*d**2*x**2*cos(a + b*x)**5/(5*b) - 4*c*d**3*x**3*sin(a + b*x)**2*
cos(a + b*x)**3/(3*b) - 8*c*d**3*x**3*cos(a + b*x)**5/(15*b) - d**4*x**4*si
n(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d**4*x**4*cos(a + b*x)**5/(15*b) +
```

```
104*c**3*d*sin(a + b*x)**5/(225*b**2) + 52*c**3*d*sin(a + b*x)**3*cos(a + b
*x)**2/(45*b**2) + 8*c**3*d*sin(a + b*x)*cos(a + b*x)**4/(15*b**2) + 104*c*
*2*d**2*x*sin(a + b*x)**5/(75*b**2) + 52*c**2*d**2*x*sin(a + b*x)**3*cos(a
+ b*x)**2/(15*b**2) + 8*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**4/(5*b**2) +
 104*c*d**3*x**2*sin(a + b*x)**5/(75*b**2) + 52*c*d**3*x**2*sin(a + b*x)**3
*cos(a + b*x)**2/(15*b**2) + 8*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**4/(5*
b**2) + 104*d**4*x**3*sin(a + b*x)**5/(225*b**2) + 52*d**4*x**3*sin(a + b*x
)**3*cos(a + b*x)**2/(45*b**2) + 8*d**4*x**3*sin(a + b*x)*cos(a + b*x)**4/(
15*b**2) + 104*c**2*d**2*sin(a + b*x)**4*cos(a + b*x)/(75*b**3) + 676*c**2*
d**2*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 1712*c**2*d**2*cos(a + b*
x)**5/(1125*b**3) + 208*c*d**3*x*sin(a + b*x)**4*cos(a + b*x)/(75*b**3) + 1
352*c*d**3*x*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 3424*c*d**3*x*cos
(a + b*x)**5/(1125*b**3) + 104*d**4*x**2*sin(a + b*x)**4*cos(a + b*x)/(75*b
**3) + 676*d**4*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 1712*d**4
*x**2*cos(a + b*x)**5/(1125*b**3) - 50272*c*d**3*sin(a + b*x)**5/(16875*b**
4) - 20456*c*d**3*sin(a + b*x)**3*cos(a + b*x)**2/(3375*b**4) - 3424*c*d**3
*sin(a + b*x)*cos(a + b*x)**4/(1125*b**4) - 50272*d**4*x*sin(a + b*x)**5/(1
6875*b**4) - 20456*d**4*x*sin(a + b*x)**3*cos(a + b*x)**2/(3375*b**4) - 342
4*d**4*x*sin(a + b*x)*cos(a + b*x)**4/(1125*b**4) - 50272*d**4*sin(a + b*x)
**4*cos(a + b*x)/(16875*b**5) - 303368*d**4*sin(a + b*x)**2*cos(a + b*x)**3
/(50625*b**5) - 760816*d**4*cos(a + b*x)**5/(253125*b**5), Ne(b, 0)), ((c**
4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*
*3*cos(a)**2, True))
```

Giac [A]
time = 0.54, size = 531, normalized size = 1.61

$$\frac{\left(625\,b^4d^4x^4+2500\,b^4cd^3x^3+3750\,b^4c^2d^2x^2+2500\,b^4c^3dx+625\,b^4c^4-300\,b^2d^4x^2-600\,b^2cd^3x-300\,b^2c^2d^2+24\,d^4\right)\cos\left(5\,bx+5\,a\right)}{50000\,b^5}-\frac{\left(27\,b^4d^4x^4+108\,b^4cd^3x^3+162\,b^4c^2d^2x^2+108\,b^4c^3dx+27\,b^4c^4-36\,b^2d^4x^2-72\,b^2cd^3x-36\,b^2c^2d^2+8\,d^4\right)\cos\left(3\,bx+3\,a\right)}{1296\,b^5}-\frac{\left(b^4d^4x^4+4\,b^4cd^3x^3+6\,b^4c^2d^2x^2+4\,b^4c^3dx+b^4c^4-12\,b^2d^4x^2-24\,b^2cd^3x-12\,b^2c^2d^2+24\,d^4\right)\cos\left(bx+a\right)}{8\,b^5}-\frac{\left(25\,b^3d^4x^3+75\,b^3cd^3x^2+75\,b^3c^2d^2x+25\,b^3c^3d-6\,bd^4x-6\,bcd^3\right)\sin\left(5\,bx+5\,a\right)}{2500\,b^5}+\frac{\left(3\,b^3d^4x^3+9\,b^3cd^3x^2+9\,b^3c^2d^2x+3\,b^3c^3d-2\,bd^4x-2\,bcd^3\right)\sin\left(3\,bx+3\,a\right)}{108\,b^5}+\frac{\left(b^3d^4x^3+3\,b^3cd^3x^2+3\,b^3c^2d^2x+b^3c^3d-6\,bd^4x-6\,bcd^3\right)\sin\left(bx+a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/50000*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 3750*b^4*c^2*d^2*x^2 + 2500
*b^4*c^3*d*x + 625*b^4*c^4 - 300*b^2*d^4*x^2 - 600*b^2*c*d^3*x - 300*b^2*c^
2*d^2 + 24*d^4)*cos(5*b*x + 5*a)/b^5 - 1/1296*(27*b^4*d^4*x^4 + 108*b^4*c*d
^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^
2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8*d^4)*cos(3*b*x + 3*a)/b^5 - 1/8*(b^
4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 -
 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^2*c^2*d^2 + 24*d^4)*cos(b*x + a)/b^
5 - 1/2500*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 75*b^3*c^2*d^2*x + 25*b^3*c
^3*d - 6*b*d^4*x - 6*b*c*d^3)*sin(5*b*x + 5*a)/b^5 + 1/108*(3*b^3*d^4*x^3 +
 9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*s
in(3*b*x + 3*a)/b^5 + 1/2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x
+ b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

Mupad [B]

time = 4.61, size = 816, normalized size = 2.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^4,x)
```

```
[Out] -(3*d^4*cos(a + b*x) + (d^4*cos(3*a + 3*b*x))/162 - (3*d^4*cos(5*a + 5*b*x)
)/6250 + (b^4*c^4*cos(a + b*x))/8 + (b^4*c^4*cos(3*a + 3*b*x))/48 - (b^4*c^
4*cos(5*a + 5*b*x))/80 - (3*b^2*c^2*d^2*cos(a + b*x))/2 - (b^3*c^3*d*sin(3*
a + 3*b*x))/36 + (b^3*c^3*d*sin(5*a + 5*b*x))/100 - (3*b^2*d^4*x^2*cos(a +
b*x))/2 + (b^4*d^4*x^4*cos(a + b*x))/8 - (b^3*d^4*x^3*sin(a + b*x))/2 + 3*b
*c*d^3*sin(a + b*x) - (b^2*c^2*d^2*cos(3*a + 3*b*x))/36 + (3*b^2*c^2*d^2*co
s(5*a + 5*b*x))/500 + 3*b*d^4*x*sin(a + b*x) - (b^2*d^4*x^2*cos(3*a + 3*b*x
))/36 + (3*b^2*d^4*x^2*cos(5*a + 5*b*x))/500 + (b^4*d^4*x^4*cos(3*a + 3*b*x
))/48 - (b^4*d^4*x^4*cos(5*a + 5*b*x))/80 - (b^3*d^4*x^3*sin(3*a + 3*b*x))/
36 + (b^3*d^4*x^3*sin(5*a + 5*b*x))/100 + (b*c*d^3*sin(3*a + 3*b*x))/54 - (
3*b*c*d^3*sin(5*a + 5*b*x))/1250 - (b^3*c^3*d*sin(a + b*x))/2 + (b*d^4*x*si
n(3*a + 3*b*x))/54 - (3*b*d^4*x*sin(5*a + 5*b*x))/1250 - 3*b^2*c*d^3*x*cos(
a + b*x) + (b^4*c^3*d*x*cos(a + b*x))/2 + (b^4*c^2*d^2*x^2*cos(3*a + 3*b*x)
)/8 - (3*b^4*c^2*d^2*x^2*cos(5*a + 5*b*x))/40 - (b^2*c*d^3*x*cos(3*a + 3*b*
x))/18 + (b^4*c^3*d*x*cos(3*a + 3*b*x))/12 + (3*b^2*c*d^3*x*cos(5*a + 5*b*x
))/250 - (b^4*c^3*d*x*cos(5*a + 5*b*x))/20 + (b^4*c*d^3*x^3*cos(a + b*x))/2
 - (3*b^3*c^2*d^2*x*sin(a + b*x))/2 - (3*b^3*c*d^3*x^2*sin(a + b*x))/2 + (b
^4*c*d^3*x^3*cos(3*a + 3*b*x))/12 - (b^4*c*d^3*x^3*cos(5*a + 5*b*x))/20 + (
3*b^4*c^2*d^2*x^2*cos(a + b*x))/4 - (b^3*c^2*d^2*x*sin(3*a + 3*b*x))/12 - (
b^3*c*d^3*x^2*sin(3*a + 3*b*x))/12 + (3*b^3*c^2*d^2*x*sin(5*a + 5*b*x))/100
 + (3*b^3*c*d^3*x^2*sin(5*a + 5*b*x))/100)/b^5
```

3.90 $\int (c+dx)^3 \cos^2(a+bx) \sin^3(a+bx)\,dx$

Optimal. Leaf size=259

$$\frac{3d^2(c+dx)\cos(a+bx)}{4b^3}-\frac{(c+dx)^3\cos(a+bx)}{8b}+\frac{d^2(c+dx)\cos(3a+3bx)}{72b^3}-\frac{(c+dx)^3\cos(3a+3bx)}{48b}-\frac{3d^2(c}{}$$

```
[Out] 3/4*d^2*(d*x+c)*cos(b*x+a)/b^3-1/8*(d*x+c)^3*cos(b*x+a)/b+1/72*d^2*(d*x+c)*
cos(3*b*x+3*a)/b^3-1/48*(d*x+c)^3*cos(3*b*x+3*a)/b-3/1000*d^2*(d*x+c)*cos(5
*b*x+5*a)/b^3+1/80*(d*x+c)^3*cos(5*b*x+5*a)/b-3/4*d^3*sin(b*x+a)/b^4+3/8*d*
(d*x+c)^2*sin(b*x+a)/b^2-1/216*d^3*sin(3*b*x+3*a)/b^4+1/48*d*(d*x+c)^2*sin(
3*b*x+3*a)/b^2+3/5000*d^3*sin(5*b*x+5*a)/b^4-3/400*d*(d*x+c)^2*sin(5*b*x+5*
a)/b^2
```

Rubi [A]
time = 0.21, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4491, 3377, 2717}

$$-\frac{3d^3\sin(a+bx)}{4b^4}-\frac{d^3\sin(3a+3bx)}{216b^4}+\frac{3d^3\sin(5a+5bx)}{5000b^4}+\frac{3d^2(c+dx)\cos(a+bx)}{4b^3}+\frac{d^2(c+dx)\cos(3a+3bx)}{72b^3}-\frac{3d^2(c+dx)\cos(5a+5bx)}{1000b^3}+\frac{3d(c+dx)^2\sin(a+bx)}{8b^2}+\frac{d(c+dx)^2\sin(3a+3bx)}{48b^2}-\frac{3d(c+dx)^2\sin(5a+5bx)}{400b^2}-\frac{(c+dx)^3\cos(a+bx)}{8b}-\frac{(c+dx)^3\cos(3a+3bx)}{48b}+\frac{(c+dx)^3\cos(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (3*d^2*(c + d*x)*Cos[a + b*x])/(4*b^3) - ((c + d*x)^3*Cos[a + b*x])/(8*b) +
 (d^2*(c + d*x)*Cos[3*a + 3*b*x])/(72*b^3) - ((c + d*x)^3*Cos[3*a + 3*b*x])
/(48*b) - (3*d^2*(c + d*x)*Cos[5*a + 5*b*x])/(1000*b^3) + ((c + d*x)^3*Cos[
5*a + 5*b*x])/(80*b) - (3*d^3*Sin[a + b*x])/(4*b^4) + (3*d*(c + d*x)^2*Sin[
a + b*x])/(8*b^2) - (d^3*Sin[3*a + 3*b*x])/(216*b^4) + (d*(c + d*x)^2*Sin[3
*a + 3*b*x])/(48*b^2) + (3*d^3*Sin[5*a + 5*b*x])/(5000*b^4) - (3*d*(c + d*x
)^2*Sin[5*a + 5*b*x])/(400*b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^3\cos^2(a+bx)\sin^3(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)^3\sin(a+bx)+\frac{1}{16}(c+dx)^3\sin(3a+3bx)-\frac{1}{16}(c+\right.\\
&= \frac{1}{16}\int (c+dx)^3\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^3\sin(5a+5bx)\,d\\
&= -\frac{(c+dx)^3\cos(a+bx)}{8b}-\frac{(c+dx)^3\cos(3a+3bx)}{48b}+\frac{(c+dx)^3\operatorname{co}}{80}\\
&= -\frac{(c+dx)^3\cos(a+bx)}{8b}-\frac{(c+dx)^3\cos(3a+3bx)}{48b}+\frac{(c+dx)^3\operatorname{co}}{80}\\
&= \frac{3d^2(c+dx)\cos(a+bx)}{4b^3}-\frac{(c+dx)^3\cos(a+bx)}{8b}+\frac{d^2(c+dx)\cos}{72b}\\
&= \frac{3d^2(c+dx)\cos(a+bx)}{4b^3}-\frac{(c+dx)^3\cos(a+bx)}{8b}+\frac{d^2(c+dx)\cos}{72b}
\end{aligned}
$$

Mathematica [**A**]
time = 1.53, size = 369, normalized size = 1.42

$$\frac{-33750b(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\cos(a+bx)-1875b(c+dx)\left(-2d^2+3b^2(c+dx)^2\right)\cos(3(a+bx))+3375b^3c^3\cos(5(a+bx))-810bcd^2\cos(5(a+bx))+10125b^3c^2dx\cos(5(a+bx))-810bd^3x\cos(5(a+bx))+10125b^3cd^2x^2\cos(5(a+bx))+3375b^3d^3x^3\cos(5(a+bx))+101250b^2c^2d\sin(a+bx)-202500d^3\sin(a+bx)+202500b^2cd^2x\sin(a+bx)+101250b^2d^3x^2\sin(a+bx)+5625b^2c^2d\sin(3(a+bx))-1250d^3\sin(3(a+bx))+11250b^2cd^2x\sin(3(a+bx))+5625b^2d^3x^2\sin(3(a+bx))-2025b^2c^2d\sin(5(a+bx))+162d^3\sin(5(a+bx))-4050b^2cd^2x\sin(5(a+bx))-2025b^2d^3x^2\sin(5(a+bx))}{270000b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-33750*b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 1875*b*(c + d
*x)*(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 3375*b^3*c^3*Cos[5*(a +
 b*x)] - 810*b*c*d^2*Cos[5*(a + b*x)] + 10125*b^3*c^2*d*x*Cos[5*(a + b*x)]
- 810*b*d^3*x*Cos[5*(a + b*x)] + 10125*b^3*c*d^2*x^2*Cos[5*(a + b*x)] + 337
5*b^3*d^3*x^3*Cos[5*(a + b*x)] + 101250*b^2*c^2*d*Sin[a + b*x] - 202500*d^3
*Sin[a + b*x] + 202500*b^2*c*d^2*x*Sin[a + b*x] + 101250*b^2*d^3*x^2*Sin[a
+ b*x] + 5625*b^2*c^2*d*Sin[3*(a + b*x)] - 1250*d^3*Sin[3*(a + b*x)] + 1125
0*b^2*c*d^2*x*Sin[3*(a + b*x)] + 5625*b^2*d^3*x^2*Sin[3*(a + b*x)] - 2025*b
^2*c^2*d*Sin[5*(a + b*x)] + 162*d^3*Sin[5*(a + b*x)] - 4050*b^2*c*d^2*x*Sin
[5*(a + b*x)] - 2025*b^2*d^3*x^2*Sin[5*(a + b*x)])/(270000*b^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 991 vs. $2(235) = 470$.
time = 0.24, size = 992, normalized size = 3.83

method	result
risch	$-\frac{(b^2d^3x^3+3b^2c\,d^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6c\,d^2)\cos(bx+a)}{8b^3}+\frac{3d(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\sin(bx+a)}{8b^4}+\frac{(25}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b^3*a^3*d^3*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)+3/b^
2*a^2*c*d^2*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)+3/b^3*a^2*d^
3*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+
a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+
a)^5)-3/b*a*c^2*d*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)-6/b^2*
a*c*d^2*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*si
n(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*si
n(b*x+a)^5)-3/b^3*a*d^3*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/15*co
s(b*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+2/135*(2+sin(b*x
+a)^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x
+a)-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos
(b*x+a))+c^3*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)+3/b*c^2*d*(
-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+
1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^
5)+3/b^2*c*d^2*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/15*cos(b*x+a)+
4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+2/135*(2+sin(b*x+a)^2)*co
s(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-2/25*
(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a))+
1/b^3*d^3*(-1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*cos(b*x+a)+2/5*(b*x+a)^2*sin(b*x
+a)-856/1125*sin(b*x+a)+4/5*(b*x+a)*cos(b*x+a)+1/15*(b*x+a)^2*sin(b*x+a)^3+
2/45*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+22/3375*sin(b*x+a)^3+1/5*(b*x+a)^3
*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-3/25*(b*x+a)^2*sin(b*x+a)^5
-6/125*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)+6/625*sin(b*x
+a)^5))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(235) = 470$.
time = 0.30, size = 766, normalized size = 2.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/270000*(18000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c^3 - 54000*(3*cos(b*
x + a)^5 - 5*cos(b*x + a)^3)*a*c^2*d/b + 54000*(3*cos(b*x + a)^5 - 5*cos(b*
```

```
x + a)^3)*a^2*c*d^2/b^2 - 18000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*a^3*d
^3/b^3 + 225*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a)
 - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) +
450*sin(b*x + a))*c^2*d/b - 450*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x +
a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*
sin(3*b*x + 3*a) + 450*sin(b*x + a))*a*c*d^2/b^2 + 225*(45*(b*x + a)*cos(5*
b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9
*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a^2*d^3/b^3 + 1
5*(27*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3
*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x
 + 5*a) + 750*(b*x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*c*
d^2/b^2 - 15*(27*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2
 - 2)*cos(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a
)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b
*x + a))*a*d^3/b^3 + (135*(25*(b*x + a)^3 - 6*b*x - 6*a)*cos(5*b*x + 5*a) -
 1875*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) - 33750*((b*x + a)^3 -
 6*b*x - 6*a)*cos(b*x + a) - 81*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 625
*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 101250*((b*x + a)^2 - 2)*sin(b*x +
a))*d^3/b^3)/b
```

Fricas [A]
time = 1.72, size = 296, normalized size = 1.14

$$\frac{135\,(25\,b^3d^3x^3+75\,b^3cd^2x^2+25\,b^3c^3-6\,bcd^2+3\,(25\,b^3c^2d-2\,bd^3)x)\cos(bx+a)^5-75\,(75\,b^3d^3x^3+225\,b^3cd^2x^2+75\,b^3c^3-26\,bcd^2+(225\,b^3c^2d-26\,bd^3)x)\cos(bx+a)^3+11700\,(bd^3x+bcd^2)\cos(bx+a)+(5850\,b^2d^3x^2+11700\,b^2cd^2x+5850\,b^2c^2d-81\,(25\,b^2d^3x^2+50\,b^2cd^2x+25\,b^2c^2d-2\,d^3)\cos(bx+a)^4-12568\,d^3+(2925\,b^2d^3x^2+5850\,b^2cd^2x+2925\,b^2c^2d-434\,d^3)\cos(bx+a)^2)\sin(bx+a)}{16875\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16875*(135*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 25*b^3*c^3 - 6*b*c*d^2 +
3*(25*b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^5 - 75*(75*b^3*d^3*x^3 + 225*b^3
*c*d^2*x^2 + 75*b^3*c^3 - 26*b*c*d^2 + (225*b^3*c^2*d - 26*b*d^3)*x)*cos(b*
x + a)^3 + 11700*(b*d^3*x + b*c*d^2)*cos(b*x + a) + (5850*b^2*d^3*x^2 + 117
00*b^2*c*d^2*x + 5850*b^2*c^2*d - 81*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*
b^2*c^2*d - 2*d^3)*cos(b*x + a)^4 - 12568*d^3 + (2925*b^2*d^3*x^2 + 5850*b^
2*c*d^2*x + 2925*b^2*c^2*d - 434*d^3)*cos(b*x + a)^2)*sin(b*x + a))/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(255) = 510$.
time = 1.00, size = 690, normalized size = 2.66

$$\begin{cases} -\frac{c^3\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{2c^3\cos^5(a+bx)}{15b}-\frac{c^2dx\sin^2(a+bx)\cos^3(a+bx)}{b}-\frac{2c^2dx\cos^5(a+bx)}{5b}-\frac{cd^2x^2\sin^2(a+bx)\cos^3(a+bx)}{b}-\frac{2cd^2x^2\cos^5(a+bx)}{5b}-\frac{d^3x^3\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{2d^3x^3\cos^5(a+bx)}{15b}+\frac{26c^2d\sin^5(a+bx)}{75b^2}+\frac{13c^2d\sin^3(a+bx)\cos^2(a+bx)}{15b^2}+\frac{2c^2d\sin(a+bx)\cos^4(a+bx)}{5b^2}+\frac{52cd^2x\sin^5(a+bx)}{75b^2}+\frac{26cd^2x\sin^3(a+bx)\cos^2(a+bx)}{15b^2}+\frac{4cd^2x\sin(a+bx)\cos^4(a+bx)}{5b^2}+\frac{26d^3x^2\sin^5(a+bx)}{75b^2}+\frac{13d^3x^2\sin^3(a+bx)\cos^2(a+bx)}{15b^2}+\frac{2d^3x^2\sin(a+bx)\cos^4(a+bx)}{5b^2}+\frac{52cd^2\sin^4(a+bx)\cos(a+bx)}{75b^3}+\frac{338cd^2\sin^2(a+bx)\cos^3(a+bx)}{225b^3}+\frac{856cd^2\cos^5(a+bx)}{1125b^3}+\frac{52d^3x\sin^4(a+bx)\cos(a+bx)}{75b^3}+\frac{338d^3x\sin^2(a+bx)\cos^3(a+bx)}{225b^3}+\frac{856d^3x\cos^5(a+bx)}{1125b^3}-\frac{12568d^3\sin^5(a+bx)}{16875b^4}-\frac{5114d^3\sin^3(a+bx)\cos^2(a+bx)}{3375b^4}-\frac{856d^3\sin(a+bx)\cos^4(a+bx)}{1125b^4} & \text{for}\ b\neq 0 \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sin^3(a)\cos^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**3*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**3*cos(a + b*x
)**5/(15*b) - c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**3/b - 2*c**2*d*x*cos(a
 + b*x)**5/(5*b) - c*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**3/b - 2*c*d**2
*x**2*cos(a + b*x)**5/(5*b) - d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**3/(3*
b) - 2*d**3*x**3*cos(a + b*x)**5/(15*b) + 26*c**2*d*sin(a + b*x)**5/(75*b**
2) + 13*c**2*d*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 2*c**2*d*sin(a +
 b*x)*cos(a + b*x)**4/(5*b**2) + 52*c*d**2*x*sin(a + b*x)**5/(75*b**2) + 26
*c*d**2*x*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 4*c*d**2*x*sin(a + b*
x)*cos(a + b*x)**4/(5*b**2) + 26*d**3*x**2*sin(a + b*x)**5/(75*b**2) + 13*d
**3*x**2*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 2*d**3*x**2*sin(a + b*
x)*cos(a + b*x)**4/(5*b**2) + 52*c*d**2*sin(a + b*x)**4*cos(a + b*x)/(75*b*
*3) + 338*c*d**2*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 856*c*d**2*co
s(a + b*x)**5/(1125*b**3) + 52*d**3*x*sin(a + b*x)**4*cos(a + b*x)/(75*b**3
) + 338*d**3*x*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 856*d**3*x*cos(
a + b*x)**5/(1125*b**3) - 12568*d**3*sin(a + b*x)**5/(16875*b**4) - 5114*d*
*3*sin(a + b*x)**3*cos(a + b*x)**2/(3375*b**4) - 856*d**3*sin(a + b*x)*cos(
a + b*x)**4/(1125*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**
3 + d**3*x**4/4)*sin(a)**3*cos(a)**2, True))
```

Giac [A]
time = 0.50, size = 351, normalized size = 1.36

$$\frac{\left(25 b^{3} d^{3} x^{3}+75 b^{3} c d^{2} x^{2}+75 b^{3} c^{2} d x+25 b^{3} c^{3}-6 b d^{3} x-6 b c d^{2}\right) \cos \left(5 b x+5 a\right)}{2000 b^{4}}-\frac{\left(3 b^{3} d^{3} x^{3}+9 b^{3} c d^{2} x^{2}+9 b^{3} c^{2} d x+3 b^{3} c^{3}-2 b d^{3} x-2 b c d^{2}\right) \cos \left(3 b x+3 a\right)}{144 b^{4}}-\frac{\left(b^{3} d^{3} x^{3}+3 b^{3} c d^{2} x^{2}+3 b^{3} c^{2} d x+b^{3} c^{3}-6 b d^{3} x-6 b c d^{2}\right) \cos \left(b x+a\right)}{8 b^{4}}-\frac{3 \left(25 b^{2} d^{3} x^{2}+50 b^{2} c d^{2} x+25 b^{2} c^{2} d-2 d^{3}\right) \sin \left(5 b x+5 a\right)}{10000 b^{4}}+\frac{\left(9 b^{2} d^{3} x^{2}+18 b^{2} c d^{2} x+9 b^{2} c^{2} d-2 d^{3}\right) \sin \left(3 b x+3 a\right)}{432 b^{4}}+\frac{3 \left(b^{2} d^{3} x^{2}+2 b^{2} c d^{2} x+b^{2} c^{2} d-2 d^{3}\right) \sin \left(b x+a\right)}{8 b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2000*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 75*b^3*c^2*d*x + 25*b^3*c^3 - 6
*b*d^3*x - 6*b*c*d^2)*cos(5*b*x + 5*a)/b^4 - 1/144*(3*b^3*d^3*x^3 + 9*b^3*c
*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^3*x - 2*b*c*d^2)*cos(3*b*x + 3
*a)/b^4 - 1/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*
b*d^3*x - 6*b*c*d^2)*cos(b*x + a)/b^4 - 3/10000*(25*b^2*d^3*x^2 + 50*b^2*c*
d^2*x + 25*b^2*c^2*d - 2*d^3)*sin(5*b*x + 5*a)/b^4 + 1/432*(9*b^2*d^3*x^2 +
 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*sin(3*b*x + 3*a)/b^4 + 3/8*(b^2*d^3*
x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a)/b^4
```

Mupad [B]
time = 2.57, size = 516, normalized size = 1.99

$$-\frac{\frac{3 d^{3} \sin(a+b x)}{4}+\frac{d^{3} \sin(3 a+3 b x)}{216}-\frac{3 d^{3} \sin(5 a+5 b x)}{5000}+\frac{b^{3} c^{3} \cos(a+b x)}{8}+\frac{b^{3} c^{3} \cos(3 a+3 b x)}{48}-\frac{b^{3} c^{3} \cos(5 a+5 b x)}{80}-\frac{b^{2} c^{2} d \sin(3 a+3 b x)}{48}+\frac{3 b^{2} c^{2} d \sin(5 a+5 b x)}{400}+\frac{b^{3} d^{3} x^{3} \cos(a+b x)}{8}-\frac{3 b^{2} d^{3} x^{2} \sin(a+b x)}{8}-\frac{3 b c d^{2} \cos(a+b x)}{4}-\frac{3 b d^{3} x \cos(a+b x)}{4}+\frac{b^{3} d^{3} x^{3} \cos(3 a+3 b x)}{48}-\frac{b^{3} d^{3} x^{3} \cos(5 a+5 b x)}{80}-\frac{b^{2} d^{3} x^{2} \sin(3 a+3 b x)}{48}+\frac{3 b^{2} d^{3} x^{2} \sin(5 a+5 b x)}{400}-\frac{b c d^{2} \cos(3 a+3 b x)}{72}+\frac{3 b c d^{2} \cos(5 a+5 b x)}{1000}-\frac{3 b^{2} c^{2} d \sin(a+b x)}{8}-\frac{b d^{3} x \cos(3 a+3 b x)}{72}+\frac{3 b d^{3} x \cos(5 a+5 b x)}{1000}+\frac{3 b^{3} c^{2} d x \cos(a+b x)}{8}-\frac{3 b^{2} c d^{2} x \sin(a+b x)}{4}+\frac{b^{3} c^{2} d x \cos(3 a+3 b x)}{16}-\frac{3 b^{3} c^{2} d x \cos(5 a+5 b x)}{80}+\frac{3 b^{3} c d^{2} x^{2} \cos(a+b x)}{8}-\frac{b^{2} c d^{2} x \sin(3 a+3 b x)}{24}+\frac{3 b^{2} c d^{2} x \sin(5 a+5 b x)}{200}+\frac{b^{3} c d^{2} x^{2} \cos(3 a+3 b x)}{16}-\frac{3 b^{3} c d^{2} x^{2} \cos(5 a+5 b x)}{80}}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] -((3*d^3*sin(a + b*x))/4 + (d^3*sin(3*a + 3*b*x))/216 - (3*d^3*sin(5*a + 5*
b*x))/5000 + (b^3*c^3*cos(a + b*x))/8 + (b^3*c^3*cos(3*a + 3*b*x))/48 - (b^
```

```
3*c^3*cos(5*a + 5*b*x))/80 - (b^2*c^2*d*sin(3*a + 3*b*x))/48 + (3*b^2*c^2*d
*sin(5*a + 5*b*x))/400 + (b^3*d^3*x^3*cos(a + b*x))/8 - (3*b^2*d^3*x^2*sin(
a + b*x))/8 - (3*b*c*d^2*cos(a + b*x))/4 - (3*b*d^3*x*cos(a + b*x))/4 + (b^
3*d^3*x^3*cos(3*a + 3*b*x))/48 - (b^3*d^3*x^3*cos(5*a + 5*b*x))/80 - (b^2*d
^3*x^2*sin(3*a + 3*b*x))/48 + (3*b^2*d^3*x^2*sin(5*a + 5*b*x))/400 - (b*c*d
^2*cos(3*a + 3*b*x))/72 + (3*b*c*d^2*cos(5*a + 5*b*x))/1000 - (3*b^2*c^2*d*
sin(a + b*x))/8 - (b*d^3*x*cos(3*a + 3*b*x))/72 + (3*b*d^3*x*cos(5*a + 5*b*
x))/1000 + (3*b^3*c^2*d*x*cos(a + b*x))/8 - (3*b^2*c*d^2*x*sin(a + b*x))/4
+ (b^3*c^2*d*x*cos(3*a + 3*b*x))/16 - (3*b^3*c^2*d*x*cos(5*a + 5*b*x))/80 +
 (3*b^3*c*d^2*x^2*cos(a + b*x))/8 - (b^2*c*d^2*x*sin(3*a + 3*b*x))/24 + (3*
b^2*c*d^2*x*sin(5*a + 5*b*x))/200 + (b^3*c*d^2*x^2*cos(3*a + 3*b*x))/16 - (
3*b^3*c*d^2*x^2*cos(5*a + 5*b*x))/80)/b^4
```

3.91 $\int (c+dx)^2 \cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=184

$$\frac{d^2\cos(a+bx)}{4b^3}-\frac{(c+dx)^2\cos(a+bx)}{8b}+\frac{d^2\cos(3a+3bx)}{216b^3}-\frac{(c+dx)^2\cos(3a+3bx)}{48b}-\frac{d^2\cos(5a+5bx)}{1000b^3}+\frac{(c+$$

```
[Out] 1/4*d^2*cos(b*x+a)/b^3-1/8*(d*x+c)^2*cos(b*x+a)/b+1/216*d^2*cos(3*b*x+3*a)/
b^3-1/48*(d*x+c)^2*cos(3*b*x+3*a)/b-1/1000*d^2*cos(5*b*x+5*a)/b^3+1/80*(d*x
+c)^2*cos(5*b*x+5*a)/b+1/4*d*(d*x+c)*sin(b*x+a)/b^2+1/72*d*(d*x+c)*sin(3*b*
x+3*a)/b^2-1/200*d*(d*x+c)*sin(5*b*x+5*a)/b^2
```

Rubi [A]
time = 0.14, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4491, 3377, 2718}

$$\frac{d^2\cos(a+bx)}{4b^3}+\frac{d^2\cos(3a+3bx)}{216b^3}-\frac{d^2\cos(5a+5bx)}{1000b^3}+\frac{d(c+dx)\sin(a+bx)}{4b^2}+\frac{d(c+dx)\sin(3a+3bx)}{72b^2}-\frac{d(c+dx)\sin(5a+5bx)}{200b^2}-\frac{(c+dx)^2\cos(a+bx)}{8b}-\frac{(c+dx)^2\cos(3a+3bx)}{48b}+\frac{(c+dx)^2\cos(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (d^2*Cos[a + b*x])/(4*b^3) - ((c + d*x)^2*Cos[a + b*x])/(8*b) + (d^2*Cos[3*
a + 3*b*x])/(216*b^3) - ((c + d*x)^2*Cos[3*a + 3*b*x])/(48*b) - (d^2*Cos[5*
a + 5*b*x])/(1000*b^3) + ((c + d*x)^2*Cos[5*a + 5*b*x])/(80*b) + (d*(c + d*
x)*Sin[a + b*x])/(4*b^2) + (d*(c + d*x)*Sin[3*a + 3*b*x])/(72*b^2) - (d*(c
+ d*x)*Sin[5*a + 5*b*x])/(200*b^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^2\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^2\sin(a+bx)+\frac{1}{16}(c+dx)^2\sin(3a+3bx)-\frac{1}{16}(c+\right.$$
$$= \frac{1}{16}\int (c+dx)^2\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^2\sin(5a+5bx)\,d$$
$$= -\frac{(c+dx)^2\cos(a+bx)}{8b}-\frac{(c+dx)^2\cos(3a+3bx)}{48b}+\frac{(c+dx)^2\,\mathrm{co}}{80}$$
$$= -\frac{(c+dx)^2\cos(a+bx)}{8b}-\frac{(c+dx)^2\cos(3a+3bx)}{48b}+\frac{(c+dx)^2\,\mathrm{co}}{80}$$
$$= \frac{d^2\cos(a+bx)}{4b^3}-\frac{(c+dx)^2\cos(a+bx)}{8b}+\frac{d^2\cos(3a+3bx)}{216b^3}-\frac{(c+}{}$$

Mathematica [**A**]
time = 0.88, size = 127, normalized size = 0.69

$$\frac{-6750(-2d^2+b^2(c+dx)^2)\cos(a+bx)-125(-2d^2+9b^2(c+dx)^2)\cos(3(a+bx))+27(-2d^2+25b^2(c+dx)^2)\cos(5(a+bx))+30bd(c+dx)(450\sin(a+bx)+25\sin(3(a+bx))-9\sin(5(a+bx)))}{54000b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-6750*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 125*(-2*d^2 + 9*b^2*(c + d
*x)^2)*Cos[3*(a + b*x)] + 27*(-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*(a + b*x)]
 + 30*b*d*(c + d*x)*(450*Sin[a + b*x] + 25*Sin[3*(a + b*x)] - 9*Sin[5*(a +
b*x)]))/(54000*b^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(166) = 332$.
time = 0.18, size = 466, normalized size = 2.53 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/b^2*a^2*d^2*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)-2/b*a
*c*d*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)-2/b^2*a*d^2*(-1/3*(
b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b
*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5)+c^2
*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)+2/b*c*d*(-1/3*(b*x+a)*(
2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8
/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5)+1/b^2*d^2*(
-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/15*cos(b*x+a)+4/15*(b*x+a)*sin
(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+2/135*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5*(b*
x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-2/25*(b*x+a)*sin(b*x+
a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(166) = 332$.
time = 0.30, size = 375, normalized size = 2.04

$$\frac{3600\left(3\cos\left(bx+a\right)^{5}-5\cos\left(bx+a\right)^{3}\right)c^{2}-\frac{7200\left(3\cos(bx+a)^{5}-5\cos(bx+a)^{3}\right)acd}{b}+\frac{3600\left(3\cos(bx+a)^{5}-5\cos(bx+a)^{3}\right)a^{2}d^{2}}{b^{2}}+\frac{30\left(45\left(bx+a\right)\cos(5bx+5a)-75\left(bx+a\right)\cos(3bx+3a)-450\left(bx+a\right)\cos(bx+a)-9\sin(5bx+5a)+25\sin(3bx+3a)+450\sin(bx+a)\right)cd}{b}-\frac{30\left(45\left(bx+a\right)\cos(5bx+5a)-75\left(bx+a\right)\cos(3bx+3a)-450\left(bx+a\right)\cos(bx+a)-9\sin(5bx+5a)+25\sin(3bx+3a)+450\sin(bx+a)\right)ad^{2}}{b^{2}}+\frac{\left(27\left(25\left(bx+a\right)^{2}-2\right)\cos(5bx+5a)-125\left(9\left(bx+a\right)^{2}-2\right)\cos(3bx+3a)-6750\left(\left(bx+a\right)^{2}-2\right)\cos(bx+a)-270\left(bx+a\right)\sin(5bx+5a)+750\left(bx+a\right)\sin(3bx+3a)+13500\left(bx+a\right)\sin(bx+a)\right)d^{2}}{b^{2}}}{54000\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/54000*(3600*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c^2 - 7200*(3*cos(b*x +
 a)^5 - 5*cos(b*x + a)^3)*a*c*d/b + 3600*(3*cos(b*x + a)^5 - 5*cos(b*x + a)
^3)*a^2*d^2/b^2 + 30*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*
x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x +
 3*a) + 450*sin(b*x + a))*c*d/b - 30*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b
*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a)
+ 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a*d^2/b^2 + (27*(25*(b*x + a)^2 -
 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 6750*((b*
x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*x + a)
*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*d^2/b^2)/b
```

Fricas [A]
time = 2.45, size = 166, normalized size = 0.90

$$\frac{27\left(25\,b^{2}d^{2}x^{2}+50\,b^{2}cdx+25\,b^{2}c^{2}-2\,d^{2}\right)\cos\left(bx+a\right)^{5}-5\left(225\,b^{2}d^{2}x^{2}+450\,b^{2}cdx+225\,b^{2}c^{2}-26\,d^{2}\right)\cos\left(bx+a\right)^{3}+780\,d^{2}\cos\left(bx+a\right)-30\left(9\left(bd^{2}x+bcd\right)\cos\left(bx+a\right)^{4}-26\,bd^{2}x-26\,bcd-13\left(bd^{2}x+bcd\right)\cos\left(bx+a\right)^{2}\right)\sin\left(bx+a\right)}{3375\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3375*(27*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(b*x + a
)^5 - 5*(225*b^2*d^2*x^2 + 450*b^2*c*d*x + 225*b^2*c^2 - 26*d^2)*cos(b*x +
a)^3 + 780*d^2*cos(b*x + a) - 30*(9*(b*d^2*x + b*c*d)*cos(b*x + a)^4 - 26*b
*d^2*x - 26*b*c*d - 13*(b*d^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(172) = 344$.
time = 0.66, size = 382, normalized size = 2.08

$$\begin{cases} -\frac{c^{2}\sin^{2}(a+bx)\cos^{3}(a+bx)}{3b}-\frac{2c^{2}\cos^{5}(a+bx)}{15b}-\frac{2cdx\sin^{2}(a+bx)\cos^{3}(a+bx)}{3b}-\frac{4cdx\cos^{5}(a+bx)}{15b}-\frac{d^{2}x^{2}\sin^{2}(a+bx)\cos^{3}(a+bx)}{3b}-\frac{2d^{2}x^{2}\cos^{5}(a+bx)}{15b}+\frac{52cd\sin^{5}(a+bx)}{225b^{2}}+\frac{26cd\sin^{3}(a+bx)\cos^{2}(a+bx)}{45b^{2}}+\frac{4cd\sin(a+bx)\cos^{4}(a+bx)}{15b^{2}}+\frac{52d^{2}x\sin^{5}(a+bx)}{225b^{2}}+\frac{26d^{2}x\sin^{3}(a+bx)\cos^{2}(a+bx)}{45b^{2}}+\frac{4d^{2}x\sin(a+bx)\cos^{4}(a+bx)}{15b^{2}}+\frac{52d^{2}\sin^{4}(a+bx)\cos(a+bx)}{225b^{3}}+\frac{338d^{2}\sin^{2}(a+bx)\cos^{3}(a+bx)}{675b^{3}}+\frac{856d^{2}\cos^{5}(a+bx)}{3375b^{3}} & \text{for } b \neq 0 \\ \left(c^{2}x+cdx^{2}+\frac{d^{2}x^{3}}{3}\right)\sin^{3}(a)\cos^{2}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**2*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**2*cos(a + b*x
)**5/(15*b) - 2*c*d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 4*c*d*x*cos(a
 + b*x)**5/(15*b) - d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d**
```

```
2*x**2*cos(a + b*x)**5/(15*b) + 52*c*d*sin(a + b*x)**5/(225*b**2) + 26*c*d*
sin(a + b*x)**3*cos(a + b*x)**2/(45*b**2) + 4*c*d*sin(a + b*x)*cos(a + b*x)
**4/(15*b**2) + 52*d**2*x*sin(a + b*x)**5/(225*b**2) + 26*d**2*x*sin(a + b*
x)**3*cos(a + b*x)**2/(45*b**2) + 4*d**2*x*sin(a + b*x)*cos(a + b*x)**4/(15
*b**2) + 52*d**2*sin(a + b*x)**4*cos(a + b*x)/(225*b**3) + 338*d**2*sin(a +
 b*x)**2*cos(a + b*x)**3/(675*b**3) + 856*d**2*cos(a + b*x)**5/(3375*b**3),
 Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**3*cos(a)**2, True))
```

Giac [A]
time = 0.48, size = 209, normalized size = 1.14

$$\frac{(25\,b^2d^2x^2+50\,b^2cdx+25\,b^2c^2-2\,d^2)\cos(5\,bx+5\,a)}{2000\,b^3}-\frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2)\cos(3\,bx+3\,a)}{432\,b^3}-\frac{(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2)\cos(bx+a)}{8\,b^3}-\frac{(bd^2x+bcd)\sin(5\,bx+5\,a)}{200\,b^3}+\frac{(bd^2x+bcd)\sin(3\,bx+3\,a)}{72\,b^3}+\frac{(bd^2x+bcd)\sin(bx+a)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2000*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(5*b*x + 5*a
)/b^3 - 1/432*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(3*b*x
+ 3*a)/b^3 - 1/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)
/b^3 - 1/200*(b*d^2*x + b*c*d)*sin(5*b*x + 5*a)/b^3 + 1/72*(b*d^2*x + b*c*d
)*sin(3*b*x + 3*a)/b^3 + 1/4*(b*d^2*x + b*c*d)*sin(b*x + a)/b^3
```

Mupad [B]
time = 0.81, size = 249, normalized size = 1.35

$$\frac{780\,d^2\,\cos(a+b\,x)+130\,d^2\cos(a+b\,x)^3-54\,d^2\cos(a+b\,x)^5-1125\,b^2\,c^2\cos(a+b\,x)^3+675\,b^2\,c^2\cos(a+b\,x)^5+780\,b\,d^2\,x\,\sin(a+b\,x)-1125\,b^2\,d^2\,x^2\cos(a+b\,x)^3+675\,b^2\,d^2\,x^2\cos(a+b\,x)^5+780\,b\,c\,d\,\sin(a+b\,x)-2250\,b^2\,c\,d\,x\cos(a+b\,x)^3+1350\,b^2\,c\,d\,x\cos(a+b\,x)^5+390\,b\,d^2\,x\cos(a+b\,x)^2\,\sin(a+b\,x)-270\,b\,d^2\,x\cos(a+b\,x)^4\,\sin(a+b\,x)+390\,b\,c\,d\cos(a+b\,x)^2\,\sin(a+b\,x)-270\,b\,c\,d\cos(a+b\,x)^4\,\sin(a+b\,x)}{3375\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] (780*d^2*cos(a + b*x) + 130*d^2*cos(a + b*x)^3 - 54*d^2*cos(a + b*x)^5 - 11
25*b^2*c^2*cos(a + b*x)^3 + 675*b^2*c^2*cos(a + b*x)^5 + 780*b*d^2*x*sin(a
+ b*x) - 1125*b^2*d^2*x^2*cos(a + b*x)^3 + 675*b^2*d^2*x^2*cos(a + b*x)^5 +
 780*b*c*d*sin(a + b*x) - 2250*b^2*c*d*x*cos(a + b*x)^3 + 1350*b^2*c*d*x*co
s(a + b*x)^5 + 390*b*d^2*x*cos(a + b*x)^2*sin(a + b*x) - 270*b*d^2*x*cos(a
+ b*x)^4*sin(a + b*x) + 390*b*c*d*cos(a + b*x)^2*sin(a + b*x) - 270*b*c*d*c
os(a + b*x)^4*sin(a + b*x))/(3375*b^3)
```

3.92 $\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=109

$$-\frac{(c+dx)\cos(a+bx)}{8b}-\frac{(c+dx)\cos(3a+3bx)}{48b}+\frac{(c+dx)\cos(5a+5bx)}{80b}+\frac{d\sin(a+bx)}{8b^2}+\frac{d\sin(3a+3bx)}{144b^2}-$$

```
[Out] -1/8*(d*x+c)*cos(b*x+a)/b-1/48*(d*x+c)*cos(3*b*x+3*a)/b+1/80*(d*x+c)*cos(5*
b*x+5*a)/b+1/8*d*sin(b*x+a)/b^2+1/144*d*sin(3*b*x+3*a)/b^2-1/400*d*sin(5*b*
x+5*a)/b^2
```

Rubi [A]
time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4491, 3377, 2717}

$$\frac{d\sin(a+bx)}{8b^2}+\frac{d\sin(3a+3bx)}{144b^2}-\frac{d\sin(5a+5bx)}{400b^2}-\frac{(c+dx)\cos(a+bx)}{8b}-\frac{(c+dx)\cos(3a+3bx)}{48b}+\frac{(c+dx)\cos(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -1/8*((c + d*x)*Cos[a + b*x])/b - ((c + d*x)*Cos[3*a + 3*b*x])/(48*b) + ((c
 + d*x)*Cos[5*a + 5*b*x])/(80*b) + (d*Sin[a + b*x])/(8*b^2) + (d*Sin[3*a +
3*b*x])/(144*b^2) - (d*Sin[5*a + 5*b*x])/(400*b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)\sin(a+bx) + \frac{1}{16}(c+dx)\sin(3a+3bx) - \frac{1}{16}(c+d\right.$$
$$= \frac{1}{16}\int (c+dx)\sin(3a+3bx)\,dx - \frac{1}{16}\int (c+dx)\sin(5a+5bx)\,dx +$$
$$= -\frac{(c+dx)\cos(a+bx)}{8b} - \frac{(c+dx)\cos(3a+3bx)}{48b} + \frac{(c+dx)\cos(5a}{80b}$$
$$= -\frac{(c+dx)\cos(a+bx)}{8b} - \frac{(c+dx)\cos(3a+3bx)}{48b} + \frac{(c+dx)\cos(5a}{80b}$$

Mathematica [A]
time = 0.33, size = 94, normalized size = 0.86

$$\frac{-450b(c+dx)\cos(a+bx) - 75b(c+dx)\cos(3(a+bx)) + 45bc\cos(5(a+bx)) + 45bdx\cos(5(a+bx)) + 450d\sin(a+bx) + 25d\sin(3(a+bx)) - 9d\sin(5(a+bx))}{3600b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
[Out] (-450*b*(c + d*x)*Cos[a + b*x] - 75*b*(c + d*x)*Cos[3*(a + b*x)] + 45*b*c*C
os[5*(a + b*x)] + 45*b*d*x*Cos[5*(a + b*x)] + 450*d*Sin[a + b*x] + 25*d*Sin
[3*(a + b*x)] - 9*d*Sin[5*(a + b*x)])/(3600*b^2)
```

Maple [A]
time = 0.17, size = 163, normalized size = 1.50

method	result
risch	$-\frac{(dx+c)\cos(bx+a)}{8b} - \frac{(dx+c)\cos(3bx+3a)}{48b} + \frac{(dx+c)\cos(5bx+5a)}{80b} + \frac{d\sin(bx+a)}{8b^2} + \frac{d\sin(3bx+3a)}{144b^2} - \frac{d\sin(5bx+}{400b^2}$
derivativedivides	$\frac{-\frac{da\left(-\frac{\left(\sin^2(bx+a)\right)\left(\cos^3(bx+a)\right)}{5}-\frac{2\left(\cos^3(bx+a)\right)}{15}\right)}{b}+c\left(-\frac{\left(\sin^2(bx+a)\right)\left(\cos^3(bx+a)\right)}{5}-\frac{2\left(\cos^3(bx+a)\right)}{15}\right)+\frac{d\left(-\frac{(bx+a)\left(2+\sin^2\right.}{}\right.}{}}{b}$
default	$\frac{-\frac{da\left(-\frac{\left(\sin^2(bx+a)\right)\left(\cos^3(bx+a)\right)}{5}-\frac{2\left(\cos^3(bx+a)\right)}{15}\right)}{b}+c\left(-\frac{\left(\sin^2(bx+a)\right)\left(\cos^3(bx+a)\right)}{5}-\frac{2\left(\cos^3(bx+a)\right)}{15}\right)+\frac{d\left(-\frac{(bx+a)\left(2+\sin^2\right.}{}\right.}{}}{b}$
norman	$\frac{-\frac{4c}{15b}+\frac{4d\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{15b^2}+\frac{56d\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{45b^2}+\frac{152d\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{225b^2}+\frac{56d\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{45b^2}+\frac{4d\left(\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{15b^2}-\frac{2dx}{15b}-\frac{4c\left(\tan^6\left(\frac{b}{}\right.\right.}{b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b*d*a*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)+c*(-1/5*si
n(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)+1/b*d*(-1/3*(b*x+a)*(2+sin(b*x+a
)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+
a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5))
```

Maxima [A]
time = 0.27, size = 139, normalized size = 1.28

$$\frac{240\left(3\cos\left(bx+a\right)^{5}-5\cos\left(bx+a\right)^{3}\right)c-\frac{240\left(3\cos(bx+a)^{5}-5\cos(bx+a)^{3}\right)ad}{b}+\frac{(45\,(bx+a)\cos(5\,bx+5\,a)-75\,(bx+a)\cos(3\,bx+3\,a)-450\,(bx+a)\cos(bx+a)-9\,\sin(5\,bx+5\,a)+25\,\sin(3\,bx+3\,a)+450\,\sin(bx+a))d}{b}}{3600\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3600*(240*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c - 240*(3*cos(b*x + a)^5
 - 5*cos(b*x + a)^3)*a*d/b + (45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*
cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin
(3*b*x + 3*a) + 450*sin(b*x + a))*d/b)/b
```

Fricas [A]
time = 3.55, size = 76, normalized size = 0.70

$$\frac{45\left(bdx+bc\right)\cos\left(bx+a\right)^{5}-75\left(bdx+bc\right)\cos\left(bx+a\right)^{3}-\left(9\,d\cos\left(bx+a\right)^{4}-13\,d\cos\left(bx+a\right)^{2}-26\,d\right)\sin\left(bx+a\right)}{225\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/225*(45*(b*d*x + b*c)*cos(b*x + a)^5 - 75*(b*d*x + b*c)*cos(b*x + a)^3 -
(9*d*cos(b*x + a)^4 - 13*d*cos(b*x + a)^2 - 26*d)*sin(b*x + a))/b^2
```

Sympy [A]
time = 0.41, size = 163, normalized size = 1.50

$$\begin{cases}-\frac{c\sin^{2}\left(a+bx\right)\cos^{3}\left(a+bx\right)}{3b}-\frac{2c\cos^{5}\left(a+bx\right)}{15b}-\frac{dx\sin^{2}\left(a+bx\right)\cos^{3}\left(a+bx\right)}{3b}-\frac{2dx\cos^{5}\left(a+bx\right)}{15b}+\frac{26d\sin^{5}\left(a+bx\right)}{225b^{2}}+\frac{13d\sin^{3}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{45b^{2}}+\frac{2d\sin\left(a+bx\right)\cos^{4}\left(a+bx\right)}{15b^{2}} & \text{for}\ b\neq 0\\ \left(cx+\frac{dx^{2}}{2}\right)\sin^{3}\left(a\right)\cos^{2}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c*cos(a + b*x)**5/(
15*b) - d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d*x*cos(a + b*x)**5/(
15*b) + 26*d*sin(a + b*x)**5/(225*b**2) + 13*d*sin(a + b*x)**3*cos(a + b*x)
**2/(45*b**2) + 2*d*sin(a + b*x)*cos(a + b*x)**4/(15*b**2), Ne(b, 0)), ((c*
x + d*x**2/2)*sin(a)**3*cos(a)**2, True))
```

Giac [A]
time = 0.45, size = 106, normalized size = 0.97

$$\frac{(bdx+bc)\cos(5\,bx+5\,a)}{80\,b^2}-\frac{(bdx+bc)\cos(3\,bx+3\,a)}{48\,b^2}-\frac{(bdx+bc)\cos(bx+a)}{8\,b^2}-\frac{d\sin(5\,bx+5\,a)}{400\,b^2}+\frac{d\sin(3\,bx+3\,a)}{144\,b^2}+\frac{d\sin(bx+a)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/80*(b*d*x + b*c)*cos(5*b*x + 5*a)/b^2 - 1/48*(b*d*x + b*c)*cos(3*b*x + 3*
a)/b^2 - 1/8*(b*d*x + b*c)*cos(b*x + a)/b^2 - 1/400*d*sin(5*b*x + 5*a)/b^2
+ 1/144*d*sin(3*b*x + 3*a)/b^2 + 1/8*d*sin(b*x + a)/b^2
```

Mupad [B]
time = 1.24, size = 99, normalized size = 0.91

$$\frac{26\,d\,\sin\left(a+b\,x\right)-75\,b\,c\cos\left(a+b\,x\right)^3+45\,b\,c\cos\left(a+b\,x\right)^5+13\,d\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)-9\,d\cos\left(a+b\,x\right)^4\,\sin\left(a+b\,x\right)-75\,b\,d\,x\cos\left(a+b\,x\right)^3+45\,b\,d\,x\cos\left(a+b\,x\right)^5}{225\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x),x)
```

```
[Out] (26*d*sin(a + b*x) - 75*b*c*cos(a + b*x)^3 + 45*b*c*cos(a + b*x)^5 + 13*d*c
os(a + b*x)^2*sin(a + b*x) - 9*d*cos(a + b*x)^4*sin(a + b*x) - 75*b*d*x*cos
(a + b*x)^3 + 45*b*d*x*cos(a + b*x)^5)/(225*b^2)
```

3.93 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=185

$$-\frac{\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)\sin\left(5a-\frac{5bc}{d}\right)}{16d}+\frac{\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{16d}+\frac{\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(\right.}{8d}$$

```
[Out] 1/8*cos(a-b*c/d)*Si(b*c/d+b*x)/d+1/16*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d-
1/16*cos(5*a-5*b*c/d)*Si(5*b*c/d+5*b*x)/d-1/16*Ci(5*b*c/d+5*b*x)*sin(5*a-5*
b*c/d)/d+1/16*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d+1/8*Ci(b*c/d+b*x)*sin(a-
b*c/d)/d
```

Rubi [A]
time = 0.25, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4491, 3384, 3380, 3383}

$$-\frac{\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d}+\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{16d}-\frac{\cos\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] -1/16*(CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/d + (CosIntegra
l[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(16*d) + (CosIntegral[(b*c)/d +
b*x]*Sin[a - (b*c)/d])/(8*d) + (Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x]
)/(8*d) + (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d) - (C
os[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx &= \int \left(\frac{\sin(a+bx)}{8(c+dx)}+\frac{\sin(3a+3bx)}{16(c+dx)}-\frac{\sin(5a+5bx)}{16(c+dx)}\right)dx \\
&= \frac{1}{16}\int \frac{\sin(3a+3bx)}{c+dx}\,dx-\frac{1}{16}\int \frac{\sin(5a+5bx)}{c+dx}\,dx+\frac{1}{8}\int \frac{\sin(a+bx)}{c+dx}\,dx \\
&= -\left(\frac{1}{16}\cos\left(5a-\frac{5bc}{d}\right)\int \frac{\sin\left(\frac{5bc}{d}+5bx\right)}{c+dx}\,dx\right)+\frac{1}{16}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\text{si}
\\
&= -\frac{\text{Ci}\left(\frac{5bc}{d}+5bx\right)\sin\left(5a-\frac{5bc}{d}\right)}{16d}+\frac{\text{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{16d}+\frac{\text{Ci}\left(\frac{bc}{d}+b
\end{aligned}
$$

Mathematica [A]
time = 0.46, size = 154, normalized size = 0.83

$$
\frac{-\text{CosIntegral}\left(\frac{5b(c+dx)}{d}\right)\sin\left(5a-\frac{5bc}{d}\right)+\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)+2\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right)+2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)+\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right)-\cos\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5b(c+dx)}{d}\right)}{16d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] (-(CosIntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d]) + CosIntegral[(3*b*
(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + 2*CosIntegral[b*(c/d + x)]*Sin[a - (b*
c)/d] + 2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Cos[3*a - (3*b*c)/d]*
SinIntegral[(3*b*(c + d*x))/d] - Cos[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c +
 d*x))/d])/(16*d)
```

Maple [A]
time = 0.16, size = 258, normalized size = 1.39

method	result
derivativedivides	$\frac{-\frac{b\left(-\frac{5\,\text{sinIntegral}\left(-5bx-5a-\frac{5(-ad+cb)}{d}\right)\cos\left(\frac{-5ad+5cb}{d}\right)}{d}-\frac{5\,\text{cosineIntegral}\left(5bx+5a+\frac{-5ad+5cb}{d}\right)\sin\left(\frac{-5ad+5cb}{d}\right)}{d}\right)}{80}+\frac{b\left(-\frac{\text{sinIn}}{}\right.}{}}{}$

default	$-\frac{b\left(-\frac{5\,\text{sinIntegral}\left(-5bx-5a-\frac{5(-ad+cb)}{d}\right)\cos\left(\frac{-5ad+5cb}{d}\right)}{d}-\frac{5\,\text{cosineIntegral}\left(5bx+5a+\frac{-5ad+5cb}{d}\right)\sin\left(\frac{-5ad+5cb}{d}\right)}{d}\right)}{80}+\frac{b\left(-\frac{\sin\cdots}{\cdots}\right.}{\cdots}$
risch	$\frac{i e^{-\frac{5i(ad-cb)}{d}}\,\text{expIntegral}\left(1,5ibx+5ia-\frac{5i(ad-cb)}{d}\right)}{32d}-\frac{i e^{-\frac{3i(ad-cb)}{d}}\,\text{expIntegral}\left(1,3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{32d}-\frac{i e^{-\frac{i(ad-\cdots}{d}}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/80*b*(-5*Si(-5*b*x-5*a-5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d-5*Ci(5
*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)+1/8*b*(-Si(-b*x-a-(-a*d+b*c
)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)+1/48*b
*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(
-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.37, size = 414, normalized size = 2.24

$$-\frac{2\,b\left(i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b\left(i\,E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_1\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)-b\left(-i\,E_1\left(\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_1\left(-\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{5\,(bc-ad)}{d}\right)+2\,b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+b\left(E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)-b\left(E_1\left(\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{5\,(bc-ad)}{d}\right)}{32\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/32*(2*b*(I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_
integral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*
(I*exp_integral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral
_e(1, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b*(-I
*exp_integral_e(1, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integral_e
(1, -5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/d) + 2*b*(ex
p_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(1, -(I*
b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(exp_integral_e(1,
 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -3*(-I*b*c - I*(
b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b*(exp_integral_e(1, 5*(-I*
b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -5*(-I*b*c - I*(b*x + a
)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/(b*d)
```

Fricas [A]
time = 2.72, size = 228, normalized size = 1.23

$$\frac{2\left(\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+\left(\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)-\left(\mathrm{Ci}\left(\frac{5\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{5\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{5\,(bc-ad)}{d}\right)-2\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{5\,(bdx+bc)}{d}\right)+2\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right)+4\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/32*(2*(cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*si
n(-(b*c - a*d)/d) + (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d
*x + b*c)/d))*sin(-3*(b*c - a*d)/d) - (cos_integral(5*(b*d*x + b*c)/d) + co
s_integral(-5*(b*d*x + b*c)/d))*sin(-5*(b*c - a*d)/d) - 2*cos(-5*(b*c - a*d
)/d)*sin_integral(5*(b*d*x + b*c)/d) + 2*cos(-3*(b*c - a*d)/d)*sin_integral
(3*(b*d*x + b*c)/d) + 4*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/
d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.43, size = 46675, normalized size = 252.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/32*(imag_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*t
an(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(
cos_integral(3*b*x + 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5
/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(b*
x + b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2
*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(-b*x - b*c/d))*tan(5/
2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 + imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a
)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_
part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2
*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(5*(b*d
*x + b*c)/d)*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral(3*(b*d*x + b*c)/d)*tan(5/2*a)^
2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 - 4*sin_integral((b*d*x + b*c)/d)*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^
2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 4*real_part(cos_inte
gral(b*x + b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*real_part(cos_integral(-b*x - b*c/d))*t
an(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan
```

(1/2*b*c/d) - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*real_part(cos_integral(b*x + b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*real_part(cos_integral(-b*x - b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*a)*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*a)*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2 - imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2 + 2*imag_part(cos_integral(b*x + b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2 - 2*imag_part(cos_integral(-b*x - b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2 + imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2 - imag_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2 + 2*sin_integral(5*(b*d*x + b*c)/d)*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2 - 2*sin_integral(3*(b*d*x + b*c)/d)*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2 + 4*sin_integral((b*d*x + b*c)/d)*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2 - 8*imag_part(cos_integral(b*x + b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 8*imag_part(cos_integral(-b*x - b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 16*sin_integral((b*d*x + b*c)/d)*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + imag_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(b*x + b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(-b*x - b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*a)^2*tan(3

```
/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(5*(
b*d*x + b*c)/d)*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan
(1/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/...
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x),x)
[Out] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x), x)
```

3.94 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=257

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{5b\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegra}}{16d^2}$$

```
[Out] -5/16*b*Ci(5*b*c/d+5*b*x)*cos(5*a-5*b*c/d)/d^2+3/16*b*Ci(3*b*c/d+3*b*x)*cos
(3*a-3*b*c/d)/d^2+1/8*b*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^2+5/16*b*Si(5*b*c/d+5*
b*x)*sin(5*a-5*b*c/d)/d^2-3/16*b*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^2-1/8
*b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2-1/8*sin(b*x+a)/d/(d*x+c)-1/16*sin(3*b*x+3
*a)/d/(d*x+c)+1/16*sin(5*b*x+5*a)/d/(d*x+c)
```

Rubi [A]
time = 0.28, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{5b\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^2}-\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{16d^2}+\frac{5b\sin\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{16d^2}-\frac{\sin(a+bx)}{8d(c+dx)}-\frac{\sin(3a+3bx)}{16d(c+dx)}+\frac{\sin(5a+5bx)}{16d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d^2) + (3*b*Cos[3*a - (3
*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) - (5*b*Cos[5*a - (5*b*c)/
d]*CosIntegral[(5*b*c)/d + 5*b*x])/(16*d^2) - Sin[a + b*x]/(8*d*(c + d*x))
- Sin[3*a + 3*b*x]/(16*d*(c + d*x)) + Sin[5*a + 5*b*x]/(16*d*(c + d*x)) - (
b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^2) - (3*b*Sin[3*a - (3*
b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) + (5*b*Sin[5*a - (5*b*c)/d
]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{\sin(a+bx)}{8(c+dx)^2}+\frac{\sin(3a+3bx)}{16(c+dx)^2}-\frac{\sin(5a+5bx)}{16(c+dx)^2}\right)dx\\
&= \frac{1}{16}\int \frac{\sin(3a+3bx)}{(c+dx)^2}\,dx-\frac{1}{16}\int \frac{\sin(5a+5bx)}{(c+dx)^2}\,dx+\frac{1}{8}\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx\\
&= -\frac{\sin(a+bx)}{8d(c+dx)}-\frac{\sin(3a+3bx)}{16d(c+dx)}+\frac{\sin(5a+5bx)}{16d(c+dx)}+\frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{8d}+\frac{(3b)\int
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\sin(a+bx)}{8d(c+dx)}-\frac{\sin(3a+3bx)}{16d(c+dx)}+\frac{\sin(5a+5bx)}{16d(c+dx)}-\frac{\left(5b\cos\left(5a-\frac{5bc}{d}\right)\right)\int \frac{\cos}{}}{16d}\\
&= \frac{b\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{8d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{5b\cos\left(5a-\right.}{}
\end{aligned}$$

Mathematica [A]
time = 1.54, size = 213, normalized size = 0.83

$$\frac{2b\cos\left(a-\frac{bc}{d}\right)\mathrm{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+3b\cos\left(3a-\frac{3bc}{d}\right)\mathrm{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)-5b\cos\left(5a-\frac{5bc}{d}\right)\mathrm{CosIntegral}\left(\frac{5b(c+dx)}{d}\right)-\frac{2d\sin(a+bx)}{c+dx}-\frac{d\sin(3(a+bx))}{c+dx}+\frac{d\sin(5(a+bx))}{c+dx}-2b\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}+x\right)\right)-3b\sin\left(3a-\frac{3bc}{d}\right)\mathrm{Si}\left(\frac{3b(c+dx)}{d}\right)+5b\sin\left(5a-\frac{5bc}{d}\right)\mathrm{Si}\left(\frac{5b(c+dx)}{d}\right)}{16d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (2*b*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + 3*b*Cos[3*a - (3*b*c)/d]*C
osIntegral[(3*b*(c + d*x))/d] - 5*b*Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*(
c + d*x))/d] - (2*d*Sin[a + b*x])/(c + d*x) - (d*Sin[3*(a + b*x)])/(c + d*x
```

```
) + (d*Sin[5*(a + b*x)])/(c + d*x) - 2*b*Sin[a - (b*c)/d]*SinIntegral[b*(c/
d + x)] - 3*b*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] + 5*b*Sin
[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c + d*x))/d])/(16*d^2)
```

Maple [A]
time = 0.11, size = 370, normalized size = 1.44

method	result
derivativedivides	$-\frac{b^2\left(-\frac{5\sin(5bx+5a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{25\,\mathrm{sinIntegral}\left(-5bx-5a-\frac{5(-ad+cb)}{d}\right)\sin\left(\frac{-5ad+5cb}{d}\right)}{d}+\frac{25\,\mathrm{cosineIntegral}\left(5bx+5a+\frac{-5ad+5cb}{d}\right)}{d}}{d}\right.}{80}$
default	$-\frac{b^2\left(-\frac{5\sin(5bx+5a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{25\,\mathrm{sinIntegral}\left(-5bx-5a-\frac{5(-ad+cb)}{d}\right)\sin\left(\frac{-5ad+5cb}{d}\right)}{d}+\frac{25\,\mathrm{cosineIntegral}\left(5bx+5a+\frac{-5ad+5cb}{d}\right)}{d}}{d}\right.}{80}$
risch	$\frac{5b\,\mathrm{e}^{-\frac{5i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,5ibx+5ia-\frac{5i(ad-cb)}{d}\right)}{32d^2}-\frac{3b\,\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{32d^2}-\frac{b\,\mathrm{e}^{-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/80*b^2*(-5*sin(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))/d+5*(-5*Si(-5*b*x-5*
a-5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(
5*(-a*d+b*c)/d)/d)/d)+1/8*b^2*(-sin(b*x+a)/(-a*d+c*b+d*(b*x+a))/d+(-Si(-b*x
-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/
d)/d)/d)+1/48*b^2*(-3*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d+3*(-3*Si(-3*b*x
-3*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*c
os(3*(-a*d+b*c)/d)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.46, size = 445, normalized size = 1.73

$$-\frac{2b^2\left(iE_2\left(\frac{ibc+i(bx+a)d-iad}{d}\right)-iE_2\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^2\left(iE_2\left(\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)-iE_2\left(-\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{3(bc-ad)}{d}\right)-b^2\left(-iE_2\left(\frac{5(-ibc-i(bx+a)d+iad)}{d}\right)+iE_2\left(-\frac{5(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{5(bc-ad)}{d}\right)+2b^2\left(E_2\left(\frac{ibc+i(bx+a)d-iad}{d}\right)+E_2\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+b^2\left(E_2\left(\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)+E_2\left(-\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)\right)\sin\left(-\frac{3(bc-ad)}{d}\right)-b^2\left(E_2\left(\frac{5(-ibc-i(bx+a)d+iad)}{d}\right)+E_2\left(-\frac{5(-ibc-i(bx+a)d+iad)}{d}\right)\right)\sin\left(-\frac{5(bc-ad)}{d}\right)}{32\,(bcd+(bx+a)d^2-ad^2)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(2*b^2*(I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*ex
p_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) -
b^2*(I*exp_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_inte
gral_e(2, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b
^2*(-I*exp_integral_e(2, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_inte
gral_e(2, -5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/d) + 2
*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e
```

```
(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^2*(exp_int
egral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(2, -3*(-I
*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b^2*(exp_integral
_e(2, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(2, -5*(-I*b*c
- I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2
 - a*d^2)*b)
```

Fricas [**A**]
time = 2.93, size = 347, normalized size = 1.35

$$\frac{10\,(bdx+bc)\sin\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{5\,(bdx+bc)}{d}\right)-6\,(bdx+bc)\sin\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-4\,(bdx+bc)\sin\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)+2\left((bdx+bc)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+3\left((bdx+bc)\,\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)-5\left((bdx+bc)\,\mathrm{Ci}\left(\frac{5\,(bdx+bc)}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{5\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{5\,(bc-ad)}{d}\right)+32\left(d\cos(bx+a)^4-d\cos(bx+a)^2\right)\sin(bx+a)}{32\,(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(10*(b*d*x + b*c)*sin(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d
) - 6*(b*d*x + b*c)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) -
 4*(b*d*x + b*c)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 2*((b*
d*x + b*c)*cos_integral((b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-(b*d
*x + b*c)/d))*cos(-(b*c - a*d)/d) + 3*((b*d*x + b*c)*cos_integral(3*(b*d*x
+ b*c)/d) + (b*d*x + b*c)*cos_integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a
*d)/d) - 5*((b*d*x + b*c)*cos_integral(5*(b*d*x + b*c)/d) + (b*d*x + b*c)*c
os_integral(-5*(b*d*x + b*c)/d))*cos(-5*(b*c - a*d)/d) + 32*(d*cos(b*x + a)
^4 - d*cos(b*x + a)^2)*sin(b*x + a))/(d^3*x + c*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x)**2, x)
```

Giac [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 37.94, size = 1014406, normalized size = 3947.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/32*(5*b*d*x*real_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)^2*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*
```

c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b*d*x*real_part(cos_integral(3
*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*
tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 - 2*b*d*x*real_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b*d*x*real_part(cos_integral(-b*x - b
*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)
^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b*d*
x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*t
an(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 5*b*d*x*real_part(cos_integral(-5*b*x - 5*b*
c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^
2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*b*d*x
*imag_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c
/d)^2*tan(1/2*b*c/d) - 4*b*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(5/
2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)
^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 8*b*d*x*sin_integral(
(b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*
tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
+ 6*b*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b
*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^
2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 6*b*d*x*imag_part(cos_integral(-3*b*x -
3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/
2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 12*b
*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/
d)*tan(1/2*b*c/d)^2 - 10*b*d*x*imag_part(cos_integral(5*b*x + 5*b*c/d))*tan
(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2
*a)^2*tan(5/2*b*c/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 10*b*d*x*imag_part
(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d)^2 - 20*b*d*x*sin_integral(5*(b*d*x + b*c)/d)*tan(5/2*b*x)^2*t
an(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2
*b*c/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 4*b*d*x*imag_part(cos_integral(
b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan
(3/2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4
*b*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*
tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d)^2 - 8*b*d*x*sin_integral((b*d*x + b*c)/d)*tan(5/2
*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)*
tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b*d*x*imag_part(cos_
integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan
(5/2*a)^2*tan(3/2*a)*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d)^2 + 6*b*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*b*x)^

```
2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)*tan(1/2*a)^2*tan(5/
2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b*d*x*sin_integral(3*(b*d
*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(
3/2*a)*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 10
*b*d*x*imag_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)
^2*tan(1/2*b*x)^2*tan(5/2*a)*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 10*b*d*x*imag_part(cos_integral(-5*b*x - 5
*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)*tan(3/2*a)
^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 20*b*d
*x*sin_integral(5*(b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*
x)^2*tan(5/2*a)*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d)^2 + 5*b*c*real_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*
b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2
*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b*c*real_part(cos_i
ntegral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(
5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/
2*b*c/d)^2 - 2*b*c*real_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*
c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b*c*real_part(cos_integral(-b*
x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (a+b\,x)^2\,\sin (a+b\,x)^3}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^2,x)
[Out] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^2, x)
```

3.95 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=338

$$-\frac{b\cos(a+bx)}{16d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{32d^2(c+dx)}+\frac{5b\cos(5a+5bx)}{32d^2(c+dx)}+\frac{25b^2\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)\sin\left(5a-\frac{5bc}{d}\right)}{32d^3}-\frac{9b^2\text{CosI}$$

```
[Out] -1/16*b*cos(b*x+a)/d^2/(d*x+c)-3/32*b*cos(3*b*x+3*a)/d^2/(d*x+c)+5/32*b*cos
(5*b*x+5*a)/d^2/(d*x+c)-1/16*b^2*cos(a-b*c/d)*Si(b*c/d+b*x)/d^3-9/32*b^2*co
s(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^3+25/32*b^2*cos(5*a-5*b*c/d)*Si(5*b*c/d+
5*b*x)/d^3+25/32*b^2*Ci(5*b*c/d+5*b*x)*sin(5*a-5*b*c/d)/d^3-9/32*b^2*Ci(3*b
*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^3-1/16*b^2*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^3-1/
16*sin(b*x+a)/d/(d*x+c)^2-1/32*sin(3*b*x+3*a)/d/(d*x+c)^2+1/32*sin(5*b*x+5*
a)/d/(d*x+c)^2
```

Rubi [A]
time = 0.34, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$\frac{25b^2\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{32d^3}-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^3}-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{16d^3}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{16d^3}-\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^3}+\frac{25b^2\cos\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{32d^3}-\frac{b\cos(a+bx)}{16d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{32d^2(c+dx)}+\frac{5b\cos(5a+5bx)}{32d^2(c+dx)}-\frac{\sin(a+bx)}{16d(c+dx)^2}-\frac{\sin(3a+3bx)}{32d(c+dx)^2}+\frac{\sin(5a+5bx)}{32d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] -1/16*(b*Cos[a + b*x])/(d^2*(c + d*x)) - (3*b*Cos[3*a + 3*b*x])/(32*d^2*(c
+ d*x)) + (5*b*Cos[5*a + 5*b*x])/(32*d^2*(c + d*x)) + (25*b^2*CosIntegral[(
5*b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/(32*d^3) - (9*b^2*CosIntegral[(3*b*
c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(32*d^3) - (b^2*CosIntegral[(b*c)/d + b
*x]*Sin[a - (b*c)/d])/(16*d^3) - Sin[a + b*x]/(16*d*(c + d*x)^2) - Sin[3*a
+ 3*b*x]/(32*d*(c + d*x)^2) + Sin[5*a + 5*b*x]/(32*d*(c + d*x)^2) - (b^2*Co
s[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(16*d^3) - (9*b^2*Cos[3*a - (3*b
*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) + (25*b^2*Cos[5*a - (5*b*c)
/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(32*d^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx &= \int \left(\frac{\sin(a+bx)}{8(c+dx)^3}+\frac{\sin(3a+3bx)}{16(c+dx)^3}-\frac{\sin(5a+5bx)}{16(c+dx)^3}\right)dx \\
&= \frac{1}{16}\int \frac{\sin(3a+3bx)}{(c+dx)^3}\,dx-\frac{1}{16}\int \frac{\sin(5a+5bx)}{(c+dx)^3}\,dx+\frac{1}{8}\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx \\
&= -\frac{\sin(a+bx)}{16d(c+dx)^2}-\frac{\sin(3a+3bx)}{32d(c+dx)^2}+\frac{\sin(5a+5bx)}{32d(c+dx)^2}+\frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{16d}+\frac{(3b)}{} \\
&= -\frac{b\cos(a+bx)}{16d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{32d^2(c+dx)}+\frac{5b\cos(5a+5bx)}{32d^2(c+dx)}-\frac{\sin(a+bx)}{16d(c+dx)^2}- \\
&= -\frac{b\cos(a+bx)}{16d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{32d^2(c+dx)}+\frac{5b\cos(5a+5bx)}{32d^2(c+dx)}-\frac{\sin(a+bx)}{16d(c+dx)^2}- \\
&= -\frac{b\cos(a+bx)}{16d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{32d^2(c+dx)}+\frac{5b\cos(5a+5bx)}{32d^2(c+dx)}+\frac{25b^2\text{Ci}\left(\frac{5bc}{d}+5bx\right)}{32}
\end{aligned}
$$

Mathematica [A]
time = 4.14, size = 279, normalized size = 0.83

$$
\frac{25b^2\text{CosIntegral}\left(\frac{5b(c+dx)}{d}\right)\sin\left(5a-\frac{5bc}{d}\right)-9b^2\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)-\frac{d(3b(c+dx)\cos(3(a+bx))+d\sin(3(a+bx)))}{(c+dx)^2}+\frac{d(5b(c+dx)\cos(5(a+bx))+d\sin(5(a+bx)))}{(c+dx)^2}-2\left(b^2\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right)+\frac{d(b(c+dx)\cos(a+bx)+d\sin(a+bx))}{(c+dx)^2}+b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)\right)-9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right)+25b^2\cos\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5b(c+dx)}{d}\right)}{32d^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] (25*b^2*CosIntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] - 9*b^2*CosInte
gral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] - (d*(3*b*(c + d*x)*Cos[3*(a +
 b*x)] + d*Sin[3*(a + b*x)]))/(c + d*x)^2 + (d*(5*b*(c + d*x)*Cos[5*(a + b*
x)] + d*Sin[5*(a + b*x)]))/(c + d*x)^2 - 2*(b^2*CosIntegral[b*(c/d + x)]*Si
n[a - (b*c)/d] + (d*(b*(c + d*x)*Cos[a + b*x] + d*Sin[a + b*x]))/(c + d*x)^
2 + b^2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)]) - 9*b^2*Cos[3*a - (3*b*c
)/d]*SinIntegral[(3*b*(c + d*x))/d] + 25*b^2*Cos[5*a - (5*b*c)/d]*SinIntegr
al[(5*b*(c + d*x))/d])/(32*d^3)
```

Maple [**A**]
time = 0.16, size = 480, normalized size = 1.42 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/80*b^3*(-5/2*sin(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))^2/d+5/2*(-5*cos(5*
b*x+5*a)/(-a*d+c*b+d*(b*x+a))/d-5*(-5*Si(-5*b*x-5*a-5*(-a*d+b*c)/d)*cos(5*(
-a*d+b*c)/d)/d-5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)/d)/d)+
1/8*b^3*(-1/2*sin(b*x+a)/(-a*d+c*b+d*(b*x+a))^2/d+1/2*(-cos(b*x+a)/(-a*d+c*
b+d*(b*x+a))/d-(-Si(-b*x-a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d
+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)/d)+1/48*b^3*(-3/2*sin(3*b*x+3*a)/(-a*d+c*b
+d*(b*x+a))^2/d+3/2*(-3*cos(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d-3*(-3*Si(-3*b
*x-3*a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)
*sin(3*(-a*d+b*c)/d)/d)/d)/d))
```

Maxima [**C**] Result contains complex when optimal does not.
time = 0.56, size = 480, normalized size = 1.42

$$-\frac{2\,b^{3}\left(i\,E_{3}\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_{3}\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^{3}\left(i\,E_{3}\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_{3}\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)-b^{3}\left(-i\,E_{3}\left(\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_{3}\left(-\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{5\,(bc-ad)}{d}\right)+2\,b^{3}\left(E_{3}\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_{3}\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+b^{3}\left(E_{3}\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_{3}\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)-b^{3}\left(E_{3}\left(\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_{3}\left(-\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{5\,(bc-ad)}{d}\right)}{32\,\left(b^{2}c^{2}d-2\,abcd^{2}+(bx+a)^{2}d^{3}+a^{2}d^{3}+2\,(bcd^{2}-ad^{3})(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(2*b^3*(I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*ex
p_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) -
b^3*(I*exp_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_inte
gral_e(3, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b
^3*(-I*exp_integral_e(3, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_inte
gral_e(3, -5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/d) + 2
*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e
(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^3*(exp_int
egral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(3, -3*(-I
*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b^3*(exp_integral
_e(3, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(3, -5*(-I*b*c
```

```
- I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d
^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Fricas [A]
time = 2.64, size = 585, normalized size = 1.73

$$\frac{160\,(bd^2x+bcd)\cos(bx+a)^5-224\,(bd^2x+bcd)\cos(bx+a)^3+50\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{5\,(bdx+bc)}{d}\right)-18\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-4\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)+64\,(bd^2x+bcd)\cos(bx+a)+32\,(d^2\cos(bx+a)^4-d^2\cos(bx+a)^2)\sin(bx+a)-2\,\left((b^2d^2x^2+2\,b^2cdx+b^2c^2)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+(b^2d^2x^2+2\,b^2cdx+b^2c^2)\,\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)-9\left((b^2d^2x^2+2\,b^2cdx+b^2c^2)\,\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+(b^2d^2x^2+2\,b^2cdx+b^2c^2)\,\mathrm{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)+25\left((b^2d^2x^2+2\,b^2cdx+b^2c^2)\,\mathrm{Ci}\left(\frac{5\,(bdx+bc)}{d}\right)+(b^2d^2x^2+2\,b^2cdx+b^2c^2)\,\mathrm{Ci}\left(-\frac{5\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{5\,(bc-ad)}{d}\right)}{64\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(160*(b*d^2*x + b*c*d)*cos(b*x + a)^5 - 224*(b*d^2*x + b*c*d)*cos(b*x
+ a)^3 + 50*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-5*(b*c - a*d)/d)*sin
_integral(5*(b*d*x + b*c)/d) - 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos
(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 4*(b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 64*(b
*d^2*x + b*c*d)*cos(b*x + a) + 32*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)
*sin(b*x + a) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*
x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-(b*d*x +
b*c)/d))*sin(-(b*c - a*d)/d) - 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos
_integral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_in
tegral(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d) + 25*((b^2*d^2*x^2 + 2*b^
2*c*d*x + b^2*c^2)*cos_integral(5*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*cos_integral(-5*(b*d*x + b*c)/d))*sin(-5*(b*c - a*d)/d))/(d
^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos^2(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x)**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 78.21, size = 1737414, normalized size = 5140.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/64*(25*b^2*d^2*x^2*imag_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)^
2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(
```

5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*imag_part(co
s_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*t
an(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(5/2*
b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2
*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*x^2*imag_pa
rt(cos_integral(-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2
*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*t
an(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 25*b^2*d^2*x^2
*imag_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*ta
n(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d)^2 + 50*b^2*d^2*x^2*sin_integral(5*(b*d*x + b*c)/d
)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*
x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b
*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d
)^2*tan(1/2*b*c/d)^2 - 4*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(5/2*
b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2
*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 4*b^2*d^2*x^2*real_pa
rt(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*
tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*ta
n(1/2*b*c/d) - 4*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(5/2*
b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2
*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 18*b^2*d^2*x^2*real_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)*t
an(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*
tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(
1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 50*b^2*d^2*x^2*
real_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(
1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)*tan(3/2*b*
c/d)^2*tan(1/2*b*c/d)^2 + 50*b^2*d^2*x^2*real_part(cos_integral(-5*b*x - 5*
b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a
)^2*tan(1/2*a)^2*tan(5/2*b*c/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*b^2*d
^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*t
an(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(3/2
*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*
c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^
2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*d^
2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^
2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*real_part(cos_integral(-3*b*
x - 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan

```
(3/2*a)*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 5
0*b^2*d^2*x^2*real_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)^2*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d
)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 50*b^2*d^2*x^2*real_part(cos_integr
al(-5*b*x - 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*
a)*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + 50*b^2*c*d*x*imag_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)^2
*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5
/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*imag_part(cos_
integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan
(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1
/2*b*c/d)^2 - 4*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)
^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan
(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*b^2*c*d*x*imag_part(cos
_integral(-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5
/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d)^2 + 18*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*
b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2
*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 50*b^2*c*d*x*imag_par
t(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*b*x)^...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^3, x)
```

3.96 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=413

$$-\frac{b\cos(a+bx)}{48d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{32d^2(c+dx)^2}+\frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2}-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{48d^4}-\frac{9b^3\cos\left(3a-\frac{3}{}\right.}{}$$

```
[Out] 125/96*b^3*Ci(5*b*c/d+5*b*x)*cos(5*a-5*b*c/d)/d^4-9/32*b^3*Ci(3*b*c/d+3*b*x
)*cos(3*a-3*b*c/d)/d^4-1/48*b^3*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^4-1/48*b*cos(b
*x+a)/d^2/(d*x+c)^2-1/32*b*cos(3*b*x+3*a)/d^2/(d*x+c)^2+5/96*b*cos(5*b*x+5*
a)/d^2/(d*x+c)^2-125/96*b^3*Si(5*b*c/d+5*b*x)*sin(5*a-5*b*c/d)/d^4+9/32*b^3
*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^4+1/48*b^3*Si(b*c/d+b*x)*sin(a-b*c/d)
/d^4-1/24*sin(b*x+a)/d/(d*x+c)^3+1/48*b^2*sin(b*x+a)/d^3/(d*x+c)-1/48*sin(3
*b*x+3*a)/d/(d*x+c)^3+3/32*b^2*sin(3*b*x+3*a)/d^3/(d*x+c)+1/48*sin(5*b*x+5*
a)/d/(d*x+c)^3-25/96*b^2*sin(5*b*x+5*a)/d^3/(d*x+c)
```

Rubi [A]
time = 0.39, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{48d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^4}+\frac{125b^3\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{96d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{48d^4}+\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^4}-\frac{125b^3\sin\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{96d^4}+\frac{b^2\sin(a+bx)}{48d^3(c+dx)}+\frac{3b^2\sin(3a+3bx)}{32d^3(c+dx)}-\frac{25b^2\sin(5a+5bx)}{96d^3(c+dx)}-\frac{b\cos(a+bx)}{48d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{32d^2(c+dx)^2}+\frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2}-\frac{\sin(a+bx)}{24d(c+dx)^3}-\frac{\sin(3a+3bx)}{48d(c+dx)^3}+\frac{\sin(5a+5bx)}{48d(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] -1/48*(b*Cos[a + b*x])/(d^2*(c + d*x)^2) - (b*Cos[3*a + 3*b*x])/(32*d^2*(c
+ d*x)^2) + (5*b*Cos[5*a + 5*b*x])/(96*d^2*(c + d*x)^2) - (b^3*Cos[a - (b*c
)/d]*CosIntegral[(b*c)/d + b*x])/(48*d^4) - (9*b^3*Cos[3*a - (3*b*c)/d]*Cos
Integral[(3*b*c)/d + 3*b*x])/(32*d^4) + (125*b^3*Cos[5*a - (5*b*c)/d]*CosIn
tegral[(5*b*c)/d + 5*b*x])/(96*d^4) - Sin[a + b*x]/(24*d*(c + d*x)^3) + (b^
2*Sin[a + b*x])/(48*d^3*(c + d*x)) - Sin[3*a + 3*b*x]/(48*d*(c + d*x)^3) +
(3*b^2*Sin[3*a + 3*b*x])/(32*d^3*(c + d*x)) + Sin[5*a + 5*b*x]/(48*d*(c + d
*x)^3) - (25*b^2*Sin[5*a + 5*b*x])/(96*d^3*(c + d*x)) + (b^3*Sin[a - (b*c)/
d]*SinIntegral[(b*c)/d + b*x])/(48*d^4) + (9*b^3*Sin[3*a - (3*b*c)/d]*SinIn
tegral[(3*b*c)/d + 3*b*x])/(32*d^4) - (125*b^3*Sin[5*a - (5*b*c)/d]*SinInte
gral[(5*b*c)/d + 5*b*x])/(96*d^4)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx &= \int \left(\frac{\sin(a+bx)}{8(c+dx)^4}+\frac{\sin(3a+3bx)}{16(c+dx)^4}-\frac{\sin(5a+5bx)}{16(c+dx)^4}\right)dx \\
&= \frac{1}{16}\int \frac{\sin(3a+3bx)}{(c+dx)^4}\,dx-\frac{1}{16}\int \frac{\sin(5a+5bx)}{(c+dx)^4}\,dx+\frac{1}{8}\int \frac{\sin(a+bx)}{(c+dx)^4}\,dx \\
&= -\frac{\sin(a+bx)}{24d(c+dx)^3}-\frac{\sin(3a+3bx)}{48d(c+dx)^3}+\frac{\sin(5a+5bx)}{48d(c+dx)^3}+\frac{b\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx}{24d}+\frac{b\int \cdots}{\cdots} \\
&= -\frac{b\cos(a+bx)}{48d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{32d^2(c+dx)^2}+\frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2}-\frac{\sin(a+bx)}{24d(c+dx)^3}-\cdots \\
&= -\frac{b\cos(a+bx)}{48d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{32d^2(c+dx)^2}+\frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2}-\frac{\sin(a+bx)}{24d(c+dx)^3}+\cdots \\
&= -\frac{b\cos(a+bx)}{48d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{32d^2(c+dx)^2}+\frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2}-\frac{\sin(a+bx)}{24d(c+dx)^3}+\cdots \\
&= -\frac{b\cos(a+bx)}{48d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{32d^2(c+dx)^2}+\frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2}-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\mathrm{C}\cdots}{48d^4}
\end{aligned}$$

Mathematica [A]
time = 2.68, size = 457, normalized size = 1.11

$$\frac{-d\cos(3bx)(3bd(c+dx)\cos(3a)-(-2d^2+9b^2(c+dx)^2)\sin(3a))+d\cos(5bx)(5bd(c+dx)\cos(5a)-(-2d^2+25b^2(c+dx)^2)\sin(5a))+d((-2d^2+9b^2(c+dx)^2)\cos(3a)+3bd(c+dx)\sin(3a))\sin(3bx)-d((-2d^2+25b^2(c+dx)^2)\cos(5a)+5bd(c+dx)\sin(5a))\sin(5bx)-2(d\cos(bx)(bd(c+dx)\cos(a)-(-2d^2+b^2(c+dx)^2)\sin(a))-d((-2d^2+b^2(c+dx)^2)\cos(a)+bd(c+dx)\sin(a))\sin(bx)+b^3(c+dx)^3\left(\cos\left(a-\frac{bc}{d}\right)\mathrm{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)-\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}+x\right)\right)\right))-27b^3(c+dx)^3\left(\cos\left(3a-\frac{3bc}{d}\right)\mathrm{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)-\sin\left(3a-\frac{3bc}{d}\right)\mathrm{Si}\left(\frac{3b(c+dx)}{d}\right)\right)+125b^3(c+dx)^3\left(\cos\left(5a-\frac{5bc}{d}\right)\mathrm{CosIntegral}\left(\frac{5b(c+dx)}{d}\right)-\sin\left(5a-\frac{5bc}{d}\right)\mathrm{Si}\left(\frac{5b(c+dx)}{d}\right)\right)}{96d^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] (-(d*Cos[3*b*x]*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)*Si
n[3*a])) + d*Cos[5*b*x]*(5*b*d*(c + d*x)*Cos[5*a] - (-2*d^2 + 25*b^2*(c + d
*x)^2)*Sin[5*a]) + d*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*
x)*Sin[3*a])*Sin[3*b*x] - d*((-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*a] + 5*b*d
*(c + d*x)*Sin[5*a])*Sin[5*b*x] - 2*(d*Cos[b*x]*(b*d*(c + d*x)*Cos[a] - (-2
*d^2 + b^2*(c + d*x)^2)*Sin[a]) - d*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*
d*(c + d*x)*Sin[a])*Sin[b*x] + b^3*(c + d*x)^3*(Cos[a - (b*c)/d]*CosIntegra
l[b*(c/d + x)] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)])) - 27*b^3*(c +
d*x)^3*(Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] - Sin[3*a - (3*
b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]) + 125*b^3*(c + d*x)^3*(Cos[5*a - (5
*b*c)/d]*CosIntegral[(5*b*(c + d*x))/d] - Sin[5*a - (5*b*c)/d]*SinIntegral[
(5*b*(c + d*x))/d]))/(96*d^4*(c + d*x)^3)
```

Maple [A]
time = 0.21, size = 585, normalized size = 1.42 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/80*b^4*(-5/3*sin(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))^3/d+5/3*(-5/2*cos(
5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))^2/d-5/2*(-5*sin(5*b*x+5*a)/(-a*d+c*b+d*(b*x
+a))/d+5*(-5*Si(-5*b*x-5*a-5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*b*x
+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d)/d)/d)/d)+1/8*b^4*(-1/3*sin(b*x+
a)/(-a*d+c*b+d*(b*x+a))^3/d+1/3*(-1/2*cos(b*x+a)/(-a*d+c*b+d*(b*x+a))^2/d-1
/2*(-sin(b*x+a)/(-a*d+c*b+d*(b*x+a))/d+(-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+
b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)/d)+1/48*b^4*(-s
in(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))^3/d+(-3/2*cos(3*b*x+3*a)/(-a*d+c*b+d*(b*
x+a))^2/d-3/2*(-3*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d+3*(-3*Si(-3*b*x-3*a
-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3
*(-a*d+b*c)/d)/d)/d)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.78, size = 530, normalized size = 1.28

$$-\frac{2b^4\left(iE_4\left(\frac{ibc+i(bx+a)d-iad}{d}\right)-iE_4\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^4\left(iE_4\left(\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)-iE_4\left(-\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{3(bc-ad)}{d}\right)-b^4\left(-iE_4\left(\frac{5(-ibc-i(bx+a)d+iad)}{d}\right)+iE_4\left(-\frac{5(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{5(bc-ad)}{d}\right)+2b^4\left(E_4\left(\frac{ibc+i(bx+a)d-iad}{d}\right)+E_4\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+b^4\left(E_4\left(\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)+E_4\left(-\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)\right)\sin\left(-\frac{3(bc-ad)}{d}\right)-b^4\left(E_4\left(\frac{5(-ibc-i(bx+a)d+iad)}{d}\right)+E_4\left(-\frac{5(-ibc-i(bx+a)d+iad)}{d}\right)\right)\sin\left(-\frac{5(bc-ad)}{d}\right)}{32\left(b^3c^3d-3ab^2c^2d^2+3a^2bcd^3+(bx+a)^3d^4-a^3d^4+3(bcd^3-ad^4)(bx+a)^2+3(b^2c^2d^2-2abcd^3+a^2d^4)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/32*(2*b^4*(I*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*ex
p_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) -
b^4*(I*exp_integral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_inte
gral_e(4, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b
^4*(-I*exp_integral_e(4, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_inte
gral_e(4, -5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/d) + 2
*b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e
(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^4*(exp_int
egral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(4, -3*(-I
*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b^4*(exp_integral
_e(4, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(4, -5*(-I*b*c
- I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c
^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b
*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 824 vs. $2(383) = 766$.
time = 2.53, size = 824, normalized size = 2.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/192*(160*(b*d^3*x + b*c*d^2)*cos(b*x + a)^5 - 224*(b*d^3*x + b*c*d^2)*cos
(b*x + a)^3 - 250*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)
*sin(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) + 54*(b^3*d^3*x^3 +
3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-3*(b*c - a*d)/d)*sin_integr
al(3*(b*d*x + b*c)/d) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x +
b^3*c^3)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 64*(b*d^3*x +
b*c*d^2)*cos(b*x + a) - 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x +
 b^3*c^3)*cos_integral((b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d
) - 27*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integ
ral(3*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b
^3*c^3)*cos_integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d) + 125*((b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(5*(b*d*x
 + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_
integral(-5*(b*d*x + b*c)/d))*cos(-5*(b*c - a*d)/d) - 32*(2*b^2*d^3*x^2 + 4
*b^2*c*d^2*x + 2*b^2*c^2*d + (25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*
d - 2*d^3)*cos(b*x + a)^4 - (21*b^2*d^3*x^2 + 42*b^2*c*d^2*x + 21*b^2*c^2*d
 - 2*d^3)*cos(b*x + a)^2)*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*
x + c^3*d^4)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos^2(a+bx)}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x)**4, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 148.79, size = 2449286, normalized size = 5930.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/192*(125*b^3*d^3*x^3*real_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x
)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 27*b^3*d^3*x^3*real_part
(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^
2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*
tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))*tan(5
/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a
)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*real
_part(cos_integral(-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d)^2 - 27*b^3*d^3*x^3*real_part(cos_integral(-3*b*x - 3*b*c/d
))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*t
an(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 125*b^3*d^
3*x^3*real_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)
^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*b^3*d^3*x^3*imag_part(cos_integral(b*x
 + b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/
2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*b^
3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)
^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d) + 8*b^3*d^3*x^3*sin_integral((b*d*x + b*c)/d
)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 54*b^3*d^3*x^
3*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*ta
n(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/
2*b*c/d)*tan(1/2*b*c/d)^2 - 54*b^3*d^3*x^3*imag_part(cos_integral(-3*b*x -
3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2
```

```
*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 108*b
^3*d^3*x^3*sin_integral(3*(b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*ta
n(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/
2*b*c/d)*tan(1/2*b*c/d)^2 - 250*b^3*d^3*x^3*imag_part(cos_integral(5*b*x +
5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2
*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 250*b
^3*d^3*x^3*imag_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2
*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d
)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 500*b^3*d^3*x^3*sin_integral(5*(b*d*x
 + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/
2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 4*b^
3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^
2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*b^3*d^3*x^3*imag_part(cos_integral(-b*x -
 b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*
a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 8*b^3*
d^3*x^3*sin_integral((b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(3/2*b*c/d
)^2*tan(1/2*b*c/d)^2 - 54*b^3*d^3*x^3*imag_part(cos_integral(3*b*x + 3*b*c/
d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)*ta
n(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 54*b^3*d^3*
x^3*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2
*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 108*b^3*d^3*x^3*sin_integral(3*(b*d*x + b*c)
/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)*ta
n(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 250*b^3*d^3
*x^3*imag_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2
*tan(1/2*b*x)^2*tan(5/2*a)*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 250*b^3*d^3*x^3*imag_part(cos_integral(-5*b*
x - 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)*tan(3
/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 5
00*b^3*d^3*x^3*sin_integral(5*(b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^
2*tan(1/2*b*x)^2*tan(5/2*a)*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 375*b^3*c*d^2*x^2*real_part(cos_integral(5*
b*x + 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 81*b^3*c*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2
*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5
/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^3*c*d^2*x^2*real_part(c
os_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^4,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^4, x)
```

3.97 $\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=144

$$\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+\right.}{2b}$$

```
[Out] 1/2*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d
)^m)+1/2*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*x+c
)/d)^m)+Unintegrable((d*x+c)^m*csc(b*x+a),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(2*b*(
((-I)*b*(c + d*x))/d)^m) + ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(2
*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Defer[Int][(c + d*x)^m*Csc[
a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx &= \int (c+dx)^m \csc(a+bx)\,dx-\int (c+dx)^m \sin(a+bx)\,dx\\ &= -\left(\frac{1}{2}i\int e^{-i(a+bx)}(c+dx)^m\,dx\right)+\frac{1}{2}i\int e^{i(a+bx)}(c+dx)^m\,dx+\int (\\ &= \frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{e^{-i\left(a-\frac{bc}{d}\right)}}{}\end{aligned}$$

Mathematica [A]
time = 6.76, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x], x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int (dx+c)^{m} \cos (bx+a) \cot (bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x)
[Out] int((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{m} \cos (a+bx) \cot (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*cot(b*x+a),x)
[Out] Integral((c + d*x)**m*cos(a + b*x)*cot(a + b*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\cot(a+b\,x)\,\left(c+d\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^m,x)
[Out] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^m, x)
```

3.98 $\int (c+dx)^4 \cos(a+bx) \cot(a+bx)\, dx$

Optimal. Leaf size=333

$$-\frac{2(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{24d^4\cos(a+bx)}{b^5}-\frac{12d^2(c+dx)^2\cos(a+bx)}{b^3}+\frac{(c+dx)^4\cos(a+bx)}{b}+\frac{4id(c$$

```
[Out] -2*(d*x+c)^4*arctanh(exp(I*(b*x+a)))/b+24*d^4*cos(b*x+a)/b^5-12*d^2*(d*x+c)
^2*cos(b*x+a)/b^3+(d*x+c)^4*cos(b*x+a)/b+4*I*d*(d*x+c)^3*polylog(2,-exp(I*(
b*x+a)))/b^2-4*I*d*(d*x+c)^3*polylog(2,exp(I*(b*x+a)))/b^2-12*d^2*(d*x+c)^2
*polylog(3,-exp(I*(b*x+a)))/b^3+12*d^2*(d*x+c)^2*polylog(3,exp(I*(b*x+a)))/
b^3-24*I*d^3*(d*x+c)*polylog(4,-exp(I*(b*x+a)))/b^4+24*I*d^3*(d*x+c)*polylo
g(4,exp(I*(b*x+a)))/b^4+24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*polylo
g(5,exp(I*(b*x+a)))/b^5+24*d^3*(d*x+c)*sin(b*x+a)/b^4-4*d*(d*x+c)^3*sin(b*x
+a)/b^2
```

Rubi [A]
time = 0.20, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4493, 3377, 2718, 4268, 2611, 6744, 2320, 6724}

$$\frac{24d^4\text{Li}_5\left(-e^{i(a+bx)}\right)}{b^5}-\frac{24d^4\text{Li}_5\left(e^{i(a+bx)}\right)}{b^5}+\frac{24d^4\cos(a+bx)}{b^5}-\frac{24id^3(c+dx)\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}+\frac{24id^3(c+dx)\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}+\frac{24d^3(c+dx)\sin(a+bx)}{b^4}-\frac{12d^2(c+dx)^2\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{12d^2(c+dx)^2\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{12d^2(c+dx)^2\cos(a+bx)}{b^3}+\frac{4id(c+dx)^3\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{4id(c+dx)^3\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{4d(c+dx)^3\sin(a+bx)}{b^2}+\frac{(c+dx)^4\cos(a+bx)}{b}-\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^4*ArcTanh[E^(I*(a + b*x))])/b + (24*d^4*Cos[a + b*x])/b^5 - (
12*d^2*(c + d*x)^2*Cos[a + b*x])/b^3 + ((c + d*x)^4*Cos[a + b*x])/b + ((4*I
)*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((4*I)*d*(c + d*x)^3*Po
lyLog[2, E^(I*(a + b*x))])/b^2 - (12*d^2*(c + d*x)^2*PolyLog[3, -E^(I*(a +
b*x))])/b^3 + (12*d^2*(c + d*x)^2*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((24*I
)*d^3*(c + d*x)*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((24*I)*d^3*(c + d*x)*P
olyLog[4, E^(I*(a + b*x))])/b^4 + (24*d^4*PolyLog[5, -E^(I*(a + b*x))])/b^5
 - (24*d^4*PolyLog[5, E^(I*(a + b*x))])/b^5 + (24*d^3*(c + d*x)*Sin[a + b*x
])/b^4 - (4*d*(c + d*x)^3*Sin[a + b*x])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4\cos(a+bx)\cot(a+bx)\,dx=\int (c+dx)^4\csc(a+bx)\,dx-\int (c+dx)^4\sin(a+bx)\,dx$$
$$=-\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{(c+dx)^4\cos(a+bx)}{b}-\frac{(4d)\int(c$$
$$=-\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{(c+dx)^4\cos(a+bx)}{b}+\frac{4id(c+$$
$$=-\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{12d^2(c+dx)^2\cos(a+bx)}{b^3}+\frac{(c+$$
$$=-\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{12d^2(c+dx)^2\cos(a+bx)}{b^3}+\frac{(c+$$
$$=-\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{24d^4\cos(a+bx)}{b^5}-\frac{12d^2(c+dx)}{b}$$
$$=-\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{24d^4\cos(a+bx)}{b^5}-\frac{12d^2(c+dx)}{b}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 812 vs. $2(333) = 666$.
time = 1.64, size = 812, normalized size = 2.44

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*b^4*c^4*ArcTanh[E^(I*(a + b*x))] + b^4*c^4*Cos[a + b*x] - 12*b^2*c^2*d^
2*Cos[a + b*x] + 24*d^4*Cos[a + b*x] + 4*b^4*c^3*d*x*Cos[a + b*x] - 24*b^2*
c*d^3*x*Cos[a + b*x] + 6*b^4*c^2*d^2*x^2*Cos[a + b*x] - 12*b^2*d^4*x^2*Cos[
a + b*x] + 4*b^4*c*d^3*x^3*Cos[a + b*x] + b^4*d^4*x^4*Cos[a + b*x] + 4*b^4*
c^3*d*x*Log[1 - E^(I*(a + b*x))] + 6*b^4*c^2*d^2*x^2*Log[1 - E^(I*(a + b*x)
)] + 4*b^4*c*d^3*x^3*Log[1 - E^(I*(a + b*x))] + b^4*d^4*x^4*Log[1 - E^(I*(a
 + b*x))] - 4*b^4*c^3*d*x*Log[1 + E^(I*(a + b*x))] - 6*b^4*c^2*d^2*x^2*Log[
1 + E^(I*(a + b*x))] - 4*b^4*c*d^3*x^3*Log[1 + E^(I*(a + b*x))] - b^4*d^4*x
^4*Log[1 + E^(I*(a + b*x))] + (4*I)*b^3*d*(c + d*x)^3*PolyLog[2, -E^(I*(a +
 b*x))] - (4*I)*b^3*d*(c + d*x)^3*PolyLog[2, E^(I*(a + b*x))] - 12*b^2*c^2*
d^2*PolyLog[3, -E^(I*(a + b*x))] - 24*b^2*c*d^3*x*PolyLog[3, -E^(I*(a + b*x
))] - 12*b^2*d^4*x^2*PolyLog[3, -E^(I*(a + b*x))] + 12*b^2*c^2*d^2*PolyLog[
3, E^(I*(a + b*x))] + 24*b^2*c*d^3*x*PolyLog[3, E^(I*(a + b*x))] + 12*b^2*d
^4*x^2*PolyLog[3, E^(I*(a + b*x))] - (24*I)*b*c*d^3*PolyLog[4, -E^(I*(a + b
*x))] - (24*I)*b*d^4*x*PolyLog[4, -E^(I*(a + b*x))] + (24*I)*b*c*d^3*PolyLo
```

```
g[4, E^(I*(a + b*x))] + (24*I)*b*d^4*x*PolyLog[4, E^(I*(a + b*x))] + 24*d^4
*PolyLog[5, -E^(I*(a + b*x))] - 24*d^4*PolyLog[5, E^(I*(a + b*x))] - 4*b^3*
c^3*d*Sin[a + b*x] + 24*b*c*d^3*Sin[a + b*x] - 12*b^3*c^2*d^2*x*Sin[a + b*x
] + 24*b*d^4*x*Sin[a + b*x] - 12*b^3*c*d^3*x^2*Sin[a + b*x] - 4*b^3*d^4*x^3
*Sin[a + b*x])/b^5
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1294 vs. $2(311) = 622$.
time = 0.22, size = 1295, normalized size = 3.89

method	result	size
risch	Expression too large to display	1295

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3-4/b
*c*d^3*ln(exp(I*(b*x+a))+1)*x^3-12*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*
x-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2+12*I/b^2*c^2*d^2*polylog(2,-
exp(I*(b*x+a)))*x+12*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-4/b^4*c*d^3
*ln(exp(I*(b*x+a))+1)*a^3-4*I/b^2*d^4*polylog(2,exp(I*(b*x+a)))*x^3+24*I/b^
4*d^4*polylog(4,exp(I*(b*x+a)))*x-4*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))+2
4*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))+1/2*(d^4*x^4*b^4+4*b^4*c*d^3*x^3+6*
b^4*c^2*d^2*x^2+4*b^4*c^3*d*x+4*I*b^3*d^4*x^3+b^4*c^4-12*b^2*d^4*x^2+12*I*b
^3*c*d^3*x^2-24*b^2*c*d^3*x+12*I*b^3*c^2*d^2*x-12*b^2*c^2*d^2+4*I*b^3*c^3*d
-24*I*b*d^4*x+24*d^4-24*I*b*c*d^3)/b^5*exp(I*(b*x+a))+1/2*(d^4*x^4*b^4+4*b^
4*c*d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^4*c^3*d*x-4*I*b^3*d^4*x^3+b^4*c^4-12*b^2*
d^4*x^2-12*I*b^3*c*d^3*x^2-24*b^2*c*d^3*x-12*I*b^3*c^2*d^2*x-12*b^2*c^2*d^2
-4*I*b^3*c^3*d+24*I*b*d^4*x+24*d^4+24*I*b*c*d^3)/b^5*exp(-I*(b*x+a))-2/b*c^
4*arctanh(exp(I*(b*x+a)))+24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*poly
log(5,exp(I*(b*x+a)))/b^5+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))-12/b^3*c
^2*d^2*polylog(3,-exp(I*(b*x+a)))-1/b^5*d^4*a^4*ln(1-exp(I*(b*x+a)))-12/b^3
*d^4*polylog(3,-exp(I*(b*x+a)))*x^2+12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^
2+8/b^2*c^3*d*a*arctanh(exp(I*(b*x+a)))+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))+1)-
2/b^5*d^4*a^4*arctanh(exp(I*(b*x+a)))-4/b^2*c^3*d*ln(exp(I*(b*x+a))+1)*a+6/
b^3*c^2*d^2*ln(exp(I*(b*x+a))+1)*a^2+8/b^4*c*d^3*a^3*arctanh(exp(I*(b*x+a))
)-12/b^3*c^2*d^2*a^2*arctanh(exp(I*(b*x+a)))+4*I/b^2*d^4*polylog(2,-exp(I*(
b*x+a)))*x^3-24*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x-24*I/b^4*c*d^3*polyl
og(4,-exp(I*(b*x+a)))+4*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))+4/b*c^3*d*ln
(1-exp(I*(b*x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*(b*x+a)))*a-4/b*c^3*d*ln(exp(I*
(b*x+a))+1)*x+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4-1/b*d^4*ln(exp(I*(b*x+a))+1)
*x^4-6/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2+6/b*c^2*d^2*ln(1-exp(I*(b*x+a)))*
x^2-6/b^3*c^2*d^2*ln(1-exp(I*(b*x+a)))*a^2-24/b^3*c*d^3*polylog(3,-exp(I*(b
*x+a)))*x+24/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1548 vs. $2(305) = 610$.
time = 0.45, size = 1548, normalized size = 4.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^4*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 4*a*c^3*d*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + 6*a^2*c^2*d^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x +
a) - 1))/b^2 - 4*a^3*c*d^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(co
s(b*x + a) - 1))/b^3 + a^4*d^4*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + lo
g(cos(b*x + a) - 1))/b^4 + (48*d^4*polylog(5, -e^(I*b*x + I*a)) - 48*d^4*po
lylog(5, e^(I*b*x + I*a)) + 2*(-I*(b*x + a)^4*d^4 + 4*(-I*b*c*d^3 + I*a*d^4
)*(b*x + a)^3 + 6*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a)^2
+ 4*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b*c*d^3 + I*a^3*d^4)*(b*x +
 a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 2*(-I*(b*x + a)^4*d^4 + 4*(-
I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^
2*d^4)*(b*x + a)^2 + 4*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b*c*d^3
+ I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 2*((b*x
+ a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*(a^2 - 2)*d^4 + 4*(b*c*d^3
- a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x +
 a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*
d^4)*(b*x + a))*cos(b*x + a) + 8*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2
*b*c*d^3 + I*(b*x + a)^3*d^4 - I*a^3*d^4 + 3*(I*b*c*d^3 - I*a*d^4)*(b*x + a
)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a))*dilog(-e^(I*
b*x + I*a)) + 8*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b*c*d^3 - I*(b*
x + a)^3*d^4 + I*a^3*d^4 + 3*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^2 + 3*(-I*b^2
*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a))*dilog(e^(I*b*x + I*a)) - (
(b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*
c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d
^3 - a^3*d^4)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x +
a) + 1) + ((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d
^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 +
3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2
*cos(b*x + a) + 1) + 48*(-I*b*c*d^3 - I*(b*x + a)*d^4 + I*a*d^4)*polylog(4,
 -e^(I*b*x + I*a)) + 48*(I*b*c*d^3 + I*(b*x + a)*d^4 - I*a*d^4)*polylog(4,
e^(I*b*x + I*a)) - 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^
4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, -e^(I*b*x + I*a)) + 24*(b^2*c
^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x
 + a))*polylog(3, e^(I*b*x + I*a)) - 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x
+ a)^3*d^4 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b
```

```
*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*sin(b*
x + a))/b^4)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1367 vs. $2(305) = 610$.
time = 2.65, size = 1367, normalized size = 4.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(24*d^4*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*polylog(5,
cos(b*x + a) - I*sin(b*x + a)) - 24*d^4*polylog(5, -cos(b*x + a) + I*sin(b*
x + a)) - 24*d^4*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) - 2*(b^4*d^4*x^
4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 24*d^4 + 6*(b^4*c^2*d^2 -
2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a) + 4*(I*b^3*d^4
*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(cos(b*x +
 a) + I*sin(b*x + a)) + 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2
*d^2*x - I*b^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + 4*(I*b^3*d^4*x
^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(-cos(b*x +
a) + I*sin(b*x + a)) + 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*
d^2*x - I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b^4*d^4*x^4 +
 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x
 + a) + I*sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^
2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (
b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(
-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^4*c^4 - 4*a*b^3*c^3*d +
6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/2*cos(b*x + a) - 1/2*I*
sin(b*x + a) + 1/2) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 +
4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4
)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3
+ 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4
*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 24*(-I*b*
d^4*x - I*b*c*d^3)*polylog(4, cos(b*x + a) + I*sin(b*x + a)) + 24*(I*b*d^4*
x + I*b*c*d^3)*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 24*(-I*b*d^4*x -
 I*b*c*d^3)*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) + 24*(I*b*d^4*x + I*
b*c*d^3)*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b
^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 12*(b
^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, cos(b*x + a) - I*sin(b
*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b
*x + a) + I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*
polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 8*(b^3*d^4*x^3 + 3*b^3*c*d^3*x
^2 + b^3*c^3*d - 6*b*c*d^3 + 3*(b^3*c^2*d^2 - 2*b*d^4)*x)*sin(b*x + a))/b^5
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \cos(a+bx) \cot(a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*cot(b*x+a),x)
[Out] Integral((c + d*x)**4*cos(a + b*x)*cot(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^4*cos(b*x + a)*cot(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\;\cot(a+b\,x)\;(c+d\,x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^4,x)
[Out] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^4, x)
```

3.99 $\int (c+dx)^3 \cos(a+bx) \cot(a+bx)\, dx$

Optimal. Leaf size=254

$$-\frac{2(c+dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{6d^2(c+dx)\cos(a+bx)}{b^3}+\frac{(c+dx)^3\cos(a+bx)}{b}+\frac{3id(c+dx)^2\text{PolyLog}(2,-e}{b^2}$$

```
[Out] -2*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b-6*d^2*(d*x+c)*cos(b*x+a)/b^3+(d*x+c)
^3*cos(b*x+a)/b+3*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^2-3*I*d*(d*x+c
)^2*polylog(2,exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/
b^3+6*d^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-6*I*d^3*polylog(4,-exp(I*(b
*x+a)))/b^4+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+6*d^3*sin(b*x+a)/b^4-3*d*
(d*x+c)^2*sin(b*x+a)/b^2
```

Rubi [A]
time = 0.14, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4493, 3377, 2717, 4268, 2611, 6744, 2320, 6724}

$$-\frac{6id^3\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}+\frac{6id^3\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\sin(a+bx)}{b^4}-\frac{6d^2(c+dx)\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{6d^2(c+dx)\cos(a+bx)}{b^3}+\frac{3id(c+dx)^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{3d(c+dx)^2\sin(a+bx)}{b^2}+\frac{(c+dx)^3\cos(a+bx)}{b}-\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (6*d^2*(c + d*x)*Cos[a + b*x]
)/b^3 + ((c + d*x)^3*Cos[a + b*x])/b + ((3*I)*d*(c + d*x)^2*PolyLog[2, -E^(
I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^2
- (6*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^2*(c + d*x)*Pol
yLog[3, E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^
4 + ((6*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + (6*d^3*Sin[a + b*x])/b^4
- (3*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \cos(a+bx) \cot(a+bx)\, dx = \int (c+dx)^3 \csc(a+bx)\, dx - \int (c+dx)^3 \sin(a+bx)\, dx$$
$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^3 \cos(a+bx)}{b} - \frac{(3d)\int (c+$$
$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^3 \cos(a+bx)}{b} + \frac{3id(c+dx$$
$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx$$
$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx$$
$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 512 vs. 2(254) = 508.
time = 0.96, size = 512, normalized size = 2.02

$$\frac{-2b^3c^3\tanh^{-1}\left(e^{i(a+bx)}\right)+b^3c^3\cos(a+bx)-6bcd^2\cos(a+bx)+3b^3c^2dx\cos(a+bx)-6bd^3x\cos(a+bx)+3b^3cd^2x^2\cos(a+bx)+b^3d^3x^3\cos(a+bx)+3b^3c^2dx\log\left(1-e^{i(a+bx)}\right)+3b^3cd^2x^2\log\left(1-e^{i(a+bx)}\right)+b^3d^3x^3\log\left(1-e^{i(a+bx)}\right)-3b^3c^2dx\log\left(1+e^{i(a+bx)}\right)-3b^3cd^2x^2\log\left(1+e^{i(a+bx)}\right)-b^3d^3x^3\log\left(1+e^{i(a+bx)}\right)+3ib^2d(c+dx)^2\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)-3ib^2d(c+dx)^2\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)-6bcd^2\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)-6bd^3x\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)+6bcd^2\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)+6bd^3x\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)-6id^3\mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)+6id^3\mathrm{PolyLog}\left(4,e^{i(a+bx)}\right)-3b^2c^2d\sin(a+bx)+6d^3\sin(a+bx)-6b^2cd^2x\sin(a+bx)-3b^2d^3x^2\sin(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*b^3*c^3*ArcTanh[E^(I*(a + b*x))] + b^3*c^3*Cos[a + b*x] - 6*b*c*d^2*Cos
[a + b*x] + 3*b^3*c^2*d*x*Cos[a + b*x] - 6*b*d^3*x*Cos[a + b*x] + 3*b^3*c*d
^2*x^2*Cos[a + b*x] + b^3*d^3*x^3*Cos[a + b*x] + 3*b^3*c^2*d*x*Log[1 - E^(I
*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1
 - E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + E^(I*(a + b*x))] - 3*b^3*c*d^2*
x^2*Log[1 + E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + E^(I*(a + b*x))] + (3*I)
*b^2*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))] - (3*I)*b^2*d*(c + d*x)^2*P
olyLog[2, E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, -E^(I*(a + b*x))] - 6*b*d
^3*x*PolyLog[3, -E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog[3, E^(I*(a + b*x))] +
 6*b*d^3*x*PolyLog[3, E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, -E^(I*(a + b*
x))] + (6*I)*d^3*PolyLog[4, E^(I*(a + b*x))] - 3*b^2*c^2*d*Sin[a + b*x] + 6
*d^3*Sin[a + b*x] - 6*b^2*c*d^2*x*Sin[a + b*x] - 3*b^2*d^3*x^2*Sin[a + b*x]
)/b^4
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 846 vs. 2(236) = 472.
time = 0.16, size = 847, normalized size = 3.33

method	result
risch	$-\frac{6icd^2 \operatorname{polylog}(2,e^{i(bx+a)})x}{b^2}+\frac{6icd^2 \operatorname{polylog}(2,-e^{i(bx+a)})x}{b^2}-\frac{d^3\ln(e^{i(bx+a)}+1)a^3}{b^4}+\frac{2d^3a^3\operatorname{arctanh}(e^{i(bx+a)})}{b^4}+\frac{3cd^2a^2\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 6/b^2*c^2*d*a*arctanh(exp(I*(b*x+a)))-6/b^3*c*d^2*a^2*arctanh(exp(I*(b*x+a)
))+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))+1)-1/b^4*d^3*ln(exp(I*(b*x+a))+1)*a^3+
6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))-6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a))
)-6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,exp(I*(b*x+a))
)*x-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x+2/b^4*d^3*a^3*arctanh(exp(I*(
b*x+a)))+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+1/2*(d^3*x^3*b^3+3*b^3*c*d^2
*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^
2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))-1/b*d^3*ln(exp(I*(b*x+a))+1)*
x^3+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+1/2
*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^3
*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))-2/b
*c^3*arctanh(exp(I*(b*x+a)))-3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(
1-exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-3/b*c*d^2*ln(exp(I*(
b*x+a))+1)*x^2+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/b^3*c*d^2*ln(1-exp(I*(b
*x+a)))*a^2-3*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,e
xp(I*(b*x+a)))*x^2-3/b^2*c^2*d*ln(exp(I*(b*x+a))+1)*a+3*I/b^2*c^2*d*polylog
(2,-exp(I*(b*x+a)))+3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+6*I/b^2*c*d^
2*polylog(2,-exp(I*(b*x+a)))*x-6*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(230) = 460$.
time = 0.38, size = 929, normalized size = 3.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 3*a*c^2*d*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + 3*a^2*c*d^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a)
 - 1))/b^2 - a^3*d^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x
+ a) - 1))/b^3 - (12*I*d^3*polylog(4, -e^(I*b*x + I*a)) - 12*I*d^3*polylog(
4, e^(I*b*x + I*a)) - 2*(-I*(b*x + a)^3*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x
 + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a))*arctan2(s
in(b*x + a), cos(b*x + a) + 1) - 2*(-I*(b*x + a)^3*d^3 + 3*(-I*b*c*d^2 + I*
a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a)
```

```
)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*((b*x + a)^3*d^3 - 6*b*c*d^2
 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 +
 (a^2 - 2)*d^3)*(b*x + a))*cos(b*x + a) - 6*(I*b^2*c^2*d - 2*I*a*b*c*d^2 +
I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*dilog(-e
^(I*b*x + I*a)) - 6*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 - I*a
^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*dilog(e^(I*b*x + I*a)) + ((b*x
 + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2
+ a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a)
+ 1) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d -
2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*c
os(b*x + a) + 1) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, -e^(I*b*
x + I*a)) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, e^(I*b*x + I*a)
) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d
^2 - a*d^3)*(b*x + a))*sin(b*x + a))/b^3)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(230) = 460$.
time = 2.85, size = 925, normalized size = 3.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4,
 cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*polylog(4, -cos(b*x + a) + I*sin(
b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 2*(b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*c
os(b*x + a) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(cos(b
*x + a) + I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2
*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2
*x + I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2
 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) +
 I*sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3
*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3
*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) +
 (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a)
- 1/2*I*sin(b*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x
+ a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 6*(b*d
^3*x + b*c*d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*
c*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*po
lylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3,
```

```
 -cos(b*x + a) - I*sin(b*x + a)) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2
*d - 2*d^3)*sin(b*x + a))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cos(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*cot(b*x+a),x)
[Out] Integral((c + d*x)**3*cos(a + b*x)*cot(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^3*cos(b*x + a)*cot(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\cot(a+b\,x)\,{(c+d\,x)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^3,x)
[Out] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^3, x)
```

3.100 $\int (c+dx)^2 \cos(a+bx) \cot(a+bx) \, dx$

Optimal. Leaf size=171

$$-\frac{2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{2d^2 \cos(a+bx)}{b^3}+\frac{(c+dx)^2 \cos(a+bx)}{b}+\frac{2id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-$$

```
[Out] -2*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-2*d^2*cos(b*x+a)/b^3+(d*x+c)^2*cos(b
*x+a)/b+2*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylog(
2,exp(I*(b*x+a)))/b^2-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,
exp(I*(b*x+a)))/b^3-2*d*(d*x+c)*sin(b*x+a)/b^2
```

Rubi [A]
time = 0.10, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {4493, 3377, 2718, 4268, 2611, 2320, 6724}

$$-\frac{2d^2\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{2d^2\cos(a+bx)}{b^3}+\frac{2id(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{2d(c+dx)\sin(a+bx)}{b^2}+\frac{(c+dx)^2\cos(a+bx)}{b}-\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (2*d^2*Cos[a + b*x])/b^3 + ((
c + d*x)^2*Cos[a + b*x])/b + ((2*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))
])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (2*d^2*PolyL
og[3, -E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3 - (2
*d*(c + d*x)*Sin[a + b*x])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)^2 \csc(a+bx)\,dx - \int (c+dx)^2 \sin(a+bx)\,dx$$

$$= -\frac{2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^2\cos(a+bx)}{b} - \frac{(2d)\int(c$$

$$= -\frac{2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^2\cos(a+bx)}{b} + \frac{2id(c+$$

$$= -\frac{2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{(c+dx)^2\cos($$

$$= -\frac{2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{(c+dx)^2\cos($$

Mathematica [A]
time = 0.59, size = 288, normalized size = 1.68

$$\frac{-2b^2c^2\tanh^{-1}\left(e^{i(a+bx)}\right)+b^2c^2\cos(a+bx)-2d^2\cos(a+bx)+2b^2cdx\cos(a+bx)+b^2d^2x^2\cos(a+bx)+2b^2cdx\log\left(1-e^{i(a+bx)}\right)+b^2d^2x^2\log\left(1-e^{i(a+bx)}\right)-2b^2cdx\log\left(1+e^{i(a+bx)}\right)-b^2d^2x^2\log\left(1+e^{i(a+bx)}\right)+2ibd(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)-2ibd(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)-2d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)+2d^2\text{PolyLog}\left(3,e^{i(a+bx)}\right)-2bcd\sin(a+bx)-2bd^2x\sin(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*b^2*c^2*ArcTanh[E^(I*(a + b*x))] + b^2*c^2*Cos[a + b*x] - 2*d^2*Cos[a +
 b*x] + 2*b^2*c*d*x*Cos[a + b*x] + b^2*d^2*x^2*Cos[a + b*x] + 2*b^2*c*d*x*L
og[1 - E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] - 2*b^2*c*d*
x*Log[1 + E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + E^(I*(a + b*x))] + (2*I)*b
*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2,
E^(I*(a + b*x))] - 2*d^2*PolyLog[3, -E^(I*(a + b*x))] + 2*d^2*PolyLog[3, E^
(I*(a + b*x))] - 2*b*c*d*Sin[a + b*x] - 2*b*d^2*x*Sin[a + b*x])/b^3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(159) = 318.
time = 0.14, size = 479, normalized size = 2.80

method	result
risch	$\frac{\left(x^2d^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2\right)\mathrm{e}^{i(bx+a)}}{2b^3}+\frac{\left(x^2d^2b^2+2b^2cdx-2ib\,d^2x+b^2c^2-2ibcd-2d^2\right)\mathrm{e}^{-i(bx+a)}}{2b^3}-\frac{2c^2\operatorname{arctanh}\left(\mathrm{e}^{i(\right.}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(I
*(b*x+a))+1/2*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)
/b^3*exp(-I*(b*x+a))-2/b*c^2*arctanh(exp(I*(b*x+a)))+1/b*d^2*ln(1-exp(I*(b*
x+a)))*x^2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2-2*I/b^2*d^2*polylog(2,exp(I*(
b*x+a)))*x-1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+1/b^3*d^2*ln(exp(I*(b*x+a))+1)*
a^2+2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-2/b^3*d^2*a^2*arctanh(exp(I*(b*x
+a)))+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a-2/b*c
*d*ln(exp(I*(b*x+a))+1)*x-2/b^2*c*d*ln(exp(I*(b*x+a))+1)*a+2*d^2*polylog(3,
exp(I*(b*x+a)))/b^3-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+4/b^2*c*d*a*arctan
h(exp(I*(b*x+a)))-2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+2*I/b^2*d^2*polylog
(2,-exp(I*(b*x+a)))*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(155) = 310.
time = 0.34, size = 513, normalized size = 3.00

$$\frac{c^2\left(2\cos(bx+a)-\log(\cos(bx+a)+1)+\log(\cos(bx+a)-1)\right)-\frac{2acd\left(2\cos(bx+a)-\log(\cos(bx+a)+1)+\log(\cos(bx+a)-1)\right)}{b}+\frac{a^2d^2\left(2\cos(bx+a)-\log(\cos(bx+a)+1)+\log(\cos(bx+a)-1)\right)}{b^2}-\frac{4d^2\mathrm{Li}_3\left(-e^{(ibx+ia)}\right)-4d^2\mathrm{Li}_3\left(e^{(ibx+ia)}\right)-2\left(-i(bx+a)^2d^2+2(-ibcd+iad^2)(bx+a)\right)\arctan\left(\sin(bx+a),\cos(bx+a)+1\right)-2\left(-i(bx+a)^2d^2+2(-ibcd+iad^2)(bx+a)\right)\arctan\left(\sin(bx+a),-\cos(bx+a)+1\right)-2\left((bx+a)^2d^2+2(bcd-ad^2)(bx+a)-2d^2\right)\cos(bx+a)-4\left(ibcd+i(bx+a)d^2-iad^2\right)\mathrm{Li}_2\left(-e^{(ibx+ia)}\right)-4\left(-ibcd-i(bx+a)d^2+iad^2\right)\mathrm{Li}_2\left(e^{(ibx+ia)}\right)+\left((bx+a)^2d^2+2(bcd-ad^2)(bx+a)\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2+2\cos(bx+a)+1\right)-\left((bx+a)^2d^2+2(bcd-ad^2)(bx+a)\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2-2\cos(bx+a)+1\right)+4\left(bcd+(bx+a)d^2-ad^2\right)\sin(bx+a)}{b^2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 2*a*c*d*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
 + a^2*d^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b^2 - (4*d^2*polylog(3, -e^(I*b*x + I*a)) - 4*d^2*polylog(3, e^(I*b*x + I*
a)) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2(sin
(b*x + a), cos(b*x + a) + 1) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^
2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(b*x + a) - 4*(I*b*c*d + I*(b*x
+ a)*d^2 - I*a*d^2)*dilog(-e^(I*b*x + I*a)) - 4*(-I*b*c*d - I*(b*x + a)*d^2
 + I*a*d^2)*dilog(e^(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(
b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - ((b*x
 + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a
)^2 - 2*cos(b*x + a) + 1) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(b*x + a))
/b^2)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(155) = 310$.
time = 2.73, size = 562, normalized size = 3.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, cos
(b*x + a) - I*sin(b*x + a)) - 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x +
a)) - 2*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 2*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) - 2*(I*b*d^2*x + I*b*c*d)*dilog(
cos(b*x + a) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(cos(b*x + a
) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(-cos(b*x + a) + I*sin(b
*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) -
 (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*sin(b*x + a) +
1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) - I*sin(b*x + a
) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(
b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a) - 1
/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2
)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*
a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 4*(b*d^2*x + b
*c*d)*sin(b*x + a))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \cos (a + bx) \cot (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)*cot(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)*cot(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)\,\mathrm{cot}(a+b\,x)\,{\left(c+d\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^2,x)
```

```
[Out] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^2, x)
```

3.101 $\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=94

$$-\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{(c+dx)\cos(a+bx)}{b}+\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{d\,\text{si}}{}$$

```
[Out] -2*(d*x+c)*arctanh(exp(I*(b*x+a)))/b+(d*x+c)*cos(b*x+a)/b+I*d*polylog(2,-ex
p(I*(b*x+a)))/b^2-I*d*polylog(2,exp(I*(b*x+a)))/b^2-d*sin(b*x+a)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4493, 3377, 2717, 4268, 2317, 2438}

$$\frac{id\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{d\sin(a+bx)}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}-\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + ((c + d*x)*Cos[a + b*x])/b + (I
*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/b^
2 - (d*Sin[a + b*x])/b^2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)\csc(a+bx)\,dx - \int (c+dx)\sin(a+bx)\,dx$$

$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)\cos(a+bx)}{b} - \frac{d\int \cos(a+b}{b}$$

$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)\cos(a+bx)}{b} - \frac{d\sin(a+bx)}{b^2}$$

$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)\cos(a+bx)}{b} + \frac{id\mathrm{Li}_2\left(-e^{i(a+}\right.}{b^2}$$

Mathematica [A]
time = 0.22, size = 176, normalized size = 1.87

$$\frac{c\cos(a+bx)}{b} - \frac{c\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{c\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{d\left((a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)-a\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)+i\left(\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)-\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)\right)\right)}{b^2} + \frac{d\cos(bx)(bx\cos(a)-\sin(a))}{b^2} - \frac{d(\cos(a)+bx\sin(a))\sin(bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (c*Cos[a + b*x])/b - (c*Log[Cos[(a + b*x)/2]])/b + (c*Log[Sin[(a + b*x)/2]]
)/b + (d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) -
 a*Log[Tan[(a + b*x)/2]] + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(
I*(a + b*x))])))/b^2 + (d*Cos[b*x]*(b*x*Cos[a] - Sin[a]))/b^2 - (d*(Cos[a]
+ b*x*Sin[a])*Sin[b*x])/b^2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(86) = 172.
time = 0.10, size = 177, normalized size = 1.88

method	result
default	$\frac{-\frac{da\cos(bx+a)}{b}+c\cos(bx+a)-\frac{d(\sin(bx+a)-(bx+a)\cos(bx+a))}{b}}{b}+\frac{-\frac{da\ln(\csc(bx+a)-\cot(bx+a))}{b}+c\ln(\csc(bx+a)-\cot(bx+a))+\frac{d\left((bx+a)\cdots\right.}{}}{}$
risch	$\frac{(dxb+cb+id)\mathrm{e}^{i(bx+a)}}{2b^2}+\frac{(dxb+cb-id)\mathrm{e}^{-i(bx+a)}}{2b^2}-\frac{2c\,\mathrm{arctanh}\left(\mathrm{e}^{i(bx+a)}\right)}{b}+\frac{d\ln\left(1-\mathrm{e}^{i(bx+a)}\right)x}{b}+\frac{d\ln\left(1-\mathrm{e}^{i(bx+a)}\right)a}{b^2}-\frac{id\,\mathrm{poly}\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*cot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b*d*a*cos(b*x+a)+c*cos(b*x+a)-1/b*d*(sin(b*x+a)-(b*x+a)*cos(b*x+a))
)+1/b*(-1/b*d*a*ln(csc(b*x+a)-cot(b*x+a))+c*ln(csc(b*x+a)-cot(b*x+a))+1/b*d
*((b*x+a)*ln(1-exp(I*(b*x+a)))-(b*x+a)*ln(exp(I*(b*x+a))+1)+I*dilog(exp(I*(
b*x+a))+1)-I*dilog(1-exp(I*(b*x+a)))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(82) = 164$.
time = 0.34, size = 200, normalized size = 2.13

$$-\frac{2i\,bdx\arctan\left(\sin\left(bx+a\right),-\cos\left(bx+a\right)+1\right)-2i\,bc\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)-1\right)-2\left(-i\,bdx-i\,bc\right)\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)+1\right)-2\left(bdx+bc\right)\cos\left(bx+a\right)-2i\,d\mathrm{Li}_2\left(-e^{(i\,bx+i\,a)}\right)+2i\,d\mathrm{Li}_2\left(e^{(i\,bx+i\,a)}\right)+\left(bdx+bc\right)\log\left(\cos\left(bx+a\right)^2+\sin\left(bx+a\right)^2+2\,\cos\left(bx+a\right)+1\right)-\left(bdx+bc\right)\log\left(\cos\left(bx+a\right)^2+\sin\left(bx+a\right)^2-2\,\cos\left(bx+a\right)+1\right)+2\,d\sin\left(bx+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*I*b*c*arctan2(
sin(b*x + a), cos(b*x + a) - 1) - 2*(-I*b*d*x - I*b*c)*arctan2(sin(b*x + a)
, cos(b*x + a) + 1) - 2*(b*d*x + b*c)*cos(b*x + a) - 2*I*d*dilog(-e^(I*b*x
+ I*a)) + 2*I*d*dilog(e^(I*b*x + I*a)) + (b*d*x + b*c)*log(cos(b*x + a)^2 +
 sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (b*d*x + b*c)*log(cos(b*x + a)^2 +
sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 2*d*sin(b*x + a))/b^2
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(82) = 164$.
time = 5.13, size = 277, normalized size = 2.95

$$\frac{2\left(bdx+bc\right)\cos\left(bx+a\right)-i\,d\mathrm{Li}_2(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right))+i\,d\mathrm{Li}_2(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right))-i\,d\mathrm{Li}_2(-\cos\left(bx+a\right)+i\,\sin\left(bx+a\right))+i\,d\mathrm{Li}_2(-\cos\left(bx+a\right)-i\,\sin\left(bx+a\right))-\left(bdx+bc\right)\log\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)+1\right)-\left(bdx+bc\right)\log\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)+1\right)+\left(bc-ad\right)\log\left(-\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}i\,\sin\left(bx+a\right)+\frac{1}{2}\right)+\left(bc-ad\right)\log\left(-\frac{1}{2}\,\cos\left(bx+a\right)-\frac{1}{2}i\,\sin\left(bx+a\right)+\frac{1}{2}\right)+\left(bdx+ad\right)\log\left(-\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)+1\right)+\left(bdx+ad\right)\log\left(-\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)+1\right)-2\,d\sin\left(bx+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b*d*x + b*c)*cos(b*x + a) - I*d*dilog(cos(b*x + a) + I*sin(b*x + a)
) + I*d*dilog(cos(b*x + a) - I*sin(b*x + a)) - I*d*dilog(-cos(b*x + a) + I*
sin(b*x + a)) + I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b*d*x + b*c)*l
og(cos(b*x + a) + I*sin(b*x + a) + 1) - (b*d*x + b*c)*log(cos(b*x + a) - I*
sin(b*x + a) + 1) + (b*c - a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a)
```

```
+ 1/2) + (b*c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b
*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b*d*x + a*d)*log(-co
s(b*x + a) - I*sin(b*x + a) + 1) - 2*d*sin(b*x + a))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \cos (a + bx) \cot (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x)
[Out] Integral((c + d*x)*cos(a + b*x)*cot(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)*cos(b*x + a)*cot(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos (a + b\,x)\,\cot(a + b\,x)\,(c + d\,x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)*(c + d*x),x)
[Out] int(cos(a + b*x)*cot(a + b*x)*(c + d*x), x)
```

3.102 $\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=70

$$-\frac{\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\text{Int}\left(\frac{\csc(a+bx)}{c+dx},x\right)$$

```
[Out] -cos(a-b*c/d)*Si(b*c/d+b*x)/d-Ci(b*c/d+b*x)*sin(a-b*c/d)/d+Unintegrable(csc
(b*x+a)/(d*x+c),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x),x]
```

```
[Out] -((CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d) - (Cos[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d + Defer[Int][Csc[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx &= \int \frac{\csc(a+bx)}{c+dx}\,dx-\int \frac{\sin(a+bx)}{c+dx}\,dx\\ &= -\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\right)-\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\\ &= -\frac{\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\int \frac{\csc(a+bx)}{c+dx}\,dx\end{aligned}$$

Mathematica [A]
time = 8.22, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x), x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\cos \left(b x +a \right) \cot \left(b x +a \right)}{d x +c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)
[Out] int(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] 1/2*((I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(1, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + 2*d*integrate(sin(b*x + a)/((d*x + c
)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*cos(b*x + a
) + c), x) + 2*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x +
c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) + (exp_integral
_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-
(b*c - a*d)/d))/d
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(cos(b*x + a)*cot(b*x + a)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos{\left(a + b x \right)} \cot{\left(a + b x \right)}}{c + d x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*cot(a + b*x)/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(cos(b*x + a)*cot(b*x + a)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,\mathrm{cot}(a+b\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*cot(a + b*x))/(c + d*x),x)
[Out] int((cos(a + b*x)*cot(a + b*x))/(c + d*x), x)
```

3.103 $\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=88

$$-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\sin(a+bx)}{d(c+dx)}+\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\text{Int}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] -b*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^2+b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2+sin(b*x+
a)/d/(d*x+c)+Unintegrable(csc(b*x+a)/(d*x+c)^2,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -((b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2) + Sin[a + b*x]/(d*(c
 + d*x)) + (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int]
[Csc[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx &= \int \frac{\csc(a+bx)}{(c+dx)^2}\,dx-\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(a+bx)}{d(c+dx)}-\frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{d}+\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(a+bx)}{d(c+dx)}-\frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}}{d}\\
&= -\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\sin(a+bx)}{d(c+dx)}+\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\int
\end{aligned}$$

Mathematica [A]
time = 3.87, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2, x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\cos \left(bx + a\right) \cot \left(bx + a\right)}{\left(dx + c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(2, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + 2*(d^2*x + c*d)*integrate(sin(b*x +
a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2
 + 2*c*d*x + c^2)*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*
x + a)), x) + 2*(d^2*x + c*d)*integrate(sin(b*x + a)/(d^2*x^2 + 2*c*d*x + (
d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(b*x
 + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x) + (exp_integr
al_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin
(-(b*c - a*d)/d))/(d^2*x + c*d)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*cot(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos \left(a + bx\right) \cot \left(a + bx\right)}{\left(c + dx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)*cot(a + b*x)/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*cot(b*x + a)/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,\mathrm{cot}\left(a+b\,x\right)}{{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*cot(a + b*x))/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)*cot(a + b*x))/(c + d*x)^2, x)
```

3.104 $\int (c+dx)^m \cot^2(a+bx)\,dx$

Optimal. Leaf size=19

$$\text{Int}\left((c+dx)^m\cot^2(a+bx),x\right)$$

```
[Out] Unintegrable((d*x+c)^m*cot(b*x+a)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m\cot^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x]^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m\cot^2(a+bx)\,dx = \int (c+dx)^m\cot^2(a+bx)\,dx$$

Mathematica [A]
time = 1.27, size = 0, normalized size = 0.00

$$\int (c+dx)^m\cot^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Cot[a + b*x]^2, x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int (dx+c)^m\left(\cot^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cot(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*cot(b*x+a)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cot(b*x + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*cot(b*x + a)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot^2 (a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cot(b*x+a)**2,x)
[Out] Integral((c + d*x)**m*cot(a + b*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*cot(b*x + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \cot(a+b\,x)^2\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2*(c + d*x)^m,x)
[Out] int(cot(a + b*x)^2*(c + d*x)^m, x)
```

3.105 $\int (c+dx)^4 \cot^2(a+bx)\,dx$

Optimal. Leaf size=155

$$-\frac{i(c+dx)^4}{b}-\frac{(c+dx)^5}{5d}-\frac{(c+dx)^4\cot(a+bx)}{b}+\frac{4d(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{6id^2(c+dx)^2\mathrm{PolyLog}(2,e$$

```
[Out] -I*(d*x+c)^4/b-1/5*(d*x+c)^5/d-(d*x+c)^4*cot(b*x+a)/b+4*d*(d*x+c)^3*ln(1-ex
p(2*I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^3+6*d^3
*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^4+3*I*d^4*polylog(4,exp(2*I*(b*x+a))
)/b^5
```

Rubi [A]
time = 0.16, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3801, 3798, 2221, 2611, 6744, 2320, 6724, 32}

$$\frac{3id^4\mathrm{Li}_4\left(e^{2i(a+bx)}\right)}{b^5}+\frac{6d^3(c+dx)\mathrm{Li}_3\left(e^{2i(a+bx)}\right)}{b^4}-\frac{6id^2(c+dx)^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^3}+\frac{4d(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^4\cot(a+bx)}{b}-\frac{i(c+dx)^4}{b}-\frac{(c+dx)^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^4)/b - (c + d*x)^5/(5*d) - ((c + d*x)^4*Cot[a + b*x])/b + (
4*d*(c + d*x)^3*Log[1 - E^((2*I)*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)^2*
PolyLog[2, E^((2*I)*(a + b*x))])/b^3 + (6*d^3*(c + d*x)*PolyLog[3, E^((2*I)
*(a + b*x))])/b^4 + ((3*I)*d^4*PolyLog[4, E^((2*I)*(a + b*x))])/b^5
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \cot^2(a+bx)\,dx = -\frac{(c+dx)^4 \cot(a+bx)}{b} + \frac{(4d)\int (c+dx)^3 \cot(a+bx)\,dx}{b} - \int (c+dx)^4\,dx$$

$$= -\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4 \cot(a+bx)}{b} - \frac{(8id)\int \frac{e^{2i(a+bx)}(c+dx)^3}{1-e^{2i(a+bx)}}\,dx}{b}$$

$$= -\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4 \cot(a+bx)}{b} + \frac{4d(c+dx)^3 \log\left(1-e^{2i(\right.}{b^2}$$

$$= -\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4 \cot(a+bx)}{b} + \frac{4d(c+dx)^3 \log\left(1-e^{2i(\right.}{b^2}$$

$$= -\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4 \cot(a+bx)}{b} + \frac{4d(c+dx)^3 \log\left(1-e^{2i(\right.}{b^2}$$

$$= -\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4 \cot(a+bx)}{b} + \frac{4d(c+dx)^3 \log\left(1-e^{2i(\right.}{b^2}$$

$$= -\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4 \cot(a+bx)}{b} + \frac{4d(c+dx)^3 \log\left(1-e^{2i(\right.}{b^2}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 592 vs. $2(155) = 310$.
time = 6.56, size = 592, normalized size = 3.82

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]^2,x]
```

```
[Out] -1/5*(x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4)) - (c
*d^3*Csc[a]*(2*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(-1 + E^((2*I)*a))*Log[1
- E^((2*I)*(a + b*x))]) + 6*b*(-1 + E^((2*I)*a))*x*PolyLog[2, E^((2*I)*(a +
 b*x))] + (3*I)*(-1 + E^((2*I)*a))*PolyLog[3, E^((2*I)*(a + b*x))]))/(b^4*E
^(I*a)) - (d^4*E^(I*a)*Csc[a]*(x^4 + (-1 + E^((-2*I)*a))*x^4 + ((-1 + E^((2
*I)*a))*(2*b^4*x^4 + (4*I)*b^3*x^3*Log[1 - E^((2*I)*(a + b*x))] + 6*b^2*x^2
*PolyLog[2, E^((2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[3, E^((2*I)*(a + b*x))
] - 3*PolyLog[4, E^((2*I)*(a + b*x))]))/(2*b^4*E^((2*I)*a))))/b + (4*c^3*d*
Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^
2*(Cos[a]^2 + Sin[a]^2)) + (Csc[a]*Csc[a + b*x]*(c^4*Sin[b*x] + 4*c^3*d*x*S
in[b*x] + 6*c^2*d^2*x^2*Sin[b*x] + 4*c*d^3*x^3*Sin[b*x] + d^4*x^4*Sin[b*x])
)/b - (6*c^2*d^2*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi
 + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]]
)*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[T
an[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan
[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^3*Sqrt[Sec[a]^2*(Cos[a]^2 + Si
n[a]^2)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 920 vs. 2(142) = 284.
time = 0.14, size = 921, normalized size = 5.94

method	result
risch	$-\frac{6 i d^{4} a^{4}}{b^{5}}-\frac{2 i d^{4} x^{4}}{b}-\frac{4 d^{4} a^{3} \ln \left(\mathrm{e}^{i(b x+a)}-1\right)}{b^{5}}-\frac{8 d c^{3} \ln \left(\mathrm{e}^{i(b x+a)}\right)}{b^{2}}+\frac{4 d c^{3} \ln \left(\mathrm{e}^{i(b x+a)}+1\right)}{b^{2}}+\frac{4 d c^{3} \ln \left(\mathrm{e}^{i(b x+a)}-1\right)}{b^{2}}-\frac{2 i(d^{4} x^{4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 24*I*d^4*polylog(4,exp(I*(b*x+a)))/b^5-12*I*d^2/b^3*c^2*polylog(2,-exp(I*(b
*x+a)))+16*I*d^3/b^4*c*a^3-8*I*d^4/b^4*a^3*x-12*I*d^2/b*c^2*x^2-12*I*d^2/b^
3*c^2*a^2-8*I*d^3/b*c*x^3-12*I*d^4/b^3*polylog(2,-exp(I*(b*x+a)))*x^2-6*I*d
^4/b^5*a^4+24*I*d^4/b^5*polylog(4,-exp(I*(b*x+a)))-2*I*d^4/b*x^4-1/5*d^4*x^
5-1/5/d*c^5-4*d^4/b^5*a^3*ln(exp(I*(b*x+a))-1)-8*d/b^2*c^3*ln(exp(I*(b*x+a)
))+4*d/b^2*c^3*ln(exp(I*(b*x+a))+1)+4*d/b^2*c^3*ln(exp(I*(b*x+a))-1)-d^3*c*
x^4-2*d^2*c^2*x^3-2*d*c^3*x^2-c^4*x-2*I*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x^2+
4*c^3*d*x+c^4)/b/(exp(2*I*(b*x+a))-1)+24*d^3/b^4*c*polylog(3,exp(I*(b*x+a))
)+24*d^3/b^4*c*polylog(3,-exp(I*(b*x+a)))+24*d^4/b^4*polylog(3,exp(I*(b*x+a
)))*x+24*d^4/b^4*polylog(3,-exp(I*(b*x+a)))*x-24*I*d^2/b^2*c^2*a*x-24*I*d^3
/b^3*c*polylog(2,exp(I*(b*x+a)))*x-24*I*d^3/b^3*c*polylog(2,-exp(I*(b*x+a))
)*x+24*I*d^3/b^3*c*a^2*x+12*d^2/b^2*c^2*ln(exp(I*(b*x+a))+1)*x+12*d^2/b^2*c
^2*ln(1-exp(I*(b*x+a)))*x+12*d^2/b^3*c^2*ln(1-exp(I*(b*x+a)))*a+4*d^4/b^2*l
n(exp(I*(b*x+a))+1)*x^3+4*d^4/b^2*ln(1-exp(I*(b*x+a)))*x^3+4*d^4/b^5*ln(1-e
xp(I*(b*x+a)))*a^3-12*I*d^2/b^3*c^2*polylog(2,exp(I*(b*x+a)))-12*I*d^4/b^3*
polylog(2,exp(I*(b*x+a)))*x^2+12*d^3/b^2*c*ln(exp(I*(b*x+a))+1)*x^2+12*d^3/
b^2*c*ln(1-exp(I*(b*x+a)))*x^2-12*d^3/b^4*c*ln(1-exp(I*(b*x+a)))*a^2+12*d^3
/b^4*c*a^2*ln(exp(I*(b*x+a))-1)-24*d^3/b^4*c*a^2*ln(exp(I*(b*x+a)))-12*d^2/
b^3*c^2*a*ln(exp(I*(b*x+a))-1)+24*d^2/b^3*c^2*a*ln(exp(I*(b*x+a)))+8*d^4/b^
5*a^3*ln(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3242 vs. 2(138) = 276.
time = 0.80, size = 3242, normalized size = 20.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -((b*x + a + 1/tan(b*x + a))*c^4 - 4*(b*x + a + 1/tan(b*x + a))*a*c^3*d/b +
 6*(b*x + a + 1/tan(b*x + a))*a^2*c^2*d^2/b^2 - 4*(b*x + a + 1/tan(b*x + a)
)*a^3*c*d^3/b^3 + (b*x + a + 1/tan(b*x + a))*a^4*d^4/b^4 + 2*((b*x + a)^2*c
os(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*
```

x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*c^3*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b) - 6*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a*c^2*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^2) + 6*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*c*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^3) - 2*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a^3*d^4/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^4) - (-I*(b*x + a)^5*d^4 - 5*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^4 - 10*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a)^3 - 20*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) - ((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (-I*(b*x + a)^3*d^4 + 3*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^2 + 3*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 20*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) - ((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (I*(b*x + a)^3*d^4 + 3*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (I*(b*x + a)^5*d^4 - 5*(-I*b*c*d^3 + (I*a + 2)*d^4)*(b*x + a)^4 - 10*(-I*b^2*c^2*d^2 + 2*(I*a + 2)*b*c*d^3 + (-I*a^2 - 4*a)*d^4)*(b*x + a)^3 - 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2)*cos(2*b*x + 2*a) + 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a) - (b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*(b*x + a)^2*d^4 + I*a^2*d^4 + 2*(I*b*c*d^3 - I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x

```
 + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a) - (b^2*c^2*d^2 - 2*a*
b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*
b*x + 2*a) - (I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*(b*x + a)^2*d^4 + I*a^2*d^4
 + 2*(I*b*c*d^3 - I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*
a)) - 10*(-I*(b*x + a)^3*d^4 + 3*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^2 + 3*(-I
*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a) + (I*(b*x + a)^3*d^4 +
3*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*
a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^
4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b
*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - 10*(
-I*(b*x + a)^3*d^4 + 3*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^2 + 3*(-I*b^2*c^2*d
^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a) + (I*(b*x + a)^3*d^4 + 3*(I*b*c*d
^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(
b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x +
a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))
*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 120*(d^4*cos(2
*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a) - d^4)*polylog(4, -e^(I*b*x + I*a)) +
120*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 856 vs. 2(138) = 276.
time = 2.10, size = 856, normalized size = 5.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/10*(10*b^4*d^4*x^4 + 40*b^4*c*d^3*x^3 + 60*b^4*c^2*d^2*x^2 + 40*b^4*c^3*
d*x + 10*b^4*c^4 - 15*I*d^4*polylog(4, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a
))*sin(2*b*x + 2*a) + 15*I*d^4*polylog(4, cos(2*b*x + 2*a) - I*sin(2*b*x +
2*a))*sin(2*b*x + 2*a) + 30*(I*b^2*d^4*x^2 + 2*I*b^2*c*d^3*x + I*b^2*c^2*d^
2)*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) + 30*(-I*b
^2*d^4*x^2 - 2*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*dilog(cos(2*b*x + 2*a) - I*si
n(2*b*x + 2*a))*sin(2*b*x + 2*a) - 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*
b*c*d^3 - a^3*d^4)*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2
)*sin(2*b*x + 2*a) - 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*
d^4)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x +
2*a) - 20*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^
2 - 3*a^2*b*c*d^3 + a^3*d^4)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1
)*sin(2*b*x + 2*a) - 20*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x +
3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*log(-cos(2*b*x + 2*a) - I*sin(2*
b*x + 2*a) + 1)*sin(2*b*x + 2*a) - 30*(b*d^4*x + b*c*d^3)*polylog(3, cos(2*
b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 30*(b*d^4*x + b*c*d^3)*
polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) + 10*(b^
```

```
4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*
cos(2*b*x + 2*a) + 2*(b^5*d^4*x^5 + 5*b^5*c*d^3*x^4 + 10*b^5*c^2*d^2*x^3 +
10*b^5*c^3*d*x^2 + 5*b^5*c^4*x)*sin(2*b*x + 2*a))/(b^5*sin(2*b*x + 2*a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^4 \cot^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**4*cot(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cot(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(a + b\,x)^2\,(c + d\,x)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2*(c + d*x)^4,x)
```

```
[Out] int(cot(a + b*x)^2*(c + d*x)^4, x)
```

3.106 $\int (c+dx)^3 \cot^2(a+bx)\,dx$

Optimal. Leaf size=127

$$-\frac{i(c+dx)^3}{b}-\frac{(c+dx)^4}{4d}-\frac{(c+dx)^3\cot(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{3id^2(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}$$

```
[Out] -I*(d*x+c)^3/b-1/4*(d*x+c)^4/d-(d*x+c)^3*cot(b*x+a)/b+3*d*(d*x+c)^2*ln(1-ex
p(2*I*(b*x+a)))/b^2-3*I*d^2*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^3+3/2*d^3
*polylog(3,exp(2*I*(b*x+a)))/b^4
```

Rubi [A]

time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3801, 3798, 2221, 2611, 2320, 6724, 32}

$$\frac{3d^3\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^2(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^3}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{i(c+dx)^3}{b}-\frac{(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^3)/b - (c + d*x)^4/(4*d) - ((c + d*x)^3*Cot[a + b*x])/b + (
3*d*(c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))])/b^2 - ((3*I)*d^2*(c + d*x)*Po
lyLog[2, E^((2*I)*(a + b*x))])/b^3 + (3*d^3*PolyLog[3, E^((2*I)*(a + b*x))]
)/(2*b^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\cot^2(a+bx)\,dx &= -\frac{(c+dx)^3\cot(a+bx)}{b}+\frac{(3d)\int (c+dx)^2\cot(a+bx)\,dx}{b}-\int (c+dx)^3\,dx\\
&= -\frac{i(c+dx)^3}{b}-\frac{(c+dx)^4}{4d}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(6id)\int \frac{e^{2i(a+bx)}(c+dx)^2}{1-e^{2i(a+bx)}}\,dx}{b}\\
&= -\frac{i(c+dx)^3}{b}-\frac{(c+dx)^4}{4d}-\frac{(c+dx)^3\cot(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2i(\right.}}{b^2}\\
&= -\frac{i(c+dx)^3}{b}-\frac{(c+dx)^4}{4d}-\frac{(c+dx)^3\cot(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2i(\right.}}{b^2}\\
&= -\frac{i(c+dx)^3}{b}-\frac{(c+dx)^4}{4d}-\frac{(c+dx)^3\cot(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2i(\right.}}{b^2}\\
&= -\frac{i(c+dx)^3}{b}-\frac{(c+dx)^4}{4d}-\frac{(c+dx)^3\cot(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2i(\right.}}{b^2}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 311 vs. $2(127) = 254$.
time = 6.05, size = 311, normalized size = 2.45

$$-\frac{1}{4}x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)-\frac{3c^2d(bx\cot(a)-\log(\sin(a+bx)))}{b^2}+\frac{3cd^2\left(ibx(\pi-2\text{ArcTan}(\tan(a)))+\pi\log\left(1+e^{-2ibx}\right)+2(bx+\text{ArcTan}(\tan(a)))\log\left(1-e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)-\pi\log(\cos(bx))-2\text{ArcTan}(\tan(a))\log(\sin(bx+\text{ArcTan}(\tan(a))))-i\text{PolyLog}\left(2,e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)-b^2e^{i\text{ArcTan}(\tan(a))}x^2\cot(a)\sqrt{\sec^2(a)}\right)}{b^3}-\frac{d^3e^{-ia}(i+\cot(a))\left(2b^2x^2\left(ibx+bx\cot(a)-3\log\left(1-e^{2i(a+bx)}\right)\right)+6ibx\text{PolyLog}\left(2,e^{2i(a+bx)}\right)-3\text{PolyLog}\left(3,e^{2i(a+bx)}\right)\right)\sin(a)}{2b^4}+\frac{(c+dx)^3\csc(a)\csc(a+bx)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]^2,x]
```

```
[Out] -1/4*(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)) - (3*c^2*d*(b*x*Cot[a]
 - Log[Sin[a + b*x]]))/b^2 + (3*c*d^2*(I*b*x*(Pi - 2*ArcTan[Tan[a]]) + Pi*L
og[1 + E^((-2*I)*b*x)] + 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + A
rcTan[Tan[a]]))] - Pi*Log[Cos[b*x]] - 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan
[Tan[a]]]] - I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))] - b^2*E^(I*ArcT
an[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^2]))/b^3 - (d^3*(I + Cot[a])*(2*b^2*x^2*
(I*b*x + b*x*Cot[a] - 3*Log[1 - E^((2*I)*(a + b*x))]) + (6*I)*b*x*PolyLog[2
, E^((2*I)*(a + b*x))] - 3*PolyLog[3, E^((2*I)*(a + b*x))])*Sin[a])/(2*b^4*
E^(I*a)) + ((c + d*x)^3*Csc[a]*Csc[a + b*x]*Sin[b*x])/b
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(115) = 230$.
time = 0.11, size = 581, normalized size = 4.57

method	result
risch	$-\frac{d^3x^4}{4}-\frac{c^4}{4d}+\frac{6id^3a^2x}{b^3}-\frac{6id^2cx^2}{b}+\frac{12d^2ca\ln\left(e^{i(bx+a)}\right)}{b^3}-\frac{6d^2ca\ln\left(e^{i(bx+a)}-1\right)}{b^3}-\frac{6id^2ca^2}{b^3}-\frac{6id^3\operatorname{polylog}\left(2,e^{i(bx+a)}\right)}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d^3*x^4-1/4/d*c^4-6*I*d^3/b^3*polylog(2,-exp(I*(b*x+a)))*x-6*I*d^2/b^3
*c*polylog(2,-exp(I*(b*x+a)))+6*I*d^3/b^3*a^2*x-6*I*d^2/b*c*x^2+6*d^2/b^3*c
*ln(1-exp(I*(b*x+a)))*a+6*d^2/b^2*c*ln(exp(I*(b*x+a))+1)*x-6*I*d^2/b^3*c*po
lylog(2,exp(I*(b*x+a)))-6*I*d^3/b^3*polylog(2,exp(I*(b*x+a)))*x+6*d^2/b^2*c
*ln(1-exp(I*(b*x+a)))*x+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4-12*I*d^2/b^2*c*
a*x+12*d^2/b^3*c*a*ln(exp(I*(b*x+a)))-6*d^2/b^3*c*a*ln(exp(I*(b*x+a))-1)-6*
I*d^2/b^3*c*a^2-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(exp(2*I*(b*x+a))
-1)-d^2*c*x^3-3/2*d*c^2*x^2-c^3*x+3*d/b^2*c^2*ln(exp(I*(b*x+a))-1)+3*d/b^2*
c^2*ln(exp(I*(b*x+a))+1)-6*d/b^2*c^2*ln(exp(I*(b*x+a)))+3*d^3/b^4*a^2*ln(ex
p(I*(b*x+a))-1)-6*d^3/b^4*a^2*ln(exp(I*(b*x+a)))+4*I*d^3/b^4*a^3-2*I*d^3/b*
x^3+3*d^3/b^2*ln(1-exp(I*(b*x+a)))*x^2-3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2+3
*d^3/b^2*ln(exp(I*(b*x+a))+1)*x^2+6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1953 vs. $2(112) = 224$.

time = 0.61, size = 1953, normalized size = 15.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b*x + a + 1/tan(b*x + a))*c^3 - 6*(b*x + a + 1/tan(b*x + a))*a*c^2
*d/b + 6*(b*x + a + 1/tan(b*x + a))*a^2*c*d^2/b^2 - 2*(b*x + a + 1/tan(b*x
+ a))*a^3*d^3/b^3 + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b
*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2
*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + s
in(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a
)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b
*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*c^2*d/((cos(2*b*x + 2*a)^2 + s
in(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b) - 6*((b*x + a)^2*cos(2*b*x +
 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) +
 (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a
) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b
*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a
*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*
b^2) + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 -
 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2
 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2
*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1)
+ 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x +
2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^3) - 2*(-I*(b*x + a)^4*d^3 - 4*(I*b*c*d^
2 - I*a*d^3)*(b*x + a)^3 - 12*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x +
 a) - ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) +
(-I*(b*x + a)^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))
*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 12*((b*x + a)^2*d^3 + 2*(b*c*d^2
 - a*d^3)*(b*x + a) - ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos
(2*b*x + 2*a) - (I*(b*x + a)^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin
(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (I*(b*x + a)^4*d^
3 - 4*(-I*b*c*d^2 + (I*a + 2)*d^3)*(b*x + a)^3 - 24*(b*c*d^2 - a*d^3)*(b*x
+ a)^2)*cos(2*b*x + 2*a) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3 - (b*c*d^2 +
 (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (I*b*c*d^2 + I*(b*x + a)*d^3 - I
*a*d^3)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + 24*(b*c*d^2 + (b*x + a)
*d^3 - a*d^3 - (b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (I*b*c*
d^2 + I*(b*x + a)*d^3 - I*a*d^3)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) -
 6*(-I*(b*x + a)^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a) + (I*(b*x + a)^
2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^2*
d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 +
```

```
 sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - 6*(-I*(b*x + a)^2*d^3 + 2*(-I*b*c*d
^2 + I*a*d^3)*(b*x + a) + (I*(b*x + a)^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x
 + a))*cos(2*b*x + 2*a) - ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))
*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1
) - 24*(I*d^3*cos(2*b*x + 2*a) - d^3*sin(2*b*x + 2*a) - I*d^3)*polylog(3, -
e^(I*b*x + I*a)) - 24*(I*d^3*cos(2*b*x + 2*a) - d^3*sin(2*b*x + 2*a) - I*d^
3)*polylog(3, e^(I*b*x + I*a)) - ((b*x + a)^4*d^3 + 4*(b*c*d^2 - (a - 2*I)*
d^3)*(b*x + a)^3 + 24*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2)*sin(2*b*x + 2*a))/
(-4*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(2*b*x + 2*a) + 4*I*b^3))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(112) = 224$.
time = 1.57, size = 599, normalized size = 4.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 4*b^3*c^3 - 3*d^3
*polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 3*d^3
*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) + 6*(I*
b*d^3*x + I*b*c*d^2)*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x
 + 2*a) + 6*(-I*b*d^3*x - I*b*c*d^2)*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x +
 2*a))*sin(2*b*x + 2*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*co
s(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) - 6*(b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x +
2*a) + 1/2)*sin(2*b*x + 2*a) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2
 - a^2*d^3)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1)*sin(2*b*x + 2*a
) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(2*b*x
+ 2*a) - I*sin(2*b*x + 2*a) + 1)*sin(2*b*x + 2*a) + 4*(b^3*d^3*x^3 + 3*b^3*
c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(2*b*x + 2*a) + (b^4*d^3*x^4 + 4*b^
4*c*d^2*x^3 + 6*b^4*c^2*d*x^2 + 4*b^4*c^3*x)*sin(2*b*x + 2*a))/(b^4*sin(2*b
*x + 2*a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cot^2 (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*cot(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cot(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(a+b\,x)^2\,(c+d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] int(cot(a + b*x)^2*(c + d*x)^3, x)
```

3.107 $\int (c+dx)^2 \cot^2(a+bx)\,dx$

Optimal. Leaf size=97

$$-\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}-\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{id^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}$$

```
[Out] -I*(d*x+c)^2/b-1/3*(d*x+c)^3/d-(d*x+c)^2*cot(b*x+a)/b+2*d*(d*x+c)*ln(1-exp(
2*I*(b*x+a)))/b^2-I*d^2*polylog(2,exp(2*I*(b*x+a)))/b^3
```

Rubi [A]

time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3801, 3798, 2221, 2317, 2438, 32}

$$-\frac{id^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^2)/b - (c + d*x)^3/(3*d) - ((c + d*x)^2*Cot[a + b*x])/b + (
2*d*(c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b^2 - (I*d^2*PolyLog[2, E^((2*I
)*(a + b*x))])/b^3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^2 \cot^2(a+bx)\,dx &= -\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{(2d)\int (c+dx)\cot(a+bx)\,dx}{b}-\int (c+dx)^2\,dx\\
&= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(4id)\int \frac{e^{2i(a+bx)}(c+dx)}{1-e^{2i(a+bx)}}\,dx}{b}\\
&= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}-\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2i(a}\right.}{b^2}\\
&= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}-\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2i(a}\right.}{b^2}\\
&= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}-\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2i(a}\right.}{b^2}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 268 vs. $2(97) = 194$.
time = 6.36, size = 268, normalized size = 2.76

$$
-\frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right)+\frac{2cd\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b^2\left(\cos^2(a)+\sin^2(a)\right)}+\frac{\csc(a)\csc(a+bx)\left(c^2\sin(bx)+2cdx\sin(bx)+d^2x^2\sin(bx)\right)}{b}-\frac{d^2\csc(a)\sec(a)\left(b^2e^{i\text{ArcTan}(\tan(a))}x^2+\frac{\left(ibx(-\pi+2\text{ArcTan}(\tan(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx+\text{ArcTan}(\tan(a)))\log\left(1-e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)+\pi\log(\cos(bx))+2\text{ArcTan}(\tan(a))\log(\sin(bx+\text{ArcTan}(\tan(a))))+i\text{PolyLog}\left(2,e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)\right)\tan(a)}{\sqrt{1+\tan^2(a)}}\right)}{b^3\sqrt{\sec^2(a)\left(\cos^2(a)+\sin^2(a)\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]^2,x]
```

```
[Out] -1/3*(x*(3*c^2 + 3*c*d*x + d^2*x^2)) + (2*c*d*Csc[a]*(-(b*x*Cos[a]) + Log[C
os[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^2*(Cos[a]^2 + Sin[a]^2)) + (C
sc[a]*Csc[a + b*x]*(c^2*Sin[b*x] + 2*c*d*x*Sin[b*x] + d^2*x^2*Sin[b*x]))/b
- (d^2*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTa
```

```
n[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 -
E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log
[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))
])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^3*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(89) = 178$.
time = 0.10, size = 305, normalized size = 3.14

method	result
risch	$-\frac{d^2x^3}{3} - cd\,x^2 - c^2x - \frac{c^3}{3d} - \frac{2id^2x^2}{b} + \frac{2dc\ln(e^{i(bx+a)}-1)}{b^2} + \frac{2dc\ln(e^{i(bx+a)}+1)}{b^2} - \frac{4dc\ln(e^{i(bx+a)})}{b^2} - \frac{2id^2a^2}{b^3} - \frac{2id}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*d^2*x^3-c*d*x^2-c^2*x-1/3/d*c^3-2*I*d^2/b*x^2+2*d/b^2*c*ln(exp(I*(b*x+
a))-1)+2*d/b^2*c*ln(exp(I*(b*x+a))+1)-4*d/b^2*c*ln(exp(I*(b*x+a)))-2*I*d^2/
b^3*polylog(2,-exp(I*(b*x+a)))-2*I*d^2/b^3*a^2-2*I*d^2*polylog(2,exp(I*(b*x
+a)))/b^3+2*d^2/b^2*ln(1-exp(I*(b*x+a)))*x+2*d^2/b^3*ln(1-exp(I*(b*x+a)))*a
-4*I*d^2/b^2*a*x+2*d^2/b^2*ln(exp(I*(b*x+a))+1)*x-2*I*(d^2*x^2+2*c*d*x+c^2)
/b/(exp(2*I*(b*x+a))-1)-2*d^2/b^3*a*ln(exp(I*(b*x+a))-1)+4*d^2/b^3*a*ln(exp
(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 645 vs. $2(86) = 172$.
time = 0.59, size = 645, normalized size = 6.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (-I*b^3*d^2*x^3 - 3*I*b^3*c*d*x^2 - 3*I*b^3*c^2*x - 6*b^2*c^2 - 6*(b*d^2*x
+ b*c*d - (b*d^2*x + b*c*d)*cos(2*b*x + 2*a) + (-I*b*d^2*x - I*b*c*d)*sin(2
*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 6*(b*c*d*cos(2*b*x +
 2*a) + I*b*c*d*sin(2*b*x + 2*a) - b*c*d)*arctan2(sin(b*x + a), cos(b*x + a
) - 1) - 6*(b*d^2*x*cos(2*b*x + 2*a) + I*b*d^2*x*sin(2*b*x + 2*a) - b*d^2*x
)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (I*b^3*d^2*x^3 - 3*(-I*b^3*c*d
 + 2*b^2*d^2)*x^2 - 3*(-I*b^3*c^2 + 4*b^2*c*d)*x)*cos(2*b*x + 2*a) - 6*(d^2
*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(-e^(I*b*x + I*a)) -
 6*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(e^(I*b*x + I
*a)) - 3*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(2*b*x + 2*a) - (
b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*
cos(b*x + a) + 1) - 3*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(2*b
*x + 2*a) - (b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*
```

```
x + a)^2 - 2*cos(b*x + a) + 1) - (b^3*d^2*x^3 + 3*(b^3*c*d + 2*I*b^2*d^2)*x
^2 + 3*(b^3*c^2 + 4*I*b^2*c*d)*x)*sin(2*b*x + 2*a))/(-3*I*b^3*cos(2*b*x + 2
*a) + 3*b^3*sin(2*b*x + 2*a) + 3*I*b^3)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(86) = 172$.
time = 1.51, size = 384, normalized size = 3.96

$$-\frac{6\,b^2d^2x^2+12\,b^2cdx+6\,b^2c^2+3i\,d^2\mathrm{Li}_2(\cos\left(2\,bx+2\,a\right)+i\,\sin\left(2\,bx+2\,a\right))\sin\left(2\,bx+2\,a\right)-3i\,d^2\mathrm{Li}_2(\cos\left(2\,bx+2\,a\right)-i\,\sin\left(2\,bx+2\,a\right))\sin\left(2\,bx+2\,a\right)-6\,(bcd-ad^2)\log\left(-\frac{1}{2}\,\cos\left(2\,bx+2\,a\right)+\frac{1}{2}i\,\sin\left(2\,bx+2\,a\right)+\frac{1}{2}\right)\sin\left(2\,bx+2\,a\right)-6\,(bcd-ad^2)\log\left(-\frac{1}{2}\,\cos\left(2\,bx+2\,a\right)-\frac{1}{2}i\,\sin\left(2\,bx+2\,a\right)+\frac{1}{2}\right)\sin\left(2\,bx+2\,a\right)-6\,(bd^2x+ad^2)\log\left(-\cos\left(2\,bx+2\,a\right)+i\,\sin\left(2\,bx+2\,a\right)+1\right)\sin\left(2\,bx+2\,a\right)-6\,(bd^2x+ad^2)\log\left(-\cos\left(2\,bx+2\,a\right)-i\,\sin\left(2\,bx+2\,a\right)+1\right)\sin\left(2\,bx+2\,a\right)+6\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\cos\left(2\,bx+2\,a\right)+2\,(b^3d^2x^3+3\,b^3cdx^2+3\,b^3c^2x)\sin\left(2\,bx+2\,a\right)}{6\,b^3\sin\left(2\,bx+2\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^2*d^2*x^2 + 12*b^2*c*d*x + 6*b^2*c^2 + 3*I*d^2*dilog(cos(2*b*x +
2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 3*I*d^2*dilog(cos(2*b*x + 2*a
) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 6*(b*c*d - a*d^2)*log(-1/2*cos(2
*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) - 6*(b*c*d - a
*d^2)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x +
 2*a) - 6*(b*d^2*x + a*d^2)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1)
*sin(2*b*x + 2*a) - 6*(b*d^2*x + a*d^2)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x
 + 2*a) + 1)*sin(2*b*x + 2*a) + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos
(2*b*x + 2*a) + 2*(b^3*d^2*x^3 + 3*b^3*c*d*x^2 + 3*b^3*c^2*x)*sin(2*b*x + 2
*a))/(b^3*sin(2*b*x + 2*a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2\cot^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*cot(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cot(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(a+b\,x)^2\,(c+d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] int(cot(a + b*x)^2*(c + d*x)^2, x)
```

3.108 $\int (c+dx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=41

$$-cx-\frac{dx^2}{2}-\frac{(c+dx)\cot(a+bx)}{b}+\frac{d\log(\sin(a+bx))}{b^2}$$

```
[Out] -c*x-1/2*d*x^2-(d*x+c)*cot(b*x+a)/b+d*ln(sin(b*x+a))/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3801, 3556}

$$\frac{d\log(\sin(a+bx))}{b^2}-\frac{(c+dx)\cot(a+bx)}{b}-cx-\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]^2,x]
```

```
[Out] -(c*x) - (d*x^2)/2 - ((c + d*x)*Cot[a + b*x])/b + (d*Log[Sin[a + b*x]])/b^2
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cot^2(a+bx)\,dx&=-\frac{(c+dx)\cot(a+bx)}{b}+\frac{d\int \cot(a+bx)\,dx}{b}-\int (c+dx)\,dx\\&=-cx-\frac{dx^2}{2}-\frac{(c+dx)\cot(a+bx)}{b}+\frac{d\log(\sin(a+bx))}{b^2}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.56, size = 82, normalized size = 2.00

$$-\frac{c\cot(a+bx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(a+bx)\right)}{b}+\frac{d\log(\sin(a+bx))}{b^2}-\frac{dx\csc(a)(2\cos(a)+bx\sin(a))}{2b}+\frac{dx\csc(a)\csc(a+bx)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]^2,x]
```

```
[Out] -((c*Cot[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[a + b*x]^2])/b) + (d
*Log[Sin[a + b*x]])/b^2 - (d*x*Csc[a]*(2*Cos[a] + b*x*Sin[a]))/(2*b) + (d*x
*Csc[a]*Csc[a + b*x]*Sin[b*x])/b
```

Maple [A]
time = 0.08, size = 64, normalized size = 1.56

method	result	size
default	$-\frac{d\,x^{2}}{2}-c x +\frac{\frac{d a \cot \left(b x +a \right)}{b}-c \cot \left(b x +a \right)+\frac{d \left(-\left(b x +a \right) \cot \left(b x +a \right)+\ln \left(\sin \left(b x +a \right)\right)\right)}{b}}{b}$	64
risch	$-\frac{d\,x^{2}}{2}-c x -\frac{2 i d x}{b}-\frac{2 i d a}{b^{2}}-\frac{2 i \left(d x +c \right)}{b \left({\mathrm e}^{2 i \left(b x +a \right)}-1\right)}+\frac{d \ln \left({\mathrm e}^{2 i \left(b x +a \right)}-1\right)}{b^{2}}$	69
norman	$\frac{-\frac{c}{b}-c x \tan \left(b x +a \right)-\frac{d x}{b}-\frac{d\,x^{2} \tan \left(b x +a \right)}{2}}{\tan \left(b x +a \right)}+\frac{d \ln \left(\tan \left(b x +a \right)\right)}{b^{2}}-\frac{d \ln \left(1+\tan^{2}\left(b x +a \right)\right)}{2 b^{2}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d*x^2-c*x+1/b*(1/b*d*a*cot(b*x+a)-c*cot(b*x+a)+1/b*d*(-(b*x+a)*cot(b*x
+a)+ln(sin(b*x+a))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(39) = 78$.
time = 0.49, size = 292, normalized size = 7.12

$$-\frac{2 \left(b x + a + \frac{1}{\tan\left(b x + a\right)}\right) c - \frac{2 \left(b x + a + \frac{1}{\tan\left(b x + a\right)}\right) a d}{b} + \frac{{\left({\left(b x + a\right)}^{2} \cos\left(2 \, b x + 2 \, a\right)^{2} + {\left(b x + a\right)}^{2} \sin\left(2 \, b x + 2 \, a\right)^{2} - 2 \, {\left(b x + a\right)}^{2} \cos\left(2 \, b x + 2 \, a\right) + {\left(b x + a\right)}^{2} - {\left(\cos\left(2 \, b x + 2 \, a\right)^{2} + \sin\left(2 \, b x + 2 \, a\right)^{2} - 2 \, \cos\left(2 \, b x + 2 \, a\right) + 1\right)} \log\left(\cos\left(b x + a\right)^{2} + \sin\left(b x + a\right)^{2} + 2 \, \cos\left(b x + a\right) + 1\right) - {\left(\cos\left(2 \, b x + 2 \, a\right)^{2} + \sin\left(2 \, b x + 2 \, a\right)^{2} - 2 \, \cos\left(2 \, b x + 2 \, a\right) + 1\right)} \log\left(\cos\left(b x + a\right)^{2} + \sin\left(b x + a\right)^{2} - 2 \, \cos\left(b x + a\right) + 1\right) + 4 \, {\left(b x + a\right)} \sin\left(2 \, b x + 2 \, a\right)\right)} d}{{\left(\cos\left(2 \, b x + 2 \, a\right)^{2} + \sin\left(2 \, b x + 2 \, a\right)^{2} - 2 \, \cos\left(2 \, b x + 2 \, a\right) + 1\right)} b}}{2 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b*x + a + 1/tan(b*x + a))*c - 2*(b*x + a + 1/tan(b*x + a))*a*d/b +
 ((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x
+ a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2
*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos
(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2
*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x
 + a)*sin(2*b*x + 2*a))*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos
(2*b*x + 2*a) + 1)*b))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(39) = 78$.
time = 2.41, size = 97, normalized size = 2.37

$$-\frac{2 \, b d x - d \log\left(-\frac{1}{2} \, \cos\left(2 \, b x + 2 \, a\right) + \frac{1}{2}\right) \sin\left(2 \, b x + 2 \, a\right) + 2 \, b c + 2 \, {\left(b d x + b c\right)} \cos\left(2 \, b x + 2 \, a\right) + {\left(b^{2} d x^{2} + 2 \, b^{2} c x\right)} \sin\left(2 \, b x + 2 \, a\right)}{2 \, b^{2} \sin\left(2 \, b x + 2 \, a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x - d*log(-1/2*cos(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) + 2*b*c
 + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d*x^2 + 2*b^2*c*x)*sin(2*b*x + 2
*a))/(b^2*sin(2*b*x + 2*a))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(36) = 72$.
time = 0.23, size = 104, normalized size = 2.54

$$\begin{cases} \tilde{\infty}\left(cx + \frac{dx^2}{2}\right) & \text{for } a = 0 \wedge b = 0 \\ \left(cx + \frac{dx^2}{2}\right)\cot^2(a) & \text{for } b = 0 \\ \tilde{\infty}\left(cx + \frac{dx^2}{2}\right) & \text{for } a = -bx \\ -cx - \frac{dx^2}{2} - \frac{c}{b\tan(a+bx)} - \frac{dx}{b\tan(a+bx)} - \frac{d\log(\tan^2(a+bx)+1)}{2b^2} + \frac{d\log(\tan(a+bx))}{b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*(c*x + d*x**2/2), Eq(a, 0) & Eq(b, 0)), ((c*x + d*x**2/2)*co
t(a)**2, Eq(b, 0)), (zoo*(c*x + d*x**2/2), Eq(a, -b*x)), (-c*x - d*x**2/2 -
 c/(b*tan(a + b*x)) - d*x/(b*tan(a + b*x)) - d*log(tan(a + b*x)**2 + 1)/(2*
b**2) + d*log(tan(a + b*x))/b**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1375 vs. $2(39) = 78$.
time = 0.70, size = 1375, normalized size = 33.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*d*x^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*d*x^2*tan(1/2*b*x)*tan(1/2*
a)^2 + 2*b^2*c*x*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*c*x*tan(1/2*b*x)*tan(1/2
*a)^2 - b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*d*x^2*tan(1/2*b*x) - b^2*d*
x^2*tan(1/2*a) - b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*c*x*tan(1/2*b*x) +
 b*d*x*tan(1/2*b*x)^2 - 2*b^2*c*x*tan(1/2*a) + 4*b*d*x*tan(1/2*b*x)*tan(1/2
*a) - d*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3
 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*
x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/
2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*ta
```

```
n(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 +
2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a
)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2
*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan
(1/2*a) + b*d*x*tan(1/2*a)^2 - d*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*ta
n(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*
tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3
+ 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2
*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1
/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2
*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2
 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2
 + 1))*tan(1/2*b*x)*tan(1/2*a)^2 + b*c*tan(1/2*b*x)^2 + 4*b*c*tan(1/2*b*x)*
tan(1/2*a) + b*c*tan(1/2*a)^2 - b*d*x + d*log(16*(tan(1/2*b*x)^8*tan(1/2*a)
^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/
2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(
1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^
5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^
3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1
/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1
/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan
(1/2*a)^2 + 1))*tan(1/2*b*x) + d*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*ta
n(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*
tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3
+ 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2
*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1
/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2
*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2
 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2
 + 1))*tan(1/2*a) - b*c)/(b^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*tan(1/2*b*x)*
tan(1/2*a)^2 - b^2*tan(1/2*b*x) - b^2*tan(1/2*a))
```

Mupad [B]
time = 1.57, size = 67, normalized size = 1.63

$$-\frac{d\,x^2}{2}+\frac{d\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-1\right)}{b^2}-\frac{\left(c+d\,x\right)\,2\mathrm{i}}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}-\frac{x\,\left(b\,c+d\,2\mathrm{i}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2*(c + d*x),x)
[Out] (d*log(exp(a*2i)*exp(b*x*2i) - 1))/b^2 - (d*x^2)/2 - ((c + d*x)*2i)/(b*(exp
(a*2i + b*x*2i) - 1)) - (x*(d*2i + b*c))/b
```

3.109 $\int \frac{\cot^2(a+bx)}{c+dx} dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\cot^2(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^2/(d*x+c),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^2(a+bx)}{c+dx} dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][Cot[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)}{c+dx} dx = \int \frac{\cot^2(a+bx)}{c+dx} dx$$

Mathematica [A]
time = 4.29, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)}{c+dx} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^2/(c + d*x),x]
```

```
[Out] Integrate[Cot[a + b*x]^2/(c + d*x), x]
```

Maple [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(cot(b*x+a)^2/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] ((b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x + b
*c*d^2)*sin(2*b*x + 2*a)^2 - 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*integr
ate(sin(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x +
b*c^2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 + 2*
(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) - (b*d^3*x + b*c*d^2 + (b
*d^3*x + b*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x + b*c*d^2)*sin(2*b*x + 2*a)
^2 - 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^2*
x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 +
(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x +
 b*c^2)*cos(b*x + a)), x) - (b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*
d*x + b*c)*sin(2*b*x + 2*a)^2 + b*c - 2*(b*d*x + b*c)*cos(2*b*x + 2*a))*log
(d*x + c) - 2*d*sin(2*b*x + 2*a))/(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(
2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^2*x + b*c*d)
*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^2/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(cot(a + b*x)**2/(c + d*x), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(cot(b*x + a)^2/(d*x + c), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cot(a+b\,x)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2/(c + d*x),x)
[Out] int(cot(a + b*x)^2/(c + d*x), x)
```

3.110 $\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\cot^2(a+bx)}{(c+dx)^2}, x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^2/(d*x+c)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Cot[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx = \int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$$

Mathematica [A]
time = 2.41, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Integrate[Cot[a + b*x]^2/(c + d*x)^2, x]
```

Maple [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(cot(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)^
2 + b*c - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + 2*(b*d^4*x^2 + 2*b*c*d^3*x + b
*c^2*d^2 + (b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*cos(2*b*x + 2*a)^2 + (b*d^
4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*sin(2*b*x + 2*a)^2 - 2*(b*d^4*x^2 + 2*b*c*
d^3*x + b*c^2*d^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^3*x^3 + 3*
b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*
x + b*c^3)*cos(b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^
3)*sin(b*x + a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos
(b*x + a)), x) - 2*(b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2 + (b*d^4*x^2 + 2*b*
c*d^3*x + b*c^2*d^2)*cos(2*b*x + 2*a)^2 + (b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*
d^2)*sin(2*b*x + 2*a)^2 - 2*(b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*cos(2*b*x
 + 2*a))*integrate(sin(b*x + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x +
b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)^2 +
(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(b*x + a)^2 - 2*(b*d^3
*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)), x) - 2*d*sin(2*b
*x + 2*a))/(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x +
b*c^2*d)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(2*b*x
 + 2*a)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a))
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(cot(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cot(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2/(c + d*x)^2,x)
```

```
[Out] int(cot(a + b*x)^2/(c + d*x)^2, x)
```

3.111 $\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=39

$$-\text{Int}((c+dx)^m\csc(a+bx),x)+\text{Int}\big((c+dx)^m\csc^3(a+bx),x\big)$$

```
[Out] -Unintegrable((d*x+c)^m*csc(b*x+a),x)+Unintegrable((d*x+c)^m*csc(b*x+a)^3,x
)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m\cot^2(a+bx)\csc(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] -Defer[Int][(c + d*x)^m*Csc[a + b*x], x] + Defer[Int][(c + d*x)^m*Csc[a + b
*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m\cot^2(a+bx)\csc(a+bx)\,dx = -\int (c+dx)^m\csc(a+bx)\,dx + \int (c+dx)^m\csc^3(a+bx)\,dx$$

Mathematica [A]
time = 35.42, size = 0, normalized size = 0.00

$$\int (c+dx)^m\cot^2(a+bx)\csc(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x], x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dx+c)^m\left(\cot^2(bx+a)\right)\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot^2{(a+bx)} \csc{(a+bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cot(b*x+a)**2*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*cot(a + b*x)**2*csc(a + b*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot(a+b\,x)^2\,(c+d\,x)^m}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(a + b*x)^2*(c + d*x)^m)/sin(a + b*x),x)
[Out] int((cot(a + b*x)^2*(c + d*x)^m)/sin(a + b*x), x)
```

3.112 $\int (c+dx)^4 \cot^2(a+bx) \csc(a+bx) \, dx$

Optimal. Leaf size=416

$$-\frac{12d^2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3}+\frac{(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{2d(c+dx)^3 \csc(a+bx)}{b^2}-\frac{(c+dx)^4 \cot(a+b}{2b}$$

```
[Out] -12*d^2*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^3+(d*x+c)^4*arctanh(exp(I*(b*x+
a)))/b-2*d*(d*x+c)^3*csc(b*x+a)/b^2-1/2*(d*x+c)^4*cot(b*x+a)*csc(b*x+a)/b+1
2*I*d^3*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^4-12*I*d^3*(d*x+c)*polylog(2,e
xp(I*(b*x+a)))/b^4+2*I*d*(d*x+c)^3*polylog(2,exp(I*(b*x+a)))/b^2-2*I*d*(d*x
+c)^3*polylog(2,-exp(I*(b*x+a)))/b^2-12*d^4*polylog(3,-exp(I*(b*x+a)))/b^5+
6*d^2*(d*x+c)^2*polylog(3,-exp(I*(b*x+a)))/b^3+12*d^4*polylog(3,exp(I*(b*x+
a)))/b^5-6*d^2*(d*x+c)^2*polylog(3,exp(I*(b*x+a)))/b^3+12*I*d^3*(d*x+c)*pol
ylog(4,-exp(I*(b*x+a)))/b^4-12*I*d^3*(d*x+c)*polylog(4,exp(I*(b*x+a)))/b^4-
12*d^4*polylog(5,-exp(I*(b*x+a)))/b^5+12*d^4*polylog(5,exp(I*(b*x+a)))/b^5
```

Rubi [A]
time = 0.37, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {4500, 4268, 2611, 6744, 2320, 6724, 4271}

$$-\frac{12d^4\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^5}+\frac{12d^4\text{Li}_3\left(e^{i(a+bx)}\right)}{b^5}-\frac{12d^4\text{Li}_5\left(-e^{i(a+bx)}\right)}{b^5}+\frac{12d^4\text{Li}_5\left(e^{i(a+bx)}\right)}{b^5}+\frac{12id^3(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^4}-\frac{12id^3(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^4}+\frac{12id^3(c+dx)\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}-\frac{12id^3(c+dx)\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}+\frac{6d^2(c+dx)^2\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}-\frac{6d^2(c+dx)^2\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{12d^2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3}-\frac{2id(c+dx)^3\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}+\frac{2id(c+dx)^3\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{2d(c+dx)^3\csc(a+bx)}{b^2}+\frac{(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)^4\cot(a+bx)\csc(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] (-12*d^2*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^3 + ((c + d*x)^4*ArcTanh[E
^(I*(a + b*x))])/b - (2*d*(c + d*x)^3*Csc[a + b*x])/b^2 - ((c + d*x)^4*Cot[
a + b*x]*Csc[a + b*x])/(2*b) + ((12*I)*d^3*(c + d*x)*PolyLog[2, -E^(I*(a +
b*x))])/b^4 - ((2*I)*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((12
*I)*d^3*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^4 + ((2*I)*d*(c + d*x)^3*P
olyLog[2, E^(I*(a + b*x))])/b^2 - (12*d^4*PolyLog[3, -E^(I*(a + b*x))])/b^5
 + (6*d^2*(c + d*x)^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (12*d^4*PolyLog[3
, E^(I*(a + b*x))])/b^5 - (6*d^2*(c + d*x)^2*PolyLog[3, E^(I*(a + b*x))])/b
^3 + ((12*I)*d^3*(c + d*x)*PolyLog[4, -E^(I*(a + b*x))])/b^4 - ((12*I)*d^3*
(c + d*x)*PolyLog[4, E^(I*(a + b*x))])/b^4 - (12*d^4*PolyLog[5, -E^(I*(a +
b*x))])/b^5 + (12*d^4*PolyLog[5, E^(I*(a + b*x))])/b^5
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4500

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx = -\int (c+dx)^4 \csc(a+bx)\,dx + \int (c+dx)^4 \csc^3(a+bx)\,dx$$

$$= \frac{2(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)^3\csc(a+bx)}{b^2} - \frac{(c+dx)}{}$$

$$= -\frac{12d^2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - $$

$$= -\frac{12d^2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - $$

$$= -\frac{12d^2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - $$

$$= -\frac{12d^2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - $$

$$= -\frac{12d^2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - $$

$$= -\frac{12d^2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - $$

Mathematica [A]
time = 6.97, size = 800, normalized size = 1.92

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] -1/2*(-2*b^4*c^4*ArcTanh[E^(I*(a + b*x))] + 24*b^2*c^2*d^2*ArcTanh[E^(I*(a
+ b*x))] + b^3*(c + d*x)^3*(4*d + b*(c + d*x)*Cot[a + b*x])*Csc[a + b*x] +
4*b^4*c^3*d*x*Log[1 - E^(I*(a + b*x))] - 24*b^2*c*d^3*x*Log[1 - E^(I*(a + b
*x))] + 6*b^4*c^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] - 12*b^2*d^4*x^2*Log[1 -
 E^(I*(a + b*x))] + 4*b^4*c*d^3*x^3*Log[1 - E^(I*(a + b*x))] + b^4*d^4*x^4*
Log[1 - E^(I*(a + b*x))] - 4*b^4*c^3*d*x*Log[1 + E^(I*(a + b*x))] + 24*b^2*
c*d^3*x*Log[1 + E^(I*(a + b*x))] - 6*b^4*c^2*d^2*x^2*Log[1 + E^(I*(a + b*x)
)] + 12*b^2*d^4*x^2*Log[1 + E^(I*(a + b*x))] - 4*b^4*c*d^3*x^3*Log[1 + E^(I
*(a + b*x))] - b^4*d^4*x^4*Log[1 + E^(I*(a + b*x))] + (4*I)*b*d*(c + d*x)*(
-6*d^2 + b^2*(c + d*x)^2)*PolyLog[2, -E^(I*(a + b*x))] - (4*I)*b*d*(c + d*x
)*(-6*d^2 + b^2*(c + d*x)^2)*PolyLog[2, E^(I*(a + b*x))] - 12*b^2*c^2*d^2*P
olyLog[3, -E^(I*(a + b*x))] + 24*d^4*PolyLog[3, -E^(I*(a + b*x))] - 24*b^2*
c*d^3*x*PolyLog[3, -E^(I*(a + b*x))] - 12*b^2*d^4*x^2*PolyLog[3, -E^(I*(a +
```

```
 b*x))] + 12*b^2*c^2*d^2*PolyLog[3, E^(I*(a + b*x))] - 24*d^4*PolyLog[3, E^
(I*(a + b*x))] + 24*b^2*c*d^3*x*PolyLog[3, E^(I*(a + b*x))] + 12*b^2*d^4*x^
2*PolyLog[3, E^(I*(a + b*x))] - (24*I)*b*c*d^3*PolyLog[4, -E^(I*(a + b*x))]
 - (24*I)*b*d^4*x*PolyLog[4, -E^(I*(a + b*x))] + (24*I)*b*c*d^3*PolyLog[4,
E^(I*(a + b*x))] + (24*I)*b*d^4*x*PolyLog[4, E^(I*(a + b*x))] + 24*d^4*Poly
Log[5, -E^(I*(a + b*x))] - 24*d^4*PolyLog[5, E^(I*(a + b*x))])/b^5
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1672 vs. $2(380) = 760$.
time = 0.20, size = 1673, normalized size = 4.02

method	result	size
risch	Expression too large to display	1673

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -6*I/b^2*polylog(2,-exp(I*(b*x+a)))*c^2*d^2*x-6*I/b^2*c*d^3*polylog(2,-exp(
I*(b*x+a)))*x^2+6*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2-2/b*c*d^3*ln(1-
exp(I*(b*x+a)))*x^3-2/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3+2/b*c*d^3*ln(exp(I
*(b*x+a))+1)*x^3+2/b^4*c*d^3*ln(exp(I*(b*x+a))+1)*a^3-12*d^4*polylog(3,-exp
(I*(b*x+a)))/b^5+12*d^4*polylog(3,exp(I*(b*x+a)))/b^5+24/b^4*c*d^3*a*arctan
h(exp(I*(b*x+a)))-12*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-2*I/b^2*d^4*poly
log(2,-exp(I*(b*x+a)))*x^3+12*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x+12*I/b
^4*d^4*polylog(2,-exp(I*(b*x+a)))*x+6*I/b^2*polylog(2,exp(I*(b*x+a)))*c^2*d
^2*x+1/b*c^4*arctanh(exp(I*(b*x+a)))+2*I/b^2*d^4*polylog(2,exp(I*(b*x+a)))*
x^3+12*I/b^4*c*d^3*polylog(2,-exp(I*(b*x+a)))-2*I/b^2*c^3*d*polylog(2,-exp(
I*(b*x+a)))+2*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))-12*I/b^4*c*d^3*polylog(
4,exp(I*(b*x+a)))+12*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))+1/b^2/(exp(2*I*
(b*x+a))-1)^2*(d^4*x^4*b*exp(3*I*(b*x+a))+4*c*d^3*x^3*b*exp(3*I*(b*x+a))+6*
c^2*d^2*x^2*b*exp(3*I*(b*x+a))+d^4*x^4*b*exp(I*(b*x+a))+4*c^3*d*x*b*exp(3*I
*(b*x+a))+4*c*d^3*x^3*b*exp(I*(b*x+a))-4*I*d^4*x^3*exp(3*I*(b*x+a))+c^4*b*e
xp(3*I*(b*x+a))+6*c^2*d^2*x^2*b*exp(I*(b*x+a))+4*I*d^4*x^3*exp(I*(b*x+a))+4
*c^3*d*x*b*exp(I*(b*x+a))-4*I*c^3*d*exp(3*I*(b*x+a))+4*I*c^3*d*exp(I*(b*x+a
))+c^4*b*exp(I*(b*x+a))+12*I*c^2*d^2*x*exp(I*(b*x+a))-12*I*c^2*d^2*x*exp(3*
I*(b*x+a))-12*I*c*d^3*x^2*exp(3*I*(b*x+a))+12*I*c*d^3*x^2*exp(I*(b*x+a)))-1
2*d^4*polylog(5,-exp(I*(b*x+a)))/b^5+12*d^4*polylog(5,exp(I*(b*x+a)))/b^5-6
/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))+6/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+
a)))+1/2/b^5*d^4*a^4*ln(1-exp(I*(b*x+a)))+6/b^3*d^4*polylog(3,-exp(I*(b*x+a
)))*x^2-6/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^2+6*d^4/b^3*ln(1-exp(I*(b*x+a
)))*x^2-6*d^4/b^5*ln(1-exp(I*(b*x+a)))*a^2-6*d^4/b^3*ln(exp(I*(b*x+a))+1)*x
^2-12/b^3*d^2*c^2*arctanh(exp(I*(b*x+a)))-12/b^5*d^4*a^2*arctanh(exp(I*(b*x
+a)))+6/b^5*d^4*ln(exp(I*(b*x+a))+1)*a^2-4/b^2*c^3*d*a*arctanh(exp(I*(b*x+a
)))-12*d^3/b^3*c*ln(exp(I*(b*x+a))+1)*x+12*d^3/b^3*c*ln(1-exp(I*(b*x+a)))*x
+12*d^3/b^4*c*ln(1-exp(I*(b*x+a)))*a-12*I*d^3/b^4*c*polylog(2,exp(I*(b*x+a)
```

```
))-12*I*d^4/b^4*polylog(2,exp(I*(b*x+a)))*x-1/2/b^5*d^4*a^4*ln(exp(I*(b*x+a
))+1)+1/b^5*d^4*a^4*arctanh(exp(I*(b*x+a)))-12/b^4*c*d^3*ln(exp(I*(b*x+a))+
1)*a+2/b^2*c^3*d*ln(exp(I*(b*x+a))+1)*a-3/b^3*c^2*d^2*ln(exp(I*(b*x+a))+1)*
a^2-4/b^4*c*d^3*a^3*arctanh(exp(I*(b*x+a)))+6/b^3*c^2*d^2*a^2*arctanh(exp(I
*(b*x+a)))-2/b*c^3*d*ln(1-exp(I*(b*x+a)))*x-2/b^2*c^3*d*ln(1-exp(I*(b*x+a))
)*a+2/b*c^3*d*ln(exp(I*(b*x+a))+1)*x-1/2/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+1/2
/b*d^4*ln(exp(I*(b*x+a))+1)*x^4+3/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2-3/b*c^
2*d^2*ln(1-exp(I*(b*x+a)))*x^2+3/b^3*c^2*d^2*ln(1-exp(I*(b*x+a)))*a^2+12/b^
3*c*d^3*polylog(3,-exp(I*(b*x+a)))*x-12/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))
*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7004 vs. $2(370) = 740$.
time = 3.92, size = 7004, normalized size = 16.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(c^4*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log
(cos(b*x + a) - 1)) - 4*a*c^3*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(
cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b + 6*a^2*c^2*d^2*(2*cos(b*x + a
)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^2
 - 4*a^3*c*d^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1)
 - log(cos(b*x + a) - 1))/b^3 + a^4*d^4*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1
) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^4 + 4*(2*((b*x + a)^4*
d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x
 + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^
3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a
) + ((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b*c*
d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b
*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6
*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 2*((b*x + a)^4*d^4 - 12*b^2*c^2*d^2
+ 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*
d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2
*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) -
 (-I*(b*x + a)^4*d^4 + 12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4 + 4
*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 + (
-I*a^2 + 2*I)*d^4)*(b*x + a)^2 + 4*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 + 3*(-
I*a^2 + 2*I)*b*c*d^3 + (I*a^3 - 6*I*a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 2
*(I*(b*x + a)^4*d^4 - 12*I*b^2*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*a^2*d^4 + 4*
(I*b*c*d^3 - I*a*d^4)*(b*x + a)^3 + 6*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + (I*a
^2 - 2*I)*d^4)*(b*x + a)^2 + 4*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*(I*a^2
- 2*I)*b*c*d^3 + (-I*a^3 + 6*I*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2
```

```
(sin(b*x + a), cos(b*x + a) + 1) + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4
+ (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(4*b*x + 4*a) - 2*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*cos(2*b*x + 2*a) - (-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3
- I*a^2*d^4)*sin(4*b*x + 4*a) - 2*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^
4)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*((b*x + a)
^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (
a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*
c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a) + ((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)
*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 +
4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*
x + a))*cos(4*b*x + 4*a) - 2*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x +
a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c
^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*
cos(2*b*x + 2*a) - (-I*(b*x + a)^4*d^4 + 4*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)
^3 + 6*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 + (-I*a^2 + 2*I)*d^4)*(b*x + a)^2 +
4*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 + 3*(-I*a^2 + 2*I)*b*c*d^3 + (I*a^3 - 6
*I*a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 2*(I*(b*x + a)^4*d^4 + 4*(I*b*c*d^
3 - I*a*d^4)*(b*x + a)^3 + 6*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + (I*a^2 - 2*I)
*d^4)*(b*x + a)^2 + 4*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*(I*a^2 - 2*I)*b*
c*d^3 + (-I*a^3 + 6*I*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x
+ a), -cos(b*x + a) + 1) - 4*(I*(b*x + a)^4*d^4 + 4*b^3*c^3*d - 12*a*b^2*c^
2*d^2 + 12*a^2*b*c*d^3 - 4*a^3*d^4 + 4*(I*b*c*d^3 + (-I*a + 1)*d^4)*(b*x +
a)^3 + 6*(I*b^2*c^2*d^2 + 2*(-I*a + 1)*b*c*d^3 + (I*a^2 - 2*a)*d^4)*(b*x +
a)^2 + 4*(I*b^3*c^3*d + 3*(-I*a + 1)*b^2*c^2*d^2 + 3*(I*a^2 - 2*a)*b*c*d^3
+ (-I*a^3 + 3*a^2)*d^4)*(b*x + a))*cos(3*b*x + 3*a) - 4*(I*(b*x + a)^4*d^4
- 4*b^3*c^3*d + 12*a*b^2*c^2*d^2 - 12*a^2*b*c*d^3 + 4*a^3*d^4 + 4*(I*b*c*d^
3 + (-I*a - 1)*d^4)*(b*x + a)^3 + 6*(I*b^2*c^2*d^2 + 2*(-I*a - 1)*b*c*d^3 +
 (I*a^2 + 2*a)*d^4)*(b*x + a)^2 + 4*(I*b^3*c^3*d + 3*(-I*a - 1)*b^2*c^2*d^2
 + 3*(I*a^2 + 2*a)*b*c*d^3 + (-I*a^3 - 3*a^2)*d^4)*(b*x + a))*cos(b*x + a)
- 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*(a^2 - 2)*b*c*d^3 -
(a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*
c*d^3 + (a^2 - 2)*d^4)*(b*x + a) + (b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)
^3*d^4 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x +
 a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*cos(4*b*x
+ 4*a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*(a^2 - 2)*b*c
*d^3 - (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (I*b^3*c^3*d -
3*I*a*b^2*c^2*d^2 + I*(b*x + a)^3*d^4 + 3*(I*a^2 - 2*I)*b*c*d^3 + (-I*a^3 +
 6*I*a)*d^4 + 3*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*
a*b*c*d^3 + (I*a^2 - 2*I)*d^4)*(b*x + a))*sin(4*b*x + 4*a) + 2*(-I*b^3*c^3*
d + 3*I*a*b^2*c^2*d^2 - I*(b*x + a)^3*d^4 + 3*(-I*a^2 + 2*I)*b*c*d^3 + (I*a
^3 - 6*I*a)*d^4 + 3*(-I*b*c*d^3 + I*a*d^4)*(b*x...
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2770 vs. $2(370) = 740$.

time = 3.09, size = 2770, normalized size = 6.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")

[Out] 1/4*(2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*cos(b*x + a) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + I*b^3*c^3*d - 6*I*b*c*d^3 + (-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - I*b^3*c^3*d + 6*I*b*c*d^3 - 3*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 + 3*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - I*b^3*c^3*d + 6*I*b*c*d^3 + (I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + I*b^3*c^3*d - 6*I*b*c*d^3 + 3*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 - 3*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + I*b^3*c^3*d - 6*I*b*c*d^3 + (-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - I*b^3*c^3*d + 6*I*b*c*d^3 - 3*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 + 3*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - I*b^3*c^3*d + 6*I*b*c*d^3 + (I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + I*b^3*c^3*d - 6*I*b*c*d^3 + 3*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 - 3*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 2)*b^2*c^2*d^2 - 4*(a^3 - 6*a)*b*c*d^3 + (a^4 - 12*a^2)*d^4 - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 2)*b^2*c^2*d^2 - 4*(a^3 - 6*a)*b*c*d^3 + (a^4 - 12*a^2)*d^4)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 2)*b^2*c^2*d^2 - 4*(a^3 - 6*a)*b*c*d^3 + (a^4 - 12*a^2)*d^4 - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 2)*b^2*c^2*d^2 - 4*(a^3 - 6*a)*b*c*d^3 + (a^4 - 12*a^2)*d^4)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a^2)*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a^2)*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a^2)*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*

```
b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a^2)*d^4
+ 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x
+ a)^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*log(-cos(b*x + a) - I*sin(b*x + a)
+ 1) + 24*(d^4*cos(b*x + a)^2 - d^4)*polylog(5, cos(b*x + a) + I*sin(b*x +
a)) + 24*(d^4*cos(b*x + a)^2 - d^4)*polylog(5, cos(b*x + a) - I*sin(b*x + a
)) - 24*(d^4*cos(b*x + a)^2 - d^4)*polylog(5, -cos(b*x + a) + I*sin(b*x + a
)) - 24*(d^4*cos(b*x + a)^2 - d^4)*polylog(5, -cos(b*x + a) - I*sin(b*x + a
)) - 24*(-I*b*d^4*x - I*b*c*d^3 + (I*b*d^4*x + I*b*c*d^3)*cos(b*x + a)^2)*p
olylog(4, cos(b*x + a) + I*sin(b*x + a)) - 24*(I*b*d^4*x + I*b*c*d^3 + (-I*
b*d^4*x - I*b*c*d^3)*cos(b*x + a)^2)*polylog(4, cos(b*x + a) - I*sin(b*x +
a)) - 24*(-I*b*d^4*x - I*b*c*d^3 + (I*b*d^4*x + I*b*c*d^3)*cos(b*x + a)^2)*
polylog(4, -cos(b*x + a) + I*sin(b*x + a)) - 24*(I*b*d^4*x + I*b*c*d^3 + (-
I*b*d^4*x - I*b*c*d^3)*cos(b*x + a)^2)*polylog(4, -cos(b*x + a) - I*sin(b*x
 + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4 - (b^2*d^4*x
^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4)*cos(b*x + a)^2)*polylog(3, cos(b*
x + a) + I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 -
2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4)*cos(b*x + a)^2)
*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*
x + b^2*c^2*d^2 - 2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^
4)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 12*(b^2*d^4
*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x +
 b^2*c^2*d^2 - 2*d^4)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) - I*sin(b*x
+ a)) + 8*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*sin
(b*x + a))/(b^5*cos(b*x + a)^2 - b^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^4 \cot^2(a + bx) \csc(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cot(b*x+a)**2*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**4*cot(a + b*x)**2*csc(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cot(b*x + a)^2*csc(b*x + a), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(a + b*x)^2*(c + d*x)^4)/sin(a + b*x),x)
[Out] \text{Hanged}
```

3.113 $\int (c+dx)^3 \cot^2(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=308

$$-\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3}+\frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx}{2b}$$

```
[Out] -6*d^2*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^3+(d*x+c)^3*arctanh(exp(I*(b*x+a))
)/b-3/2*d*(d*x+c)^2*csc(b*x+a)/b^2-1/2*(d*x+c)^3*cot(b*x+a)*csc(b*x+a)/b+3*
I*d^3*polylog(2,-exp(I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*
x+a)))/b^2-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4+3/2*I*d*(d*x+c)^2*polylog(
2,exp(I*(b*x+a)))/b^2+3*d^2*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^3-3*d^2*(d
*x+c)*polylog(3,exp(I*(b*x+a)))/b^3+3*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4-
3*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4
```

Rubi [A]

time = 0.26, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {4500, 4268, 2611, 6744, 2320, 6724, 4271, 2317, 2438}

$$\frac{3id^3\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{Li}_2\left(e^{i(a+bx)}\right)}{b^4}+\frac{3id^3\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}+\frac{3d^2(c+dx)\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}-\frac{3d^2(c+dx)\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3}-\frac{3id(c+dx)^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{2b^2}+\frac{3id(c+dx)^2\text{Li}_2\left(e^{i(a+bx)}\right)}{2b^2}-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}+\frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] (-6*d^2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^3 + ((c + d*x)^3*ArcTanh[E^(I
*(a + b*x))])/b - (3*d*(c + d*x)^2*Csc[a + b*x])/(2*b^2) - ((c + d*x)^3*Cot
[a + b*x]*Csc[a + b*x])/(2*b) + ((3*I)*d^3*PolyLog[2, -E^(I*(a + b*x))])/b^
4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d^3
*PolyLog[2, E^(I*(a + b*x))])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^(
I*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 - (
3*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 + ((3*I)*d^3*PolyLog[4, -E
^(I*(a + b*x))])/b^4 - ((3*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4500

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \cot^2(a+bx)\csc(a+bx)\,dx = -\int (c+dx)^3\csc(a+bx)\,dx + \int (c+dx)^3\csc^3(a+bx)\,dx$$

$$= \frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\csc(a+bx)}{2b^2} - \frac{(c+dx}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3}{}$$

Mathematica [A]
time = 4.74, size = 478, normalized size = 1.55

$$-\frac{-2b^3c^3\tanh^{-1}\left(e^{i(a+bx)}\right)+12bcd^2\tanh^{-1}\left(e^{i(a+bx)}\right)+b^2(c+dx)^2(3d+b(c+dx)\cot(a+bx))\csc(a+bx)+3b^3c^2dx\log\left(1-e^{i(a+bx)}\right)-6bd^3x\log\left(1-e^{i(a+bx)}\right)+3b^3cd^2x^2\log\left(1-e^{i(a+bx)}\right)+b^3d^3x^3\log\left(1-e^{i(a+bx)}\right)-3b^3c^2dx\log\left(1+e^{i(a+bx)}\right)+6bd^3x\log\left(1+e^{i(a+bx)}\right)-3b^3cd^2x^2\log\left(1+e^{i(a+bx)}\right)-b^3d^3x^3\log\left(1+e^{i(a+bx)}\right)+3id(-2d^2+b^2(c+dx)^2)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)-3id(-2d^2+b^2(c+dx)^2)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)-6bcd^2\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)-6bd^3x\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)+6bcd^2\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)+6bd^3x\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)-6id^3\mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)+6id^3\mathrm{PolyLog}\left(4,e^{i(a+bx)}\right)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] -1/2*(-2*b^3*c^3*ArcTanh[E^(I*(a + b*x))] + 12*b*c*d^2*ArcTanh[E^(I*(a + b*
x))] + b^2*(c + d*x)^2*(3*d + b*(c + d*x)*Cot[a + b*x])*Csc[a + b*x] + 3*b^
3*c^2*d*x*Log[1 - E^(I*(a + b*x))] - 6*b*d^3*x*Log[1 - E^(I*(a + b*x))] + 3
*b^3*c*d^2*x^2*Log[1 - E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1 - E^(I*(a + b*x
))] - 3*b^3*c^2*d*x*Log[1 + E^(I*(a + b*x))] + 6*b*d^3*x*Log[1 + E^(I*(a +
b*x))] - 3*b^3*c*d^2*x^2*Log[1 + E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + E^(
I*(a + b*x))] + (3*I)*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, -E^(I*(a + b*
```

```
x))] - (3*I)*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, E^(I*(a + b*x))] - 6*b
*c*d^2*PolyLog[3, -E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, -E^(I*(a + b*x))
] + 6*b*c*d^2*PolyLog[3, E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, E^(I*(a +
b*x))] - (6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, E^(I
*(a + b*x))])/b^4
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1055 vs. $2(274) = 548$.
time = 0.15, size = 1056, normalized size = 3.43

method	result	size
risch	Expression too large to display	1056

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -3/b^2*c^2*d*a*arctanh(exp(I*(b*x+a)))+3/b^3*c*d^2*a^2*arctanh(exp(I*(b*x+a
)))-3/2/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))+1)+1/2/b^4*d^3*ln(exp(I*(b*x+a))+1)
*a^3+3*I/b^2*polylog(2,exp(I*(b*x+a)))*c*d^2*x-3*I/b^2*polylog(2,-exp(I*(b*
x+a)))*c*d^2*x-3/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+3/b^3*c*d^2*polylog(3,
-exp(I*(b*x+a)))+3/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x-3/b^3*d^3*polylog(3
,exp(I*(b*x+a)))*x-1/b^4*d^3*a^3*arctanh(exp(I*(b*x+a)))+3*I*d^3*polylog(2,
-exp(I*(b*x+a)))/b^4+3*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4+6/b^4*d^3*a*arc
tanh(exp(I*(b*x+a)))-6/b^3*d^2*c*arctanh(exp(I*(b*x+a)))-3/b^4*d^3*ln(exp(I
*(b*x+a))+1)*a+3*d^3/b^3*ln(1-exp(I*(b*x+a)))*x+3*d^3/b^4*ln(1-exp(I*(b*x+a
)))*a-3*d^3/b^3*ln(exp(I*(b*x+a))+1)*x+1/2/b*d^3*ln(exp(I*(b*x+a))+1)*x^3-1
/2/b*d^3*ln(1-exp(I*(b*x+a)))*x^3-1/2/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+1/b*
c^3*arctanh(exp(I*(b*x+a)))+1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^3*x^3*b*exp(3*I
*(b*x+a))+3*c*d^2*x^2*b*exp(3*I*(b*x+a))+3*c^2*d*x*b*exp(3*I*(b*x+a))+d^3*x
^3*b*exp(I*(b*x+a))+c^3*b*exp(3*I*(b*x+a))+3*c*d^2*x^2*b*exp(I*(b*x+a))-3*I
*d^3*x^2*exp(3*I*(b*x+a))+3*c^2*d*x*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(3*I*(b
*x+a))+c^3*b*exp(I*(b*x+a))-3*I*c^2*d*exp(3*I*(b*x+a))+3*I*d^3*x^2*exp(I*(b
*x+a))+6*I*c*d^2*x*exp(I*(b*x+a))+3*I*c^2*d*exp(I*(b*x+a)))+3/2/b*c^2*d*ln(
exp(I*(b*x+a))+1)*x-3/2/b*c^2*d*ln(1-exp(I*(b*x+a)))*x-3/2/b^2*c^2*d*ln(1-e
xp(I*(b*x+a)))*a+3/2/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-3/2/b*c*d^2*ln(1-exp(
I*(b*x+a)))*x^2+3/2/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2+3/2/b^2*c^2*d*ln(exp
(I*(b*x+a))+1)*a-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-3*I*d^3*polylog(4,ex
p(I*(b*x+a)))/b^4+3/2*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-3/2*I/b^2*d^3
*polylog(2,-exp(I*(b*x+a)))*x^2-3/2*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))+
3/2*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3887 vs. $2(264) = 528$.
time = 1.38, size = 3887, normalized size = 12.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(c^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log
(cos(b*x + a) - 1)) - 3*a*c^2*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(
cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*cos(b*x + a)/
(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^2 -
 a^3*d^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log
(cos(b*x + a) - 1))/b^3 + 4*(2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*
(b*x + a) + ((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b
*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(4*b*
x + 4*a) - 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(
b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(2*b
*x + 2*a) - (-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + 3*(-I*b*c*d^2 +
 I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 + 2*I)*d^
3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a
*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2
 + (I*a^2 - 2*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), co
s(b*x + a) + 1) + 12*(b*c*d^2 - a*d^3 + (b*c*d^2 - a*d^3)*cos(4*b*x + 4*a)
- 2*(b*c*d^2 - a*d^3)*cos(2*b*x + 2*a) - (-I*b*c*d^2 + I*a*d^3)*sin(4*b*x +
 4*a) - 2*(I*b*c*d^2 - I*a*d^3)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos
(b*x + a) - 1) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(
b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a) + ((b*x + a)^3*d^3 + 3*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*
(b*x + a))*cos(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x
 + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(2*b*x
+ 2*a) - (-I*(b*x + a)^3*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I
*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 + 2*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a
) - 2*(I*(b*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c
^2*d - 2*I*a*b*c*d^2 + (I*a^2 - 2*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arct
an2(sin(b*x + a), -cos(b*x + a) + 1) - 4*(I*(b*x + a)^3*d^3 + 3*b^2*c^2*d -
 6*a*b*c*d^2 + 3*a^2*d^3 + 3*(I*b*c*d^2 + (-I*a + 1)*d^3)*(b*x + a)^2 + 3*(
I*b^2*c^2*d + 2*(-I*a + 1)*b*c*d^2 + (I*a^2 - 2*a)*d^3)*(b*x + a))*cos(3*b*
x + 3*a) - 4*(I*(b*x + a)^3*d^3 - 3*b^2*c^2*d + 6*a*b*c*d^2 - 3*a^2*d^3 + 3
*(I*b*c*d^2 + (-I*a - 1)*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d + 2*(-I*a - 1)*b
*c*d^2 + (I*a^2 + 2*a)*d^3)*(b*x + a))*cos(b*x + a) - 6*(b^2*c^2*d - 2*a*b*
c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) + (
b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*
d^3)*(b*x + a))*cos(4*b*x + 4*a) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2
*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (I
*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + (I*a^2 - 2*I)*d^3 + 2*(I*b
*c*d^2 - I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*(-I*b^2*c^2*d + 2*I*a*b*c
*d^2 - I*(b*x + a)^2*d^3 + (-I*a^2 + 2*I)*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b
```

```
*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + 6*(b^2*c^2*d - 2*a*b*c
*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) + (b
^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d
^3)*(b*x + a))*cos(4*b*x + 4*a) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*
d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-I
*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 + (-I*a^2 + 2*I)*d^3 + 2*(-I
*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(I*b^2*c^2*d - 2*I*a*b*
c*d^2 + I*(b*x + a)^2*d^3 + (I*a^2 - 2*I)*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*
x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b*x + a)^3*d^3 + 6*
I*b*c*d^2 - 6*I*a*d^3 - 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 - 3*(I*b^2*c^2*
d - 2*I*a*b*c*d^2 + (I*a^2 - 2*I)*d^3)*(b*x + a) + (-I*(b*x + a)^3*d^3 + 6*
I*b*c*d^2 - 6*I*a*d^3 - 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 - 3*(I*b^2*c^2*
d - 2*I*a*b*c*d^2 + (I*a^2 - 2*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 2*(-I*
(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x +
 a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 + 2*I)*d^3)*(b*x + a))*co
s(2*b*x + 2*a) + ((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^
3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*sin
(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d
^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*si
n(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) +
 (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 - 3*(-I*b*c*d^2 + I*a*d^3)*(b
*x + a)^2 - 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 + 2*I)*d^3)*(b*x + a)
 + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 - 3*(-I*b*c*d^2 + I*a*d^3)*
(b*x + a)^2 - 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 + 2*I)*d^3)*(b*x +
a))*cos(4*b*x + 4*a) - 2*(I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + 3*(
I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2...
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1742 vs. $2(264) = 528$.
time = 2.68, size = 1742, normalized size = 5.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x +
a) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I*d^3 + (-I*b^2*d
^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^2)*dilog(cos
(b*x + a) + I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c
^2*d + 2*I*d^3 + (I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I*d^3)*
cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2
*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I*d^3 + (-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x
- I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x +
a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d + 2*I*d^3 + (I*b^2*
```

```
d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^2)*dilog(-c
os(b*x + a) - I*sin(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 -
6*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c
^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*log(cos(b*x
+ a) + I*sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b
*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*
d - 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*log(cos(b*x + a
) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 -
(a^3 - 6*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6
*a)*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)
+ (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3 - (b^3*c
^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2)
*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c
*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 - (b^3*d^3*x^3 +
 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3
*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*log(-cos(b
*x + a) + I*sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*
cos(b*x + a)^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*log(-cos(b*x + a) - I*sin(b*x +
 a) + 1) - 6*(I*d^3*cos(b*x + a)^2 - I*d^3)*polylog(4, cos(b*x + a) + I*sin
(b*x + a)) - 6*(-I*d^3*cos(b*x + a)^2 + I*d^3)*polylog(4, cos(b*x + a) - I*
sin(b*x + a)) - 6*(I*d^3*cos(b*x + a)^2 - I*d^3)*polylog(4, -cos(b*x + a) +
 I*sin(b*x + a)) - 6*(-I*d^3*cos(b*x + a)^2 + I*d^3)*polylog(4, -cos(b*x +
a) - I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x +
 a)^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2 - (
b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)
) - 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -
cos(b*x + a) + I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)
*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 6*(b^2*d^3*x^
2 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(b*x + a))/(b^4*cos(b*x + a)^2 - b^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cot^2(a+bx)\csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cot(b*x+a)**2*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*cot(a + b*x)**2*csc(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cot(b*x + a)^2*csc(b*x + a), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(a + b*x)^2*(c + d*x)^3)/sin(a + b*x),x)
```

```
[Out] \text{Hanged}
```

3.114 $\int (c+dx)^2 \cot^2(a+bx) \csc(a+bx) \, dx$

Optimal. Leaf size=179

$$\frac{(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d^2 \tanh^{-1}(\cos(a+bx))}{b^3}-\frac{d(c+dx)\csc(a+bx)}{b^2}-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

```
[Out] (d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-d^2*arctanh(cos(b*x+a))/b^3-d*(d*x+c)*c
sc(b*x+a)/b^2-1/2*(d*x+c)^2*cot(b*x+a)*csc(b*x+a)/b-I*d*(d*x+c)*polylog(2,-
exp(I*(b*x+a)))/b^2+I*d*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^2+d^2*polylog(3
,-exp(I*(b*x+a)))/b^3-d^2*polylog(3,exp(I*(b*x+a)))/b^3
```

Rubi [A]
time = 0.17, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {4500, 4268, 2611, 2320, 6724, 4271, 3855}

$$\frac{d^2\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}-\frac{d^2\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{id(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}+\frac{id(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\csc(a+bx)}{b^2}+\frac{(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] ((c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (d^2*ArcTanh[Cos[a + b*x]])/b^3
- (d*(c + d*x)*Csc[a + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]*Csc[a + b*x])/
(2*b) - (I*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 + (I*d*(c + d*x)*P
olyLog[2, E^(I*(a + b*x))])/b^2 + (d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 -
(d^2*PolyLog[3, E^(I*(a + b*x))])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4500

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \cot^2(a+bx)\csc(a+bx)\,dx = -\int (c+dx)^2 \csc(a+bx)\,dx + \int (c+dx)^2 \csc^3(a+bx)\,dx$$
$$= \frac{2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d(c+dx)\csc(a+bx)}{b^2} - \frac{(c+dx)^2}{}$$
$$= \frac{(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2 \tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx}{}$$
$$= \frac{(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2 \tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx}{}$$
$$= \frac{(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2 \tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx}{}$$
$$= \frac{(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2 \tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 423 vs. $2(179) = 358$.
time = 6.92, size = 423, normalized size = 2.36

$$-\frac{d(c+dx)\csc(a)}{b^2} + \frac{(-c^2-2cdx-d^2x^2)\csc^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} + \frac{2b^2c^2\tanh^{-1}\left(e^{i(a+bx)}\right) - 4d^2\tanh^{-1}\left(e^{i(a+bx)}\right) - 2b^2cdx\log\left(1-e^{i(a+bx)}\right) - b^2d^2x^2\log\left(1-e^{i(a+bx)}\right) + 2b^2cdx\log\left(1+e^{i(a+bx)}\right) + b^2d^2x^2\log\left(1+e^{i(a+bx)}\right) - 2ibd(c+dx)\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right) + 2ibd(c+dx)\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right) + 2d^2\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right) - 2d^2\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)}{2b^3} + \frac{(c^2+2cdx+d^2x^2)\sec^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} + \frac{\sec\left(\frac{a}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\left(-cd\sin\left(\frac{bx}{2}\right)-d^2x\sin\left(\frac{bx}{2}\right)\right)}{2b^2} + \frac{\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\left(cd\sin\left(\frac{bx}{2}\right)+d^2x\sin\left(\frac{bx}{2}\right)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] -((d*(c + d*x)*Csc[a])/b^2) + ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a/2 + (b*x)/2
]^2)/(8*b) + (2*b^2*c^2*ArcTanh[E^(I*(a + b*x))] - 4*d^2*ArcTanh[E^(I*(a +
b*x))] - 2*b^2*c*d*x*Log[1 - E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 - E^(I*(a
 + b*x))] + 2*b^2*c*d*x*Log[1 + E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 + E^(I
*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + (2*I)*b*d
*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] + 2*d^2*PolyLog[3, -E^(I*(a + b*x))]
 - 2*d^2*PolyLog[3, E^(I*(a + b*x))])/(2*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*
Sec[a/2 + (b*x)/2]^2)/(8*b) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c*d*Sin[(b*x)
/2]) - d^2*x*Sin[(b*x)/2]))/(2*b^2) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*(c*d*Sin
[(b*x)/2] + d^2*x*Sin[(b*x)/2]))/(2*b^2)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(165) = 330$.
time = 0.12, size = 546, normalized size = 3.05

method	result
risch	$\frac{d^2x^2b\,e^{3i(bx+a)}+2cdxb\,e^{3i(bx+a)}+c^2b\,e^{3i(bx+a)}+d^2x^2b\,e^{i(bx+a)}+2cdxb\,e^{i(bx+a)}-2id^2x\,e^{3i(bx+a)}+c^2b\,e^{i(bx+a)}-2idc\,e^{3i(bx+a)}+2id}{b^2\left(e^{2i(bx+a)}-1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x,method=_RETURNVERBOSE)

[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^2*x^2*b*exp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I*(b*x+a))+c^2*b*exp(3*I*(b*x+a))+d^2*x^2*b*exp(I*(b*x+a))+2*c*d*x*b*exp(I*(b*x+a))-2*I*d^2*x*exp(3*I*(b*x+a))+c^2*b*exp(I*(b*x+a))-2*I*d*c*exp(3*I*(b*x+a))+2*I*d^2*x*exp(I*(b*x+a))+2*I*d*c*exp(I*(b*x+a)))-d^2*polylog(3,exp(I*(b*x+a)))/b^3+d^2*polylog(3,-exp(I*(b*x+a)))/b^3-2/b^3*d^2*arctanh(exp(I*(b*x+a)))+1/2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2-1/2/b^3*d^2*ln(exp(I*(b*x+a))+1)*a^2+1/b*c^2*arctanh(exp(I*(b*x+a)))+1/b^3*d^2*a^2*arctanh(exp(I*(b*x+a)))-1/2/b*d^2*ln(1-exp(I*(b*x+a)))*x^2+I/b^2*polylog(2,exp(I*(b*x+a)))*d^2*x+1/2/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+I/b^2*c*d*polylog(2,exp(I*(b*x+a)))-1/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+1/b*c*d*ln(exp(I*(b*x+a))+1)*x+1/b^2*c*d*ln(exp(I*(b*x+a))+1)*a-1/b*c*d*ln(1-exp(I*(b*x+a)))*x-2/b^2*c*d*a*arctanh(exp(I*(b*x+a)))-I/b^2*polylog(2,-exp(I*(b*x+a)))*d^2*x-I/b^2*c*d*polylog(2,-exp(I*(b*x+a))))

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1938 vs. $2(161) = 322$.
time = 0.55, size = 1938, normalized size = 10.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")

[Out] 1/4*(c^2*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1)) - 2*a*c*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b + a^2*d^2*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^2 + 4*(2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2 + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(4*b*x + 4*a) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 4*(d^2*cos(4*b*x + 4*a) - 2*d^2*cos(2*b*x + 2*a) + I*d^2*sin(4*b*x + 4*a) - 2*I*d^2*sin(2*b*x + 2*a) + d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*(I*(b*x + a)^2*d^2 + 2*b

```
*c*d - 2*a*d^2 + 2*(I*b*c*d + (-I*a + 1)*d^2)*(b*x + a))*cos(3*b*x + 3*a) -
 4*(I*(b*x + a)^2*d^2 - 2*b*c*d + 2*a*d^2 + 2*(I*b*c*d + (-I*a - 1)*d^2)*(b
*x + a))*cos(b*x + a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x +
a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*
b*x + 2*a) + (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(4*b*x + 4*a) + 2*(-I
*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a
)) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos
(4*b*x + 4*a) - 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-I*b*
c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(4*b*x + 4*a) - 2*(I*b*c*d + I*(b*x + a
)*d^2 - I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b*x + a)^2
*d^2 - 2*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2 + (-I*(b*x + a)^2*d^2 - 2*
(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*cos(4*b*x + 4*a) - 2*(-I*(b*x + a)
^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a) + 2*I*d^2)*cos(2*b*x + 2*a) + ((b
*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*sin(4*b*x + 4*a) - 2*(
(b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*sin(2*b*x + 2*a))*lo
g(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*(b*x + a)^2*d^
2 - 2*(-I*b*c*d + I*a*d^2)*(b*x + a) - 2*I*d^2 + (I*(b*x + a)^2*d^2 - 2*(-I
*b*c*d + I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(4*b*x + 4*a) - 2*(I*(b*x + a)^2*
d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(2*b*x + 2*a) - ((b*x +
 a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*sin(4*b*x + 4*a) + 2*((b*x
 + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*sin(2*b*x + 2*a))*log(co
s(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*(I*d^2*cos(4*b*x +
4*a) - 2*I*d^2*cos(2*b*x + 2*a) - d^2*sin(4*b*x + 4*a) + 2*d^2*sin(2*b*x +
2*a) + I*d^2)*polylog(3, -e^(I*b*x + I*a)) - 4*(-I*d^2*cos(4*b*x + 4*a) + 2
*I*d^2*cos(2*b*x + 2*a) + d^2*sin(4*b*x + 4*a) - 2*d^2*sin(2*b*x + 2*a) - I
*d^2)*polylog(3, e^(I*b*x + I*a)) + 4*((b*x + a)^2*d^2 - 2*I*b*c*d + 2*I*a*
d^2 + 2*(b*c*d - (a + I)*d^2)*(b*x + a))*sin(3*b*x + 3*a) + 4*((b*x + a)^2*
d^2 + 2*I*b*c*d - 2*I*a*d^2 + 2*(b*c*d - (a - I)*d^2)*(b*x + a))*sin(b*x +
a))/(-4*I*b^2*cos(4*b*x + 4*a) + 8*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(4*b*x
 + 4*a) - 8*b^2*sin(2*b*x + 2*a) - 4*I*b^2))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 970 vs. $2(161) = 322$.
time = 2.71, size = 970, normalized size = 5.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a) - 2*(I*b*d^2*x +
I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x + a) + I*sin
(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2
)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d + (-I*b*d^2
*x - I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 2*(-I
```

```
*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x +
 a) - I*sin(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - 2*d^2)*log(cos(b*x + a)
+ I*sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - 2*d^2)*log(cos(b*x + a)
- I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2 - (b^2*c^2 - 2
*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin
(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2 - (b^2*c^2 - 2*a*b*
c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x
+ a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x
^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) +
 I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (
b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-cos(b
*x + a) - I*sin(b*x + a) + 1) - 2*(d^2*cos(b*x + a)^2 - d^2)*polylog(3, cos
(b*x + a) + I*sin(b*x + a)) - 2*(d^2*cos(b*x + a)^2 - d^2)*polylog(3, cos(b
*x + a) - I*sin(b*x + a)) + 2*(d^2*cos(b*x + a)^2 - d^2)*polylog(3, -cos(b*
x + a) + I*sin(b*x + a)) + 2*(d^2*cos(b*x + a)^2 - d^2)*polylog(3, -cos(b*x
 + a) - I*sin(b*x + a)) + 4*(b*d^2*x + b*c*d)*sin(b*x + a))/(b^3*cos(b*x +
a)^2 - b^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \cot^2(a+bx) \csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cot(b*x+a)**2*csc(b*x+a),x)
[Out] Integral((c + d*x)**2*cot(a + b*x)**2*csc(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^2*cot(b*x + a)^2*csc(b*x + a), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(a + b*x)^2*(c + d*x)^2)/sin(a + b*x),x)
[Out] \text{Hanged}
```

3.115 $\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=108

$$\frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}-\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}+\frac{id\text{P}}{}$$

```
[Out] (d*x+c)*arctanh(exp(I*(b*x+a)))/b-1/2*d*csc(b*x+a)/b^2-1/2*(d*x+c)*cot(b*x+
a)*csc(b*x+a)/b-1/2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2+1/2*I*d*polylog(2,ex
p(I*(b*x+a)))/b^2
```

Rubi [A]
time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4500, 4268, 2317, 2438, 4270}

$$-\frac{id\text{Li}_2\left(-e^{i(a+bx)}\right)}{2b^2}+\frac{id\text{Li}_2\left(e^{i(a+bx)}\right)}{2b^2}-\frac{d\csc(a+bx)}{2b^2}+\frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] ((c + d*x)*ArcTanh[E^(I*(a + b*x))])/b - (d*Csc[a + b*x])/(2*b^2) - ((c + d
*x)*Cot[a + b*x]*Csc[a + b*x])/(2*b) - ((I/2)*d*PolyLog[2, -E^(I*(a + b*x))
])/b^2 + ((I/2)*d*PolyLog[2, E^(I*(a + b*x))])/b^2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4500

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rubi steps

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx = -\int (c+dx)\csc(a+bx)\,dx + \int (c+dx)\csc^3(a+bx)\,dx$$

$$= \frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\,c}{2b}$$

$$= \frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\,\mathrm{cs}}{2b}$$

$$= \frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\,\mathrm{cs}}{2b}$$

$$= \frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\,\mathrm{cs}}{2b}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 260 vs. $2(108) = 216$.
time = 1.82, size = 260, normalized size = 2.41

$$-\frac{d\cot\left(\frac{1}{2}(a+bx)\right)}{4b^2} - \frac{c\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{dx\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{c\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{c\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{ad\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)}{2b^2} - \frac{d\left((a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)+i\left(\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)-\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)\right)\right)}{2b^2} + \frac{c\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{dx\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{d\tan\left(\frac{1}{2}(a+bx)\right)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] -1/4*(d*Cot[(a + b*x)/2])/b^2 - (c*Csc[(a + b*x)/2]^2)/(8*b) - (d*x*Csc[(a
+ b*x)/2]^2)/(8*b) + (c*Log[Cos[(a + b*x)/2]])/(2*b) - (c*Log[Sin[(a + b*x)
/2]])/(2*b) + (a*d*Log[Tan[(a + b*x)/2]])/(2*b^2) - (d*((a + b*x)*(Log[1 -
E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a + b*x
))] - PolyLog[2, E^(I*(a + b*x))])))/(2*b^2) + (c*Sec[(a + b*x)/2]^2)/(8*b)
 + (d*x*Sec[(a + b*x)/2]^2)/(8*b) - (d*Tan[(a + b*x)/2])/(4*b^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(92) = 184$.
time = 0.10, size = 246, normalized size = 2.28

method	result
risch	$\frac{dxb\,\mathrm{e}^{3i(bx+a)}+cb\,\mathrm{e}^{3i(bx+a)}+dxb\,\mathrm{e}^{i(bx+a)}+cb\,\mathrm{e}^{i(bx+a)}-id\,\mathrm{e}^{3i(bx+a)}+id\,\mathrm{e}^{i(bx+a)}}{b^2\left(\mathrm{e}^{2i(bx+a)}-1\right)^2}+\frac{c\,\mathrm{arctanh}\left(\mathrm{e}^{i(bx+a)}\right)}{b}-\frac{d\ln\left(1-\mathrm{e}^{i(bx+a)}\right)x}{2b}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2*(d*x*b*exp(3*I*(b*x+a))+c*b*exp(3*I*(b*x+a))+d
*x*b*exp(I*(b*x+a))+c*b*exp(I*(b*x+a))-I*d*exp(3*I*(b*x+a))+I*d*exp(I*(b*x+
a)))+1/b*c*arctanh(exp(I*(b*x+a)))-1/2/b*d*ln(1-exp(I*(b*x+a)))*x-1/2/b^2*d
*ln(1-exp(I*(b*x+a)))*a+1/2*I*d*polylog(2,exp(I*(b*x+a)))/b^2+1/2/b*d*ln(ex
p(I*(b*x+a))+1)*x+1/2/b^2*d*ln(exp(I*(b*x+a))+1)*a-1/2*I*d*polylog(2,-exp(I
*(b*x+a)))/b^2-1/b^2*d*a*arctanh(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(88) = 176$.
time = 0.40, size = 762, normalized size = 7.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] (2*(b*d*x + b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) - 2*(b*d*x + b*c)*cos(2*b*
x + 2*a) - (-I*b*d*x - I*b*c)*sin(4*b*x + 4*a) - 2*(I*b*d*x + I*b*c)*sin(2*
b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(b*c*cos(4*b*x + 4*
a) - 2*b*c*cos(2*b*x + 2*a) + I*b*c*sin(4*b*x + 4*a) - 2*I*b*c*sin(2*b*x +
2*a) + b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*(b*d*x*cos(4*b*x +
4*a) - 2*b*d*x*cos(2*b*x + 2*a) + I*b*d*x*sin(4*b*x + 4*a) - 2*I*b*d*x*sin(
2*b*x + 2*a) + b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*(I*b*d*x
 + I*b*c + d)*cos(3*b*x + 3*a) - 4*(I*b*d*x + I*b*c - d)*cos(b*x + a) - 2*(
d*cos(4*b*x + 4*a) - 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*si
n(2*b*x + 2*a) + d)*dilog(-e^(I*b*x + I*a)) + 2*(d*cos(4*b*x + 4*a) - 2*d*c
os(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x + 2*a) + d)*dilog(
e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(4*b*x + 4*a)
- 2*(-I*b*d*x - I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*b*x + 4*a) -
2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*c
os(b*x + a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(4*b*x + 4*a) -
2*(I*b*d*x + I*b*c)*cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*b*x + 4*a) + 2*(
b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(
b*x + a) + 1) + 4*(b*d*x + b*c - I*d)*sin(3*b*x + 3*a) + 4*(b*d*x + b*c + I
```

```
*d)*sin(b*x + a))/(-4*I*b^2*cos(4*b*x + 4*a) + 8*I*b^2*cos(2*b*x + 2*a) + 4
*b^2*sin(4*b*x + 4*a) - 8*b^2*sin(2*b*x + 2*a) - 4*I*b^2)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(88) = 176$.
time = 3.16, size = 454, normalized size = 4.20

$$\frac{2\,(bdx+bc)\cos(bx+a)+\left(i\,d\cos(bx+a)^2-i\,d\right)\mathrm{Li}_2(\cos(bx+a)+i\,\sin(bx+a))+\left(-i\,d\cos(bx+a)^2+i\,d\right)\mathrm{Li}_2(\cos(bx+a)-i\,\sin(bx+a))+\left(i\,d\cos(bx+a)^2-i\,d\right)\mathrm{Li}_2(-\cos(bx+a)+i\,\sin(bx+a))+\left(-i\,d\cos(bx+a)^2+i\,d\right)\mathrm{Li}_2(-\cos(bx+a)-i\,\sin(bx+a))-\left(bdx-(bdx+bc)\cos(bx+a)^2+bc\right)\log(\cos(bx+a)+i\,\sin(bx+a)+1)-\left(bdx-(bdx+bc)\cos(bx+a)^2+bc\right)\log(\cos(bx+a)-i\,\sin(bx+a)+1)-\left((bc-ad)\cos(bx+a)^2-bc+ad\right)\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}i\,\sin(bx+a)+\frac{1}{2}\right)-\left((bc-ad)\cos(bx+a)^2-bc+ad\right)\log\left(-\frac{1}{2}\cos(bx+a)-\frac{1}{2}i\,\sin(bx+a)+\frac{1}{2}\right)+\left(bdx-(bdx+ad)\cos(bx+a)^2+ad\right)\log(-\cos(bx+a)+i\,\sin(bx+a)+1)+\left(bdx-(bdx+ad)\cos(bx+a)^2+ad\right)\log(-\cos(bx+a)-i\,\sin(bx+a)+1)+2\,d\sin(bx+a)}{4\,(b^2\cos(bx+a)^2-b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*d*x + b*c)*cos(b*x + a) + (I*d*cos(b*x + a)^2 - I*d)*dilog(cos(b*
x + a) + I*sin(b*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(cos(b*x + a) -
 I*sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(-cos(b*x + a) + I*sin(b
*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)
) - (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a) + I*sin(b
*x + a) + 1) - (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a
) - I*sin(b*x + a) + 1) - ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(-1/2
*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - ((b*c - a*d)*cos(b*x + a)^2 - b
*c + a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b*d*x - (b*d
*x + a*d)*cos(b*x + a)^2 + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (
b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*log(-cos(b*x + a) - I*sin(b*x +
 a) + 1) + 2*d*sin(b*x + a))/(b^2*cos(b*x + a)^2 - b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)**2*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)*cot(a + b*x)**2*csc(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*cot(b*x + a)^2*csc(b*x + a), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

$$\text{Hanged}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(a + b*x)^2*(c + d*x))/sin(a + b*x),x)
[Out] \text{Hanged}
```

3.116 $\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=39

$$-\mathrm{Int}\left(\frac{\csc(a+bx)}{c+dx},x\right)+\mathrm{Int}\left(\frac{\csc^3(a+bx)}{c+dx},x\right)$$

```
[Out] -Unintegrable(csc(b*x+a)/(d*x+c),x)+Unintegrable(csc(b*x+a)^3/(d*x+c),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x),x]
```

```
[Out] -Defer[Int][Csc[a + b*x]/(c + d*x), x] + Defer[Int][Csc[a + b*x]^3/(c + d*x
), x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx=-\int \frac{\csc(a+bx)}{c+dx}\,dx+\int \frac{\csc^3(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 35.64, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x), x]
```

Maple [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(\cot^2(bx+a)\right)\csc(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x)
```

```
[Out] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] (((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - d*sin(3*b*x
 + 3*a) + d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x + b*c)
*cos(2*b*x + 2*a) - 2*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d*x + b*
c)*cos(b*x + a) + d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c)*cos(b*x
+ a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2
*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 -
4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) +
 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b
*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2
*b*x + 2*a))*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*si
n(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)^2 + (b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x + a)^2 + 2*(b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)), x) -
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x +
2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*
a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x +
2*a))*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x +
 a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)^2 + (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x + a)^2 - 2*(b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)), x) + (d*cos(
3*b*x + 3*a) - d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a) + (b*d*x + b
*c)*sin(b*x + a))*sin(4*b*x + 4*a) + (2*d*cos(2*b*x + 2*a) - 2*(b*d*x + b*c
)*sin(2*b*x + 2*a) - d)*sin(3*b*x + 3*a) + 2*(d*cos(b*x + a) - (b*d*x + b*c
)*sin(b*x + a))*sin(2*b*x + 2*a) + d*sin(b*x + a))/(b^2*d^2*x^2 + 2*b^2*c*d
```

```
*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4
*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4
*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(cot(b*x + a)^2*csc(b*x + a)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2*csc(b*x+a)/(d*x+c),x)
[Out] Integral(cot(a + b*x)**2*csc(a + b*x)/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(cot(b*x + a)^2*csc(b*x + a)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot(a+b\,x)^2}{\sin\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)),x)
[Out] int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)), x)
```

3.117 $\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=39

$$-\text{Int}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right)+\text{Int}\left(\frac{\csc^3(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] -Unintegrable(csc(b*x+a)/(d*x+c)^2,x)+Unintegrable(csc(b*x+a)^3/(d*x+c)^2,x
)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -Defer[Int][Csc[a + b*x]/(c + d*x)^2, x] + Defer[Int][Csc[a + b*x]^3/(c + d
*x)^2, x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = -\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx + \int \frac{\csc^3(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 41.32, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2, x]
```

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(\cot^2(bx+a)\right)\csc(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - 2*d*sin(3*b
*x + 3*a) + 2*d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x +
b*c)*cos(2*b*x + 2*a) - 4*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d*x
+ b*c)*cos(b*x + a) + 2*d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c)*co
s(b*x + a) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 +
 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*
a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x
+ 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(
4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^
2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(1/2*(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2 - 6*d^2)*sin(b*x + a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x
^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d
^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)^2 + (b^2
*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*s
in(b*x + a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^
2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^
3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*
c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4
*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)
)*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 6*d^2)*sin(b*x + a)/
(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^
4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^
```

```
2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2
+ 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2 - 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^
3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) + (2*d*c
os(3*b*x + 3*a) - 2*d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a) + (b*d*
x + b*c)*sin(b*x + a))*sin(4*b*x + 4*a) + 2*(2*d*cos(2*b*x + 2*a) - (b*d*x
+ b*c)*sin(2*b*x + 2*a) - d)*sin(3*b*x + 3*a) + 2*(2*d*cos(b*x + a) - (b*d*
x + b*c)*sin(b*x + a))*sin(2*b*x + 2*a) + 2*d*sin(b*x + a))/(b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
+ 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^
2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*
c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a
) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x +
 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b
^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*c
os(4*b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^
3)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(cot(b*x + a)^2*csc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2*csc(b*x+a)/(d*x+c)**2,x)
[Out] Integral(cot(a + b*x)**2*csc(a + b*x)/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^2*csc(b*x + a)/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot(a+b\,x)^2}{\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)^2),x)
[Out] int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)^2), x)
```

3.118 $\int (c+dx)^{5/2} \cos^2(a+bx) \sin(a+bx) \, dx$

Optimal. Leaf size=406

$$\frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\,\cos(3a+3bx)}{144b^3}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}$$

```
[Out] -1/4*(d*x+c)^(5/2)*cos(b*x+a)/b-1/12*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b+5/8*d*(
d*x+c)^(3/2)*sin(b*x+a)/b^2+5/72*d*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b^2-5/864*d
^(5/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)+5/864*d^(5/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)-15/32*
d^(5/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2
))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d^(5/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/16*d^2*cos
(b*x+a)*(d*x+c)^(1/2)/b^3+5/144*d^2*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.58, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}+\frac{5d^2\sqrt{c+dx}\,\cos(3a+3bx)}{144b^3}+\frac{5d(c+dx)^{3/2}\sin(a+bx)}{8b^2}+\frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{72b^2}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x
])/(4*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^(5
/2)*Cos[3*a + 3*b*x])/(12*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*Fres
nelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)
*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x]
)/Sqrt[d]])/(144*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/
2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a -
(b*c)/d])/(16*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x])/(8*b^2) + (5*d*
(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(72*b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{5/2}\sin(3a+3bx)\right)\,dx$$

$$= \frac{1}{4}\int (c+dx)^{5/2}\sin(a+bx)\,dx+\frac{1}{4}\int (c+dx)^{5/2}\sin(3a+3bx)\,d$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}+\frac{(5d)\int}{}$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}+\frac{5d(c+}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.41, size = 1168, normalized size = 2.88

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (c^2*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (c^2*(2*Sqrt[3]*Sqrt
[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*Fres
nelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sq
rt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(24*Sqrt[3]*b*Sqrt[b/d]) - (
c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d
*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]
*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d])
+ 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(8*b^
3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*
```

```
x]]*((4*b^2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d]) - S
qrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (
b*c)/d] + (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) - 2*Sqrt[b/d]*d*Sqrt[c + d
*x]*(d*(-15 + 4*b^2*x^2)*Cos[a + b*x] + 2*b*(c - 5*d*x)*Sin[a + b*x])))/(32
*b^5) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d
*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*S
in[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a
+ b*x)] - Sin[3*(a + b*x)])))/(24*Sqrt[3]*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*P
i]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c^2 - 5*d^2)*Cos[3
*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b*c)/d] + (12*b^2*c
^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(d*
(5 - 12*b^2*x^2)*Cos[3*(a + b*x)] - 2*b*(c - 5*d*x)*Sin[3*(a + b*x)])))/(28
8*Sqrt[3]*b^5)
```

Maple [A]
time = 0.07, size = 476, normalized size = 1.17

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$

default	$$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{}}{}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+5/8/b*d*(1/2/b*d
*(d*x+c)^(3/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/
2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos
((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))-1/
24/b*d*(d*x+c)^(5/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+5/24/b*d*(1/6/b*d*(d*
x+c)^(3/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/2)
*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*b*(d*x+c)^(1/2)/d)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 547, normalized size = 1.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3456*(240*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d) + 2160*(
d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d) - 24*(12*(d*x + c)^(5/2
)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(3*((d*x + c)*b - b*c + a*d)/d) - 216*(
4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(((d*x + c)*b - b*c +
a*d)/d) - 5*(-(I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3
*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*si
n(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 405*(-(I - 1)*sqrt(
```

```
2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(2)*sqr
t(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b
/d)) - 405*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)
/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*e
rf(sqrt(d*x + c)*sqrt(-I*b/d)) - 5*((I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*
(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*
d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)
))*d/b^5
```

Fricas [**A**]
time = 2.93, size = 341, normalized size = 0.84

$$-\frac{5\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+405\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-5\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)-24\left(30\,bd^2\cos(bx+a)-(12\,b^3d^2x^2+24\,b^3cdx+12\,b^3c^2-5\,bd^2)\cos(bx+a)^3+10\left(2\,b^2d^2x+2\,b^2cd+(b^2d^2x+b^2cd)\cos(bx+a)^2\right)\sin(bx+a)\right)\sqrt{dx+c}}{864\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(s
qrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*s
qrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) - 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(
6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(30*b*d^2*cos(b
*x + a) - (12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x +
a)^3 + 10*(2*b^2*d^2*x + 2*b^2*c*d + (b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*
sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [**C**] Result contains complex when optimal does not.
time = 1.06, size = 2479, normalized size = 6.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/1728*(72*(3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
```

(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b
d)(-I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)
/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*(9*(I*sqrt(2)*sqrt(p
i)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 +
(-I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d
- 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*x + c)*b + I*b*
c - I*a*d)/d)/b^2)/d^2 + 9*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*
d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*
(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e
^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + (I*sqrt(6)*sqrt(pi)*(12*b^2
*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sq
rt(d*x + c)*d^2)*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) + d^3*(
27*(-I*sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3
)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(
d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^
2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d
*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + (I*sqrt(6)*s
qrt(pi)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqr
t(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(
5/2)*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d -
10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3
)*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 27*(I*sqrt(2)*sqrt(p
i)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)*b^2
*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x
+ c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I
*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 + (-I*sqrt(6)*sqrt(pi)*(72*b^3*c^
3 + 36*I*b^2*c^2*d - 18*b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)*b^2*d - 36*I*(

```
d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*
b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^(-3*(I*(d*x + c
)*b - I*b*c + I*a*d)/d)/b^3)/d^3) + 36*(-9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)
*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(6)*sqr
t(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt
(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-
1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*
b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 18*sqrt(d*x + c)*
d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^(-3*(I*(d*x
 + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b
*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)
/d)/b)*c^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2),x)
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2), x)
```

3.119 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=353

$$-\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{d^{3}}{}$$

```
[Out] -1/4*(d*x+c)^(3/2)*cos(b*x+a)/b-1/12*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b-1/144*d
^(3/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/144*d^(3/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d
^(3/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*2^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d^(3/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/8*d*sin(b*x+a
)*(d*x+c)^(1/2)/b^2+1/24*d*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]

time = 0.41, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{8b^2}+\frac{d\sqrt{c+dx}\sin(3a+3bx)}{24b^2}-\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -1/4*((c + d*x)^(3/2)*Cos[a + b*x])/b - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(12*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b
*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2)) -
 (d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[3*a - (3*b*c)/d])/(24*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + (3*d*Sq
rt[c + d*x]*Sin[a + b*x])/(8*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(24*
b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(a+bx) + \frac{1}{4}(c+dx)^{3/2}\sin(3a+3bx)\right)\,dx$$

$$= \frac{1}{4}\int (c+dx)^{3/2}\sin(a+bx)\,dx + \frac{1}{4}\int (c+dx)^{3/2}\sin(3a+3bx)\,$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} + \frac{d\int\sqrt{c}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} + \frac{3d\sqrt{c}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} + \frac{3d\sqrt{c}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} + \frac{3d\sqrt{c}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} - \frac{3d^{3/2}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.55, size = 676, normalized size = 1.92

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (c*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-
I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqr
t[(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (c*(2*Sqrt[3]*Sqrt[b/d
]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC
[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6
/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(24*Sqrt[3]*b*Sqrt[b/d]) - (Sqrt
[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d*Cos[a
 - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[
2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*Sq
rt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(16*b^3) -
(Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Co
s[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (
```

```
3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a + b*x)] -
Sin[3*(a + b*x)])))/(48*Sqrt[3]*b^3)
```

Maple [A]
time = 0.06, size = 384, normalized size = 1.09

method	result
derivativedivides	$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}}\right.\right.}{}\right.}{4b}}{}$
default	$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}}\right.\right.}{}\right.}{4b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+3/8/b*d*(1/2/b*d
*(d*x+c)^(1/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)))-1/24/b*d*(d*x+c)^(3/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/8/b*d*(1
/6/b*d*(d*x+c)^(1/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(
1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1
/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(
1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 499, normalized size = 1.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/576*(48*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 144*(
d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d - 24*sqrt(d*x + c)*b^
```

```
2*sin(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*sin(((d*x + c)
*b - b*c + a*d)/d) - (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)
*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/
4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + 27*((I + 1)*sq
rt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(2)*sq
rt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/
d)) + 27*(-(I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)
 + (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(-I*b/d)) - ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)
^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^
2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*d/b^4
```

Fricas [A]
time = 2.81, size = 280, normalized size = 0.79

$$-\frac{\sqrt{6}\,\pi d^{2}\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\,\sqrt{2}\,\pi d^{2}\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\,\sqrt{2}\,\pi d^{2}\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+\sqrt{6}\,\pi d^{2}\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)+24\,\left(2\,(b^{2}dx+b^{2}c)\cos\left(bx+a\right)^{3}-\left(bd\cos\left(bx+a\right)^{2}+2\,bd\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{144\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqr
t(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-
(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(
2)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*
sin(-(b*c - a*d)/d) + sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqr
t(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(2*(b^2*d*x + b^2*c)*
cos(b*x + a)^3 - (b*d*cos(b*x + a)^2 + 2*b*d)*sin(b*x + a))*sqrt(d*x + c))/
b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin\left(a+bx\right)\cos^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x)**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.96, size = 1548, normalized size = 4.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
[Out] -1/288*(12*(3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x +
 c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt
(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/
(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(9*(I*sqrt(2)*sqrt(pi)*(4*
b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3
*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (-I*s
qrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*
sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*x + c)*b + I*b*c - I*
a*d)/d)/b^2)/d^2 + 9*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d
*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(
d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(
d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + (I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 +
 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x
 + c)*d^2)*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) + 4*(-9*I*sqr
t(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)*b) + I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*
erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(6)*sqr
t(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2)
+ 1)*b) + 18*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*sq
rt(d*x + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)
*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^(-3*(-I*(
d*x + c)*b + I*b*c - I*a*d)/d)/b)*c)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2), x)
```

3.120 $\int \sqrt{c+dx}\,\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=304

$$-\frac{\sqrt{c+dx}\,\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b}+\frac{\sqrt{d}\,\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\cdots$$

```
[Out] 1/72*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1/72*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1
/8*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^
(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/4*cos(b*x+
a)*(d*x+c)^(1/2)/b-1/12*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.33, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{d}\,\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\,\sqrt{d}\,\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\,\sqrt{d}\,\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\,\sqrt{\frac{6}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{d}\,\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{c+dx}\,\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -1/4*(Sqrt[c + d*x]*Cos[a + b*x])/b - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(12*
b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]
*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (Sqrt
[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
 - (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\,\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\,\sin(a+bx)+\frac{1}{4}\sqrt{c+dx}\,\sin(3a+3bx)\right)\,dx$$

$$= \frac{1}{4}\int \sqrt{c+dx}\,\sin(a+bx)\,dx + \frac{1}{4}\int \sqrt{c+dx}\,\sin(3a+3bx)\,dx$$

$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{4b} - \frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b} + \frac{d\int \frac{\cos(3a+3b\cdots}{\sqrt{c+dx}}}{24b}$$

$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{4b} - \frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b} + \frac{\left(d\cos\left(3a-\cdots\right.\right.}{\cdots}$$

$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{4b} - \frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b} + \frac{\cos\left(3a-\frac{3b\cdots}{d}\right.}{\cdots}$$

$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{4b} - \frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b} + \frac{\sqrt{d}\,\sqrt{\frac{\pi}{2}}\,\text{co}\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.11, size = 278, normalized size = 0.91

$$\frac{e^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(-\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{8b}-\frac{2\sqrt{3}\,\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\,\cos(3(a+bx))-\sqrt{2\pi}\,\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{b}{d}}\,\sqrt{\frac{6}{\pi}}\,\sqrt{c+dx}\right)+\sqrt{2\pi}\,S\left(\sqrt{\frac{b}{d}}\,\sqrt{\frac{6}{\pi}}\,\sqrt{c+dx}\right)\sin\left(3a-\frac{3bc}{d}\right)}{24\sqrt{3}\,b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)
*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[
(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (2*Sqrt[3]*Sqrt[b/d]*Sqr
t[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*
Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d])/(24*Sqrt[3]*b*Sqrt[b/d])
```

Maple [A]
time = 0.06, size = 296, normalized size = 0.97

method	result

derivativedivides	$$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}}{\sqrt{}}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{}$$
default	$$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}}{\sqrt{}}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+1/16/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d))-1/24/b*d*(d*x+c)^(1/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/
d)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresn
elC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)
/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.50, size = 424, normalized size = 1.39

$$-\frac{\left(\frac{24\sqrt{dx+c}\,b^2\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{72\sqrt{dx+c}\,b^2\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+\left((i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)+(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{3ib}{d}}\right)-9\left(-(i-1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)-(i+1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)-9\left((i+1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i-1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)+\left(-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)-(i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{3ib}{d}}\right)\right)d}{288\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/288*(24*sqrt(d*x + c)*b^2*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 72*sqrt
(d*x + c)*b^2*cos(((d*x + c)*b - b*c + a*d)/d)/d + ((I - 1)*9^(1/4)*sqrt(2)
*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)
*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3
*I*b/d)) - 9*(-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/
d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(I*b/d)) - 9*((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*co
s(-(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c -
a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(p
i)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sqrt(p
i)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d
)))*d/b^3
```

Fricas [A]
time = 2.48, size = 235, normalized size = 0.77

$$\frac{\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+9\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)-24\sqrt{dx+c}\,b\cos\left(bx+a\right)^{3}}{72\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d))) + 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c -
a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*
sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c
- a*d)/d) - sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*sqrt(d*x + c)*b*cos(b*x + a)^3)/b
^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin\left(a+bx\right)\cos^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.67, size = 848, normalized size = 2.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/144*(-9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*s
qrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*
(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b)
 - I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)*b) + 6*(3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b
```

```
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(
b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*s
qrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(
-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I
*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c + 18*sqrt(d*x + c
)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^(-3*(I*(d
*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I
*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*
d)/d)/b)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\,\sin(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2),x)
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2), x)
```

3.121 $\int \sqrt{c+dx} \cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=304

$$-\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\cdot$$

```
[Out] 1/72*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1/72*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1
/8*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^
(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/4*cos(b*x+
a)*(d*x+c)^(1/2)/b-1/12*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.32, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -1/4*(Sqrt[c + d*x]*Cos[a + b*x])/b - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(12*
b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]
*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (Sqrt
[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
 - (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\,\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\,\sin(a+bx)+\frac{1}{4}\sqrt{c+dx}\,\sin(3a+3bx)\right)\,dx$$

$$= \frac{1}{4}\int \sqrt{c+dx}\,\sin(a+bx)\,dx+\frac{1}{4}\int \sqrt{c+dx}\,\sin(3a+3bx)\,dx$$

$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b}+\frac{d\int \frac{\cos(3a+3b}{\sqrt{c+dx}}}{24b}$$

$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b}+\frac{\left(d\cos\left(3a-\right.\right.}{}$$

$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b}+\frac{\cos\left(3a-\frac{3b}{d}\right.}{}$$

$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\,\mathrm{co}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.00, size = 264, normalized size = 0.87

$$\frac{9e^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(-\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)+\frac{-6\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(3(a+bx))+\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)-\sqrt{6\pi}\,S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\sin\left(3a-\frac{3bc}{d}\right)}{\sqrt{\frac{b}{d}}}}{72b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] ((9*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((
-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sq
rt[(I*b*(c + d*x))/d]))/E^((I*(b*c + a*d))/d) + (-6*Sqrt[b/d]*Sqrt[c + d*x]
*Cos[3*(a + b*x)] + Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt
[6/Pi]*Sqrt[c + d*x]] - Sqrt[6*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d
*x]]*Sin[3*a - (3*b*c)/d])/Sqrt[b/d])/(72*b)
```

Maple [A]
time = 0.00, size = 296, normalized size = 0.97

method	result

derivativedivides	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}}{\sqrt{}}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{}$
default	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}}{\sqrt{}}\right.\right.}{8b\sqrt{\frac{b}{d}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+1/16/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d))-1/24/b*d*(d*x+c)^(1/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/
d)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresn
elC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)
/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 424, normalized size = 1.39

$$-\frac{\left(\frac{24\sqrt{dx+c}\,b^2\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{72\sqrt{dx+c}\,b^2\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+\left((i-1)\cdot 9^{\frac14}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\cos\left(-\frac{3(bc-ad)}{d}\right)+(i+1)\cdot 9^{\frac14}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{3ib}{d}}\right)-9\left(-(i-1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\cos\left(-\frac{bc-ad}{d}\right)-(i+1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)-9\left((i+1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\cos\left(-\frac{bc-ad}{d}\right)+(i-1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)+\left(-(i+1)\cdot 9^{\frac14}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\cos\left(-\frac{3(bc-ad)}{d}\right)-(i-1)\cdot 9^{\frac14}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac14}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{3ib}{d}}\right)\right)d}{288\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/288*(24*sqrt(d*x + c)*b^2*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 72*sqrt
(d*x + c)*b^2*cos(((d*x + c)*b - b*c + a*d)/d)/d + ((I - 1)*9^(1/4)*sqrt(2)
*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)
*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3
*I*b/d)) - 9*(-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/
d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(I*b/d)) - 9*((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*co
s(-(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c -
a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(p
i)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sqrt(p
i)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d
)))*d/b^3
```

Fricas [A]
time = 6.88, size = 235, normalized size = 0.77

$$\frac{\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+9\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)-24\sqrt{dx+c}\,b\cos\left(bx+a\right)^3}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d))) + 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c -
a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*
sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c
- a*d)/d) - sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*sqrt(d*x + c)*b*cos(b*x + a)^3)/b
^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c+dx}\,\sin\left(a+bx\right)\cos^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.68, size = 848, normalized size = 2.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/144*(-9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*s
qrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*
(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b)
 - I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)*b) + 6*(3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b
```

```
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(
b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*s
qrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(
-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I
*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c + 18*sqrt(d*x + c
)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^(-3*(I*(d
*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I
*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*
d)/d)/b)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\,\sin(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2),x)
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2), x)
```

3.122 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=353

$$-\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{d^{3/2}$$

```
[Out] -1/4*(d*x+c)^(3/2)*cos(b*x+a)/b-1/12*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b-1/144*d
^(3/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/144*d^(3/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d
^(3/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*2^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d^(3/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/8*d*sin(b*x+a
)*(d*x+c)^(1/2)/b^2+1/24*d*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.36, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{8b^2}+\frac{d\sqrt{c+dx}\sin(3a+3bx)}{24b^2}-\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -1/4*((c + d*x)^(3/2)*Cos[a + b*x])/b - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(12*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b
*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2)) -
 (d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[3*a - (3*b*c)/d])/(24*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + (3*d*Sq
rt[c + d*x]*Sin[a + b*x])/(8*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(24*
b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{3/2}\sin(3a+3bx)\right)\,dx$$

$$= \frac{1}{4}\int (c+dx)^{3/2}\sin(a+bx)\,dx + \frac{1}{4}\int (c+dx)^{3/2}\sin(3a+3bx)\,dx$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} + \frac{d\int \sqrt{c\cdots}}{\cdots}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} + \frac{3d\sqrt{c+\cdots}}{\cdots}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} + \frac{3d\sqrt{c+\cdots}}{\cdots}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} + \frac{3d\sqrt{c+\cdots}}{\cdots}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} - \frac{3d^{3/2}\sqrt{\cdots}}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.07, size = 676, normalized size = 1.92

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (c*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-
I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqr
t[(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (c*(2*Sqrt[3]*Sqrt[b/d
]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC
[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6
/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(24*Sqrt[3]*b*Sqrt[b/d]) - (Sqrt
[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d*Cos[a
 - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[
2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*Sq
rt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(16*b^3) -
(Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Co
s[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (
```

```
3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a + b*x)] -
Sin[3*(a + b*x)])))/(48*Sqrt[3]*b^3)
```

Maple [A]
time = 0.00, size = 384, normalized size = 1.09

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{d}}{\sqrt{\pi}\,\sqrt{}}\right.\right.}{}\right.}{4b}$
default	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{d}}{\sqrt{\pi}\,\sqrt{}}\right.\right.}{}\right.}{4b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+3/8/b*d*(1/2/b*d
*(d*x+c)^(1/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)))-1/24/b*d*(d*x+c)^(3/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/8/b*d*(1
/6/b*d*(d*x+c)^(1/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(
1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1
/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(
1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 499, normalized size = 1.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/576*(48*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 144*(
d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d - 24*sqrt(d*x + c)*b^
```

```
2*sin(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*sin(((d*x + c)
*b - b*c + a*d)/d) - (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)
*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/
4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + 27*((I + 1)*sq
rt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(2)*sq
rt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/
d)) + 27*(-(I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)
 + (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(-I*b/d)) - ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)
^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^
2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*d/b^4
```

Fricas [A]
time = 6.12, size = 280, normalized size = 0.79

$$-\frac{\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)+24\left(2\left(b^2dx+b^2c\right)\cos\left(bx+a\right)^3-\left(bd\cos\left(bx+a\right)^2+2\,bd\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{144\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqr
t(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-
(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(
2)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*
sin(-(b*c - a*d)/d) + sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqr
t(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(2*(b^2*d*x + b^2*c)*
cos(b*x + a)^3 - (b*d*cos(b*x + a)^2 + 2*b*d)*sin(b*x + a))*sqrt(d*x + c))/
b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin(a+bx)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x)**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.94, size = 1548, normalized size = 4.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/288*(12*(3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x +
 c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt
(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/
(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(9*(I*sqrt(2)*sqrt(pi)*(4*
b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3
*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (-I*s
qrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*
sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*x + c)*b + I*b*c - I*
a*d)/d)/b^2)/d^2 + 9*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d
*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(
d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(
d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + (I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 +
 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x
 + c)*d^2)*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) + 4*(-9*I*sqr
t(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)*b) + I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*
erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(6)*sqr
t(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2)
+ 1)*b) + 18*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*sq
rt(d*x + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)
*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^(-3*(-I*(
d*x + c)*b + I*b*c - I*a*d)/d)/b)*c)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2), x)
```

3.123 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=406

$$\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{144b^3}-\frac{(c+dx)^{5/2}\cos(3a+3bx}{12b}$$

```
[Out] -1/4*(d*x+c)^(5/2)*cos(b*x+a)/b-1/12*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b+5/8*d*(
d*x+c)^(3/2)*sin(b*x+a)/b^2+5/72*d*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b^2-5/864*d
^(5/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)+5/864*d^(5/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)-15/32*
d^(5/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2
))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d^(5/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/16*d^2*cos
(b*x+a)*(d*x+c)^(1/2)/b^3+5/144*d^2*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.44, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{16b^3}+\frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{144b^3}+\frac{5d(c+dx)^{3/2}\sin(a+bx)}{8b^2}+\frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{72b^2}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x
])/(4*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^(5
/2)*Cos[3*a + 3*b*x])/(12*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*Fres
nelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)
*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x]
)/Sqrt[d]])/(144*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/
2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a -
(b*c)/d])/(16*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x])/(8*b^2) + (5*d*
(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(72*b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{5/2}\sin(3a+3bx)\right)\,dx$$

$$= \frac{1}{4}\int (c+dx)^{5/2}\sin(a+bx)\,dx+\frac{1}{4}\int (c+dx)^{5/2}\sin(3a+3bx)\,\ldots$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}+\frac{(5d)\int \ldots}{\ldots}$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}+\frac{5d(c+\ldots}{\ldots}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c\ldots}}{\ldots}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c\ldots}}{\ldots}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c\ldots}}{\ldots}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c\ldots}}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.05, size = 1168, normalized size = 2.88

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (c^2*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (c^2*(2*Sqrt[3]*Sqrt
[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*Fres
nelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sq
rt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(24*Sqrt[3]*b*Sqrt[b/d]) - (
c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d
*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]
*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d])
+ 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(8*b^
3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*
```

```
x]]*((4*b^2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d]) - S
qrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (
b*c)/d] + (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) - 2*Sqrt[b/d]*d*Sqrt[c + d
*x]*(d*(-15 + 4*b^2*x^2)*Cos[a + b*x] + 2*b*(c - 5*d*x)*Sin[a + b*x])))/(32
*b^5) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d
*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*S
in[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a
+ b*x)] - Sin[3*(a + b*x)])))/(24*Sqrt[3]*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*P
i]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c^2 - 5*d^2)*Cos[3
*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b*c)/d] + (12*b^2*c
^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(d*
(5 - 12*b^2*x^2)*Cos[3*(a + b*x)] - 2*b*(c - 5*d*x)*Sin[3*(a + b*x)])))/(28
8*Sqrt[3]*b^5)
```

Maple [A]

time = 0.00, size = 476, normalized size = 1.17

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}}{}\right)}{}\right)}{}$

default	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+5/8/b*d*(1/2/b*d
*(d*x+c)^(3/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/
2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos
((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))-1/
24/b*d*(d*x+c)^(5/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+5/24/b*d*(1/6/b*d*(d*
x+c)^(3/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/2)
*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*b*(d*x+c)^(1/2)/d)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.54, size = 547, normalized size = 1.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3456*(240*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d) + 2160*(
d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d) - 24*(12*(d*x + c)^(5/2
)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(3*((d*x + c)*b - b*c + a*d)/d) - 216*(
4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(((d*x + c)*b - b*c +
a*d)/d) - 5*(-(I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3
*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*si
n(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 405*(-(I - 1)*sqrt(
```

```
2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(2)*sqr
t(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b
/d)) - 405*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)
/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*e
rf(sqrt(d*x + c)*sqrt(-I*b/d)) - 5*((I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*
(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*
d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)
))*d/b^5
```

Fricas [**A**]
time = 1.56, size = 341, normalized size = 0.84

$$-\frac{5\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+405\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-405\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-5\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)-24\left(30\,bd^2\cos(bx+a)-(12\,b^3d^2x^2+24\,b^3cdx+12\,b^3c^2-5\,bd^2)\cos(bx+a)^3+10\left(2\,b^2d^2x+2\,b^2cd+(b^2d^2x+b^2cd)\cos(bx+a)^2\right)\sin(bx+a)\right)\sqrt{dx+c}}{864\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(s
qrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*s
qrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) - 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(
6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(30*b*d^2*cos(b
*x + a) - (12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x +
a)^3 + 10*(2*b^2*d^2*x + 2*b^2*c*d + (b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*
sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [**C**] Result contains complex when optimal does not.
time = 1.07, size = 2479, normalized size = 6.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/1728*(72*(3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
```

(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b
d)(-I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)
/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*(9*(I*sqrt(2)*sqrt(p
i)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 +
 (-I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d
- 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*x + c)*b + I*b*
c - I*a*d)/d)/b^2)/d^2 + 9*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*
d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*
(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e
^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + (I*sqrt(6)*sqrt(pi)*(12*b^2
*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sq
rt(d*x + c)*d^2)*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) + d^3*(
27*(-I*sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3
)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(
d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^
2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d
*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + (I*sqrt(6)*s
qrt(pi)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqr
t(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(
5/2)*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d -
10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3
)*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 27*(I*sqrt(2)*sqrt(p
i)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)*b^2
*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x
+ c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I
*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 + (-I*sqrt(6)*sqrt(pi)*(72*b^3*c^
3 + 36*I*b^2*c^2*d - 18*b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)*b^2*d - 36*I*(

```
d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*
b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^(-3*(I*(d*x + c
)*b - I*b*c + I*a*d)/d)/b^3)/d^3) + 36*(-9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)
*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(6)*sqr
t(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt
(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-
1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*
b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 18*sqrt(d*x + c)*
d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^(-3*(I*(d*x
 + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b
*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)
/d)/b)*c^2)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos (a+b\,x)^2\,\sin (a+b\,x)\,{(c+d\,x)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2), x)
```

3.124 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=228

$$\frac{(c+dx)^{7/2}}{28d}-\frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2}-\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cdots}{\cdots}$$

```
[Out] 1/28*(d*x+c)^(7/2)/d-5/256*d*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b^2-1/32*(d*x+c)^
(5/2)*sin(4*b*x+4*a)/b-15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*
2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)-15/8192*d^
(5/2)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*
b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/2048*d^2*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^
3
```

Rubi [A]
time = 0.29, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sin(4a+4bx)}{2048b^3}-\frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2}-\frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b}+\frac{(c+dx)^{7/2}}{28d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(7/2)/(28*d) - (5*d*(c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(256*b^2) -
 (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]
*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4096*b
^(7/2)) + (15*d^2*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(2048*b^3) - ((c + d*x)^(
5/2)*Sin[4*a + 4*b*x])/(32*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)^{5/2}-\frac{1}{8}(c+dx)^{5/2}\cos(4a+4bx)\right)\,dx \\
&= \frac{(c+dx)^{7/2}}{28d}-\frac{1}{8}\int (c+dx)^{5/2}\cos(4a+4bx)\,dx \\
&= \frac{(c+dx)^{7/2}}{28d}-\frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b}+\frac{(5d)\int (c+dx)^{3/2}\,\mathrm{si}}{64b} \\
&= \frac{(c+dx)^{7/2}}{28d}-\frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2}-\frac{(c+dx)^{5/2}\sin(4}{32b} \\
&= \frac{(c+dx)^{7/2}}{28d}-\frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2}+\frac{15d^2\sqrt{c+dx}\,\sin}{2048b^3} \\
&= \frac{(c+dx)^{7/2}}{28d}-\frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2}+\frac{15d^2\sqrt{c+dx}\,\sin}{2048b^3} \\
&= \frac{(c+dx)^{7/2}}{28d}-\frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2}+\frac{15d^2\sqrt{c+dx}\,\sin}{2048b^3} \\
&= \frac{(c+dx)^{7/2}}{28d}-\frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2}-\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(\right.}{}
\end{aligned}$$

Mathematica [A]
time = 3.60, size = 206, normalized size = 0.90

$$\frac{\sqrt{\frac{b}{d}}\left(-105d^3\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)-105d^3\sqrt{2\pi}\,\mathrm{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(4a-\frac{4bc}{d}\right)+4\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(512b^3(c+dx)^3-280bd^2(c+dx)\cos(4(a+bx))-7d(-15d^2+64b^2(c+dx)^2)\sin(4(a+bx))\right)\right)}{57344b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(-105*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*S
qrt[2/Pi]*Sqrt[c + d*x]] - 105*d^3*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(512*b^3
*(c + d*x)^3 - 280*b*d^2*(c + d*x)*Cos[4*(a + b*x)] - 7*d*(-15*d^2 + 64*b^2
*(c + d*x)^2)*Sin[4*(a + b*x)])))/(57344*b^4)
```

Maple [A]
time = 0.08, size = 251, normalized size = 1.10

method	result

derivativedivides	$\frac{\frac{(dx+c)^{\frac{7}{2}}}{28}-\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}\ldots\right.}{8b}\ldots\right.}{\ldots}\right.}{\ldots}}{d\ldots}$
default	$\frac{\frac{(dx+c)^{\frac{7}{2}}}{28}-\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}\ldots\right.}{8b}\ldots\right.}{\ldots}\right.}{\ldots}}{d\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/56*(d*x+c)^(7/2)-1/64/b*d*(d*x+c)^(5/2)*sin(4/d*b*(d*x+c)+4*(a*d-b*c
)/d)+5/64/b*d*(-1/8/b*d*(d*x+c)^(3/2)*cos(4/d*b*(d*x+c)+4*(a*d-b*c)/d)+3/8/
b*d*(1/8/b*d*(d*x+c)^(1/2)*sin(4/d*b*(d*x+c)+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2
)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d
)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(
b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 285, normalized size = 1.25

$$\frac{\sqrt{2}\left(\frac{4096\sqrt{2}(dx+c)^{\frac{7}{2}}b^4}{d}-2240\sqrt{2}(dx+c)^{\frac{3}{2}}b^2d\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)+105\left(-(i+1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)+(i-1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4(bc-ad)}{d}\right)\right)\mathrm{erf}\left(2\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)+105\left((i-1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)-(i+1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4(bc-ad)}{d}\right)\right)\mathrm{erf}\left(2\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)-56\left(64\sqrt{2}(dx+c)^{\frac{5}{2}}b^3-15\sqrt{2}\sqrt{dx+c}\,bd^2\right)\sin\left(\frac{4((dx+c)b-bc+ad)}{d}\right)\right)}{229376\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/229376*sqrt(2)*(4096*sqrt(2)*(d*x + c)^(7/2)*b^4/d - 2240*sqrt(2)*(d*x +
c)^(3/2)*b^2*d*cos(4*((d*x + c)*b - b*c + a*d)/d) + 105*(-(I + 1)*sqrt(pi)*
d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(pi)*d^3*(b^2/d^2)^
(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + 105*((I - 1
)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I + 1)*sqrt(pi)*d^3
*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) -
 56*(64*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c)*b*d^2)*sin(4
*((d*x + c)*b - b*c + a*d)/d))/b^4
```

Fricas [**A**]
time = 5.23, size = 347, normalized size = 1.52

$$-\frac{105\,\sqrt{2}\,\pi d^4\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+105\,\sqrt{2}\,\pi d^4\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)-16\,\left(128\,b^4d^3x^3+384\,b^4cd^2x^2+128\,b^4c^3-70\,b^2cd^2-560\,(b^2d^3x+b^2cd^2)\cos\left(bx+a\right)^4+560\,(b^2d^3x+b^2cd^2)\cos\left(bx+a\right)^2+2\,(192\,b^4c^2d-35\,b^2d^3)x-7\,\left(2\,(64\,b^3d^3x^2+128\,b^3cd^2x+64\,b^3c^2d-15\,bd^3)\cos\left(bx+a\right)^3-(64\,b^3d^3x^2+128\,b^3cd^2x+64\,b^3c^2d-15\,bd^3)\cos\left(bx+a\right)\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{57344\,b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/57344*(105*sqrt(2)*pi*d^4*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_s
in(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 105*sqrt(2)*pi*d^4*sqrt(b/(pi*
d))*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/
d) - 16*(128*b^4*d^3*x^3 + 384*b^4*c*d^2*x^2 + 128*b^4*c^3 - 70*b^2*c*d^2 -
 560*(b^2*d^3*x + b^2*c*d^2)*cos(b*x + a)^4 + 560*(b^2*d^3*x + b^2*c*d^2)*c
os(b*x + a)^2 + 2*(192*b^4*c^2*d - 35*b^2*d^3)*x - 7*(2*(64*b^3*d^3*x^2 + 1
28*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a)^3 - (64*b^3*d^3*x^2
+ 128*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a))*sin(b*x + a))*sq
rt(d*x + c))/(b^4*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [**C**] Result contains complex when optimal does not.
time = 1.10, size = 1372, normalized size = 6.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/573440*(17920*(sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) + 8*sqrt(d*x + c))*c^3 + 56*c*d^2*(512*(3*(d*x + c)^(5/2)
 - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)/d^2 + 15*(sqrt(2)*sqrt(pi)*
(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)*b^2) - 4*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^
2 + 15*(sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*s
qrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/
d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*(8*I*(d*x + c)^(3/2)*b*d -
 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b
*c + I*a*d)/d)/b^2)/d^2) + d^3*(4096*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2
)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)/d^3 - 35*(sqrt(2)*sqrt
(pi)*(512*b^3*c^3 - 192*I*b^2*c^2*d - 72*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 4*(-64*I*(d*x + c)^(5/2)*
b^2*d + 192*I*(d*x + c)^(3/2)*b^2*c*d - 192*I*sqrt(d*x + c)*b^2*c^2*d + 40*
(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*
e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 - 35*(sqrt(2)*sqrt(pi)*(
512*b^3*c^3 + 192*I*b^2*c^2*d - 72*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(2)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(
sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 4*(64*I*(d*x + c)^(5/2)*b^2*d -
192*I*(d*x + c)^(3/2)*b^2*c*d + 192*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c
)^(3/2)*b*d^2 - 72*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-4*(I
*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3) - 2240*(3*sqrt(2)*sqrt(pi)*(8*b*
c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 3*sq
rt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)*b) - 64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c + 12*I*sqrt(d*x + c
)*d*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 12*I*sqrt(d*x + c)*d*e^(-4
*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c^2)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos (a+b\,x)^2\,\sin (a+b\,x)^2\,{\left(c+d\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

3.125 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=200

$$\frac{(c+dx)^{5/2}}{20d}-\frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2}+\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d^{3/2}}{}$$

```
[Out] 1/20*(d*x+c)^(5/2)/d-1/32*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b+3/1024*d^(3/2)*cos
(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^
(1/2)*Pi^(1/2)/b^(5/2)-3/1024*d^(3/2)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)-3/256*d*cos
(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.23, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2}-\frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b}+\frac{(c+dx)^{5/2}}{20d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(5/2)/(20*d) - (3*d*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(256*b^2) + (
3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(2*Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)
) - ((c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(32*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2} - \frac{1}{8}(c+dx)^{3/2}\cos(4a+4bx)\right)\,dx$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{1}{8}\int (c+dx)^{3/2}\cos(4a+4bx)\,dx$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b} + \frac{(3d)\int \sqrt{c+dx}\,\sin}{64b}$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a}{32b}$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a}{32b}$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a}{32b}$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\cos\left(4a\right.}{}$$

Mathematica [A]
time = 3.06, size = 187, normalized size = 0.94

$$\frac{\sqrt{\frac{b}{d}}\left(15d^2\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)-15d^2\sqrt{2\pi}\,S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(4a-\frac{4bc}{d}\right)+4\sqrt{\frac{b}{d}}\sqrt{c+dx}\,(-15d^2\cos(4(a+bx))+8b(c+dx)(8b(c+dx)-5d\sin(4(a+bx))))\right)}{5120b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(15*d^2*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqr
t[2/Pi]*Sqrt[c + d*x]] - 15*d^2*Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*
Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(-15*d^2*Co
s[4*(a + b*x)] + 8*b*(c + d*x)*(8*b*(c + d*x) - 5*d*Sin[4*(a + b*x)]))))/(5
120*b^3)
```

Maple [A]
time = 0.07, size = 206, normalized size = 1.03

method	result

derivativedivides	$\frac{\frac{(dx+c)^{\frac{5}{2}}}{20}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4ad-4cb}{d}\right)\cdots\right.}{\cdots}\right.}{\cdots}}{d}$
default	$\frac{\frac{(dx+c)^{\frac{5}{2}}}{20}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4ad-4cb}{d}\right)\cdots\right.}{\cdots}\right.}{\cdots}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/40*(d*x+c)^(5/2)-1/64/b*d*(d*x+c)^(3/2)*sin(4/d*b*(d*x+c)+4*(a*d-b*c
)/d)+3/64/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d*b*(d*x+c)+4*(a*d-b*c)/d)+1/32
/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)
/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.50, size = 264, normalized size = 1.32

$$\frac{\sqrt{2}\left(\frac{512\sqrt{2}\,(dx+c)^{\frac{5}{2}}b^3}{d}-320\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2\sin\left(\frac{4((dx+c)b-bc+ad)}{d}\right)-120\sqrt{2}\,\sqrt{dx+c}\,bd\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)+15\left(-(i-1)\sqrt{\pi}\,d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)-(i+1)\sqrt{\pi}\,d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4(bc-ad)}{d}\right)\right)\mathrm{erf}\left(2\sqrt{dx+c}\,\sqrt{\frac{ib}{d}}\right)+15\left((i+1)\sqrt{\pi}\,d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)+(i-1)\sqrt{\pi}\,d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4(bc-ad)}{d}\right)\right)\mathrm{erf}\left(2\sqrt{dx+c}\,\sqrt{-\frac{ib}{d}}\right)\right)}{20480\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/20480*sqrt(2)*(512*sqrt(2)*(d*x + c)^(5/2)*b^3/d - 320*sqrt(2)*(d*x + c)^
(3/2)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 120*sqrt(2)*sqrt(d*x + c)*b*
d*cos(4*((d*x + c)*b - b*c + a*d)/d) + 15*(-(I - 1)*sqrt(pi)*d^2*(b^2/d^2)^
(1/4)*cos(-4*(b*c - a*d)/d) - (I + 1)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-4*(
b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + 15*((I + 1)*sqrt(pi)*d^2*
(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(pi)*d^2*(b^2/d^2)^(1/4
)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^3
```

Fricas [A]
time = 3.26, size = 249, normalized size = 1.24

$$\frac{15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+4\left(64b^3d^2x^2-120bd^2\cos(bx+a)^4+128b^3cdx+64b^3c^2+120bd^2\cos(bx+a)^2-15bd^2-160\left(2(b^2d^2x+b^2cd)\cos(bx+a)^3-(b^2d^2x+b^2cd)\cos(bx+a)\right)\sin(bx+a)\right)\sqrt{dx+c}}{5120\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")

[Out] 1/5120*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 4*(64*b^3*d^2*x^2 - 120*b*d^2*cos(b*x + a)^4 + 128*b^3*c*d*x + 64*b^3*c^2 + 120*b*d^2*cos(b*x + a)^2 - 15*b*d^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3 - (b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/(b^3*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}} \sin^2(a+bx)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)

[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x)**2, x)

Giac [C] Result contains complex when optimal does not.
time = 0.87, size = 852, normalized size = 4.26

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")

[Out] 1/30720*(960*(sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 8*sqrt(d*x + c))*c^2 + d^2*(512*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)/d^2 + 15*(sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 15*(sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) - 80*(3*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt

```
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 3*sqrt(2)*sqrt(pi)*(8*b*c + I*d)
*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*
(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 64*(d*x + c)^
(3/2) + 192*sqrt(d*x + c)*c + 12*I*sqrt(d*x + c)*d*e^(-4*(I*(d*x + c)*b - I
*b*c + I*a*d)/d)/b - 12*I*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b + I*b*c - I
*a*d)/d)/b)*c)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\sin(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

3.126 $\int \sqrt{c+dx} \cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=174

$$\frac{(c+dx)^{3/2}}{12d}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}$$

```
[Out] 1/12*(d*x+c)^(3/2)/d+1/128*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(
1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/128*FresnelC
(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)
*2^(1/2)*Pi^(1/2)/b^(3/2)-1/32*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.18, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{(c+dx)^{3/2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(3/2)/(12*d) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt
[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*
b*c)/d])/(64*b^(3/2)) - (Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(32*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx} - \frac{1}{8}\sqrt{c+dx}\cos(4a+4bx)\right)dx$$
$$= \frac{(c+dx)^{3/2}}{12d} - \frac{1}{8}\int \sqrt{c+dx}\cos(4a+4bx)\,dx$$
$$= \frac{(c+dx)^{3/2}}{12d} - \frac{\sqrt{c+dx}\sin(4a+4bx)}{32b} + \frac{d\int \frac{\sin(4a+4bx)}{\sqrt{c+dx}}dx}{64b}$$
$$= \frac{(c+dx)^{3/2}}{12d} - \frac{\sqrt{c+dx}\sin(4a+4bx)}{32b} + \frac{\left(d\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\sin}{\sqrt{}}}{64b}$$
$$= \frac{(c+dx)^{3/2}}{12d} - \frac{\sqrt{c+dx}\sin(4a+4bx)}{32b} + \frac{\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Subst}\Big(}{}$$
$$= \frac{(c+dx)^{3/2}}{12d} + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}$$

Mathematica [A]
time = 0.82, size = 161, normalized size = 0.93

$$\frac{3d\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+3d\sqrt{2\pi}\,\mathrm{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(4a-\frac{4bc}{d}\right)+4\sqrt{\frac{b}{d}}\sqrt{c+dx}\,(8b(c+dx)-3d\sin(4(a+bx)))}{384\left(\frac{b}{d}\right)^{3/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c
 + d*x]] + 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Si
n[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(8*b*(c + d*x) - 3*d*Sin[4*(
a + b*x)]))/(384*(b/d)^(3/2)*d^2)
```

Maple [A]
time = 0.07, size = 159, normalized size = 0.91

method	result

derivativedivides	$\frac{\frac{(dx+c)^{\frac{3}{2}}}{12}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{4ad-4cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{4ad-4cb}{d}\right)\right)}{128b\sqrt{\frac{b}{d}}}}{d}$
default	$\frac{\frac{(dx+c)^{\frac{3}{2}}}{12}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{4ad-4cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{4ad-4cb}{d}\right)\right)}{128b\sqrt{\frac{b}{d}}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/24*(d*x+c)^(3/2)-1/64/b*d*(d*x+c)^(1/2)*sin(4/d*b*(d*x+c)+4*(a*d-b*c
)/d)+1/256/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*
2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC
(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 219, normalized size = 1.26

$$\frac{\sqrt{2}\left(\frac{64\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^{2}}{d}-24\sqrt{2}\,\sqrt{dx+c}\,b\sin\left(\frac{4((dx+c)b-bc+ad)}{d}\right)+3\left((i+1)\,\sqrt{\pi}\,d\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)-(i-1)\,\sqrt{\pi}\,d\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\sin\left(-\frac{4(bc-ad)}{d}\right)\right)\mathrm{erf}\left(2\sqrt{dx+c}\,\sqrt{\frac{ib}{d}}\right)+3\left(-(i-1)\,\sqrt{\pi}\,d\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)+(i+1)\,\sqrt{\pi}\,d\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\sin\left(-\frac{4(bc-ad)}{d}\right)\right)\mathrm{erf}\left(2\sqrt{dx+c}\,\sqrt{-\frac{ib}{d}}\right)\right)}{1536\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*sqrt(2)*(64*sqrt(2)*(d*x + c)^(3/2)*b^2/d - 24*sqrt(2)*sqrt(d*x + c)
*b*sin(4*((d*x + c)*b - b*c + a*d)/d) + 3*((I + 1)*sqrt(pi)*d*(b^2/d^2)^(1/
4)*cos(-4*(b*c - a*d)/d) - (I - 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-4*(b*c -
 a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + 3*(-(I - 1)*sqrt(pi)*d*(b^2/d^
2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I + 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-4*
(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^2
```

Fricas [A]
time = 3.40, size = 175, normalized size = 1.01

$$\frac{3\sqrt{2}\,\pi d^{2}\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\,\pi d^{2}\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+16\left(2b^{2}dx+2b^{2}c-3\left(2bd\cos(bx+a)^{3}-bd\cos(bx+a)\right)\sin(bx+a)\right)\sqrt{dx+c}}{384\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2*
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fre
snel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 16
*(2*b^2*d*x + 2*b^2*c - 3*(2*b*d*cos(b*x + a)^3 - b*d*cos(b*x + a))*sin(b*x
 + a))*sqrt(d*x + c))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^2\left(a+bx\right)\cos^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.69, size = 458, normalized size = 2.63

$$-\frac{\frac{3\sqrt{2}\sqrt{\pi}(8bc-id)d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{4(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{3\sqrt{2}\sqrt{\pi}(8bc+id)d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{4(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-24\left(\frac{\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{4(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}+\frac{\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{4(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}+8\sqrt{dx+c}\right)c-64(dx+c)^{\frac{3}{2}}+192\sqrt{dx+c}\,c+\frac{12i\sqrt{dx+c}\,de^{\left(-\frac{4(i(dx+c)b-ibc+iad)}{d}\right)}}{b}-\frac{12i\sqrt{dx+c}\,de^{\left(-\frac{4(-i(dx+c)b+ibc-iad)}{d}\right)}}{b}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/768*(3*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b) + 3*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d
)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 24*(sqrt(2)*sqrt(pi)*d*erf(-
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c
- I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf
(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*
c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 8*sqrt(d*x + c))*c -
64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c + 12*I*sqrt(d*x + c)*d*e^(-4*(I*(d
*x + c)*b - I*b*c + I*a*d)/d)/b - 12*I*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*
b + I*b*c - I*a*d)/d)/b)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(a+b\,x\right)}^2\,{\sin\left(a+b\,x\right)}^2\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

3.127 $\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=174

$$\frac{(c+dx)^{3/2}}{12d}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\,\mathrm{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}$$

```
[Out] 1/12*(d*x+c)^(3/2)/d+1/128*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(
1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/128*FresnelC
(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)
*2^(1/2)*Pi^(1/2)/b^(3/2)-1/32*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.17, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{(c+dx)^{3/2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(3/2)/(12*d) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt
[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*
b*c)/d])/(64*b^(3/2)) - (Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(32*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\, \cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx} - \frac{1}{8}\sqrt{c+dx}\,\cos(4a+4bx)\right)dx$$
$$= \frac{(c+dx)^{3/2}}{12d} - \frac{1}{8}\int \sqrt{c+dx}\,\cos(4a+4bx)\,dx$$
$$= \frac{(c+dx)^{3/2}}{12d} - \frac{\sqrt{c+dx}\,\sin(4a+4bx)}{32b} + \frac{d\int \frac{\sin(4a+4bx)}{\sqrt{c+dx}}dx}{64b}$$
$$= \frac{(c+dx)^{3/2}}{12d} - \frac{\sqrt{c+dx}\,\sin(4a+4bx)}{32b} + \frac{\left(d\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\sin(}{\sqrt{}}}{64b}$$
$$= \frac{(c+dx)^{3/2}}{12d} - \frac{\sqrt{c+dx}\,\sin(4a+4bx)}{32b} + \frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Subst}\left(\int\right.}{}$$
$$= \frac{(c+dx)^{3/2}}{12d} + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}$$

Mathematica [A]
time = 0.05, size = 161, normalized size = 0.93

$$\frac{3d\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+3d\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(4a-\frac{4bc}{d}\right)+4\sqrt{\frac{b}{d}}\sqrt{c+dx}\,(8b(c+dx)-3d\sin(4(a+bx)))}{384\left(\frac{b}{d}\right)^{3/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c
 + d*x]] + 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Si
n[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(8*b*(c + d*x) - 3*d*Sin[4*(
a + b*x)]))/(384*(b/d)^(3/2)*d^2)
```

Maple [A]
time = 0.00, size = 159, normalized size = 0.91

method	result

derivativedivides	$\frac{\frac{(dx+c)^{\frac{3}{2}}}{12}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4ad-4cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{4ad-4c}{d}\right.\right.}{128b\sqrt{\frac{b}{d}}}}{d}$
default	$\frac{\frac{(dx+c)^{\frac{3}{2}}}{12}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4ad-4cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{4ad-4c}{d}\right.\right.}{128b\sqrt{\frac{b}{d}}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/24*(d*x+c)^(3/2)-1/64/b*d*(d*x+c)^(1/2)*sin(4/d*b*(d*x+c)+4*(a*d-b*c
)/d)+1/256/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*
2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC
(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.49, size = 219, normalized size = 1.26

$$\frac{\sqrt{2}\left(\frac{64\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2}{d}-24\sqrt{2}\,\sqrt{dx+c}\,b\sin\left(\frac{4((dx+c)b-bc+ad)}{d}\right)+3\left((i+1)\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)-(i-1)\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4(bc-ad)}{d}\right)\right)\mathrm{erf}\left(2\sqrt{dx+c}\,\sqrt{\frac{ib}{d}}\right)+3\left(-(i-1)\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)+(i+1)\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4(bc-ad)}{d}\right)\right)\mathrm{erf}\left(2\sqrt{dx+c}\,\sqrt{-\frac{ib}{d}}\right)\right)}{1536\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*sqrt(2)*(64*sqrt(2)*(d*x + c)^(3/2)*b^2/d - 24*sqrt(2)*sqrt(d*x + c)
*b*sin(4*((d*x + c)*b - b*c + a*d)/d) + 3*((I + 1)*sqrt(pi)*d*(b^2/d^2)^(1/
4)*cos(-4*(b*c - a*d)/d) - (I - 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-4*(b*c -
 a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + 3*(-(I - 1)*sqrt(pi)*d*(b^2/d^
2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I + 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-4*
(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^2
```

Fricas [A]
time = 3.10, size = 175, normalized size = 1.01

$$\frac{3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+16\left(2b^2dx+2b^2c-3\left(2bd\cos\left(bx+a\right)^3-bd\cos\left(bx+a\right)\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{384\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2*
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fre
snel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 16
*(2*b^2*d*x + 2*b^2*c - 3*(2*b*d*cos(b*x + a)^3 - b*d*cos(b*x + a))*sin(b*x
 + a))*sqrt(d*x + c))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^2\left(a+bx\right)\cos^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.69, size = 458, normalized size = 2.63

$$-\frac{\frac{3\sqrt{2}\sqrt{\pi}(8bc-id)d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{4(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{3\sqrt{2}\sqrt{\pi}(8bc+id)d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{4(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-24\left(\frac{\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{4(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}+\frac{\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{4(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}+8\sqrt{dx+c}\right)c-64(dx+c)^{\frac{3}{2}}+192\sqrt{dx+c}\,c+\frac{12i\sqrt{dx+c}\,de^{\left(-\frac{4(i(dx+c)b-ibc+iad)}{d}\right)}}{b}-\frac{12i\sqrt{dx+c}\,de^{\left(-\frac{4(-i(dx+c)b+ibc-iad)}{d}\right)}}{b}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/768*(3*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b) + 3*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d
)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 24*(sqrt(2)*sqrt(pi)*d*erf(-
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c
- I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf
(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*
c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 8*sqrt(d*x + c))*c -
64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c + 12*I*sqrt(d*x + c)*d*e^(-4*(I*(d
*x + c)*b - I*b*c + I*a*d)/d)/b - 12*I*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*
b + I*b*c - I*a*d)/d)/b)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(a+b\,x\right)}^2\,{\sin\left(a+b\,x\right)}^2\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

3.128 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=200

$$\frac{(c+dx)^{5/2}}{20d}-\frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2}+\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d^{3/2}\cdots}{}$$

```
[Out] 1/20*(d*x+c)^(5/2)/d-1/32*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b+3/1024*d^(3/2)*cos
(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^
(1/2)*Pi^(1/2)/b^(5/2)-3/1024*d^(3/2)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)-3/256*d*cos
(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.21, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2}-\frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b}+\frac{(c+dx)^{5/2}}{20d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(5/2)/(20*d) - (3*d*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(256*b^2) + (
3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(2*Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)
) - ((c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(32*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2} - \frac{1}{8}(c+dx)^{3/2}\cos(4a+4bx)\right)\,dx$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{1}{8}\int (c+dx)^{3/2}\cos(4a+4bx)\,dx$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b} + \frac{(3d)\int \sqrt{c+dx}\,\sin}{64b}$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a}{32b}$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a}{32b}$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a}{32b}$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\cos\left(4a\right.}{}$$

Mathematica [A]
time = 1.07, size = 187, normalized size = 0.94

$$\frac{\sqrt{\frac{b}{d}}\left(15d^2\sqrt{2\pi}\,\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)-15d^2\sqrt{2\pi}\,S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(4a-\frac{4bc}{d}\right)+4\sqrt{\frac{b}{d}}\sqrt{c+dx}\,(-15d^2\cos(4(a+bx))+8b(c+dx)(8b(c+dx)-5d\sin(4(a+bx))))\right)}{5120b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(15*d^2*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqr
t[2/Pi]*Sqrt[c + d*x]] - 15*d^2*Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*
Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(-15*d^2*Co
s[4*(a + b*x)] + 8*b*(c + d*x)*(8*b*(c + d*x) - 5*d*Sin[4*(a + b*x)]))))/(5
120*b^3)
```

Maple [A]
time = 0.00, size = 206, normalized size = 1.03

method	result

derivativedivides	$\frac{\frac{(dx+c)^{\frac{5}{2}}}{20}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{4ad-4cb}{d}\right)\cdots\right.}{\cdots}\right.}{\cdots}}{d}$
default	$\frac{\frac{(dx+c)^{\frac{5}{2}}}{20}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{4ad-4cb}{d}\right)\cdots\right.}{\cdots}\right.}{\cdots}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/40*(d*x+c)^(5/2)-1/64/b*d*(d*x+c)^(3/2)*sin(4/d*b*(d*x+c)+4*(a*d-b*c
)/d)+3/64/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d*b*(d*x+c)+4*(a*d-b*c)/d)+1/32
/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)
/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 264, normalized size = 1.32

$$\frac{\sqrt{2}\left(\frac{512\sqrt{2}(dx+c)^{\frac{5}{2}}b^3}{d}-320\sqrt{2}(dx+c)^{\frac{3}{2}}b^2\sin\left(\frac{4((dx+c)b-bc+ad)}{d}\right)-120\sqrt{2}\sqrt{dx+c}\,bd\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)+15\left(-(i-1)\sqrt{\pi}d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)-(i+1)\sqrt{\pi}d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4(bc-ad)}{d}\right)\right)\operatorname{erf}\left(2\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)+15\left((i+1)\sqrt{\pi}d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)+(i-1)\sqrt{\pi}d^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4(bc-ad)}{d}\right)\right)\operatorname{erf}\left(2\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)\right)}{20480b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/20480*sqrt(2)*(512*sqrt(2)*(d*x + c)^(5/2)*b^3/d - 320*sqrt(2)*(d*x + c)^
(3/2)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 120*sqrt(2)*sqrt(d*x + c)*b*
d*cos(4*((d*x + c)*b - b*c + a*d)/d) + 15*(-(I - 1)*sqrt(pi)*d^2*(b^2/d^2)^
(1/4)*cos(-4*(b*c - a*d)/d) - (I + 1)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-4*(
b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + 15*((I + 1)*sqrt(pi)*d^2*
(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(pi)*d^2*(b^2/d^2)^(1/4
)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^3
```

Fricas [A]
time = 3.98, size = 249, normalized size = 1.24

$$\frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\operatorname{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\operatorname{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+4\left(64b^3d^2x^2-120bd^2\cos(bx+a)^4+128b^3cdx+64b^3c^2+120bd^2\cos(bx+a)^2-15bd^2-160\left(2(b^2d^2x+b^2cd)\cos(bx+a)^3-(b^2d^2x+b^2cd)\cos(bx+a)\right)\sin(bx+a)\right)\sqrt{dx+c}}{5120b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5120*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
 4*(64*b^3*d^2*x^2 - 120*b*d^2*cos(b*x + a)^4 + 128*b^3*c*d*x + 64*b^3*c^2
+ 120*b*d^2*cos(b*x + a)^2 - 15*b*d^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*
x + a)^3 - (b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))
/(b^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}} \sin^2(a+bx)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.90, size = 852, normalized size = 4.26

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/30720*(960*(sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b
^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) + 8*sqrt(d*x + c))*c^2 + d^2*(512*(3*(d*x + c)^(5/2) - 10*(d
*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)/d^2 + 15*(sqrt(2)*sqrt(pi)*(64*b^2*
c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^
2) + 1)*b^2) - 4*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d + 3*s
qrt(d*x + c)*d^2)*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 15*(
sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sq
rt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a
*d)/d)/b^2)/d^2) - 80*(3*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt
```

```
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 3*sqrt(2)*sqrt(pi)*(8*b*c + I*d)
*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*
(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 64*(d*x + c)^
(3/2) + 192*sqrt(d*x + c)*c + 12*I*sqrt(d*x + c)*d*e^(-4*(I*(d*x + c)*b - I
*b*c + I*a*d)/d)/b - 12*I*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b + I*b*c - I
*a*d)/d)/b)*c)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\sin(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

3.129 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=228

$$\frac{(c+dx)^{7/2}}{28d}-\frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2}-\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\ \cdots}{\cdots}$$

```
[Out] 1/28*(d*x+c)^(7/2)/d-5/256*d*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b^2-1/32*(d*x+c)^
(5/2)*sin(4*b*x+4*a)/b-15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*
2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)-15/8192*d^
(5/2)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*
b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/2048*d^2*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^
3
```

Rubi [A]

time = 0.26, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sin(4a+4bx)}{2048b^3}-\frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2}-\frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b}+\frac{(c+dx)^{7/2}}{28d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(7/2)/(28*d) - (5*d*(c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(256*b^2) -
 (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]
*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4096*b
^(7/2)) + (15*d^2*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(2048*b^3) - ((c + d*x)^(
5/2)*Sin[4*a + 4*b*x])/(32*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2} - \frac{1}{8}(c+dx)^{5/2}\cos(4a+4bx)\right)\,dx$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{1}{8}\int (c+dx)^{5/2}\cos(4a+4bx)\,dx$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b} + \frac{(5d)\int(c+dx)^{3/2}\text{si}}{64b}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{5/2}\sin(4}{32b}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} + \frac{15d^2\sqrt{c+dx}\,\text{sin}}{2048b^3}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} + \frac{15d^2\sqrt{c+dx}\,\text{sin}}{2048b^3}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} + \frac{15d^2\sqrt{c+dx}\,\text{sin}}{2048b^3}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(\right.}{}$$

Mathematica [A]
time = 2.05, size = 206, normalized size = 0.90

$$\frac{\sqrt{\frac{b}{d}}\left(-105d^3\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)-105d^3\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(4a-\frac{4bc}{d}\right)+4\sqrt{\frac{b}{d}}\sqrt{c+dx}\,(512b^3(c+dx)^3-280bd^2(c+dx)\cos(4(a+bx))-7d(-15d^2+64b^2(c+dx)^2)\sin(4(a+bx)))\right)}{57344b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(-105*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*S
qrt[2/Pi]*Sqrt[c + d*x]] - 105*d^3*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(512*b^3
*(c + d*x)^3 - 280*b*d^2*(c + d*x)*Cos[4*(a + b*x)] - 7*d*(-15*d^2 + 64*b^2
*(c + d*x)^2)*Sin[4*(a + b*x)])))/(57344*b^4)
```

Maple [A]
time = 0.00, size = 251, normalized size = 1.10

method	result

derivativedivides	$\frac{\frac{(dx+c)^{\frac{7}{2}}}{28}-\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}\right.}{8b}\right.}{}\right)}{}}{d}$
default	$\frac{\frac{(dx+c)^{\frac{7}{2}}}{28}-\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}\right.}{8b}\right.}{}\right)}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/56*(d*x+c)^(7/2)-1/64/b*d*(d*x+c)^(5/2)*sin(4/d*b*(d*x+c)+4*(a*d-b*c
)/d)+5/64/b*d*(-1/8/b*d*(d*x+c)^(3/2)*cos(4/d*b*(d*x+c)+4*(a*d-b*c)/d)+3/8/
b*d*(1/8/b*d*(d*x+c)^(1/2)*sin(4/d*b*(d*x+c)+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2
)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d
)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(
b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 285, normalized size = 1.25

$$\frac{\sqrt{2}\left(\frac{4096\sqrt{2}(dx+c)^{\frac{7}{2}}b^4}{d}-2240\sqrt{2}(dx+c)^{\frac{3}{2}}b^2d\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)+105\left(-(i+1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)+(i-1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4(bc-ad)}{d}\right)\right)\operatorname{erf}\left(2\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)+105\left((i-1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)-(i+1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4(bc-ad)}{d}\right)\right)\operatorname{erf}\left(2\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)-56\left(64\sqrt{2}(dx+c)^{\frac{5}{2}}b^3-15\sqrt{2}\sqrt{dx+c}\,bd^2\right)\sin\left(\frac{4((dx+c)b-bc+ad)}{d}\right)\right)}{229376\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/229376*sqrt(2)*(4096*sqrt(2)*(d*x + c)^(7/2)*b^4/d - 2240*sqrt(2)*(d*x +
c)^(3/2)*b^2*d*cos(4*((d*x + c)*b - b*c + a*d)/d) + 105*(-(I + 1)*sqrt(pi)*
d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(pi)*d^3*(b^2/d^2)^
(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + 105*((I - 1
)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I + 1)*sqrt(pi)*d^3
*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) -
 56*(64*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c)*b*d^2)*sin(4
*((d*x + c)*b - b*c + a*d)/d))/b^4
```

Fricas [**A**]
time = 3.21, size = 347, normalized size = 1.52

$$-\frac{105\sqrt{2}\,\pi d^4\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+105\sqrt{2}\,\pi d^4\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-16\left(128\,b^4d^3x^3+384\,b^4cd^2x^2+128\,b^4c^3-70\,b^2cd^2-560\left(b^2d^3x+b^2cd^2\right)\cos\left(bx+a\right)^4+560\left(b^2d^3x+b^2cd^2\right)\cos\left(bx+a\right)^2+2\left(192\,b^4c^2d-35\,b^2d^3\right)x-7\left(2\left(64\,b^3d^3x^2+128\,b^3cd^2x+64\,b^3c^2d-15\,bd^3\right)\cos\left(bx+a\right)^3-\left(64\,b^3d^3x^2+128\,b^3cd^2x+64\,b^3c^2d-15\,bd^3\right)\cos\left(bx+a\right)\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{57344\,b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/57344*(105*sqrt(2)*pi*d^4*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_s
in(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 105*sqrt(2)*pi*d^4*sqrt(b/(pi*
d))*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/
d) - 16*(128*b^4*d^3*x^3 + 384*b^4*c*d^2*x^2 + 128*b^4*c^3 - 70*b^2*c*d^2 -
 560*(b^2*d^3*x + b^2*c*d^2)*cos(b*x + a)^4 + 560*(b^2*d^3*x + b^2*c*d^2)*c
os(b*x + a)^2 + 2*(192*b^4*c^2*d - 35*b^2*d^3)*x - 7*(2*(64*b^3*d^3*x^2 + 1
28*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a)^3 - (64*b^3*d^3*x^2
+ 128*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a))*sin(b*x + a))*sq
rt(d*x + c))/(b^4*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [**C**] Result contains complex when optimal does not.
time = 1.11, size = 1372, normalized size = 6.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/573440*(17920*(sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) + 8*sqrt(d*x + c))*c^3 + 56*c*d^2*(512*(3*(d*x + c)^(5/2)
 - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)/d^2 + 15*(sqrt(2)*sqrt(pi)*
(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)*b^2) - 4*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^
2 + 15*(sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*s
qrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/
d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*(8*I*(d*x + c)^(3/2)*b*d -
 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b
*c + I*a*d)/d)/b^2)/d^2) + d^3*(4096*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2
)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)/d^3 - 35*(sqrt(2)*sqrt
(pi)*(512*b^3*c^3 - 192*I*b^2*c^2*d - 72*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 4*(-64*I*(d*x + c)^(5/2)*
b^2*d + 192*I*(d*x + c)^(3/2)*b^2*c*d - 192*I*sqrt(d*x + c)*b^2*c^2*d + 40*
(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*
e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 - 35*(sqrt(2)*sqrt(pi)*(
512*b^3*c^3 + 192*I*b^2*c^2*d - 72*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(2)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(
sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 4*(64*I*(d*x + c)^(5/2)*b^2*d -
192*I*(d*x + c)^(3/2)*b^2*c*d + 192*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c
)^(3/2)*b*d^2 - 72*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-4*(I
*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3) - 2240*(3*sqrt(2)*sqrt(pi)*(8*b*
c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 3*sq
rt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)*b) - 64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c + 12*I*sqrt(d*x + c
)*d*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 12*I*sqrt(d*x + c)*d*e^(-4
*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c^2)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\sin(a+b\,x)^2\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2),x)
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

3.130 $\int (c+dx)^{5/2} \cos^2(a+bx) \sin^3(a+bx) \, dx$

Optimal. Leaf size=615

$$\frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}\,\cos(3a+3bx)}{576b^3}-\frac{(c+dx)^{5/2}\cos(3a+3bx}{48b}$$

[Out] -1/8*(d*x+c)^(5/2)*cos(b*x+a)/b-1/48*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b+1/80*(d*x+c)^(5/2)*cos(5*b*x+5*a)/b+5/16*d*(d*x+c)^(3/2)*sin(b*x+a)/b^2+5/288*d*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b^2-1/160*d*(d*x+c)^(3/2)*sin(5*b*x+5*a)/b^2+3/16000*d^(5/2)*cos(5*a-5*b*c/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(7/2)-3/16000*d^(5/2)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(7/2)-5/3456*d^(5/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)+5/3456*d^(5/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)-15/64*d^(5/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^(5/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d^2*cos(b*x+a)*(d*x+c)^(1/2)/b^3+5/576*d^2*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b^3-3/1600*d^2*cos(5*b*x+5*a)*(d*x+c)^(1/2)/b^3

Rubi [A]
time = 0.86, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3}+\frac{5d^2\sqrt{c+dx}\,\cos(3a+3bx)}{576b^3}-\frac{3d^2\sqrt{c+dx}\,\cos(5a+5bx)}{1600b^3}+\frac{5d(c+dx)^{3/2}\sin(a+bx)}{16b^2}+\frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{288b^2}-\frac{d(c+dx)^{3/2}\sin(5a+5bx)}{160b^2}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{5/2}\cos(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]

[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(32*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x])/(8*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5/2)*Cos[3*a + 3*b*x])/(48*b) - (3*d^2*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(1600*b^3) + ((c + d*x)^(5/2)*Cos[5*a + 5*b*x])/(80*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (3*d^(5/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/

```
6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) + (5*d*(c + d*x)^(3/2)*S
in[a + b*x])/(16*b^2) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(288*b^2) -
(d*(c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(160*b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2}\sin(a+bx) + \frac{1}{16}(c+dx)^{5/2}\sin(3a+3bx) -\right.$$

$$= \frac{1}{16}\int (c+dx)^{5/2}\sin(3a+3bx)\,dx - \frac{1}{16}\int (c+dx)^{5/2}\sin(5a+$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{8b} - \frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b} + \frac{(c+}{}$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{8b} - \frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b} + \frac{(c+}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3} - \frac{(c+dx)^{5/2}\cos(a+bx)}{8b} + \frac{5d^2\sqrt{c}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3} - \frac{(c+dx)^{5/2}\cos(a+bx)}{8b} + \frac{5d^2\sqrt{c}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3} - \frac{(c+dx)^{5/2}\cos(a+bx)}{8b} + \frac{5d^2\sqrt{c}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3} - \frac{(c+dx)^{5/2}\cos(a+bx)}{8b} + \frac{5d^2\sqrt{c}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 23.19, size = 3348, normalized size = 5.44

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (c^2*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (c^2*(2*Sqrt[5]*Sqr
t[b/d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*Fre
snelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*
Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d])
 - (c^2*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Co
s[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi
]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*S
```

qrt[3]*b*Sqrt[b/d]) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/
Pi]*Sqrt[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2
*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] +
 3*d*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] -
3*Sin[a + b*x])))/(16*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d
]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c
*d*Sin[a - (b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*
x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) -
2*Sqrt[b/d]*d*Sqrt[c + d*x]*(d*(-15 + 4*b^2*x^2)*Cos[a + b*x] + 2*b*(c - 5*
d*x)*Sin[a + b*x])))/(64*b^5) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/
d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b
*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Co
s[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c
 + d*x]*(2*b*x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) + ((
b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((1
2*b^2*c^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) -
Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a
- (3*b*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[
b/d]*d*Sqrt[c + d*x]*(d*(5 - 12*b^2*x^2)*Cos[3*(a + b*x)] - 2*b*(c - 5*d*x)
Sin[3(a + b*x)])))/(1152*Sqrt[3]*b^5) + (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*Fresne
lS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(3*d*Cos[5*a - (5*b*c)/d] - 10*b*c*
Sin[5*a - (5*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c +
d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*
Sqrt[b/d]*d*Sqrt[c + d*x]*(10*b*x*Cos[5*(a + b*x)] - 3*Sin[5*(a + b*x)])))/
(800*Sqrt[5]*b^3) - (d^2*(Sin[5*a]*((c^2*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]
*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c
 + d*x]])*Sin[(5*b*c)/d])/(5*Sqrt[5]*(b/d)^(3/2)*d^3) + (c^2*Cos[(5*b*c)/d]
*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqr
t[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/(5*Sqrt[5]*(b/d)^(3/2)*d^3) -
 (2*c*Cos[(5*b*c)/d]*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c +
d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 +
5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c + d*x))/d]))/(25*Sqrt[5]*
(b/d)^(5/2)*d^3) - (2*c*Sin[(5*b*c)/d]*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3
/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi
]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d])
)/2))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) + (Sin[(5*b*c)/d]*(-25*Sqrt[5]*(b/d)^(5/
2)*(c + d*x)^(5/2)*Cos[(5*b*(c + d*x))/d] + (5*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sq
rt[c + d*x]*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10
/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c
 + d*x))/d]))/2))/(125*Sqrt[5]*(b/d)^(7/2)*d^3) + (Cos[(5*b*c)/d]*(25*Sqrt[
5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Sin[(5*b*(c + d*x))/d] - (5*(-5*Sqrt[5]*(b/d
)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[
Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin
[(5*b*(c + d*x))/d]))/2))/2))/(125*Sqrt[5]*(b/d)^(7/2)*d^3)) + Cos[5*a]*((c
^2*Cos[(5*b*c)/d]*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]

```
) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/(5*Sqrt[5]*(
b/d)^(3/2)*d^3) - (c^2*Sin[(5*b*c)/d]*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x
))/d]))/(5*Sqrt[5]*(b/d)^(3/2)*d^3) + (2*c*Sin[(5*b*c)/d]*((-3*(-(Sqrt[5]*S
qrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b
/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*
Sin[(5*b*(c + d*x))/d]))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) - (2*c*Cos[(5*b*c)/d]
*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqr
t[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*
Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/2))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) + (
Cos[(5*b*c)/d]*(-25*Sqrt[5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Cos[(5*b*(c + d*x))
/d] + (5*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]) +
Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[...
```

Maple [A]

time = 0.07, size = 719, normalized size = 1.17

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{\ldots}}{\ldots}\right.}{\ldots}\right.}{\ldots}$
default	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{\ldots}}{\ldots}\right.}{\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+5/16/b*d*(1/2/b
*d*(d*x+c)^(3/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(
1/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(c
os((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-si
n((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-
1/96/b*d*(d*x+c)^(5/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+5/96/b*d*(1/6/b*d*(
d*x+c)^(3/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/
2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)
^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*
(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)
^(1/2)*b*(d*x+c)^(1/2)/d))))+1/160/b*d*(d*x+c)^(5/2)*cos(5/d*b*(d*x+c)+5*(a
*d-b*c)/d)-1/32/b*d*(1/10/b*d*(d*x+c)^(3/2)*sin(5/d*b*(d*x+c)+5*(a*d-b*c)/d
)-3/10/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5/d*b*(d*x+c)+5*(a*d-b*c)/d)+1/100/
b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(5*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.55, size = 826, normalized size = 1.34

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/1728000*sqrt(2)*(5400*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b - b
*c + a*d)/d)/d - 15000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b*c
 + a*d)/d)/d - 270000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*c +
a*d)/d)/d - 540*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 3*sqrt(2)*sqrt(d*x +
c)*b^3)*cos(5*((d*x + c)*b - b*c + a*d)/d) + 1500*(12*sqrt(2)*(d*x + c)^(5/
2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*cos(3*((d*x + c)*b - b*c + a*d)/d
) + 27000*(4*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 15*sqrt(2)*sqrt(d*x + c)*b^3
)*cos(((d*x + c)*b - b*c + a*d)/d) - 81*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*d*(
b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (I + 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2
/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 625*(
(I - 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I +
 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(
d*x + c)*sqrt(3*I*b/d)) - 101250*((I - 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*co
s(-(b*c - a*d)/d) + (I + 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)
/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 101250*(-(I + 1)*sqrt(pi)*b^2*d*(b^2/
```

```
d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin
(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - 625*(-(I + 1)*9^(1/4)*s
qrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(
pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*
I*b/d)) - 81*((I + 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c -
 a*d)/d) + (I - 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*
d)/d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^6
```

Fricas [A]
time = 2.89, size = 521, normalized size = 0.85

$$\frac{81\,\sqrt{10}\,\pi d^{3}\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5\,(bc-ad)}{d}\right)C\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-625\,\sqrt{6}\,\pi d^{3}\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)C\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-101250\,\sqrt{2}\,\pi d^{3}\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+101250\,\sqrt{2}\,\pi d^{3}\sqrt{\frac{b}{\pi d}}\,S\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+625\,\sqrt{6}\,\pi d^{3}\sqrt{\frac{b}{\pi d}}\,S\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)-81\,\sqrt{10}\,\pi d^{3}\sqrt{\frac{b}{\pi d}}\,S\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{5\,(bc-ad)}{d}\right)+480\,\left(9\,(20\,b^{3}d^{2}x^{2}+40\,b^{3}cdx+20\,b^{3}c^{2}-3\,bd^{2})\cos\left(bx+a\right)^{5}+390\,bd^{2}\cos\left(bx+a\right)-5\,(60\,b^{3}d^{2}x^{2}+120\,b^{3}cdx+60\,b^{3}c^{2}-13\,bd^{2})\cos\left(bx+a\right)^{3}+10\,\left(26\,b^{2}d^{2}x-9\,(b^{2}d^{2}x+b^{2}cd)\cos\left(bx+a\right)^{4}+26\,b^{2}cd+13\,(b^{2}d^{2}x+b^{2}cd)\cos\left(bx+a\right)^{2}\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{432000\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_c
os(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 625*sqrt(6)*pi*d^3*sqrt(b/(pi*d
))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))
- 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fr
esnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 625*s
qrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-3*(b*c - a*d)/d) - 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(9*(20*b^
3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*cos(b*x + a)^5 + 390*b*d^2
*cos(b*x + a) - 5*(60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 - 13*b*d^2)*
cos(b*x + a)^3 + 10*(26*b^2*d^2*x - 9*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^4
+ 26*b^2*c*d + 13*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(
d*x + c))/b^4
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep
```

Giac [C] Result contains complex when optimal does not.
time = 1.75, size = 3717, normalized size = 6.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")

[Out] -1/864000*(1800*(30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sq
rt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)*d*erf(-1
/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c
 + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*
d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(10)
*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^
3 + 18*c*d^2*(2250*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*er
f(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b
*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x +
c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x +
 c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 125*(-I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 -
 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*
x + c)*d^2)*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 9*(I*sqrt(
10)*sqrt(pi)*(100*b^2*c^2 - 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*
d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I*(-10*I*(d*x + c)^(3/2)*b*d +
 20*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-5*(-I*(d*x + c)*b + I*
b*c - I*a*d)/d)/b^2)/d^2 + 2250*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d
 - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) -
 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d
^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 125*(I*sqrt(6)*sqrt(pi
)*(12*b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b
*c*d + sqrt(d*x + c)*d^2)*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2
 + 9*(-I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sq
rt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c +
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I*(-10*I*(d*x + c
)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-5*(I*(d*x
 + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) + d^3*(6750*(-I*sqrt(2)*sqrt(pi)*(8*b
^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b
*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)*b^2*d - 12*I*
```

```
(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)
*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c
)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 125*(I*sqrt(6)*sqrt(pi)*(72*b^3*c^3 - 36
*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)*b^2*d - 36*I*(d*x +
 c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2
 + 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^(-3*(-I*(d*x + c)*b
+ I*b*c - I*a*d)/d)/b^3)/d^3 + 27*(-I*sqrt(10)*sqrt(pi)*(200*b^3*c^3 - 60*I
*b^2*c^2*d - 18*b*c*d^2 + 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x +
 c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)*b^3) - 10*I*(-20*I*(d*x + c)^(5/2)*b^2*d + 60*I*(d*x
+ c)^(3/2)*b^2*c*d - 60*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^
2 - 18*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c)*d^3)*e^(-5*(-I*(d*x + c)*b
 + I*b*c - I*a*d)/d)/b^3)/d^3 + 6750*(I*sqrt(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*
b^2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(
3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18
*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b*c
+ I*a*d)/d)/b^3)/d^3 + 125*(-I*sqrt(6)*sqrt(pi)*(72*b^3*c^3 + 36*I*b^2*c^2*
d - 18*b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)*b^2...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\sin(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

3.131 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=534

$$-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\cos(5a+5bx)}{80b}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\cdots\right.}{16b^{5/2}}$$

```
[Out] -1/8*(d*x+c)^(3/2)*cos(b*x+a)/b-1/48*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b+1/80*(d
*x+c)^(3/2)*cos(5*b*x+5*a)/b+3/8000*d^(3/2)*cos(5*a-5*b*c/d)*FresnelS(b^(1/
2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(5/2)+3/800
0*d^(3/2)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a
-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(5/2)-1/576*d^(3/2)*cos(3*a-3*b*c/d)*FresnelS
(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1
/576*d^(3/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3
*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*cos(a-b*c/d)*FresnelS(b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-3/32*
d^(3/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/
d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d*sin(b*x+a)*(d*x+c)^(1/2)/b^2+1/96*d*sin(
3*b*x+3*a)*(d*x+c)^(1/2)/b^2-3/800*d*sin(5*b*x+5*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.64, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\cos\left(5a-\frac{5bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{16b^2}+\frac{d\sqrt{c+dx}\sin(3a+3bx)}{96b^2}-\frac{3d\sqrt{c+dx}\sin(5a+5bx)}{800b^2}-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\cos(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -1/8*((c + d*x)^(3/2)*Cos[a + b*x])/b - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(48*b) + ((c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(80*b) - (3*d^(3/2)*Sqrt[Pi/2]*
Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*
b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[
6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*Cos[5*
a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(800*
b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*
x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Fres
nelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(96*
b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[a + b*x]
)/(16*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(96*b^2) - (3*d*Sqrt[c + d*
x]*Sin[5*a + 5*b*x])/(800*b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2}\sin(a+bx)+\frac{1}{16}(c+dx)^{3/2}\sin(3a+3bx)-\frac{1}{1}\right.$$

$$=\frac{1}{16}\int (c+dx)^{3/2}\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^{3/2}\sin(5a+5$$

$$=-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx}{}$$

$$=-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx}{}$$

$$=-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx}{}$$

$$=-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx}{}$$

$$=-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.38, size = 1041, normalized size = 1.95

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (c*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-
I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqr
t[(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (c*(2*Sqrt[5]*Sqrt[b/
d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*Fresnel
C[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (
c*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a
- (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*Fres
nelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*Sqrt[3]
*b*Sqrt[b/d]) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt
[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin
[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a
```

```
+ b*x])))/(32*b^3) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]
*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqr
t[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*
c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*
x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(192*Sqrt[3]*b^3) + (Sqrt[b/d]*d*(
Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(3*d*Cos[5*a - (5*
b*c)/d] - 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Sin[5*a - (5*b*c)
/d]) + 2*Sqrt[5]*Sqrt[b/d]*d*Sqrt[c + d*x]*(10*b*x*Cos[5*(a + b*x)] - 3*Sin
[5*(a + b*x)])))/(1600*Sqrt[5]*b^3)
```

Maple [A]
time = 0.07, size = 580, normalized size = 1.09

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{d}}{\sqrt{\pi}\,\sqrt{}}\right.\right.}{}\right.}{8b}$
default	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{d}}{\sqrt{\pi}\,\sqrt{}}\right.\right.}{}\right.}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+3/16/b*d*(1/2/b
*d*(d*x+c)^(1/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b
/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)
^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)))-1/96/b*d*(d*x+c)^(3/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/32/b*d
*(1/6/b*d*(d*x+c)^(1/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3
^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))+1/160/b*d*(d*x+c)^(3/2)*co
s(5/d*b*(d*x+c)+5*(a*d-b*c)/d)-3/160/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5/d*b*
(d*x+c)+5*(a*d-b*c)/d)-1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(
```

5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2
)/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*
x+c)^(1/2)/d))))

Maxima [C] Result contains complex when optimal does not.
time = 0.55, size = 760, normalized size = 1.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/288000*sqrt(2)*(1800*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b - b*c
 + a*d)/d)/d^2 - 3000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b*c
+ a*d)/d)/d^2 - 18000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c +
a*d)/d)/d^2 - 540*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c + a*d)
/d)/d + 1500*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d)/d
 + 27000*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d + 27*
((I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (I -
 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d
*x + c)*sqrt(5*I*b/d)) + 125*(-(I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)
*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-
3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + 6750*(-(I + 1)*sqrt(pi
)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(pi)*b^2*(b^2/d^2)^
(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + 6750*((I - 1)*s
qrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(pi)*b^2*(b^2
/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + 125*((I
- 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9
^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(-3*I*b/d)) + 27*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(
-5*(b*c - a*d)/d) + (I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b
*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^5
```

Fricas [A]
time = 3.66, size = 427, normalized size = 0.80

$$\frac{27\sqrt{10}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)S\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 125\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 6750\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 6750\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right) - 125\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right) + 27\sqrt{10}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,C\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{5(bc-ad)}{d}\right) + 480\left(30\left(b^2dx+b^2c\right)\cos\left(bx+a\right)^5 - 50\left(b^2dx+b^2c\right)\cos\left(bx+a\right)^3 - \left(9\,bd\cos\left(bx+a\right)^4 - 13\,bd\cos\left(bx+a\right)^2 - 26\,bd\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{72000\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_si
n(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d)
)*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) -
 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d))) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel
```

```
_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6
)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-3*(b*c - a*d)/d) + 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(1
0)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(30*(b^2*d*x +
 b^2*c)*cos(b*x + a)^5 - 50*(b^2*d*x + b^2*c)*cos(b*x + a)^3 - (9*b*d*cos(b
*x + a)^4 - 13*b*d*cos(b*x + a)^2 - 26*b*d)*sin(b*x + a))*sqrt(d*x + c))/b^
3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}} \sin^3(a+bx)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**3*cos(a + b*x)**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 1.27, size = 2320, normalized size = 4.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/144000*(300*(30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/
2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c
+ I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d
*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(10)*
sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^2
 + d^2*(2250*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2
*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/
2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b
+ I*b*c - I*a*d)/d)/b^2)/d^2 + 125*(-I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b
*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
```

```
) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^
2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)
*d^2)*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 9*(I*sqrt(10)*sq
rt(pi)*(100*b^2*c^2 - 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*
sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-5*(-I*(d*x + c)*b + I*b*c -
I*a*d)/d)/b^2)/d^2 + 2250*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d
^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(
2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^
((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 125*(I*sqrt(6)*sqrt(pi)*(12*
b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d +
 sqrt(d*x + c)*d^2)*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 9*(
-I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d
)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I*(-10*I*(d*x + c)^(3/2
)*b*d + 20*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-5*(I*(d*x + c)*
b - I*b*c + I*a*d)/d)/b^2)/d^2) + 20*(-450*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)
*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*
sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d
^2) + 1)*b) - 9*I*sqrt(10)*sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*sqrt(10)*sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*sqrt(pi)*(2*b*c -
I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*s
qrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)*b) + 9*I*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10
)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 900*sqrt(d*x + c)*d*e^((I*(
d*x + c)*b - I*b*c + I*a*d)/d)/b + 150*sqrt(d*x + c)*d*e^(-3*(I*(d*x + c)*b
 - I*b*c + I*a*d)/d)/b - 90*sqrt(d*x + c)*d*e^(-5*(I*(d*x + c)*b - I*b*c +
I*a*d)/d)/b + 900*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b
+ 150*sqrt(d*x + c)*d*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b - 90*sqrt
(d*x + c)*d*e^(-5*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\,\sin(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

3.132 $\int \sqrt{c+dx} \cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=459

$$-\frac{\sqrt{c+dx}\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\cos(5a+5bx)}{80b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}}{8b^{3/2}}$$

```
[Out] -1/800*cos(5*a-5*b*c/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(3/2)+1/800*FresnelS(b^(1/2)*10^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*d^(1/2)*10^(1/2)*Pi^(1/2)/b^
(3/2)+1/288*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2
)/d^(1/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1/288*FresnelS(b^(1/2)*6^(1/2)/
Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b
^(3/2)+1/16*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(b^(1/2)*2^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/
8*cos(b*x+a)*(d*x+c)^(1/2)/b-1/48*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b+1/80*cos(5
*b*x+5*a)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.50, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\cos(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -1/8*(Sqrt[c + d*x]*Cos[a + b*x])/b - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(48*
b) + (Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(80*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a -
(b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(3/2))
+ (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/10]*Cos[5*a - (5*b*c)
/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(80*b^(3/2)) + (
Sqrt[d]*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[5*a - (5*b*c)/d])/(80*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(48*b^(3/2)) - (Sqr
t[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
- (b*c)/d])/(8*b^(3/2))
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\,\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx}\,\sin(a+bx) + \frac{1}{16}\sqrt{c+dx}\,\sin(3a+3bx) - \frac{1}{16}\sqrt{c}\right.$$

$$= \frac{1}{16}\int \sqrt{c+dx}\,\sin(3a+3bx)\,dx - \frac{1}{16}\int \sqrt{c+dx}\,\sin(5a+5bx)$$

$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{8b} - \frac{\sqrt{c+dx}\,\cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx}\,\mathrm{c}}{8}$$

$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{8b} - \frac{\sqrt{c+dx}\,\cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx}\,\mathrm{c}}{8}$$

$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{8b} - \frac{\sqrt{c+dx}\,\cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx}\,\mathrm{c}}{8}$$

$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{8b} - \frac{\sqrt{c+dx}\,\cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx}\,\mathrm{c}}{8}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.97, size = 432, normalized size = 0.94

$$\frac{e^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(-\frac{e^{2ia}\mathrm{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\mathrm{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b}+\frac{2\sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx}\,\cos(5(a+bx))-\sqrt{2\pi}\,\cos\left(5a-\frac{5bc}{d}\right)\mathrm{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right)+\sqrt{2\pi}\,S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right)\sin\left(5a-\frac{5bc}{d}\right)}{160\sqrt{5}\,b\sqrt{\frac{b}{d}}}-\frac{2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\,\cos(3(a+bx))-\sqrt{2\pi}\,\cos\left(3a-\frac{3bc}{d}\right)\mathrm{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+\sqrt{2\pi}\,S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\sin\left(3a-\frac{3bc}{d}\right)}{96\sqrt{3}\,b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)
*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[
(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (2*Sqrt[5]*Sqrt[b/d]*Sq
rt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelC[Sqr
t[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/P
i]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d])/(160*Sqrt[5]*b*Sqrt[b/d]) - (2*Sqrt
[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)
/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d])/(96*Sqrt[3]*b*Sqrt[b/
d])
```

Maple [A]
time = 0.07, size = 447, normalized size = 0.97

method	result
derivativedivides	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}}{\sqrt{}}\right.\right.}{16b\sqrt{\frac{b}{d}}}}{}$
default	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}}{\sqrt{}}\right.\right.}{16b\sqrt{\frac{b}{d}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+1/32/b*d*2^(1/2
)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d))-1/96/b*d*(d*x+c)^(1/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)
/d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fres
nelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c
)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))+1/16
0/b*d*(d*x+c)^(1/2)*cos(5/d*b*(d*x+c)+5*(a*d-b*c)/d)-1/1600/b*d*2^(1/2)*Pi^
(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(
1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 680, normalized size = 1.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/28800*sqrt(2)*(180*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c + a
*d)/d)/d^2 - 300*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/
d)/d^2 - 1800*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d^
2 - 9*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)
/d - (I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d)
*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 25*((I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d
^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d + (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^
(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 450*((I -
```

```
 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d + (I + 1)*sqrt(pi)*b
^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(I*b/d)) -
450*(-(I + 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d - (I - 1)*
sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(
-I*b/d)) - 25*(-(I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-3*(b*c -
a*d)/d)/d - (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)
/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - 9*((I + 1)*25^(1/4)*sqrt(pi)*b^2
*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d + (I - 1)*25^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d
^2/b^4
```

Fricas [A]
time = 2.62, size = 356, normalized size = 0.78

$$-\frac{9\,\sqrt{10}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5\,(bc-ad)}{d}\right)C\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-25\,\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)C\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-450\,\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+450\,\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,S\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+25\,\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\,S\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)-9\,\sqrt{10}\,\pi d\sqrt{\frac{b}{\pi d}}\,S\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{5\,(bc-ad)}{d}\right)-480\,\left(3\,b\cos\left(bx+a\right)^{5}-5\,b\cos\left(bx+a\right)^{3}\right)\sqrt{dx+c}}{7200\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_cos(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(
-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*s
qrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b
/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*
d)/d) - 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(10)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^5 - 5*b*cos(b*
x + a)^3)*sqrt(d*x + c))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^{3}\left(a+bx\right)\cos^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x)**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.89, size = 1270, normalized size = 2.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/14400*(-450*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf
(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(
I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(10)*s
qrt(pi)*(10*b*c - I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*
d^2) + 1)*b) + 450*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*(6*b*c + I*d
)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt
(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)*b) + 30*(30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)
*(I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(
b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(
sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sq
rt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)*d
*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sq
rt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*s
qrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
1)))*c + 900*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 150*
sqrt(d*x + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 90*sqrt(d*x +
c)*d*e^(-5*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 900*sqrt(d*x + c)*d*e^((-
I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*sqrt(d*x + c)*d*e^(-3*(-I*(d*x +
c)*b + I*b*c - I*a*d)/d)/b - 90*sqrt(d*x + c)*d*e^(-5*(-I*(d*x + c)*b + I*b
*c - I*a*d)/d)/b)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\sin(a+b\,x)^3\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

3.133 $\int \sqrt{c+dx}\,\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=459

$$-\frac{\sqrt{c+dx}\,\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\,\cos(5a+5bx)}{80b}+\frac{\sqrt{d}\,\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}}{8b^{3/2}}$$

```
[Out] -1/800*cos(5*a-5*b*c/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(3/2)+1/800*FresnelS(b^(1/2)*10^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*d^(1/2)*10^(1/2)*Pi^(1/2)/b^
(3/2)+1/288*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2
)/d^(1/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1/288*FresnelS(b^(1/2)*6^(1/2)/
Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b
^(3/2)+1/16*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(b^(1/2)*2^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/
8*cos(b*x+a)*(d*x+c)^(1/2)/b-1/48*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b+1/80*cos(5
*b*x+5*a)*(d*x+c)^(1/2)/b
```

Rubi [A]

time = 0.50, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\cos(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -1/8*(Sqrt[c + d*x]*Cos[a + b*x])/b - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(48*
b) + (Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(80*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a -
(b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(3/2))
+ (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/10]*Cos[5*a - (5*b*c)
/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(80*b^(3/2)) + (
Sqrt[d]*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[5*a - (5*b*c)/d])/(80*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(48*b^(3/2)) - (Sqr
t[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
- (b*c)/d])/(8*b^(3/2))
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\,\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx}\,\sin(a+bx)+\frac{1}{16}\sqrt{c+dx}\,\sin(3a+3bx)-\frac{1}{16}\sqrt{c}\right.$$

$$=\frac{1}{16}\int\sqrt{c+dx}\,\sin(3a+3bx)\,dx-\frac{1}{16}\int\sqrt{c+dx}\,\sin(5a+5bx)$$

$$=-\frac{\sqrt{c+dx}\,\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\,\text{c}}{8}$$

$$=-\frac{\sqrt{c+dx}\,\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\,\text{c}}{8}$$

$$=-\frac{\sqrt{c+dx}\,\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\,\text{c}}{8}$$

$$=-\frac{\sqrt{c+dx}\,\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\,\text{c}}{8}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.83, size = 432, normalized size = 0.94

$$\frac{e^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(-\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b}+\frac{2\sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx}\,\cos(5(a+bx))-\sqrt{2\pi}\,\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right)+\sqrt{2\pi}\,S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right)\sin\left(5a-\frac{5bc}{d}\right)}{160\sqrt{5}\,b\sqrt{\frac{b}{d}}}-\frac{2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\,\cos(3(a+bx))-\sqrt{2\pi}\,\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+\sqrt{2\pi}\,S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\sin\left(3a-\frac{3bc}{d}\right)}{96\sqrt{3}\,b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)
*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[
(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (2*Sqrt[5]*Sqrt[b/d]*Sq
rt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelC[Sqr
t[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/P
i]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d])/(160*Sqrt[5]*b*Sqrt[b/d]) - (2*Sqrt
[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)
/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d])/(96*Sqrt[3]*b*Sqrt[b/
d])
```

Maple [A]
time = 0.00, size = 447, normalized size = 0.97

method	result
derivativedivides	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}}{\sqrt{}}\right.\right.}{16b\sqrt{\frac{b}{d}}}}{}$
default	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}}{\sqrt{}}\right.\right.}{16b\sqrt{\frac{b}{d}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+1/32/b*d*2^(1/2
)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d))-1/96/b*d*(d*x+c)^(1/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)
/d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fres
nelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c
)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))+1/16
0/b*d*(d*x+c)^(1/2)*cos(5/d*b*(d*x+c)+5*(a*d-b*c)/d)-1/1600/b*d*2^(1/2)*Pi^
(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(
1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 680, normalized size = 1.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/28800*sqrt(2)*(180*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c + a
*d)/d)/d^2 - 300*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/
d)/d^2 - 1800*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d^
2 - 9*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)
/d - (I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d)
*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 25*((I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d
^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d + (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^
(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 450*((I -
```

```
1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d + (I + 1)*sqrt(pi)*b
^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(I*b/d)) -
450*(-(I + 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d - (I - 1)*
sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(
-I*b/d)) - 25*(-(I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-3*(b*c -
a*d)/d)/d - (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)
/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - 9*((I + 1)*25^(1/4)*sqrt(pi)*b^2
*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d + (I - 1)*25^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d
^2/b^4
```

Fricas [A]
time = 3.44, size = 356, normalized size = 0.78

$$-\frac{9\sqrt{10}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-25\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-450\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+450\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+25\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)-9\sqrt{10}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{5(bc-ad)}{d}\right)-480\left(3b\cos(bx+a)^5-5b\cos(bx+a)^3\right)\sqrt{dx+c}}{7200\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_cos(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(
-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*s
qrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b
/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*
d)/d) - 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(10)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^5 - 5*b*cos(b*
x + a)^3)*sqrt(d*x + c))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^3(a+bx)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x)**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.89, size = 1270, normalized size = 2.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/14400*(-450*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf
(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(
I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(10)*s
qrt(pi)*(10*b*c - I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*
d^2) + 1)*b) + 450*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*(6*b*c + I*d
)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt
(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)*b) + 30*(30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)
*(I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(
b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(
sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sq
rt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)*d
*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sq
rt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*s
qrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
1)))*c + 900*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 150*
sqrt(d*x + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 90*sqrt(d*x +
c)*d*e^(-5*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 900*sqrt(d*x + c)*d*e^((-
I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*sqrt(d*x + c)*d*e^(-3*(-I*(d*x +
c)*b + I*b*c - I*a*d)/d)/b - 90*sqrt(d*x + c)*d*e^(-5*(-I*(d*x + c)*b + I*b
*c - I*a*d)/d)/b)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\sin(a+b\,x)^3\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

3.134 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=534

$$-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\cos(5a+5bx)}{80b}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\right.}{16b^{5/2}}$$

```
[Out] -1/8*(d*x+c)^(3/2)*cos(b*x+a)/b-1/48*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b+1/80*(d
*x+c)^(3/2)*cos(5*b*x+5*a)/b+3/8000*d^(3/2)*cos(5*a-5*b*c/d)*FresnelS(b^(1/
2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(5/2)+3/800
0*d^(3/2)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a
-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(5/2)-1/576*d^(3/2)*cos(3*a-3*b*c/d)*FresnelS
(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1
/576*d^(3/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3
*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*cos(a-b*c/d)*FresnelS(b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-3/32*
d^(3/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/
d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d*sin(b*x+a)*(d*x+c)^(1/2)/b^2+1/96*d*sin(
3*b*x+3*a)*(d*x+c)^(1/2)/b^2-3/800*d*sin(5*b*x+5*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.58, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\cos\left(5a-\frac{5bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{16b^2}+\frac{d\sqrt{c+dx}\sin(3a+3bx)}{96b^2}-\frac{3d\sqrt{c+dx}\sin(5a+5bx)}{800b^2}-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\cos(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -1/8*((c + d*x)^(3/2)*Cos[a + b*x])/b - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(48*b) + ((c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(80*b) - (3*d^(3/2)*Sqrt[Pi/2]*
Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*
b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[
6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*Cos[5*
a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(800*
b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*
x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Fres
nelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(96*
b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[a + b*x]
)/(16*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(96*b^2) - (3*d*Sqrt[c + d*
x]*Sin[5*a + 5*b*x])/(800*b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2}\sin(a+bx) + \frac{1}{16}(c+dx)^{3/2}\sin(3a+3bx) - \frac{1}{1}\right.$$

$$= \frac{1}{16}\int (c+dx)^{3/2}\sin(3a+3bx)\,dx - \frac{1}{16}\int (c+dx)^{3/2}\sin(5a+5$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b} + \frac{(c+dx}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b} + \frac{(c+dx}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b} + \frac{(c+dx}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b} + \frac{(c+dx}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b} + \frac{(c+dx}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.15, size = 1041, normalized size = 1.95

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (c*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-
I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqr
t[(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (c*(2*Sqrt[5]*Sqrt[b/
d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*Fresnel
C[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (
c*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a
- (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*Fres
nelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*Sqrt[3]
*b*Sqrt[b/d]) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt
[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin
[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a
```

```
+ b*x])))/(32*b^3) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]
*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqr
t[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*
c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*
x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(192*Sqrt[3]*b^3) + (Sqrt[b/d]*d*(
Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(3*d*Cos[5*a - (5*
b*c)/d] - 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Sin[5*a - (5*b*c)
/d]) + 2*Sqrt[5]*Sqrt[b/d]*d*Sqrt[c + d*x]*(10*b*x*Cos[5*(a + b*x)] - 3*Sin
[5*(a + b*x)])))/(1600*Sqrt[5]*b^3)
```

Maple [A]
time = 0.00, size = 580, normalized size = 1.09

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{d}}{\sqrt{\pi}\,\sqrt{}}\right.\right.}{}\right.}{8b}$
default	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{d}}{\sqrt{\pi}\,\sqrt{}}\right.\right.}{}\right.}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+3/16/b*d*(1/2/b
*d*(d*x+c)^(1/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b
/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)
^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)))-1/96/b*d*(d*x+c)^(3/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/32/b*d
*(1/6/b*d*(d*x+c)^(1/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3
^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))+1/160/b*d*(d*x+c)^(3/2)*co
s(5/d*b*(d*x+c)+5*(a*d-b*c)/d)-3/160/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5/d*b*
(d*x+c)+5*(a*d-b*c)/d)-1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(
```

```
5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2
)/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*
x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.54, size = 760, normalized size = 1.42

$$\frac{\sqrt{2}\left(\frac{1800\sqrt{2}(dx+c)^{\frac{3}{2}}b^4\cos\left(\frac{5((dx+c)b-bc+ad)}{d}\right)}{d^2}-\frac{3000\sqrt{2}(dx+c)^{\frac{3}{2}}b^4\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d^2}-\frac{18000\sqrt{2}(dx+c)^{\frac{3}{2}}b^4\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d^2}-\frac{540\sqrt{2}\sqrt{dx+c}\,b^3\sin\left(\frac{5((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{1500\sqrt{2}\sqrt{dx+c}\,b^3\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{27000\sqrt{2}\sqrt{dx+c}\,b^3\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+27\left((i+1)\cdot 25^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{5(bc-ad)}{d}\right)-(i-1)\cdot 25^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{5(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{5ib}{d}}\right)+125\left(-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)+(i-1)\cdot 9^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{3ib}{d}}\right)+6750\left(-(i+1)\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i-1)\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)+6750\left((i-1)\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)-(i+1)\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)+125\left((i-1)\cdot 9^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{3ib}{d}}\right)+27\left(-(i-1)\cdot 25^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{5(bc-ad)}{d}\right)+(i+1)\cdot 25^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{5(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{5ib}{d}}\right)\right)d^2}{288000\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/288000*sqrt(2)*(1800*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b - b*c
 + a*d)/d)/d^2 - 3000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b*c
+ a*d)/d)/d^2 - 18000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c +
a*d)/d)/d^2 - 540*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c + a*d)
/d)/d + 1500*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d)/d
 + 27000*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d + 27*
((I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (I -
 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d
*x + c)*sqrt(5*I*b/d)) + 125*(-(I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)
*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-
3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + 6750*(-(I + 1)*sqrt(pi
)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(pi)*b^2*(b^2/d^2)^
(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + 6750*((I - 1)*s
qrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(pi)*b^2*(b^2
/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + 125*((I
- 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9
^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(-3*I*b/d)) + 27*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(
-5*(b*c - a*d)/d) + (I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b
*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^5
```

Fricas [A]
time = 3.22, size = 427, normalized size = 0.80

$$\frac{27\sqrt{10}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)S\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-125\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-6750\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-6750\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-125\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)+27\sqrt{10}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,C\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{5(bc-ad)}{d}\right)+480\left(30\left(b^2dx+b^2c\right)\cos(bx+a)^5-50\left(b^2dx+b^2c\right)\cos(bx+a)^3-\left(9bd\cos(bx+a)^4-13bd\cos(bx+a)^2-26bd\right)\sin(bx+a)\right)\sqrt{dx+c}}{72000\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_si
n(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d)
)*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) -
 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d))) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel
```

```
_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6
)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-3*(b*c - a*d)/d) + 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(1
0)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(30*(b^2*d*x +
 b^2*c)*cos(b*x + a)^5 - 50*(b^2*d*x + b^2*c)*cos(b*x + a)^3 - (9*b*d*cos(b
*x + a)^4 - 13*b*d*cos(b*x + a)^2 - 26*b*d)*sin(b*x + a))*sqrt(d*x + c))/b^
3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin^3(a+bx)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**3*cos(a + b*x)**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 1.23, size = 2320, normalized size = 4.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/144000*(300*(30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/
2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c
+ I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d
*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(10)*
sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^2
 + d^2*(2250*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2
*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/
2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b
+ I*b*c - I*a*d)/d)/b^2)/d^2 + 125*(-I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b
*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
```

```
) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^
2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)
*d^2)*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 9*(I*sqrt(10)*sq
rt(pi)*(100*b^2*c^2 - 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*
sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-5*(-I*(d*x + c)*b + I*b*c -
I*a*d)/d)/b^2)/d^2 + 2250*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d
^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(
2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^
((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 125*(I*sqrt(6)*sqrt(pi)*(12*
b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d +
 sqrt(d*x + c)*d^2)*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 9*(
-I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d
)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I*(-10*I*(d*x + c)^(3/2
)*b*d + 20*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-5*(I*(d*x + c)*
b - I*b*c + I*a*d)/d)/b^2)/d^2) + 20*(-450*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)
*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*
sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d
^2) + 1)*b) - 9*I*sqrt(10)*sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*sqrt(10)*sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*sqrt(pi)*(2*b*c -
I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*s
qrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)*b) + 9*I*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10
)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 900*sqrt(d*x + c)*d*e^((I*(
d*x + c)*b - I*b*c + I*a*d)/d)/b + 150*sqrt(d*x + c)*d*e^(-3*(I*(d*x + c)*b
 - I*b*c + I*a*d)/d)/b - 90*sqrt(d*x + c)*d*e^(-5*(I*(d*x + c)*b - I*b*c +
I*a*d)/d)/b + 900*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b
+ 150*sqrt(d*x + c)*d*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b - 90*sqrt
(d*x + c)*d*e^(-5*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c)/d
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

3.135 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=615

$$\frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}\,\cos(3a+3bx)}{576b^3}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}$$

[Out] -1/8*(d*x+c)^(5/2)*cos(b*x+a)/b-1/48*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b+1/80*(d*x+c)^(5/2)*cos(5*b*x+5*a)/b+5/16*d*(d*x+c)^(3/2)*sin(b*x+a)/b^2+5/288*d*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b^2-1/160*d*(d*x+c)^(3/2)*sin(5*b*x+5*a)/b^2+3/16000*d^(5/2)*cos(5*a-5*b*c/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(7/2)-3/16000*d^(5/2)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(7/2)-5/3456*d^(5/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)+5/3456*d^(5/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)-15/64*d^(5/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^(5/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d^2*cos(b*x+a)*(d*x+c)^(1/2)/b^3+5/576*d^2*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b^3-3/1600*d^2*cos(5*b*x+5*a)*(d*x+c)^(1/2)/b^3

Rubi [A]
time = 0.76, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{3\sqrt{\frac{\pi}{10}}\,d^{5/2}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{3\sqrt{\frac{\pi}{10}}\,d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3}+\frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{576b^3}-\frac{3d^2\sqrt{c+dx}\cos(5a+5bx)}{1600b^3}+\frac{5d(c+dx)^{3/2}\sin(a+bx)}{16b^2}+\frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{288b^2}-\frac{d(c+dx)^{3/2}\sin(5a+5bx)}{160b^2}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{5/2}\cos(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]

[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(32*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x])/(8*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5/2)*Cos[3*a + 3*b*x])/(48*b) - (3*d^2*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(1600*b^3) + ((c + d*x)^(5/2)*Cos[5*a + 5*b*x])/(80*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (3*d^(5/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/

```
6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) + (5*d*(c + d*x)^(3/2)*S
in[a + b*x])/(16*b^2) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(288*b^2) -
(d*(c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(160*b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2}\sin(a+bx)+\frac{1}{16}(c+dx)^{5/2}\sin(3a+3bx)-\frac{1}{1}\right.$$

$$=\frac{1}{16}\int (c+dx)^{5/2}\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^{5/2}\sin(5a+5$$

$$=-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx}{}$$

$$=-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx}{}$$

$$=\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+}}{}$$

$$=\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+}}{}$$

$$=\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+}}{}$$

$$=\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 22.01, size = 3348, normalized size = 5.44

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (c^2*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (c^2*(2*Sqrt[5]*Sqr
t[b/d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*Fre
snelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*
Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d])
 - (c^2*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Co
s[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi
]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*S
```

```
qrt[3]*b*Sqrt[b/d]) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/
Pi]*Sqrt[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2
*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] +
 3*d*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] -
3*Sin[a + b*x])))/(16*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d
]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c
*d*Sin[a - (b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*
x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) -
2*Sqrt[b/d]*d*Sqrt[c + d*x]*(d*(-15 + 4*b^2*x^2)*Cos[a + b*x] + 2*b*(c - 5*
d*x)*Sin[a + b*x])))/(64*b^5) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/
d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b
*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Co
s[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c
 + d*x]*(2*b*x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) + ((
b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((1
2*b^2*c^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) -
Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a
- (3*b*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[
b/d]*d*Sqrt[c + d*x]*(d*(5 - 12*b^2*x^2)*Cos[3*(a + b*x)] - 2*b*(c - 5*d*x)
*Sin[3*(a + b*x)])))/(1152*Sqrt[3]*b^5) + (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*Fresne
lS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(3*d*Cos[5*a - (5*b*c)/d] - 10*b*c*
Sin[5*a - (5*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c +
d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*
Sqrt[b/d]*d*Sqrt[c + d*x]*(10*b*x*Cos[5*(a + b*x)] - 3*Sin[5*(a + b*x)])))/
(800*Sqrt[5]*b^3) - (d^2*(Sin[5*a]*((c^2*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]
*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c
 + d*x]])*Sin[(5*b*c)/d])/(5*Sqrt[5]*(b/d)^(3/2)*d^3) + (c^2*Cos[(5*b*c)/d]
*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqr
t[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/(5*Sqrt[5]*(b/d)^(3/2)*d^3) -
 (2*c*Cos[(5*b*c)/d]*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c +
d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 +
5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c + d*x))/d]))/(25*Sqrt[5]*
(b/d)^(5/2)*d^3) - (2*c*Sin[(5*b*c)/d]*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3
/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi
]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d])
)/2))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) + (Sin[(5*b*c)/d]*(-25*Sqrt[5]*(b/d)^(5/
2)*(c + d*x)^(5/2)*Cos[(5*b*(c + d*x))/d] + (5*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sq
rt[c + d*x]*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10
/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c
 + d*x))/d]))/2))/(125*Sqrt[5]*(b/d)^(7/2)*d^3) + (Cos[(5*b*c)/d]*(25*Sqrt[
5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Sin[(5*b*(c + d*x))/d] - (5*(-5*Sqrt[5]*(b/d
)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[
Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin
[(5*b*(c + d*x))/d]))/2))/2))/(125*Sqrt[5]*(b/d)^(7/2)*d^3)) + Cos[5*a]*((c
^2*Cos[(5*b*c)/d]*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]
```

```
) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/(5*Sqrt[5]*(
b/d)^(3/2)*d^3) - (c^2*Sin[(5*b*c)/d]*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x
))/d]))/(5*Sqrt[5]*(b/d)^(3/2)*d^3) + (2*c*Sin[(5*b*c)/d]*((-3*(-(Sqrt[5]*S
qrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b
/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*
Sin[(5*b*(c + d*x))/d]))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) - (2*c*Cos[(5*b*c)/d]
*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqr
t[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*
Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/2))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) + (
Cos[(5*b*c)/d]*(-25*Sqrt[5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Cos[(5*b*(c + d*x))
/d] + (5*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]) +
Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[...
```

Maple [A]

time = 0.00, size = 719, normalized size = 1.17

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}$
default	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+5/16/b*d*(1/2/b
*d*(d*x+c)^(3/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(
1/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(c
os((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-si
n((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-
1/96/b*d*(d*x+c)^(5/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+5/96/b*d*(1/6/b*d*(
d*x+c)^(3/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/
2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)
^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*
(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)
^(1/2)*b*(d*x+c)^(1/2)/d))))+1/160/b*d*(d*x+c)^(5/2)*cos(5/d*b*(d*x+c)+5*(a
*d-b*c)/d)-1/32/b*d*(1/10/b*d*(d*x+c)^(3/2)*sin(5/d*b*(d*x+c)+5*(a*d-b*c)/d
)-3/10/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5/d*b*(d*x+c)+5*(a*d-b*c)/d)+1/100/
b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(5*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.56, size = 826, normalized size = 1.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/1728000*sqrt(2)*(5400*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b - b
*c + a*d)/d)/d - 15000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b*c
 + a*d)/d)/d - 270000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*c +
a*d)/d)/d - 540*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 3*sqrt(2)*sqrt(d*x +
c)*b^3)*cos(5*((d*x + c)*b - b*c + a*d)/d) + 1500*(12*sqrt(2)*(d*x + c)^(5/
2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*cos(3*((d*x + c)*b - b*c + a*d)/d
) + 27000*(4*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 15*sqrt(2)*sqrt(d*x + c)*b^3
)*cos(((d*x + c)*b - b*c + a*d)/d) - 81*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*d*(
b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (I + 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2
/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 625*(
(I - 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I +
 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(
d*x + c)*sqrt(3*I*b/d)) - 101250*((I - 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*co
s(-(b*c - a*d)/d) + (I + 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)
/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 101250*(-(I + 1)*sqrt(pi)*b^2*d*(b^2/
```

```
d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin
(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - 625*(-(I + 1)*9^(1/4)*s
qrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(
pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*
I*b/d)) - 81*((I + 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c -
 a*d)/d) + (I - 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*
d)/d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^6
```

Fricas [A]
time = 3.07, size = 521, normalized size = 0.85

$$\frac{81\sqrt{10}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)C\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-625\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-101250\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+101250\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+625\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)-81\sqrt{10}\,\pi d^3\sqrt{\frac{b}{\pi d}}S\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{5(bc-ad)}{d}\right)+480\left(9\left(20b^3d^2x^2+40b^3cdx+20b^3c^2-3bd^2\right)\cos(bx+a)^5+390bd^2\cos(bx+a)-5\left(60b^3d^2x^2+120b^3cdx+60b^3c^2-13bd^2\right)\cos(bx+a)^3+10\left(26b^2d^2x-9\left(b^2d^2x+b^2cd\right)\cos(bx+a)^4+26b^2cd+13\left(b^2d^2x+b^2cd\right)\cos(bx+a)^2\right)\sin(bx+a)\right)\sqrt{dx+c}}{432000\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_c
os(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 625*sqrt(6)*pi*d^3*sqrt(b/(pi*d
))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))
- 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fr
esnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 625*s
qrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-3*(b*c - a*d)/d) - 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(9*(20*b^
3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*cos(b*x + a)^5 + 390*b*d^2
*cos(b*x + a) - 5*(60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 - 13*b*d^2)*
cos(b*x + a)^3 + 10*(26*b^2*d^2*x - 9*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^4
+ 26*b^2*c*d + 13*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(
d*x + c))/b^4
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep
```

Giac [C] Result contains complex when optimal does not.
time = 1.72, size = 3717, normalized size = 6.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")

[Out] -1/864000*(1800*(30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*(2250*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 125*(-I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 9*(I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 - 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-5*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 2250*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 125*(I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 9*(-I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-5*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) + d^3*(6750*(-I*sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)*b^2*d - 12*I*

```
(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)
*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c
)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 125*(I*sqrt(6)*sqrt(pi)*(72*b^3*c^3 - 36
*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)*b^2*d - 36*I*(d*x +
 c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2
 + 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^(-3*(-I*(d*x + c)*b
+ I*b*c - I*a*d)/d)/b^3)/d^3 + 27*(-I*sqrt(10)*sqrt(pi)*(200*b^3*c^3 - 60*I
*b^2*c^2*d - 18*b*c*d^2 + 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x +
 c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)*b^3) - 10*I*(-20*I*(d*x + c)^(5/2)*b^2*d + 60*I*(d*x
+ c)^(3/2)*b^2*c*d - 60*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^
2 - 18*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c)*d^3)*e^(-5*(-I*(d*x + c)*b
 + I*b*c - I*a*d)/d)/b^3)/d^3 + 6750*(I*sqrt(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*
b^2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(
3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18
*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b*c
+ I*a*d)/d)/b^3)/d^3 + 125*(-I*sqrt(6)*sqrt(pi)*(72*b^3*c^3 + 36*I*b^2*c^2*
d - 18*b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)*b^2...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\sin(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

3.136 $\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=273

$$-\frac{2^{-4-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-4-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right.}{b}$$

```
[Out] -2^(-4-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*b
*(d*x+c)/d)^m)-2^(-4-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a-b
*c/d))/((I*b*(d*x+c)/d)^m)-exp(4*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-4*I*b*(d
*x+c)/d)/(2^(6+2*m))/b/((-I*b*(d*x+c)/d)^m)-(d*x+c)^m*GAMMA(1+m,4*I*b*(d*x+
c)/d)/(2^(6+2*m))/b/exp(4*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.22, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4491, 3389, 2212}

$$-\frac{2^{-m-4}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{4ib(c+dx)}{d}\right)}{b}-\frac{2^{-m-4}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-2(m+3)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{4ib(c+dx)}{d}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -((2^(-4 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c
 + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-4 - m)*(c + d*x)^m*Gamma[
1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/
d)^m) - (E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-4*I)*b*(c + d*
x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 +
 m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*E^((4*I)*(a - (b*c)/d))*((I*b*
(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx &= \int \left(\frac{1}{4}(c+dx)^m \sin(2a+2bx)+\frac{1}{8}(c+dx)^m \sin(4a+4bx)\right)\,dx \\
&= \frac{1}{8}\int (c+dx)^m \sin(4a+4bx)\,dx+\frac{1}{4}\int (c+dx)^m \sin(2a+2bx)\,dx \\
&= \frac{1}{16}i\int e^{-i(4a+4bx)}(c+dx)^m\,dx-\frac{1}{16}i\int e^{i(4a+4bx)}(c+dx)^m\,dx+\frac{1}{8} \\
&= -\frac{2^{-4-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}-
\end{aligned}
$$

Mathematica [A]
time = 1.19, size = 362, normalized size = 1.33

$$
\frac{2^{-2(3+m)}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-2m}\left(-2^{2+m}\left(\frac{b^2(c+dx)^2}{d^2}\right)^m\left(\left(-\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(1+m,\frac{2ib(c+dx)}{d}\right)\left(\cos\left(2a-\frac{2bc}{d}\right)-i\sin\left(2a-\frac{2bc}{d}\right)\right)+\left(\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(1+m,-\frac{2ib(c+dx)}{d}\right)\left(\cos\left(2a-\frac{2bc}{d}\right)+i\sin\left(2a-\frac{2bc}{d}\right)\right)\right)-e^{-\frac{4ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\left(\left(\frac{b^2(c+dx)^2}{d^2}\right)^m\text{Gamma}\left(1+m,\frac{4ib(c+dx)}{d}\right)\left(\cos\left(4a-\frac{4bc}{d}\right)-i\sin\left(4a-\frac{4bc}{d}\right)\right)+\left(\frac{ib(c+dx)}{d}\right)^{2m}\text{Gamma}\left(1+m,-\frac{4ib(c+dx)}{d}\right)\left(\cos\left(4a-\frac{4bc}{d}\right)+i\sin\left(4a-\frac{4bc}{d}\right)\right)\right)\left(\cos\left(\frac{4bc}{d}\right)+i\sin\left(\frac{4bc}{d}\right)\right)\right)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] ((c + d*x)^m*(-(2^(2 + m)*((b^2*(c + d*x)^2)/d^2)^m*((((-I)*b*(c + d*x))/d)
^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d]*(Cos[2*a - (2*b*c)/d] - I*Sin[2*a -
(2*b*c)/d]) + ((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d]*(C
os[2*a - (2*b*c)/d] + I*Sin[2*a - (2*b*c)/d]))) - ((((-I)*b*(c + d*x))/d)^m
*(((b^2*(c + d*x)^2)/d^2)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d]*(Cos[4*a -
(4*b*c)/d] - I*Sin[4*a - (4*b*c)/d]) + ((I*b*(c + d*x))/d)^(2*m)*Gamma[1 +
m, ((-4*I)*b*(c + d*x))/d]*(Cos[4*a - (4*b*c)/d] + I*Sin[4*a - (4*b*c)/d]))
*(Cos[(4*b*c)/d] + I*Sin[(4*b*c)/d]))/E^(((4*I)*b*c)/d)))/(2^(2*(3 + m))*b*
((b^2*(c + d*x)^2)/d^2)^(2*m))
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$
\int (dx+c)^m \left(\cos^3(bx+a)\right)\sin(bx+a)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a), x)
```

Fricas [A]
time = 0.63, size = 188, normalized size = 0.69

$$-\frac{4\,e^{\left(-\frac{dm\log\left(-\frac{2i\,b}{d}\right)+2i\,bc-2i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{2\,(i\,bdx+i\,bc)}{d}\right)+e^{\left(-\frac{dm\log\left(-\frac{4i\,b}{d}\right)+4i\,bc-4i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{4\,(i\,bdx+i\,bc)}{d}\right)+4\,e^{\left(-\frac{dm\log\left(\frac{2i\,b}{d}\right)-2i\,bc+2i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{2\,(-i\,bdx-i\,bc)}{d}\right)+e^{\left(-\frac{dm\log\left(\frac{4i\,b}{d}\right)-4i\,bc+4i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{4\,(-i\,bdx-i\,bc)}{d}\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/64*(4*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m + 1, -2*(I*
b*d*x + I*b*c)/d) + e^(-(d*m*log(-4*I*b/d) + 4*I*b*c - 4*I*a*d)/d)*gamma(m
+ 1, -4*(I*b*d*x + I*b*c)/d) + 4*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)
/d)*gamma(m + 1, -2*(-I*b*d*x - I*b*c)/d) + e^(-(d*m*log(4*I*b/d) - 4*I*b*c
 + 4*I*a*d)/d)*gamma(m + 1, -4*(-I*b*d*x - I*b*c)/d))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos{(a+b\,x)}^3\,\sin{(a+b\,x)}\,\left(c+d\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^m,x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^m, x)
```

3.137 $\int (c+dx)^4 \cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=260

$$-\frac{45cd^3x}{64b^3}-\frac{45d^4x^2}{128b^3}+\frac{3(c+dx)^4}{32b}-\frac{45d^4\cos^2(a+bx)}{128b^5}+\frac{9d^2(c+dx)^2\cos^2(a+bx)}{16b^3}-\frac{3d^4\cos^4(a+bx)}{128b^5}+\frac{3d^2(c+$$

```
[Out] -45/64*c*d^3*x/b^3-45/128*d^4*x^2/b^3+3/32*(d*x+c)^4/b-45/128*d^4*cos(b*x+a
)^2/b^5+9/16*d^2*(d*x+c)^2*cos(b*x+a)^2/b^3-3/128*d^4*cos(b*x+a)^4/b^5+3/16
*d^2*(d*x+c)^2*cos(b*x+a)^4/b^3-1/4*(d*x+c)^4*cos(b*x+a)^4/b-45/64*d^3*(d*x
+c)*cos(b*x+a)*sin(b*x+a)/b^4+3/8*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2-3/3
2*d^3*(d*x+c)*cos(b*x+a)^3*sin(b*x+a)/b^4+1/4*d*(d*x+c)^3*cos(b*x+a)^3*sin(
b*x+a)/b^2
```

Rubi [A]
time = 0.17, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4490, 3392, 32, 3391}

$$-\frac{3d^4\cos^4(a+bx)}{128b^5}-\frac{45d^4\cos^2(a+bx)}{128b^5}-\frac{3d^3(c+dx)\sin(a+bx)\cos^3(a+bx)}{32b^4}-\frac{45d^3(c+dx)\sin(a+bx)\cos(a+bx)}{64b^4}+\frac{3d^2(c+dx)^2\cos^4(a+bx)}{16b^3}+\frac{9d^2(c+dx)^2\cos^2(a+bx)}{16b^3}+\frac{d(c+dx)^3\sin(a+bx)\cos^3(a+bx)}{4b^2}+\frac{3d(c+dx)^3\sin(a+bx)\cos(a+bx)}{8b^2}-\frac{(c+dx)^4\cos^4(a+bx)}{4b}-\frac{45cd^3x}{64b^3}-\frac{45d^4x^2}{128b^3}+\frac{3(c+dx)^4}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-45*c*d^3*x)/(64*b^3) - (45*d^4*x^2)/(128*b^3) + (3*(c + d*x)^4)/(32*b) -
(45*d^4*Cos[a + b*x]^2)/(128*b^5) + (9*d^2*(c + d*x)^2*Cos[a + b*x]^2)/(16*
b^3) - (3*d^4*Cos[a + b*x]^4)/(128*b^5) + (3*d^2*(c + d*x)^2*Cos[a + b*x]^4
)/(16*b^3) - ((c + d*x)^4*Cos[a + b*x]^4)/(4*b) - (45*d^3*(c + d*x)*Cos[a +
 b*x]*Sin[a + b*x])/(64*b^4) + (3*d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/
(8*b^2) - (3*d^3*(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x])/(32*b^4) + (d*(c +
d*x)^3*Cos[a + b*x]^3*Sin[a + b*x])/(4*b^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^4\cos^3(a+bx)\sin(a+bx)\,dx &= -\frac{(c+dx)^4\cos^4(a+bx)}{4b}+\frac{d\int (c+dx)^3\cos^4(a+bx)\,dx}{b}\\ &= \frac{3d^2(c+dx)^2\cos^4(a+bx)}{16b^3}-\frac{(c+dx)^4\cos^4(a+bx)}{4b}+\frac{d(c+dx)^3\,c\ldots}{}\\ &= \frac{9d^2(c+dx)^2\cos^2(a+bx)}{16b^3}-\frac{3d^4\cos^4(a+bx)}{128b^5}+\frac{3d^2(c+dx)^2\cos^4(\ldots}{16b^3}\\ &= \frac{3(c+dx)^4}{32b}-\frac{45d^4\cos^2(a+bx)}{128b^5}+\frac{9d^2(c+dx)^2\cos^2(a+bx)}{16b^3}-\frac{3d^{\ldots}}{}\\ &= -\frac{45cd^3x}{64b^3}-\frac{45d^4x^2}{128b^3}+\frac{3(c+dx)^4}{32b}-\frac{45d^4\cos^2(a+bx)}{128b^5}+\frac{9d^2(c+d\ldots}{}\end{aligned}$$

Mathematica [A]
time = 1.90, size = 158, normalized size = 0.61

$$-\frac{64(3d^4-6b^2d^2(c+dx)^2+2b^4(c+dx)^4)\cos(2(a+bx))+(3d^4-24b^2d^2(c+dx)^2+32b^4(c+dx)^4)\cos(4(a+bx))-8bd(c+dx)\left(16(-3d^2+2b^2(c+dx)^2)+(-3d^2+8b^2(c+dx)^2)\cos(2(a+bx))\right)\sin(2(a+bx))}{1024b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -1/1024*(64*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a +
b*x)] + (3*d^4 - 24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Cos[4*(a + b*
x)] - 8*b*d*(c + d*x)*(16*(-3*d^2 + 2*b^2*(c + d*x)^2) + (-3*d^2 + 8*b^2*(c
 + d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)])/b^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1133 vs. $2(236) = 472$.
time = 0.28, size = 1134, normalized size = 4.36

method	result
risch	$-\frac{(32d^4x^4b^4+128b^4c\,d^3x^3+192b^4c^2d^2x^2+128b^4c^3dx+32b^4c^4-24b^2d^4x^2-48b^2c\,d^3x-24b^2c^2d^2+3d^4)\cos(4bx+4a)}{1024b^5}+$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4/b^4*a^4*d^4*cos(b*x+a)^4+1/b^3*a^3*c*d^3*cos(b*x+a)^4-4/b^4*a^3*d
^4*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)
+3/32*b*x+3/32*a)-3/2/b^2*a^2*c^2*d^2*cos(b*x+a)^4+12/b^3*a^2*c*d^3*(-1/4*(
b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+
3/32*a)+6/b^4*a^2*d^4*(-1/4*(b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*
x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/128*(2*co
s(b*x+a)^2+3)^2)+1/b*a*c^3*d*cos(b*x+a)^4-12/b^2*a*c^2*d^2*(-1/4*(b*x+a)*co
s(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)-1
2/b^3*a*c*d^3*(-1/4*(b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3
/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/128*(2*cos(b*x+a)
^2+3)^2)-4/b^4*a*d^4*(-1/4*(b*x+a)^3*cos(b*x+a)^4+3/4*(b*x+a)^2*(1/4*(cos(b
*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)*cos(b*x+a)^4
-3/128*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)-45/256*b*x-45/256*a+9/32*(b
*x+a)*cos(b*x+a)^2-9/64*cos(b*x+a)*sin(b*x+a)-3/16*(b*x+a)^3)-1/4*c^4*cos(b
*x+a)^4+4/b*c^3*d*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x
+a))*sin(b*x+a)+3/32*b*x+3/32*a)+6/b^2*c^2*d^2*(-1/4*(b*x+a)^2*cos(b*x+a)^4
+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-3
/32*(b*x+a)^2+1/128*(2*cos(b*x+a)^2+3)^2)+4/b^3*c*d^3*(-1/4*(b*x+a)^3*cos(b
*x+a)^4+3/4*(b*x+a)^2*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x
+3/8*a)+3/32*(b*x+a)*cos(b*x+a)^4-3/128*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b
*x+a)-45/256*b*x-45/256*a+9/32*(b*x+a)*cos(b*x+a)^2-9/64*cos(b*x+a)*sin(b*x
+a)-3/16*(b*x+a)^3)+1/b^4*d^4*(-1/4*(b*x+a)^4*cos(b*x+a)^4+(b*x+a)^3*(1/4*(
cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3/16*(b*x+a)^2*cos(b
*x+a)^4-3/8*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3
/8*a)+45/128*(b*x+a)^2-3/512*(2*cos(b*x+a)^2+3)^2+9/16*(b*x+a)^2*cos(b*x+a)
^2-9/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+9/32*sin(b*x+a)^2-
9/32*(b*x+a)^4))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 967 vs. $2(236) = 472$.
time = 0.31, size = 967, normalized size = 3.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/1024*(256*c^4*cos(b*x + a)^4 - 1024*a*c^3*d*cos(b*x + a)^4/b + 1536*a^2*
c^2*d^2*cos(b*x + a)^4/b^2 - 1024*a^3*c*d^3*cos(b*x + a)^4/b^3 + 256*a^4*d^
4*cos(b*x + a)^4/b^4 + 32*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(
2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*c^3*d/b - 96*(4*(b*x
+ a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) -
8*sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 96*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(
b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*a^2*c*d^
3/b^3 - 32*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) -
sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*a^3*d^4/b^4 + 24*((8*(b*x + a)^2 - 1
)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*
sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 - 48*((8*(b*x
 + a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4
*(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/b^3 +
24*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x
 + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*a^2
*d^4/b^4 + 4*(4*(8*(b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) + 64*(2*(b*x
 + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x +
 4*a) - 96*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 - 4*(4*(8*(b*x +
 a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) + 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*co
s(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) - 96*(2*(b*x + a)^2
 - 1)*sin(2*b*x + 2*a))*a*d^4/b^4 + ((32*(b*x + a)^4 - 24*(b*x + a)^2 + 3)*
cos(4*b*x + 4*a) + 64*(2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*cos(2*b*x + 2*a)
- 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a) - 128*(2*(b*x + a)^3 - 3
*b*x - 3*a)*sin(2*b*x + 2*a))*d^4/b^4)/b
```

Fricas [A]
time = 4.33, size = 378, normalized size = 1.45

$$\frac{12\,b^4d^4x^4+48\,b^4cd^3x^3-(32\,b^4d^4x^4+128\,b^4cd^3x^3+32\,b^4c^4-24\,b^2c^2d^2+3\,d^4+24\,(8\,b^4c^2d^2-b^2d^4)x^2+16\,(8\,b^4c^3d-3\,b^2cd^3)x)\cos(bx+a)^4+9\,(8\,b^4c^2d^2-5\,b^2d^4)x^2+9\,(8\,b^2d^4x^2+16\,b^2cd^3x+8\,b^2c^2d^2-5\,d^4)\cos(bx+a)^2+6\,(8\,b^4c^3d-15\,b^2cd^3)x+2\,(2\,(8\,b^3d^4x^3+24\,b^3cd^3x^2+8\,b^3c^3d-3\,bcd^3+3\,(8\,b^3c^2d^2-bd^4)x)\cos(bx+a)^3+3\,(8\,b^3d^4x^3+24\,b^3cd^3x^2+8\,b^3c^3d-15\,bcd^3+3\,(8\,b^3c^2d^2-5\,bd^4)x)\cos(bx+a))\sin(bx+a)}{128\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/128*(12*b^4*d^4*x^4 + 48*b^4*c*d^3*x^3 - (32*b^4*d^4*x^4 + 128*b^4*c*d^3*
x^3 + 32*b^4*c^4 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^
2 + 16*(8*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^4 + 9*(8*b^4*c^2*d^2 - 5
*b^2*d^4)*x^2 + 9*(8*b^2*d^4*x^2 + 16*b^2*c*d^3*x + 8*b^2*c^2*d^2 - 5*d^4)*
cos(b*x + a)^2 + 6*(8*b^4*c^3*d - 15*b^2*c*d^3)*x + 2*(2*(8*b^3*d^4*x^3 + 2
4*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^3*c^2*d^2 - b*d^4)*x)*co
s(b*x + a)^3 + 3*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 15*b*c*d
^3 + 3*(8*b^3*c^2*d^2 - 5*b*d^4)*x)*cos(b*x + a))*sin(b*x + a))/b^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(262) = 524$.

time = 1.13, size = 935, normalized size = 3.60

$$\begin{cases} -\frac{c^4\cos^4(a+bx)}{4b} + \frac{3c^3dx\sin^4(a+bx)}{8b} + \frac{3c^3dx\sin^2(a+bx)\cos^2(a+bx)}{4b} - \frac{5c^3dx\cos^4(a+bx)}{8b} + \frac{9c^2d^2x^2\sin^4(a+bx)}{16b} + \frac{9c^2d^2x^2\sin^2(a+bx)\cos^2(a+bx)}{8b} - \frac{15c^2d^2x^2\cos^4(a+bx)}{16b} + \frac{3cd^3x^3\sin^4(a+bx)}{8b} + \frac{3cd^3x^3\sin^2(a+bx)\cos^2(a+bx)}{4b} - \frac{5cd^3x^3\cos^4(a+bx)}{8b} + \frac{3d^4x^4\sin^4(a+bx)}{32b} + \frac{3d^4x^4\sin^2(a+bx)\cos^2(a+bx)}{16b} - \frac{5d^4x^4\cos^4(a+bx)}{32b} + \frac{3c^3d\sin^3(a+bx)\cos(a+bx)}{8b^2} + \frac{5c^3d\sin(a+bx)\cos^3(a+bx)}{8b^2} + \frac{9c^2d^2x\sin^3(a+bx)\cos(a+bx)}{8b^2} + \frac{15c^2d^2x\sin(a+bx)\cos^3(a+bx)}{8b^2} + \frac{9cd^3x^2\sin^3(a+bx)\cos(a+bx)}{8b^2} + \frac{15cd^3x^2\sin(a+bx)\cos^3(a+bx)}{8b^2} + \frac{3d^4x^3\sin^3(a+bx)\cos(a+bx)}{8b^2} + \frac{5d^4x^3\sin(a+bx)\cos^3(a+bx)}{8b^2} - \frac{9c^2d^2\sin^4(a+bx)}{32b^3} + \frac{15c^2d^2\cos^4(a+bx)}{32b^3} - \frac{45cd^3x\sin^4(a+bx)}{64b^3} - \frac{9cd^3x\sin^2(a+bx)\cos^2(a+bx)}{32b^3} + \frac{51cd^3x\cos^4(a+bx)}{64b^3} - \frac{45d^4x^2\sin^4(a+bx)}{128b^3} - \frac{9d^4x^2\sin^2(a+bx)\cos^2(a+bx)}{64b^3} + \frac{51d^4x^2\cos^4(a+bx)}{128b^3} - \frac{45cd^3\sin^3(a+bx)\cos(a+bx)}{64b^4} - \frac{51cd^3\sin(a+bx)\cos^3(a+bx)}{64b^4} - \frac{45d^4x\sin^3(a+bx)\cos(a+bx)}{64b^4} - \frac{51d^4x\sin(a+bx)\cos^3(a+bx)}{64b^4} + \frac{45d^4\sin^4(a+bx)}{256b^5} - \frac{51d^4\cos^4(a+bx)}{256b^5} & \text{for}\ b \neq 0 \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sin(a)\cos^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**4*cos(a + b*x)**4/(4*b) + 3*c**3*d*x*sin(a + b*x)**4/(8*b) +
 3*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(4*b) - 5*c**3*d*x*cos(a + b*x)
**4/(8*b) + 9*c**2*d**2*x**2*sin(a + b*x)**4/(16*b) + 9*c**2*d**2*x**2*sin(
a + b*x)**2*cos(a + b*x)**2/(8*b) - 15*c**2*d**2*x**2*cos(a + b*x)**4/(16*b
) + 3*c*d**3*x**3*sin(a + b*x)**4/(8*b) + 3*c*d**3*x**3*sin(a + b*x)**2*cos
(a + b*x)**2/(4*b) - 5*c*d**3*x**3*cos(a + b*x)**4/(8*b) + 3*d**4*x**4*sin(
a + b*x)**4/(32*b) + 3*d**4*x**4*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 5
*d**4*x**4*cos(a + b*x)**4/(32*b) + 3*c**3*d*sin(a + b*x)**3*cos(a + b*x)/(
8*b**2) + 5*c**3*d*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 9*c**2*d**2*x*si
n(a + b*x)**3*cos(a + b*x)/(8*b**2) + 15*c**2*d**2*x*sin(a + b*x)*cos(a + b
*x)**3/(8*b**2) + 9*c*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 15*
c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 3*d**4*x**3*sin(a + b*x
)**3*cos(a + b*x)/(8*b**2) + 5*d**4*x**3*sin(a + b*x)*cos(a + b*x)**3/(8*b*
*2) - 9*c**2*d**2*sin(a + b*x)**4/(32*b**3) + 15*c**2*d**2*cos(a + b*x)**4/
(32*b**3) - 45*c*d**3*x*sin(a + b*x)**4/(64*b**3) - 9*c*d**3*x*sin(a + b*x)
**2*cos(a + b*x)**2/(32*b**3) + 51*c*d**3*x*cos(a + b*x)**4/(64*b**3) - 45*
d**4*x**2*sin(a + b*x)**4/(128*b**3) - 9*d**4*x**2*sin(a + b*x)**2*cos(a +
b*x)**2/(64*b**3) + 51*d**4*x**2*cos(a + b*x)**4/(128*b**3) - 45*c*d**3*sin
(a + b*x)**3*cos(a + b*x)/(64*b**4) - 51*c*d**3*sin(a + b*x)*cos(a + b*x)**
3/(64*b**4) - 45*d**4*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**4) - 51*d**4*x*
sin(a + b*x)*cos(a + b*x)**3/(64*b**4) + 45*d**4*sin(a + b*x)**4/(256*b**5)
 - 51*d**4*cos(a + b*x)**4/(256*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2
+ 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*cos(a)**3, True))
```

Giac [A]
time = 0.47, size = 361, normalized size = 1.39

$$-\frac{\left(32\,b^4d^4x^4 + 128\,b^4cd^3x^3 + 192\,b^4c^2d^2x^2 + 128\,b^4c^3dx + 32\,b^4c^4 - 24\,b^2d^4x^2 - 48\,b^2cd^3x - 24\,b^2c^2d^2 + 3\,d^4\right)\cos(4\,bx + 4\,a)}{1024\,b^5} - \frac{\left(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 12\,b^4c^2d^2x^2 + 8\,b^4c^3dx + 2\,b^4c^4 - 6\,b^2d^4x^2 - 12\,b^2cd^3x - 6\,b^2c^2d^2 + 3\,d^4\right)\cos(2\,bx + 2\,a)}{16\,b^5} + \frac{\left(8\,b^3d^4x^3 + 24\,b^3cd^3x^2 + 24\,b^3c^2d^2x + 8\,b^3c^3d - 3\,bd^4x - 3\,bcd^3\right)\sin(4\,bx + 4\,a)}{256\,b^5} + \frac{\left(2\,b^3d^4x^3 + 6\,b^3cd^3x^2 + 6\,b^3c^2d^2x + 2\,b^3c^3d - 3\,bd^4x - 3\,bcd^3\right)\sin(2\,bx + 2\,a)}{8\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/1024*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^4
*c^3*d*x + 32*b^4*c^4 - 24*b^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2 +
3*d^4)*cos(4*b*x + 4*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^
4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x
- 6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 + 1/256*(8*b^3*d^4*x^3 + 24*b
^3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(
```

```
4*b*x + 4*a)/b^5 + 1/8*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x +
 2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

Mupad [B]
time = 1.34, size = 576, normalized size = 2.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^4,x)
```

```
[Out] -(192*d^4*cos(2*a + 2*b*x) + 3*d^4*cos(4*a + 4*b*x) + 128*b^4*c^4*cos(2*a +
 2*b*x) + 32*b^4*c^4*cos(4*a + 4*b*x) - 256*b^3*c^3*d*sin(2*a + 2*b*x) - 32
*b^3*c^3*d*sin(4*a + 4*b*x) - 384*b^2*c^2*d^2*cos(2*a + 2*b*x) - 24*b^2*c^2
*d^2*cos(4*a + 4*b*x) - 384*b^2*d^4*x^2*cos(2*a + 2*b*x) - 24*b^2*d^4*x^2*c
os(4*a + 4*b*x) + 128*b^4*d^4*x^4*cos(2*a + 2*b*x) + 32*b^4*d^4*x^4*cos(4*a
 + 4*b*x) - 256*b^3*d^4*x^3*sin(2*a + 2*b*x) - 32*b^3*d^4*x^3*sin(4*a + 4*b
*x) + 384*b*c*d^3*sin(2*a + 2*b*x) + 12*b*c*d^3*sin(4*a + 4*b*x) + 384*b*d^
4*x*sin(2*a + 2*b*x) + 12*b*d^4*x*sin(4*a + 4*b*x) + 768*b^4*c^2*d^2*x^2*co
s(2*a + 2*b*x) + 192*b^4*c^2*d^2*x^2*cos(4*a + 4*b*x) - 768*b^2*c*d^3*x*cos
(2*a + 2*b*x) + 512*b^4*c^3*d*x*cos(2*a + 2*b*x) - 48*b^2*c*d^3*x*cos(4*a +
 4*b*x) + 128*b^4*c^3*d*x*cos(4*a + 4*b*x) + 512*b^4*c*d^3*x^3*cos(2*a + 2*
b*x) + 128*b^4*c*d^3*x^3*cos(4*a + 4*b*x) - 768*b^3*c^2*d^2*x*sin(2*a + 2*b
*x) - 768*b^3*c*d^3*x^2*sin(2*a + 2*b*x) - 96*b^3*c^2*d^2*x*sin(4*a + 4*b*x
) - 96*b^3*c*d^3*x^2*sin(4*a + 4*b*x))/(1024*b^5)
```

3.138 $\int (c+dx)^3 \cos^3(a+bx) \sin(a+bx)\,dx$

Optimal. Leaf size=196

$$-\frac{45d^3x}{256b^3}+\frac{3(c+dx)^3}{32b}+\frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3}+\frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3}-\frac{(c+dx)^3\cos^4(a+bx)}{4b}-\frac{45d^3\,\mathrm{c}}{}$$

```
[Out] -45/256*d^3*x/b^3+3/32*(d*x+c)^3/b+9/32*d^2*(d*x+c)*cos(b*x+a)^2/b^3+3/32*d
^2*(d*x+c)*cos(b*x+a)^4/b^3-1/4*(d*x+c)^3*cos(b*x+a)^4/b-45/256*d^3*cos(b*x
+a)*sin(b*x+a)/b^4+9/32*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^2-3/128*d^3*cos
(b*x+a)^3*sin(b*x+a)/b^4+3/16*d*(d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)/b^2
```

Rubi [A]
time = 0.11, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4490, 3392, 32, 2715, 8}

$$-\frac{3d^3\sin(a+bx)\cos^3(a+bx)}{128b^4}-\frac{45d^3\sin(a+bx)\cos(a+bx)}{256b^4}+\frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3}+\frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3}+\frac{3d(c+dx)^2\sin(a+bx)\cos^3(a+bx)}{16b^2}+\frac{9d(c+dx)^2\sin(a+bx)\cos(a+bx)}{32b^2}-\frac{(c+dx)^3\cos^4(a+bx)}{4b}-\frac{45d^3x}{256b^3}+\frac{3(c+dx)^3}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-45*d^3*x)/(256*b^3) + (3*(c + d*x)^3)/(32*b) + (9*d^2*(c + d*x)*Cos[a + b
*x]^2)/(32*b^3) + (3*d^2*(c + d*x)*Cos[a + b*x]^4)/(32*b^3) - ((c + d*x)^3*
Cos[a + b*x]^4)/(4*b) - (45*d^3*Cos[a + b*x]*Sin[a + b*x])/(256*b^4) + (9*d
*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(32*b^2) - (3*d^3*Cos[a + b*x]^3*Si
n[a + b*x])/(128*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x])/(16*b
^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\cos^3(a+bx)\sin(a+bx)\,dx &= -\frac{(c+dx)^3\cos^4(a+bx)}{4b}+\frac{(3d)\int (c+dx)^2\cos^4(a+bx)\,dx}{4b}\\ &= \frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3}-\frac{(c+dx)^3\cos^4(a+bx)}{4b}+\frac{3d(c+dx)^2\,c}{}\\ &= \frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3}+\frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3}-\frac{(c+dx)^3\,c}{4}\\ &= \frac{3(c+dx)^3}{32b}+\frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3}+\frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3}\\ &= -\frac{45d^3x}{256b^3}+\frac{3(c+dx)^3}{32b}+\frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3}+\frac{3d^2(c+dx)\,c}{32b}\end{aligned}$$

Mathematica [A]
time = 0.97, size = 135, normalized size = 0.69

$$\frac{-64b(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\cos(2(a+bx))-4b(c+dx)\left(-3d^2+8b^2(c+dx)^2\right)\cos(4(a+bx))+6d\left(16\left(-d^2+2b^2(c+dx)^2\right)+\left(-d^2+8b^2(c+dx)^2\right)\cos(2(a+bx))\right)\sin(2(a+bx))}{1024b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-64*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] - 4*b*(c + d
*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] + 6*d*(16*(-d^2 + 2*b^2*(
c + d*x)^2) + (-d^2 + 8*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)]
)/(1024*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 585 vs. 2(178) = 356.
time = 0.18, size = 586, normalized size = 2.99 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4/b^3*a^3*d^3*cos(b*x+a)^4-3/4/b^2*a^2*c*d^2*cos(b*x+a)^4+3/b^3*a^2*
d^3*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a
)+3/32*b*x+3/32*a)+3/4/b*a*c^2*d*cos(b*x+a)^4-6/b^2*a*c*d^2*(-1/4*(b*x+a)*c
os(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)-
3/b^3*a*d^3*(-1/4*(b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2
*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/128*(2*cos(b*x+a)^2
+3)^2)-1/4*c^3*cos(b*x+a)^4+3/b*c^2*d*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(
b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)+3/b^2*c*d^2*(-1/4*(b*x
+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a
)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/128*(2*cos(b*x+a)^2+3)^2)+1/b^3*d^3*(-1/4
*(b*x+a)^3*cos(b*x+a)^4+3/4*(b*x+a)^2*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*si
n(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)*cos(b*x+a)^4-3/128*(cos(b*x+a)^3+3/2*c
os(b*x+a))*sin(b*x+a)-45/256*b*x-45/256*a+9/32*(b*x+a)*cos(b*x+a)^2-9/64*co
s(b*x+a)*sin(b*x+a)-3/16*(b*x+a)^3))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(178) = 356$.
time = 0.30, size = 549, normalized size = 2.80

$$-\frac{256\,c^{3}\cos\left(bx+a\right)^{4} - \frac{768\,ac^{2}d\cos\left(bx+a\right)^{4}}{b} + \frac{768\,a^{2}cd^{2}\cos\left(bx+a\right)^{4}}{b^{2}} - \frac{256\,a^{3}d^{3}\cos\left(bx+a\right)^{4}}{b^{3}} + \frac{24\,\left(4\,(bx+a)\cos\left(4\,bx+4\,a\right)+16\,(bx+a)\cos\left(2\,bx+2\,a\right)-\sin\left(4\,bx+4\,a\right)-8\,\sin\left(2\,bx+2\,a\right)\right)c^{2}d}{b} - \frac{48\,\left(4\,(bx+a)\cos\left(4\,bx+4\,a\right)+16\,(bx+a)\cos\left(2\,bx+2\,a\right)-\sin\left(4\,bx+4\,a\right)-8\,\sin\left(2\,bx+2\,a\right)\right)acd^{2}}{b^{2}} + \frac{24\,\left(4\,(bx+a)\cos\left(4\,bx+4\,a\right)+16\,(bx+a)\cos\left(2\,bx+2\,a\right)-\sin\left(4\,bx+4\,a\right)-8\,\sin\left(2\,bx+2\,a\right)\right)a^{2}d^{3}}{b^{3}} + \frac{12\,\left(\left(8\,(bx+a)^{2}-1\right)\cos\left(4\,bx+4\,a\right)+16\,\left(2\,(bx+a)^{2}-1\right)\cos\left(2\,bx+2\,a\right)-4\,(bx+a)\sin\left(4\,bx+4\,a\right)-32\,(bx+a)\sin\left(2\,bx+2\,a\right)\right)cd^{2}}{b^{2}} - \frac{12\,\left(\left(8\,(bx+a)^{2}-1\right)\cos\left(4\,bx+4\,a\right)+16\,\left(2\,(bx+a)^{2}-1\right)\cos\left(2\,bx+2\,a\right)-4\,(bx+a)\sin\left(4\,bx+4\,a\right)-32\,(bx+a)\sin\left(2\,bx+2\,a\right)\right)ad^{3}}{b^{3}} + \frac{\left(4\,\left(8\,(bx+a)^{3}-3\,bx-3\,a\right)\cos\left(4\,bx+4\,a\right)+64\,\left(2\,(bx+a)^{3}-3\,bx-3\,a\right)\cos\left(2\,bx+2\,a\right)-3\,\left(8\,(bx+a)^{2}-1\right)\sin\left(4\,bx+4\,a\right)-96\,\left(2\,(bx+a)^{2}-1\right)\sin\left(2\,bx+2\,a\right)\right)d^{3}}{b^{3}}}{1024\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/1024*(256*c^3*cos(b*x + a)^4 - 768*a*c^2*d*cos(b*x + a)^4/b + 768*a^2*c*
d^2*cos(b*x + a)^4/b^2 - 256*a^3*d^3*cos(b*x + a)^4/b^3 + 24*(4*(b*x + a)*c
os(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(
2*b*x + 2*a))*c^2*d/b - 48*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos
(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*a*c*d^2/b^2 + 24*(4*
(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*
a) - 8*sin(2*b*x + 2*a))*a^2*d^3/b^3 + 12*((8*(b*x + a)^2 - 1)*cos(4*b*x +
4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*
a) - 32*(b*x + a)*sin(2*b*x + 2*a))*c*d^2/b^2 - 12*((8*(b*x + a)^2 - 1)*cos
(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4
*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*a*d^3/b^3 + (4*(8*(b*x + a)^3
- 3*b*x - 3*a)*cos(4*b*x + 4*a) + 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*
x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) - 96*(2*(b*x + a)^2 - 1)*
sin(2*b*x + 2*a))*d^3/b^3)/b
```

Fricas [A]
time = 3.31, size = 238, normalized size = 1.21

$$\frac{24\,b^{3}d^{3}x^{3} + 72\,b^{3}cd^{2}x^{2} - 8\,(8\,b^{3}d^{3}x^{3} + 24\,b^{3}cd^{2}x^{2} + 8\,b^{3}c^{3} - 3\,bcd^{2} + 3\,(8\,b^{3}c^{2}d - bd^{3})x)\cos\left(bx+a\right)^{4} + 72\,(bd^{3}x + bcd^{2})\cos\left(bx+a\right)^{2} + 9\,(8\,b^{3}c^{2}d - 5\,bd^{3})x + 3\,\left(2\,(8\,b^{2}d^{3}x^{2} + 16\,b^{2}cd^{2}x + 8\,b^{2}c^{2}d - d^{3})\cos\left(bx+a\right)^{3} + 3\,(8\,b^{2}d^{3}x^{2} + 16\,b^{2}cd^{2}x + 8\,b^{2}c^{2}d - 5\,d^{3})\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{256\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/256*(24*b^3*d^3*x^3 + 72*b^3*c*d^2*x^2 - 8*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*
x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^4 + 7
2*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 + 9*(8*b^3*c^2*d - 5*b*d^3)*x + 3*(2*(
8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3 + 3*(8*b
^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - 5*d^3)*cos(b*x + a))*sin(b*x +
a))/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(197) = 394$.
time = 0.74, size = 602, normalized size = 3.07

$$\begin{cases} -\frac{c^3\cos^4(a+bx)}{4b} + \frac{9c^2dx\sin^4(a+bx)}{32b} + \frac{9c^2dx\sin^2(a+bx)\cos^2(a+bx)}{16b} - \frac{15c^2dx\cos^4(a+bx)}{32b} + \frac{9cd^2x^2\sin^4(a+bx)}{32b} + \frac{9cd^2x^2\sin^2(a+bx)\cos^2(a+bx)}{16b} - \frac{15cd^2x^2\cos^4(a+bx)}{32b} + \frac{3d^3x^3\sin^4(a+bx)}{32b} + \frac{3d^3x^3\sin^2(a+bx)\cos^2(a+bx)}{16b} - \frac{5d^3x^3\cos^4(a+bx)}{32b} + \frac{9c^2d\sin^3(a+bx)\cos(a+bx)}{32b^2} + \frac{15c^2d\sin(a+bx)\cos^3(a+bx)}{32b^2} + \frac{9cd^2x\sin^3(a+bx)\cos(a+bx)}{16b^2} + \frac{15cd^2x\sin(a+bx)\cos^3(a+bx)}{16b^2} + \frac{9d^3x^2\sin^3(a+bx)\cos(a+bx)}{32b^2} + \frac{15d^3x^2\sin(a+bx)\cos^3(a+bx)}{32b^2} - \frac{9cd^2\sin^4(a+bx)}{64b^3} + \frac{15cd^2\cos^4(a+bx)}{64b^3} - \frac{45d^3x\sin^4(a+bx)}{256b^3} - \frac{9d^3x\sin^2(a+bx)\cos^2(a+bx)}{128b^3} + \frac{51d^3x\cos^4(a+bx)}{256b^3} - \frac{45d^3\sin^3(a+bx)\cos(a+bx)}{256b^4} - \frac{51d^3\sin(a+bx)\cos^3(a+bx)}{256b^4} & \text{for}\ b \neq 0 \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin(a)\cos^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**3*cos(a + b*x)**4/(4*b) + 9*c**2*d*x*sin(a + b*x)**4/(32*b)
+ 9*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 15*c**2*d*x*cos(a + b
*x)**4/(32*b) + 9*c*d**2*x**2*sin(a + b*x)**4/(32*b) + 9*c*d**2*x**2*sin(a
+ b*x)**2*cos(a + b*x)**2/(16*b) - 15*c*d**2*x**2*cos(a + b*x)**4/(32*b) +
3*d**3*x**3*sin(a + b*x)**4/(32*b) + 3*d**3*x**3*sin(a + b*x)**2*cos(a + b*
x)**2/(16*b) - 5*d**3*x**3*cos(a + b*x)**4/(32*b) + 9*c**2*d*sin(a + b*x)**
3*cos(a + b*x)/(32*b**2) + 15*c**2*d*sin(a + b*x)*cos(a + b*x)**3/(32*b**2)
 + 9*c*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 15*c*d**2*x*sin(a +
b*x)*cos(a + b*x)**3/(16*b**2) + 9*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(
32*b**2) + 15*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(32*b**2) - 9*c*d**2*s
in(a + b*x)**4/(64*b**3) + 15*c*d**2*cos(a + b*x)**4/(64*b**3) - 45*d**3*x*
sin(a + b*x)**4/(256*b**3) - 9*d**3*x*sin(a + b*x)**2*cos(a + b*x)**2/(128*
b**3) + 51*d**3*x*cos(a + b*x)**4/(256*b**3) - 45*d**3*sin(a + b*x)**3*cos(
a + b*x)/(256*b**4) - 51*d**3*sin(a + b*x)*cos(a + b*x)**3/(256*b**4), Ne(b
, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)*cos(a
)**3, True))
```

Giac [A]
time = 0.48, size = 241, normalized size = 1.23

$$-\frac{(8\,b^3d^3x^3 + 24\,b^3cd^2x^2 + 24\,b^3c^2dx + 8\,b^3c^3 - 3\,bd^3x - 3\,bcd^2)\cos(4\,bx + 4\,a)}{256\,b^4} - \frac{(2\,b^3d^3x^3 + 6\,b^3cd^2x^2 + 6\,b^3c^2dx + 2\,b^3c^3 - 3\,bd^3x - 3\,bcd^2)\cos(2\,bx + 2\,a)}{16\,b^4} + \frac{3\,(8\,b^2d^3x^2 + 16\,b^2cd^2x + 8\,b^2c^2d - d^3)\sin(4\,bx + 4\,a)}{1024\,b^4} + \frac{3\,(2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d - d^3)\sin(2\,bx + 2\,a)}{32\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*b
*d^3*x - 3*b*c*d^2)*cos(4*b*x + 4*a)/b^4 - 1/16*(2*b^3*d^3*x^3 + 6*b^3*c*d^
```

```
2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)
/b^4 + 3/1024*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*sin(4*b*
x + 4*a)/b^4 + 3/32*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin
(2*b*x + 2*a)/b^4
```

Mupad [B]
time = 2.06, size = 366, normalized size = 1.87

$$-\frac{24\,d^3\,\sin\left(2\,a+2\,b\,x\right)+\frac{3\,d^3\,\sin\left(4\,a+4\,b\,x\right)}{4}+32\,b^3\,c^3\,\cos\left(2\,a+2\,b\,x\right)+8\,b^3\,c^3\,\cos\left(4\,a+4\,b\,x\right)-48\,b^2\,c^2\,d\,\sin\left(2\,a+2\,b\,x\right)-6\,b^2\,c^2\,d\,\sin\left(4\,a+4\,b\,x\right)+32\,b^3\,d^3\,x^3\,\cos\left(2\,a+2\,b\,x\right)+8\,b^3\,d^3\,x^3\,\cos\left(4\,a+4\,b\,x\right)-48\,b^2\,d^3\,x^2\,\sin\left(2\,a+2\,b\,x\right)-6\,b^2\,d^3\,x^2\,\sin\left(4\,a+4\,b\,x\right)-48\,b\,c\,d^2\,\cos\left(2\,a+2\,b\,x\right)-3\,b\,c\,d^2\,\cos\left(4\,a+4\,b\,x\right)-48\,b\,d^3\,x\,\cos\left(2\,a+2\,b\,x\right)-3\,b\,d^3\,x\,\cos\left(4\,a+4\,b\,x\right)+96\,b^3\,c^2\,d\,x\,\cos\left(2\,a+2\,b\,x\right)+24\,b^3\,c^2\,d\,x\,\cos\left(4\,a+4\,b\,x\right)-96\,b^2\,c\,d^2\,x\,\sin\left(2\,a+2\,b\,x\right)-12\,b^2\,c\,d^2\,x\,\sin\left(4\,a+4\,b\,x\right)+96\,b^3\,c\,d^2\,x^2\,\cos\left(2\,a+2\,b\,x\right)+24\,b^3\,c\,d^2\,x^2\,\cos\left(4\,a+4\,b\,x\right)}{256\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^3,x)
```

```
[Out] -(24*d^3*sin(2*a + 2*b*x) + (3*d^3*sin(4*a + 4*b*x))/4 + 32*b^3*c^3*cos(2*a
 + 2*b*x) + 8*b^3*c^3*cos(4*a + 4*b*x) - 48*b^2*c^2*d*sin(2*a + 2*b*x) - 6*
b^2*c^2*d*sin(4*a + 4*b*x) + 32*b^3*d^3*x^3*cos(2*a + 2*b*x) + 8*b^3*d^3*x^
3*cos(4*a + 4*b*x) - 48*b^2*d^3*x^2*sin(2*a + 2*b*x) - 6*b^2*d^3*x^2*sin(4*
a + 4*b*x) - 48*b*c*d^2*cos(2*a + 2*b*x) - 3*b*c*d^2*cos(4*a + 4*b*x) - 48*
b*d^3*x*cos(2*a + 2*b*x) - 3*b*d^3*x*cos(4*a + 4*b*x) + 96*b^3*c^2*d*x*cos(
2*a + 2*b*x) + 24*b^3*c^2*d*x*cos(4*a + 4*b*x) - 96*b^2*c*d^2*x*sin(2*a + 2
*b*x) - 12*b^2*c*d^2*x*sin(4*a + 4*b*x) + 96*b^3*c*d^2*x^2*cos(2*a + 2*b*x)
 + 24*b^3*c*d^2*x^2*cos(4*a + 4*b*x))/(256*b^4)
```

3.139 $\int (c+dx)^2 \cos^3(a+bx) \sin(a+bx) \, dx$

Optimal. Leaf size=134

$$\frac{3cdx}{16b}+\frac{3d^2x^2}{32b}+\frac{3d^2\cos^2(a+bx)}{32b^3}+\frac{d^2\cos^4(a+bx)}{32b^3}-\frac{(c+dx)^2\cos^4(a+bx)}{4b}+\frac{3d(c+dx)\cos(a+bx)\sin(a+bx)}{16b^2}$$

```
[Out] 3/16*c*d*x/b+3/32*d^2*x^2/b+3/32*d^2*cos(b*x+a)^2/b^3+1/32*d^2*cos(b*x+a)^4
/b^3-1/4*(d*x+c)^2*cos(b*x+a)^4/b+3/16*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2+
1/8*d*(d*x+c)*cos(b*x+a)^3*sin(b*x+a)/b^2
```

Rubi [A]

time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4490, 3391}

$$\frac{d^2\cos^4(a+bx)}{32b^3}+\frac{3d^2\cos^2(a+bx)}{32b^3}+\frac{d(c+dx)\sin(a+bx)\cos^3(a+bx)}{8b^2}+\frac{3d(c+dx)\sin(a+bx)\cos(a+bx)}{16b^2}-\frac{(c+dx)^2\cos^4(a+bx)}{4b}+\frac{3cdx}{16b}+\frac{3d^2x^2}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (3*c*d*x)/(16*b) + (3*d^2*x^2)/(32*b) + (3*d^2*Cos[a + b*x]^2)/(32*b^3) + (
d^2*Cos[a + b*x]^4)/(32*b^3) - ((c + d*x)^2*Cos[a + b*x]^4)/(4*b) + (3*d*(c
 + d*x)*Cos[a + b*x]*Sin[a + b*x])/(16*b^2) + (d*(c + d*x)*Cos[a + b*x]^3*S
in[a + b*x])/(8*b^2)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^2 \cos^3(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^2\cos^4(a+bx)}{4b} + \frac{d\int (c+dx)\cos^4(a+bx)\,dx}{2b}$$
$$= \frac{d^2\cos^4(a+bx)}{32b^3} - \frac{(c+dx)^2\cos^4(a+bx)}{4b} + \frac{d(c+dx)\cos^3(a+bx)}{8b^2}$$
$$= \frac{3d^2\cos^2(a+bx)}{32b^3} + \frac{d^2\cos^4(a+bx)}{32b^3} - \frac{(c+dx)^2\cos^4(a+bx)}{4b} + \frac{3}{}$$
$$= \frac{3cdx}{16b} + \frac{3d^2x^2}{32b} + \frac{3d^2\cos^2(a+bx)}{32b^3} + \frac{d^2\cos^4(a+bx)}{32b^3} - \frac{(c+dx)^2}{}$$

Mathematica [A]
time = 0.47, size = 89, normalized size = 0.66

$$\frac{-16(-d^2+2b^2(c+dx)^2)\cos(2(a+bx)) + (d^2-8b^2(c+dx)^2)\cos(4(a+bx)) + 4bd(c+dx)(8\sin(2(a+bx)) + \sin(4(a+bx)))}{256b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x],x]
[Out] (-16*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (d^2 - 8*b^2*(c + d*x)^2
)*Cos[4*(a + b*x)] + 4*b*d*(c + d*x)*(8*Sin[2*(a + b*x)] + Sin[4*(a + b*x)]
))/(256*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(120) = 240.
time = 0.19, size = 256, normalized size = 1.91

method	result
risch	$-\frac{(8x^2d^2b^2+16b^2cdx+8b^2c^2-d^2)\cos(4bx+4a)}{256b^3} + \frac{d(dx+c)\sin(4bx+4a)}{64b^2} - \frac{(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2)\cos(2bx+}{16b^3}$
derivativedivides	$\frac{-\frac{a^2d^2(\cos^4(bx+a))}{4b^2} + \frac{acd(\cos^4(bx+a))}{2b} - \frac{2a\,d^2\left(-\frac{(bx+a)(\cos^4(bx+a))}{4} + \frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{16} + \frac{3bx}{32} + \frac{3a}{32}\right)}{b^2} -}{}$
default	$\frac{-\frac{a^2d^2(\cos^4(bx+a))}{4b^2} + \frac{acd(\cos^4(bx+a))}{2b} - \frac{2a\,d^2\left(-\frac{(bx+a)(\cos^4(bx+a))}{4} + \frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{16} + \frac{3bx}{32} + \frac{3a}{32}\right)}{b^2} -}{}$
norman	$\frac{-\frac{5d^2x^2}{32b} - \frac{3d^2\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b^3} + \frac{(16b^2c^2-5d^2)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b^3} + \frac{(16b^2c^2-5d^2)\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b^3} + \frac{5cd\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b^2} - \frac{3cd(\tan^3(}{8b^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4/b^2*a^2*d^2*cos(b*x+a)^4+1/2/b*a*c*d*cos(b*x+a)^4-2/b^2*a*d^2*(-1
/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*
b*x+3/32*a)-1/4*c^2*cos(b*x+a)^4+2/b*c*d*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(c
os(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)+1/b^2*d^2*(-1/4*(b*
x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+
a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/128*(2*cos(b*x+a)^2+3)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(120) = 240$.
time = 0.28, size = 263, normalized size = 1.96

$$-\frac{64\,c^2\cos\left(bx+a\right)^4-\frac{128\,acd\cos(bx+a)^4}{b}+\frac{64\,a^2d^2\cos(bx+a)^4}{b^2}+\frac{4\,(4\,(bx+a)\cos(4\,bx+4\,a)+16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)-8\,\sin(2\,bx+2\,a))cd}{b}-\frac{4\,(4\,(bx+a)\cos(4\,bx+4\,a)+16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)-8\,\sin(2\,bx+2\,a))ad^2}{b^2}+\frac{\left(\left(8\,(bx+a)^2-1\right)\cos(4\,bx+4\,a)+16\,\left(2\,(bx+a)^2-1\right)\cos(2\,bx+2\,a)-4\,(bx+a)\sin(4\,bx+4\,a)-32\,(bx+a)\sin(2\,bx+2\,a)\right)d^2}{b^2}}{256\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/256*(64*c^2*cos(b*x + a)^4 - 128*a*c*d*cos(b*x + a)^4/b + 64*a^2*d^2*cos
(b*x + a)^4/b^2 + 4*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x
+ 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*c*d/b - 4*(4*(b*x + a)*cos(
4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b
*x + 2*a))*a*d^2/b^2 + ((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x +
 a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x + a)*s
in(2*b*x + 2*a))*d^2/b^2)/b
```

Fricas [A]
time = 2.65, size = 130, normalized size = 0.97

$$\frac{3\,b^2d^2x^2+6\,b^2cdx-\left(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2\right)\cos\left(bx+a\right)^4+3\,d^2\cos\left(bx+a\right)^2+2\,\left(2\,\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3+3\,\left(bd^2x+bcd\right)\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{32\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/32*(3*b^2*d^2*x^2 + 6*b^2*c*d*x - (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c
^2 - d^2)*cos(b*x + a)^4 + 3*d^2*cos(b*x + a)^2 + 2*(2*(b*d^2*x + b*c*d)*co
s(b*x + a)^3 + 3*(b*d^2*x + b*c*d)*cos(b*x + a))*sin(b*x + a))/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(129) = 258$.
time = 0.48, size = 320, normalized size = 2.39

$$\begin{cases} -\frac{c^2\cos^4(a+bx)}{4b}+\frac{3cdx\sin^4(a+bx)}{16b}+\frac{3cdx\sin^2(a+bx)\cos^2(a+bx)}{8b}-\frac{5cdx\cos^4(a+bx)}{16b}+\frac{3d^2x^2\sin^4(a+bx)}{32b}+\frac{3d^2x^2\sin^2(a+bx)\cos^2(a+bx)}{16b}-\frac{5d^2x^2\cos^4(a+bx)}{32b}+\frac{3cd\sin^3(a+bx)\cos(a+bx)}{16b^2}+\frac{5cd\sin(a+bx)\cos^3(a+bx)}{16b^2}+\frac{3d^2x\sin^3(a+bx)\cos(a+bx)}{16b^2}+\frac{5d^2x\sin(a+bx)\cos^3(a+bx)}{16b^2}-\frac{3d^2\sin^4(a+bx)}{64b^3}+\frac{5d^2\cos^4(a+bx)}{64b^3} & \text{for}\ b\neq 0 \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin(a)\cos^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**2*cos(a + b*x)**4/(4*b) + 3*c*d*x*sin(a + b*x)**4/(16*b) + 3
*c*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(8*b) - 5*c*d*x*cos(a + b*x)**4/(16*
b) + 3*d**2*x**2*sin(a + b*x)**4/(32*b) + 3*d**2*x**2*sin(a + b*x)**2*cos(a
 + b*x)**2/(16*b) - 5*d**2*x**2*cos(a + b*x)**4/(32*b) + 3*c*d*sin(a + b*x)
**3*cos(a + b*x)/(16*b**2) + 5*c*d*sin(a + b*x)*cos(a + b*x)**3/(16*b**2) +
 3*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 5*d**2*x*sin(a + b*x)*co
s(a + b*x)**3/(16*b**2) - 3*d**2*sin(a + b*x)**4/(64*b**3) + 5*d**2*cos(a +
 b*x)**4/(64*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)*co
s(a)**3, True))
```

Giac [A]
time = 0.46, size = 145, normalized size = 1.08

$$-\frac{(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2)\cos{(4\,bx+4\,a)}}{256\,b^3}-\frac{(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\cos{(2\,bx+2\,a)}}{16\,b^3}+\frac{(bd^2x+bcd)\sin{(4\,bx+4\,a)}}{64\,b^3}+\frac{(bd^2x+bcd)\sin{(2\,bx+2\,a)}}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(4*b*x + 4*a)/b^
3 - 1/16*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a)/b
^3 + 1/64*(b*d^2*x + b*c*d)*sin(4*b*x + 4*a)/b^3 + 1/8*(b*d^2*x + b*c*d)*si
n(2*b*x + 2*a)/b^3
```

Mupad [B]
time = 1.63, size = 202, normalized size = 1.51

$$\frac{8\,d^2\,\cos\left(2\,a+2\,b\,x\right)+\frac{d^2\,\cos\left(4\,a+4\,b\,x\right)}{2}-16\,b^2\,c^2\,\cos\left(2\,a+2\,b\,x\right)-4\,b^2\,c^2\,\cos\left(4\,a+4\,b\,x\right)+16\,b\,c\,d\,\sin\left(2\,a+2\,b\,x\right)+2\,b\,c\,d\,\sin\left(4\,a+4\,b\,x\right)-16\,b^2\,d^2\,x^2\,\cos\left(2\,a+2\,b\,x\right)-4\,b^2\,d^2\,x^2\,\cos\left(4\,a+4\,b\,x\right)+16\,b\,d^2\,x\,\sin\left(2\,a+2\,b\,x\right)+2\,b\,d^2\,x\,\sin\left(4\,a+4\,b\,x\right)-32\,b^2\,c\,d\,x\,\cos\left(2\,a+2\,b\,x\right)-8\,b^2\,c\,d\,x\,\cos\left(4\,a+4\,b\,x\right)}{128\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^2,x)
```

```
[Out] (8*d^2*cos(2*a + 2*b*x) + (d^2*cos(4*a + 4*b*x))/2 - 16*b^2*c^2*cos(2*a + 2
*b*x) - 4*b^2*c^2*cos(4*a + 4*b*x) + 16*b*c*d*sin(2*a + 2*b*x) + 2*b*c*d*si
n(4*a + 4*b*x) - 16*b^2*d^2*x^2*cos(2*a + 2*b*x) - 4*b^2*d^2*x^2*cos(4*a +
4*b*x) + 16*b*d^2*x*sin(2*a + 2*b*x) + 2*b*d^2*x*sin(4*a + 4*b*x) - 32*b^2*
c*d*x*cos(2*a + 2*b*x) - 8*b^2*c*d*x*cos(4*a + 4*b*x))/(128*b^3)
```

3.140 $\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=72

$$\frac{3dx}{32b}-\frac{(c+dx)\cos^4(a+bx)}{4b}+\frac{3d\cos(a+bx)\sin(a+bx)}{32b^2}+\frac{d\cos^3(a+bx)\sin(a+bx)}{16b^2}$$

```
[Out] 3/32*d*x/b-1/4*(d*x+c)*cos(b*x+a)^4/b+3/32*d*cos(b*x+a)*sin(b*x+a)/b^2+1/16
*d*cos(b*x+a)^3*sin(b*x+a)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {4490, 2715, 8}

$$\frac{d\sin(a+bx)\cos^3(a+bx)}{16b^2}+\frac{3d\sin(a+bx)\cos(a+bx)}{32b^2}-\frac{(c+dx)\cos^4(a+bx)}{4b}+\frac{3dx}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (3*d*x)/(32*b) - ((c + d*x)*Cos[a + b*x]^4)/(4*b) + (3*d*Cos[a + b*x]*Sin[a
 + b*x])/(32*b^2) + (d*Cos[a + b*x]^3*Sin[a + b*x])/(16*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)\cos^4(a+bx)}{4b}+\frac{d\int\cos^4(a+bx)\,dx}{4b}$$
$$= -\frac{(c+dx)\cos^4(a+bx)}{4b}+\frac{d\cos^3(a+bx)\sin(a+bx)}{16b^2}+\frac{(3d)\int\cos^2$$
$$= -\frac{(c+dx)\cos^4(a+bx)}{4b}+\frac{3d\cos(a+bx)\sin(a+bx)}{32b^2}+\frac{d\cos^3(a+$$
$$= \frac{3dx}{32b}-\frac{(c+dx)\cos^4(a+bx)}{4b}+\frac{3d\cos(a+bx)\sin(a+bx)}{32b^2}+\frac{d\cos$$

Mathematica [**A**]
time = 0.13, size = 75, normalized size = 1.04

$$-\frac{c\cos^4(a+bx)}{4b}+\frac{d(-2bx\cos(2(a+bx))+\sin(2(a+bx)))}{16b^2}+\frac{d(-4bx\cos(4(a+bx))+\sin(4(a+bx)))}{128b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -1/4*(c*Cos[a + b*x]^4)/b + (d*(-2*b*x*Cos[2*(a + b*x)] + Sin[2*(a + b*x)])
)/(16*b^2) + (d*(-4*b*x*Cos[4*(a + b*x)] + Sin[4*(a + b*x)]))/(128*b^2)
```

Maple [**A**]
time = 0.12, size = 85, normalized size = 1.18

method	result
risch	$-\frac{(dx+c)\cos(4bx+4a)}{32b}+\frac{d\sin(4bx+4a)}{128b^2}-\frac{(dx+c)\cos(2bx+2a)}{8b}+\frac{d\sin(2bx+2a)}{16b^2}$
derivativedivides	$\frac{\frac{da\left(\cos^4(bx+a)\right)}{4b}-\frac{c\left(\cos^4(bx+a)\right)}{4}+\frac{d\left(-\frac{(bx+a)\left(\cos^4(bx+a)\right)}{4}+\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{16}+\frac{3bx}{32}+\frac{3a}{32}\right)}{b}}{b}$
default	$\frac{\frac{da\left(\cos^4(bx+a)\right)}{4b}-\frac{c\left(\cos^4(bx+a)\right)}{4}+\frac{d\left(-\frac{(bx+a)\left(\cos^4(bx+a)\right)}{4}+\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{16}+\frac{3bx}{32}+\frac{3a}{32}\right)}{b}}{b}$
norman	$\frac{\frac{5d\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{16b^2}-\frac{3d\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{16b^2}+\frac{3d\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{16b^2}-\frac{5d\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{16b^2}-\frac{5dx}{32b}+\frac{2c\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{2c\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4/b*d*a*cos(b*x+a)^4-1/4*c*cos(b*x+a)^4+1/b*d*(-1/4*(b*x+a)*cos(b*x+
a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a))
```

Maxima [A]
time = 0.27, size = 92, normalized size = 1.28

$$-\frac{32\,c\cos\left(bx+a\right)^4-\frac{32\,ad\cos(bx+a)^4}{b}+\frac{(4\,(bx+a)\cos(4\,bx+4\,a)+16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)-8\,\sin(2\,bx+2\,a))d}{b}}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/128*(32*c*cos(b*x + a)^4 - 32*a*d*cos(b*x + a)^4/b + (4*(b*x + a)*cos(4*
b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x
 + 2*a))*d/b)/b
```

Fricas [A]
time = 2.14, size = 58, normalized size = 0.81

$$-\frac{8\,(bdx+bc)\cos\left(bx+a\right)^4-3\,bdx-\left(2\,d\cos\left(bx+a\right)^3+3\,d\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/32*(8*(b*d*x + b*c)*cos(b*x + a)^4 - 3*b*d*x - (2*d*cos(b*x + a)^3 + 3*d
*cos(b*x + a))*sin(b*x + a))/b^2
```

Sympy [A]
time = 0.30, size = 138, normalized size = 1.92

$$\begin{cases} -\frac{c\cos^4(a+bx)}{4b}+\frac{3dx\sin^4(a+bx)}{32b}+\frac{3dx\sin^2(a+bx)\cos^2(a+bx)}{16b}-\frac{5dx\cos^4(a+bx)}{32b}+\frac{3d\sin^3(a+bx)\cos(a+bx)}{32b^2}+\frac{5d\sin(a+bx)\cos^3(a+bx)}{32b^2} & \text{for } b\neq 0\\ \left(cx+\frac{dx^2}{2}\right)\sin(a)\cos^3(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((-c*cos(a + b*x)**4/(4*b) + 3*d*x*sin(a + b*x)**4/(32*b) + 3*d*x*
sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 5*d*x*cos(a + b*x)**4/(32*b) + 3*d
*sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 5*d*sin(a + b*x)*cos(a + b*x)**3/
(32*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(a)*cos(a)**3, True))
```

Giac [A]
time = 0.46, size = 75, normalized size = 1.04

$$-\frac{(bdx+bc)\cos\left(4\,bx+4\,a\right)}{32\,b^2}-\frac{(bdx+bc)\cos\left(2\,bx+2\,a\right)}{8\,b^2}+\frac{d\sin\left(4\,bx+4\,a\right)}{128\,b^2}+\frac{d\sin\left(2\,bx+2\,a\right)}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/32*(b*d*x + b*c)*cos(4*b*x + 4*a)/b^2 - 1/8*(b*d*x + b*c)*cos(2*b*x + 2*
a)/b^2 + 1/128*d*sin(4*b*x + 4*a)/b^2 + 1/16*d*sin(2*b*x + 2*a)/b^2
```

Mupad [B]
time = 0.31, size = 94, normalized size = 1.31

$$\frac{4\,d\,\sin\left(2\,a+2\,b\,x\right)+\frac{d\,\sin\left(4\,a+4\,b\,x\right)}{2}+4\,b\,c\,\sin\left(2\,a+2\,b\,x\right)^2+16\,b\,c\,\sin\left(a+b\,x\right)^2+8\,b\,d\,x\,\left(2\,\sin\left(a+b\,x\right)^2-1\right)+2\,b\,d\,x\,\left(2\,\sin\left(2\,a+2\,b\,x\right)^2-1\right)}{64\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x),x)
```

```
[Out] (4*d*sin(2*a + 2*b*x) + (d*sin(4*a + 4*b*x))/2 + 4*b*c*sin(2*a + 2*b*x)^2 +
 16*b*c*sin(a + b*x)^2 + 8*b*d*x*(2*sin(a + b*x)^2 - 1) + 2*b*d*x*(2*sin(2*
a + 2*b*x)^2 - 1))/(64*b^2)
```

3.141 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=129

$$\frac{\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{8d}+\frac{\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}+$$

```
[Out] 1/4*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d+1/8*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*
b*x)/d+1/8*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d+1/4*Ci(2*b*c/d+2*b*x)*sin(2
*a-2*b*c/d)/d
```

Rubi [A]
time = 0.17, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4491, 3384, 3380, 3383}

$$\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(8*d) + (CosIntegral[
(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(4*d) + (Cos[2*a - (2*b*c)/d]*SinI
ntegral[(2*b*c)/d + 2*b*x])/(4*d) + (Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*
c)/d + 4*b*x])/(8*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)}+\frac{\sin(4a+4bx)}{8(c+dx)}\right)dx\\ &= \frac{1}{8}\int \frac{\sin(4a+4bx)}{c+dx}\,dx+\frac{1}{4}\int \frac{\sin(2a+2bx)}{c+dx}\,dx\\ &= \frac{1}{8}\cos\left(4a-\frac{4bc}{d}\right)\int \frac{\sin\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx+\frac{1}{4}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+\right.}{c+dx}\\ &= \frac{\operatorname{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{8d}+\frac{\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{4d}+\frac{\cos\left(2a-\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 110, normalized size = 0.85

$$\frac{\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)\sin\left(4a-\frac{4bc}{d}\right)+2\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right)+2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)+\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + 2*CosIntegral[(2*b*(
c + d*x))/d]*Sin[2*a - (2*b*c)/d] + 2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b
*(c + d*x))/d] + Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(8*d)
```

Maple [A]
time = 0.13, size = 178, normalized size = 1.38

method	result
derivativedivides	$\frac{\frac{b\left(-\frac{4\,\text{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}-\frac{4\,\text{cosineIntegral}\left(4bx+4a+\frac{-4ad+4cb}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}\right)}{32}+\frac{b\left(-\frac{2\,\text{sinI}}{}\right.}{}}{b}$
default	$\frac{\frac{b\left(-\frac{4\,\text{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}-\frac{4\,\text{cosineIntegral}\left(4bx+4a+\frac{-4ad+4cb}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}\right)}{32}+\frac{b\left(-\frac{2\,\text{sinI}}{}\right.}{}}{b}$
risch	$-\frac{i\mathrm{e}^{-\frac{4i(ad-cb)}{d}}\,\text{expIntegral}\left(1,4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{16d}-\frac{i\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\text{expIntegral}\left(1,2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{8d}+\frac{i\mathrm{e}^{\frac{2i(a}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/32*b*(-4*Si(-4*b*x-4*a-4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*
b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)+1/8*b*(-2*Si(-2*b*x-2*a-2*(-
a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*
d+b*c)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.35, size = 281, normalized size = 2.18

$$\frac{2\,b\left(i\,E_1\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_1\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b\left(-i\,E_1\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_1\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)-2\,b\left(E_1\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-b\left(E_1\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)}{16\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/16*(2*b*(I*exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*ex
p_integral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/
d) - b*(-I*exp_integral_e(1, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_
integral_e(1, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a*d)/d)
 - 2*b*(exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integ
ral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d) - b*
(exp_integral_e(1, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1
, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d))/(b*d)
```

Fricas [A]
time = 2.47, size = 155, normalized size = 1.20

$$\frac{2\left(\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+\left(\mathrm{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{4\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)+2\,\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{4\,(bdx+bc)}{d}\right)+4\,\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/16*(2*(cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d)
)*sin(-2*(b*c - a*d)/d) + (cos_integral(4*(b*d*x + b*c)/d) + cos_integral(-
4*(b*d*x + b*c)/d))*sin(-4*(b*c - a*d)/d) + 2*cos(-4*(b*c - a*d)/d)*sin_int
egral(4*(b*d*x + b*c)/d) + 4*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x +
b*c)/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+bx\right)\cos^{3}\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.59, size = 6046, normalized size = 46.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/16*(imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*
c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2
*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2
*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_in
tegral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 +
 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b
*c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d
)^2*tan(b*c/d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*ta
n(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(cos_integra
l(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 2*real_
part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b
*c/d)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(
2*b*c/d)^2*tan(b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2
*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_integral(4*b*x +
 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_
integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 +
 imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^
2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*
c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*ta
n(2*b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^
2*tan(2*b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*ta
n(2*b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*
b*c/d)^2 + 8*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan
(2*b*c/d)^2*tan(b*c/d) - 8*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*
a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) + 16*sin_integral(2*(b*d*x + b*c)/d)*
tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b
*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + imag_part(cos_integra
l(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*sin_integral(4*(b
*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*sin_integral(2*(b*d*x +
```

```
 b*c)/d)*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*imag_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 4*imag_part(cos_i
ntegral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 8*
sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2
 + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2*tan(b
*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d
)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*t
an(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(
2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(
2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(
2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d
))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x +
2*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(-
2*b*x - 2*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + imag_part(cos_inte
gral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*sin_integr
al(4*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*sin_integral
(2*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*real_part(cos_
integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d) + 2*real_part(c
os_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d) + 4*real_pa
rt(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2 + 4*real
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2 - 2*
real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2 -
 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)
^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c
/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b
*c/d) - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^
2*tan(b*c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(2
*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*
tan(2*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 4*real_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(2*a)^2*tan(a)*tan(b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(2*a)^2*tan(a)*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4*b*c
/d))*tan(2*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(-4*b*x - 4*b
*c/d))*tan(2*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4*
b*c/d))*tan(2*a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + ...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x),x)
[Out] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x), x)
```

3.142 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=179

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{\sin(2a+2bx)}{4d(c+dx)}-\frac{\sin(4a+}{8d(c+}$$

```
[Out] 1/2*b*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^2+1/2*b*Ci(2*b*c/d+2*b*x)*cos(2*
a-2*b*c/d)/d^2-1/2*b*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^2-1/2*b*Si(2*b*c/
d+2*b*x)*sin(2*a-2*b*c/d)/d^2-1/4*sin(2*b*x+2*a)/d/(d*x+c)-1/8*sin(4*b*x+4*
a)/d/(d*x+c)
```

Rubi [A]
time = 0.20, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^2}-\frac{b\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{\sin(2a+2bx)}{4d(c+dx)}-\frac{\sin(4a+4bx)}{8d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) + (b*Cos[4*
a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(2*d^2) - Sin[2*a + 2*b*x]/(
4*d*(c + d*x)) - Sin[4*a + 4*b*x]/(8*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]
*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) - (b*Sin[4*a - (4*b*c)/d]*SinInteg
ral[(4*b*c)/d + 4*b*x])/(2*d^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^2}+\frac{\sin(4a+4bx)}{8(c+dx)^2}\right)dx\\
&= \frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^2}\,dx+\frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx\\
&= -\frac{\sin(2a+2bx)}{4d(c+dx)}-\frac{\sin(4a+4bx)}{8d(c+dx)}+\frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{2d}+\frac{b\int \frac{\cos(4a+4bx)}{c+dx}\,dx}{2d}\\
&= -\frac{\sin(2a+2bx)}{4d(c+dx)}-\frac{\sin(4a+4bx)}{8d(c+dx)}+\frac{\left(b\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\cos\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx}{2d}+\frac{(b
\\
&= \frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{\sin(2a+
}{4d(c+
\end{aligned}$$

Mathematica [A]
time = 1.71, size = 151, normalized size = 0.84

$$-\frac{-4b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)-4b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)+\frac{2d\sin(2(a+bx))}{c+dx}+\frac{d\sin(4(a+bx))}{c+dx}+4b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)+4b\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right)}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -1/8*(-4*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 4*b*Cos[4*
a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] + (2*d*Sin[2*(a + b*x)])/(c +
 d*x) + (d*Sin[4*(a + b*x)])/(c + d*x) + 4*b*Sin[2*a - (2*b*c)/d]*SinIntegr
al[(2*b*(c + d*x))/d] + 4*b*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x)
)/d])/d^2
```

Maple [A]
time = 0.22, size = 256, normalized size = 1.43

method	result
derivativedivides	$\frac{b^{2}\left(-\frac{4\sin(4bx+4a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{16\,\mathrm{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}+\frac{16\,\mathrm{cosineIntegral}\left(4bx+4a+\frac{-4ad+4cb}{d}\right)\mathrm{co}}{d}}{d}\right.}{32}$ (cut off)
default	$\frac{b^{2}\left(-\frac{4\sin(4bx+4a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{16\,\mathrm{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}+\frac{16\,\mathrm{cosineIntegral}\left(4bx+4a+\frac{-4ad+4cb}{d}\right)\mathrm{co}}{d}}{d}\right.}{32}$ (cut off)
risch	$-\frac{b\,\mathrm{e}^{-\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{4d^{2}}-\frac{b\,\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{4d^{2}}-\frac{b\,\mathrm{e}^{2i}}{}$ (cut off)

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/32*b^2*(-4*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))/d+4*(-4*Si(-4*b*x-4*a
-4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4
*(-a*d+b*c)/d)/d)/d)+1/8*b^2*(-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2*(-
2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*
d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.40, size = 308, normalized size = 1.72

$$\frac{2\,b^{2}\left(i\,E_{2}\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_{2}\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^{2}\left(-i\,E_{2}\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_{2}\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)-2\,b^{2}\left(E_{2}\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_{2}\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-b^{2}\left(E_{2}\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_{2}\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)}{16\,(bcd+(bx+a)d^{2}-ad^{2})b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*b^2*(I*exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*
exp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d
)/d) - b^2*(-I*exp_integral_e(2, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*
exp_integral_e(2, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a*d
)/d) - 2*b^2*(exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp
_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d
) - b^2*(exp_integral_e(2, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_inte
gral_e(2, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d))/((
b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A]
time = 2.50, size = 235, normalized size = 1.31

$$-\frac{4\,d\cos(bx+a)^{3}\sin(bx+a)+2\,(bdx+bc)\sin\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{4\,(bdx+bc)}{d}\right)+2\,(bdx+bc)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)-\left((bdx+bc)\,\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-\left((bdx+bc)\,\mathrm{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{4\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)}{4\,(d^{3}x+cd^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*d*cos(b*x + a)^3*sin(b*x + a) + 2*(b*d*x + b*c)*sin(-4*(b*c - a*d)/
d)*sin_integral(4*(b*d*x + b*c)/d) + 2*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*
sin_integral(2*(b*d*x + b*c)/d) - ((b*d*x + b*c)*cos_integral(2*(b*d*x + b*
c)/d) + (b*d*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/
d) - ((b*d*x + b*c)*cos_integral(4*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_int
egral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\cos^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x)**2, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.15, size = 63510, normalized size = 354.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2
*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + b*d*x*real_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*
c/d)^2*tan(b*c/d)^2 + b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2
*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + b*d*x*
real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^
2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(2*b
*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*t
an(b*c/d) + 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*
tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 4*b*d*x*sin_inte
gral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b
*c/d)^2*tan(b*c/d) - 2*b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2
*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 2*b*d*x*
imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^
2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 4*b*d*x*sin_integral(4*(b*d*x + b*c)
/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 +
```

2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*b*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + b*c*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + b*c*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 - b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 - b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + 4*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) + 4*b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - 2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 2*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 4*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 4*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*b*c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 2*b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 4*b*c*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - b*d*x*real_part(cos_integral(2*b*x + 2

```
*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - b
*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(
2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + b*d*x*real_part(cos_integral(-4*b*x -
4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 +
2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(
2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)}{\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^2, x)
```

3.143 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=231

$$-\frac{b\cos(2a+2bx)}{4d^2(c+dx)}-\frac{b\cos(4a+4bx)}{4d^2(c+dx)}-\frac{b^2\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{d^3}-\frac{b^2\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin}{2d^3}$$

```
[Out] -1/4*b*cos(2*b*x+2*a)/d^2/(d*x+c)-1/4*b*cos(4*b*x+4*a)/d^2/(d*x+c)-1/2*b^2*
cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^3-b^2*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*b*
x)/d^3-b^2*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^3-1/2*b^2*Ci(2*b*c/d+2*b*x)
*sin(2*a-2*b*c/d)/d^3-1/8*sin(2*b*x+2*a)/d/(d*x+c)^2-1/16*sin(4*b*x+4*a)/d/
(d*x+c)^2
```

Rubi [A]
time = 0.25, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$-\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b\cos(2a+2bx)}{4d^2(c+dx)}-\frac{b\cos(4a+4bx)}{4d^2(c+dx)}-\frac{\sin(2a+2bx)}{8d(c+dx)^2}-\frac{\sin(4a+4bx)}{16d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -1/4*(b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)) - (b*Cos[4*a + 4*b*x])/(4*d^2*(c
+ d*x)) - (b^2*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/d^3 - (
b^2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d^3) - Sin[2*a
+ 2*b*x]/(8*d*(c + d*x)^2) - Sin[4*a + 4*b*x]/(16*d*(c + d*x)^2) - (b^2*Cos
[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^3) - (b^2*Cos[4*a -
(4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/d^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^3}+\frac{\sin(4a+4bx)}{8(c+dx)^3}\right)dx \\
&= \frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^3}\,dx+\frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx \\
&= -\frac{\sin(2a+2bx)}{8d(c+dx)^2}-\frac{\sin(4a+4bx)}{16d(c+dx)^2}+\frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{4d}+\frac{b\int \frac{\cos(4a+4bx)}{(c+dx)^2}\,dx}{4d} \\
&= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)}-\frac{b\cos(4a+4bx)}{4d^2(c+dx)}-\frac{\sin(2a+2bx)}{8d(c+dx)^2}-\frac{\sin(4a+4bx)}{16d(c+dx)^2}-\frac{b^2}{} \\
&= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)}-\frac{b\cos(4a+4bx)}{4d^2(c+dx)}-\frac{\sin(2a+2bx)}{8d(c+dx)^2}-\frac{\sin(4a+4bx)}{16d(c+dx)^2}-\frac{(b}{} \\
&= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)}-\frac{b\cos(4a+4bx)}{4d^2(c+dx)}-\frac{b^2\operatorname{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{d^3}-\frac{b^2\mathrm{C}}{}
\end{aligned}$$

Mathematica [A]
time = 3.84, size = 197, normalized size = 0.85

$$-\frac{16b^2\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)\sin\left(4a-\frac{4bc}{d}\right)+8b^2\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right)+\frac{2d(2b(c+dx)\cos(2(a+bx))+d\sin(2(a+bx)))}{(c+dx)^2}+\frac{d(4b(c+dx)\cos(4(a+bx))+d\sin(4(a+bx)))}{(c+dx)^2}+8b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)+16b^2\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -1/16*(16*b^2*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + 8*b^2*C
osIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (2*d*(2*b*(c + d*x)*Co
s[2*(a + b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + (d*(4*b*(c + d*x)*Cos[4
*(a + b*x)] + d*Sin[4*(a + b*x)]))/(c + d*x)^2 + 8*b^2*Cos[2*a - (2*b*c)/d]
*SinIntegral[(2*b*(c + d*x))/d] + 16*b^2*Cos[4*a - (4*b*c)/d]*SinIntegral[(
4*b*(c + d*x))/d])/d^3
```

Maple [A]
time = 0.15, size = 329, normalized size = 1.42

method	result
derivativedivides	$\frac{b^3\left(-\frac{2\sin(4bx+4a)}{(-ad+cb+d(bx+a))^2 d}+\frac{-\frac{8\cos(4bx+4a)}{(-ad+cb+d(bx+a))d}-\frac{8\left(-\frac{4\,\mathrm{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}-\frac{4\,\mathrm{cosineInteg}\ldots}{}\right)}{d}}{d}\ldots\right)}{32}\ldots$
default	$\frac{b^3\left(-\frac{2\sin(4bx+4a)}{(-ad+cb+d(bx+a))^2 d}+\frac{-\frac{8\cos(4bx+4a)}{(-ad+cb+d(bx+a))d}-\frac{8\left(-\frac{4\,\mathrm{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}-\frac{4\,\mathrm{cosineInteg}\ldots}{}\right)}{d}}{d}\ldots\right)}{32}\ldots$
risch	$\frac{ib^2 e^{-\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{2d^3}+\frac{ib^2 e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{4d^3}-\frac{ib^2 e\ldots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/32*b^3*(-2*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))^2/d+2*(-4*cos(4*b*x+4
*a)/(-a*d+c*b+d*(b*x+a))/d-4*(-4*Si(-4*b*x-4*a-4*(-a*d+b*c)/d)*cos(4*(-a*d+
b*c)/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)/d)/d)+1/8*b
^3*(-sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d+(-2*cos(2*b*x+2*a)/(-a*d+c*b+d
*(b*x+a))/d-2*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(
2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.46, size = 343, normalized size = 1.48

$$\frac{2\,b^3\left(i\,E_3\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_3\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^3\left(-i\,E_3\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_3\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)-2\,b^3\left(E_3\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_3\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-b^3\left(E_3\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_3\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)}{16\,\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\,(bcd^2-ad^3)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(2*b^3*(I*exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*
exp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d
)/d) - b^3*(-I*exp_integral_e(3, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*
exp_integral_e(3, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a*d
)/d) - 2*b^3*(exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp
_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d
) - b^3*(exp_integral_e(3, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_inte
gral_e(3, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d))/((
b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(
b*x + a))*b)
```

Fricas [A]
time = 3.32, size = 397, normalized size = 1.72

$$-\frac{2\,d^2\cos\left(bx+a\right)^3\sin\left(bx+a\right)+8\left(bd^2x+bcd\right)\cos\left(bx+a\right)^4-6\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2+4\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{4\,(bdx+bc)}{d}\right)+2\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+2\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(-\frac{4\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)}{4\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^2*cos(b*x + a)^3*sin(b*x + a) + 8*(b*d^2*x + b*c*d)*cos(b*x + a)^
4 - 6*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*cos(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*
c)/d) + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(2*(b*d*x + b*c)
/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-2*(b*d*x + b*c)/d
))*sin(-2*(b*c - a*d)/d) + 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_int
egral(4*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integr
al(-4*(b*d*x + b*c)/d))*sin(-4*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d
^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+bx\right)\cos^3\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x)**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.65, size = 111694, normalized size = 483.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")

[Out] -1/8*(4*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*t
an(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*i
mag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*
tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d
)^2*tan(b*c/d)^2 - 4*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*
tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 8
*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*
a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*b^2*d^2*x^2*sin_integral(2*(b
*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*t
an(b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*
b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 4*b^2*d^2
*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(
2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 8*b^2*d^2*x^2*real_part(cos_int
egral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b
*c/d)*tan(b*c/d)^2 + 8*b^2*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d)
)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 4
*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)
^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*b^2*d^2*x^2*real_part(
cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*t
an(2*b*c/d)^2*tan(b*c/d)^2 - 8*b^2*d^2*x^2*real_part(cos_integral(4*b*x + 4
*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d
)^2 - 8*b^2*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*
tan(b*x)^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 8*b^2*c*d*x*imag
_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan
(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*b^2*c*d*x*imag_part(cos_integral(2*b*
x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*ta
n(b*c/d)^2 - 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*
x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 8*b^2*c*d
*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*
a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 16*b^2*c*d*x*sin_integral(4*(b*
d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*ta
n(b*c/d)^2 + 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b
*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*b^2*d^2*x^2*imag_
part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(
a)^2*tan(2*b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + 2*b^2*d^2*x^
2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a
)^2*tan(a)^2*tan(2*b*c/d)^2 - 4*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x -
 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + 8*b
^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)
^2*tan(a)^2*tan(2*b*c/d)^2 - 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*
tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + 8*b^2*d^2*x^2*
```

```
imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2
*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - 8*b^2*d^2*x^2*imag_part(cos_integral(-2
*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*t
an(b*c/d) + 16*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan
(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) + 8*b^2*c*d*x*real_part
(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2
*tan(2*b*c/d)^2*tan(b*c/d) + 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*
b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/
d) - 4*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*ta
n(b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*imag_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c
/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^
2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*b^2*d^2*x^2*imag_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*ta
n(b*c/d)^2 - 8*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan
(b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*b^2*d^2*x^2*sin_integral(2*(b*
d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 16
*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)
^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 16*b^2*d^2*x^2*imag_part(c
os_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*ta
n(2*b*c/d)*tan(b*c/d)^2 + 32*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*ta
n(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 16*b^2*
c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(
2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 16*...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a+bx)^3 \sin(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^3,x)
[Out] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^3, x)
```

3.144 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=287

$$-\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}-\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegr}}{3d^4}$$

```
[Out] -4/3*b^3*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^4-1/3*b^3*Ci(2*b*c/d+2*b*x)*c
os(2*a-2*b*c/d)/d^4-1/12*b*cos(2*b*x+2*a)/d^2/(d*x+c)^2-1/12*b*cos(4*b*x+4*
a)/d^2/(d*x+c)^2+4/3*b^3*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^4+1/3*b^3*Si(
2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^4-1/12*sin(2*b*x+2*a)/d/(d*x+c)^3+1/6*b^2
*sin(2*b*x+2*a)/d^3/(d*x+c)-1/24*sin(4*b*x+4*a)/d/(d*x+c)^3+1/3*b^2*sin(4*b
*x+4*a)/d^3/(d*x+c)
```

Rubi [A]
time = 0.28, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{4b^3\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^2\sin(2a+2bx)}{6d^3(c+dx)}+\frac{b^2\sin(4a+4bx)}{3d^3(c+dx)}-\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}-\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{12d(c+dx)^3}-\frac{\sin(4a+4bx)}{24d(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] -1/12*(b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)^2) - (b*Cos[4*a + 4*b*x])/(12*d^2
*(c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(
3*d^4) - (4*b^3*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(3*d^4
) - Sin[2*a + 2*b*x]/(12*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(6*d^3*(c
+ d*x)) - Sin[4*a + 4*b*x]/(24*d*(c + d*x)^3) + (b^2*Sin[4*a + 4*b*x])/(3*d
^3*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(
3*d^4) + (4*b^3*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(3*d^4
)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^4}+\frac{\sin(4a+4bx)}{8(c+dx)^4}\right)dx \\
&= \frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^4}\,dx+\frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^4}\,dx \\
&= -\frac{\sin(2a+2bx)}{12d(c+dx)^3}-\frac{\sin(4a+4bx)}{24d(c+dx)^3}+\frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^3}\,dx}{6d}+\frac{b\int \frac{\cos(4a+4bx)}{(c+dx)^3}\,dx}{6d} \\
&= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}-\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{12d(c+dx)^3}-\frac{\sin(4a+4bx)}{24d(c+dx)^3}-b^2\cdots \\
&= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}-\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{12d(c+dx)^3}+\frac{b^2\sin(2a+2bx)}{6d^3(c+dx)}-\cdots \\
&= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}-\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{12d(c+dx)^3}+\frac{b^2\sin(2a+2bx)}{6d^3(c+dx)}-\cdots \\
&= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}-\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-4b\cdots
\end{aligned}$$

Mathematica [A]
time = 2.26, size = 316, normalized size = 1.10

$$-\frac{2d\cos(2bx)\left(bd(c+dx)\cos(2a)+\left(d^2-2b^2(c+dx)^2\right)\sin(2a)\right)+d\cos(4bx)\left(2bd(c+dx)\cos(4a)+\left(d^2-8b^2(c+dx)^2\right)\sin(4a)\right)-2d\left(\left(-d^2+2b^2(c+dx)^2\right)\cos(2a)+bd(c+dx)\sin(2a)\right)\sin(2bx)-d\left(\left(-d^2+8b^2(c+dx)^2\right)\cos(4a)+2bd(c+dx)\sin(4a)\right)\sin(4bx)+8b^3(c+dx)^3\left(\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)-\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right)\right)+32b^3(c+dx)^3\left(\cos\left(4a-\frac{4bc}{d}\right)\operatorname{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)-\sin\left(4a-\frac{4bc}{d}\right)\operatorname{Si}\left(\frac{4b(c+dx)}{d}\right)\right)}{24d^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] -1/24*(2*d*Cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*S
in[2*a]) + d*Cos[4*b*x]*(2*b*d*(c + d*x)*Cos[4*a] + (d^2 - 8*b^2*(c + d*x)^
2)*Sin[4*a]) - 2*d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*Sin
[2*a])*Sin[2*b*x] - d*((-d^2 + 8*b^2*(c + d*x)^2)*Cos[4*a] + 2*b*d*(c + d*x
)*Sin[4*a])*Sin[4*b*x] + 8*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIntegra
l[(2*b*(c + d*x))/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])
 + 32*b^3*(c + d*x)^3*(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d]
- Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d]))/(d^4*(c + d*x)^3)
```

Maple [A]
time = 0.22, size = 404, normalized size = 1.41

method	result
derivativedivides	$\frac{b^4\left(-\frac{4\sin(4bx+4a)}{3(-ad+cb+d(bx+a))^3 d}+\frac{-\frac{8\cos(4bx+4a)}{3(-ad+cb+d(bx+a))^2 d}-\frac{8\left(-\frac{4\sin(4bx+4a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{16\,\mathrm{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\mathrm{si}\ldots}{d}}{\ldots}\right)}{\ldots}}{d}\ldots\right)}{32}\ldots$
default	$\frac{b^4\left(-\frac{4\sin(4bx+4a)}{3(-ad+cb+d(bx+a))^3 d}+\frac{-\frac{8\cos(4bx+4a)}{3(-ad+cb+d(bx+a))^2 d}-\frac{8\left(-\frac{4\sin(4bx+4a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{16\,\mathrm{sinIntegral}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\mathrm{si}\ldots}{d}}{\ldots}\right)}{\ldots}}{d}\ldots\right)}{32}\ldots$
risch	$\frac{2b^3 e^{-\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{3d^4}+\frac{b^3 e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{6d^4}+\frac{b^3 e^{\ldots}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/32*b^4*(-4/3*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))^3/d+4/3*(-2*cos(4*b
*x+4*a)/(-a*d+c*b+d*(b*x+a))^2/d-2*(-4*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))/
d+4*(-4*Si(-4*b*x-4*a-4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4*a+
4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)/d)/d)/d)+1/8*b^4*(-2/3*sin(2*b*x+2*a
)/(-a*d+c*b+d*(b*x+a))^3/d+2/3*(-cos(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d-(-
2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)
*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d
)/d)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.60, size = 393, normalized size = 1.37

$$\frac{2\,b^4\left(i\,E_4\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_4\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^4\left(-i\,E_4\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_4\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)-2\,b^4\left(E_4\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_4\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-b^4\left(E_4\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_4\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)}{16\,\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\,(bcd^3-ad^4)(bx+a)^2+3\,(b^2c^2d^2-2\,abcd^3+a^2d^4)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/16*(2*b^4*(I*exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*
exp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d
)/d) - b^4*(-I*exp_integral_e(4, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*
exp_integral_e(4, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a*d
)/d) - 2*b^4*(exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp
_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d
) - b^4*(exp_integral_e(4, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_inte
gral_e(4, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d))/((
b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3
*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b
*x + a))*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(267) = 534$.
time = 2.71, size = 568, normalized size = 1.98

$$-\frac{4\,(bd^3x+bcd^2)\cos(bx+a)^4-3\,(bd^3x+bcd^2)\cos(bx+a)^2-8\,(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\sin\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{4\,(bdx+bc)}{d}\right)-2\,(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+\left((b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\,\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\,\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+4\left((b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\,\mathrm{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)+(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\,\mathrm{Ci}\left(-\frac{4\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)-2\left((8\,b^2d^3x^2+16\,b^2cd^2x+8\,b^2c^2d-d^3)\cos(bx+a)^3-3\,(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d)\cos(bx+a)\right)\sin(bx+a)}{6\,(d^7x^3+3\,cd^6x^2+3\,c^2d^5x+c^3d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(4*(b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - 3*(b*d^3*x + b*c*d^2)*cos(b*x
+ a)^2 - 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-4
*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) - 2*(b^3*d^3*x^3 + 3*b^3*c*
d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*
d*x + b*c)/d) + ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*
cos_integral(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^
2*d*x + b^3*c^3)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d) +
4*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(4
*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^
3)*cos_integral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c - a*d)/d) - 2*((8*b^2*d^3*
x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3 - 3*(b^2*d^3*x^2 +
 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a))*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*
x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\cos^3(a+bx)}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**4,x)

[Out] Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x)**4, x)

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.56, size = 157526, normalized size = 548.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")

[Out] -1/12*(8*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 8*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 16*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 16*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 32*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 16*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 16*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 32*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 24*b^3*c*d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)

```
^2*tan(b*c/d)^2 + 24*b^3*c*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d)
)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 +
 8*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*
x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 - 2*b^3*d^3*x^3*real_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c
/d)^2 - 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^
2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + 8*b^3*d^3*x^3*real_part(c
os_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*
tan(2*b*c/d)^2 + 8*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan
(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) + 8*b^3*d^
3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan
(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2
*b*c/d)^2*tan(b*c/d) + 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b
*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d
) - 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^
2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 8*b^3*d^3*x^3*real_part(c
os_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*t
an(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*
b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_par
t(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)
^2*tan(b*c/d)^2 - 8*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*t
an(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 32*b^3*d^3*x^3*re
al_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan
(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 32*b^3*d^3*x^3*real_part(cos_integral(-4*
b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(
b*c/d)^2 - 48*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*
b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 48*b^3*c*
d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*t
an(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 96*b^3*c*d^2*x^2*sin_integra
l(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/
d)*tan(b*c/d)^2 + 8*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*ta
n(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^
3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)
^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3*...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos{(a+b\,x)}^3\,\sin{(a+b\,x)}}{{(c+d\,x)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^4,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^4, x)
```

3.145 $\int (c+dx)^m \cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=419

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{ib(c+dx)}{d}\right)}{16b}+\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}$$

```
[Out] -1/16*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+
c)/d)^m)+1/16*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b
*(d*x+c)/d)^m)+1/32*I*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*
b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/32*I*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I
*b*(d*x+c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)+1/32*I*5^(-1-m)*exp(
5*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-5*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)
-1/32*I*5^(-1-m)*(d*x+c)^m*GAMMA(1+m,5*I*b*(d*x+c)/d)/b/exp(5*I*(a-b*c/d))/
((I*b*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.31, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4491, 3388, 2212}

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b}+\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{32b}+\frac{i5^{-m-1}e^{5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{5ib(c+dx)}{d}\right)}{32b}+\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{ib(c+dx)}{d}\right)}{16b}-\frac{i3^{-m-1}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{3ib(c+dx)}{d}\right)}{32b}-\frac{i5^{-m-1}e^{-5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{5ib(c+dx)}{d}\right)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/
d])/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/16)*(c + d*x)^m*Gamma[1 + m, (I*b*(c
 + d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + ((I/32)*3^(-1
- m)*E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/
d])/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/32)*3^(-1 - m)*(c + d*x)^m*Gamma[1 +
 m, ((3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^
m) + ((I/32)*5^(-1 - m)*E^((5*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((
-5*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/32)*5^(-1 - m)*(c
 + d*x)^m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d])/(b*E^((5*I)*(a - (b*c)/d))*(
(I*b*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^m \cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^m\cos(a+bx) - \frac{1}{16}(c+dx)^m\cos(3a+3bx) - \frac{1}{16}(\right.$$
$$= -\left(\frac{1}{16}\int (c+dx)^m\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^m\cos(5a$$
$$= -\left(\frac{1}{32}\int e^{-i(3a+3bx)}(c+dx)^m\,dx\right) - \frac{1}{32}\int e^{i(3a+3bx)}(c+dx)^m\,dx$$
$$= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{16b} + \frac{ie^{-i(}}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1622 vs. $2(419) = 838$.
time = 51.14, size = 1622, normalized size = 3.87

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*(c + d*x)^m*((E^((2*I)*a)*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/((
(-I)*b*(c + d*x))/d)^m - (E^(((2*I)*b*c)/d)*Gamma[1 + m, (I*b*(c + d*x))/d]
)/((I*b*(c + d*x))/d)^m))/(b*E^((I*(b*c + a*d))/d)) - ((I/32)*3^(-1 - m)*(c
 + d*x)^m*(((-I)*b*(c + d*x))/d)^m*(-(E^((6*I)*a)*((I*b*(c + d*x))/d)^(2*m)
*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d]) + E^(((6*I)*b*c)/d)*((b^2*(c + d*x)^
2)/d^2)^m*Gamma[1 + m, ((3*I)*b*(c + d*x))/d]))/(b*E^(((3*I)*(b*c + a*d))/d
)*((b^2*(c + d*x)^2)/d^2)^(2*m)) - ((I/256)*15^(-1 - m)*(c + d*x)^m*(-(15^m
*E^(((15*I)*b*c)/d)*((I*b*(c + d*x))/d)^m*((b^2*(c + d*x)^2)/d^2)^m) + 15^m
*E^(((5*I)*(3*b*c + 2*a*d))/d)*((I*b*(c + d*x))/d)^m*((b^2*(c + d*x)^2)/d^2
```

```
)^m - 3^(1 + m)*5^m*E^(((5*I)*b*(3*c + 2*d*x))/d)*((I*b*(c + d*x))/d)^m*((b
^2*(c + d*x)^2)/d^2)^m + 15^m*E^(((15*I)*(b*c + 2*a*d + 2*b*d*x))/d)*((I*b*
(c + d*x))/d)^m*((b^2*(c + d*x)^2)/d^2)^m + 3^(1 + m)*5^m*E^(((5*I)*(3*b*c
+ 2*a*d + 2*b*d*x))/d)*((I*b*(c + d*x))/d)^m*((b^2*(c + d*x)^2)/d^2)^m - 3^
(1 + m)*5^m*E^(((5*I)*(3*b*c + 4*a*d + 4*b*d*x))/d)*((I*b*(c + d*x))/d)^m*(
(b^2*(c + d*x)^2)/d^2)^m + 3^(1 + m)*5^m*E^(((5*I)*(3*b*c + 6*a*d + 4*b*d*x
))/d)*((I*b*(c + d*x))/d)^m*((b^2*(c + d*x)^2)/d^2)^m - 15^m*E^(((5*I)*(3*b
*c + 4*a*d + 6*b*d*x))/d)*((I*b*(c + d*x))/d)^m*((b^2*(c + d*x)^2)/d^2)^m +
 3^(1 + m)*E^(((5*I)*(2*b*c + 4*a*d + 3*b*d*x))/d)*(-1 + E^((10*I)*a))*m*((
I*b*(c + d*x))/d)^(2*m)*Gamma[m, ((-5*I)*b*(c + d*x))/d] + 3^(1 + m)*E^(((5
*I)*b*(4*c + 3*d*x))/d)*(-1 + E^((10*I)*a))*m*((b^2*(c + d*x)^2)/d^2)^m*Gam
ma[m, ((5*I)*b*(c + d*x))/d] + E^((15*I)*(2*a + b*x))*m*((I*b*(c + d*x))/d)
^(2*m)*Gamma[m, ((-15*I)*b*(c + d*x))/d] - E^((5*I)*(4*a + 3*b*x))*m*((I*b*
(c + d*x))/d)^(2*m)*Gamma[m, ((-15*I)*b*(c + d*x))/d] - E^(((15*I)*b*(2*c +
 d*x))/d)*m*((b^2*(c + d*x)^2)/d^2)^m*Gamma[m, ((15*I)*b*(c + d*x))/d] + E^
(((5*I)*(6*b*c + 2*a*d + 3*b*d*x))/d)*m*((b^2*(c + d*x)^2)/d^2)^m*Gamma[m,
((15*I)*b*(c + d*x))/d] - 7*3^(1 + m)*E^(((5*I)*(2*b*c + 4*a*d + 3*b*d*x))/
d)*((I*b*(c + d*x))/d)^(2*m)*Gamma[1 + m, ((-5*I)*b*(c + d*x))/d] - 3^(1 +
m)*E^(((5*I)*(2*b*c + 6*a*d + 3*b*d*x))/d)*((I*b*(c + d*x))/d)^(2*m)*Gamma[
1 + m, ((-5*I)*b*(c + d*x))/d] + 3^(1 + m)*E^(((5*I)*b*(4*c + 3*d*x))/d)*((
b^2*(c + d*x)^2)/d^2)^m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d] + 7*3^(1 + m)*E
^(((5*I)*(4*b*c + 2*a*d + 3*b*d*x))/d)*((b^2*(c + d*x)^2)/d^2)^m*Gamma[1 +
m, ((5*I)*b*(c + d*x))/d] - E^((15*I)*(2*a + b*x))*((I*b*(c + d*x))/d)^(2*m
)*Gamma[1 + m, ((-15*I)*b*(c + d*x))/d] + E^((5*I)*(4*a + 3*b*x))*((I*b*(c
+ d*x))/d)^(2*m)*Gamma[1 + m, ((-15*I)*b*(c + d*x))/d] + E^(((15*I)*b*(2*c
+ d*x))/d)*((b^2*(c + d*x)^2)/d^2)^m*Gamma[1 + m, ((15*I)*b*(c + d*x))/d] -
 E^(((5*I)*(6*b*c + 2*a*d + 3*b*d*x))/d)*((b^2*(c + d*x)^2)/d^2)^m*Gamma[1
+ m, ((15*I)*b*(c + d*x))/d]))/(b*E^(((15*I)*(a*d + b*(c + d*x)))/d)*((I*b*
(c + d*x))/d)^m*((b^2*(c + d*x)^2)/d^2)^m)
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cos^3(bx+a)\right) \left(\sin^2(bx+a)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x)
[Out] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^2, x)
```

Fricas [A]
time = 0.84, size = 280, normalized size = 0.67

$$\frac{30i\,e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-i\,bc+i\,ad}{d}\right)}\Gamma\left(m+1,\frac{i\,bdx+i\,bc}{d}\right)+5i\,e^{\left(-\frac{dm\log\left(-\frac{3i\,b}{d}\right)+3i\,bc-3i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{3\,(i\,bdx+i\,bc)}{d}\right)+3i\,e^{\left(-\frac{dm\log\left(-\frac{5i\,b}{d}\right)+5i\,bc-5i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{5\,(i\,bdx+i\,bc)}{d}\right)-30i\,e^{\left(-\frac{dm\log\left(-\frac{ib}{d}\right)+i\,bc-i\,ad}{d}\right)}\Gamma\left(m+1,\frac{-i\,bdx-i\,bc}{d}\right)-5i\,e^{\left(-\frac{dm\log\left(\frac{3i\,b}{d}\right)-3i\,bc+3i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{3\,(-i\,bdx-i\,bc)}{d}\right)-3i\,e^{\left(-\frac{dm\log\left(\frac{5i\,b}{d}\right)-5i\,bc+5i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{5\,(-i\,bdx-i\,bc)}{d}\right)}{480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/480*(30*I*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x +
 I*b*c)/d) + 5*I*e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m + 1
, -3*(I*b*d*x + I*b*c)/d) + 3*I*e^(-(d*m*log(-5*I*b/d) + 5*I*b*c - 5*I*a*d)
/d)*gamma(m + 1, -5*(I*b*d*x + I*b*c)/d) - 30*I*e^(-(d*m*log(-I*b/d) + I*b*
c - I*a*d)/d)*gamma(m + 1, (-I*b*d*x - I*b*c)/d) - 5*I*e^(-(d*m*log(3*I*b/d
) - 3*I*b*c + 3*I*a*d)/d)*gamma(m + 1, -3*(-I*b*d*x - I*b*c)/d) - 3*I*e^(-(
d*m*log(5*I*b/d) - 5*I*b*c + 5*I*a*d)/d)*gamma(m + 1, -5*(-I*b*d*x - I*b*c)
/d))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2\,{\left(c+d\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^m, x)
```

3.146 $\int (c+dx)^4 \cos^3(a+bx) \sin^2(a+bx) \, dx$

Optimal. Leaf size=330

$$-\frac{3d^3(c+dx)\cos(a+bx)}{b^4}+\frac{d(c+dx)^3\cos(a+bx)}{2b^2}+\frac{d^3(c+dx)\cos(3a+3bx)}{54b^4}-\frac{d(c+dx)^3\cos(3a+3bx)}{36b^2}+$$

[Out] -3*d^3*(d*x+c)*cos(b*x+a)/b^4+1/2*d*(d*x+c)^3*cos(b*x+a)/b^2+1/54*d^3*(d*x+c)*cos(3*b*x+3*a)/b^4-1/36*d*(d*x+c)^3*cos(3*b*x+3*a)/b^2+3/1250*d^3*(d*x+c)*cos(5*b*x+5*a)/b^4-1/100*d*(d*x+c)^3*cos(5*b*x+5*a)/b^2+3*d^4*sin(b*x+a)/b^5-3/2*d^2*(d*x+c)^2*sin(b*x+a)/b^3+1/8*(d*x+c)^4*sin(b*x+a)/b-1/162*d^4*sin(3*b*x+3*a)/b^5+1/36*d^2*(d*x+c)^2*sin(3*b*x+3*a)/b^3-1/48*(d*x+c)^4*sin(3*b*x+3*a)/b-3/6250*d^4*sin(5*b*x+5*a)/b^5+3/500*d^2*(d*x+c)^2*sin(5*b*x+5*a)/b^3-1/80*(d*x+c)^4*sin(5*b*x+5*a)/b

Rubi [A]
time = 0.27, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4491, 3377, 2717}

$$\frac{3d^4\sin(a+bx)}{b^5}-\frac{d^4\sin(3a+3bx)}{162b^5}-\frac{3d^4\sin(5a+5bx)}{6250b^5}-\frac{3d^3(c+dx)\cos(a+bx)}{b^4}+\frac{d^3(c+dx)\cos(3a+3bx)}{54b^4}+\frac{3d^3(c+dx)\cos(5a+5bx)}{1250b^4}-\frac{3d^2(c+dx)^2\sin(a+bx)}{2b^3}+\frac{d^2(c+dx)^2\sin(3a+3bx)}{36b^3}+\frac{3d^2(c+dx)^2\sin(5a+5bx)}{500b^3}+\frac{d(c+dx)^3\cos(a+bx)}{2b^2}-\frac{d(c+dx)^3\cos(3a+3bx)}{36b^2}-\frac{d(c+dx)^3\cos(5a+5bx)}{100b^2}+\frac{(c+dx)^4\sin(a+bx)}{8b}-\frac{(c+dx)^4\sin(3a+3bx)}{48b}-\frac{(c+dx)^4\sin(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^2,x]

[Out] (-3*d^3*(c + d*x)*Cos[a + b*x])/b^4 + (d*(c + d*x)^3*Cos[a + b*x])/(2*b^2) + (d^3*(c + d*x)*Cos[3*a + 3*b*x])/(54*b^4) - (d*(c + d*x)^3*Cos[3*a + 3*b*x])/(36*b^2) + (3*d^3*(c + d*x)*Cos[5*a + 5*b*x])/(1250*b^4) - (d*(c + d*x)^3*Cos[5*a + 5*b*x])/(100*b^2) + (3*d^4*Sin[a + b*x])/b^5 - (3*d^2*(c + d*x)^2*Sin[a + b*x])/(2*b^3) + ((c + d*x)^4*Sin[a + b*x])/(8*b) - (d^4*Sin[3*a + 3*b*x])/(162*b^5) + (d^2*(c + d*x)^2*Sin[3*a + 3*b*x])/(36*b^3) - ((c + d*x)^4*Sin[3*a + 3*b*x])/(48*b) - (3*d^4*Sin[5*a + 5*b*x])/(6250*b^5) + (3*d^2*(c + d*x)^2*Sin[5*a + 5*b*x])/(500*b^3) - ((c + d*x)^4*Sin[5*a + 5*b*x])/(80*b)

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^4\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^4\cos(a+bx) - \frac{1}{16}(c+dx)^4\cos(3a+3bx) - \frac{1}{16}(c\cdots\right.$$

$$= -\left(\frac{1}{16}\int (c+dx)^4\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^4\cos(5a+\cdots$$

$$= \frac{(c+dx)^4\sin(a+bx)}{8b} - \frac{(c+dx)^4\sin(3a+3bx)}{48b} - \frac{(c+dx)^4\sin(5\cdots}{80b}$$

$$= \frac{d(c+dx)^3\cos(a+bx)}{2b^2} - \frac{d(c+dx)^3\cos(3a+3bx)}{36b^2} - \frac{d(c+dx)^3\cdots}{10\cdots}$$

$$= \frac{d(c+dx)^3\cos(a+bx)}{2b^2} - \frac{d(c+dx)^3\cos(3a+3bx)}{36b^2} - \frac{d(c+dx)^3\cdots}{10\cdots}$$

$$= -\frac{3d^3(c+dx)\cos(a+bx)}{b^4} + \frac{d(c+dx)^3\cos(a+bx)}{2b^2} + \frac{d^3(c+dx)\cdots}{5\cdots}$$

$$= -\frac{3d^3(c+dx)\cos(a+bx)}{b^4} + \frac{d(c+dx)^3\cos(a+bx)}{2b^2} + \frac{d^3(c+dx)\cdots}{5\cdots}$$

Mathematica [A]
time = 3.81, size = 563, normalized size = 1.71

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -1/4050000*(-506250*b^4*c^4*Sin[a + b*x] - 2025000*b^3*c^3*d*(Cos[a + b*x]
+ b*x*Sin[a + b*x]) - 2025000*b*c*d^3*(3*(-2 + b^2*x^2)*Cos[a + b*x] + b*x*
(-6 + b^2*x^2)*Sin[a + b*x]) - 3037500*b^2*c^2*d^2*(2*b*x*Cos[a + b*x] + (-
2 + b^2*x^2)*Sin[a + b*x]) - 506250*d^4*(4*b*x*(-6 + b^2*x^2)*Cos[a + b*x]
+ (24 - 12*b^2*x^2 + b^4*x^4)*Sin[a + b*x]) + 84375*b^4*c^4*Sin[3*(a + b*x)
] + 112500*b^3*c^3*d*(Cos[3*(a + b*x)] + 3*b*x*Sin[3*(a + b*x)]) + 37500*b*
c*d^3*((-2 + 9*b^2*x^2)*Cos[3*(a + b*x)] + 3*b*x*(-2 + 3*b^2*x^2)*Sin[3*(a
+ b*x)]) + 56250*b^2*c^2*d^2*(6*b*x*Cos[3*(a + b*x)] + (-2 + 9*b^2*x^2)*Sin
[3*(a + b*x)]) + 3125*d^4*(12*b*x*(-2 + 3*b^2*x^2)*Cos[3*(a + b*x)] + (8 -
36*b^2*x^2 + 27*b^4*x^4)*Sin[3*(a + b*x)]) + 50625*b^4*c^4*Sin[5*(a + b*x)]
```

```
 + 40500*b^3*c^3*d*(Cos[5*(a + b*x)] + 5*b*x*Sin[5*(a + b*x)]) + 1620*b*c*d
^3*((-6 + 75*b^2*x^2)*Cos[5*(a + b*x)] + 5*b*x*(-6 + 25*b^2*x^2)*Sin[5*(a +
 b*x)]) + 12150*b^2*c^2*d^2*(10*b*x*Cos[5*(a + b*x)] + (-2 + 25*b^2*x^2)*Si
n[5*(a + b*x)]) + 81*d^4*(20*b*x*(-6 + 25*b^2*x^2)*Cos[5*(a + b*x)] + (24 -
 300*b^2*x^2 + 625*b^4*x^4)*Sin[5*(a + b*x)]))/b^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1841 vs. $2(304) = 608$.
time = 0.35, size = 1842, normalized size = 5.58

method	result
risch	$\frac{d(b^2d^3x^3+3b^2c\,d^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6d^2c)\cos(bx+a)}{2b^4}+\frac{(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-12b}{8b^5}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/b^4*a^4*d^4*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)*sin(
b*x+a))-4/b^3*a^3*c*d^3*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)
*sin(b*x+a))-4/b^4*a^3*d^4*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*co
s(b*x+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*
sin(b*x+a)-1/25*cos(b*x+a)^5)+6/b^2*a^2*c^2*d^2*(-1/5*sin(b*x+a)*cos(b*x+a)
^4+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))+12/b^3*a^2*c*d^3*(1/3*(b*x+a)*(2+cos(b
*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(
b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)+6/b^4*a^2*d^4*(1/3
*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/15*sin(b*x+a)+4/15*(b*x+a)*cos(b*x
+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/135*(2+cos(b*x+a)^2)*sin(b*x+a)-1/5*(b*x+a)
^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-2/25*(b*x+a)*cos(b*x+a)^5
+2/125*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a))-4/b*a*c^3*d*(-1/5*si
n(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))-12/b^2*a*c^2*d^2*(1
/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/
5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)
-12/b^3*a*c*d^3*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/15*sin(b*x+a)+
4/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/135*(2+cos(b*x+a)^2)*si
n(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-2/25*
(b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a))-
4/b^4*a*d^4*(1/3*(b*x+a)^3*(2+cos(b*x+a)^2)*sin(b*x+a)+2/5*(b*x+a)^2*cos(b*
x+a)-856/1125*cos(b*x+a)-4/5*(b*x+a)*sin(b*x+a)+1/15*(b*x+a)^2*cos(b*x+a)^3
-2/45*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+22/3375*cos(b*x+a)^3-1/5*(b*x+a)^
3*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-3/25*(b*x+a)^2*cos(b*x+a)^
5+6/125*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)+6/625*cos(b*
x+a)^5)+c^4*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))
+4/b*c^3*d*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*
```

```
cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*
cos(b*x+a)^5)+6/b^2*c^2*d^2*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/15
*sin(b*x+a)+4/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/135*(2+cos(
b*x+a)^2)*sin(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(
b*x+a)-2/25*(b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*
sin(b*x+a))+4/b^3*c*d^3*(1/3*(b*x+a)^3*(2+cos(b*x+a)^2)*sin(b*x+a)+2/5*(b*x
+a)^2*cos(b*x+a)-856/1125*cos(b*x+a)-4/5*(b*x+a)*sin(b*x+a)+1/15*(b*x+a)^2*
cos(b*x+a)^3-2/45*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+22/3375*cos(b*x+a)^3-
1/5*(b*x+a)^3*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-3/25*(b*x+a)^2
*cos(b*x+a)^5+6/125*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)+
6/625*cos(b*x+a)^5)+1/b^4*d^4*(1/3*(b*x+a)^4*(2+cos(b*x+a)^2)*sin(b*x+a)+8/
15*(b*x+a)^3*cos(b*x+a)-8/5*(b*x+a)^2*sin(b*x+a)+3424/1125*sin(b*x+a)-3424/
1125*(b*x+a)*cos(b*x+a)+4/45*(b*x+a)^3*cos(b*x+a)^3-4/45*(b*x+a)^2*(2+cos(b
*x+a)^2)*sin(b*x+a)+88/3375*(b*x+a)*cos(b*x+a)^3-88/10125*(2+cos(b*x+a)^2)*
sin(b*x+a)-1/5*(b*x+a)^4*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-4/2
5*(b*x+a)^3*cos(b*x+a)^5+12/125*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^
2)*sin(b*x+a)+24/625*(b*x+a)*cos(b*x+a)^5-24/3125*(8/3+cos(b*x+a)^4+4/3*cos
(b*x+a)^2)*sin(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1339 vs. $2(304) = 608$.
time = 0.33, size = 1339, normalized size = 4.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4050000*(270000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c^4 - 1080000*(3*s
in(b*x + a)^5 - 5*sin(b*x + a)^3)*a*c^3*d/b + 1620000*(3*sin(b*x + a)^5 - 5
*sin(b*x + a)^3)*a^2*c^2*d^2/b^2 - 1080000*(3*sin(b*x + a)^5 - 5*sin(b*x +
a)^3)*a^3*c*d^3/b^3 + 270000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*a^4*d^4/
b^4 + 4500*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) -
 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 45
0*cos(b*x + a))*c^3*d/b - 13500*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x +
a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*
cos(3*b*x + 3*a) - 450*cos(b*x + a))*a*c^2*d^2/b^2 + 13500*(45*(b*x + a)*si
n(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a)
 + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a^2*c*d^3/b
^3 - 4500*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) -
450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 450
*cos(b*x + a))*a^3*d^4/b^4 + 450*(270*(b*x + a)*cos(5*b*x + 5*a) + 750*(b*x
 + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*x + a) + 27*(25*(b*x + a)^2
- 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 6750*((b
*x + a)^2 - 2)*sin(b*x + a))*c^2*d^2/b^2 - 900*(270*(b*x + a)*cos(5*b*x + 5
```

```
*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*x + a) + 27*(2
5*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3
*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*a*c*d^3/b^3 + 450*(270*(b*x + a)
*cos(5*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*
x + a) + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)
*sin(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*a^2*d^4/b^4 + 60*(
81*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*cos(3*b*
x + 3*a) - 101250*((b*x + a)^2 - 2)*cos(b*x + a) + 135*(25*(b*x + a)^3 - 6*
b*x - 6*a)*sin(5*b*x + 5*a) + 1875*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x
+ 3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*c*d^3/b^3 - 60*(81
*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*cos(3*b*x
+ 3*a) - 101250*((b*x + a)^2 - 2)*cos(b*x + a) + 135*(25*(b*x + a)^3 - 6*b*
x - 6*a)*sin(5*b*x + 5*a) + 1875*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x +
3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*a*d^4/b^4 + (1620*(2
5*(b*x + a)^3 - 6*b*x - 6*a)*cos(5*b*x + 5*a) + 37500*(3*(b*x + a)^3 - 2*b*
x - 2*a)*cos(3*b*x + 3*a) - 2025000*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a
) + 81*(625*(b*x + a)^4 - 300*(b*x + a)^2 + 24)*sin(5*b*x + 5*a) + 3125*(27
*(b*x + a)^4 - 36*(b*x + a)^2 + 8)*sin(3*b*x + 3*a) - 506250*((b*x + a)^4 -
 12*(b*x + a)^2 + 24)*sin(b*x + a))*d^4/b^4)/b
```

Fricas [**A**]
time = 2.93, size = 527, normalized size = 1.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/253125*(1620*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 25*b^3*c^3*d - 6*b*c*d
^3 + 3*(25*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^5 - 300*(75*b^3*d^4*x^3 +
 225*b^3*c*d^3*x^2 + 75*b^3*c^3*d + 22*b*c*d^3 + (225*b^3*c^2*d^2 + 22*b*d^
4)*x)*cos(b*x + a)^3 - 1800*(75*b^3*d^4*x^3 + 225*b^3*c*d^3*x^2 + 75*b^3*c^
3*d - 428*b*c*d^3 + (225*b^3*c^2*d^2 - 428*b*d^4)*x)*cos(b*x + a) - (33750*
b^4*d^4*x^4 + 135000*b^4*c*d^3*x^3 + 33750*b^4*c^4 - 385200*b^2*c^2*d^2 - 8
1*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 625*b^4*c^4 - 300*b^2*c^2*d^2 + 2
4*d^4 + 150*(25*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 100*(25*b^4*c^3*d - 6*b^2*c*
d^3)*x)*cos(b*x + a)^4 + 760816*d^4 + 900*(225*b^4*c^2*d^2 - 428*b^2*d^4)*x
^2 + (16875*b^4*d^4*x^4 + 67500*b^4*c*d^3*x^3 + 16875*b^4*c^4 + 9900*b^2*c^
2*d^2 - 4792*d^4 + 450*(225*b^4*c^2*d^2 + 22*b^2*d^4)*x^2 + 900*(75*b^4*c^3
*d + 22*b^2*c*d^3)*x)*cos(b*x + a)^2 + 1800*(75*b^4*c^3*d - 428*b^2*c*d^3)*
x)*sin(b*x + a))/b^5
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(325) = 650$.
time = 1.54, size = 1098, normalized size = 3.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((2*c**4*sin(a + b*x)**5/(15*b) + c**4*sin(a + b*x)**3*cos(a + b*x
)**2/(3*b) + 8*c**3*d*x*sin(a + b*x)**5/(15*b) + 4*c**3*d*x*sin(a + b*x)**3
*cos(a + b*x)**2/(3*b) + 4*c**2*d**2*x**2*sin(a + b*x)**5/(5*b) + 2*c**2*d*
*2*x**2*sin(a + b*x)**3*cos(a + b*x)**2/b + 8*c*d**3*x**3*sin(a + b*x)**5/(
15*b) + 4*c*d**3*x**3*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 2*d**4*x**4*s
in(a + b*x)**5/(15*b) + d**4*x**4*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 8
*c**3*d*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 52*c**3*d*sin(a + b*x)**2*
cos(a + b*x)**3/(45*b**2) + 104*c**3*d*cos(a + b*x)**5/(225*b**2) + 8*c**2*
d**2*x*sin(a + b*x)**4*cos(a + b*x)/(5*b**2) + 52*c**2*d**2*x*sin(a + b*x)*
*2*cos(a + b*x)**3/(15*b**2) + 104*c**2*d**2*x*cos(a + b*x)**5/(75*b**2) +
8*c*d**3*x**2*sin(a + b*x)**4*cos(a + b*x)/(5*b**2) + 52*c*d**3*x**2*sin(a
+ b*x)**2*cos(a + b*x)**3/(15*b**2) + 104*c*d**3*x**2*cos(a + b*x)**5/(75*b
**2) + 8*d**4*x**3*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 52*d**4*x**3*si
n(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 104*d**4*x**3*cos(a + b*x)**5/(22
5*b**2) - 1712*c**2*d**2*sin(a + b*x)**5/(1125*b**3) - 676*c**2*d**2*sin(a
+ b*x)**3*cos(a + b*x)**2/(225*b**3) - 104*c**2*d**2*sin(a + b*x)*cos(a + b
*x)**4/(75*b**3) - 3424*c*d**3*x*sin(a + b*x)**5/(1125*b**3) - 1352*c*d**3*
x*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 208*c*d**3*x*sin(a + b*x)*co
s(a + b*x)**4/(75*b**3) - 1712*d**4*x**2*sin(a + b*x)**5/(1125*b**3) - 676*
d**4*x**2*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 104*d**4*x**2*sin(a
+ b*x)*cos(a + b*x)**4/(75*b**3) - 3424*c*d**3*sin(a + b*x)**4*cos(a + b*x)
/(1125*b**4) - 20456*c*d**3*sin(a + b*x)**2*cos(a + b*x)**3/(3375*b**4) - 5
0272*c*d**3*cos(a + b*x)**5/(16875*b**4) - 3424*d**4*x*sin(a + b*x)**4*cos(
a + b*x)/(1125*b**4) - 20456*d**4*x*sin(a + b*x)**2*cos(a + b*x)**3/(3375*b
**4) - 50272*d**4*x*cos(a + b*x)**5/(16875*b**4) + 760816*d**4*sin(a + b*x)
**5/(253125*b**5) + 303368*d**4*sin(a + b*x)**3*cos(a + b*x)**2/(50625*b**5
) + 50272*d**4*sin(a + b*x)*cos(a + b*x)**4/(16875*b**5), Ne(b, 0)), ((c**4
*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**
2*cos(a)**3, True))
```

Giac [A]
time = 0.51, size = 531, normalized size = 1.61

$$-\frac{\left(25\,b^3d^4x^3+75\,b^3cd^3x^2+75\,b^3c^2d^2x+25\,b^3c^3d-6\,bd^4x-6\,bcd^3\right)\cos\left(5\,bx+5\,a\right)}{2500\,b^5}-\frac{\left(3\,b^3d^4x^3+9\,b^3cd^3x^2+9\,b^3c^2d^2x+3\,b^3c^3d-2\,bd^4x-2\,bcd^3\right)\cos\left(3\,bx+3\,a\right)}{108\,b^5}+\frac{\left(b^3d^4x^3+3\,b^3cd^3x^2+3\,b^3c^2d^2x+b^3c^3d-6\,bd^4x-6\,bcd^3\right)\cos\left(bx+a\right)}{2\,b^5}-\frac{\left(625\,b^4d^4x^4+2500\,b^4cd^3x^3+3750\,b^4c^2d^2x^2+2500\,b^4c^3dx+625\,b^4c^4-300\,b^2d^4x^2-600\,b^2cd^3x-300\,b^2c^2d^2+24\,d^4\right)\sin\left(5\,bx+5\,a\right)}{50000\,b^5}-\frac{\left(27\,b^4d^4x^4+108\,b^4cd^3x^3+162\,b^4c^2d^2x^2+108\,b^4c^3dx+27\,b^4c^4-36\,b^2d^4x^2-72\,b^2cd^3x-36\,b^2c^2d^2+8\,d^4\right)\sin\left(3\,bx+3\,a\right)}{1296\,b^5}+\frac{\left(b^4d^4x^4+4\,b^4cd^3x^3+6\,b^4c^2d^2x^2+4\,b^4c^3dx+b^4c^4-12\,b^2d^4x^2-24\,b^2cd^3x-12\,b^2c^2d^2+24\,d^4\right)\sin\left(bx+a\right)}{8\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2500*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 75*b^3*c^2*d^2*x + 25*b^3*c^3*
d - 6*b*d^4*x - 6*b*c*d^3)*cos(5*b*x + 5*a)/b^5 - 1/108*(3*b^3*d^4*x^3 + 9*
b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*cos(
3*b*x + 3*a)/b^5 + 1/2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b
```

```
^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*cos(b*x + a)/b^5 - 1/50000*(625*b^4*d^4*x
^4 + 2500*b^4*c*d^3*x^3 + 3750*b^4*c^2*d^2*x^2 + 2500*b^4*c^3*d*x + 625*b^4
*c^4 - 300*b^2*d^4*x^2 - 600*b^2*c*d^3*x - 300*b^2*c^2*d^2 + 24*d^4)*sin(5*
b*x + 5*a)/b^5 - 1/1296*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d
^2*x^2 + 108*b^4*c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 3
6*b^2*c^2*d^2 + 8*d^4)*sin(3*b*x + 3*a)/b^5 + 1/8*(b^4*d^4*x^4 + 4*b^4*c*d^
3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b
^2*c*d^3*x - 12*b^2*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

Mupad [B]
time = 4.38, size = 816, normalized size = 2.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^4,x)
```

```
[Out] -((d^4*sin(3*a + 3*b*x))/162 - 3*d^4*sin(a + b*x) + (3*d^4*sin(5*a + 5*b*x)
)/6250 - (b^4*c^4*sin(a + b*x))/8 + (b^4*c^4*sin(3*a + 3*b*x))/48 + (b^4*c^
4*sin(5*a + 5*b*x))/80 + (b^3*c^3*d*cos(3*a + 3*b*x))/36 + (b^3*c^3*d*cos(5
*a + 5*b*x))/100 + (3*b^2*c^2*d^2*sin(a + b*x))/2 - (b^3*d^4*x^3*cos(a + b*
x))/2 + (3*b^2*d^4*x^2*sin(a + b*x))/2 - (b^4*d^4*x^4*sin(a + b*x))/8 + 3*b
*c*d^3*cos(a + b*x) + 3*b*d^4*x*cos(a + b*x) - (b^2*c^2*d^2*sin(3*a + 3*b*x
))/36 - (3*b^2*c^2*d^2*sin(5*a + 5*b*x))/500 + (b^3*d^4*x^3*cos(3*a + 3*b*x
))/36 + (b^3*d^4*x^3*cos(5*a + 5*b*x))/100 - (b^2*d^4*x^2*sin(3*a + 3*b*x))
/36 - (3*b^2*d^4*x^2*sin(5*a + 5*b*x))/500 + (b^4*d^4*x^4*sin(3*a + 3*b*x))
/48 + (b^4*d^4*x^4*sin(5*a + 5*b*x))/80 - (b*c*d^3*cos(3*a + 3*b*x))/54 - (
3*b*c*d^3*cos(5*a + 5*b*x))/1250 - (b^3*c^3*d*cos(a + b*x))/2 - (b*d^4*x*co
s(3*a + 3*b*x))/54 - (3*b*d^4*x*cos(5*a + 5*b*x))/1250 + 3*b^2*c*d^3*x*sin(
a + b*x) - (b^4*c^3*d*x*sin(a + b*x))/2 + (b^4*c^2*d^2*x^2*sin(3*a + 3*b*x)
)/8 + (3*b^4*c^2*d^2*x^2*sin(5*a + 5*b*x))/40 - (3*b^3*c^2*d^2*x*cos(a + b*
x))/2 - (3*b^3*c*d^3*x^2*cos(a + b*x))/2 - (b^2*c*d^3*x*sin(3*a + 3*b*x))/1
8 + (b^4*c^3*d*x*sin(3*a + 3*b*x))/12 - (3*b^2*c*d^3*x*sin(5*a + 5*b*x))/25
0 + (b^4*c^3*d*x*sin(5*a + 5*b*x))/20 - (b^4*c*d^3*x^3*sin(a + b*x))/2 + (b
^3*c^2*d^2*x*cos(3*a + 3*b*x))/12 + (b^3*c*d^3*x^2*cos(3*a + 3*b*x))/12 + (
3*b^3*c^2*d^2*x*cos(5*a + 5*b*x))/100 + (3*b^3*c*d^3*x^2*cos(5*a + 5*b*x))/
100 + (b^4*c*d^3*x^3*sin(3*a + 3*b*x))/12 + (b^4*c*d^3*x^3*sin(5*a + 5*b*x)
)/20 - (3*b^4*c^2*d^2*x^2*sin(a + b*x))/4)/b^5
```

3.147 $\int (c+dx)^3 \cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=259

$$-\frac{3d^3\cos(a+bx)}{4b^4}+\frac{3d(c+dx)^2\cos(a+bx)}{8b^2}+\frac{d^3\cos(3a+3bx)}{216b^4}-\frac{d(c+dx)^2\cos(3a+3bx)}{48b^2}+\frac{3d^3\cos(5a+5bx)}{5000b^4}$$

```
[Out] -3/4*d^3*cos(b*x+a)/b^4+3/8*d*(d*x+c)^2*cos(b*x+a)/b^2+1/216*d^3*cos(3*b*x+
3*a)/b^4-1/48*d*(d*x+c)^2*cos(3*b*x+3*a)/b^2+3/5000*d^3*cos(5*b*x+5*a)/b^4-
3/400*d*(d*x+c)^2*cos(5*b*x+5*a)/b^2-3/4*d^2*(d*x+c)*sin(b*x+a)/b^3+1/8*(d*
x+c)^3*sin(b*x+a)/b+1/72*d^2*(d*x+c)*sin(3*b*x+3*a)/b^3-1/48*(d*x+c)^3*sin(
3*b*x+3*a)/b+3/1000*d^2*(d*x+c)*sin(5*b*x+5*a)/b^3-1/80*(d*x+c)^3*sin(5*b*x
+5*a)/b
```

Rubi [A]
time = 0.21, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4491, 3377, 2718}

$$-\frac{3d^3\cos(a+bx)}{4b^4}+\frac{d^3\cos(3a+3bx)}{216b^4}+\frac{3d^3\cos(5a+5bx)}{5000b^4}-\frac{3d^2(c+dx)\sin(a+bx)}{4b^3}+\frac{d^2(c+dx)\sin(3a+3bx)}{72b^3}+\frac{3d^2(c+dx)\sin(5a+5bx)}{1000b^3}+\frac{3d(c+dx)^2\cos(a+bx)}{8b^2}-\frac{d(c+dx)^2\cos(3a+3bx)}{48b^2}-\frac{3d(c+dx)^2\cos(5a+5bx)}{400b^2}+\frac{(c+dx)^3\sin(a+bx)}{8b}-\frac{(c+dx)^3\sin(3a+3bx)}{48b}-\frac{(c+dx)^3\sin(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (-3*d^3*Cos[a + b*x])/(4*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x])/(8*b^2) + (d
^3*Cos[3*a + 3*b*x])/(216*b^4) - (d*(c + d*x)^2*Cos[3*a + 3*b*x])/(48*b^2)
+ (3*d^3*Cos[5*a + 5*b*x])/(5000*b^4) - (3*d*(c + d*x)^2*Cos[5*a + 5*b*x])/
(400*b^2) - (3*d^2*(c + d*x)*Sin[a + b*x])/(4*b^3) + ((c + d*x)^3*Sin[a + b
*x])/(8*b) + (d^2*(c + d*x)*Sin[3*a + 3*b*x])/(72*b^3) - ((c + d*x)^3*Sin[3
*a + 3*b*x])/(48*b) + (3*d^2*(c + d*x)*Sin[5*a + 5*b*x])/(1000*b^3) - ((c +
 d*x)^3*Sin[5*a + 5*b*x])/(80*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^3\cos^3(a+bx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)^3\cos(a+bx)-\frac{1}{16}(c+dx)^3\cos(3a+3bx)-\frac{1}{16}(c \\
&= -\left(\frac{1}{16}\int (c+dx)^3\cos(3a+3bx)\,dx\right)-\frac{1}{16}\int (c+dx)^3\cos(5a+ \\
&= \frac{(c+dx)^3\sin(a+bx)}{8b}-\frac{(c+dx)^3\sin(3a+3bx)}{48b}-\frac{(c+dx)^3\sin}{80b} \\
&= \frac{3d(c+dx)^2\cos(a+bx)}{8b^2}-\frac{d(c+dx)^2\cos(3a+3bx)}{48b^2}-\frac{3d(c+d}{} \\
&= \frac{3d(c+dx)^2\cos(a+bx)}{8b^2}-\frac{d(c+dx)^2\cos(3a+3bx)}{48b^2}-\frac{3d(c+d}{} \\
&= -\frac{3d^3\cos(a+bx)}{4b^4}+\frac{3d(c+dx)^2\cos(a+bx)}{8b^2}+\frac{d^3\cos(3a+3bx)}{216b^4}
\end{aligned}
$$

Mathematica [A]
time = 2.35, size = 195, normalized size = 0.75

$$
-\frac{-101250d(-2d^2+b^2(c+dx)^2)\cos(a+bx)+625d(-2d^2+9b^2(c+dx)^2)\cos(3(a+bx))+81d(-2d^2+25b^2(c+dx)^2)\cos(5(a+bx))+30b(c+dx)\left(-825b^2c^2+6598d^2-1650b^2cdx-825b^2d^2x^2+8(-38d^2+75b^2(c+dx)^2)\cos(2(a+bx))+9(-6d^2+25b^2(c+dx)^2)\cos(4(a+bx))\right)\sin(a+bx)}{270000b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -1/270000*(-101250*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + 625*d*(-2*d^
2 + 9*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 81*d*(-2*d^2 + 25*b^2*(c + d*x)^2
)*Cos[5*(a + b*x)] + 30*b*(c + d*x)*(-825*b^2*c^2 + 6598*d^2 - 1650*b^2*c*d
*x - 825*b^2*d^2*x^2 + 8*(-38*d^2 + 75*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] +
9*(-6*d^2 + 25*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[a + b*x])/b^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(235) = 470$.
time = 0.25, size = 1016, normalized size = 3.92

method	result
risch	$\frac{3d(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\cos(bx+a)}{8b^4}+\frac{(b^2d^3x^3+3b^2c\,d^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6d^2c)\sin(bx+a)}{8b^3}-\frac{3d(25a}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b^3*a^3*d^3*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)*sin
(b*x+a))+3/b^2*a^2*c*d^2*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2
)*sin(b*x+a))+3/b^3*a^2*d^3*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*c
os(b*x+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)
*sin(b*x+a)-1/25*cos(b*x+a)^5)-3/b*a*c^2*d*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/
15*(2+cos(b*x+a)^2)*sin(b*x+a))-6/b^2*a*c*d^2*(1/3*(b*x+a)*(2+cos(b*x+a)^2)
*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4
+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)-3/b^3*a*d^3*(1/3*(b*x+a)^2
*(2+cos(b*x+a)^2)*sin(b*x+a)-4/15*sin(b*x+a)+4/15*(b*x+a)*cos(b*x+a)+2/45*(
b*x+a)*cos(b*x+a)^3-2/135*(2+cos(b*x+a)^2)*sin(b*x+a)-1/5*(b*x+a)^2*(8/3+co
s(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-2/25*(b*x+a)*cos(b*x+a)^5+2/125*(8/
3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a))+c^3*(-1/5*sin(b*x+a)*cos(b*x+a
)^4+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))+3/b*c^2*d*(1/3*(b*x+a)*(2+cos(b*x+a)^
2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)
^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)+3/b^2*c*d^2*(1/3*(b*x+a)
^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/15*sin(b*x+a)+4/15*(b*x+a)*cos(b*x+a)+2/45
*(b*x+a)*cos(b*x+a)^3-2/135*(2+cos(b*x+a)^2)*sin(b*x+a)-1/5*(b*x+a)^2*(8/3+
cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-2/25*(b*x+a)*cos(b*x+a)^5+2/125*(
8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a))+1/b^3*d^3*(1/3*(b*x+a)^3*(2+
cos(b*x+a)^2)*sin(b*x+a)+2/5*(b*x+a)^2*cos(b*x+a)-856/1125*cos(b*x+a)-4/5*(
b*x+a)*sin(b*x+a)+1/15*(b*x+a)^2*cos(b*x+a)^3-2/45*(b*x+a)*(2+cos(b*x+a)^2)
*sin(b*x+a)+22/3375*cos(b*x+a)^3-1/5*(b*x+a)^3*(8/3+cos(b*x+a)^4+4/3*cos(b*
x+a)^2)*sin(b*x+a)-3/25*(b*x+a)^2*cos(b*x+a)^5+6/125*(b*x+a)*(8/3+cos(b*x+a
)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)+6/625*cos(b*x+a)^5))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(235) = 470$.
time = 0.29, size = 766, normalized size = 2.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/270000*(18000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c^3 - 54000*(3*sin(b
*x + a)^5 - 5*sin(b*x + a)^3)*a*c^2*d/b + 54000*(3*sin(b*x + a)^5 - 5*sin(b
*x + a)^3)*a^2*c*d^2/b^2 - 18000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*a^3*
d^3/b^3 + 225*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a
) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) -
 450*cos(b*x + a))*c^2*d/b - 450*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x +
```

```
 a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25
*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a*c*d^2/b^2 + 225*(45*(b*x + a)*sin(5
*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) +
9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a^2*d^3/b^3 +
15*(270*(b*x + a)*cos(5*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500
*(b*x + a)*cos(b*x + a) + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9
*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*c
*d^2/b^2 - 15*(270*(b*x + a)*cos(5*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3
*a) - 13500*(b*x + a)*cos(b*x + a) + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*
a) + 125*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(
b*x + a))*a*d^3/b^3 + (81*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) + 625*(9*(b
*x + a)^2 - 2)*cos(3*b*x + 3*a) - 101250*((b*x + a)^2 - 2)*cos(b*x + a) + 1
35*(25*(b*x + a)^3 - 6*b*x - 6*a)*sin(5*b*x + 5*a) + 1875*(3*(b*x + a)^3 -
2*b*x - 2*a)*sin(3*b*x + 3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x +
 a))*d^3/b^3)/b
```

Fricas [**A**]
time = 2.95, size = 342, normalized size = 1.32

$$-\frac{81\,(25\,b^2d^3x^2+50\,b^2cd^2x+25\,b^2c^2d-2\,d^3)\cos(bx+a)^5-5\,(225\,b^2d^3x^2+450\,b^2cd^2x+225\,b^2c^2d+22\,d^3)\cos(bx+a)^3-30\,(225\,b^2d^3x^2+450\,b^2cd^2x+225\,b^2c^2d-428\,d^3)\cos(bx+a)-15\,(150\,b^3d^3x^3+450\,b^3cd^2x^2+150\,b^3c^3-9\,(25\,b^3d^3x^3+75\,b^3cd^2x^2+25\,b^3c^3-6\,bcd^2+3\,(25\,b^3c^2d-2\,bd^3)x)\cos(bx+a)^4-856\,bcd^2+(75\,b^3d^3x^3+225\,b^3cd^2x^2+75\,b^3c^3+22\,bcd^2+(225\,b^3c^2d+22\,bd^3)x)\cos(bx+a)^2+2\,(225\,b^3c^2d-428\,bd^3)x)\sin(bx+a)}{16875\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16875*(81*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(b
*x + a)^5 - 5*(225*b^2*d^3*x^2 + 450*b^2*c*d^2*x + 225*b^2*c^2*d + 22*d^3)*
cos(b*x + a)^3 - 30*(225*b^2*d^3*x^2 + 450*b^2*c*d^2*x + 225*b^2*c^2*d - 42
8*d^3)*cos(b*x + a) - 15*(150*b^3*d^3*x^3 + 450*b^3*c*d^2*x^2 + 150*b^3*c^3
 - 9*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 25*b^3*c^3 - 6*b*c*d^2 + 3*(25*b^
3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^4 - 856*b*c*d^2 + (75*b^3*d^3*x^3 + 225*
b^3*c*d^2*x^2 + 75*b^3*c^3 + 22*b*c*d^2 + (225*b^3*c^2*d + 22*b*d^3)*x)*cos
(b*x + a)^2 + 2*(225*b^3*c^2*d - 428*b*d^3)*x)*sin(b*x + a))/b^4
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(255) = 510$.
time = 1.08, size = 690, normalized size = 2.66

$$\begin{cases} \frac{2c^3\sin^5(a+bx)}{15b}+\frac{c^3\sin^3(a+bx)\cos^2(a+bx)}{3b}+\frac{2c^2dx\sin^5(a+bx)}{5b}+\frac{c^2dx\sin^3(a+bx)\cos^2(a+bx)}{b}+\frac{2cd^2x^2\sin^5(a+bx)}{5b}+\frac{cd^2x^2\sin^3(a+bx)\cos^2(a+bx)}{b}+\frac{2d^3x^3\sin^5(a+bx)}{15b}+\frac{d^3x^3\sin^3(a+bx)\cos^2(a+bx)}{3b}+\frac{2c^2d\sin^4(a+bx)\cos(a+bx)}{5b^2}+\frac{13c^2d\sin^2(a+bx)\cos^3(a+bx)}{15b^2}+\frac{26c^2d\cos^5(a+bx)}{75b^2}+\frac{4cd^2x\sin^4(a+bx)\cos(a+bx)}{5b^2}+\frac{26cd^2x\sin^2(a+bx)\cos^3(a+bx)}{15b^2}+\frac{52cd^2x\cos^5(a+bx)}{75b^2}+\frac{2d^3x^2\sin^4(a+bx)\cos(a+bx)}{5b^2}+\frac{13d^3x^2\sin^2(a+bx)\cos^3(a+bx)}{15b^2}+\frac{26d^3x^2\cos^5(a+bx)}{75b^2}-\frac{856cd^2\sin^5(a+bx)}{1125b^3}-\frac{338cd^2\sin^3(a+bx)\cos^2(a+bx)}{225b^3}-\frac{52cd^2\sin(a+bx)\cos^4(a+bx)}{75b^3}-\frac{856d^3x\sin^5(a+bx)}{1125b^3}-\frac{338d^3x\sin^3(a+bx)\cos^2(a+bx)}{225b^3}-\frac{52d^3x\sin(a+bx)\cos^4(a+bx)}{75b^3}-\frac{856d^3\sin^4(a+bx)\cos(a+bx)}{1125b^4}-\frac{5114d^3\sin^2(a+bx)\cos^3(a+bx)}{3375b^4}-\frac{12568d^3\cos^5(a+bx)}{16875b^4} & \text{for}\ b \neq 0 \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sin^2(a)\cos^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((2*c**3*sin(a + b*x)**5/(15*b) + c**3*sin(a + b*x)**3*cos(a + b*x
)**2/(3*b) + 2*c**2*d*x*sin(a + b*x)**5/(5*b) + c**2*d*x*sin(a + b*x)**3*co
s(a + b*x)**2/b + 2*c*d**2*x**2*sin(a + b*x)**5/(5*b) + c*d**2*x**2*sin(a +
 b*x)**3*cos(a + b*x)**2/b + 2*d**3*x**3*sin(a + b*x)**5/(15*b) + d**3*x**3
```

```
*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 2*c**2*d*sin(a + b*x)**4*cos(a + b
*x)/(5*b**2) + 13*c**2*d*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 26*c**
2*d*cos(a + b*x)**5/(75*b**2) + 4*c*d**2*x*sin(a + b*x)**4*cos(a + b*x)/(5*
b**2) + 26*c*d**2*x*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 52*c*d**2*x
*cos(a + b*x)**5/(75*b**2) + 2*d**3*x**2*sin(a + b*x)**4*cos(a + b*x)/(5*b*
*2) + 13*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 26*d**3*x**2
*cos(a + b*x)**5/(75*b**2) - 856*c*d**2*sin(a + b*x)**5/(1125*b**3) - 338*c
*d**2*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 52*c*d**2*sin(a + b*x)*c
os(a + b*x)**4/(75*b**3) - 856*d**3*x*sin(a + b*x)**5/(1125*b**3) - 338*d**
3*x*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 52*d**3*x*sin(a + b*x)*cos
(a + b*x)**4/(75*b**3) - 856*d**3*sin(a + b*x)**4*cos(a + b*x)/(1125*b**4)
- 5114*d**3*sin(a + b*x)**2*cos(a + b*x)**3/(3375*b**4) - 12568*d**3*cos(a
+ b*x)**5/(16875*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3
 + d**3*x**4/4)*sin(a)**2*cos(a)**3, True))
```

Giac [A]
time = 0.48, size = 351, normalized size = 1.36

$$-\frac{3\,(25\,b^2d^3x^2+50\,b^2cd^2x+25\,b^2c^2d-2\,d^3)\cos(5\,bx+5\,a)}{10000\,b^4}-\frac{(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3)\cos(3\,bx+3\,a)}{432\,b^4}+\frac{3\,(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-2\,d^3)\cos(bx+a)}{8\,b^4}-\frac{(25\,b^3d^3x^3+75\,b^3cd^2x^2+75\,b^3c^2dx+25\,b^3c^3-6\,bd^3x-6\,bcd^2)\sin(5\,bx+5\,a)}{2000\,b^4}-\frac{(3\,b^3d^3x^3+9\,b^3cd^2x^2+9\,b^3c^2dx+3\,b^3c^3-2\,bd^3x-2\,bcd^2)\sin(3\,bx+3\,a)}{144\,b^4}+\frac{(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3-6\,bd^3x-6\,bcd^2)\sin(bx+a)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -3/10000*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(5*b*x
 + 5*a)/b^4 - 1/432*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*
cos(3*b*x + 3*a)/b^4 + 3/8*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3
)*cos(b*x + a)/b^4 - 1/2000*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 75*b^3*c^2
*d*x + 25*b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(5*b*x + 5*a)/b^4 - 1/144*(3*
b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^3*x - 2*b
*c*d^2)*sin(3*b*x + 3*a)/b^4 + 1/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c
^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x + a)/b^4
```

Mupad [B]
time = 2.43, size = 516, normalized size = 1.99

$$-\frac{\frac{3d^3\cos(a+bx)}{4}-\frac{d^3\cos(3a+3bx)}{216}-\frac{3d^3\cos(5a+5bx)}{5000}-\frac{b^3c^3\sin(a+bx)}{8}+\frac{b^3c^3\sin(3a+3bx)}{48}+\frac{b^3c^3\sin(5a+5bx)}{80}+\frac{b^2c^2d\cos(3a+3bx)}{48}+\frac{3b^2c^2d\cos(5a+5bx)}{400}-\frac{3b^2d^3x^2\cos(a+bx)}{8}-\frac{b^3d^3x^3\sin(a+bx)}{8}+\frac{3bcd^2\sin(a+bx)}{4}+\frac{3bd^3x\sin(a+bx)}{4}+\frac{b^2d^3x^2\cos(3a+3bx)}{48}+\frac{3b^2d^3x^2\cos(5a+5bx)}{400}+\frac{b^3d^3x^3\sin(3a+3bx)}{48}+\frac{b^3d^3x^3\sin(5a+5bx)}{80}-\frac{3b^2c^2d\cos(a+bx)}{8}-\frac{bcd^2\sin(3a+3bx)}{72}-\frac{3bcd^2\sin(5a+5bx)}{1000}-\frac{bd^3x\sin(3a+3bx)}{72}-\frac{3bd^3x\sin(5a+5bx)}{1000}-\frac{3b^2cd^2x\cos(a+bx)}{4}-\frac{3b^3c^2dx\sin(a+bx)}{8}+\frac{b^2cd^2x\cos(3a+3bx)}{24}+\frac{3b^2cd^2x\cos(5a+5bx)}{200}+\frac{b^3c^2dx\sin(3a+3bx)}{16}+\frac{3b^3c^2dx\sin(5a+5bx)}{80}-\frac{3b^3cd^2x^2\sin(a+bx)}{8}+\frac{b^3cd^2x^2\sin(3a+3bx)}{16}+\frac{3b^3cd^2x^2\sin(5a+5bx)}{80}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] -((3*d^3*cos(a + b*x))/4 - (d^3*cos(3*a + 3*b*x))/216 - (3*d^3*cos(5*a + 5*
b*x))/5000 - (b^3*c^3*sin(a + b*x))/8 + (b^3*c^3*sin(3*a + 3*b*x))/48 + (b^
3*c^3*sin(5*a + 5*b*x))/80 + (b^2*c^2*d*cos(3*a + 3*b*x))/48 + (3*b^2*c^2*d
*cos(5*a + 5*b*x))/400 - (3*b^2*d^3*x^2*cos(a + b*x))/8 - (b^3*d^3*x^3*sin(
a + b*x))/8 + (3*b*c*d^2*sin(a + b*x))/4 + (3*b*d^3*x*sin(a + b*x))/4 + (b^
2*d^3*x^2*cos(3*a + 3*b*x))/48 + (3*b^2*d^3*x^2*cos(5*a + 5*b*x))/400 + (b^
```

```
3*d^3*x^3*sin(3*a + 3*b*x))/48 + (b^3*d^3*x^3*sin(5*a + 5*b*x))/80 - (3*b^2
*c^2*d*cos(a + b*x))/8 - (b*c*d^2*sin(3*a + 3*b*x))/72 - (3*b*c*d^2*sin(5*a
 + 5*b*x))/1000 - (b*d^3*x*sin(3*a + 3*b*x))/72 - (3*b*d^3*x*sin(5*a + 5*b*
x))/1000 - (3*b^2*c*d^2*x*cos(a + b*x))/4 - (3*b^3*c^2*d*x*sin(a + b*x))/8
+ (b^2*c*d^2*x*cos(3*a + 3*b*x))/24 + (3*b^2*c*d^2*x*cos(5*a + 5*b*x))/200
+ (b^3*c^2*d*x*sin(3*a + 3*b*x))/16 + (3*b^3*c^2*d*x*sin(5*a + 5*b*x))/80 -
 (3*b^3*c*d^2*x^2*sin(a + b*x))/8 + (b^3*c*d^2*x^2*sin(3*a + 3*b*x))/16 + (
3*b^3*c*d^2*x^2*sin(5*a + 5*b*x))/80)/b^4
```

3.148 $\int (c+dx)^2 \cos^3(a+bx) \sin^2(a+bx)\, dx$

Optimal. Leaf size=184

$$\frac{d(c+dx)\cos(a+bx)}{4b^2}-\frac{d(c+dx)\cos(3a+3bx)}{72b^2}-\frac{d(c+dx)\cos(5a+5bx)}{200b^2}-\frac{d^2\sin(a+bx)}{4b^3}+\frac{(c+dx)^2\sin(a+}{8b}$$

```
[Out] 1/4*d*(d*x+c)*cos(b*x+a)/b^2-1/72*d*(d*x+c)*cos(3*b*x+3*a)/b^2-1/200*d*(d*x
+c)*cos(5*b*x+5*a)/b^2-1/4*d^2*sin(b*x+a)/b^3+1/8*(d*x+c)^2*sin(b*x+a)/b+1/
216*d^2*sin(3*b*x+3*a)/b^3-1/48*(d*x+c)^2*sin(3*b*x+3*a)/b+1/1000*d^2*sin(5
*b*x+5*a)/b^3-1/80*(d*x+c)^2*sin(5*b*x+5*a)/b
```

Rubi [A]
time = 0.14, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4491, 3377, 2717}

$$-\frac{d^2\sin(a+bx)}{4b^3}+\frac{d^2\sin(3a+3bx)}{216b^3}+\frac{d^2\sin(5a+5bx)}{1000b^3}+\frac{d(c+dx)\cos(a+bx)}{4b^2}-\frac{d(c+dx)\cos(3a+3bx)}{72b^2}-\frac{d(c+dx)\cos(5a+5bx)}{200b^2}+\frac{(c+dx)^2\sin(a+bx)}{8b}-\frac{(c+dx)^2\sin(3a+3bx)}{48b}-\frac{(c+dx)^2\sin(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (d*(c + d*x)*Cos[a + b*x])/(4*b^2) - (d*(c + d*x)*Cos[3*a + 3*b*x])/(72*b^2
) - (d*(c + d*x)*Cos[5*a + 5*b*x])/(200*b^2) - (d^2*Sin[a + b*x])/(4*b^3) +
 ((c + d*x)^2*Sin[a + b*x])/(8*b) + (d^2*Sin[3*a + 3*b*x])/(216*b^3) - ((c
+ d*x)^2*Sin[3*a + 3*b*x])/(48*b) + (d^2*Sin[5*a + 5*b*x])/(1000*b^3) - ((c
 + d*x)^2*Sin[5*a + 5*b*x])/(80*b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^2 \cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^2\cos(a+bx) - \frac{1}{16}(c+dx)^2\cos(3a+3bx) - \frac{1}{16}(c$$

$$= -\left(\frac{1}{16}\int (c+dx)^2\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^2\cos(5a+$$

$$= \frac{(c+dx)^2\sin(a+bx)}{8b} - \frac{(c+dx)^2\sin(3a+3bx)}{48b} - \frac{(c+dx)^2\sin}{80b}$$

$$= \frac{d(c+dx)\cos(a+bx)}{4b^2} - \frac{d(c+dx)\cos(3a+3bx)}{72b^2} - \frac{d(c+dx)\cos}{200}$$

$$= \frac{d(c+dx)\cos(a+bx)}{4b^2} - \frac{d(c+dx)\cos(3a+3bx)}{72b^2} - \frac{d(c+dx)\cos}{200}$$

Mathematica [**A**]
time = 1.03, size = 252, normalized size = 1.37

$$-\frac{-13500bd(c+dx)\cos(a+bx)+750bd(c+dx)\cos(3(a+bx))+270bcd\cos(5(a+bx))+270bd^2x\cos(5(a+bx))-6750b^2c^2\sin(a+bx)+13500d^2\sin(a+bx)-13500b^2cdx\sin(a+bx)-6750b^2d^2x^2\sin(a+bx)+1125b^2c^2\sin(3(a+bx))-250d^2\sin(3(a+bx))+2250b^2cdx\sin(3(a+bx))+1125b^2d^2x^2\sin(3(a+bx))+675b^2c^2\sin(5(a+bx))-54d^2\sin(5(a+bx))+1350b^2cdx\sin(5(a+bx))+675b^2d^2x^2\sin(5(a+bx))}{54000b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -1/54000*(-13500*b*d*(c + d*x)*Cos[a + b*x] + 750*b*d*(c + d*x)*Cos[3*(a +
b*x)] + 270*b*c*d*Cos[5*(a + b*x)] + 270*b*d^2*x*Cos[5*(a + b*x)] - 6750*b^
2*c^2*Sin[a + b*x] + 13500*d^2*Sin[a + b*x] - 13500*b^2*c*d*x*Sin[a + b*x]
- 6750*b^2*d^2*x^2*Sin[a + b*x] + 1125*b^2*c^2*Sin[3*(a + b*x)] - 250*d^2*S
in[3*(a + b*x)] + 2250*b^2*c*d*x*Sin[3*(a + b*x)] + 1125*b^2*d^2*x^2*Sin[3*
(a + b*x)] + 675*b^2*c^2*Sin[5*(a + b*x)] - 54*d^2*Sin[5*(a + b*x)] + 1350*
b^2*c*d*x*Sin[5*(a + b*x)] + 675*b^2*d^2*x^2*Sin[5*(a + b*x)])/b^3
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(166) = 332$.
time = 0.18, size = 484, normalized size = 2.63 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/b^2*a^2*d^2*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)*sin(
b*x+a))-2/b*a*c*d*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)*sin(b
*x+a))-2/b^2*a*d^2*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)
^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+
a)-1/25*cos(b*x+a)^5)+c^2*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^
2)*sin(b*x+a))+2/b*c*d*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*
x+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(
b*x+a)-1/25*cos(b*x+a)^5)+1/b^2*d^2*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x
```

```
+a)-4/15*sin(b*x+a)+4/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/135
*(2+cos(b*x+a)^2)*sin(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)
^2)*sin(b*x+a)-2/25*(b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(b*
x+a)^2)*sin(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(166) = 332$.
time = 0.29, size = 375, normalized size = 2.04

$$-\frac{3600\left(3\sin\left(bx+a\right)^{5}-5\sin\left(bx+a\right)^{3}\right)c^{2}-\frac{7200\left(3\sin(bx+a)^{5}-5\sin(bx+a)^{3}\right)acd}{b}+\frac{3600\left(3\sin(bx+a)^{5}-5\sin(bx+a)^{3}\right)a^{2}d^{2}}{b^{2}}+\frac{30\left(45\left(bx+a\right)\sin(5bx+5a)+75\left(bx+a\right)\sin(3bx+3a)-450\left(bx+a\right)\sin(bx+a)+9\cos(5bx+5a)+25\cos(3bx+3a)-450\cos(bx+a)\right)cd}{b}-\frac{30\left(45\left(bx+a\right)\sin(5bx+5a)+75\left(bx+a\right)\sin(3bx+3a)-450\left(bx+a\right)\sin(bx+a)+9\cos(5bx+5a)+25\cos(3bx+3a)-450\cos(bx+a)\right)ad^{2}}{b^{2}}+\frac{\left(270\left(bx+a\right)\cos(5bx+5a)+750\left(bx+a\right)\cos(3bx+3a)-13500\left(bx+a\right)\cos(bx+a)+27\left(25\left(bx+a\right)^{2}-2\right)\sin(5bx+5a)+125\left(9\left(bx+a\right)^{2}-2\right)\sin(3bx+3a)-6750\left(\left(bx+a\right)^{2}-2\right)\sin(bx+a)\right)d^{2}}{b^{2}}}{54000\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/54000*(3600*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c^2 - 7200*(3*sin(b*x
+ a)^5 - 5*sin(b*x + a)^3)*a*c*d/b + 3600*(3*sin(b*x + a)^5 - 5*sin(b*x + a
)^3)*a^2*d^2/b^2 + 30*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b
*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x
+ 3*a) - 450*cos(b*x + a))*c*d/b - 30*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(
b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a)
 + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a*d^2/b^2 + (270*(b*x + a)*cos(5
*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*x + a)
 + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)*sin(3
*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*d^2/b^2)/b
```

Fricas [A]
time = 2.37, size = 193, normalized size = 1.05

$$-\frac{270\left(bd^{2}x+bcd\right)\cos\left(bx+a\right)^{5}-150\left(bd^{2}x+bcd\right)\cos\left(bx+a\right)^{3}-900\left(bd^{2}x+bcd\right)\cos\left(bx+a\right)-\left(450\,b^{2}d^{2}x^{2}+900\,b^{2}cdx-27\left(25\,b^{2}d^{2}x^{2}+50\,b^{2}cdx+25\,b^{2}c^{2}-2\,d^{2}\right)\cos\left(bx+a\right)^{4}+450\,b^{2}c^{2}+\left(225\,b^{2}d^{2}x^{2}+450\,b^{2}cdx+225\,b^{2}c^{2}+22\,d^{2}\right)\cos\left(bx+a\right)^{2}-856\,d^{2}\right)\sin\left(bx+a\right)}{3375\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3375*(270*(b*d^2*x + b*c*d)*cos(b*x + a)^5 - 150*(b*d^2*x + b*c*d)*cos(b
*x + a)^3 - 900*(b*d^2*x + b*c*d)*cos(b*x + a) - (450*b^2*d^2*x^2 + 900*b^2
*c*d*x - 27*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(b*x +
a)^4 + 450*b^2*c^2 + (225*b^2*d^2*x^2 + 450*b^2*c*d*x + 225*b^2*c^2 + 22*d^
2)*cos(b*x + a)^2 - 856*d^2)*sin(b*x + a))/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(172) = 344$.
time = 0.70, size = 382, normalized size = 2.08

$$\begin{cases} \frac{2c^{2}\sin^{5}\left(a+bx\right)}{15b}+\frac{c^{2}\sin^{3}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{3b}+\frac{4cdx\sin^{5}\left(a+bx\right)}{15b}+\frac{2cdx\sin^{3}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{3b}+\frac{2d^{2}x^{2}\sin^{5}\left(a+bx\right)}{15b}+\frac{d^{2}x^{2}\sin^{3}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{3b}+\frac{4cd\sin^{4}\left(a+bx\right)\cos\left(a+bx\right)}{15b^{2}}+\frac{26cd\sin^{2}\left(a+bx\right)\cos^{3}\left(a+bx\right)}{45b^{2}}+\frac{52cd\cos^{5}\left(a+bx\right)}{225b^{2}}+\frac{4d^{2}x\sin^{4}\left(a+bx\right)\cos\left(a+bx\right)}{15b^{2}}+\frac{26d^{2}x\sin^{2}\left(a+bx\right)\cos^{3}\left(a+bx\right)}{45b^{2}}+\frac{52d^{2}x\cos^{5}\left(a+bx\right)}{225b^{2}}-\frac{856d^{2}\sin^{5}\left(a+bx\right)}{3375b^{3}}-\frac{338d^{2}\sin^{3}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{675b^{3}}-\frac{52d^{2}\sin\left(a+bx\right)\cos^{4}\left(a+bx\right)}{225b^{3}} & \text{for}\: b \neq 0 \\ \left(c^{2}x+cdx^{2}+\frac{d^{2}x^{3}}{3}\right)\sin^{2}\left(a\right)\cos^{3}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((2*c**2*sin(a + b*x)**5/(15*b) + c**2*sin(a + b*x)**3*cos(a + b*x
)**2/(3*b) + 4*c*d*x*sin(a + b*x)**5/(15*b) + 2*c*d*x*sin(a + b*x)**3*cos(a
 + b*x)**2/(3*b) + 2*d**2*x**2*sin(a + b*x)**5/(15*b) + d**2*x**2*sin(a + b
*x)**3*cos(a + b*x)**2/(3*b) + 4*c*d*sin(a + b*x)**4*cos(a + b*x)/(15*b**2)
 + 26*c*d*sin(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 52*c*d*cos(a + b*x)**
5/(225*b**2) + 4*d**2*x*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 26*d**2*x*
sin(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 52*d**2*x*cos(a + b*x)**5/(225*
b**2) - 856*d**2*sin(a + b*x)**5/(3375*b**3) - 338*d**2*sin(a + b*x)**3*cos
(a + b*x)**2/(675*b**3) - 52*d**2*sin(a + b*x)*cos(a + b*x)**4/(225*b**3),
Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**2*cos(a)**3, True))
```

Giac [A]
time = 0.50, size = 209, normalized size = 1.14

$$-\frac{(bd^2x+bcd)\cos(5bx+5a)}{200b^3}-\frac{(bd^2x+bcd)\cos(3bx+3a)}{72b^3}+\frac{(bd^2x+bcd)\cos(bx+a)}{4b^3}-\frac{(25b^2d^2x^2+50b^2cdx+25b^2c^2-2d^2)\sin(5bx+5a)}{2000b^3}-\frac{(9b^2d^2x^2+18b^2cdx+9b^2c^2-2d^2)\sin(3bx+3a)}{432b^3}+\frac{(b^2d^2x^2+2b^2cdx+b^2c^2-2d^2)\sin(bx+a)}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/200*(b*d^2*x + b*c*d)*cos(5*b*x + 5*a)/b^3 - 1/72*(b*d^2*x + b*c*d)*cos(
3*b*x + 3*a)/b^3 + 1/4*(b*d^2*x + b*c*d)*cos(b*x + a)/b^3 - 1/2000*(25*b^2*
d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*sin(5*b*x + 5*a)/b^3 - 1/432*(
9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*sin(3*b*x + 3*a)/b^3 + 1/
8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x + a)/b^3
```

Mupad [B]
time = 0.85, size = 295, normalized size = 1.60

$$\frac{52d^2x\cos(a+bx)^5}{225b^2}-\frac{52d^2\cos(a+bx)^4\sin(a+bx)}{225b^3}-\frac{\cos(a+bx)^2\sin(a+bx)^3(338d^2-225b^2c^2)}{675b^3}-\frac{2\sin(a+bx)^5(428d^2-225b^2c^2)}{3375b^3}+\frac{2d^2x^2\sin(a+bx)^5}{15b}+\frac{52cd\cos(a+bx)^5}{225b^2}+\frac{4cd\cos(a+bx)\sin(a+bx)^4}{15b^2}+\frac{4cdx\sin(a+bx)^5}{15b}+\frac{d^2x^2\cos(a+bx)^2\sin(a+bx)^3}{3b}+\frac{26cd\cos(a+bx)^3\sin(a+bx)^2}{45b^2}+\frac{4d^2x\cos(a+bx)\sin(a+bx)^4}{15b^2}+\frac{26d^2x\cos(a+bx)^3\sin(a+bx)^2}{45b^2}+\frac{2cdx\cos(a+bx)^2\sin(a+bx)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] (52*d^2*x*cos(a + b*x)^5)/(225*b^2) - (52*d^2*cos(a + b*x)^4*sin(a + b*x))/
(225*b^3) - (cos(a + b*x)^2*sin(a + b*x)^3*(338*d^2 - 225*b^2*c^2))/(675*b^
3) - (2*sin(a + b*x)^5*(428*d^2 - 225*b^2*c^2))/(3375*b^3) + (2*d^2*x^2*sin
(a + b*x)^5)/(15*b) + (52*c*d*cos(a + b*x)^5)/(225*b^2) + (4*c*d*cos(a + b*
x)*sin(a + b*x)^4)/(15*b^2) + (4*c*d*x*sin(a + b*x)^5)/(15*b) + (d^2*x^2*co
s(a + b*x)^2*sin(a + b*x)^3)/(3*b) + (26*c*d*cos(a + b*x)^3*sin(a + b*x)^2)
/(45*b^2) + (4*d^2*x*cos(a + b*x)*sin(a + b*x)^4)/(15*b^2) + (26*d^2*x*cos(
a + b*x)^3*sin(a + b*x)^2)/(45*b^2) + (2*c*d*x*cos(a + b*x)^2*sin(a + b*x)^
3)/(3*b)
```

3.149 $\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=109

$$\frac{d\cos(a+bx)}{8b^2}-\frac{d\cos(3a+3bx)}{144b^2}-\frac{d\cos(5a+5bx)}{400b^2}+\frac{(c+dx)\sin(a+bx)}{8b}-\frac{(c+dx)\sin(3a+3bx)}{48b}-\frac{(c+dx)\sin(5a+5bx)}{80b}$$

```
[Out] 1/8*d*cos(b*x+a)/b^2-1/144*d*cos(3*b*x+3*a)/b^2-1/400*d*cos(5*b*x+5*a)/b^2+
1/8*(d*x+c)*sin(b*x+a)/b-1/48*(d*x+c)*sin(3*b*x+3*a)/b-1/80*(d*x+c)*sin(5*b
*x+5*a)/b
```

Rubi [A]
time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4491, 3377, 2718}

$$\frac{d\cos(a+bx)}{8b^2}-\frac{d\cos(3a+3bx)}{144b^2}-\frac{d\cos(5a+5bx)}{400b^2}+\frac{(c+dx)\sin(a+bx)}{8b}-\frac{(c+dx)\sin(3a+3bx)}{48b}-\frac{(c+dx)\sin(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (d*Cos[a + b*x])/(8*b^2) - (d*Cos[3*a + 3*b*x])/(144*b^2) - (d*Cos[5*a + 5*
b*x])/(400*b^2) + ((c + d*x)*Sin[a + b*x])/(8*b) - ((c + d*x)*Sin[3*a + 3*b
*x])/(48*b) - ((c + d*x)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)\cos(a+bx) - \frac{1}{16}(c+dx)\cos(3a+3bx) - \frac{1}{16}(c+$$
$$= -\left(\frac{1}{16}\int (c+dx)\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)\cos(5a+5b$$
$$= \frac{(c+dx)\sin(a+bx)}{8b} - \frac{(c+dx)\sin(3a+3bx)}{48b} - \frac{(c+dx)\sin(5a}{80b}$$
$$= \frac{d\cos(a+bx)}{8b^2} - \frac{d\cos(3a+3bx)}{144b^2} - \frac{d\cos(5a+5bx)}{400b^2} + \frac{(c+dx)\sin}{8b}$$

Mathematica [A]
time = 0.38, size = 110, normalized size = 1.01

$$-\frac{-450d\cos(a+bx)+25d\cos(3(a+bx))+9d\cos(5(a+bx))-450bc\sin(a+bx)-450bdx\sin(a+bx)+75bc\sin(3(a+bx))+75bdx\sin(3(a+bx))+45bc\sin(5(a+bx))+45bdx\sin(5(a+bx))}{3600b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
[Out] -1/3600*(-450*d*Cos[a + b*x] + 25*d*Cos[3*(a + b*x)] + 9*d*Cos[5*(a + b*x)]
 - 450*b*c*Sin[a + b*x] - 450*b*d*x*Sin[a + b*x] + 75*b*c*Sin[3*(a + b*x)]
+ 75*b*d*x*Sin[3*(a + b*x)] + 45*b*c*Sin[5*(a + b*x)] + 45*b*d*x*Sin[5*(a +
 b*x)])/b^2
```

Maple [A]
time = 0.17, size = 175, normalized size = 1.61

method	result
risch	$\frac{d\cos(bx+a)}{8b^2} - \frac{d\cos(3bx+3a)}{144b^2} - \frac{d\cos(5bx+5a)}{400b^2} + \frac{(dx+c)\sin(bx+a)}{8b} - \frac{(dx+c)\sin(3bx+3a)}{48b} - \frac{(dx+c)\sin(5bx+}{80b}$
derivativedivides	$\frac{-\frac{da\left(-\frac{\sin(bx+a)\left(\cos^4(bx+a)\right)}{5}+\frac{\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{15}\right)}{b}+c\left(-\frac{\sin(bx+a)\left(\cos^4(bx+a)\right)}{5}+\frac{\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{15}\right)+- \quad d}{}$
default	$\frac{-\frac{da\left(-\frac{\sin(bx+a)\left(\cos^4(bx+a)\right)}{5}+\frac{\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{15}\right)}{b}+c\left(-\frac{\sin(bx+a)\left(\cos^4(bx+a)\right)}{5}+\frac{\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{15}\right)+- \quad d}{}$
norman	$\frac{\frac{52d}{225b^2}+\frac{8c\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}-\frac{16c\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{15b}+\frac{8c\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{4d\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b^2}+\frac{44d\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{45b^2}+\frac{52d\left(\tan^2\left(\frac{bx}{2}\right.\right.}{45b^2}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b*d*a*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)*sin(b*x+a
))+c*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))+1/b*d*
(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*cos(b*x+a)-
1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^
5))
```

Maxima [A]
time = 0.26, size = 139, normalized size = 1.28

$$-\frac{240\left(3\sin\left(bx+a\right)^{5}-5\sin\left(bx+a\right)^{3}\right)c-\frac{240\left(3\sin(bx+a)^{5}-5\sin(bx+a)^{3}\right)ad}{b}+\frac{\left(45\left(bx+a\right)\sin(5bx+5a)+75\left(bx+a\right)\sin(3bx+3a)-450\left(bx+a\right)\sin(bx+a)+9\cos(5bx+5a)+25\cos(3bx+3a)-450\cos(bx+a)\right)d}{b}}{3600\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3600*(240*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c - 240*(3*sin(b*x + a)^
5 - 5*sin(b*x + a)^3)*a*d/b + (45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)
*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*co
s(3*b*x + 3*a) - 450*cos(b*x + a))*d/b)/b
```

Fricas [A]
time = 2.12, size = 91, normalized size = 0.83

$$-\frac{9\,d\cos\left(bx+a\right)^{5}-5\,d\cos\left(bx+a\right)^{3}-30\,d\cos\left(bx+a\right)+15\left(3\left(bdx+bc\right)\cos\left(bx+a\right)^{4}-2\,bdx-\left(bdx+bc\right)\cos\left(bx+a\right)^{2}-2\,bc\right)\sin\left(bx+a\right)}{225\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/225*(9*d*cos(b*x + a)^5 - 5*d*cos(b*x + a)^3 - 30*d*cos(b*x + a) + 15*(3
*(b*d*x + b*c)*cos(b*x + a)^4 - 2*b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 - 2*
b*c)*sin(b*x + a))/b^2
```

Sympy [A]
time = 0.42, size = 163, normalized size = 1.50

$$\begin{cases} \frac{2c\sin^{5}\left(a+bx\right)}{15b}+\frac{c\sin^{3}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{3b}+\frac{2dx\sin^{5}\left(a+bx\right)}{15b}+\frac{dx\sin^{3}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{3b}+\frac{2d\sin^{4}\left(a+bx\right)\cos\left(a+bx\right)}{15b^{2}}+\frac{13d\sin^{2}\left(a+bx\right)\cos^{3}\left(a+bx\right)}{45b^{2}}+\frac{26d\cos^{5}\left(a+bx\right)}{225b^{2}} & \text{for}\ b\neq 0 \\ \left(cx+\frac{dx^{2}}{2}\right)\sin^{2}\left(a\right)\cos^{3}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((2*c*sin(a + b*x)**5/(15*b) + c*sin(a + b*x)**3*cos(a + b*x)**2/(
3*b) + 2*d*x*sin(a + b*x)**5/(15*b) + d*x*sin(a + b*x)**3*cos(a + b*x)**2/(
3*b) + 2*d*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 13*d*sin(a + b*x)**2*co
s(a + b*x)**3/(45*b**2) + 26*d*cos(a + b*x)**5/(225*b**2), Ne(b, 0)), ((c*x
 + d*x**2/2)*sin(a)**2*cos(a)**3, True))
```

Giac [A]
time = 0.47, size = 106, normalized size = 0.97

$$-\frac{d\cos\left(5\,bx+5\,a\right)}{400\,b^2}-\frac{d\cos\left(3\,bx+3\,a\right)}{144\,b^2}+\frac{d\cos\left(bx+a\right)}{8\,b^2}-\frac{\left(bdx+bc\right)\sin\left(5\,bx+5\,a\right)}{80\,b^2}-\frac{\left(bdx+bc\right)\sin\left(3\,bx+3\,a\right)}{48\,b^2}+\frac{\left(bdx+bc\right)\sin\left(bx+a\right)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/400*d*cos(5*b*x + 5*a)/b^2 - 1/144*d*cos(3*b*x + 3*a)/b^2 + 1/8*d*cos(b*
x + a)/b^2 - 1/80*(b*d*x + b*c)*sin(5*b*x + 5*a)/b^2 - 1/48*(b*d*x + b*c)*s
in(3*b*x + 3*a)/b^2 + 1/8*(b*d*x + b*c)*sin(b*x + a)/b^2
```

Mupad [B]
time = 0.47, size = 119, normalized size = 1.09

$$\frac{26\,d\cos\left(a+b\,x\right)^5+65\,d\cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^2+30\,d\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^4+30\,b\,c\sin\left(a+b\,x\right)^5+30\,b\,d\,x\sin\left(a+b\,x\right)^5+75\,b\,c\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^3+75\,b\,d\,x\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^3}{225\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x),x)
```

```
[Out] (26*d*cos(a + b*x)^5 + 65*d*cos(a + b*x)^3*sin(a + b*x)^2 + 30*d*cos(a + b*
x)*sin(a + b*x)^4 + 30*b*c*sin(a + b*x)^5 + 30*b*d*x*sin(a + b*x)^5 + 75*b*
c*cos(a + b*x)^2*sin(a + b*x)^3 + 75*b*d*x*cos(a + b*x)^2*sin(a + b*x)^3)/(
225*b^2)
```

3.150 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=185

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d}-\frac{\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+\right.}{16d}$$

```
[Out] -1/16*Ci(5*b*c/d+5*b*x)*cos(5*a-5*b*c/d)/d-1/16*Ci(3*b*c/d+3*b*x)*cos(3*a-3
*b*c/d)/d+1/8*Ci(b*c/d+b*x)*cos(a-b*c/d)/d+1/16*Si(5*b*c/d+5*b*x)*sin(5*a-5
*b*c/d)/d+1/16*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d-1/8*Si(b*c/d+b*x)*sin(a
-b*c/d)/d
```

Rubi [A]
time = 0.24, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4491, 3384, 3380, 3383}

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d}-\frac{\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d}+\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{16d}+\frac{\sin\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d) - (Cos[3*a - (3*b*c)/d]
*CosIntegral[(3*b*c)/d + 3*b*x])/(16*d) - (Cos[5*a - (5*b*c)/d]*CosIntegral
[(5*b*c)/d + 5*b*x])/(16*d) - (Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])
/(8*d) + (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d) + (Si
n[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx &= \int \left(\frac{\cos(a+bx)}{8(c+dx)}-\frac{\cos(3a+3bx)}{16(c+dx)}-\frac{\cos(5a+5bx)}{16(c+dx)}\right)dx\\
&= -\left(\frac{1}{16}\int\frac{\cos(3a+3bx)}{c+dx}\,dx\right)-\frac{1}{16}\int\frac{\cos(5a+5bx)}{c+dx}\,dx+\frac{1}{8}\int\frac{\cos(a+bx)}{c+dx}\\
&= -\left(\frac{1}{16}\cos\left(5a-\frac{5bc}{d}\right)\int\frac{\cos\left(\frac{5bc}{d}+5bx\right)}{c+dx}\,dx\right)-\frac{1}{16}\cos\left(3a-\frac{3bc}{d}\right)\int\\
&= \frac{\cos\left(a-\frac{bc}{d}\right)\operatorname{Ci}\left(\frac{bc}{d}+bx\right)}{8d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\operatorname{Ci}\left(\frac{3bc}{d}+3bx\right)}{16d}-\frac{\cos\left(5a-\frac{5bc}{d}\right)}{1}
\end{aligned}$$

Mathematica [A]

time = 0.46, size = 154, normalized size = 0.83

$$\frac{2\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)-\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)-\cos\left(5a-\frac{5bc}{d}\right)\operatorname{CosIntegral}\left(\frac{5b(c+dx)}{d}\right)-2\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(b\left(\frac{c}{d}+x\right)\right)+\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3b(c+dx)}{d}\right)+\sin\left(5a-\frac{5bc}{d}\right)\operatorname{Si}\left(\frac{5b(c+dx)}{d}\right)}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x),x]
[Out] (2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - Cos[3*a - (3*b*c)/d]*CosInte
gral[(3*b*(c + d*x))/d] - Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*(c + d*x))/
d] - 2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Sin[3*a - (3*b*c)/d]*Sin
Integral[(3*b*(c + d*x))/d] + Sin[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c + d*
x))/d])/(16*d)
```

Maple [A]

time = 0.16, size = 257, normalized size = 1.39

method	result
derivativedivides	$\frac{b\left(-\frac{\operatorname{sinIntegral}\left(-bx-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\operatorname{cosineIntegral}\left(bx+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}\right)}{8}-\frac{b\left(-\frac{5\operatorname{sinIntegral}\left(-5bx-5a\right.}{}\right.}{}$

default	$\frac{b\left(-\frac{\text{sinIntegral}\left(-bx-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\text{cosineIntegral}\left(bx+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}\right)}{8}-\frac{b\left(-\frac{5\,\text{sinIntegral}\left(-5bx-5a-\right.}{}\right.}{}$
risch	$\frac{e^{-\frac{5i(ad-cb)}{d}}\,\text{expIntegral}\left(1,5ibx+5ia-\frac{5i(ad-cb)}{d}\right)}{32d}+\frac{e^{-\frac{3i(ad-cb)}{d}}\,\text{expIntegral}\left(1,3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{32d}-\frac{e^{-\frac{i(ad-cb)}{d}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/8*b*(-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c
)/d)*cos((-a*d+b*c)/d)/d)-1/80*b*(-5*Si(-5*b*x-5*a-5*(-a*d+b*c)/d)*sin(5*(-
a*d+b*c)/d)/d+5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d)-1/48*b*
(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-
a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.36, size = 413, normalized size = 2.23

$$-\frac{2\,b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b\left(E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)-b\left(E_1\left(\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{5\,(bc-ad)}{d}\right)-2\,b\left(i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+b\left(-i\,E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_1\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)+b\left(-i\,E_1\left(\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_1\left(-\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{5\,(bc-ad)}{d}\right)}{32\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/32*(2*b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_inte
gral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*(exp
_integral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -3
*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b*(exp_integr
al_e(1, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -5*(-I*b*
c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/d) - 2*b*(I*exp_integral_
e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(1, -(I*b*c + I*(
b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(-I*exp_integral_e(1, 3*(-I
*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integral_e(1, -3*(-I*b*c - I*(b*x
+ a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) + b*(-I*exp_integral_e(1, 5*(-I*b
*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integral_e(1, -5*(-I*b*c - I*(b*x +
a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/(b*d)
```

Fricas [A]
time = 1.60, size = 229, normalized size = 1.24

$$\frac{2\left(\text{Ci}\left(\frac{bdx+bc}{d}\right)+\text{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-\left(\text{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\text{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)-\left(\text{Ci}\left(\frac{5\,(bdx+bc)}{d}\right)+\text{Ci}\left(-\frac{5\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{5\,(bc-ad)}{d}\right)+2\,\sin\left(-\frac{5\,(bc-ad)}{d}\right)\text{Si}\left(\frac{5\,(bdx+bc)}{d}\right)+2\,\sin\left(-\frac{3\,(bc-ad)}{d}\right)\text{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-4\,\sin\left(-\frac{bc-ad}{d}\right)\text{Si}\left(\frac{bdx+bc}{d}\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/32*(2*(cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*co
s(-(b*c - a*d)/d) - (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d
*x + b*c)/d))*cos(-3*(b*c - a*d)/d) - (cos_integral(5*(b*d*x + b*c)/d) + co
s_integral(-5*(b*d*x + b*c)/d))*cos(-5*(b*c - a*d)/d) + 2*sin(-5*(b*c - a*d
)/d)*sin_integral(5*(b*d*x + b*c)/d) + 2*sin(-3*(b*c - a*d)/d)*sin_integral
(3*(b*d*x + b*c)/d) - 4*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/
d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\cos^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x), x)
```

Giac [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.51, size = 44961, normalized size = 243.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/32*(real_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*t
an(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + real_part(
cos_integral(3*b*x + 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5
/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(b*
x + b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2
*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-b*x - b*c/d))*tan(5/
2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a
)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + real_
part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2
*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*imag_part(cos_integ
ral(b*x + b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*imag_part(cos_integral(-b*x - b*c/d))*ta
n(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d) + 8*sin_integral((b*d*x + b*c)/d)*tan(5/2*a)^2*tan(3/2*a)^2*tan(
1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos
_integral(3*b*x + 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*
b*c/d)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(-3*b*x
- 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2
```

*b*c/d)*tan(1/2*b*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(5/2*a)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 -
2*imag_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(5/2*b*c/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_
integral(-5*b*x - 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*
b*c/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 4*sin_integral(5*(b*d*x + b*c)/d
)*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)*tan(3/2*b*c/d)^2*ta
n(1/2*b*c/d)^2 - 4*imag_part(cos_integral(b*x + b*c/d))*tan(5/2*a)^2*tan(3/
2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*im
ag_part(cos_integral(-b*x - b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)*ta
n(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 8*sin_integral((b*d*x +
b*c)/d)*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(3/2*b*c/d
)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*a
)^2*tan(3/2*a)*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 - 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)*t
an(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*sin_inte
gral(3*(b*d*x + b*c)/d)*tan(5/2*a)^2*tan(3/2*a)*tan(1/2*a)^2*tan(5/2*b*c/d)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(5*b*x + 5*b
*c/d))*tan(5/2*a)*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)
^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*a
)*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 + 4*sin_integral(5*(b*d*x + b*c)/d)*tan(5/2*a)*tan(3/2*a)^2*tan(1/2*a)^
2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + real_part(cos_integr
al(5*b*x + 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^
2*tan(3/2*b*c/d)^2 + real_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*a)^2*
tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2 + 2*real_part(c
os_integral(b*x + b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*
c/d)^2*tan(3/2*b*c/d)^2 + 2*real_part(cos_integral(-b*x - b*c/d))*tan(5/2*a
)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2 + real_part
(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan
(5/2*b*c/d)^2*tan(3/2*b*c/d)^2 + real_part(cos_integral(-5*b*x - 5*b*c/d))*
tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2 -
8*real_part(cos_integral(b*x + b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)
*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 8*real_part(cos_integra
l(-b*x - b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d) + real_part(cos_integral(5*b*x + 5*b*c/d))*tan(
5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real
_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2
*tan(5/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(b*x + b*c/d))
*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(1/2*b*c/d)^2 -
2*real_part(cos_integral(-b*x - b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(5/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(-3*b*x - 3*
b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(1/2*b*c
/d)^2 + real_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2
*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*real_part(cos_integral(

```
3*b*x + 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan
(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 4*real_part(cos_integral(-3*b*x - 3*b*c/d))*
tan(5/2*a)^2*tan(3/2*a)*tan(1/2*a)^2*tan(5/2*b*...
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2}{c+d\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x),x)
[Out] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x), x)
```

3.151 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=257

$$-\frac{\cos(a+bx)}{8d(c+dx)}+\frac{\cos(3a+3bx)}{16d(c+dx)}+\frac{\cos(5a+5bx)}{16d(c+dx)}+\frac{5b\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)\sin\left(5a-\frac{5bc}{d}\right)}{16d^2}+\frac{3b\text{CosIntegral}\left(\frac{3bc}{d}\right.}{1}$$

```
[Out] -1/8*cos(b*x+a)/d/(d*x+c)+1/16*cos(3*b*x+3*a)/d/(d*x+c)+1/16*cos(5*b*x+5*a)
/d/(d*x+c)-1/8*b*cos(a-b*c/d)*Si(b*c/d+b*x)/d^2+3/16*b*cos(3*a-3*b*c/d)*Si(
3*b*c/d+3*b*x)/d^2+5/16*b*cos(5*a-5*b*c/d)*Si(5*b*c/d+5*b*x)/d^2+5/16*b*Ci(
5*b*c/d+5*b*x)*sin(5*a-5*b*c/d)/d^2+3/16*b*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/
d)/d^2-1/8*b*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^2
```

Rubi [A]
time = 0.27, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$\frac{5b\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d^2}+\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^2}-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{16d^2}+\frac{5b\cos\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{16d^2}-\frac{\cos(a+bx)}{8d(c+dx)}+\frac{\cos(3a+3bx)}{16d(c+dx)}+\frac{\cos(5a+5bx)}{16d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -1/8*Cos[a + b*x]/(d*(c + d*x)) + Cos[3*a + 3*b*x]/(16*d*(c + d*x)) + Cos[5
*a + 5*b*x]/(16*d*(c + d*x)) + (5*b*CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a
- (5*b*c)/d])/(16*d^2) + (3*b*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b
*c)/d])/(16*d^2) - (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d^2)
- (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^2) + (3*b*Cos[3*a -
(3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) + (5*b*Cos[5*a - (5*b*c
)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{\cos(a+bx)}{8(c+dx)^2} - \frac{\cos(3a+3bx)}{16(c+dx)^2} - \frac{\cos(5a+5bx)}{16(c+dx)^2}\right)dx \\
&= -\left(\frac{1}{16}\int \frac{\cos(3a+3bx)}{(c+dx)^2}\,dx\right) - \frac{1}{16}\int \frac{\cos(5a+5bx)}{(c+dx)^2}\,dx + \frac{1}{8}\int \frac{\cos(a+b}{(c+dx)^2} \\
&= -\frac{\cos(a+bx)}{8d(c+dx)} + \frac{\cos(3a+3bx)}{16d(c+dx)} + \frac{\cos(5a+5bx)}{16d(c+dx)} - \frac{b\int \frac{\sin(a+bx)}{c+dx}\,dx}{8d} + \frac{(3b)}{} \\
&= -\frac{\cos(a+bx)}{8d(c+dx)} + \frac{\cos(3a+3bx)}{16d(c+dx)} + \frac{\cos(5a+5bx)}{16d(c+dx)} + \frac{\left(5b\cos\left(5a-\frac{5bc}{d}\right)\right)\int}{16d} \\
&= -\frac{\cos(a+bx)}{8d(c+dx)} + \frac{\cos(3a+3bx)}{16d(c+dx)} + \frac{\cos(5a+5bx)}{16d(c+dx)} + \frac{5b\text{Ci}\left(\frac{5bc}{d}+5bx\right)\sin\left(5\right.}{16d^2}
\end{aligned}$$

Mathematica [A]
time = 2.04, size = 212, normalized size = 0.82

$$\frac{\frac{d\cos(3(a+bx))}{c+dx} + \frac{d\cos(5(a+bx))}{c+dx} + 5b\text{CosIntegral}\left(\frac{5b(c+dx)}{d}\right)\sin\left(5a-\frac{5bc}{d}\right) + 3b\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right) - 2\left(\frac{d\cos(a+bx)}{c+dx} + b\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right) + b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)\right) + 3b\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right) + 5b\cos\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5b(c+dx)}{d}\right)}{16d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] ((d*Cos[3*(a + b*x)])/(c + d*x) + (d*Cos[5*(a + b*x)])/(c + d*x) + 5*b*CosI
ntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] + 3*b*CosIntegral[(3*b*(c +
 d*x))/d]*Sin[3*a - (3*b*c)/d] - 2*((d*Cos[a + b*x])/(c + d*x) + b*CosInteg
```

```
ral[b*(c/d + x)]*Sin[a - (b*c)/d] + b*Cos[a - (b*c)/d]*SinIntegral[b*(c/d +
 x)]) + 3*b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] + 5*b*Cos[5
*a - (5*b*c)/d]*SinIntegral[(5*b*(c + d*x))/d])/(16*d^2)
```

Maple [A]
time = 0.12, size = 372, normalized size = 1.45

method	result
derivativedivides	$\frac{b^2\left(-\frac{\cos(bx+a)}{(-ad+cb+d(bx+a))d}-\frac{-\frac{\text{sinIntegral}\left(-bx-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\frac{\text{cosineIntegral}\left(bx+a+\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}}{d}\right)}{8}-\frac{b^2}{}$
default	$\frac{b^2\left(-\frac{\cos(bx+a)}{(-ad+cb+d(bx+a))d}-\frac{-\frac{\text{sinIntegral}\left(-bx-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\frac{\text{cosineIntegral}\left(bx+a+\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}}{d}\right)}{8}-\frac{b^2}{}$
risch	$-\frac{5ib\,e^{-\frac{5i(ad-cb)}{d}}\,\text{expIntegral}\left(1,5ibx+5ia-\frac{5i(ad-cb)}{d}\right)}{32d^2}-\frac{3ib\,e^{-\frac{3i(ad-cb)}{d}}\,\text{expIntegral}\left(1,3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{32d^2}+\frac{ib}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/8*b^2*(-cos(b*x+a)/(-a*d+c*b+d*(b*x+a))/d-(-Si(-b*x-a-(-a*d+b*c)/d)*
cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)-1/80*b^2
*(-5*cos(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))/d-5*(-5*Si(-5*b*x-5*a-5*(-a*d+b*c)
/d)*cos(5*(-a*d+b*c)/d)/d-5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d
)/d)/d)-1/48*b^2*(-3*cos(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d-3*(-3*Si(-3*b*x-
3*a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*si
n(3*(-a*d+b*c)/d)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.44, size = 446, normalized size = 1.74

$$-\frac{2\,b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^2\left(E_2\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_2\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)-b^2\left(E_2\left(\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_2\left(-\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{5\,(bc-ad)}{d}\right)-2\,b^2\left(i\,E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)-b^2\left(i\,E_2\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_2\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)-b^2\left(i\,E_2\left(\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_2\left(-\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{5\,(bc-ad)}{d}\right)}{32\,(bcd+(bx+a)d^2-ad^2)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(2*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_in
tegral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^2*
(exp_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(2
, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b^2*(exp_
```

```
integral_e(2, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(2, -5*
(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/d) - 2*b^2*(I*exp_i
ntegral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(2, -(I*b
*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^2*(I*exp_integral_e
(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(2, -3*(-I*b*c
- I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b^2*(I*exp_integral_e(
2, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(2, -5*(-I*b*c -
 I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2
- a*d^2)*b)
```

Fricas [A]
time = 1.66, size = 339, normalized size = 1.32

$$\frac{32\,d\cos\left(bx+a\right)^{5}-32\,d\cos\left(bx+a\right)^{3}+10\,\left(bdx+bc\right)\cos\left(-\frac{5\,\left(bc-ad\right)}{d}\right)\operatorname{Si}\left(\frac{5\,\left(bdx+bc\right)}{d}\right)+6\,\left(bdx+bc\right)\cos\left(-\frac{3\,\left(bc-ad\right)}{d}\right)\operatorname{Si}\left(\frac{3\,\left(bdx+bc\right)}{d}\right)-4\,\left(bdx+bc\right)\cos\left(-\frac{bc-ad}{d}\right)\operatorname{Si}\left(\frac{bdx+bc}{d}\right)-2\,\left(\left(bdx+bc\right)\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)+\left(bdx+bc\right)\operatorname{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+3\,\left(\left(bdx+bc\right)\operatorname{Ci}\left(\frac{3\,\left(bdx+bc\right)}{d}\right)+\left(bdx+bc\right)\operatorname{Ci}\left(-\frac{3\,\left(bdx+bc\right)}{d}\right)\right)\sin\left(-\frac{3\,\left(bc-ad\right)}{d}\right)+5\,\left(\left(bdx+bc\right)\operatorname{Ci}\left(\frac{5\,\left(bdx+bc\right)}{d}\right)+\left(bdx+bc\right)\operatorname{Ci}\left(-\frac{5\,\left(bdx+bc\right)}{d}\right)\right)\sin\left(-\frac{5\,\left(bc-ad\right)}{d}\right)}{32\,\left(d^{3}x+cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(32*d*cos(b*x + a)^5 - 32*d*cos(b*x + a)^3 + 10*(b*d*x + b*c)*cos(-5*(
b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) + 6*(b*d*x + b*c)*cos(-3*(b*c
 - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 4*(b*d*x + b*c)*cos(-(b*c - a*
d)/d)*sin_integral((b*d*x + b*c)/d) - 2*((b*d*x + b*c)*cos_integral((b*d*x
+ b*c)/d) + (b*d*x + b*c)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/
d) + 3*((b*d*x + b*c)*cos_integral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_i
ntegral(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d) + 5*((b*d*x + b*c)*cos_i
ntegral(5*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-5*(b*d*x + b*c)/d)
)*sin(-5*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}\left(a+bx\right)\cos^{3}\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x)**2, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 37.49, size = 1022022, normalized size = 3976.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/32*(5*b*d*x*imag_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)^2*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c
/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b*d*x*imag_part(cos_integral(3*
b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 2*b*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)
^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b*d*x*imag_part(cos_integral(-b*x - b*
c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^
2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b*d*x
*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*ta
n(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d)^2 - 5*b*d*x*imag_part(cos_integral(-5*b*x - 5*b*c
/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2
*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 10*b*d*x
*sin_integral(5*(b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d)^2 + 6*b*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(5/2*b*x)^2*
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/
2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 4*b*d*x*sin_integral((b*d*x
+ b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2
*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 4*b
*d*x*real_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan
(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2
*b*c/d)^2*tan(1/2*b*c/d) - 4*b*d*x*real_part(cos_integral(-b*x - b*c/d))*ta
n(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/
2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b*d*x*real_part
(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^
2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)*ta
n(1/2*b*c/d)^2 + 6*b*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*
b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2
*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 10*b*d*x*real_part(cos_
integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan
(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d)^2 + 10*b*d*x*real_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*b*x)
^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan
(5/2*b*c/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*b*d*x*real_part(cos_integ
ral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2
*tan(3/2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 + 4*b*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b*d*x*real_part(cos_integral(3*b*x + 3*
b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a
)*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b*d*x
*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*ta
```

```
n(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d)^2 - 10*b*d*x*real_part(cos_integral(5*b*x + 5*b*c/d
))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)*tan(3/2*a)^2*tan
(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 10*b*d*x*rea
l_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/
2*b*x)^2*tan(5/2*a)*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/
d)^2*tan(1/2*b*c/d)^2 + 5*b*c*imag_part(cos_integral(5*b*x + 5*b*c/d))*tan(
5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b*c*imag_part(c
os_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*
tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*ta
n(1/2*b*c/d)^2 - 2*b*c*imag_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/
2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b*c*imag_part(cos_integral
(-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 3*b*c*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b
*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^
2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 5*b*c*imag_part(cos_integral(-5*b*x -
 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/
2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 10
*b*c*sin_integral(5*(b*d*x + b*c)/d)*tan(5/2*b*...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (a+b\,x)^3\,\sin (a+b\,x)^2}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^2, x)
```

3.152 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=338

$$-\frac{\cos(a+bx)}{16d(c+dx)^2}+\frac{\cos(3a+3bx)}{32d(c+dx)^2}+\frac{\cos(5a+5bx)}{32d(c+dx)^2}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{16d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Cos}}{32}$$

```
[Out] 25/32*b^2*Ci(5*b*c/d+5*b*x)*cos(5*a-5*b*c/d)/d^3+9/32*b^2*Ci(3*b*c/d+3*b*x)
*cos(3*a-3*b*c/d)/d^3-1/16*b^2*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^3-1/16*cos(b*x+
a)/d/(d*x+c)^2+1/32*cos(3*b*x+3*a)/d/(d*x+c)^2+1/32*cos(5*b*x+5*a)/d/(d*x+c
)^2-25/32*b^2*Si(5*b*c/d+5*b*x)*sin(5*a-5*b*c/d)/d^3-9/32*b^2*Si(3*b*c/d+3*
b*x)*sin(3*a-3*b*c/d)/d^3+1/16*b^2*Si(b*c/d+b*x)*sin(a-b*c/d)/d^3+1/16*b*si
n(b*x+a)/d^2/(d*x+c)-3/32*b*sin(3*b*x+3*a)/d^2/(d*x+c)-5/32*b*sin(5*b*x+5*a
)/d^2/(d*x+c)
```

Rubi [A]
time = 0.34, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{16d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^3}+\frac{25b^2\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{32d^3}+\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{16d^3}-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^3}-\frac{25b^2\sin\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{32d^3}+\frac{b\sin(a+bx)}{16d^2(c+dx)}-\frac{3b\sin(3a+3bx)}{32d^2(c+dx)}-\frac{5b\sin(5a+5bx)}{32d^2(c+dx)}-\frac{\cos(a+bx)}{16d(c+dx)^2}+\frac{\cos(3a+3bx)}{32d(c+dx)^2}+\frac{\cos(5a+5bx)}{32d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] -1/16*Cos[a + b*x]/(d*(c + d*x)^2) + Cos[3*a + 3*b*x]/(32*d*(c + d*x)^2) +
Cos[5*a + 5*b*x]/(32*d*(c + d*x)^2) - (b^2*Cos[a - (b*c)/d]*CosIntegral[(b*
c)/d + b*x])/(16*d^3) + (9*b^2*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d +
 3*b*x])/(32*d^3) + (25*b^2*Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*c)/d + 5*
b*x])/(32*d^3) + (b*Sin[a + b*x])/(16*d^2*(c + d*x)) - (3*b*Sin[3*a + 3*b*x
])/(32*d^2*(c + d*x)) - (5*b*Sin[5*a + 5*b*x])/(32*d^2*(c + d*x)) + (b^2*Si
n[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(16*d^3) - (9*b^2*Sin[3*a - (3*b
*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) - (25*b^2*Sin[5*a - (5*b*c)
/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(32*d^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx &= \int \left(\frac{\cos(a+bx)}{8(c+dx)^3} - \frac{\cos(3a+3bx)}{16(c+dx)^3} - \frac{\cos(5a+5bx)}{16(c+dx)^3}\right) dx \\
&= -\left(\frac{1}{16}\int \frac{\cos(3a+3bx)}{(c+dx)^3}\,dx\right) - \frac{1}{16}\int \frac{\cos(5a+5bx)}{(c+dx)^3}\,dx + \frac{1}{8}\int \frac{\cos(a+b \cdots}{(c+dx)^3 \cdots} \\
&= -\frac{\cos(a+bx)}{16d(c+dx)^2} + \frac{\cos(3a+3bx)}{32d(c+dx)^2} + \frac{\cos(5a+5bx)}{32d(c+dx)^2} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx}{16d} + \frac{(3b \cdots}{\cdots} \\
&= -\frac{\cos(a+bx)}{16d(c+dx)^2} + \frac{\cos(3a+3bx)}{32d(c+dx)^2} + \frac{\cos(5a+5bx)}{32d(c+dx)^2} + \frac{b\sin(a+bx)}{16d^2(c+dx)} - \frac{3b\sin \cdots}{32 \cdots} \\
&= -\frac{\cos(a+bx)}{16d(c+dx)^2} + \frac{\cos(3a+3bx)}{32d(c+dx)^2} + \frac{\cos(5a+5bx)}{32d(c+dx)^2} + \frac{b\sin(a+bx)}{16d^2(c+dx)} - \frac{3b\sin \cdots}{32 \cdots} \\
&= -\frac{\cos(a+bx)}{16d(c+dx)^2} + \frac{\cos(3a+3bx)}{32d(c+dx)^2} + \frac{\cos(5a+5bx)}{32d(c+dx)^2} - \frac{b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{Ci}\left(\frac{bc}{d} \cdots\right.}{16d^3}
\end{aligned}
$$

Mathematica [A]
time = 3.10, size = 283, normalized size = 0.84

$$
\frac{\frac{d^2\cos(3(a+bx))}{(c+dx)^2} + \frac{d^2\cos(5(a+bx))}{(c+dx)^2} - 2b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right) + 9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) + 25b^2\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5b(c+dx)}{d}\right) + \frac{2d(-d\cos(a+bx)+b(c+dx)\sin(a+bx))}{(c+dx)^2} - \frac{3bd\sin(3(a+bx))}{c+dx} - \frac{5bd\sin(5(a+bx))}{c+dx} + 2b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right) - 9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right) - 25b^2\sin\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5b(c+dx)}{d}\right)}{32d^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] ((d^2*Cos[3*(a + b*x)])/(c + d*x)^2 + (d^2*Cos[5*(a + b*x)])/(c + d*x)^2 -
2*b^2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + 9*b^2*Cos[3*a - (3*b*c)/d
]*CosIntegral[(3*b*(c + d*x))/d] + 25*b^2*Cos[5*a - (5*b*c)/d]*CosIntegral[
(5*b*(c + d*x))/d] + (2*d*(-(d*Cos[a + b*x]) + b*(c + d*x)*Sin[a + b*x]))/(
c + d*x)^2 - (3*b*d*Sin[3*(a + b*x)])/(c + d*x) - (5*b*d*Sin[5*(a + b*x)])/
(c + d*x) + 2*b^2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 9*b^2*Sin[3*a
 - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] - 25*b^2*Sin[5*a - (5*b*c)/d]*
SinIntegral[(5*b*(c + d*x))/d])/(32*d^3)
```

Maple [**A**]
time = 0.20, size = 478, normalized size = 1.41 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/8*b^3*(-1/2*cos(b*x+a)/(-a*d+c*b+d*(b*x+a))^2/d-1/2*(-sin(b*x+a)/(-a
*d+c*b+d*(b*x+a))/d+(-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+
(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)-1/80*b^3*(-5/2*cos(5*b*x+5*a)/(-a*
d+c*b+d*(b*x+a))^2/d-5/2*(-5*sin(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))/d+5*(-5*Si
(-5*b*x-5*a-5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*b*x+5*a+5*(-a*d+b*
c)/d)*cos(5*(-a*d+b*c)/d)/d)/d)/d)-1/48*b^3*(-3/2*cos(3*b*x+3*a)/(-a*d+c*b+
d*(b*x+a))^2/d-3/2*(-3*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d+3*(-3*Si(-3*b*
x-3*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*
cos(3*(-a*d+b*c)/d)/d)/d)/d))
```

Maxima [**C**] Result contains complex when optimal does not.
time = 0.54, size = 481, normalized size = 1.42

$$-\frac{2\,b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^3\left(E_3\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_3\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)-b^3\left(E_3\left(\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_3\left(-\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{5\,(bc-ad)}{d}\right)-2\,b^3\left(i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)-b^3\left(i\,E_3\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_3\left(-\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)-b^3\left(i\,E_3\left(\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_3\left(-\frac{5\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{5\,(bc-ad)}{d}\right)}{32\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\,(bcd^2-ad^3)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(2*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_in
tegral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^3*
(exp_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(3
, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b^3*(exp_
integral_e(3, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(3, -5*
(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/d) - 2*b^3*(I*exp_i
ntegral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(3, -(I*b
*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^3*(I*exp_integral_e
(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(3, -3*(-I*b*c
- I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b^3*(I*exp_integral_e(
3, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(3, -5*(-I*b*c -
```

```
 I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^
2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Fricas [A]
time = 2.47, size = 567, normalized size = 1.68

$$\frac{32\,d^2\cos\left(bx+a\right)^5-32\,d^2\cos\left(bx+a\right)^3-50\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{5\,(bdx+bc)}{d}\right)-18\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right)+4\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)-2\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+9\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)+25\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ci}\left(\frac{5\,(bdx+bc)}{d}\right)+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ci}\left(-\frac{5\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{5\,(bc-ad)}{d}\right)-32\left(5\left(bd^2x+bcd\right)\cos\left(bx+a\right)^4-3\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2\right)\sin\left(bx+a\right)}{64\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(32*d^2*cos(b*x + a)^5 - 32*d^2*cos(b*x + a)^3 - 50*(b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)*sin(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) -
 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-3*(b*c - a*d)/d)*sin_integra
l(3*(b*d*x + b*c)/d) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-(b*c -
a*d)/d)*sin_integral((b*d*x + b*c)/d) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*cos_integral((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2)*cos_integral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*cos_integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d) +
 25*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(5*(b*d*x + b*c)/d)
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-5*(b*d*x + b*c)/d))*c
os(-5*(b*c - a*d)/d) - 32*(5*(b*d^2*x + b*c*d)*cos(b*x + a)^4 - 3*(b*d^2*x
+ b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(a+bx\right)\cos^3\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x)**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 76.71, size = 1708998, normalized size = 5056.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/64*(25*b^2*d^2*x^2*real_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)^
2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(
```

5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 25*b^2*d^2*x^2*real_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 8*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 50*b^2*d^2*x^2*imag_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 50*b^2*d^2*x^2*imag_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 100*b^2*d^2*x^2*sin_integral(5*(b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 4*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 8*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)*tan(1/2*a)

```
^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag
_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(5/2*a)^2*tan(3/2*a)*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d
)^2*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(5
/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)*tan(1/2*a)^
2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 50*b^2*d^2*x^2*imag_
part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b
*x)^2*tan(5/2*a)*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d)^2 - 50*b^2*d^2*x^2*imag_part(cos_integral(-5*b*x - 5*b*c/d
))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)*tan(3/2*a)^2*tan
(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 100*b^2*d^2*
x^2*sin_integral(5*(b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b
*x)^2*tan(5/2*a)*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d)^2 + 50*b^2*c*d*x*real_part(cos_integral(5*b*x + 5*b*c/d))*
tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(
1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*
real_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(
1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d)^2 - 4*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d)
)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a+b\,x)^3\,\sin(a+b\,x)^2}{(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^3, x)
```

3.153 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=413

$$-\frac{\cos(a+bx)}{24d(c+dx)^3}+\frac{b^2\cos(a+bx)}{48d^3(c+dx)}+\frac{\cos(3a+3bx)}{48d(c+dx)^3}-\frac{3b^2\cos(3a+3bx)}{32d^3(c+dx)}+\frac{\cos(5a+5bx)}{48d(c+dx)^3}-\frac{25b^2\cos(5a+5bx)}{96d^3(c+dx)}-12$$

```
[Out] -1/24*cos(b*x+a)/d/(d*x+c)^3+1/48*b^2*cos(b*x+a)/d^3/(d*x+c)+1/48*cos(3*b*x
+3*a)/d/(d*x+c)^3-3/32*b^2*cos(3*b*x+3*a)/d^3/(d*x+c)+1/48*cos(5*b*x+5*a)/d
/(d*x+c)^3-25/96*b^2*cos(5*b*x+5*a)/d^3/(d*x+c)+1/48*b^3*cos(a-b*c/d)*Si(b*
c/d+b*x)/d^4-9/32*b^3*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^4-125/96*b^3*cos
(5*a-5*b*c/d)*Si(5*b*c/d+5*b*x)/d^4-125/96*b^3*Ci(5*b*c/d+5*b*x)*sin(5*a-5*
b*c/d)/d^4-9/32*b^3*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^4+1/48*b^3*Ci(b*c/
d+b*x)*sin(a-b*c/d)/d^4+1/48*b*sin(b*x+a)/d^2/(d*x+c)^2-1/32*b*sin(3*b*x+3*
a)/d^2/(d*x+c)^2-5/96*b*sin(5*b*x+5*a)/d^2/(d*x+c)^2
```

Rubi [A]
time = 0.42, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$-\frac{125b^3\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{96d^4}-\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{48d^4}+\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{48d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^4}-\frac{125b^3\cos\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{96d^4}+\frac{b^2\cos(a+bx)}{48d^3(c+dx)}-\frac{3b^2\cos(3a+3bx)}{32d^3(c+dx)}-\frac{25b^2\cos(5a+5bx)}{96d^3(c+dx)}+\frac{b\sin(a+bx)}{48d^2(c+dx)^2}-\frac{b\sin(3a+3bx)}{32d^2(c+dx)^2}-\frac{5b\sin(5a+5bx)}{96d^2(c+dx)^2}-\frac{\cos(a+bx)}{24d(c+dx)^3}+\frac{\cos(3a+3bx)}{48d(c+dx)^3}+\frac{\cos(5a+5bx)}{48d(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -1/24*Cos[a + b*x]/(d*(c + d*x)^3) + (b^2*Cos[a + b*x])/(48*d^3*(c + d*x))
+ Cos[3*a + 3*b*x]/(48*d*(c + d*x)^3) - (3*b^2*Cos[3*a + 3*b*x])/(32*d^3*(c
 + d*x)) + Cos[5*a + 5*b*x]/(48*d*(c + d*x)^3) - (25*b^2*Cos[5*a + 5*b*x])/
(96*d^3*(c + d*x)) - (125*b^3*CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a - (5*b
*c)/d])/(96*d^4) - (9*b^3*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/
d])/(32*d^4) + (b^3*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(48*d^4) +
 (b*Sin[a + b*x])/(48*d^2*(c + d*x)^2) - (b*Sin[3*a + 3*b*x])/(32*d^2*(c +
d*x)^2) - (5*b*Sin[5*a + 5*b*x])/(96*d^2*(c + d*x)^2) + (b^3*Cos[a - (b*c)/
d]*SinIntegral[(b*c)/d + b*x])/(48*d^4) - (9*b^3*Cos[3*a - (3*b*c)/d]*SinIn
tegral[(3*b*c)/d + 3*b*x])/(32*d^4) - (125*b^3*Cos[5*a - (5*b*c)/d]*SinInte
gral[(5*b*c)/d + 5*b*x])/(96*d^4)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx &= \int \left(\frac{\cos(a+bx)}{8(c+dx)^4} - \frac{\cos(3a+3bx)}{16(c+dx)^4} - \frac{\cos(5a+5bx)}{16(c+dx)^4}\right) dx \\
&= -\left(\frac{1}{16}\int \frac{\cos(3a+3bx)}{(c+dx)^4}\,dx\right) - \frac{1}{16}\int \frac{\cos(5a+5bx)}{(c+dx)^4}\,dx + \frac{1}{8}\int \frac{\cos(a+b
\end{aligned}$$

$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} + \frac{\cos(5a+5bx)}{48d(c+dx)^3} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx}{24d} + \frac{b\int}{}$$

$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} + \frac{\cos(5a+5bx)}{48d(c+dx)^3} + \frac{b\sin(a+bx)}{48d^2(c+dx)^2} - \frac{b\,\mathrm{si}}{32}$$

$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{b^2\cos(a+bx)}{48d^3(c+dx)} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{32d^3(c+dx)} +$$

$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{b^2\cos(a+bx)}{48d^3(c+dx)} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{32d^3(c+dx)} +$$

$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{b^2\cos(a+bx)}{48d^3(c+dx)} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{32d^3(c+dx)} +$$

Mathematica [A]
time = 2.81, size = 451, normalized size = 1.09

$$-\frac{d\cos(3bx)\left((-2d^2+9b^2(c+dx)^2)\cos(3a)+3bd(c+dx)\sin(3a)\right)+d\cos(5bx)\left((-2d^2+25b^2(c+dx)^2)\cos(5a)+5bd(c+dx)\sin(5a)\right)+d(3bd(c+dx)\cos(3a)-(-2d^2+9b^2(c+dx)^2)\sin(3a))\sin(3bx)+d(5bd(c+dx)\cos(5a)-(-2d^2+25b^2(c+dx)^2)\sin(5a))\sin(5bx)-2\left(d\cos(bx)\left((-2d^2+b^2(c+dx)^2)\cos(a)+bd(c+dx)\sin(a)\right)+d(bd(c+dx)\cos(a)-(-2d^2+b^2(c+dx)^2)\sin(a))\sin(bx)+b^3(c+dx)^3\left(\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right)+\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)\right)\right)+27b^3(c+dx)^3\left(\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)+\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right)\right)+125b^3(c+dx)^3\left(\text{CosIntegral}\left(\frac{5b(c+dx)}{d}\right)\sin\left(5a-\frac{5bc}{d}\right)+\cos\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5b(c+dx)}{d}\right)\right)}{96d^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -1/96*(d*Cos[3*b*x]*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*x
)*Sin[3*a]) + d*Cos[5*b*x]*((-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*a] + 5*b*d*
(c + d*x)*Sin[5*a]) + d*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*
x)^2)*Sin[3*a])*Sin[3*b*x] + d*(5*b*d*(c + d*x)*Cos[5*a] - (-2*d^2 + 25*b^2
*(c + d*x)^2)*Sin[5*a])*Sin[5*b*x] - 2*(d*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)
^2)*Cos[a] + b*d*(c + d*x)*Sin[a]) + d*(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^
2*(c + d*x)^2)*Sin[a])*Sin[b*x] + b^3*(c + d*x)^3*(CosIntegral[b*(c/d + x)]
*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)])) + 27*b^3*(c
 + d*x)^3*(CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + Cos[3*a -
(3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]) + 125*b^3*(c + d*x)^3*(CosIntegr
al[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] + Cos[5*a - (5*b*c)/d]*SinIntegr
al[(5*b*(c + d*x))/d]))/(d^4*(c + d*x)^3)
```

Maple [A]
time = 0.26, size = 588, normalized size = 1.42 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/8*b^4*(-1/3*cos(b*x+a)/(-a*d+c*b+d*(b*x+a))^3/d-1/3*(-1/2*sin(b*x+a)
/(-a*d+c*b+d*(b*x+a))^2/d+1/2*(-cos(b*x+a)/(-a*d+c*b+d*(b*x+a))/d-(-Si(-b*x
-a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/
d)/d)/d)/d)/d)-1/80*b^4*(-5/3*cos(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))^3/d-5/3*(
-5/2*sin(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))^2/d+5/2*(-5*cos(5*b*x+5*a)/(-a*d+c
*b+d*(b*x+a))/d-5*(-5*Si(-5*b*x-5*a-5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d-5
*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)/d)/d)/d)-1/48*b^4*(-co
s(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))^3/d-(-3/2*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x
+a))^2/d+3/2*(-3*cos(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d-3*(-3*Si(-3*b*x-3*a-
3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*
(-a*d+b*c)/d)/d)/d)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.79, size = 531, normalized size = 1.29

$$-\frac{2b^4\left(E_4\left(\frac{ibc+i(bx+a)d-iad}{d}\right)+E_4\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^4\left(E_4\left(\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)+E_4\left(-\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{3(bc-ad)}{d}\right)-b^4\left(E_4\left(\frac{5(-ibc-i(bx+a)d+iad)}{d}\right)+E_4\left(-\frac{5(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{5(bc-ad)}{d}\right)-2b^4\left(iE_4\left(\frac{ibc+i(bx+a)d-iad}{d}\right)-iE_4\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)-b^4\left(iE_4\left(\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)-iE_4\left(-\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)\right)\sin\left(-\frac{3(bc-ad)}{d}\right)-b^4\left(iE_4\left(\frac{5(-ibc-i(bx+a)d+iad)}{d}\right)-iE_4\left(-\frac{5(-ibc-i(bx+a)d+iad)}{d}\right)\right)\sin\left(-\frac{5(bc-ad)}{d}\right)}{32\left(b^3c^3d-3ab^2c^2d^2+3a^2bcd^3+(bx+a)^3d^4-a^3d^4+3(bcd^3-ad^4)(bx+a)^2+3(b^2c^2d^2-2abcd^3+a^2d^4)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/32*(2*b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_in
tegral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^4*
(exp_integral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(4
, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b^4*(exp_
integral_e(4, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(4, -5*
(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/d) - 2*b^4*(I*exp_i
ntegral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(4, -(I*b
*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^4*(I*exp_integral_e
(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(4, -3*(-I*b*c
- I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b^4*(I*exp_integral_e(
4, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(4, -5*(-I*b*c -
 I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^
2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*
x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(383) = 766$.
time = 1.29, size = 811, normalized size = 1.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/192*(32*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(b*x
 + a)^5 - 32*(29*b^2*d^3*x^2 + 58*b^2*c*d^2*x + 29*b^2*c^2*d - 2*d^3)*cos(b
*x + a)^3 + 250*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*c
os(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) + 54*(b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-3*(b*c - a*d)/d)*sin_integral
(3*(b*d*x + b*c)/d) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^
3*c^3)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 192*(b^2*d^3*x^2
 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a) + 32*(5*(b*d^3*x + b*c*d^2)*cos(
b*x + a)^4 - 3*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*sin(b*x + a) - 2*((b^3*d
^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral((b*d*x + b
*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_inte
gral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d) + 27*((b^3*d^3*x^3 + 3*b^3*c*d^
2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(3*(b*d*x + b*c)/d) + (b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-3*(b*d*x +
b*c)/d))*sin(-3*(b*c - a*d)/d) + 125*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + b^3*c^3)*cos_integral(5*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-5*(b*d*x + b*c)/d))*sin
(-5*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\cos^3(a+bx)}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x)**4, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 155.21, size = 2478918, normalized size = 6002.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/192*(125*b^3*d^3*x^3*imag_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*
x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*t
an(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 27*b^3*d^3*x^3*imag_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(
5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b^3*d^3*x^3*ima
g_part(cos_integral(-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*
x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)
^2*tan(1/2*b*c/d)^2 - 27*b^3*d^3*x^3*imag_part(cos_integral(-3*b*x - 3*b*c/
d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*
tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 125*b^3*d
^3*x^3*imag_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 250*b^3*d^3*x^3*sin_integral(5*(b*d*x +
 b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*
a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 54*b
^3*d^3*x^3*sin_integral(3*(b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*ta
n(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d)^2 - 4*b^3*d^3*x^3*sin_integral((b*d*x + b*c)/d)*t
an(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 4*b^3*d^3*x^3*
real_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/
d)^2*tan(1/2*b*c/d) - 4*b^3*d^3*x^3*real_part(cos_integral(-b*x - b*c/d))*t
an(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1
```

/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 54*b^3*d^3*x^3*r
eal_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b
*c/d)*tan(1/2*b*c/d)^2 + 54*b^3*d^3*x^3*real_part(cos_integral(-3*b*x - 3*b
*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)
^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 250*b^3*
d^3*x^3*real_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 250*b^3*d^3*x^3*real_part(cos_integral(-5
*b*x - 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*
tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
+ 4*b^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/2
*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^
2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*b^3*d^3*x^3*real_part(cos_integral(
-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*ta
n(3/2*a)^2*tan(1/2*a)*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 -
54*b^3*d^3*x^3*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)*tan(1/2*a)^2*tan(5/2*b*c/
d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 54*b^3*d^3*x^3*real_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2
*a)^2*tan(3/2*a)*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d)^2 - 250*b^3*d^3*x^3*real_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*
x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)*tan(3/2*a)^2*tan(1/2*a)^2*tan
(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 250*b^3*d^3*x^3*real_part
(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(5/2*a)*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d)^2 + 375*b^3*c*d^2*x^2*imag_part(cos_integral(5*b*x + 5*b*c/d)
)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 81*b^3*c*d^
2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^
2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^3*c*d^2*x^2*imag_part(cos_integral(b*
x + b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3
/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6
*b^3*c*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2
*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d
)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 81*b^3*c*d^2*x^2*imag_part(cos_inte
gral(-3*b*x - 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/
2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 - 375*b^3*c*d^2*x^2*imag_part(cos_inte...

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^{3} \sin\left(a+b\,x\right)^{2}}{\left(c+d\,x\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^4,x)
[Out] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^4, x)
```

3.154 $\int (c+dx)^m \cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=285

$$-\frac{3\ 2^{-7-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{3\ 2^{-7-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c\ldots}{}$$

```
[Out] -3*2^(-7-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I
*b*(d*x+c)/d)^m)-3*2^(-7-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*
(a-b*c/d))/((I*b*(d*x+c)/d)^m)+2^(-7-m)*3^(-1-m)*exp(6*I*(a-b*c/d))*(d*x+c)
^m*GAMMA(1+m,-6*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+2^(-7-m)*3^(-1-m)*(d*
x+c)^m*GAMMA(1+m,6*I*b*(d*x+c)/d)/b/exp(6*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.24, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4491, 3389, 2212}

$$-\frac{3\ 2^{-m-7}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-m-7}3^{-m-1}e^{6i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{6ib(c+dx)}{d}\right)}{b}-\frac{3\ 2^{-m-7}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-m-7}3^{-m-1}e^{-6i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{6ib(c+dx)}{d}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*2^(-7 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(
c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - (3*2^(-7 - m)*(c + d*x)^m*Gamm
a[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x)
)/d)^m) + (2^(-7 - m)*3^(-1 - m)*E^((6*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[
1 + m, ((-6*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (2^(-7 - m)*
3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((6*I)*b*(c + d*x))/d])/(b*E^((6*I)*(a
- (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \cos^3(a+bx)\sin^3(a+bx)\,dx &= \int \left(\frac{3}{32}(c+dx)^m \sin(2a+2bx) - \frac{1}{32}(c+dx)^m \sin(6a+6bx)\right) d\\ &= -\left(\frac{1}{32}\int (c+dx)^m \sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^m \sin(2a+\\ &= -\left(\frac{1}{64}i\int e^{-i(6a+6bx)}(c+dx)^m\,dx\right) + \frac{1}{64}i\int e^{i(6a+6bx)}(c+dx)^m\,a\\ &= -\frac{3\,2^{-7-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right.}{b}\end{aligned}$$

Mathematica [A]
time = 56.93, size = 255, normalized size = 0.89

$$\frac{2^{-7-m}3^{-1-m}e^{-\frac{6i(bc+ad)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-3^{2+m}e^{4i\left(2a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(1+m,-\frac{2ib(c+dx)}{d}\right)-3^{2+m}e^{4ia+\frac{8ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(1+m,\frac{2ib(c+dx)}{d}\right)+e^{12ia}\left(\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(1+m,-\frac{6ib(c+dx)}{d}\right)+e^{\frac{12ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(1+m,\frac{6ib(c+dx)}{d}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (2^(-7 - m)*3^(-1 - m)*(c + d*x)^m*(-(3^(2 + m)*E^((4*I)*(2*a + (b*c)/d))*(
(I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d]) - 3^(2 + m)*E^((
4*I)*a + ((8*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(c +
 d*x))/d] + E^((12*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-6*I)*b*(c +
d*x))/d] + E^(((12*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((6*I)*
b*(c + d*x))/d]))/(b*E^(((6*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m)
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cos^3(bx+a)\right)\left(\sin^3(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^3, x)
```

Fricas [A]
time = 0.31, size = 190, normalized size = 0.67

$$-\frac{9\,e^{\left(-\frac{dm\log\left(-\frac{2ib}{d}\right)+2ibc-2iad}{d}\right)}\Gamma\left(m+1,-\frac{2(ibdx+ibc)}{d}\right)-e^{\left(-\frac{dm\log\left(-\frac{6ib}{d}\right)+6ibc-6iad}{d}\right)}\Gamma\left(m+1,-\frac{6(ibdx+ibc)}{d}\right)+9\,e^{\left(-\frac{dm\log\left(\frac{2ib}{d}\right)-2ibc+2iad}{d}\right)}\Gamma\left(m+1,-\frac{2(-ibdx-ibc)}{d}\right)-e^{\left(-\frac{dm\log\left(\frac{6ib}{d}\right)-6ibc+6iad}{d}\right)}\Gamma\left(m+1,-\frac{6(-ibdx-ibc)}{d}\right)}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/384*(9*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m + 1, -2*(I
*b*d*x + I*b*c)/d) - e^(-(d*m*log(-6*I*b/d) + 6*I*b*c - 6*I*a*d)/d)*gamma(m
 + 1, -6*(I*b*d*x + I*b*c)/d) + 9*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d
)/d)*gamma(m + 1, -2*(-I*b*d*x - I*b*c)/d) - e^(-(d*m*log(6*I*b/d) - 6*I*b*
c + 6*I*a*d)/d)*gamma(m + 1, -6*(-I*b*d*x - I*b*c)/d))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos (a+b\,x)^3\,\sin (a+b\,x)^3\,{\left (c+d\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^m,x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^m, x)
```

3.155 $\int (c+dx)^4 \cos^3(a+bx) \sin^3(a+bx) \, dx$

Optimal. Leaf size=233

$$-\frac{9d^4 \cos(2a+2bx)}{128b^5}+\frac{9d^2(c+dx)^2 \cos(2a+2bx)}{64b^3}-\frac{3(c+dx)^4 \cos(2a+2bx)}{64b}+\frac{d^4 \cos(6a+6bx)}{10368b^5}-\frac{d^2(c+dx)}{}$$

```
[Out] -9/128*d^4*cos(2*b*x+2*a)/b^5+9/64*d^2*(d*x+c)^2*cos(2*b*x+2*a)/b^3-3/64*(d
*x+c)^4*cos(2*b*x+2*a)/b+1/10368*d^4*cos(6*b*x+6*a)/b^5-1/576*d^2*(d*x+c)^2
*cos(6*b*x+6*a)/b^3+1/192*(d*x+c)^4*cos(6*b*x+6*a)/b-9/64*d^3*(d*x+c)*sin(2
*b*x+2*a)/b^4+3/32*d*(d*x+c)^3*sin(2*b*x+2*a)/b^2+1/1728*d^3*(d*x+c)*sin(6*
b*x+6*a)/b^4-1/288*d*(d*x+c)^3*sin(6*b*x+6*a)/b^2
```

Rubi [A]
time = 0.20, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4491, 3377, 2718}

$$-\frac{9d^4 \cos(2a+2bx)}{128b^5}+\frac{d^4 \cos(6a+6bx)}{10368b^5}-\frac{9d^3(c+dx)\sin(2a+2bx)}{64b^4}+\frac{d^3(c+dx)\sin(6a+6bx)}{1728b^4}+\frac{9d^2(c+dx)^2\cos(2a+2bx)}{64b^3}-\frac{d^2(c+dx)^2\cos(6a+6bx)}{576b^3}+\frac{3d(c+dx)^3\sin(2a+2bx)}{32b^2}-\frac{d(c+dx)^3\sin(6a+6bx)}{288b^2}-\frac{3(c+dx)^4\cos(2a+2bx)}{64b}+\frac{(c+dx)^4\cos(6a+6bx)}{192b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-9*d^4*Cos[2*a + 2*b*x])/(128*b^5) + (9*d^2*(c + d*x)^2*Cos[2*a + 2*b*x])/
(64*b^3) - (3*(c + d*x)^4*Cos[2*a + 2*b*x])/(64*b) + (d^4*Cos[6*a + 6*b*x])
/(10368*b^5) - (d^2*(c + d*x)^2*Cos[6*a + 6*b*x])/(576*b^3) + ((c + d*x)^4*
Cos[6*a + 6*b*x])/(192*b) - (9*d^3*(c + d*x)*Sin[2*a + 2*b*x])/(64*b^4) + (
3*d*(c + d*x)^3*Sin[2*a + 2*b*x])/(32*b^2) + (d^3*(c + d*x)*Sin[6*a + 6*b*x
])/(1728*b^4) - (d*(c + d*x)^3*Sin[6*a + 6*b*x])/(288*b^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^4 \cos^3(a+bx) \sin^3(a+bx)\, dx &= \int \left(\frac{3}{32}(c+dx)^4 \sin(2a+2bx) - \frac{1}{32}(c+dx)^4 \sin(6a+6bx)\right) dx \\
&= -\left(\frac{1}{32}\int (c+dx)^4 \sin(6a+6bx)\, dx\right) + \frac{3}{32}\int (c+dx)^4 \sin(2a+2 \\
&= -\frac{3(c+dx)^4 \cos(2a+2bx)}{64b} + \frac{(c+dx)^4 \cos(6a+6bx)}{192b} - \frac{d\int (c+ \\
&= -\frac{3(c+dx)^4 \cos(2a+2bx)}{64b} + \frac{(c+dx)^4 \cos(6a+6bx)}{192b} + \frac{3d(c+d \\
&= \frac{9d^2(c+dx)^2 \cos(2a+2bx)}{64b^3} - \frac{3(c+dx)^4 \cos(2a+2bx)}{64b} - \frac{d^2(c+ \\
&= \frac{9d^2(c+dx)^2 \cos(2a+2bx)}{64b^3} - \frac{3(c+dx)^4 \cos(2a+2bx)}{64b} - \frac{d^2(c+ \\
&= -\frac{9d^4 \cos(2a+2bx)}{128b^5} + \frac{9d^2(c+dx)^2 \cos(2a+2bx)}{64b^3} - \frac{3(c+dx)^4 \cos}{64}
\end{aligned}
$$

Mathematica [A]
time = 1.34, size = 153, normalized size = 0.66

$$\frac{-243(3d^4-6b^2d^2(c+dx)^2+2b^4(c+dx)^4)\cos(2(a+bx))+(d^4-18b^2d^2(c+dx)^2+54b^4(c+dx)^4)\cos(6(a+bx))-12bd(c+dx)\left(121d^2-78b^2(c+dx)^2+(-d^2+6b^2(c+dx)^2)\cos(4(a+bx))\right)\sin(2(a+bx))}{10368b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-243*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)]
+ (d^4 - 18*b^2*d^2*(c + d*x)^2 + 54*b^4*(c + d*x)^4)*Cos[6*(a + b*x)] - 12
*b*d*(c + d*x)*(121*d^2 - 78*b^2*(c + d*x)^2 + (-d^2 + 6*b^2*(c + d*x)^2)*C
os[4*(a + b*x)])*Sin[2*(a + b*x)])/(10368*b^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2124 vs. $2(213) = 426$.
time = 0.50, size = 2125, normalized size = 9.12

method	result
risch	$\frac{(54d^4x^4b^4+216b^4c\,d^3x^3+324b^4c^2d^2x^2+216b^4c^3dx+54b^4c^4-18b^2d^4x^2-36b^2c\,d^3x-18b^2c^2d^2+d^4)\cos(6bx+6a)}{10368b^5} - \frac{d(6b}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/b*(1/b^4*a^4*d^4*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)-4/b^3*a^3*c*d^3*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)-4/b^4*a^3*d^4*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+6/b^2*a^2*c^2*d^2*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+12/b^3*a^2*c*d^3*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+6/b^4*a^2*d^4*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1/24*(b*x+a)^2-1/128*(2*sin(b*x+a)^2+3)^2-1/6*(b*x+a)^2*sin(b*x+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6+5/288*sin(b*x+a)^4+5/96*sin(b*x+a)^2)-4/b*a*c^3*d*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)-12/b^2*a*c^2*d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))-12/b^3*a*c*d^3*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1/24*(b*x+a)^2-1/128*(2*sin(b*x+a)^2+3)^2-1/6*(b*x+a)^2*sin(b*x+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6+5/288*sin(b*x+a)^4+5/96*sin(b*x+a)^2)-4/b^4*a*d^4*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/24*(b*x+a)*sin(b*x+a)^4-1/96*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/18*b*x-1/18*a+1/8*(b*x+a)*cos(b*x+a)^2-1/16*cos(b*x+a)*sin(b*x+a)+1/12*(b*x+a)^3-1/6*(b*x+a)^3*sin(b*x+a)^6+1/2*(b*x+a)^2*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/36*(b*x+a)*sin(b*x+a)^6+1/216*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+c^4*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+4/b*c^3*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+6/b^2*c^2*d^2*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1/24*(b*x+a)^2-1/128*(2*sin(b*x+a)^2+3)^2-1/6*(b*x+a)^2*sin(b*x+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6+5/288*sin(b*x+a)^4+5/96*sin(b*x+a)^2)+4/b^3*c*d^3*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/24*(b*x+a)*sin(b*x+a)^4-1/96*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/18*b*x-1/18*a+1/8*(b*x+a)*cos(b*x+a)^2-1/16*cos(b*x+a)*sin(b*x+a)+1/12*(b*x+a)^3-1/6*(b*x+a)^3*sin(b*x+a)^6+1/2*(b*x+a)^2*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/36*(b*x+a)*sin(

```
b*x+a)^6+1/216*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+
1/b^4*d^4*(1/4*(b*x+a)^4*sin(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*x+a)^3+3/2*sin
(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/12*(b*x+a)^2*sin(b*x+a)^4+1/6*(b*x+a)*
(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1/9*(b*x+a)^2
+1/384*(2*sin(b*x+a)^2+3)^2+1/4*(b*x+a)^2*cos(b*x+a)^2-1/2*(b*x+a)*(1/2*cos
(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+31/288*sin(b*x+a)^2+1/8*(b*x+a)^4-1/6*(b*
x+a)^4*sin(b*x+a)^6+2/3*(b*x+a)^3*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8
*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/18*(b*x+a)^2*sin(b*x+a)^6-1/9*(b
*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16
*b*x+5/16*a)-1/324*sin(b*x+a)^6-5/864*sin(b*x+a)^4))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1033 vs. $2(213) = 426$.
time = 0.32, size = 1033, normalized size = 4.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/10368*(864*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c^4 - 3456*(2*sin(b*x +
 a)^6 - 3*sin(b*x + a)^4)*a*c^3*d/b + 5184*(2*sin(b*x + a)^6 - 3*sin(b*x +
a)^4)*a^2*c^2*d^2/b^2 - 3456*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^3*c*d^
3/b^3 + 864*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^4*d^4/b^4 - 36*(6*(b*x
+ a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) +
27*sin(2*b*x + 2*a))*c^3*d/b + 108*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x
+ a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a*c^2*d^2/b
^2 - 108*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - si
n(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a^2*c*d^3/b^3 + 36*(6*(b*x + a)*cos(6
*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b
*x + 2*a))*a^3*d^4/b^4 - 18*((18*(b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*
(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*x
 + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 + 36*((18*(b*x + a)^2 - 1)*cos(6*b*x +
6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*
a) + 162*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/b^3 - 18*((18*(b*x + a)^2 - 1)
*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*s
in(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^4/b^4 - 6*(6*(6*(b*
x + a)^3 - b*x - a)*cos(6*b*x + 6*a) - 162*(2*(b*x + a)^3 - 3*b*x - 3*a)*co
s(2*b*x + 2*a) - (18*(b*x + a)^2 - 1)*sin(6*b*x + 6*a) + 243*(2*(b*x + a)^2
 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 + 6*(6*(6*(b*x + a)^3 - b*x - a)*cos(6*b*
x + 6*a) - 162*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - (18*(b*x +
a)^2 - 1)*sin(6*b*x + 6*a) + 243*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a*d^
4/b^4 - ((54*(b*x + a)^4 - 18*(b*x + a)^2 + 1)*cos(6*b*x + 6*a) - 243*(2*(b
*x + a)^4 - 6*(b*x + a)^2 + 3)*cos(2*b*x + 2*a) - 6*(6*(b*x + a)^3 - b*x -
a)*sin(6*b*x + 6*a) + 486*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))*d
^4/b^4)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(213) = 426$.
time = 2.75, size = 546, normalized size = 2.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/648*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 2*(54*b^4*d^4*x^4 + 216*b^4*c*d
^3*x^3 + 54*b^4*c^4 - 18*b^2*c^2*d^2 + d^4 + 18*(18*b^4*c^2*d^2 - b^2*d^4)*
x^2 + 36*(6*b^4*c^3*d - b^2*c*d^3)*x)*cos(b*x + a)^6 - 3*(54*b^4*d^4*x^4 +
216*b^4*c*d^3*x^3 + 54*b^4*c^4 - 18*b^2*c^2*d^2 + d^4 + 18*(18*b^4*c^2*d^2
- b^2*d^4)*x^2 + 36*(6*b^4*c^3*d - b^2*c*d^3)*x)*cos(b*x + a)^4 + 18*(9*b^4
*c^2*d^2 - 5*b^2*d^4)*x^2 + 18*(9*b^2*d^4*x^2 + 18*b^2*c*d^3*x + 9*b^2*c^2*
d^2 - 5*d^4)*cos(b*x + a)^2 + 36*(3*b^4*c^3*d - 5*b^2*c*d^3)*x - 12*((6*b^3
*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*c^3*d - b*c*d^3 + (18*b^3*c^2*d^2 - b*d
^4)*x)*cos(b*x + a)^5 - (6*b^3*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*c^3*d - b
*c*d^3 + (18*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^3 - 3*(3*b^3*d^4*x^3 + 9*
b^3*c*d^3*x^2 + 3*b^3*c^3*d - 5*b*c*d^3 + (9*b^3*c^2*d^2 - 5*b*d^4)*x)*cos(
b*x + a))*sin(b*x + a))/b^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1334 vs. $2(231) = 462$.
time = 2.22, size = 1334, normalized size = 5.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**4*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c**4*cos(a + b*x)*
*6/(12*b) + c**3*d*x*sin(a + b*x)**6/(6*b) + c**3*d*x*sin(a + b*x)**4*cos(a
 + b*x)**2/(2*b) - c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(2*b) - c**3*d*
x*cos(a + b*x)**6/(6*b) + c**2*d**2*x**2*sin(a + b*x)**6/(4*b) + 3*c**2*d**
2*x**2*sin(a + b*x)**4*cos(a + b*x)**2/(4*b) - 3*c**2*d**2*x**2*sin(a + b*x
)**2*cos(a + b*x)**4/(4*b) - c**2*d**2*x**2*cos(a + b*x)**6/(4*b) + c*d**3*
x**3*sin(a + b*x)**6/(6*b) + c*d**3*x**3*sin(a + b*x)**4*cos(a + b*x)**2/(2
*b) - c*d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**4/(2*b) - c*d**3*x**3*cos(a
 + b*x)**6/(6*b) + d**4*x**4*sin(a + b*x)**6/(24*b) + d**4*x**4*sin(a + b*x
)**4*cos(a + b*x)**2/(8*b) - d**4*x**4*sin(a + b*x)**2*cos(a + b*x)**4/(8*b
) - d**4*x**4*cos(a + b*x)**6/(24*b) + c**3*d*sin(a + b*x)**5*cos(a + b*x)/
(6*b**2) + 4*c**3*d*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) + c**3*d*sin(a
 + b*x)*cos(a + b*x)**5/(6*b**2) + c**2*d**2*x*sin(a + b*x)**5*cos(a + b*x)
/(2*b**2) + 4*c**2*d**2*x*sin(a + b*x)**3*cos(a + b*x)**3/(3*b**2) + c**2*d
```

```
**2*x*sin(a + b*x)*cos(a + b*x)**5/(2*b**2) + c*d**3*x**2*sin(a + b*x)**5*c
os(a + b*x)/(2*b**2) + 4*c*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)**3/(3*b**
2) + c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**5/(2*b**2) + d**4*x**3*sin(a +
b*x)**5*cos(a + b*x)/(6*b**2) + 4*d**4*x**3*sin(a + b*x)**3*cos(a + b*x)**3
/(9*b**2) + d**4*x**3*sin(a + b*x)*cos(a + b*x)**5/(6*b**2) - c**2*d**2*sin
(a + b*x)**6/(12*b**3) + c**2*d**2*sin(a + b*x)**2*cos(a + b*x)**4/(3*b**3)
 + 7*c**2*d**2*cos(a + b*x)**6/(36*b**3) - 5*c*d**3*x*sin(a + b*x)**6/(18*b
**3) - c*d**3*x*sin(a + b*x)**4*cos(a + b*x)**2/(3*b**3) + c*d**3*x*sin(a +
 b*x)**2*cos(a + b*x)**4/(3*b**3) + 5*c*d**3*x*cos(a + b*x)**6/(18*b**3) -
5*d**4*x**2*sin(a + b*x)**6/(36*b**3) - d**4*x**2*sin(a + b*x)**4*cos(a + b
*x)**2/(6*b**3) + d**4*x**2*sin(a + b*x)**2*cos(a + b*x)**4/(6*b**3) + 5*d*
*4*x**2*cos(a + b*x)**6/(36*b**3) - 5*c*d**3*sin(a + b*x)**5*cos(a + b*x)/(
18*b**4) - 31*c*d**3*sin(a + b*x)**3*cos(a + b*x)**3/(54*b**4) - 5*c*d**3*s
in(a + b*x)*cos(a + b*x)**5/(18*b**4) - 5*d**4*x*sin(a + b*x)**5*cos(a + b*
x)/(18*b**4) - 31*d**4*x*sin(a + b*x)**3*cos(a + b*x)**3/(54*b**4) - 5*d**4
*x*sin(a + b*x)*cos(a + b*x)**5/(18*b**4) + 5*d**4*sin(a + b*x)**6/(108*b**
5) - 31*d**4*sin(a + b*x)**2*cos(a + b*x)**4/(216*b**5) - 61*d**4*cos(a + b
*x)**6/(648*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 +
 c*d**3*x**4 + d**4*x**5/5)*sin(a)**3*cos(a)**3, True))
```

Giac [A]
time = 0.57, size = 359, normalized size = 1.54

$$\frac{\left(54 b^4 d^4 x^4 + 216 b^4 c d^3 x^3 + 324 b^4 c^2 d^2 x^2 + 216 b^4 c^3 d x + 54 b^4 c^4 - 18 b^2 d^4 x^2 - 36 b^2 c d^3 x - 18 b^2 c^2 d^2 + d^4\right)\cos\left(6 b x + 6 a\right)}{10368 b^5} - \frac{3 \left(2 b^4 d^4 x^4 + 8 b^4 c d^3 x^3 + 12 b^4 c^2 d^2 x^2 + 8 b^4 c^3 d x + 2 b^4 c^4 - 6 b^2 d^4 x^2 - 12 b^2 c d^3 x - 6 b^2 c^2 d^2 + 3 d^4\right)\cos\left(2 b x + 2 a\right)}{128 b^5} - \frac{\left(6 b^3 d^4 x^3 + 18 b^3 c d^3 x^2 + 18 b^3 c^2 d^2 x + 6 b^3 c^3 d - b d^4 x - b c d^3\right)\sin\left(6 b x + 6 a\right)}{1728 b^5} + \frac{3 \left(2 b^3 d^4 x^3 + 6 b^3 c d^3 x^2 + 6 b^3 c^2 d^2 x + 2 b^3 c^3 d - 3 b d^4 x - 3 b c d^3\right)\sin\left(2 b x + 2 a\right)}{64 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/10368*(54*b^4*d^4*x^4 + 216*b^4*c*d^3*x^3 + 324*b^4*c^2*d^2*x^2 + 216*b^4
*c^3*d*x + 54*b^4*c^4 - 18*b^2*d^4*x^2 - 36*b^2*c*d^3*x - 18*b^2*c^2*d^2 +
d^4)*cos(6*b*x + 6*a)/b^5 - 3/128*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4
*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x -
 6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 - 1/1728*(6*b^3*d^4*x^3 + 18*b
^3*c*d^3*x^2 + 18*b^3*c^2*d^2*x + 6*b^3*c^3*d - b*d^4*x - b*c*d^3)*sin(6*b*
x + 6*a)/b^5 + 3/64*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x + 2*
b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

Mupad [B]
time = 2.58, size = 576, normalized size = 2.47

$$-\frac{729 d^4 \cos\left(2 a+2 b x\right) - d^4 \cos\left(6 a+6 b x\right) + 486 b^4 c^4 \cos\left(2 a+2 b x\right) - 54 b^4 c^4 \cos\left(6 a+6 b x\right) - 972 b^3 c^3 d \sin\left(2 a+2 b x\right) + 36 b^3 c^3 d \sin\left(6 a+6 b x\right) - 1458 b^2 c^2 d^2 \cos\left(2 a+2 b x\right) + 18 b^2 c^2 d^2 \cos\left(6 a+6 b x\right) - 1458 b^2 d^4 x^2 \cos\left(2 a+2 b x\right) + 486 b^4 d^4 x^4 \cos\left(2 a+2 b x\right) + 18 b^2 d^4 x^2 \cos\left(6 a+6 b x\right) - 54 b^4 d^4 x^4 \cos\left(6 a+6 b x\right) - 972 b^3 d^4 x^3 \sin\left(2 a+2 b x\right) + 36 b^3 d^4 x^3 \sin\left(6 a+6 b x\right) + 1458 b c d^3 \sin\left(2 a+2 b x\right) - 6 b c d^3 \sin\left(6 a+6 b x\right) + 1458 b d^4 x \sin\left(2 a+2 b x\right) - 6 b d^4 x \sin\left(6 a+6 b x\right) + 2916 b^4 c^2 d^2 x^2 \cos\left(2 a+2 b x\right) - 324 b^4 c^2 d^2 x^2 \cos\left(6 a+6 b x\right) - 2916 b^2 c d^3 x \cos\left(2 a+2 b x\right) + 1944 b^4 c^3 d x \cos\left(2 a+2 b x\right) + 36 b^2 c d^3 x \cos\left(6 a+6 b x\right) - 216 b^4 c^3 d x \cos\left(6 a+6 b x\right) + 1944 b^4 c d^3 x^3 \cos\left(2 a+2 b x\right) - 216 b^4 c d^3 x^3 \cos\left(6 a+6 b x\right) - 2916 b^3 c^2 d^2 x \sin\left(2 a+2 b x\right) - 2916 b^3 c d^3 x^2 \sin\left(2 a+2 b x\right) + 108 b^3 c^2 d^2 x \sin\left(6 a+6 b x\right) + 108 b^3 c d^3 x^2 \sin\left(6 a+6 b x\right)}{10368 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^4,x)
```

```
[Out] -(729*d^4*cos(2*a + 2*b*x) - d^4*cos(6*a + 6*b*x) + 486*b^4*c^4*cos(2*a + 2
*b*x) - 54*b^4*c^4*cos(6*a + 6*b*x) - 972*b^3*c^3*d*sin(2*a + 2*b*x) + 36*b
^3*c^3*d*sin(6*a + 6*b*x) - 1458*b^2*c^2*d^2*cos(2*a + 2*b*x) + 18*b^2*c^2*
d^2*cos(6*a + 6*b*x) - 1458*b^2*d^4*x^2*cos(2*a + 2*b*x) + 486*b^4*d^4*x^4*
cos(2*a + 2*b*x) + 18*b^2*d^4*x^2*cos(6*a + 6*b*x) - 54*b^4*d^4*x^4*cos(6*a
 + 6*b*x) - 972*b^3*d^4*x^3*sin(2*a + 2*b*x) + 36*b^3*d^4*x^3*sin(6*a + 6*b
*x) + 1458*b*c*d^3*sin(2*a + 2*b*x) - 6*b*c*d^3*sin(6*a + 6*b*x) + 1458*b*d
^4*x*sin(2*a + 2*b*x) - 6*b*d^4*x*sin(6*a + 6*b*x) + 2916*b^4*c^2*d^2*x^2*c
os(2*a + 2*b*x) - 324*b^4*c^2*d^2*x^2*cos(6*a + 6*b*x) - 2916*b^2*c*d^3*x*c
os(2*a + 2*b*x) + 1944*b^4*c^3*d*x*cos(2*a + 2*b*x) + 36*b^2*c*d^3*x*cos(6*
a + 6*b*x) - 216*b^4*c^3*d*x*cos(6*a + 6*b*x) + 1944*b^4*c*d^3*x^3*cos(2*a
+ 2*b*x) - 216*b^4*c*d^3*x^3*cos(6*a + 6*b*x) - 2916*b^3*c^2*d^2*x*sin(2*a
+ 2*b*x) - 2916*b^3*c*d^3*x^2*sin(2*a + 2*b*x) + 108*b^3*c^2*d^2*x*sin(6*a
+ 6*b*x) + 108*b^3*c*d^3*x^2*sin(6*a + 6*b*x))/(10368*b^5)
```

3.156 $\int (c+dx)^3 \cos^3(a+bx) \sin^3(a+bx) \, dx$

Optimal. Leaf size=181

$$\frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3}-\frac{3(c+dx)^3\cos(2a+2bx)}{64b}-\frac{d^2(c+dx)\cos(6a+6bx)}{1152b^3}+\frac{(c+dx)^3\cos(6a+6bx)}{192b}-$$

```
[Out] 9/128*d^2*(d*x+c)*cos(2*b*x+2*a)/b^3-3/64*(d*x+c)^3*cos(2*b*x+2*a)/b-1/1152
*d^2*(d*x+c)*cos(6*b*x+6*a)/b^3+1/192*(d*x+c)^3*cos(6*b*x+6*a)/b-9/256*d^3*
sin(2*b*x+2*a)/b^4+9/128*d*(d*x+c)^2*sin(2*b*x+2*a)/b^2+1/6912*d^3*sin(6*b*
x+6*a)/b^4-1/384*d*(d*x+c)^2*sin(6*b*x+6*a)/b^2
```

Rubi [A]
time = 0.16, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4491, 3377, 2717}

$$-\frac{9d^3\sin(2a+2bx)}{256b^4}+\frac{d^3\sin(6a+6bx)}{6912b^4}+\frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3}-\frac{d^2(c+dx)\cos(6a+6bx)}{1152b^3}+\frac{9d(c+dx)^2\sin(2a+2bx)}{128b^2}-\frac{d(c+dx)^2\sin(6a+6bx)}{384b^2}-\frac{3(c+dx)^3\cos(2a+2bx)}{64b}+\frac{(c+dx)^3\cos(6a+6bx)}{192b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (9*d^2*(c + d*x)*Cos[2*a + 2*b*x])/(128*b^3) - (3*(c + d*x)^3*Cos[2*a + 2*b
*x])/(64*b) - (d^2*(c + d*x)*Cos[6*a + 6*b*x])/(1152*b^3) + ((c + d*x)^3*Co
s[6*a + 6*b*x])/(192*b) - (9*d^3*Sin[2*a + 2*b*x])/(256*b^4) + (9*d*(c + d*
x)^2*Sin[2*a + 2*b*x])/(128*b^2) + (d^3*Sin[6*a + 6*b*x])/(6912*b^4) - (d*(
c + d*x)^2*Sin[6*a + 6*b*x])/(384*b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^3 \cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^3\sin(2a+2bx) - \frac{1}{32}(c+dx)^3\sin(6a+6bx)\right) dx$$
$$= -\left(\frac{1}{32}\int (c+dx)^3\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^3\sin(2a+$$
$$= -\frac{3(c+dx)^3\cos(2a+2bx)}{64b} + \frac{(c+dx)^3\cos(6a+6bx)}{192b} - \frac{d\int(c+$$
$$= -\frac{3(c+dx)^3\cos(2a+2bx)}{64b} + \frac{(c+dx)^3\cos(6a+6bx)}{192b} + \frac{9d(c+$$
$$= \frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3} - \frac{3(c+dx)^3\cos(2a+2bx)}{64b} - \frac{d^2(c+$$
$$= \frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3} - \frac{3(c+dx)^3\cos(2a+2bx)}{64b} - \frac{d^2(c+$$

Mathematica [A]
time = 2.34, size = 132, normalized size = 0.73

$$\frac{-324b(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\cos(2(a+bx))+12b(c+dx)\left(-d^2+6b^2(c+dx)^2\right)\cos(6(a+bx))-4d\left(121d^2-234b^2(c+dx)^2+\left(-d^2+18b^2(c+dx)^2\right)\cos(4(a+bx))\right)\sin(2(a+bx))}{13824b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-324*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 12*b*(c +
 d*x)*(-d^2 + 6*b^2*(c + d*x)^2)*Cos[6*(a + b*x)] - 4*d*(121*d^2 - 234*b^2*
(c + d*x)^2 + (-d^2 + 18*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[2*(a + b*x)
])/(13824*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1131 vs. $2(165) = 330$.
time = 0.31, size = 1132, normalized size = 6.25

method	result
risch	$\frac{\left(6b^2d^3x^3+18b^2c\,d^2x^2+18b^2c^2dx+6b^2c^3-d^3x-d^2c\right)\cos(6bx+6a)}{1152b^3} - \frac{d\left(18x^2d^2b^2+36b^2cdx+18b^2c^2-d^2\right)\sin(6bx+6a)}{6912b^4}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b^3*a^3*d^3*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+3/b^
2*a^2*c*d^2*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+3/b^3*a^2*d^
```

```
3*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1
/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3
+15/8*sin(b*x+a))*cos(b*x+a))-3/b*a*c^2*d*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1
/12*cos(b*x+a)^4)-6/b^2*a*c*d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^
3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*
(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))-3/b^3*a*d^3*(1/
4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*co
s(b*x+a)+3/8*b*x+3/8*a)+1/24*(b*x+a)^2-1/128*(2*sin(b*x+a)^2+3)^2-1/6*(b*x+
a)^2*sin(b*x+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin
(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6+5/288*sin(b*x+a)^4+
5/96*sin(b*x+a)^2)+c^3*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+3
/b*c^2*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b
*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b
*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+3/b^2*c*d^2*(1/4*(b*x+a)^2*sin(b*x+a)^
4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)
+1/24*(b*x+a)^2-1/128*(2*sin(b*x+a)^2+3)^2-1/6*(b*x+a)^2*sin(b*x+a)^6+1/3*(
b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/1
6*b*x+5/16*a)+1/108*sin(b*x+a)^6+5/288*sin(b*x+a)^4+5/96*sin(b*x+a)^2)+1/b^
3*d^3*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin
(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/24*(b*x+a)*sin(b*x+a)^4-1/96*(sin(b*x+
a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/18*b*x-1/18*a+1/8*(b*x+a)*cos(b*x+a)^2-1/
16*cos(b*x+a)*sin(b*x+a)+1/12*(b*x+a)^3-1/6*(b*x+a)^3*sin(b*x+a)^6+1/2*(b*x
+a)^2*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16
*b*x+5/16*a)+1/36*(b*x+a)*sin(b*x+a)^6+1/216*(sin(b*x+a)^5+5/4*sin(b*x+a)^3
+15/8*sin(b*x+a))*cos(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(165) = 330$.
time = 0.28, size = 602, normalized size = 3.33

$$-\frac{576\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)c^3-\frac{1728\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)ac^2d}{b}+\frac{1728\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)a^2cd^2}{b^2}-\frac{576\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)a^3d^3}{b^3}-\cdots}{6912}$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6912*(576*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c^3 - 1728*(2*sin(b*x +
a)^6 - 3*sin(b*x + a)^4)*a*c^2*d/b + 1728*(2*sin(b*x + a)^6 - 3*sin(b*x + a
)^4)*a^2*c*d^2/b^2 - 576*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^3*d^3/b^3
- 18*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*
b*x + 6*a) + 27*sin(2*b*x + 2*a))*c^2*d/b + 36*(6*(b*x + a)*cos(6*b*x + 6*a
) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))
*a*c*d^2/b^2 - 18*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x +
2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a^2*d^3/b^3 - 6*((18*(b*x +
a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b
*x + a)*sin(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x + 2*a))*c*d^2/b^2 + 6*((
```

```
18*(b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2
*a) - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x + 2*a))*a*d^3/
b^3 - (6*(6*(b*x + a)^3 - b*x - a)*cos(6*b*x + 6*a) - 162*(2*(b*x + a)^3 -
3*b*x - 3*a)*cos(2*b*x + 2*a) - (18*(b*x + a)^2 - 1)*sin(6*b*x + 6*a) + 243
*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*d^3/b^3)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(165) = 330$.
time = 6.09, size = 349, normalized size = 1.93

$$\frac{9 b^3d^3x^3 + 27 b^3cd^2x^2 + 6 (6 b^3d^3x^3 + 18 b^3cd^2x^2 + 6 b^3c^3 - bcd^2 + (18 b^3c^2d - bd^3)x) \cos (bx + a)^6 - 9 (6 b^3d^3x^3 + 18 b^3cd^2x^2 + 6 b^3c^3 - bcd^2 + (18 b^3c^2d - bd^3)x) \cos (bx + a)^4 + 27 (bd^3x + bcd^2) \cos (bx + a)^2 + 3 (9 b^3c^2d - 5 bd^3)x - ((18 b^2d^3x^2 + 36 b^2cd^2x + 18 b^2c^2d - d^3) \cos (bx + a)^5 - (18 b^2d^3x^2 + 36 b^2cd^2x + 18 b^2c^2d - d^3) \cos (bx + a)^3 - 3 (9 b^2d^3x^2 + 18 b^2cd^2x + 9 b^2c^2d - 5 d^3) \cos (bx + a)) \sin (bx + a)}{216 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x^2 + 6*(6*b^3*d^3*x^3 + 18*b^3*c*d^2*x
^2 + 6*b^3*c^3 - b*c*d^2 + (18*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^6 - 9*(6*
b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 6*b^3*c^3 - b*c*d^2 + (18*b^3*c^2*d - b*d^
3)*x)*cos(b*x + a)^4 + 27*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 + 3*(9*b^3*c^2
*d - 5*b*d^3)*x - ((18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*c
os(b*x + a)^5 - (18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*cos(
b*x + a)^3 - 3*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 5*d^3)*cos(b
*x + a))*sin(b*x + a))/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(178) = 356$.
time = 1.51, size = 857, normalized size = 4.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**3*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c**3*cos(a + b*x)*
*6/(12*b) + c**2*d*x*sin(a + b*x)**6/(8*b) + 3*c**2*d*x*sin(a + b*x)**4*cos
(a + b*x)**2/(8*b) - 3*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) - c**
2*d*x*cos(a + b*x)**6/(8*b) + c*d**2*x**2*sin(a + b*x)**6/(8*b) + 3*c*d**2*
x**2*sin(a + b*x)**4*cos(a + b*x)**2/(8*b) - 3*c*d**2*x**2*sin(a + b*x)**2*
cos(a + b*x)**4/(8*b) - c*d**2*x**2*cos(a + b*x)**6/(8*b) + d**3*x**3*sin(a
 + b*x)**6/(24*b) + d**3*x**3*sin(a + b*x)**4*cos(a + b*x)**2/(8*b) - d**3*
x**3*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) - d**3*x**3*cos(a + b*x)**6/(24*
b) + c**2*d*sin(a + b*x)**5*cos(a + b*x)/(8*b**2) + c**2*d*sin(a + b*x)**3*
cos(a + b*x)**3/(3*b**2) + c**2*d*sin(a + b*x)*cos(a + b*x)**5/(8*b**2) + c
*d**2*x*sin(a + b*x)**5*cos(a + b*x)/(4*b**2) + 2*c*d**2*x*sin(a + b*x)**3*
cos(a + b*x)**3/(3*b**2) + c*d**2*x*sin(a + b*x)*cos(a + b*x)**5/(4*b**2) +
 d**3*x**2*sin(a + b*x)**5*cos(a + b*x)/(8*b**2) + d**3*x**2*sin(a + b*x)**
```

```
3*cos(a + b*x)**3/(3*b**2) + d**3*x**2*sin(a + b*x)*cos(a + b*x)**5/(8*b**2
) - c*d**2*sin(a + b*x)**6/(24*b**3) + c*d**2*sin(a + b*x)**2*cos(a + b*x)*
*4/(6*b**3) + 7*c*d**2*cos(a + b*x)**6/(72*b**3) - 5*d**3*x*sin(a + b*x)**6
/(72*b**3) - d**3*x*sin(a + b*x)**4*cos(a + b*x)**2/(12*b**3) + d**3*x*sin(
a + b*x)**2*cos(a + b*x)**4/(12*b**3) + 5*d**3*x*cos(a + b*x)**6/(72*b**3)
- 5*d**3*sin(a + b*x)**5*cos(a + b*x)/(72*b**4) - 31*d**3*sin(a + b*x)**3*c
os(a + b*x)**3/(216*b**4) - 5*d**3*sin(a + b*x)*cos(a + b*x)**5/(72*b**4),
Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**
3*cos(a)**3, True))
```

Giac [**A**]
time = 0.53, size = 241, normalized size = 1.33

$$\frac{(6 b^3 d^3 x^3 + 18 b^3 c d^2 x^2 + 18 b^3 c^2 d x + 6 b^3 c^3 - b d^3 x - b c d^2) \cos(6 b x + 6 a)}{1152 b^4} - \frac{3 (2 b^3 d^3 x^3 + 6 b^3 c d^2 x^2 + 6 b^3 c^2 d x + 2 b^3 c^3 - 3 b d^3 x - 3 b c d^2) \cos(2 b x + 2 a)}{128 b^4} - \frac{(18 b^2 d^3 x^2 + 36 b^2 c d^2 x + 18 b^2 c^2 d - d^3) \sin(6 b x + 6 a)}{6912 b^4} + \frac{9 (2 b^2 d^3 x^2 + 4 b^2 c d^2 x + 2 b^2 c^2 d - d^3) \sin(2 b x + 2 a)}{256 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/1152*(6*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 18*b^3*c^2*d*x + 6*b^3*c^3 - b*d
^3*x - b*c*d^2)*cos(6*b*x + 6*a)/b^4 - 3/128*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x
^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)/b^
4 - 1/6912*(18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*sin(6*b*x
 + 6*a)/b^4 + 9/256*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin
(2*b*x + 2*a)/b^4
```

Mupad [**B**]
time = 1.23, size = 366, normalized size = 2.02

$$-\frac{243 d^3 \sin(2 a + 2 b x) - d^3 \sin(6 a + 6 b x) + 324 b^3 c^3 \cos(2 a + 2 b x) - 36 b^3 c^3 \cos(6 a + 6 b x) - 486 b^2 c^2 d \sin(2 a + 2 b x) + 18 b^2 c^2 d \sin(6 a + 6 b x) + 324 b^3 d^3 x^3 \cos(2 a + 2 b x) - 36 b^3 d^3 x^3 \cos(6 a + 6 b x) - 486 b^2 d^3 x^2 \sin(2 a + 2 b x) + 18 b^2 d^3 x^2 \sin(6 a + 6 b x) - 486 b c d^2 \cos(2 a + 2 b x) + 6 b c d^2 \cos(6 a + 6 b x) - 486 b d^3 x \cos(2 a + 2 b x) + 6 b d^3 x \cos(6 a + 6 b x) + 972 b^3 c^2 d x \cos(2 a + 2 b x) - 108 b^3 c^2 d x \cos(6 a + 6 b x) - 972 b^2 c d^2 x \sin(2 a + 2 b x) + 36 b^2 c d^2 x \sin(6 a + 6 b x) + 972 b^3 c d^2 x^2 \cos(2 a + 2 b x) - 108 b^3 c d^2 x^2 \cos(6 a + 6 b x)}{6912 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] -(243*d^3*sin(2*a + 2*b*x) - d^3*sin(6*a + 6*b*x) + 324*b^3*c^3*cos(2*a + 2
*b*x) - 36*b^3*c^3*cos(6*a + 6*b*x) - 486*b^2*c^2*d*sin(2*a + 2*b*x) + 18*b
^2*c^2*d*sin(6*a + 6*b*x) + 324*b^3*d^3*x^3*cos(2*a + 2*b*x) - 36*b^3*d^3*x
^3*cos(6*a + 6*b*x) - 486*b^2*d^3*x^2*sin(2*a + 2*b*x) + 18*b^2*d^3*x^2*sin
(6*a + 6*b*x) - 486*b*c*d^2*cos(2*a + 2*b*x) + 6*b*c*d^2*cos(6*a + 6*b*x) -
 486*b*d^3*x*cos(2*a + 2*b*x) + 6*b*d^3*x*cos(6*a + 6*b*x) + 972*b^3*c^2*d*
x*cos(2*a + 2*b*x) - 108*b^3*c^2*d*x*cos(6*a + 6*b*x) - 972*b^2*c*d^2*x*sin
(2*a + 2*b*x) + 36*b^2*c*d^2*x*sin(6*a + 6*b*x) + 972*b^3*c*d^2*x^2*cos(2*a
 + 2*b*x) - 108*b^3*c*d^2*x^2*cos(6*a + 6*b*x))/(6912*b^4)
```

3.157 $\int (c+dx)^2 \cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=129

$$\frac{3d^2\cos(2a+2bx)}{128b^3}-\frac{3(c+dx)^2\cos(2a+2bx)}{64b}-\frac{d^2\cos(6a+6bx)}{3456b^3}+\frac{(c+dx)^2\cos(6a+6bx)}{192b}+\frac{3d(c+dx)\sin(2a+2bx)}{64b^2}$$

```
[Out] 3/128*d^2*cos(2*b*x+2*a)/b^3-3/64*(d*x+c)^2*cos(2*b*x+2*a)/b-1/3456*d^2*cos
(6*b*x+6*a)/b^3+1/192*(d*x+c)^2*cos(6*b*x+6*a)/b+3/64*d*(d*x+c)*sin(2*b*x+2
*a)/b^2-1/576*d*(d*x+c)*sin(6*b*x+6*a)/b^2
```

Rubi [A]
time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4491, 3377, 2718}

$$\frac{3d^2\cos(2a+2bx)}{128b^3}-\frac{d^2\cos(6a+6bx)}{3456b^3}+\frac{3d(c+dx)\sin(2a+2bx)}{64b^2}-\frac{d(c+dx)\sin(6a+6bx)}{576b^2}-\frac{3(c+dx)^2\cos(2a+2bx)}{64b}+\frac{(c+dx)^2\cos(6a+6bx)}{192b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (3*d^2*Cos[2*a + 2*b*x])/(128*b^3) - (3*(c + d*x)^2*Cos[2*a + 2*b*x])/(64*b
) - (d^2*Cos[6*a + 6*b*x])/(3456*b^3) + ((c + d*x)^2*Cos[6*a + 6*b*x])/(192
*b) + (3*d*(c + d*x)*Sin[2*a + 2*b*x])/(64*b^2) - (d*(c + d*x)*Sin[6*a + 6*
b*x])/(576*b^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^2 \cos^3(a+bx) \sin^3(a+bx)\, dx = \int \left(\frac{3}{32}(c+dx)^2 \sin(2a+2bx) - \frac{1}{32}(c+dx)^2 \sin(6a+6bx)\right) dx$$
$$= -\left(\frac{1}{32}\int (c+dx)^2 \sin(6a+6bx)\, dx\right) + \frac{3}{32}\int (c+dx)^2 \sin(2a+2$$
$$= -\frac{3(c+dx)^2\cos(2a+2bx)}{64b} + \frac{(c+dx)^2\cos(6a+6bx)}{192b} - \frac{d\int (c+}{}$$
$$= -\frac{3(c+dx)^2\cos(2a+2bx)}{64b} + \frac{(c+dx)^2\cos(6a+6bx)}{192b} + \frac{3d(c+d}{}$$
$$= \frac{3d^2\cos(2a+2bx)}{128b^3} - \frac{3(c+dx)^2\cos(2a+2bx)}{64b} - \frac{d^2\cos(6a+6bx)}{3456b^3}$$

Mathematica [A]
time = 0.52, size = 91, normalized size = 0.71

$$\frac{-81(-d^2+2b^2(c+dx)^2)\cos(2(a+bx)) + (-d^2+18b^2(c+dx)^2)\cos(6(a+bx)) - 6bd(c+dx)(-27\sin(2(a+bx)) + \sin(6(a+bx)))}{3456b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-81*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (-d^2 + 18*b^2*(c + d*x)
^2)*Cos[6*(a + b*x)] - 6*b*d*(c + d*x)*(-27*Sin[2*(a + b*x)] + Sin[6*(a + b
*x)]))/(3456*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(117) = 234.
time = 0.24, size = 514, normalized size = 3.98 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/b^2*a^2*d^2*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)-2/b*a
*c*d*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)-2/b^2*a*d^2*(1/4*(b
*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1
/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin
(b*x+a))*cos(b*x+a))+c^2*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)
+2/b*c*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b
*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b
*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+1/b^2*d^2*(1/4*(b*x+a)^2*sin(b*x+a)^4-
1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1
/24*(b*x+a)^2-1/128*(2*sin(b*x+a)^2+3)^2-1/6*(b*x+a)^2*sin(b*x+a)^6+1/3*(b*
x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*
b*x+5/16*a)+1/108*sin(b*x+a)^6+5/288*sin(b*x+a)^4+5/96*sin(b*x+a)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(117) = 234$.
time = 0.29, size = 303, normalized size = 2.35

$$-\frac{288\,\left(2\,\sin\left(bx+a\right)^{6}-3\,\sin\left(bx+a\right)^{4}\right)c^{2}-\frac{576\,\left(2\,\sin\left(bx+a\right)^{6}-3\,\sin\left(bx+a\right)^{4}\right)acd}{b}+\frac{288\,\left(2\,\sin\left(bx+a\right)^{6}-3\,\sin\left(bx+a\right)^{4}\right)a^{2}d^{2}}{b^{2}}-\frac{6\,\left(6\,\left(bx+a\right)\cos\left(6\,bx+6\,a\right)-54\,\left(bx+a\right)\cos\left(2\,bx+2\,a\right)-\sin\left(6\,bx+6\,a\right)+27\,\sin\left(2\,bx+2\,a\right)\right)cd}{b}+\frac{6\,\left(6\,\left(bx+a\right)\cos\left(6\,bx+6\,a\right)-54\,\left(bx+a\right)\cos\left(2\,bx+2\,a\right)-\sin\left(6\,bx+6\,a\right)+27\,\sin\left(2\,bx+2\,a\right)\right)ad^{2}}{b^{2}}-\frac{\left(\left(18\,\left(bx+a\right)^{2}-1\right)\cos\left(6\,bx+6\,a\right)-81\,\left(2\,\left(bx+a\right)^{2}-1\right)\cos\left(2\,bx+2\,a\right)-6\,\left(bx+a\right)\sin\left(6\,bx+6\,a\right)+162\,\left(bx+a\right)\sin\left(2\,bx+2\,a\right)\right)d^{2}}{b^{2}}}{3456\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3456*(288*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c^2 - 576*(2*sin(b*x + a
)^6 - 3*sin(b*x + a)^4)*a*c*d/b + 288*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)
*a^2*d^2/b^2 - 6*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2
*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*c*d/b + 6*(6*(b*x + a)*cos(6*
b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*
x + 2*a))*a*d^2/b^2 - ((18*(b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*(b*x +
 a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*x + a)*
sin(2*b*x + 2*a))*d^2/b^2)/b
```

Fricas [A]
time = 2.31, size = 194, normalized size = 1.50

$$\frac{2\,\left(18\,b^{2}d^{2}x^{2}+36\,b^{2}cdx+18\,b^{2}c^{2}-d^{2}\right)\cos\left(bx+a\right)^{6}+9\,b^{2}d^{2}x^{2}+18\,b^{2}cdx-3\,\left(18\,b^{2}d^{2}x^{2}+36\,b^{2}cdx+18\,b^{2}c^{2}-d^{2}\right)\cos\left(bx+a\right)^{4}+9\,d^{2}\cos\left(bx+a\right)^{2}-6\,\left(2\,\left(bd^{2}x+bcd\right)\cos\left(bx+a\right)^{5}-2\,\left(bd^{2}x+bcd\right)\cos\left(bx+a\right)^{3}-3\,\left(bd^{2}x+bcd\right)\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{216\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/216*(2*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*c^2 - d^2)*cos(b*x + a)^6
+ 9*b^2*d^2*x^2 + 18*b^2*c*d*x - 3*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*
c^2 - d^2)*cos(b*x + a)^4 + 9*d^2*cos(b*x + a)^2 - 6*(2*(b*d^2*x + b*c*d)*c
os(b*x + a)^5 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 3*(b*d^2*x + b*c*d)*co
s(b*x + a))*sin(b*x + a))/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(126) = 252$.
time = 1.01, size = 461, normalized size = 3.57

$$\begin{cases} -\frac{c^{2}\sin^{2}\left(a+bx\right)\cos^{4}\left(a+bx\right)}{4b}-\frac{c^{2}\cos^{6}\left(a+bx\right)}{12b}+\frac{cdx\sin^{6}\left(a+bx\right)}{12b}+\frac{cdx\sin^{4}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{4b}-\frac{cdx\sin^{2}\left(a+bx\right)\cos^{4}\left(a+bx\right)}{4b}-\frac{cdx\cos^{6}\left(a+bx\right)}{12b}+\frac{d^{2}x^{2}\sin^{6}\left(a+bx\right)}{24b}+\frac{d^{2}x^{2}\sin^{4}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{8b}-\frac{d^{2}x^{2}\sin^{2}\left(a+bx\right)\cos^{4}\left(a+bx\right)}{8b}-\frac{d^{2}x^{2}\cos^{6}\left(a+bx\right)}{24b}+\frac{cd\sin^{5}\left(a+bx\right)\cos\left(a+bx\right)}{12b^{2}}+\frac{2cd\sin^{3}\left(a+bx\right)\cos^{3}\left(a+bx\right)}{9b^{2}}+\frac{cd\sin\left(a+bx\right)\cos^{5}\left(a+bx\right)}{12b^{2}}+\frac{d^{2}x\sin^{5}\left(a+bx\right)\cos\left(a+bx\right)}{12b^{2}}+\frac{2d^{2}x\sin^{3}\left(a+bx\right)\cos^{3}\left(a+bx\right)}{9b^{2}}+\frac{d^{2}x\sin\left(a+bx\right)\cos^{5}\left(a+bx\right)}{12b^{2}}-\frac{d^{2}\sin^{6}\left(a+bx\right)}{72b^{3}}+\frac{d^{2}\sin^{2}\left(a+bx\right)\cos^{4}\left(a+bx\right)}{18b^{3}}+\frac{7d^{2}\cos^{6}\left(a+bx\right)}{216b^{3}} & \text{for}\: b \neq 0 \\ \left(c^{2}x+cdx^{2}+\frac{d^{2}x^{3}}{3}\right)\sin^{3}\left(a\right)\cos^{3}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**2*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c**2*cos(a + b*x)*
*6/(12*b) + c*d*x*sin(a + b*x)**6/(12*b) + c*d*x*sin(a + b*x)**4*cos(a + b*
x)**2/(4*b) - c*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c*d*x*cos(a + b
*x)**6/(12*b) + d**2*x**2*sin(a + b*x)**6/(24*b) + d**2*x**2*sin(a + b*x)**
```

```
4*cos(a + b*x)**2/(8*b) - d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) -
 d**2*x**2*cos(a + b*x)**6/(24*b) + c*d*sin(a + b*x)**5*cos(a + b*x)/(12*b*
*2) + 2*c*d*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) + c*d*sin(a + b*x)*cos
(a + b*x)**5/(12*b**2) + d**2*x*sin(a + b*x)**5*cos(a + b*x)/(12*b**2) + 2*
d**2*x*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) + d**2*x*sin(a + b*x)*cos(a
 + b*x)**5/(12*b**2) - d**2*sin(a + b*x)**6/(72*b**3) + d**2*sin(a + b*x)**
2*cos(a + b*x)**4/(18*b**3) + 7*d**2*cos(a + b*x)**6/(216*b**3), Ne(b, 0)),
 ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**3*cos(a)**3, True))
```

Giac [A]
time = 0.50, size = 145, normalized size = 1.12

$$\frac{(18\,b^2d^2x^2+36\,b^2cdx+18\,b^2c^2-d^2)\cos(6\,bx+6\,a)}{3456\,b^3}-\frac{3\,(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\cos(2\,bx+2\,a)}{128\,b^3}-\frac{(bd^2x+bcd)\sin(6\,bx+6\,a)}{576\,b^3}+\frac{3\,(bd^2x+bcd)\sin(2\,bx+2\,a)}{64\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/3456*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*c^2 - d^2)*cos(6*b*x + 6*a)/
b^3 - 3/128*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a
)/b^3 - 1/576*(b*d^2*x + b*c*d)*sin(6*b*x + 6*a)/b^3 + 3/64*(b*d^2*x + b*c*
d)*sin(2*b*x + 2*a)/b^3
```

Mupad [B]
time = 0.81, size = 202, normalized size = 1.57

$$\frac{81\,d^2\cos(2\,a+2\,b\,x)-d^2\cos(6\,a+6\,b\,x)-162\,b^2\,c^2\cos(2\,a+2\,b\,x)+18\,b^2\,c^2\cos(6\,a+6\,b\,x)+162\,b\,c\,d\sin(2\,a+2\,b\,x)-6\,b\,c\,d\sin(6\,a+6\,b\,x)-162\,b^2\,d^2\,x^2\cos(2\,a+2\,b\,x)+18\,b^2\,d^2\,x^2\cos(6\,a+6\,b\,x)+162\,b\,d^2\,x\sin(2\,a+2\,b\,x)-6\,b\,d^2\,x\sin(6\,a+6\,b\,x)-324\,b^2\,c\,d\,x\cos(2\,a+2\,b\,x)+36\,b^2\,c\,d\,x\cos(6\,a+6\,b\,x)}{3456\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] (81*d^2*cos(2*a + 2*b*x) - d^2*cos(6*a + 6*b*x) - 162*b^2*c^2*cos(2*a + 2*b
*x) + 18*b^2*c^2*cos(6*a + 6*b*x) + 162*b*c*d*sin(2*a + 2*b*x) - 6*b*c*d*si
n(6*a + 6*b*x) - 162*b^2*d^2*x^2*cos(2*a + 2*b*x) + 18*b^2*d^2*x^2*cos(6*a
+ 6*b*x) + 162*b*d^2*x*sin(2*a + 2*b*x) - 6*b*d^2*x*sin(6*a + 6*b*x) - 324*
b^2*c*d*x*cos(2*a + 2*b*x) + 36*b^2*c*d*x*cos(6*a + 6*b*x))/(3456*b^3)
```

3.158 $\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=77

$$-\frac{3(c+dx)\cos(2a+2bx)}{64b}+\frac{(c+dx)\cos(6a+6bx)}{192b}+\frac{3d\sin(2a+2bx)}{128b^2}-\frac{d\sin(6a+6bx)}{1152b^2}$$

```
[Out] -3/64*(d*x+c)*cos(2*b*x+2*a)/b+1/192*(d*x+c)*cos(6*b*x+6*a)/b+3/128*d*sin(2
*b*x+2*a)/b^2-1/1152*d*sin(6*b*x+6*a)/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4491, 3377, 2717}

$$\frac{3d\sin(2a+2bx)}{128b^2}-\frac{d\sin(6a+6bx)}{1152b^2}-\frac{3(c+dx)\cos(2a+2bx)}{64b}+\frac{(c+dx)\cos(6a+6bx)}{192b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*(c + d*x)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)*Cos[6*a + 6*b*x])/(192*
b) + (3*d*Sin[2*a + 2*b*x])/(128*b^2) - (d*Sin[6*a + 6*b*x])/(1152*b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)\sin(2a+2bx) - \frac{1}{32}(c+dx)\sin(6a+6bx)\right)\,dx$$
$$= -\left(\frac{1}{32}\int (c+dx)\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)\sin(2a+2bx)$$
$$= -\frac{3(c+dx)\cos(2a+2bx)}{64b} + \frac{(c+dx)\cos(6a+6bx)}{192b} - \frac{d\int \cos(6a}{192}$$
$$= -\frac{3(c+dx)\cos(2a+2bx)}{64b} + \frac{(c+dx)\cos(6a+6bx)}{192b} + \frac{3d\sin(2a+}{128b^2}$$

Mathematica [A]
time = 0.23, size = 63, normalized size = 0.82

$$\frac{-54b(c+dx)\cos(2(a+bx)) + 6b(c+dx)\cos(6(a+bx)) + d(27\sin(2(a+bx)) - \sin(6(a+bx)))}{1152b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-54*b*(c + d*x)*Cos[2*(a + b*x)] + 6*b*(c + d*x)*Cos[6*(a + b*x)] + d*(27*
Sin[2*(a + b*x)] - Sin[6*(a + b*x)]))/(1152*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(69) = 138.
time = 0.23, size = 176, normalized size = 2.29

method	result
risch	$-\frac{3(dx+c)\cos(2bx+2a)}{64b} + \frac{(dx+c)\cos(6bx+6a)}{192b} + \frac{3d\sin(2bx+2a)}{128b^2} - \frac{d\sin(6bx+6a)}{1152b^2}$
derivativedivides	$\frac{-\frac{da\left(-\frac{\left(\sin^2(bx+a)\right)\left(\cos^4(bx+a)\right)}{6} - \frac{\left(\cos^4(bx+a)\right)}{12}\right)}{b} + c\left(-\frac{\left(\sin^2(bx+a)\right)\left(\cos^4(bx+a)\right)}{6} - \frac{\left(\cos^4(bx+a)\right)}{12}\right) + \frac{d\left(\frac{(bx+a)\left(\sin^4(bx+a\right.}{4}\right.}{}}{}$
default	$\frac{-\frac{da\left(-\frac{\left(\sin^2(bx+a)\right)\left(\cos^4(bx+a)\right)}{6} - \frac{\left(\cos^4(bx+a)\right)}{12}\right)}{b} + c\left(-\frac{\left(\sin^2(bx+a)\right)\left(\cos^4(bx+a)\right)}{6} - \frac{\left(\cos^4(bx+a)\right)}{12}\right) + \frac{d\left(\frac{(bx+a)\left(\sin^4(bx+a\right.}{4}\right.}{}}{}$
norman	$\frac{\frac{4c\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b} + \frac{4c\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b} + \frac{d\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{12b^2} + \frac{17d\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{36b^2} - \frac{d\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b^2} + \frac{d\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b^2} - \frac{17d\left(\tan^9\left(\right.\right.}{36b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b*d*a*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+c*(-1/6*si
n(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+1/b*d*(1/4*(b*x+a)*sin(b*x+a)^4+
1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*s
in(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)
))
```

Maxima [A]
time = 0.26, size = 119, normalized size = 1.55

$$-\frac{96\left(2\sin\left(bx+a\right)^{6}-3\sin\left(bx+a\right)^{4}\right)c-\frac{96\left(2\sin(bx+a)^{6}-3\sin(bx+a)^{4}\right)ad}{b}-\frac{(6(bx+a)\cos(6bx+6a)-54(bx+a)\cos(2bx+2a)-\sin(6bx+6a)+27\sin(2bx+2a))d}{b}}{1152\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/1152*(96*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c - 96*(2*sin(b*x + a)^6
- 3*sin(b*x + a)^4)*a*d/b - (6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*co
s(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*d/b)/b
```

Fricas [A]
time = 1.23, size = 87, normalized size = 1.13

$$\frac{12\left(bdx+bc\right)\cos\left(bx+a\right)^{6}-18\left(bdx+bc\right)\cos\left(bx+a\right)^{4}+3\,bdx-\left(2\,d\cos\left(bx+a\right)^{5}-2\,d\cos\left(bx+a\right)^{3}-3\,d\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{72\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/72*(12*(b*d*x + b*c)*cos(b*x + a)^6 - 18*(b*d*x + b*c)*cos(b*x + a)^4 + 3
*b*d*x - (2*d*cos(b*x + a)^5 - 2*d*cos(b*x + a)^3 - 3*d*cos(b*x + a))*sin(b
*x + a))/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(80) = 160$.
time = 0.65, size = 201, normalized size = 2.61

$$\begin{cases}-\frac{c\sin^{2}\left(a+bx\right)\cos^{4}\left(a+bx\right)}{4b}-\frac{c\cos^{6}\left(a+bx\right)}{12b}+\frac{dx\sin^{6}\left(a+bx\right)}{24b}+\frac{dx\sin^{4}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{8b}-\frac{dx\sin^{2}\left(a+bx\right)\cos^{4}\left(a+bx\right)}{8b}-\frac{dx\cos^{6}\left(a+bx\right)}{24b}+\frac{d\sin^{5}\left(a+bx\right)\cos\left(a+bx\right)}{24b^{2}}+\frac{d\sin^{3}\left(a+bx\right)\cos^{3}\left(a+bx\right)}{9b^{2}}+\frac{d\sin\left(a+bx\right)\cos^{5}\left(a+bx\right)}{24b^{2}} & \text{for}\: b\neq 0\\\left(cx+\frac{dx^{2}}{2}\right)\sin^{3}\left(a\right)\cos^{3}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c*cos(a + b*x)**6/(12
*b) + d*x*sin(a + b*x)**6/(24*b) + d*x*sin(a + b*x)**4*cos(a + b*x)**2/(8*b
) - d*x*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) - d*x*cos(a + b*x)**6/(24*b)
+ d*sin(a + b*x)**5*cos(a + b*x)/(24*b**2) + d*sin(a + b*x)**3*cos(a + b*x)
```

```
**3/(9*b**2) + d*sin(a + b*x)*cos(a + b*x)**5/(24*b**2), Ne(b, 0)), ((c*x +
 d*x**2/2)*sin(a)**3*cos(a)**3, True))
```

Giac [A]
time = 0.46, size = 75, normalized size = 0.97

$$\frac{(bdx+bc)\cos(6\,bx+6\,a)}{192\,b^2}-\frac{3\,(bdx+bc)\cos(2\,bx+2\,a)}{64\,b^2}-\frac{d\sin(6\,bx+6\,a)}{1152\,b^2}+\frac{3\,d\sin(2\,bx+2\,a)}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/192*(b*d*x + b*c)*cos(6*b*x + 6*a)/b^2 - 3/64*(b*d*x + b*c)*cos(2*b*x + 2
*a)/b^2 - 1/1152*d*sin(6*b*x + 6*a)/b^2 + 3/128*d*sin(2*b*x + 2*a)/b^2
```

Mupad [B]
time = 0.71, size = 84, normalized size = 1.09

$$\frac{\frac{27\,d\,\sin(2\,a+2\,b\,x)}{4}-\frac{d\,\sin(6\,a+6\,b\,x)}{4}-\frac{27\,b\,c\,\cos(2\,a+2\,b\,x)}{2}+\frac{3\,b\,c\,\cos(6\,a+6\,b\,x)}{2}-\frac{27\,b\,d\,x\,\cos(2\,a+2\,b\,x)}{2}+\frac{3\,b\,d\,x\,\cos(6\,a+6\,b\,x)}{2}}{288\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x),x)
```

```
[Out] ((27*d*sin(2*a + 2*b*x))/4 - (d*sin(6*a + 6*b*x))/4 - (27*b*c*cos(2*a + 2*b
*x))/2 + (3*b*c*cos(6*a + 6*b*x))/2 - (27*b*d*x*cos(2*a + 2*b*x))/2 + (3*b*
d*x*cos(6*a + 6*b*x))/2)/(288*b^2)
```

3.159 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=129

$$-\frac{\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)\sin\left(6a-\frac{6bc}{d}\right)}{32d}+\frac{3\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{32d}+\frac{3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+\right.}{32d}$$

```
[Out] 3/32*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d-1/32*cos(6*a-6*b*c/d)*Si(6*b*c/d+
6*b*x)/d-1/32*Ci(6*b*c/d+6*b*x)*sin(6*a-6*b*c/d)/d+3/32*Ci(2*b*c/d+2*b*x)*s
in(2*a-2*b*c/d)/d
```

Rubi [A]

time = 0.19, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4491, 3384, 3380, 3383}

$$-\frac{\sin\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{32d}+\frac{3\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{32d}+\frac{3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{32d}-\frac{\cos\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6bc}{d}+6bx\right)}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] -1/32*(CosIntegral[(6*b*c)/d + 6*b*x]*Sin[6*a - (6*b*c)/d])/d + (3*CosInteg
ral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(32*d) + (3*Cos[2*a - (2*b*c)/
d]*SinIntegral[(2*b*c)/d + 2*b*x])/(32*d) - (Cos[6*a - (6*b*c)/d]*SinIntegr
al[(6*b*c)/d + 6*b*x])/(32*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx &= \int \left(\frac{3\sin(2a+2bx)}{32(c+dx)} - \frac{\sin(6a+6bx)}{32(c+dx)}\right)dx\\ &= -\left(\frac{1}{32}\int \frac{\sin(6a+6bx)}{c+dx}\,dx\right) + \frac{3}{32}\int \frac{\sin(2a+2bx)}{c+dx}\,dx\\ &= -\left(\frac{1}{32}\cos\left(6a-\frac{6bc}{d}\right)\int \frac{\sin\left(\frac{6bc}{d}+6bx\right)}{c+dx}\,dx\right) + \frac{1}{32}\left(3\cos\left(2a-\frac{2bc}{d}\right)\right)\\ &= -\frac{\text{Ci}\left(\frac{6bc}{d}+6bx\right)\sin\left(6a-\frac{6bc}{d}\right)}{32d} + \frac{3\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{32d} + \frac{3\cos(2a}{}\end{aligned}$$

Mathematica [A]
time = 0.29, size = 110, normalized size = 0.85

$$-\frac{\text{CosIntegral}\left(\frac{6b(c+dx)}{d}\right)\sin\left(6a-\frac{6bc}{d}\right) - 3\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right) - 3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) + \cos\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6b(c+dx)}{d}\right)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] -1/32*(CosIntegral[(6*b*(c + d*x))/d]*Sin[6*a - (6*b*c)/d] - 3*CosIntegral[
(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 3*Cos[2*a - (2*b*c)/d]*SinIntegra
l[(2*b*(c + d*x))/d] + Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/d])
/d
```

Maple [A]
time = 0.18, size = 178, normalized size = 1.38

method	result
derivativedivides	$-\frac{\frac{b\left(-\frac{6\,\text{sinIntegral}\left(-6bx-6a-\frac{6(-ad+cb)}{d}\right)\cos\left(\frac{-6ad+6cb}{d}\right)}{d}-\frac{6\,\text{cosineIntegral}\left(6bx+6a+\frac{-6ad+6cb}{d}\right)\sin\left(\frac{-6ad+6cb}{d}\right)}{d}\right)}{192}+\frac{3b\left(-\frac{2\sin}{}\right.}{}}{b}$
default	$-\frac{\frac{b\left(-\frac{6\,\text{sinIntegral}\left(-6bx-6a-\frac{6(-ad+cb)}{d}\right)\cos\left(\frac{-6ad+6cb}{d}\right)}{d}-\frac{6\,\text{cosineIntegral}\left(6bx+6a+\frac{-6ad+6cb}{d}\right)\sin\left(\frac{-6ad+6cb}{d}\right)}{d}\right)}{192}+\frac{3b\left(-\frac{2\sin}{}\right.}{}}{b}$

risch	$\frac{ie^{-\frac{6i(ad-cb)}{d}} \operatorname{expIntegral}\left(1,6ibx+6ia-\frac{6i(ad-cb)}{d}\right)}{64d} - \frac{3ie^{-\frac{2i(ad-cb)}{d}} \operatorname{expIntegral}\left(1,2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{64d} + \frac{3ie^{\frac{2i(a}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/192*b*(-6*Si(-6*b*x-6*a-6*(-a*d+b*c)/d)*cos(6*(-a*d+b*c)/d)/d-6*Ci(
6*b*x+6*a+6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d)+3/64*b*(-2*Si(-2*b*x-2*a-2
*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(
-a*d+b*c)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.34, size = 281, normalized size = 2.18

$$-\frac{3\,b\left(-i\,E_1\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_1\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b\left(-i\,E_1\left(\frac{6\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_1\left(-\frac{6\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{6\,(bc-ad)}{d}\right)+3\,b\left(E_1\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-b\left(E_1\left(\frac{6\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-\frac{6\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)}{64\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/64*(3*b*(-I*exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*
exp_integral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d
)/d) - b*(-I*exp_integral_e(1, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*ex
p_integral_e(1, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-6*(b*c - a*d)/
d) + 3*b*(exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_int
egral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d) -
b*(exp_integral_e(1, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e
(1, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-6*(b*c - a*d)/d))/(b*d)
```

Fricas [A]
time = 1.24, size = 156, normalized size = 1.21

$$\frac{3\left(\operatorname{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+\operatorname{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-\left(\operatorname{Ci}\left(\frac{6\,(bdx+bc)}{d}\right)+\operatorname{Ci}\left(-\frac{6\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)-2\,\cos\left(-\frac{6\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{6\,(bdx+bc)}{d}\right)+6\,\cos\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2\,(bdx+bc)}{d}\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/64*(3*(cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d)
)*sin(-2*(b*c - a*d)/d) - (cos_integral(6*(b*d*x + b*c)/d) + cos_integral(-
6*(b*d*x + b*c)/d))*sin(-6*(b*c - a*d)/d) - 2*cos(-6*(b*c - a*d)/d)*sin_int
egral(6*(b*d*x + b*c)/d) + 6*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x +
b*c)/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**3/(c + d*x), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.60, size = 6046, normalized size = 46.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/64*(imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b
*c/d)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^
2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 3*imag_part(cos_integral(-2*b*x -
2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_i
ntegral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2
+ 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(
b*c/d)^2 - 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(3*b*c/
d)^2*tan(b*c/d)^2 - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*t
an(a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 2*real_part(cos_integr
al(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 + 2*real
_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(
b*c/d)^2 + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)*tan
(3*b*c/d)^2*tan(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_integral(6*b*x
+ 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos
_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2
+ imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)
^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b
*c/d)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)^2*t
an(3*b*c/d)^2 - imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)
^2*tan(3*b*c/d)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*t
an(3*b*c/d)^2 + 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(3
*b*c/d)^2 - 12*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)*t
an(3*b*c/d)^2*tan(b*c/d) + 12*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) - 24*sin_integral(2*(b*d*x + b*c)/
d)*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(6*b
*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(
2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 + 3*imag_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 + imag_part(cos_inte
gral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*sin_integral(6
*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 - 6*sin_integral(2*(b*d*
```

```
x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*imag_part(cos_integral(6*b
*x + 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 4*imag_part(co
s_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 +
 8*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)*tan(a)^2*tan(3*b*c/d)*tan(b*c/d
)^2 + imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d)^2*ta
n(b*c/d)^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(3*b*
c/d)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^
2*tan(3*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(-6*b*x - 6*b*c/d))*t
an(3*a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*t
an(3*a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*sin_integral(2*(b*d*x + b*c)/d)*t
an(3*a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(6*b*x + 6*b*
c/d))*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 3*imag_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + imag_part(cos_i
ntegral(-6*b*x - 6*b*c/d))*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 2*sin_int
egral(6*(b*d*x + b*c)/d)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 6*sin_integ
ral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 2*real_part(c
os_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d) + 2*real_par
t(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d) - 6*real
_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2 - 6*r
eal_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2 -
 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)^
2 - 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c
/d)^2 - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(
b*c/d) - 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)^2*ta
n(b*c/d) + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(3*b*c/
d)^2*tan(b*c/d) + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*ta
n(3*b*c/d)^2*tan(b*c/d) - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)
^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 6*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(3*a)^2*tan(a)*tan(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x - 2*b
*c/d))*tan(3*a)^2*tan(a)*tan(b*c/d)^2 + 2*real_part(cos_integral(6*b*x + 6*
b*c/d))*tan(3*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(-6*b*x -
6*b*c/d))*tan(3*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(6*b*x +
 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d)*tan(b*c/d)^2...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^3\sin(a+b\,x)^3}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x),x)
[Out] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x), x)
```

3.160 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=179

$$\frac{3b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{16d^2}-\frac{3b\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{16d^2}-\frac{3\sin(2a+2bx)}{32d(c+dx)}+\frac{\sin(6a}{32d(c}$$

```
[Out] -3/16*b*Ci(6*b*c/d+6*b*x)*cos(6*a-6*b*c/d)/d^2+3/16*b*Ci(2*b*c/d+2*b*x)*cos
(2*a-2*b*c/d)/d^2+3/16*b*Si(6*b*c/d+6*b*x)*sin(6*a-6*b*c/d)/d^2-3/16*b*Si(2
*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^2-3/32*sin(2*b*x+2*a)/d/(d*x+c)+1/32*sin(6
*b*x+6*a)/d/(d*x+c)
```

Rubi [A]
time = 0.22, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$\frac{3b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{16d^2}-\frac{3b\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{16d^2}-\frac{3b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{16d^2}+\frac{3b\sin\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6bc}{d}+6bx\right)}{16d^2}-\frac{3\sin(2a+2bx)}{32d(c+dx)}+\frac{\sin(6a+6bx)}{32d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (3*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(16*d^2) - (3*b*C
os[6*a - (6*b*c)/d]*CosIntegral[(6*b*c)/d + 6*b*x])/(16*d^2) - (3*Sin[2*a +
 2*b*x])/(32*d*(c + d*x)) + Sin[6*a + 6*b*x]/(32*d*(c + d*x)) - (3*b*Sin[2*
a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(16*d^2) + (3*b*Sin[6*a - (6
*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(16*d^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{3\sin(2a+2bx)}{32(c+dx)^2}-\frac{\sin(6a+6bx)}{32(c+dx)^2}\right)dx\\ &= -\left(\frac{1}{32}\int \frac{\sin(6a+6bx)}{(c+dx)^2}\,dx\right)+\frac{3}{32}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx\\ &= -\frac{3\sin(2a+2bx)}{32d(c+dx)}+\frac{\sin(6a+6bx)}{32d(c+dx)}+\frac{(3b)\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{16d}-\frac{(3b)\int \frac{\cos(6a+\cdots}{c+dx\cdots}}{16d}\cdots\\ &= -\frac{3\sin(2a+2bx)}{32d(c+dx)}+\frac{\sin(6a+6bx)}{32d(c+dx)}-\frac{\left(3b\cos\left(6a-\frac{6bc}{d}\right)\right)\int \frac{\cos\left(\frac{6bc}{d}+6bx\right)}{c+dx}\,dx\cdots}{16d}\\ &= \frac{3b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{16d^2}-\frac{3b\cos\left(6a-\frac{6bc}{d}\right)\operatorname{Ci}\left(\frac{6bc}{d}+6bx\right)}{16d^2}-\frac{3\sin\cdots}{32\cdots}\end{aligned}$$

Mathematica [A]
time = 0.98, size = 189, normalized size = 1.06

$$\frac{6b(c+dx)\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)-6b(c+dx)\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6b(c+dx)}{d}\right)-3d\cos(2bx)\sin(2a)+d\cos(6bx)\sin(6a)-3d\cos(2a)\sin(2bx)+d\cos(6a)\sin(6bx)-6b(c+dx)\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)+6b(c+dx)\sin\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6b(c+dx)}{d}\right)}{32d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (6*b*(c + d*x)*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 6*b*(c
 + d*x)*Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*(c + d*x))/d] - 3*d*Cos[2*b*x
]*Sin[2*a] + d*Cos[6*b*x]*Sin[6*a] - 3*d*Cos[2*a]*Sin[2*b*x] + d*Cos[6*a]*S
in[6*b*x] - 6*b*(c + d*x)*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/
d] + 6*b*(c + d*x)*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/d])/(32
*d^2*(c + d*x))
```

Maple [A]
time = 0.10, size = 256, normalized size = 1.43

method	result
derivativedivides	$-\frac{b^2\left(-\frac{6\sin(6bx+6a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{36\,\mathrm{sinIntegral}\left(-6bx-6a-\frac{6(-ad+cb)}{d}\right)\sin\left(\frac{-6ad+6cb}{d}\right)}{d}+\frac{36\,\mathrm{cosineIntegral}\left(6bx+6a+\frac{-6ad+6cb}{d}\right)\cos}{d}}{d}\right.}{192}$
default	$-\frac{b^2\left(-\frac{6\sin(6bx+6a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{36\,\mathrm{sinIntegral}\left(-6bx-6a-\frac{6(-ad+cb)}{d}\right)\sin\left(\frac{-6ad+6cb}{d}\right)}{d}+\frac{36\,\mathrm{cosineIntegral}\left(6bx+6a+\frac{-6ad+6cb}{d}\right)\cos}{d}}{d}\right.}{192}$
risch	$\frac{3b\,e^{-\frac{6i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,6ibx+6ia-\frac{6i(ad-cb)}{d}\right)}{32d^2}-\frac{3b\,e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{32d^2}-\frac{3b\,e^{\frac{2i(}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/192*b^2*(-6*sin(6*b*x+6*a)/(-a*d+c*b+d*(b*x+a))/d+6*(-6*Si(-6*b*x-6
*a-6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d+6*Ci(6*b*x+6*a+6*(-a*d+b*c)/d)*cos
(6*(-a*d+b*c)/d)/d)/d)+3/64*b^2*(-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2
*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(
-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.38, size = 308, normalized size = 1.72

$$-\frac{3\,b^2\left(-i\,E_2\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_2\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^2\left(-i\,E_2\left(\frac{6\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_2\left(-\frac{6\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{6\,(bc-ad)}{d}\right)+3\,b^2\left(E_2\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_2\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-b^2\left(E_2\left(\frac{6\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_2\left(-\frac{6\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)}{64\,(bcd+(bx+a)d^2-ad^2)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/64*(3*b^2*(-I*exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
I*exp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a
*d)/d) - b^2*(-I*exp_integral_e(2, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
I*exp_integral_e(2, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-6*(b*c - a
*d)/d) + 3*b^2*(exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + e
xp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)
/d) - b^2*(exp_integral_e(2, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_in
tegral_e(2, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-6*(b*c - a*d)/d))/
((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A]
time = 1.00, size = 248, normalized size = 1.39

$$\frac{6\,(bdx+bc)\sin\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{6\,(bdx+bc)}{d}\right)-6\,(bdx+bc)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+3\left((bdx+bc)\,\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-3\left((bdx+bc)\,\mathrm{Ci}\left(\frac{6\,(bdx+bc)}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{6\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{6\,(bc-ad)}{d}\right)+32\left(d\cos(bx+a)^5-d\cos(bx+a)^3\right)\sin(bx+a)}{32\,(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(6*(b*d*x + b*c)*sin(-6*(b*c - a*d)/d)*sin_integral(6*(b*d*x + b*c)/d)
 - 6*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) +
3*((b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integr
al(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d) - 3*((b*d*x + b*c)*cos_integr
al(6*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-6*(b*d*x + b*c)/d))*cos
(-6*(b*c - a*d)/d) + 32*(d*cos(b*x + a)^5 - d*cos(b*x + a)^3)*sin(b*x + a))
/(d^3*x + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**3/(c + d*x)**2, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.94, size = 63798, normalized size = 356.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/32*(3*b*d*x*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*
x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*b*d*x*real_part(co
s_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*ta
n(3*b*c/d)^2*tan(b*c/d)^2 - 3*b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2
+ 3*b*d*x*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2
*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*b*d*x*imag_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*
b*c/d)^2*tan(b*c/d) - 6*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 12*b*d
*x*sin_integral(2*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a
)^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*b*d*x*imag_part(cos_integral(6*b*x + 6*b*
c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2
 + 6*b*d*x*imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^
2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 12*b*d*x*sin_integral(6*(
```

b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*ta
n(b*c/d)^2 - 6*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*
tan(b*x)^2*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*b*d*x*imag_par
t(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)
*tan(3*b*c/d)^2*tan(b*c/d)^2 - 12*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan
(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*b*d*
x*imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)
*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 6*b*d*x*imag_part(cos_integral(-6*b
*x - 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)*tan(a)^2*tan(3*b*c/d)^2*tan
(b*c/d)^2 + 12*b*d*x*sin_integral(6*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^
2*tan(3*a)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 3*b*c*real_part(cos_integ
ral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c
/d)^2*tan(b*c/d)^2 - 3*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b
*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*b*c*re
al_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*
tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 3*b*c*real_part(cos_integral(-6*b*x
- 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(
b*c/d)^2 + 3*b*d*x*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*ta
n(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2 + 3*b*d*x*real_part(cos_integra
l(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d
)^2 + 3*b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*
x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2 + 3*b*d*x*real_part(cos_integral(-6
*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2
 - 12*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^
2*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) - 12*b*d*x*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)*tan(3*b*c
/d)^2*tan(b*c/d) + 6*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x
)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*b*c*imag_p
art(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(
a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 12*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan
(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 3*b*d*
x*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)
^2*tan(a)^2*tan(b*c/d)^2 - 3*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))
*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 - 3*b*d*x*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a
)^2*tan(b*c/d)^2 - 3*b*d*x*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*
b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 + 12*b*d*x*real_part(cos
_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)*tan(a)^2*tan(3
*b*c/d)*tan(b*c/d)^2 + 12*b*d*x*real_part(cos_integral(-6*b*x - 6*b*c/d))*t
an(3*b*x)^2*tan(b*x)^2*tan(3*a)*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 6*b*c*
imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2
*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 + 6*b*c*imag_part(cos_integral(-6*b*x -
 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c
/d)^2 - 12*b*c*sin_integral(6*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(
3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 + 3*b*d*x*real_part(cos_integral(

```
6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(3*b*c/d)^2*tan(b*c
/d)^2 + 3*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b
*x)^2*tan(3*a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 3*b*d*x*real_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(3*b*c/d)^2*tan
(b*c/d)^2 + 3*b*d*x*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*x)^2*
tan(b*x)^2*tan(3*a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 6*b*c*imag_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^2,x)
[Out] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^2, x)
```

3.161 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=235

$$-\frac{3b\cos(2a+2bx)}{32d^2(c+dx)}+\frac{3b\cos(6a+6bx)}{32d^2(c+dx)}+\frac{9b^2\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)\sin\left(6a-\frac{6bc}{d}\right)}{16d^3}-\frac{3b^2\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{16d^3}$$

```
[Out] -3/32*b*cos(2*b*x+2*a)/d^2/(d*x+c)+3/32*b*cos(6*b*x+6*a)/d^2/(d*x+c)-3/16*b
^2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^3+9/16*b^2*cos(6*a-6*b*c/d)*Si(6*b*
c/d+6*b*x)/d^3+9/16*b^2*Ci(6*b*c/d+6*b*x)*sin(6*a-6*b*c/d)/d^3-3/16*b^2*Ci(
2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^3-3/64*sin(2*b*x+2*a)/d/(d*x+c)^2+1/64*si
n(6*b*x+6*a)/d/(d*x+c)^2
```

Rubi [A]

time = 0.26, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$\frac{9b^2\sin\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{16d^3}-\frac{3b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{16d^3}-\frac{3b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{16d^3}+\frac{9b^2\cos\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6bc}{d}+6bx\right)}{16d^3}-\frac{3b\cos(2a+2bx)}{32d^2(c+dx)}+\frac{3b\cos(6a+6bx)}{32d^2(c+dx)}-\frac{3\sin(2a+2bx)}{64d(c+dx)^2}+\frac{\sin(6a+6bx)}{64d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] (-3*b*Cos[2*a + 2*b*x])/(32*d^2*(c + d*x)) + (3*b*Cos[6*a + 6*b*x])/(32*d^2
*(c + d*x)) + (9*b^2*CosIntegral[(6*b*c)/d + 6*b*x]*Sin[6*a - (6*b*c)/d])/(
16*d^3) - (3*b^2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(16*d
^3) - (3*Sin[2*a + 2*b*x])/(64*d*(c + d*x)^2) + Sin[6*a + 6*b*x]/(64*d*(c +
 d*x)^2) - (3*b^2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(16*
d^3) + (9*b^2*Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(16*d^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx &= \int \left(\frac{3\sin(2a+2bx)}{32(c+dx)^3}-\frac{\sin(6a+6bx)}{32(c+dx)^3}\right)dx\\
&= -\left(\frac{1}{32}\int \frac{\sin(6a+6bx)}{(c+dx)^3}\,dx\right)+\frac{3}{32}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx\\
&= -\frac{3\sin(2a+2bx)}{64d(c+dx)^2}+\frac{\sin(6a+6bx)}{64d(c+dx)^2}+\frac{(3b)\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{32d}-\frac{(3b)\int \frac{\cos(6a+}{(c+dx}}{32d}\\
&= -\frac{3b\cos(2a+2bx)}{32d^2(c+dx)}+\frac{3b\cos(6a+6bx)}{32d^2(c+dx)}-\frac{3\sin(2a+2bx)}{64d(c+dx)^2}+\frac{\sin(6a+6bx}{64d(c+dx)^2}\\
&= -\frac{3b\cos(2a+2bx)}{32d^2(c+dx)}+\frac{3b\cos(6a+6bx)}{32d^2(c+dx)}-\frac{3\sin(2a+2bx)}{64d(c+dx)^2}+\frac{\sin(6a+6bx}{64d(c+dx)^2}\\
&= -\frac{3b\cos(2a+2bx)}{32d^2(c+dx)}+\frac{3b\cos(6a+6bx)}{32d^2(c+dx)}+\frac{9b^2\operatorname{Ci}\left(\frac{6bc}{d}+6bx\right)\sin\left(6a-\frac{6bc}{d}\right)}{16d^3}
\end{aligned}$$

Mathematica [A]
time = 1.07, size = 239, normalized size = 1.02

$$\frac{-3d\cos(2bx)(2b(c+dx)\cos(2a)+d\sin(2a))+d\cos(6bx)(6b(c+dx)\cos(6a)+d\sin(6a))+3d(-d\cos(2a)+2b(c+dx)\sin(2a))\sin(2bx)+d(d\cos(6a)-6b(c+dx)\sin(6a))\sin(6bx)+6b^2(c+dx)^2\left(6\text{CosIntegral}\left(\frac{6b(c+dx)}{d}\right)\sin\left(6a-\frac{6bc}{d}\right)-2\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right)-2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)+6\cos\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6b(c+dx)}{d}\right)\right)}{64d^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] (-3*d*Cos[2*b*x]*(2*b*(c + d*x)*Cos[2*a] + d*Sin[2*a]) + d*Cos[6*b*x]*(6*b*
(c + d*x)*Cos[6*a] + d*Sin[6*a]) + 3*d*(-(d*Cos[2*a]) + 2*b*(c + d*x)*Sin[2
*a])*Sin[2*b*x] + d*(d*Cos[6*a] - 6*b*(c + d*x)*Sin[6*a])*Sin[6*b*x] + 6*b^
2*(c + d*x)^2*(6*CosIntegral[(6*b*(c + d*x))/d]*Sin[6*a - (6*b*c)/d] - 2*Co
sIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 2*Cos[2*a - (2*b*c)/d]*
SinIntegral[(2*b*(c + d*x))/d] + 6*Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c
 + d*x))/d]))/(64*d^3*(c + d*x)^2)
```

Maple [A]
time = 0.15, size = 329, normalized size = 1.40

method	result
derivativedivides	$-\frac{b^{3}\left(-\frac{3\sin(6bx+6a)}{(-ad+cb+d(bx+a))^{2}d}+\frac{-\frac{18\cos(6bx+6a)}{(-ad+cb+d(bx+a))d}-\frac{18\left(-\frac{6\,\mathrm{sinIntegral}\left(-6bx-6a-\frac{6(-ad+cb)}{d}\right)\cos\left(\frac{-6ad+6cb}{d}\right)}{d}-\frac{6\,\mathrm{cosineInteg}}{}\right.}{d}}{d}\right.}{192}$
default	$-\frac{b^{3}\left(-\frac{3\sin(6bx+6a)}{(-ad+cb+d(bx+a))^{2}d}+\frac{-\frac{18\cos(6bx+6a)}{(-ad+cb+d(bx+a))d}-\frac{18\left(-\frac{6\,\mathrm{sinIntegral}\left(-6bx-6a-\frac{6(-ad+cb)}{d}\right)\cos\left(\frac{-6ad+6cb}{d}\right)}{d}-\frac{6\,\mathrm{cosineInteg}}{}\right.}{d}}{d}\right.}{192}$
risch	$-\frac{9ib^{2}\mathrm{e}^{-\frac{6i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,6ibx+6ia-\frac{6i(ad-cb)}{d}\right)}{32d^{3}}+\frac{3ib^{2}\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{32d^{3}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/192*b^3*(-3*sin(6*b*x+6*a)/(-a*d+c*b+d*(b*x+a))^2/d+3*(-6*cos(6*b*x
+6*a)/(-a*d+c*b+d*(b*x+a))/d-6*(-6*Si(-6*b*x-6*a-6*(-a*d+b*c)/d)*cos(6*(-a*
d+b*c)/d)/d-6*Ci(6*b*x+6*a+6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d)/d)/d)+3/6
4*b^3*(-sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d+(-2*cos(2*b*x+2*a)/(-a*d+c*
b+d*(b*x+a))/d-2*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*
Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.46, size = 343, normalized size = 1.46

$$-\frac{3\,b^{3}\left(-i\,E_{3}\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_{3}\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^{3}\left(-i\,E_{3}\left(\frac{6\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_{3}\left(-\frac{6\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{6\,(bc-ad)}{d}\right)+3\,b^{3}\left(E_{3}\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_{3}\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-b^{3}\left(E_{3}\left(\frac{6\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_{3}\left(-\frac{6\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)}{64\,\left(b^{2}c^{2}d-2\,abcd^{2}+(bx+a)^{2}d^{3}+a^{2}d^{3}+2\,(bcd^{2}-ad^{3})(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/64*(3*b^3*(-I*exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
I*exp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a
*d)/d) - b^3*(-I*exp_integral_e(3, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
I*exp_integral_e(3, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-6*(b*c - a
*d)/d) + 3*b^3*(exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + e
xp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)
/d) - b^3*(exp_integral_e(3, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_in
tegral_e(3, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-6*(b*c - a*d)/d))/
((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)
*(b*x + a))*b)
```

Fricas [**A**]
time = 3.09, size = 434, normalized size = 1.85

$$\frac{96\left(bd^2x+bcd\right)\cos\left(bx+a\right)^6-144\left(bd^2x+bcd\right)\cos\left(bx+a\right)^4+48\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2+18\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left(-\frac{6\left(bc-ad\right)}{d}\right)\mathrm{Si}\left(\frac{6\left(bdx+bc\right)}{d}\right)-6\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left(-\frac{2\left(bc-ad\right)}{d}\right)\mathrm{Si}\left(\frac{2\left(bdx+bc\right)}{d}\right)+16\left(d^2\cos\left(bx+a\right)^5-d^2\cos\left(bx+a\right)^3\right)\sin\left(bx+a\right)-3\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ci}\left(\frac{2\left(bdx+bc\right)}{d}\right)+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ci}\left(-\frac{2\left(bdx+bc\right)}{d}\right)\right)\sin\left(-\frac{2\left(bc-ad\right)}{d}\right)+9\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ci}\left(\frac{6\left(bdx+bc\right)}{d}\right)+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ci}\left(-\frac{6\left(bdx+bc\right)}{d}\right)\right)\sin\left(-\frac{6\left(bc-ad\right)}{d}\right)}{32\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(96*(b*d^2*x + b*c*d)*cos(b*x + a)^6 - 144*(b*d^2*x + b*c*d)*cos(b*x +
 a)^4 + 48*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 18*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*cos(-6*(b*c - a*d)/d)*sin_integral(6*(b*d*x + b*c)/d) - 6*(b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*
x + b*c)/d) + 16*(d^2*cos(b*x + a)^5 - d^2*cos(b*x + a)^3)*sin(b*x + a) - 3
*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(2*(b*d*x + b*c)/d) + (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-2*(b*d*x + b*c)/d))*sin(
-2*(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(6
*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-6*(
b*d*x + b*c)/d))*sin(-6*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3\left(a+bx\right)\cos^3\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**3/(c + d*x)**3, x)
```

Giac [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.84, size = 111694, normalized size = 475.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")

[Out] 1/32*(9*b^2*d^2*x^2*imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*t
an(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*b^2*d^2*x^2*i
mag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*
tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 3*b^2*d^2*x^2*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d
)^2*tan(b*c/d)^2 - 9*b^2*d^2*x^2*imag_part(cos_integral(-6*b*x - 6*b*c/d))*
tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 1
8*b^2*d^2*x^2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3
*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 6*b^2*d^2*x^2*sin_integral(2*(
b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*
tan(b*c/d)^2 - 6*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3
*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*b^2*d^
2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan
(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 18*b^2*d^2*x^2*real_part(cos_i
ntegral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3
*b*c/d)*tan(b*c/d)^2 + 18*b^2*d^2*x^2*real_part(cos_integral(-6*b*x - 6*b*c
/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2
+ 6*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b
*x)^2*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*b^2*d^2*x^2*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a
)*tan(3*b*c/d)^2*tan(b*c/d)^2 - 18*b^2*d^2*x^2*real_part(cos_integral(6*b*x
 + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)*tan(a)^2*tan(3*b*c/d)^2*tan(b
*c/d)^2 - 18*b^2*d^2*x^2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*
x)^2*tan(b*x)^2*tan(3*a)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 18*b^2*c*d*
x*imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)
^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 6*b^2*c*d*x*imag_part(cos_integra
l(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d
)^2*tan(b*c/d)^2 + 6*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 18*
b^2*c*d*x*imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2
*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 36*b^2*c*d*x*sin_integra
l(6*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/
d)^2*tan(b*c/d)^2 - 12*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(3*b*x)
^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 9*b^2*d^2*x
^2*imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a
)^2*tan(a)^2*tan(3*b*c/d)^2 + 3*b^2*d^2*x^2*imag_part(cos_integral(2*b*x +
2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2 - 3*b^
2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2
*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2 - 9*b^2*d^2*x^2*imag_part(cos_integral(
-6*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)
^2 + 18*b^2*d^2*x^2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2
*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2 + 6*b^2*d^2*x^2*sin_integral(2*(b*d*x +
 b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2 - 12*b^
```

```
2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*
tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) + 12*b^2*d^2*x^2*imag_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)*tan(3
*b*c/d)^2*tan(b*c/d) - 24*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(3
*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) - 12*b^2*c*d
*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a
)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 12*b^2*c*d*x*real_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/
d)^2*tan(b*c/d) - 9*b^2*d^2*x^2*imag_part(cos_integral(6*b*x + 6*b*c/d))*ta
n(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 - 3*b^2*d^2*x^2*imag
_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan
(a)^2*tan(b*c/d)^2 + 3*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 + 9*b^2*d^2*x^2*
imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^
2*tan(a)^2*tan(b*c/d)^2 - 18*b^2*d^2*x^2*sin_integral(6*(b*d*x + b*c)/d)*ta
n(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 - 6*b^2*d^2*x^2*sin_
integral(2*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan
(b*c/d)^2 + 36*b^2*d^2*x^2*imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b
*x)^2*tan(b*x)^2*tan(3*a)*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 36*b^2*d^2*x
^2*imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*
a)*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 + 72*b^2*d^2*x^2*sin_integral(6*(b*d*
x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)*tan(a)^2*tan(3*b*c/d)*tan(b*c/
d)^2 + 36*b^2*c*d*x*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*t
an(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos{(a+b\,x)}^3\,\sin{(a+b\,x)}^3}{{(c+d\,x)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^3, x)
```

3.162 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=287

$$-\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2}+\frac{b\cos(6a+6bx)}{32d^2(c+dx)^2}-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{8d^4}+\frac{9b^3\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}}{8d^4}$$

```
[Out] 9/8*b^3*Ci(6*b*c/d+6*b*x)*cos(6*a-6*b*c/d)/d^4-1/8*b^3*Ci(2*b*c/d+2*b*x)*co
s(2*a-2*b*c/d)/d^4-1/32*b*cos(2*b*x+2*a)/d^2/(d*x+c)^2+1/32*b*cos(6*b*x+6*a
)/d^2/(d*x+c)^2-9/8*b^3*Si(6*b*c/d+6*b*x)*sin(6*a-6*b*c/d)/d^4+1/8*b^3*Si(2
*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^4-1/32*sin(2*b*x+2*a)/d/(d*x+c)^3+1/16*b^2
*sin(2*b*x+2*a)/d^3/(d*x+c)+1/96*sin(6*b*x+6*a)/d/(d*x+c)^3-3/16*b^2*sin(6*
b*x+6*a)/d^3/(d*x+c)
```

Rubi [A]

time = 0.29, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4491, 3378, 3384, 3380, 3383}

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{8d^4}+\frac{9b^3\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{8d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{8d^4}-\frac{9b^3\sin\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6bc}{d}+6bx\right)}{8d^4}+\frac{b^2\sin(2a+2bx)}{16d^3(c+dx)}-\frac{3b^2\sin(6a+6bx)}{16d^3(c+dx)}-\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2}+\frac{b\cos(6a+6bx)}{32d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{32d(c+dx)^3}+\frac{\sin(6a+6bx)}{96d(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] -1/32*(b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)^2) + (b*Cos[6*a + 6*b*x])/(32*d^2
*(c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(
8*d^4) + (9*b^3*Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*c)/d + 6*b*x])/(8*d^4
) - Sin[2*a + 2*b*x]/(32*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(16*d^3*(c
 + d*x)) + Sin[6*a + 6*b*x]/(96*d*(c + d*x)^3) - (3*b^2*Sin[6*a + 6*b*x])/(
16*d^3*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x
])/(8*d^4) - (9*b^3*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(8
*d^4)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx &= \int \left(\frac{3\sin(2a+2bx)}{32(c+dx)^4} - \frac{\sin(6a+6bx)}{32(c+dx)^4}\right) dx \\
&= -\left(\frac{1}{32}\int \frac{\sin(6a+6bx)}{(c+dx)^4}\,dx\right) + \frac{3}{32}\int \frac{\sin(2a+2bx)}{(c+dx)^4}\,dx \\
&= -\frac{\sin(2a+2bx)}{32d(c+dx)^3} + \frac{\sin(6a+6bx)}{96d(c+dx)^3} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^3}\,dx}{16d} - \frac{b\int \frac{\cos(6a+6bx)}{(c+dx)^3}\,dx}{16d} \\
&= -\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2} + \frac{b\cos(6a+6bx)}{32d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{32d(c+dx)^3} + \frac{\sin(6a+6bx)}{96d(c+dx)^3} - \\
&= -\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2} + \frac{b\cos(6a+6bx)}{32d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{32d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{16d^3(c+dx)} \\
&= -\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2} + \frac{b\cos(6a+6bx)}{32d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{32d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{16d^3(c+dx)} \\
&= -\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2} + \frac{b\cos(6a+6bx)}{32d^2(c+dx)^2} - \frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{8d^4} +
\end{aligned}$$

Mathematica [A]
time = 5.13, size = 554, normalized size = 1.93

$$\frac{-3bcd^2\cos(2(a+bx)) - 3bd^3x\cos(2(a+bx)) + 3bcd^2\cos(6(a+bx)) + 3bd^3x\cos(6(a+bx)) - 12b^3(c+dx)^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) + 108b^3(c+dx)^3\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6b(c+dx)}{d}\right) + 6b^2c^2d\sin(2(a+bx)) - 3d^3\sin(2(a+bx)) + 12b^2cd^2x\sin(2(a+bx)) + 6b^2d^3x^2\sin(2(a+bx)) - 18b^2c^2d\sin(6(a+bx)) + d^3\sin(6(a+bx)) - 36b^2cd^2x\sin(6(a+bx)) - 18b^2d^3x^2\sin(6(a+bx)) + 12b^3c^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) + 36b^3c^2dx\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) + 36b^3cd^2x^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) + 12b^3d^3x^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) - 108b^3c^3\sin\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6b(c+dx)}{d}\right) - 324b^3c^2dx\sin\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6b(c+dx)}{d}\right) - 324b^3cd^2x^2\sin\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6b(c+dx)}{d}\right) - 108b^3d^3x^3\sin\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6b(c+dx)}{d}\right)}{96d^4(c+dx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^4,x]

[Out] (-3*b*c*d^2*Cos[2*(a + b*x)] - 3*b*d^3*x*Cos[2*(a + b*x)] + 3*b*c*d^2*Cos[6*(a + b*x)] + 3*b*d^3*x*Cos[6*(a + b*x)] - 12*b^3*(c + d*x)^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + 108*b^3*(c + d*x)^3*Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*(c + d*x))/d] + 6*b^2*c^2*d*Sin[2*(a + b*x)] - 3*d^3*Sin[2*(a + b*x)] + 12*b^2*c*d^2*x*Sin[2*(a + b*x)] + 6*b^2*d^3*x^2*Sin[2*(a + b*x)] - 18*b^2*c^2*d*Sin[6*(a + b*x)] + d^3*Sin[6*(a + b*x)] - 36*b^2*c*d^2*x*Sin[6*(a + b*x)] - 18*b^2*d^3*x^2*Sin[6*(a + b*x)] + 12*b^3*c^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 36*b^3*c^2*d*x*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 36*b^3*c*d^2*x^2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 12*b^3*d^3*x^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] - 108*b^3*c^3*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/d] - 324*b^3*c^2*d*x*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/d] - 324*b^3*c*d^2*x^2*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/d] - 108*b^3*d^3*x^3*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/d])/(96*d^4*(c + d*x)^3)

Maple [A]
time = 0.18, size = 404, normalized size = 1.41

method	result
derivativedivides	$-\frac{b^4\left(-\frac{2\sin(6bx+6a)}{(-ad+cb+d(bx+a))^3 d}+\frac{-\frac{6\cos(6bx+6a)}{(-ad+cb+d(bx+a))^2 d}-\frac{6\left(-\frac{6\sin(6bx+6a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{36\,\mathrm{sinIntegral}\left(-6bx-6a-\frac{6(-ad+cb)}{d}\right)\sin\left(\cdots\right.}{d}}{\cdots}\right)}{d}}{\cdots}\right)}{192}$
default	$-\frac{b^4\left(-\frac{2\sin(6bx+6a)}{(-ad+cb+d(bx+a))^3 d}+\frac{-\frac{6\cos(6bx+6a)}{(-ad+cb+d(bx+a))^2 d}-\frac{6\left(-\frac{6\sin(6bx+6a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{36\,\mathrm{sinIntegral}\left(-6bx-6a-\frac{6(-ad+cb)}{d}\right)\sin\left(\cdots\right.}{d}}{\cdots}\right)}{d}}{\cdots}\right)}{192}$
risch	$-\frac{9b^3 e^{-\frac{6i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,6ibx+6ia-\frac{6i(ad-cb)}{d}\right)}{16d^4}+\frac{b^3 e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{16d^4}+\frac{b^3 e^{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x,method=_RETURNVERBOSE)

```
[Out] 1/b*(-1/192*b^4*(-2*sin(6*b*x+6*a)/(-a*d+c*b+d*(b*x+a))^3/d+2*(-3*cos(6*b*x
+6*a)/(-a*d+c*b+d*(b*x+a))^2/d-3*(-6*sin(6*b*x+6*a)/(-a*d+c*b+d*(b*x+a))/d+
6*(-6*Si(-6*b*x-6*a-6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d+6*Ci(6*b*x+6*a+6*
(-a*d+b*c)/d)*cos(6*(-a*d+b*c)/d)/d)/d)/d)/d)+3/64*b^4*(-2/3*sin(2*b*x+2*a)
/(-a*d+c*b+d*(b*x+a))^3/d+2/3*(-cos(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d-(-2
*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*
sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)
/d)/d)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.62, size = 393, normalized size = 1.37

$$-\frac{3\,b^4\left(-i\,E_4\left(\frac{2\left(-i\,bc-i\,(bx+a)d+i\,ad\right)}{d}\right)+i\,E_4\left(-\frac{2\left(-i\,bc-i\,(bx+a)d+i\,ad\right)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^4\left(-i\,E_4\left(\frac{6\left(-i\,bc-i\,(bx+a)d+i\,ad\right)}{d}\right)+i\,E_4\left(-\frac{6\left(-i\,bc-i\,(bx+a)d+i\,ad\right)}{d}\right)\right)\cos\left(-\frac{6\,(bc-ad)}{d}\right)+3\,b^4\left(E_4\left(\frac{2\left(-i\,bc-i\,(bx+a)d+i\,ad\right)}{d}\right)+E_4\left(-\frac{2\left(-i\,bc-i\,(bx+a)d+i\,ad\right)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-b^4\left(E_4\left(\frac{6\left(-i\,bc-i\,(bx+a)d+i\,ad\right)}{d}\right)+E_4\left(-\frac{6\left(-i\,bc-i\,(bx+a)d+i\,ad\right)}{d}\right)\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)}{64\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\,(bcd^3-ad^4)(bx+a)^2+3\,(b^2c^2d^2-2\,abcd^3+a^2d^4)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/64*(3*b^4*(-I*exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
I*exp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a
*d)/d) - b^4*(-I*exp_integral_e(4, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
I*exp_integral_e(4, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-6*(b*c - a
*d)/d) + 3*b^4*(exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + e
xp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)
/d) - b^4*(exp_integral_e(4, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_in
tegral_e(4, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-6*(b*c - a*d)/d))/
((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 +
 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*
(b*x + a))*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 638 vs. 2(267) = 534.
time = 1.66, size = 638, normalized size = 2.22

$$\frac{48\,(bd^3x+bcd^2)\cos(bx+a)^6-72\,(bd^3x+bcd^2)\cos(bx+a)^4+24\,(bd^3x+bcd^2)\cos(bx+a)^2-54\,(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\sin\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{6\,(bdx+bc)}{d}\right)+6\,(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)-3\left((b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+27\left((b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\mathrm{Ci}\left(\frac{6\,(bdx+bc)}{d}\right)+(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\mathrm{Ci}\left(-\frac{6\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{6\,(bc-ad)}{d}\right)-16\left((18\,b^2d^3x^2+36\,b^2cd^2x+18\,b^2c^2d-d^3)\cos(bx+a)^5-(18\,b^2d^3x^2+36\,b^2cd^2x+18\,b^2c^2d-d^3)\cos(bx+a)^3+3\,(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d)\cos(bx+a)\right)\sin(bx+a)}{48\,(d^7x^3+3\,cd^6x^2+3\,c^2d^5x+c^3d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/48*(48*(b*d^3*x + b*c*d^2)*cos(b*x + a)^6 - 72*(b*d^3*x + b*c*d^2)*cos(b*
x + a)^4 + 24*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 - 54*(b^3*d^3*x^3 + 3*b^3*
c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-6*(b*c - a*d)/d)*sin_integral(6*(
b*d*x + b*c)/d) + 6*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^
3)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - 3*((b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(2*(b*d*x + b*c)/d
) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(
-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d) + 27*((b^3*d^3*x^3 + 3*b^3*c*d^2
```

```
*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(6*(b*d*x + b*c)/d) + (b^3*d^3*
x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-6*(b*d*x + b
*c)/d))*cos(-6*(b*c - a*d)/d) - 16*((18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b
^2*c^2*d - d^3)*cos(b*x + a)^5 - (18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*
c^2*d - d^3)*cos(b*x + a)^3 + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*c
os(b*x + a))*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.80, size = 157526, normalized size = 548.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/48*(27*b^3*d^3*x^3*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*
tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*b^3*d^3*x^3*
real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2
*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*b^3*d^3*x^3*real_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/
d)^2*tan(b*c/d)^2 + 27*b^3*d^3*x^3*real_part(cos_integral(-6*b*x - 6*b*c/d)
)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 +
 6*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*
x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*b^3*d^3*x^3*imag_par
t(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)
^2*tan(3*b*c/d)^2*tan(b*c/d) + 12*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/
d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) -
54*b^3*d^3*x^3*imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*
x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 + 54*b^3*d^3*x^3*imag_pa
rt(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a
)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 108*b^3*d^3*x^3*sin_integral(6*(b*d*x + b*c
)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2
- 6*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b
*x)^2*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*b^3*d^3*x^3*imag_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a
)*tan(3*b*c/d)^2*tan(b*c/d)^2 - 12*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)
```

/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 + 54*b^3*d^3*x^3*imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 54*b^3*d^3*x^3*imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 108*b^3*d^3*x^3*sin_integral(6*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 81*b^3*c*d^2*x^2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 9*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 9*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 81*b^3*c*d^2*x^2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 27*b^3*d^3*x^3*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2 + 3*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2 + 3*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2 + 27*b^3*d^3*x^3*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2 - 12*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) - 12*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) + 18*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 18*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 36*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 27*b^3*d^3*x^3*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 - 3*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 - 3*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 - 27*b^3*d^3*x^3*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 + 108*b^3*d^3*x^3*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 + 108*b^3*d^3*x^3*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 162*b^3*c*d^2*x^2*imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 + 162*b^3*c*d^2*x^2*imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 324*b^3*c*d^2*x^2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 + 27*b^3*d^3*x^3*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 3*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*ta

```
n(b*x)^2*tan(3*a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2...
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos{(a+b\,x)}^3\,\sin{(a+b\,x)}^3}{{(c+d\,x)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^4,x)
[Out] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^4, x)
```

3.163 $\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=152

$$\frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-3-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)}{b}$$

```
[Out] 2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*b*
(d*x+c)/d)^m)+2^(-3-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a-b*
c/d))/((I*b*(d*x+c)/d)^m)+Unintegrable((d*x+c)^m*cot(b*x+a),x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c +
 d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (2^(-3 - m)*(c + d*x)^m*Gamma[1 +
 m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^
m) + Defer[Int][(c + d*x)^m*Cot[a + b*x], x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx &= \int (c+dx)^m \cot(a+bx)\,dx - \int (c+dx)^m \cos(a+bx)\sin(a+b\\
&= \int (c+dx)^m \cot(a+bx)\,dx - \int \frac{1}{2}(c+dx)^m \sin(2a+2bx)\,dx\\
&= -\left(\frac{1}{2}\int (c+dx)^m \sin(2a+2bx)\,dx\right) + \int (c+dx)^m \cot(a+bx)\\
&= -\left(\frac{1}{4}i\int e^{-i(2a+2bx)}(c+dx)^m\,dx\right) + \frac{1}{4}i\int e^{i(2a+2bx)}(c+dx)^m\,dx\\
&= \frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}+
\end{aligned}$$

Mathematica [A]
time = 7.31, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Cot[a + b*x], x]
```

Maple [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cos^2\left(bx+a\right)\right)\cot\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cos^2\left(a+bx\right)\cot\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**2*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*cos(a + b*x)**2*cot(a + b*x), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)^2\cot(a+b\,x)\,\left(c+d\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^m,x)
[Out] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^m, x)
```

3.164 $\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=307

$$-\frac{3cd^3x}{2b^3}-\frac{3d^4x^2}{4b^3}+\frac{(c+dx)^4}{4b}-\frac{i(c+dx)^5}{5d}+\frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{2id(c+dx)^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}+\frac{3\cdots}{}$$

```
[Out] -3/2*c*d^3*x/b^3-3/4*d^4*x^2/b^3+1/4*(d*x+c)^4/b-1/5*I*(d*x+c)^5/d+(d*x+c)^
4*ln(1-exp(2*I*(b*x+a)))/b-2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^2+
3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(b*x+a)))/b^3+3*I*d^3*(d*x+c)*polylog(4,e
xp(2*I*(b*x+a)))/b^4-3/2*d^4*polylog(5,exp(2*I*(b*x+a)))/b^5+3/2*d^3*(d*x+c
)*cos(b*x+a)*sin(b*x+a)/b^4-d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2-3/4*d^4*s
in(b*x+a)^2/b^5+3/2*d^2*(d*x+c)^2*sin(b*x+a)^2/b^3-1/2*(d*x+c)^4*sin(b*x+a)
^2/b
```

Rubi [A]
time = 0.23, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4493, 4489, 3392, 32, 3391, 3798, 2221, 2611, 6744, 2320, 6724}

$$-\frac{3d^4\text{Li}_5\left(e^{2i(a+bx)}\right)}{2b^5}-\frac{3d^4\sin^2(a+bx)}{4b^5}+\frac{3id^3(c+dx)\text{Li}_4\left(e^{2i(a+bx)}\right)}{b^4}+\frac{3d^3(c+dx)\sin(a+bx)\cos(a+bx)}{2b^4}+\frac{3d^2(c+dx)^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{b^3}+\frac{3d^2(c+dx)^2\sin^2(a+bx)}{2b^3}-\frac{2id(c+dx)^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{d(c+dx)^3\sin(a+bx)\cos(a+bx)}{b^2}+\frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{3cd^3x}{2b^3}-\frac{3d^4x^2}{4b^3}+\frac{(c+dx)^4}{4b}-\frac{i(c+dx)^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (-3*c*d^3*x)/(2*b^3) - (3*d^4*x^2)/(4*b^3) + (c + d*x)^4/(4*b) - ((I/5)*(c
+ d*x)^5)/d + ((c + d*x)^4*Log[1 - E^((2*I)*(a + b*x))])/b - ((2*I)*d*(c +
d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2*PolyLog[3,
 E^((2*I)*(a + b*x))])/b^3 + ((3*I)*d^3*(c + d*x)*PolyLog[4, E^((2*I)*(a +
b*x))])/b^4 - (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5) + (3*d^3*(c +
 d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b^4) - (d*(c + d*x)^3*Cos[a + b*x]*Sin[
a + b*x])/b^2 - (3*d^4*Sin[a + b*x]^2)/(4*b^5) + (3*d^2*(c + d*x)^2*Sin[a +
 b*x]^2)/(2*b^3) - ((c + d*x)^4*Sin[a + b*x]^2)/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4\cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)^4\cot(a+bx)\,dx - \int (c+dx)^4\cos(a+bx)\sin(a+bx)\,d$$

$$= -\frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^4}{1-e^{2i(a+bx)}}\,dx$$

$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)^3\cos(a+b}{b^2}$$

$$= \frac{(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+d}{}$$

$$= -\frac{3cd^3x}{2b^3} - \frac{3d^4x^2}{4b^3} + \frac{(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{}\right.}{b}$$

$$= -\frac{3cd^3x}{2b^3} - \frac{3d^4x^2}{4b^3} + \frac{(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{}\right.}{b}$$

$$= -\frac{3cd^3x}{2b^3} - \frac{3d^4x^2}{4b^3} + \frac{(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{}\right.}{b}$$

$$= -\frac{3cd^3x}{2b^3} - \frac{3d^4x^2}{4b^3} + \frac{(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{}\right.}{b}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2486 vs. $2(307) = 614$.

time = 6.36, size = 2486, normalized size = 8.10

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] -1/2*(c^2*d^2*Csc[a]*(2*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(-1 + E^((2*I)*a
))*Log[1 - E^((2*I)*(a + b*x))]) + 6*b*(-1 + E^((2*I)*a))*x*PolyLog[2, E^((
2*I)*(a + b*x))] + (3*I)*(-1 + E^((2*I)*a))*PolyLog[3, E^((2*I)*(a + b*x))]
))/(b^3*E^(I*a)) - c*d^3*E^(I*a)*Csc[a]*(x^4 + (-1 + E^((-2*I)*a))*x^4 + ((
-1 + E^((2*I)*a))*(2*b^4*x^4 + (4*I)*b^3*x^3*Log[1 - E^((2*I)*(a + b*x))] +
 6*b^2*x^2*PolyLog[2, E^((2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[3, E^((2*I)*
(a + b*x))] - 3*PolyLog[4, E^((2*I)*(a + b*x))]))/(2*b^4*E^((2*I)*a))) - (d
^4*E^(I*a)*Csc[a]*(x^5 + (-1 + E^((-2*I)*a))*x^5 + ((-1 + E^((2*I)*a))*(4*b
^5*x^5 + (10*I)*b^4*x^4*Log[1 - E^((2*I)*(a + b*x))] + 20*b^3*x^3*PolyLog[2
, E^((2*I)*(a + b*x))] + (30*I)*b^2*x^2*PolyLog[3, E^((2*I)*(a + b*x))] - 3
0*b*x*PolyLog[4, E^((2*I)*(a + b*x))] - (15*I)*PolyLog[5, E^((2*I)*(a + b*x
))]))/(4*b^5*E^((2*I)*a))))/5 + (c^4*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*S
in[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + Csc[a]*(Cos[2
*a + 2*b*x]/(160*b^5) - ((I/160)*Sin[2*a + 2*b*x])/b^5)*(80*b^5*c^4*x*Cos[a
 + 2*b*x] + 160*b^5*c^3*d*x^2*Cos[a + 2*b*x] + 160*b^5*c^2*d^2*x^3*Cos[a +
2*b*x] + 80*b^5*c*d^3*x^4*Cos[a + 2*b*x] + 16*b^5*d^4*x^5*Cos[a + 2*b*x] +
80*b^5*c^4*x*Cos[3*a + 2*b*x] + 160*b^5*c^3*d*x^2*Cos[3*a + 2*b*x] + 160*b^
5*c^2*d^2*x^3*Cos[3*a + 2*b*x] + 80*b^5*c*d^3*x^4*Cos[3*a + 2*b*x] + 16*b^5
*d^4*x^5*Cos[3*a + 2*b*x] + (10*I)*b^4*c^4*Cos[3*a + 4*b*x] - 20*b^3*c^3*d*
Cos[3*a + 4*b*x] - (30*I)*b^2*c^2*d^2*Cos[3*a + 4*b*x] + 30*b*c*d^3*Cos[3*a
 + 4*b*x] + (15*I)*d^4*Cos[3*a + 4*b*x] + (40*I)*b^4*c^3*d*x*Cos[3*a + 4*b*
x] - 60*b^3*c^2*d^2*x*Cos[3*a + 4*b*x] - (60*I)*b^2*c*d^3*x*Cos[3*a + 4*b*x
] + 30*b*d^4*x*Cos[3*a + 4*b*x] + (60*I)*b^4*c^2*d^2*x^2*Cos[3*a + 4*b*x] -
 60*b^3*c*d^3*x^2*Cos[3*a + 4*b*x] - (30*I)*b^2*d^4*x^2*Cos[3*a + 4*b*x] +
(40*I)*b^4*c*d^3*x^3*Cos[3*a + 4*b*x] - 20*b^3*d^4*x^3*Cos[3*a + 4*b*x] + (
10*I)*b^4*d^4*x^4*Cos[3*a + 4*b*x] - (10*I)*b^4*c^4*Cos[5*a + 4*b*x] + 20*b
^3*c^3*d*Cos[5*a + 4*b*x] + (30*I)*b^2*c^2*d^2*Cos[5*a + 4*b*x] - 30*b*c*d^
3*Cos[5*a + 4*b*x] - (15*I)*d^4*Cos[5*a + 4*b*x] - (40*I)*b^4*c^3*d*x*Cos[5
*a + 4*b*x] + 60*b^3*c^2*d^2*x*Cos[5*a + 4*b*x] + (60*I)*b^2*c*d^3*x*Cos[5*
a + 4*b*x] - 30*b*d^4*x*Cos[5*a + 4*b*x] - (60*I)*b^4*c^2*d^2*x^2*Cos[5*a +
 4*b*x] + 60*b^3*c*d^3*x^2*Cos[5*a + 4*b*x] + (30*I)*b^2*d^4*x^2*Cos[5*a +
4*b*x] - (40*I)*b^4*c*d^3*x^3*Cos[5*a + 4*b*x] + 20*b^3*d^4*x^3*Cos[5*a + 4
*b*x] - (10*I)*b^4*d^4*x^4*Cos[5*a + 4*b*x] + 20*b^4*c^4*Sin[a] - (40*I)*b^
3*c^3*d*Sin[a] - 60*b^2*c^2*d^2*Sin[a] + (60*I)*b*c*d^3*Sin[a] + 30*d^4*Sin
[a] + 80*b^4*c^3*d*x*Sin[a] - (120*I)*b^3*c^2*d^2*x*Sin[a] - 120*b^2*c*d^3*
x*Sin[a] + (60*I)*b*d^4*x*Sin[a] + 120*b^4*c^2*d^2*x^2*Sin[a] - (120*I)*b^3
*c*d^3*x^2*Sin[a] - 60*b^2*d^4*x^2*Sin[a] + 80*b^4*c*d^3*x^3*Sin[a] - (40*I
)*b^3*d^4*x^3*Sin[a] + 20*b^4*d^4*x^4*Sin[a] + (80*I)*b^5*c^4*x*Sin[a + 2*b
```

```
*x] + (160*I)*b^5*c^3*d*x^2*Sin[a + 2*b*x] + (160*I)*b^5*c^2*d^2*x^3*Sin[a
+ 2*b*x] + (80*I)*b^5*c*d^3*x^4*Sin[a + 2*b*x] + (16*I)*b^5*d^4*x^5*Sin[a +
 2*b*x] + (80*I)*b^5*c^4*x*Sin[3*a + 2*b*x] + (160*I)*b^5*c^3*d*x^2*Sin[3*a
 + 2*b*x] + (160*I)*b^5*c^2*d^2*x^3*Sin[3*a + 2*b*x] + (80*I)*b^5*c*d^3*x^4
*Sin[3*a + 2*b*x] + (16*I)*b^5*d^4*x^5*Sin[3*a + 2*b*x] - 10*b^4*c^4*Sin[3*
a + 4*b*x] - (20*I)*b^3*c^3*d*Sin[3*a + 4*b*x] + 30*b^2*c^2*d^2*Sin[3*a + 4
*b*x] + (30*I)*b*c*d^3*Sin[3*a + 4*b*x] - 15*d^4*Sin[3*a + 4*b*x] - 40*b^4*
c^3*d*x*Sin[3*a + 4*b*x] - (60*I)*b^3*c^2*d^2*x*Sin[3*a + 4*b*x] + 60*b^2*c
*d^3*x*Sin[3*a + 4*b*x] + (30*I)*b*d^4*x*Sin[3*a + 4*b*x] - 60*b^4*c^2*d^2*
x^2*Sin[3*a + 4*b*x] - (60*I)*b^3*c*d^3*x^2*Sin[3*a + 4*b*x] + 30*b^2*d^4*x
^2*Sin[3*a + 4*b*x] - 40*b^4*c*d^3*x^3*Sin[3*a + 4*b*x] - (20*I)*b^3*d^4*x^
3*Sin[3*a + 4*b*x] - 10*b^4*d^4*x^4*Sin[3*a + 4*b*x] + 10*b^4*c^4*Sin[5*a +
 4*b*x] + (20*I)*b^3*c^3*d*Sin[5*a + 4*b*x] - 30*b^2*c^2*d^2*Sin[5*a + 4*b*
x] - (30*I)*b*c*d^3*Sin[5*a + 4*b*x] + 15*d^4*Sin[5*a + 4*b*x] + 40*b^4*c^3
*d*x*Sin[5*a + 4*b*x] + (60*I)*b^3*c^2*d^2*x*Sin[5*a + 4*b*x] - 60*b^2*c*d^
3*x*Sin[5*a + 4*b*x] - (30*I)*b*d^4*x*Sin[5*a + 4*b*x] + 60*b^4*c^2*d^2*x^2
*Sin[5*a + 4*b*x] + (60*I)*b^3*c*d^3*x^2*Sin[5*a + 4*b*x] - 30*b^2*d^4*x^2*
Sin[5*a + 4*b*x] + 40*b^4*c*d^3*x^3*Sin[5*a + 4*b*x] + (20*I)*b^3*d^4*x^3*S
in[5*a + 4*b*x] + 10*b^4*d^4*x^4*Sin[5*a + 4*b*x]) - (2*c^3*d*Csc[a]*Sec[a]
*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[
1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcT
an[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Ta
n[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 +
Tan[a]^2]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1334 vs. 2(276) = 552.
time = 0.62, size = 1335, normalized size = 4.35

method	result	size
risch	Expression too large to display	1335

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b^4*d*(2*b^2*d^3*x^3+6*b^2*c*d^2*x^2+6*b^2*c^2*d*x+2*b^2*c^3-3*d^3*x-3
*c*d^2)*sin(2*b*x+2*a)+4/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3+4/b^4*c*d^3*ln(1-
exp(I*(b*x+a)))*a^3+4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3-12*I/b^2*c^2*d^2*pol
ylog(2,exp(I*(b*x+a)))*x-12*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-12*I
/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-12*I/b^2*c*d^3*polylog(2,exp(I*(b
*x+a)))*x^2-8*I/b^3*c*d^3*a^3*x+12*I/b^2*d^2*c^2*a^2*x-8*I/b*c^3*d*a*x-12/b
^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)))-4/b^2*c^3*d*a*ln(exp(I*(b*x+a))-1)+8/b^2*
c^3*d*a*ln(exp(I*(b*x+a)))-4/b^4*c*d^3*a^3*ln(exp(I*(b*x+a))-1)-4*I/b^2*d^4
*polylog(2,exp(I*(b*x+a)))*x^3-4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+2
4*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x+24*I/b^4*d^4*polylog(4,exp(I*(b*x+
```

```
a)))*x+2*I/b^4*d^4*a^4*x+24*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))-4*I/b^2*
c^3*d*polylog(2,exp(I*(b*x+a)))+24*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))-4*
I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))-4*I/b^2*c^3*d*a^2-6*I/b^4*c*d^3*a^4+
8*I/b^3*d^2*c^2*a^3+1/8*(2*b^4*d^4*x^4+8*b^4*c*d^3*x^3+12*b^4*c^2*d^2*x^2+8
*b^4*c^3*d*x+2*b^4*c^4-6*b^2*d^4*x^2-12*b^2*c*d^3*x-6*b^2*c^2*d^2+3*d^4)/b^
5*cos(2*b*x+2*a)+I*c^4*x+1/5*I/d*c^5-24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-
24*d^4*polylog(5,exp(I*(b*x+a)))/b^5+1/b*c^4*ln(exp(I*(b*x+a))-1)+1/b*c^4*l
n(exp(I*(b*x+a))+1)-2/b*c^4*ln(exp(I*(b*x+a)))-1/5*I*d^4*x^5-2/b^5*d^4*a^4*
ln(exp(I*(b*x+a)))+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))+12/b^3*c^2*d^2*
polylog(3,-exp(I*(b*x+a)))+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)-1/b^5*d^4*a^4
*ln(1-exp(I*(b*x+a)))+12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2+12/b^3*d^4*
polylog(3,exp(I*(b*x+a)))*x^2+8/5*I/b^5*a^5*d^4-I*d^3*c*x^4-2*I*d^2*c^2*x^3
-2*I*d*c^3*x^2+4/b*c^3*d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*(b*x
+a)))*a+4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+1
/b*d^4*ln(exp(I*(b*x+a))+1)*x^4+8/b^4*c*d^3*a^3*ln(exp(I*(b*x+a)))+6/b^3*c^
2*d^2*a^2*ln(exp(I*(b*x+a))-1)+6/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2+6/b*c^2
*d^2*ln(1-exp(I*(b*x+a)))*x^2-6/b^3*c^2*d^2*ln(1-exp(I*(b*x+a)))*a^2+24/b^3
*c*d^3*polylog(3,-exp(I*(b*x+a)))*x+24/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*
x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1654 vs. $2(272) = 544$.
time = 0.48, size = 1654, normalized size = 5.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/40*(20*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*c^4 - 80*(sin(b*x + a)^2 -
 log(sin(b*x + a)^2))*a*c^3*d/b + 120*(sin(b*x + a)^2 - log(sin(b*x + a)^2)
)*a^2*c^2*d^2/b^2 - 80*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a^3*c*d^3/b^3
 + 20*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a^4*d^4/b^4 - (-8*I*(b*x + a)^
5*d^4 - 40*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^4 - 960*d^4*polylog(5, -e^(I*b*x
 + I*a)) - 960*d^4*polylog(5, e^(I*b*x + I*a)) - 80*(I*b^2*c^2*d^2 - 2*I*a*
b*c*d^3 + I*a^2*d^4)*(b*x + a)^3 - 80*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*
I*a^2*b*c*d^3 - I*a^3*d^4)*(b*x + a)^2 - 40*(-I*(b*x + a)^4*d^4 + 4*(-I*b*c
*d^3 + I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4
)*(b*x + a)^2 + 4*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b*c*d^3 + I*a
^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 40*(I*(b*x + a
)^4*d^4 + 4*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^3 + 6*(I*b^2*c^2*d^2 - 2*I*a*b*
c*d^3 + I*a^2*d^4)*(b*x + a)^2 + 4*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a
^2*b*c*d^3 - I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1)
 + 5*(2*(b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 3*(2*a^2 - 1)*d^4
+ 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(2*b^2*c^2*d^2 - 4*a*b*c*d^3 + (2*a^2
```

```
 - 1)*d^4)*(b*x + a)^2 + 4*(2*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 3*(2*a^2 - 1)*b
*c*d^3 - (2*a^3 - 3*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - 160*(I*b^3*c^3*d
- 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 + I*(b*x + a)^3*d^4 - I*a^3*d^4 + 3*(
I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2
*d^4)*(b*x + a))*dilog(-e^(I*b*x + I*a)) - 160*(I*b^3*c^3*d - 3*I*a*b^2*c^2
*d^2 + 3*I*a^2*b*c*d^3 + I*(b*x + a)^3*d^4 - I*a^3*d^4 + 3*(I*b*c*d^3 - I*a
*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a)
)*dilog(e^(I*b*x + I*a)) + 20*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x +
 a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d
- 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(b*x + a)^2
+ sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 20*((b*x + a)^4*d^4 + 4*(b*c*d^3 -
 a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 +
 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(c
os(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 960*(-I*b*c*d^3 - I*
(b*x + a)*d^4 + I*a*d^4)*polylog(4, -e^(I*b*x + I*a)) - 960*(-I*b*c*d^3 - I
*(b*x + a)*d^4 + I*a*d^4)*polylog(4, e^(I*b*x + I*a)) + 480*(b^2*c^2*d^2 -
2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*po
lylog(3, -e^(I*b*x + I*a)) + 480*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d
^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, e^(I*b*x + I*a)) -
 10*(2*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 2*(b*x + a)^3*d^4 + 3*(2*a^2 - 1)*b*c*
d^3 - (2*a^3 - 3*a)*d^4 + 6*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(2*b^2*c^2*d^
2 - 4*a*b*c*d^3 + (2*a^2 - 1)*d^4)*(b*x + a))*sin(2*b*x + 2*a))/b^4)/b
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1453 vs. 2(272) = 544.
time = 3.99, size = 1453, normalized size = 4.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 48*d^4*polylog(5, cos(b*x + a) + I*si
n(b*x + a)) + 48*d^4*polylog(5, cos(b*x + a) - I*sin(b*x + a)) + 48*d^4*pol
ylog(5, -cos(b*x + a) + I*sin(b*x + a)) + 48*d^4*polylog(5, -cos(b*x + a) -
 I*sin(b*x + a)) + 3*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - (2*b^4*d^4*x^4 + 8*b^4
*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^2*d^4
)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^2 + 2*(2*b^3*d^4*x^3
+ 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*
cos(b*x + a)*sin(b*x + a) + 2*(2*b^4*c^3*d - 3*b^2*c*d^3)*x + 8*(I*b^3*d^4*
x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(cos(b*x +
a) + I*sin(b*x + a)) + 8*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*
d^2*x - I*b^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + 8*(-I*b^3*d^4*x
^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog(-cos(b*x +
a) + I*sin(b*x + a)) + 8*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d
```

```
^2*x + I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 2*(b^4*d^4*x^4
+ 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*
x + a) + I*sin(b*x + a) + 1) - 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2
*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x + a) - I*sin(b*x + a) + 1)
- 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*(b^4*c^4 - 4*a*b^3*c
^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/2*cos(b*x + a) -
 1/2*I*sin(b*x + a) + 1/2) - 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d
^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3
- a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b^4*d^4*x^4 + 4*b^4
*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*
c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)
+ 48*(-I*b*d^4*x - I*b*c*d^3)*polylog(4, cos(b*x + a) + I*sin(b*x + a)) + 4
8*(I*b*d^4*x + I*b*c*d^3)*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 48*(I
*b*d^4*x + I*b*c*d^3)*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) + 48*(-I*b
*d^4*x - I*b*c*d^3)*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) - 24*(b^2*d^
4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, cos(b*x + a) + I*sin(b*x +
a)) - 24*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, cos(b*x + a
) - I*sin(b*x + a)) - 24*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylo
g(3, -cos(b*x + a) + I*sin(b*x + a)) - 24*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^
2*c^2*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/b^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2*cot(b*x+a),x)
[Out] Integral((c + d*x)**4*cos(a + b*x)**2*cot(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^4*cos(b*x + a)^2*cot(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\mathrm{cot}\left(a+b\,x\right)\,{\left(c+d\,x\right)}^4 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^4,x)
```

```
[Out] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^4, x)
```

3.165 $\int (c+dx)^3 \cos^2(a+bx) \cot(a+bx)\, dx$

Optimal. Leaf size=246

$$-\frac{3d^3x}{8b^3}+\frac{(c+dx)^3}{4b}-\frac{i(c+dx)^4}{4d}+\frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{3id(c+dx)^2\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{3d^2(c+d}{}$$

```
[Out] -3/8*d^3*x/b^3+1/4*(d*x+c)^3/b-1/4*I*(d*x+c)^4/d+(d*x+c)^3*ln(1-exp(2*I*(b*
x+a)))/b-3/2*I*d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2+3/2*d^2*(d*x+c)*
polylog(3,exp(2*I*(b*x+a)))/b^3+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4+3
/8*d^3*cos(b*x+a)*sin(b*x+a)/b^4-3/4*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^2+
3/4*d^2*(d*x+c)*sin(b*x+a)^2/b^3-1/2*(d*x+c)^3*sin(b*x+a)^2/b
```

Rubi [A]
time = 0.20, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {4493, 4489, 3392, 32, 2715, 8, 3798, 2221, 2611, 6744, 2320, 6724}

$$\frac{3id^3\mathrm{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^3\sin(a+bx)\cos(a+bx)}{8b^4}+\frac{3d^2(c+dx)\mathrm{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3}-\frac{3id(c+dx)^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{3d(c+dx)^2\sin(a+bx)\cos(a+bx)}{4b^2}+\frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{3d^3x}{8b^3}+\frac{(c+dx)^3}{4b}-\frac{i(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (-3*d^3*x)/(8*b^3) + (c + d*x)^3/(4*b) - ((I/4)*(c + d*x)^4)/d + ((c + d*x)
^3*Log[1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^
((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])
/(2*b^3) + (((3*I)/4)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4 + (3*d^3*Cos
[a + b*x]*Sin[a + b*x])/(8*b^4) - (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x
])/(4*b^2) + (3*d^2*(c + d*x)*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^3*Sin[a
+ b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
```

```
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)^3\cot(a+bx)\,dx - \int (c+dx)^3\cos(a+bx)\sin(a+bx)$$

$$= -\frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1-e^{2i(a+bx)}}\,d$$

$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\cos(a}{4b}$$

$$= \frac{(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+}{}$$

$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b}$$

$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b}$$

$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1712 vs. 2(246) = 492.

time = 6.30, size = 1712, normalized size = 6.96

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] -1/4*(c*d^2*Csc[a]*(2*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(-1 + E^((2*I)*a))
*Log[1 - E^((2*I)*(a + b*x))]) + 6*b*(-1 + E^((2*I)*a))*x*PolyLog[2, E^((2*
I)*(a + b*x))] + (3*I)*(-1 + E^((2*I)*a))*PolyLog[3, E^((2*I)*(a + b*x))]))
/(b^3*E^(I*a)) - (d^3*E^(I*a)*Csc[a]*(x^4 + (-1 + E^((-2*I)*a))*x^4 + ((-1
+ E^((2*I)*a))*(2*b^4*x^4 + (4*I)*b^3*x^3*Log[1 - E^((2*I)*(a + b*x))] + 6*
b^2*x^2*PolyLog[2, E^((2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[3, E^((2*I)*(a
+ b*x))] - 3*PolyLog[4, E^((2*I)*(a + b*x))]))/(2*b^4*E^((2*I)*a))))/4 + (c
^3*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/
(b*(Cos[a]^2 + Sin[a]^2)) + Csc[a]*(Cos[2*a + 2*b*x]/(64*b^4) - ((I/64)*Sin
[2*a + 2*b*x])/b^4)*(32*b^4*c^3*x*Cos[a + 2*b*x] + 48*b^4*c^2*d*x^2*Cos[a +
 2*b*x] + 32*b^4*c*d^2*x^3*Cos[a + 2*b*x] + 8*b^4*d^3*x^4*Cos[a + 2*b*x] +
32*b^4*c^3*x*Cos[3*a + 2*b*x] + 48*b^4*c^2*d*x^2*Cos[3*a + 2*b*x] + 32*b^4*
c*d^2*x^3*Cos[3*a + 2*b*x] + 8*b^4*d^3*x^4*Cos[3*a + 2*b*x] + (4*I)*b^3*c^3
*Cos[3*a + 4*b*x] - 6*b^2*c^2*d*Cos[3*a + 4*b*x] - (6*I)*b*c*d^2*Cos[3*a +
4*b*x] + 3*d^3*Cos[3*a + 4*b*x] + (12*I)*b^3*c^2*d*x*Cos[3*a + 4*b*x] - 12*
b^2*c*d^2*x*Cos[3*a + 4*b*x] - (6*I)*b*d^3*x*Cos[3*a + 4*b*x] + (12*I)*b^3*
c*d^2*x^2*Cos[3*a + 4*b*x] - 6*b^2*d^3*x^2*Cos[3*a + 4*b*x] + (4*I)*b^3*d^3
*x^3*Cos[3*a + 4*b*x] - (4*I)*b^3*c^3*Cos[5*a + 4*b*x] + 6*b^2*c^2*d*Cos[5*
a + 4*b*x] + (6*I)*b*c*d^2*Cos[5*a + 4*b*x] - 3*d^3*Cos[5*a + 4*b*x] - (12*
I)*b^3*c^2*d*x*Cos[5*a + 4*b*x] + 12*b^2*c*d^2*x*Cos[5*a + 4*b*x] + (6*I)*b
*d^3*x*Cos[5*a + 4*b*x] - (12*I)*b^3*c*d^2*x^2*Cos[5*a + 4*b*x] + 6*b^2*d^3
*x^2*Cos[5*a + 4*b*x] - (4*I)*b^3*d^3*x^3*Cos[5*a + 4*b*x] + 8*b^3*c^3*Sin[
a] - (12*I)*b^2*c^2*d*Sin[a] - 12*b*c*d^2*Sin[a] + (6*I)*d^3*Sin[a] + 24*b^
3*c^2*d*x*Sin[a] - (24*I)*b^2*c*d^2*x*Sin[a] - 12*b*d^3*x*Sin[a] + 24*b^3*c
*d^2*x^2*Sin[a] - (12*I)*b^2*d^3*x^2*Sin[a] + 8*b^3*d^3*x^3*Sin[a] + (32*I)
*b^4*c^3*x*Sin[a + 2*b*x] + (48*I)*b^4*c^2*d*x^2*Sin[a + 2*b*x] + (32*I)*b^
4*c*d^2*x^3*Sin[a + 2*b*x] + (8*I)*b^4*d^3*x^4*Sin[a + 2*b*x] + (32*I)*b^4*
c^3*x*Sin[3*a + 2*b*x] + (48*I)*b^4*c^2*d*x^2*Sin[3*a + 2*b*x] + (32*I)*b^4
*c*d^2*x^3*Sin[3*a + 2*b*x] + (8*I)*b^4*d^3*x^4*Sin[3*a + 2*b*x] - 4*b^3*c^
3*Sin[3*a + 4*b*x] - (6*I)*b^2*c^2*d*Sin[3*a + 4*b*x] + 6*b*c*d^2*Sin[3*a +
 4*b*x] + (3*I)*d^3*Sin[3*a + 4*b*x] - 12*b^3*c^2*d*x*Sin[3*a + 4*b*x] - (1
2*I)*b^2*c*d^2*x*Sin[3*a + 4*b*x] + 6*b*d^3*x*Sin[3*a + 4*b*x] - 12*b^3*c*d
^2*x^2*Sin[3*a + 4*b*x] - (6*I)*b^2*d^3*x^2*Sin[3*a + 4*b*x] - 4*b^3*d^3*x^
3*Sin[3*a + 4*b*x] + 4*b^3*c^3*Sin[5*a + 4*b*x] + (6*I)*b^2*c^2*d*Sin[5*a +
 4*b*x] - 6*b*c*d^2*Sin[5*a + 4*b*x] - (3*I)*d^3*Sin[5*a + 4*b*x] + 12*b^3*
c^2*d*x*Sin[5*a + 4*b*x] + (12*I)*b^2*c*d^2*x*Sin[5*a + 4*b*x] - 6*b*d^3*x*
Sin[5*a + 4*b*x] + 12*b^3*c*d^2*x^2*Sin[5*a + 4*b*x] + (6*I)*b^2*d^3*x^2*Si
n[5*a + 4*b*x] + 4*b^3*d^3*x^3*Sin[5*a + 4*b*x]) - (3*c^2*d*Csc[a]*Sec[a]*(
```

```
b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1
+ E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan
[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[
a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Ta
n[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 907 vs. 2(215) = 430.
time = 0.56, size = 908, normalized size = 3.69

method	result
risch	$-\frac{3d(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2)\sin(2bx+2a)}{16b^4}-\frac{id^3x^4}{4}+\frac{c^3\ln(e^{i(bx+a)}+1)}{b}-\frac{2c^3\ln(e^{i(bx+a)})}{b}+\frac{c^3\ln(e^{i(bx+a)}-1)}{b}+ic$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -3/16*d*(2*b^2*d^2*x^2+4*b^2*c*d*x+2*b^2*c^2-d^2)/b^4*sin(2*b*x+2*a)-3/2*I/
b^4*d^3*a^4+6*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)))-I*d^2*c*x^3-3/2*I*d*c^2*
x^2-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+2/b^4*d^3*a^3*ln(exp(I*(b*x+a)))+6/b
^3*c*d^2*polylog(3,exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+6
/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x
-1/4*I*d^3*x^4+1/b*c^3*ln(exp(I*(b*x+a))+1)-2/b*c^3*ln(exp(I*(b*x+a)))+1/b*
c^3*ln(exp(I*(b*x+a))-1)+I*c^3*x+1/4*I/d*c^4-6*I/b^2*c*d^2*polylog(2,exp(I*
(b*x+a)))*x+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+1/b*d^3*ln(exp(I*(b*x+a))
+1)*x^3+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3
+6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)+1/8/
b^3*(2*b^2*d^3*x^3+6*b^2*c*d^2*x^2+6*b^2*c^2*d*x+2*b^2*c^3-3*d^3*x-3*c*d^2)
*cos(2*b*x+2*a)+3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+
a)))*x+3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a+3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^
2+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-6
/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)-3*I/b^
2*c^2*d*polylog(2,exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-
3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2,exp(I*(b*x
+a)))*x^2+4*I/b^3*c*d^2*a^3-3*I/b^2*c^2*d*a^2-2*I/b^3*d^3*a^3*x+6*I/b^2*c*d
^2*a^2*x-6*I/b*c^2*d*a*x-6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 979 vs. 2(211) = 422.
time = 0.41, size = 979, normalized size = 3.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/16*(8*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*c^3 - 24*(sin(b*x + a)^2 -
log(sin(b*x + a)^2))*a*c^2*d/b + 24*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*
a^2*c*d^2/b^2 - 8*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a^3*d^3/b^3 - (-4*
I*(b*x + a)^4*d^3 - 16*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^3 + 96*I*d^3*polylog
(4, -e^(I*b*x + I*a)) + 96*I*d^3*polylog(4, e^(I*b*x + I*a)) - 24*(I*b^2*c^
2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b*x + a)^2 - 16*(-I*(b*x + a)^3*d^3 + 3*(
-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2
*d^3)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 16*(I*(b*x + a)^
3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^
2 + I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 2*(2*(
b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*
(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 - 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) -
 48*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c
*d^2 - I*a*d^3)*(b*x + a))*dilog(-e^(I*b*x + I*a)) - 48*(I*b^2*c^2*d - 2*I*
a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x +
a))*dilog(e^(I*b*x + I*a)) + 8*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x
+ a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^
2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 8*((b*x + a)^3*d^3 + 3*(b*c*d^2
- a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log
(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 96*(b*c*d^2 + (b*x
 + a)*d^3 - a*d^3)*polylog(3, -e^(I*b*x + I*a)) + 96*(b*c*d^2 + (b*x + a)*d
^3 - a*d^3)*polylog(3, e^(I*b*x + I*a)) - 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*
(b*x + a)^2*d^3 + (2*a^2 - 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*
x + 2*a))/b^3)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 984 vs. $2(211) = 422$.
time = 3.24, size = 984, normalized size = 4.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 24*I*d^3*polylog(4, cos(b*x + a) +
I*sin(b*x + a)) + 24*I*d^3*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 24*I
*d^3*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) - 24*I*d^3*polylog(4, -cos(
b*x + a) - I*sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3
 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b^2*d^3*x^2
 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 3*(2*b^3*
c^2*d - b*d^3)*x + 12*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog
(cos(b*x + a) + I*sin(b*x + a)) + 12*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*
b^2*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + 12*(-I*b^2*d^3*x^2 - 2*I*
b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + 12*(I*b^
2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x
```

```
+ a)) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos
(b*x + a) + I*sin(b*x + a) + 1) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*
c^2*d*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 4*(b^3*c^3 - 3*
a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*
x + a) + 1/2) - 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-
1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2
*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*
x + a) + I*sin(b*x + a) + 1) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2
*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) - I*sin(b
*x + a) + 1) - 24*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) + I*sin(b*x +
 a)) - 24*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 2
4*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 24*(b*d^
3*x + b*c*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cos^2(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2*cot(b*x+a),x)
[Out] Integral((c + d*x)**3*cos(a + b*x)**2*cot(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^3*cos(b*x + a)^2*cot(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\mathrm{cot}\left(a+b\,x\right)\,{\left(c+d\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^3,x)
[Out] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^3, x)
```

3.166 $\int (c+dx)^2 \cos^2(a+bx) \cot(a+bx) \, dx$

Optimal. Leaf size=181

$$\frac{cdx}{2b}+\frac{d^2x^2}{4b}-\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}$$

```
[Out] 1/2*c*d*x/b+1/4*d^2*x^2/b-1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1-exp(2*I*(b*x+a))
)/b-I*d*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^2+1/2*d^2*polylog(3,exp(2*I*(
b*x+a)))/b^3-1/2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2+1/4*d^2*sin(b*x+a)^2/b
^3-1/2*(d*x+c)^2*sin(b*x+a)^2/b
```

Rubi [A]
time = 0.16, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4493, 4489, 3391, 3798, 2221, 2611, 2320, 6724}

$$\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\sin^2(a+bx)}{4b^3}-\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}+\frac{cdx}{2b}+\frac{d^2x^2}{4b}-\frac{i(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (c*d*x)/(2*b) + (d^2*x^2)/(4*b) - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[
1 - E^((2*I)*(a + b*x))])/b - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
])/b^2 + (d^2*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) - (d*(c + d*x)*Cos[a
 + b*x]*Sin[a + b*x])/(2*b^2) + (d^2*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^2
*Sin[a + b*x]^2)/(2*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^2\cos^2(a+bx)\cot(a+bx)\,dx &= \int (c+dx)^2\cot(a+bx)\,dx - \int (c+dx)^2\cos(a+bx)\sin(a+bx)\,d \\
&= -\frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1-e^{2i(a+bx)}}\,dx \\
&= -\frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cos(a+bx}{2b^2} \\
&= \frac{cdx}{2b} + \frac{d^2x^2}{4b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id(c+}{} \\
&= \frac{cdx}{2b} + \frac{d^2x^2}{4b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id(c+}{} \\
&= \frac{cdx}{2b} + \frac{d^2x^2}{4b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id(c+}{}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 511 vs. $2(181) = 362$.
time = 6.32, size = 511, normalized size = 2.82

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Cot[a])/3 - (d^2*Csc[a]*(2*b^2*x^2*(2*b*E^((
2*I)*a)*x + (3*I)*(-1 + E^((2*I)*a))*Log[1 - E^((2*I)*(a + b*x))]) + 6*b*(-
1 + E^((2*I)*a))*x*PolyLog[2, E^((2*I)*(a + b*x))] + (3*I)*(-1 + E^((2*I)*a
))*PolyLog[3, E^((2*I)*(a + b*x))]))/(12*b^3*E^(I*a)) + (c^2*Csc[a]*(-(b*x*
Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Si
n[a]^2)) + (Cos[2*b*x]*(2*b^2*c^2*Cos[2*a] - d^2*Cos[2*a] + 4*b^2*c*d*x*Cos
[2*a] + 2*b^2*d^2*x^2*Cos[2*a] - 2*b*c*d*Sin[2*a] - 2*b*d^2*x*Sin[2*a]))/(8
*b^3) - ((2*b*c*d*Cos[2*a] + 2*b*d^2*x*Cos[2*a] + 2*b^2*c^2*Sin[2*a] - d^2*
Sin[2*a] + 4*b^2*c*d*x*Sin[2*a] + 2*b^2*d^2*x^2*Sin[2*a])*Sin[2*b*x])/(8*b^
3) - (c*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*Ar
cTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1
 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*
Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]
]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)
])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(159) = 318$.
time = 0.60, size = 544, normalized size = 3.01

method	result
risch	$-\frac{4icdax}{b}+\frac{4cda\ln(\mathrm{e}^{i(bx+a)})}{b^2}-\frac{2cda\ln(\mathrm{e}^{i(bx+a)}-1)}{b^2}+\frac{2id^2a^2x}{b^2}-\frac{2icd\,a^2}{b^2}-\frac{id^2x^3}{3}+\frac{(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2)\cos(2}{8b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -4*I/b*c*d*a*x-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))+1/b^3*d^2*a^2*ln(exp(I*(b*x
+a))-1)+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2
+1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+1/8*(2*b^2*d^2*x^2+4*b^2*c*d*x+2*b^2*c^2-
d^2)/b^3*cos(2*b*x+2*a)+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+2/b*c*d*ln(exp(I*(
b*x+a))+1)*x+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c*d*a*ln(exp(I*(b*x+a)))-
2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)+2*I/b^2*d^2*a^2*x-2*I/b^2*c*d*a^2-2*I/b^2*
d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-2*
I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))-
1/3*I*d^2*x^3+1/b*c^2*ln(exp(I*(b*x+a))-1)+1/b*c^2*ln(exp(I*(b*x+a))+1)-2/b
*c^2*ln(exp(I*(b*x+a)))+4/3*I/b^3*d^2*a^3-I*d*c*x^2+I*c^2*x+1/3*I/d*c^3+2*d
^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3-1/4*d
*(d*x+c)*sin(2*b*x+2*a)/b^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(156) = 312$.
time = 0.37, size = 529, normalized size = 2.92

$$-\frac{12\left(\sin\left(bx+a\right)^2-\log\left(\sin\left(bx+a\right)^2\right)\right)c^2-\frac{24\left(\sin(bx+a)^2-\log\left(\sin(bx+a)^2\right)\right)acd}{b}+\frac{12\left(\sin(bx+a)^2-\log\left(\sin(bx+a)^2\right)\right)a^2d^2}{b^2}-\frac{-8i\,(bx+a)^3d^2-24\,(i\,bcd-i\,ad^2)(bx+a)^2+48\,d^2\mathrm{Li}_3(-e^{(i\,bx+i\,a)})+48\,d^2\mathrm{Li}_3(e^{(i\,bx+i\,a)})-24\left(-i\,(bx+a)^2d^2+2\,(-i\,bcd+i\,ad^2)(bx+a)\right)\arctan(\sin(bx+a),\ldots}{\ldots}}{\ldots}$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/24*(12*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*c^2 - 24*(sin(b*x + a)^2 -
 log(sin(b*x + a)^2))*a*c*d/b + 12*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a
^2*d^2/b^2 - (-8*I*(b*x + a)^3*d^2 - 24*(I*b*c*d - I*a*d^2)*(b*x + a)^2 + 4
8*d^2*polylog(3, -e^(I*b*x + I*a)) + 48*d^2*polylog(3, e^(I*b*x + I*a)) - 2
4*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2(sin(b*x +
 a), cos(b*x + a) + 1) - 24*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x
 + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 3*(2*(b*x + a)^2*d^2 + 4*
(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(2*b*x + 2*a) - 48*(I*b*c*d + I*(b*x +
a)*d^2 - I*a*d^2)*dilog(-e^(I*b*x + I*a)) - 48*(I*b*c*d + I*(b*x + a)*d^2 -
 I*a*d^2)*dilog(e^(I*b*x + I*a)) + 12*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*
(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 12*(
(b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x
 + a)^2 - 2*cos(b*x + a) + 1) - 6*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x
 + 2*a))/b^2)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(156) = 312$.
time = 3.03, size = 594, normalized size = 3.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2
- d^2)*cos(b*x + a)^2 - 4*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 4
*d^2*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 4*d^2*polylog(3, -cos(b*x
+ a) + I*sin(b*x + a)) - 4*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) +
 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) + 4*(I*b*d^2*x + I*b*c*d)*di
log(cos(b*x + a) + I*sin(b*x + a)) + 4*(-I*b*d^2*x - I*b*c*d)*dilog(cos(b*x
 + a) - I*sin(b*x + a)) + 4*(-I*b*d^2*x - I*b*c*d)*dilog(-cos(b*x + a) + I*
sin(b*x + a)) + 4*(I*b*d^2*x + I*b*c*d)*dilog(-cos(b*x + a) - I*sin(b*x + a
)) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*sin(b*x +
 a) + 1) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) - I*sin
(b*x + a) + 1) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a) +
1/2*I*sin(b*x + a) + 1/2) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(
b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b
*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b^2*d^2*x^2 +
2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1))
/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \cos^2 (a + bx) \cot (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)**2*cot(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)^2*cot(b*x + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos{(a+b\,x)}^2\cot(a+b\,x)\,{(c+d\,x)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^2,x)
[Out] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^2, x)
```

3.167 $\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=114

$$\frac{dx}{4b}-\frac{i(c+dx)^2}{2d}+\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{d\cos(a+bx)\sin(a+bx)}{4b^2}-\frac{(c+dx)\sin^2(a+bx)}{2b}$$

```
[Out] 1/4*d*x/b-1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b-1/2*I*d*polylo
g(2,exp(2*I*(b*x+a)))/b^2-1/4*d*cos(b*x+a)*sin(b*x+a)/b^2-1/2*(d*x+c)*sin(b
*x+a)^2/b
```

Rubi [A]
time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4493, 4489, 2715, 8, 3798, 2221, 2317, 2438}

$$-\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}+\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{dx}{4b}-\frac{i(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (d*x)/(4*b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 - E^((2*I)*(a + b*x)
)])/b - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d*Cos[a + b*x]*Sin
[a + b*x])/(4*b^2) - ((c + d*x)*Sin[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule steps

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)\cot(a+bx)\,dx - \int (c+dx)\cos(a+bx)\sin(a+bx)\,dx$$

$$= -\frac{i(c+dx)^2}{2d} - \frac{(c+dx)\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1-e^{2i(a+bx)}}\,dx$$

$$= -\frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)\sin(a+$$

$$= \frac{dx}{4b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)\sin}{4b^2}$$

$$= \frac{dx}{4b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}$$

Mathematica [A]

time = 0.40, size = 131, normalized size = 1.15

$$\frac{dx\cos(2(a+bx))}{4b}+\frac{c\log(\sin(a+bx))}{b}-\frac{ad\log(\sin(a+bx))}{b^2}+\frac{d\left((a+bx)\log\left(1-e^{2i(a+bx)}\right)-\frac{1}{2}i\left((a+bx)^2+\text{PolyLog}\left(2,e^{2i(a+bx)}\right)\right)\right)}{b^2}-\frac{c\sin^2(a+bx)}{2b}-\frac{d\sin(2(a+bx))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (d*x*Cos[2*(a + b*x)])/(4*b) + (c*Log[Sin[a + b*x]])/b - (a*d*Log[Sin[a + b
*x]])/b^2 + (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x))] - (I/2)*((a + b*x)^2
 + PolyLog[2, E^((2*I)*(a + b*x))])))/b^2 - (c*Sin[a + b*x]^2)/(2*b) - (d*S
in[2*(a + b*x)])/(8*b^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(98) = 196.
time = 0.56, size = 249, normalized size = 2.18

method	result
risch	$-\frac{i d\,x^2}{2}-\frac{i d\,\mathrm{polylog}\left(2,-\mathrm{e}^{i(bx+a)}\right)}{b^2}+\frac{c\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b}+\frac{c\ln\left(\mathrm{e}^{i(bx+a)}+1\right)}{b}-\frac{2c\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b}-\frac{i d\,a^2}{b^2}-\frac{2idax}{b}+icx+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*d*x^2-I*d*polylog(2,-exp(I*(b*x+a)))/b^2+1/b*c*ln(exp(I*(b*x+a))-1)+
1/b*c*ln(exp(I*(b*x+a))+1)-2/b*c*ln(exp(I*(b*x+a)))-I/b^2*d*a^2-2*I/b*d*a*x
-I*d*polylog(2,exp(I*(b*x+a)))/b^2+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(
1-exp(I*(b*x+a)))*a+I*c*x+1/b*d*ln(exp(I*(b*x+a))+1)*x-1/b^2*d*a*ln(exp(I*(
b*x+a))-1)+2/b^2*d*a*ln(exp(I*(b*x+a)))+1/4*(d*x+c)*cos(2*b*x+2*a)/b-1/8*d*
sin(2*b*x+2*a)/b^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(95) = 190.
time = 0.35, size = 223, normalized size = 1.96

$$\frac{-4i\,b^2dx^2-8i\,b^2cx-8i\,bdx\arctan\left(\sin(bx+a),-\cos(bx+a)+1\right)+8i\,bc\arctan\left(\sin(bx+a),\cos(bx+a)-1\right)-8\left(-i\,bdx-i\,bc\right)\arctan\left(\sin(bx+a),\cos(bx+a)+1\right)+2\left(bdx+bc\right)\cos(2\,bx+2\,a)-8i\,d\mathrm{Li}_2\left(-e^{(i\,bx+i\,a)}\right)-8i\,d\mathrm{Li}_2\left(e^{(i\,bx+i\,a)}\right)+4\left(bdx+bc\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2+2\,\cos(bx+a)+1\right)+4\left(bdx+bc\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2-2\,\cos(bx+a)+1\right)-d\sin(2\,bx+2\,a)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(-4*I*b^2*d*x^2 - 8*I*b^2*c*x - 8*I*b*d*x*arctan2(sin(b*x + a), -cos(b*
x + a) + 1) + 8*I*b*c*arctan2(sin(b*x + a), cos(b*x + a) - 1) - 8*(-I*b*d*x
 - I*b*c)*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 2*(b*d*x + b*c)*cos(2*b
*x + 2*a) - 8*I*d*dilog(-e^(I*b*x + I*a)) - 8*I*d*dilog(e^(I*b*x + I*a)) +
4*(b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) +
 4*(b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1)
- d*sin(2*b*x + 2*a))/b^2
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(95) = 190$.
time = 2.24, size = 292, normalized size = 2.56

$$-\frac{bdx - 2\,(bdx + bc)\cos{(bx + a)}^2 + d\cos{(bx + a)}\sin{(bx + a)} + 2i\,d\mathrm{Li}_2(\cos{(bx + a)} + i\,\sin{(bx + a)}) - 2i\,d\mathrm{Li}_2(\cos{(bx + a)} - i\,\sin{(bx + a)}) - 2i\,d\mathrm{Li}_2(-\cos{(bx + a)} + i\,\sin{(bx + a)}) + 2i\,d\mathrm{Li}_2(-\cos{(bx + a)} - i\,\sin{(bx + a)}) - 2\,(bdx + bc)\log{(\cos{(bx + a)} + i\,\sin{(bx + a)} + 1)} - 2\,(bdx + bc)\log{(\cos{(bx + a)} - i\,\sin{(bx + a)} + 1)} - 2\,(bc - ad)\log{(-\frac{1}{2}\,\cos{(bx + a)} + \frac{1}{2}i\,\sin{(bx + a)} + \frac{1}{2})} - 2\,(bc - ad)\log{(-\frac{1}{2}\,\cos{(bx + a)} - \frac{1}{2}i\,\sin{(bx + a)} + \frac{1}{2})} - 2\,(bdx + ad)\log{(-\cos{(bx + a)} + i\,\sin{(bx + a)} + 1)} - 2\,(bdx + ad)\log{(-\cos{(bx + a)} - i\,\sin{(bx + a)} + 1)}}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a)
+ 2*I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) - 2*I*d*dilog(cos(b*x + a) - I
*sin(b*x + a)) - 2*I*d*dilog(-cos(b*x + a) + I*sin(b*x + a)) + 2*I*d*dilog(
-cos(b*x + a) - I*sin(b*x + a)) - 2*(b*d*x + b*c)*log(cos(b*x + a) + I*sin(
b*x + a) + 1) - 2*(b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 2*
(b*c - a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*(b*c - a*
d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 2*(b*d*x + a*d)*log(
-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b*d*x + a*d)*log(-cos(b*x + a) - I
*sin(b*x + a) + 1))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)\cos^2{(a + bx)}\cot{(a + bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**2*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)**2*cot(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*cos(b*x + a)^2*cot(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(a+b\,x\right)}^2\,\mathrm{cot}\left(a+b\,x\right)\,\left(c+d\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x),x)
```

```
[Out] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x), x)
```

3.168 $\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=82

$$-\frac{\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d}-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\text{Int}\left(\frac{\cot(a+bx)}{c+dx},x\right)$$

```
[Out] -1/2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d-1/2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b
*c/d)/d+Unintegrable(cot(b*x+a)/(d*x+c),x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x),x]
```

```
[Out] -1/2*(CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/d - (Cos[2*a - (
2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Defer[Int][Cot[a + b*x]/(
c + d*x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx &= \int \frac{\cot(a+bx)}{c+dx}\,dx-\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx\\
&= \int \frac{\cot(a+bx)}{c+dx}\,dx-\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx\\
&= -\left(\frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx\right)+\int \frac{\cot(a+bx)}{c+dx}\,dx\\
&= -\left(\frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx\right)-\frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\right.}{c}\\
&= -\frac{\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d}-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\int \frac{\cot(a+}{c+d}
\end{aligned}$$

Mathematica [A]
time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x), x]
```

Maple [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{2}\left(bx+a\right)\right)\cot\left(bx+a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x)
```

```
[Out] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*((I*exp_integral_e(1, 2*(-I*b*d*x - I*b*c)/d) - I*exp_integral_e(1, -2
*(-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 4*d*integrate(sin(b*x + a)/
((d*x + c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*co
s(b*x + a) + c), x) - 4*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2
+ (d*x + c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) - (exp
_integral_e(1, 2*(-I*b*d*x - I*b*c)/d) + exp_integral_e(1, -2*(-I*b*d*x - I
*b*c)/d))*sin(-2*(b*c - a*d)/d))/d
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^2*cot(b*x + a)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(a+bx\right)\cot\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*cot(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)**2*cot(a + b*x)/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2*cot(b*x + a)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^2\,\mathrm{cot}\left(a+b\,x\right)}{c+d\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x), x)
```

3.169 $\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=102

$$-\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin(2a+2bx)}{2d(c+dx)}+\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\text{Int}\left(\frac{\cot(a+bx)}{(c+dx)^2}\right.$$

```
[Out] -b*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^2+b*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c
/d)/d^2+1/2*sin(2*b*x+2*a)/d/(d*x+c)+Unintegrable(cot(b*x+a)/(d*x+c)^2,x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -((b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2) + Sin[2*a +
2*b*x]/(2*d*(c + d*x)) + (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*
b*x])/d^2 + Defer[Int][Cot[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx &= \int \frac{\cot(a+bx)}{(c+dx)^2}\,dx-\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx\\
&= \int \frac{\cot(a+bx)}{(c+dx)^2}\,dx-\int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx\\
&= -\left(\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx\right)+\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(2a+2bx)}{2d(c+dx)}-\frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d}+\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(2a+2bx)}{2d(c+dx)}-\frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(2a-\frac{2bc}{d}\right)\right)\int}{d}\\
&= -\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin(2a+2bx)}{2d(c+dx)}+\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}\right.}{d^2}
\end{aligned}$$

Mathematica [A]
time = 2.71, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2, x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2\left(bx+a\right)\right)\cot\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((I*exp_integral_e(2, 2*(-I*b*d*x - I*b*c)/d) - I*exp_integral_e(2, -2
*(-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 4*(d^2*x + c*d)*integrate(s
in(b*x + a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 +
 (d^2*x^2 + 2*c*d*x + c^2)*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^
2)*cos(b*x + a)), x) - 4*(d^2*x + c*d)*integrate(sin(b*x + a)/(d^2*x^2 + 2*
c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x + c^2
)*sin(b*x + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x) - (e
xp_integral_e(2, 2*(-I*b*d*x - I*b*c)/d) + exp_integral_e(2, -2*(-I*b*d*x -
 I*b*c)/d))*sin(-2*(b*c - a*d)/d))/(d^2*x + c*d)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^2*cot(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*cot(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)**2*cot(a + b*x)/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2*cot(b*x + a)/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^2\cot(a+b\,x)}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x)^2, x)
```

3.170 $\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=154

$$\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1\right.}{2b}$$

```
[Out] CannotIntegrate((d*x+c)^m*cot(b*x+a)*csc(b*x+a),x)+1/2*I*exp(I*(a-b*c/d))*(
d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/2*I*(d*x+c)^m*G
AMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] ((I/2)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/
(b*(((-I)*b*(c + d*x))/d)^m) - ((I/2)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d*
x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Defer[Int][(c + d*x
)^m*Cot[a + b*x]*Csc[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m\cos(a+bx)\cot^2(a+bx)\,dx &= -\int (c+dx)^m\cos(a+bx)\,dx+\int (c+dx)^m\cot(a+bx)\csc(a+b\\ &= -\left(\frac{1}{2}\int e^{-i(a+bx)}(c+dx)^m\,dx\right)-\frac{1}{2}\int e^{i(a+bx)}(c+dx)^m\,dx+\int (\\ &= \frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{-i\left(a-\right.}}{}\end{aligned}$$

Mathematica [A]
time = 8.20, size = 0, normalized size = 0.00

$$\int (c+dx)^m\cos(a+bx)\cot^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a) \left(\cot^2(bx+a)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x)
[Out] int((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cos(a+bx) \cot^2(a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*cot(b*x+a)**2,x)
[Out] Integral((c + d*x)**m*cos(a + b*x)*cot(a + b*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\cot(a+b\,x)^2\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^m, x)
```

3.171 $\int (c+dx)^4 \cos(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=299

$$-\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{24d^3(c+dx)\cos(a+bx)}{b^4}-\frac{4d(c+dx)^3\cos(a+bx)}{b^2}-\frac{(c+dx)^4\csc(a+bx)}{b}$$

```
[Out] -8*d*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b^2+24*d^3*(d*x+c)*cos(b*x+a)/b^4-4*
d*(d*x+c)^3*cos(b*x+a)/b^2-(d*x+c)^4*csc(b*x+a)/b+12*I*d^2*(d*x+c)^2*polylo
g(2,-exp(I*(b*x+a)))/b^3-12*I*d^2*(d*x+c)^2*polylog(2,exp(I*(b*x+a)))/b^3-2
4*d^3*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^4+24*d^3*(d*x+c)*polylog(3,exp(I
*(b*x+a)))/b^4-24*I*d^4*polylog(4,-exp(I*(b*x+a)))/b^5+24*I*d^4*polylog(4,e
xp(I*(b*x+a)))/b^5-24*d^4*sin(b*x+a)/b^5+12*d^2*(d*x+c)^2*sin(b*x+a)/b^3-(d
*x+c)^4*sin(b*x+a)/b
```

Rubi [A]
time = 0.21, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {4493, 3377, 2717, 4495, 4268, 2611, 6744, 2320, 6724}

$$-\frac{24id^4\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^5}+\frac{24id^4\text{Li}_4\left(e^{i(a+bx)}\right)}{b^5}-\frac{24d^4\sin(a+bx)}{b^5}-\frac{24d^3(c+dx)\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^4}+\frac{24d^3(c+dx)\text{Li}_3\left(e^{i(a+bx)}\right)}{b^4}+\frac{24d^3(c+dx)\cos(a+bx)}{b^4}+\frac{12id^2(c+dx)^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{12id^2(c+dx)^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}+\frac{12d^2(c+dx)^2\sin(a+bx)}{b^3}-\frac{4d(c+dx)^3\cos(a+bx)}{b^2}-\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{(c+dx)^4\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (-8*d*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b^2 + (24*d^3*(c + d*x)*Cos[a +
 b*x])/b^4 - (4*d*(c + d*x)^3*Cos[a + b*x])/b^2 - ((c + d*x)^4*Csc[a + b*x]
)/b + ((12*I)*d^2*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((12*I)*d
^2*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (24*d^3*(c + d*x)*PolyLog
[3, -E^(I*(a + b*x))])/b^4 + (24*d^3*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])
/b^4 - ((24*I)*d^4*PolyLog[4, -E^(I*(a + b*x))])/b^5 + ((24*I)*d^4*PolyLog[
4, E^(I*(a + b*x))])/b^5 - (24*d^4*Sin[a + b*x])/b^5 + (12*d^2*(c + d*x)^2*
Sin[a + b*x])/b^3 - ((c + d*x)^4*Sin[a + b*x])/b
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \cos(a+bx)\cot^2(a+bx)\,dx = -\int (c+dx)^4\cos(a+bx)\,dx + \int (c+dx)^4\cot(a+bx)\csc(a+b$$
$$= -\frac{(c+dx)^4\csc(a+bx)}{b} - \frac{(c+dx)^4\sin(a+bx)}{b} + \frac{(4d)\int(c+dx)}{}$$
$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{4d(c+dx)^3\cos(a+bx)}{b^2} - \frac{(c+}{}$$
$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{4d(c+dx)^3\cos(a+bx)}{b^2} - \frac{(c+}{}$$
$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^3(c+dx)\cos(a+bx)}{b^4} - \frac{4d}{}$$
$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^3(c+dx)\cos(a+bx)}{b^4} - \frac{4d}{}$$
$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^3(c+dx)\cos(a+bx)}{b^4} - \frac{4d}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 833 vs. $2(299) = 598$.
time = 2.42, size = 833, normalized size = 2.79

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (Csc[a + b*x]*(-3*b^4*c^4 + 12*b^2*c^2*d^2 - 24*d^4 - 12*b^4*c^3*d*x + 24*b
^2*c*d^3*x - 18*b^4*c^2*d^2*x^2 + 12*b^2*d^4*x^2 - 12*b^4*c*d^3*x^3 - 3*b^4
*d^4*x^4 + b^4*c^4*Cos[2*(a + b*x)] - 12*b^2*c^2*d^2*Cos[2*(a + b*x)] + 24*
d^4*Cos[2*(a + b*x)] + 4*b^4*c^3*d*x*Cos[2*(a + b*x)] - 24*b^2*c*d^3*x*Cos[
2*(a + b*x)] + 6*b^4*c^2*d^2*x^2*Cos[2*(a + b*x)] - 12*b^2*d^4*x^2*Cos[2*(a
 + b*x)] + 4*b^4*c*d^3*x^3*Cos[2*(a + b*x)] + b^4*d^4*x^4*Cos[2*(a + b*x)]
- 16*b^3*c^3*d*ArcTanh[E^(I*(a + b*x))]*Sin[a + b*x] + 24*b^3*c^2*d^2*x*Log
[1 - E^(I*(a + b*x))]*Sin[a + b*x] + 24*b^3*c*d^3*x^2*Log[1 - E^(I*(a + b*x
))]*Sin[a + b*x] + 8*b^3*d^4*x^3*Log[1 - E^(I*(a + b*x))]*Sin[a + b*x] - 24
*b^3*c^2*d^2*x*Log[1 + E^(I*(a + b*x))]*Sin[a + b*x] - 24*b^3*c*d^3*x^2*Log
[1 + E^(I*(a + b*x))]*Sin[a + b*x] - 8*b^3*d^4*x^3*Log[1 + E^(I*(a + b*x))]
```

```
*Sin[a + b*x] + (24*I)*b^2*d^2*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))]*Sin
[a + b*x] - (24*I)*b^2*d^2*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))]*Sin[a +
b*x] - 48*b*c*d^3*PolyLog[3, -E^(I*(a + b*x))]*Sin[a + b*x] - 48*b*d^4*x*Po
lyLog[3, -E^(I*(a + b*x))]*Sin[a + b*x] + 48*b*c*d^3*PolyLog[3, E^(I*(a + b
*x))]*Sin[a + b*x] + 48*b*d^4*x*PolyLog[3, E^(I*(a + b*x))]*Sin[a + b*x] -
(48*I)*d^4*PolyLog[4, -E^(I*(a + b*x))]*Sin[a + b*x] + (48*I)*d^4*PolyLog[4
, E^(I*(a + b*x))]*Sin[a + b*x] - 4*b^3*c^3*d*Sin[2*(a + b*x)] + 24*b*c*d^3
*Sin[2*(a + b*x)] - 12*b^3*c^2*d^2*x*Sin[2*(a + b*x)] + 24*b*d^4*x*Sin[2*(a
 + b*x)] - 12*b^3*c*d^3*x^2*Sin[2*(a + b*x)] - 4*b^3*d^4*x^3*Sin[2*(a + b*x
)]))/(2*b^5)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1055 vs. $2(281) = 562$.
time = 0.18, size = 1056, normalized size = 3.53

method	result	size
risch	Expression too large to display	1056

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 24*I*d^4*polylog(4,exp(I*(b*x+a)))/b^5+1/2*I*(d^4*x^4*b^4+4*b^4*c*d^3*x^3+6
*b^4*c^2*d^2*x^2+4*b^4*c^3*d*x+4*I*b^3*d^4*x^3+b^4*c^4-12*b^2*d^4*x^2+12*I*
b^3*c*d^3*x^2-24*b^2*c*d^3*x+12*I*b^3*c^2*d^2*x-12*b^2*c^2*d^2+4*I*b^3*c^3*
d-24*I*b*d^4*x+24*d^4-24*I*b*c*d^3)/b^5*exp(I*(b*x+a))-1/2*I*(d^4*x^4*b^4+4
*b^4*c*d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^4*c^3*d*x-4*I*b^3*d^4*x^3+b^4*c^4-12*b
^2*d^4*x^2-12*I*b^3*c*d^3*x^2-24*b^2*c*d^3*x-12*I*b^3*c^2*d^2*x-12*b^2*c^2*
d^2-4*I*b^3*c^3*d+24*I*b*d^4*x+24*d^4+24*I*b*c*d^3)/b^5*exp(-I*(b*x+a))-2*I
*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x^2+4*c^3*d*x+c^4)*exp(I*(b*x+a))/b/(exp(2*
I*(b*x+a))-1)-8*d/b^2*c^3*arctanh(exp(I*(b*x+a)))+8*d^4/b^5*a^3*arctanh(exp
(I*(b*x+a)))+24*d^3/b^4*c*polylog(3,exp(I*(b*x+a)))-24*d^3/b^4*c*polylog(3,
-exp(I*(b*x+a)))+24*d^4/b^4*polylog(3,exp(I*(b*x+a)))*x-24*d^4/b^4*polylog(
3,-exp(I*(b*x+a)))*x+24*I*d^3/b^3*c*polylog(2,-exp(I*(b*x+a)))*x-24*I*d^3/b
^3*c*polylog(2,exp(I*(b*x+a)))*x-12*d^2/b^2*c^2*ln(exp(I*(b*x+a))+1)*x-12*d
^2/b^3*c^2*ln(exp(I*(b*x+a))+1)*a+12*d^2/b^2*c^2*ln(1-exp(I*(b*x+a)))*x+12*
d^2/b^3*c^2*ln(1-exp(I*(b*x+a)))*a-4*d^4/b^2*ln(exp(I*(b*x+a))+1)*x^3-4*d^4
/b^5*ln(exp(I*(b*x+a))+1)*a^3+4*d^4/b^2*ln(1-exp(I*(b*x+a)))*x^3+4*d^4/b^5*
ln(1-exp(I*(b*x+a)))*a^3+12*I*d^2/b^3*c^2*polylog(2,-exp(I*(b*x+a)))-12*I*d
^2/b^3*c^2*polylog(2,exp(I*(b*x+a)))+12*I*d^4/b^3*polylog(2,-exp(I*(b*x+a))
)*x^2-12*I*d^4/b^3*polylog(2,exp(I*(b*x+a)))*x^2+24*d^2/b^3*c^2*a*arctanh(e
xp(I*(b*x+a)))-24*d^3/b^4*c*a^2*arctanh(exp(I*(b*x+a)))-12*d^3/b^2*c*ln(exp
(I*(b*x+a))+1)*x^2+12*d^3/b^4*c*ln(exp(I*(b*x+a))+1)*a^2+12*d^3/b^2*c*ln(1-
exp(I*(b*x+a)))*x^2-12*d^3/b^4*c*ln(1-exp(I*(b*x+a)))*a^2-24*I*d^4*polylog(
4,-exp(I*(b*x+a)))/b^5
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 17589 vs. 2(275) = 550.
time = 4.50, size = 17589, normalized size = 58.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^4*(1/sin(b*x + a) + sin(b*x + a)) - 8*a*c^3*d*(1/sin(b*x + a) + s
in(b*x + a))/b + 12*a^2*c^2*d^2*(1/sin(b*x + a) + sin(b*x + a))/b^2 - 8*a^3
*c*d^3*(1/sin(b*x + a) + sin(b*x + a))/b^3 + 2*a^4*d^4*(1/sin(b*x + a) + si
n(b*x + a))/b^4 - 4*(((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*co
s(3*b*x + 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin(2*b*x + 2*a)
)*sin(3*b*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2*(4*(b*x + a)*cos(b*x
+ a)*sin(2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x
 + 2*a) + 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a)^2 - ((b
*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x + a)*cos(
2*b*x + 2*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
+ 1)*cos(3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b
*x + 2*a) - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a)^2 - ((b*x + a)*sin(b
*x + a) + cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x + a)*cos(b*x + a)*sin
(b*x + a) - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^2 + sin(
b*x + a)^2 + 2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a)^2 + cos(b*x + a)^2 + (1
3*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a) + s
in(2*b*x + 2*a)^2 + sin(b*x + a)^2 + 1)*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*s
in(b*x + a)^3 + (3*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x + a) + cos(b
*x + a))*cos(2*b*x + 2*a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*c
os(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)
*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*
x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*
b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x
+ 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a)
 - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 -
2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*co
s(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)
^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + ((cos(2*b*x
 + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2
 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)
^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos
(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*co
s(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x +
 a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*
cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + si
n(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x +
```

```
a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2
 - 2*cos(b*x + a) + 1) + ((b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin(2*b*x
 + 2*a))*cos(3*b*x + 3*a)^2 + (b*x + a)*cos(2*b*x + 2*a)^2 + (b*x + a)*cos(
b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2 + 13*(b*x + a)*sin(b*x + a)^2 + b
*x + 2*(((b*x + a)*cos(b*x + a) + sin(b*x + a))*cos(2*b*x + 2*a) - (b*x + a
)*cos(b*x + a) - ((b*x + a)*sin(b*x + a) - cos(b*x + a))*sin(2*b*x + 2*a) -
 sin(b*x + a))*cos(3*b*x + 3*a) - ((b*x + a)*cos(b*x + a)^2 + 13*(b*x + a)*
sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)*cos(b*x + a)
*sin(b*x + a) - cos(b*x + a)^2 - sin(b*x + a)^2)*sin(2*b*x + 2*a) + a)*sin(
3*b*x + 3*a) - 6*((b*x + a)*cos(b*x + a)^3 + (b*x + a)*cos(b*x + a)*sin(b*x
 + a)^2)*sin(2*b*x + 2*a) - (6*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x
+ a) - cos(b*x + a))*c^3*d/(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*c
os(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)
*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*
x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*
b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x
+ 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a)
 - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 -
2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*co
s(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)
^2)*b) + 12*(((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*cos(3*b*x
+ 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin(2*b*x + 2*a))*sin(3*
b*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2*(4*(b*x + a)*cos(b*x + a)*sin
(2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)
+ 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a)^2 - ((b*x + a)*
sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x + a)*cos(2*b*x +
2*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*cos
(3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 2*a
) - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a)^2 - ((b*x + a)*sin(b*x + a)
+ cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x + a)*cos(b*x + a)*sin(b*x + a
) - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + c...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1233 vs. $2(275) = 550$.
time = 3.57, size = 1233, normalized size = 4.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 12*b^2*c^2*d^2 - 12*I*d^4*p
olylog(4, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(4,
 cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 12*I*d^4*polylog(4, -cos(b*x
 + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(4, -cos(b*x + a) -
```

```
I*sin(b*x + a))*sin(b*x + a) + 24*d^4 + 12*(b^4*c^2*d^2 - b^2*d^4)*x^2 - (b
^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 24*d^4 + 6*(b^4*c
^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4
*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d - 6*b*c*d^3 + 3*(b^3*c^2*d^2 -
2*b*d^4)*x)*cos(b*x + a)*sin(b*x + a) + 6*(I*b^2*d^4*x^2 + 2*I*b^2*c*d^3*x
+ I*b^2*c^2*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(-I*
b^2*d^4*x^2 - 2*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*dilog(cos(b*x + a) - I*sin(b
*x + a))*sin(b*x + a) + 6*(I*b^2*d^4*x^2 + 2*I*b^2*c*d^3*x + I*b^2*c^2*d^2)
*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(-I*b^2*d^4*x^2 - 2
*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b
*x + a) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*l
og(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 2*(b^3*d^4*x^3 + 3*b^3
*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*log(cos(b*x + a) - I*sin(b*x + a)
 + 1)*sin(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d
^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 2*(b^3
*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*log(-1/2*cos(b*x + a) -
 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2
+ 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*log(-cos(b*x
 + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2
 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*log(-cos(b*
x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 12*(b*d^4*x + b*c*d^3)*polylog(
3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 12*(b*d^4*x + b*c*d^3)*pol
ylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 12*(b*d^4*x + b*c*d^3
)*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*(b*d^4*x + b
*c*d^3)*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 8*(b^4*c^
3*d - 3*b^2*c*d^3)*x)/(b^5*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \cos(a+bx)\cot^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**4*cos(a + b*x)*cot(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*cot(b*x + a)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)\,\cot(a+bx)^2\,(c+dx)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^4,x)
[Out] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^4, x)
```

3.172 $\int (c+dx)^3 \cos(a+bx) \cot^2(a+bx)\,dx$

Optimal. Leaf size=216

$$-\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{6d^3\cos(a+bx)}{b^4}-\frac{3d(c+dx)^2\cos(a+bx)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{6id^2(c-$$

```
[Out] -6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^2+6*d^3*cos(b*x+a)/b^4-3*d*(d*x+c)
^2*cos(b*x+a)/b^2-(d*x+c)^3*csc(b*x+a)/b+6*I*d^2*(d*x+c)*polylog(2,-exp(I*(
b*x+a)))/b^3-6*I*d^2*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^3-6*d^3*polylog(3,
-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4+6*d^2*(d*x+c)*sin(
b*x+a)/b^3-(d*x+c)^3*sin(b*x+a)/b
```

Rubi [A]

time = 0.15, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4493, 3377, 2718, 4495, 4268, 2611, 2320, 6724}

$$-\frac{6d^3\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{Li}_3\left(e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\cos(a+bx)}{b^4}+\frac{6id^2(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\sin(a+bx)}{b^3}-\frac{3d(c+dx)^2\cos(a+bx)}{b^2}-\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (-6*d*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^2 + (6*d^3*Cos[a + b*x])/b^4
- (3*d*(c + d*x)^2*Cos[a + b*x])/b^2 - ((c + d*x)^3*Csc[a + b*x])/b + ((6*I
)*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((6*I)*d^2*(c + d*x)*Po
lyLog[2, E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog[3, -E^(I*(a + b*x))])/b^4 +
 (6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4 + (6*d^2*(c + d*x)*Sin[a + b*x])/b
^3 - ((c + d*x)^3*Sin[a + b*x])/b
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^3\cos(a+bx)\cot^2(a+bx)\,dx &= -\int (c+dx)^3\cos(a+bx)\,dx + \int (c+dx)^3\cot(a+bx)\csc(a+b \\
&= -\frac{(c+dx)^3\csc(a+bx)}{b} - \frac{(c+dx)^3\sin(a+bx)}{b} + \frac{(3d)\int (c+dx}{} \\
&= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3d(c+dx)^2\cos(a+bx)}{b^2} - \frac{(c+}{} \\
&= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3d(c+dx)^2\cos(a+bx)}{b^2} - \frac{(c+}{} \\
&= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6d^3\cos(a+bx)}{b^4} - \frac{3d(c+dx)^2}{b} \\
&= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6d^3\cos(a+bx)}{b^4} - \frac{3d(c+dx)^2}{b}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 506 vs. 2(216) = 432.
time = 1.40, size = 506, normalized size = 2.34

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (Csc[a + b*x]*(-3*b^3*c^3 + 6*b*c*d^2 - 9*b^3*c^2*d*x + 6*b*d^3*x - 9*b^3*c
*d^2*x^2 - 3*b^3*d^3*x^3 + b^3*c^3*Cos[2*(a + b*x)] - 6*b*c*d^2*Cos[2*(a +
b*x)] + 3*b^3*c^2*d*x*Cos[2*(a + b*x)] - 6*b*d^3*x*Cos[2*(a + b*x)] + 3*b^3
*c*d^2*x^2*Cos[2*(a + b*x)] + b^3*d^3*x^3*Cos[2*(a + b*x)] - 12*b^2*c^2*d*A
rcTanh[E^(I*(a + b*x))]*Sin[a + b*x] + 12*b^2*c*d^2*x*Log[1 - E^(I*(a + b*x
))]*Sin[a + b*x] + 6*b^2*d^3*x^2*Log[1 - E^(I*(a + b*x))]*Sin[a + b*x] - 12
*b^2*c*d^2*x*Log[1 + E^(I*(a + b*x))]*Sin[a + b*x] - 6*b^2*d^3*x^2*Log[1 +
E^(I*(a + b*x))]*Sin[a + b*x] + (12*I)*b*d^2*(c + d*x)*PolyLog[2, -E^(I*(a
+ b*x))]*Sin[a + b*x] - (12*I)*b*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))]*
Sin[a + b*x] - 12*d^3*PolyLog[3, -E^(I*(a + b*x))]*Sin[a + b*x] + 12*d^3*Po
lyLog[3, E^(I*(a + b*x))]*Sin[a + b*x] - 3*b^2*c^2*d*Sin[2*(a + b*x)] + 6*d
^3*Sin[2*(a + b*x)] - 6*b^2*c*d^2*x*Sin[2*(a + b*x)] - 3*b^2*d^3*x^2*Sin[2*
(a + b*x)]))/(2*b^4)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 648 vs. 2(204) = 408.
time = 0.14, size = 649, normalized size = 3.00

method	result
risch	$\frac{i\left(d^3x^3b^3+3b^3c\,d^2x^2+3ib^2d^3x^2+3b^3c^2dx+6ib^2c\,d^2x+b^3c^3+3ib^2c^2d-6b\,d^3x-6c\,d^2b-6id^3\right)\mathrm{e}^{i(bx+a)}}{2b^4}-\frac{2i\left(d^3x^3+3c\,d^2x^2+3c^2dx+c^3\right)}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -6*I*d^2/b^3*c*polylog(2,exp(I*(b*x+a)))+1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2
+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+
3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))-6*I*d^3/b^3*polylog(2,exp(I*(b*x+
a)))*x-6*d^2/b^2*c*ln(exp(I*(b*x+a))+1)*x-6*d^2/b^3*c*ln(exp(I*(b*x+a))+1)*
a+6*d^2/b^2*c*ln(1-exp(I*(b*x+a)))*x+6*d^2/b^3*c*ln(1-exp(I*(b*x+a)))*a-6*d
^3/b^4*a^2*arctanh(exp(I*(b*x+a)))+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4-6*d^
3*polylog(3,-exp(I*(b*x+a)))/b^4-6*d/b^2*c^2*arctanh(exp(I*(b*x+a)))+3*d^3/
b^2*ln(1-exp(I*(b*x+a)))*x^2-3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2-2*I*(d^3*x^
3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)-3*d^3/b^
2*ln(exp(I*(b*x+a))+1)*x^2+3*d^3/b^4*ln(exp(I*(b*x+a))+1)*a^2+6*I*d^2/b^3*c
*polylog(2,-exp(I*(b*x+a)))+12*d^2/b^3*c*a*arctanh(exp(I*(b*x+a)))+6*I*d^3/
b^3*polylog(2,-exp(I*(b*x+a)))*x-1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c
^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*
c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10994 vs. $2(200) = 400$.
time = 1.33, size = 10994, normalized size = 50.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^3*(1/sin(b*x + a) + sin(b*x + a)) - 6*a*c^2*d*(1/sin(b*x + a) + s
in(b*x + a))/b + 6*a^2*c*d^2*(1/sin(b*x + a) + sin(b*x + a))/b^2 - 2*a^3*d^
3*(1/sin(b*x + a) + sin(b*x + a))/b^3 - 3*(((b*x + a)*sin(2*b*x + 2*a) - co
s(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a)
+ a - sin(2*b*x + 2*a))*sin(3*b*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2
*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + c
os(b*x + a))*cos(2*b*x + 2*a) + 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*co
s(3*b*x + 3*a)^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)
^2 + (8*(b*x + a)*cos(2*b*x + 2*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*
a) - cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) -
 sin(b*x + a))*sin(2*b*x + 2*a) - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a
)^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x
 + a)*cos(b*x + a)*sin(b*x + a) - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) +
```

cos(b*x + a)^2 + sin(b*x + a)^2 + 2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a)^2 + cos(b*x + a)^2 + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a) + sin(2*b*x + 2*a)^2 + sin(b*x + a)^2 + 1)*cos(3*b*x + 3 *a) + 2*(3*(b*x + a)*sin(b*x + a)^3 + (3*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a) - ((cos(2*b*x + 2*a)^2 + si n(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + c os(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2* a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a) ^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) ^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin (3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2* (cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2* a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*si n(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + ((b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + (b*x + a)*cos(2*b*x + 2 *a)^2 + (b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2 + 13*(b*x + a)*sin(b*x + a)^2 + b*x + 2*(((b*x + a)*cos(b*x + a) + sin(b*x + a))*cos(2 *b*x + 2*a) - (b*x + a)*cos(b*x + a) - ((b*x + a)*sin(b*x + a) - cos(b*x + a))*sin(2*b*x + 2*a) - sin(b*x + a))*cos(3*b*x + 3*a) - ((b*x + a)*cos(b*x + a)^2 + 13*(b*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12* (b*x + a)*cos(b*x + a)*sin(b*x + a) - cos(b*x + a)^2 - sin(b*x + a)^2)*sin(2*b*x + 2*a) + a)*sin(3*b*x + 3*a) - 6*((b*x + a)*cos(b*x + a)^3 + (b*x + a)*cos(b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a) - (6*(b*x + a)*cos(b*x + a) ^2 + b*x + a)*sin(b*x + a) - cos(b*x + a))*c^2*d/(((cos(2*b*x + 2*a)^2 + si n(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + c os(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2* a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a) ^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*b) + 6*(((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin (2*b*x + 2*a))*sin(3*b*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2*(4*(b*x

```
+ a)*cos(b*x + a)*sin(2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + cos(b*x +
a))*cos(2*b*x + 2*a) + 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(3*b*x +
 3*a)^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(
b*x + a)*cos(2*b*x + 2*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*a) - cos(
2*b*x + 2*a) + 1)*cos(3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x
+ a))*sin(2*b*x + 2*a) - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a)^2 - ((b
*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x + a)*cos
(b*x + a)*sin(b*x + a) - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x
+ a)^2 + sin(b*x + a)^2 + 2)*cos(2*b*x + 2*a) +...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(200) = 400$.
time = 4.23, size = 797, normalized size = 3.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 4*b^3*c^3 - 6*d^3*polylog(3, cos(b
*x + a) + I*sin(b*x + a))*sin(b*x + a) - 6*d^3*polylog(3, cos(b*x + a) - I*
sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) + I*sin(b*x + a
))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x
+ a) - 12*b*c*d^2 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2
+ 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*
x + b^2*c^2*d - 2*d^3)*cos(b*x + a)*sin(b*x + a) + 6*(I*b*d^3*x + I*b*c*d^2
)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(-I*b*d^3*x - I*b*c
*d^2)*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*(I*b*d^3*x + I*
b*c*d^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(-I*b*d^3*x
 - I*b*c*d^2)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d
^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*
sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a)
 - I*sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)
*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3*(b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1
/2)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*
log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b
^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)
*sin(b*x + a) + 12*(b^3*c^2*d - b*d^3)*x)/(b^4*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \cos (a + bx) \cot^2 (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*cos(a + b*x)*cot(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)*cot(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)\,\mathrm{cot}(a+b\,x)^2\,\left(c+d\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^3, x)
```

3.173 $\int (c+dx)^2 \cos(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=139

$$-\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2d(c+dx)\cos(a+bx)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{2id^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-$$

```
[Out] -4*d*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^2-2*d*(d*x+c)*cos(b*x+a)/b^2-(d*x+c)
^2*csc(b*x+a)/b+2*I*d^2*polylog(2,-exp(I*(b*x+a)))/b^3-2*I*d^2*polylog(2,ex
p(I*(b*x+a)))/b^3+2*d^2*sin(b*x+a)/b^3-(d*x+c)^2*sin(b*x+a)/b
```

Rubi [A]
time = 0.09, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {4493, 3377, 2717, 4495, 4268, 2317, 2438}

$$\frac{2id^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}+\frac{2d^2\sin(a+bx)}{b^3}-\frac{2d(c+dx)\cos(a+bx)}{b^2}-\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (-4*d*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^2 - (2*d*(c + d*x)*Cos[a + b*x]
)/b^2 - ((c + d*x)^2*Csc[a + b*x])/b + ((2*I)*d^2*PolyLog[2, -E^(I*(a + b*x
))])/b^3 - ((2*I)*d^2*PolyLog[2, E^(I*(a + b*x))])/b^3 + (2*d^2*Sin[a + b*x
])/b^3 - ((c + d*x)^2*Sin[a + b*x])/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\cos(a+bx)\cot^2(a+bx)\,dx &= -\int (c+dx)^2\cos(a+bx)\,dx+\int (c+dx)^2\cot(a+bx)\csc(a+b\\ &= -\frac{(c+dx)^2\csc(a+bx)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}+\frac{(2d)\int(c+dx\\ &= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2d(c+dx)\cos(a+bx)}{b^2}-\frac{(c+d\\ &= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2d(c+dx)\cos(a+bx)}{b^2}-\frac{(c+d\\ &= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2d(c+dx)\cos(a+bx)}{b^2}-\frac{(c+d\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 310 vs. 2(139) = 278.
time = 4.13, size = 310, normalized size = 2.23

$$-\frac{8bcd\tanh^{-1}\left(\cos(a)-\sin(a)\tan\left(\frac{bx}{2}\right)\right)+2b^2(c+dx)^2\csc(a)-4d^2\left(2\text{ArcTan}(\tan(a))\tanh^{-1}\left(\cos(a)-\sin(a)\tan\left(\frac{bx}{2}\right)\right)+\frac{(bx+\text{ArcTan}(\tan(a)))\left(\log\left(1-e^{i(bx+\text{ArcTan}(\tan(a)))}\right)-\log\left(1+e^{i(bx+\text{ArcTan}(\tan(a)))}\right)\right)+i\text{PolyLog}\left(2,-e^{i(bx+\text{ArcTan}(\tan(a)))}\right)-i\text{PolyLog}\left(2,e^{i(bx+\text{ArcTan}(\tan(a)))}\right)}{\sqrt{\sec^2(a)}}\sec(a)\right)+2\cos(bx)\left(2bd(c+dx)\cos(a)+\left(-2d^2+b^2(c+dx)^2\right)\sin(a)\right)-b^2(c+dx)^2\csc\left(\frac{a}{2}\right)\csc\left(\frac{1}{2}(a+bx)\right)\sin\left(\frac{bx}{2}\right)+b^2(c+dx)^2\sec\left(\frac{a}{2}\right)\sec\left(\frac{1}{2}(a+bx)\right)\sin\left(\frac{bx}{2}\right)+2\left(\left(-2d^2+b^2(c+dx)^2\right)\cos(a)-2bd(c+dx)\sin(a)\right)\sin(bx)}{2b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] -1/2*(8*b*c*d*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + 2*b^2*(c + d*x)^2*Csc
[a] - 4*d^2*(2*ArcTan[Tan[a]]*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + (((b*
x + ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(I*(
b*x + ArcTan[Tan[a]]))]) + I*PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))] - I*
PolyLog[2, E^(I*(b*x + ArcTan[Tan[a]]))])*Sec[a])/Sqrt[Sec[a]^2]) + 2*Cos[b
*x]*(2*b*d*(c + d*x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a]) - b^2*(c +
 d*x)^2*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2] + b^2*(c + d*x)^2*Sec[a/2]*S
ec[(a + b*x)/2]*Sin[(b*x)/2] + 2*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] - 2*b*d
*(c + d*x)*Sin[a])*Sin[b*x])/b^3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(131) = 262.
time = 0.14, size = 332, normalized size = 2.39

method	result
risch	$\frac{i\left(x^2 d^2 b^2+2 b^2 c d x+2 i b\, d^2 x+b^2 c^2+2 i b c d-2 d^2\right) \mathrm{e}^{i(b x+a)}}{2 b^3}-\frac{i\left(x^2 d^2 b^2+2 b^2 c d x-2 i b\, d^2 x+b^2 c^2-2 i b c d-2 d^2\right) \mathrm{e}^{-i(b x+a)}}{2 b^3}-\frac{2 i\left(x^2 d^2+2 c d x\right.}{b\left(\mathrm{e}^{2 i(b x\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp
(I*(b*x+a))-1/2*I*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*
c*d)/b^3*exp(-I*(b*x+a))-2*I*(d^2*x^2+2*c*d*x+c^2)*exp(I*(b*x+a))/b/(exp(2*
I*(b*x+a))-1)-4*d/b^2*c*arctanh(exp(I*(b*x+a)))+2*d^2/b^2*ln(1-exp(I*(b*x+a
)))*x+2*d^2/b^3*ln(1-exp(I*(b*x+a)))*a-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^
3-2*d^2/b^2*ln(exp(I*(b*x+a))+1)*x-2*d^2/b^3*ln(exp(I*(b*x+a))+1)*a+2*I*d^2
*polylog(2,-exp(I*(b*x+a)))/b^3+4*d^2/b^3*a*arctanh(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3199 vs. 2(127) = 254.
time = 1.15, size = 3199, normalized size = 23.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*d^2*x^2*(-I*cos(a) + sin(a)) + b^2*c^2*(-I*cos(a) + sin(a)) - 2*b*
c*d*(cos(a) + I*sin(a)) - 2*d^2*(-I*cos(a) + sin(a)) - 2*(b^2*c*d*(I*cos(a)
 - sin(a)) + b*d^2*(cos(a) + I*sin(a)))*x - 4*((b*d^2*x*(-I*cos(a) + sin(a)
) + b*c*d*(-I*cos(a) + sin(a)) + (I*b*d^2*x + I*b*c*d)*cos(2*b*x + 3*a) - (
```

b*d^2*x + b*c*d)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + ((-I*b*d^2*x - I*b*c*
d)*cos(b*x + a) + (b*d^2*x + b*c*d)*sin(b*x + a))*cos(2*b*x + 3*a) + (b*d^2
x(I*cos(a) - sin(a)) + b*c*d*(I*cos(a) - sin(a)))*cos(b*x + a) + (b*d^2*x
*(cos(a) + I*sin(a)) + b*c*d*(cos(a) + I*sin(a)) - (b*d^2*x + b*c*d)*cos(2*
b*x + 3*a) + (-I*b*d^2*x - I*b*c*d)*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) + ((
b*d^2*x + b*c*d)*cos(b*x + a) + (I*b*d^2*x + I*b*c*d)*sin(b*x + a))*sin(2*b
*x + 3*a) - (b*d^2*x*(cos(a) + I*sin(a)) + b*c*d*(cos(a) + I*sin(a)))*sin(b
*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 4*(b*c*d*(-I*cos(a) + si
n(a))*cos(b*x + a) + b*c*d*(cos(a) + I*sin(a))*sin(b*x + a) + (b*c*d*(I*cos
(a) - sin(a)) - I*b*c*d*cos(2*b*x + 3*a) + b*c*d*sin(2*b*x + 3*a))*cos(3*b*
x + 3*a) + (I*b*c*d*cos(b*x + a) - b*c*d*sin(b*x + a))*cos(2*b*x + 3*a) - (
b*c*d*(cos(a) + I*sin(a)) - b*c*d*cos(2*b*x + 3*a) - I*b*c*d*sin(2*b*x + 3*
a))*sin(3*b*x + 3*a) - (b*c*d*cos(b*x + a) + I*b*c*d*sin(b*x + a))*sin(2*b*
x + 3*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) - 4*(b*d^2*x*(I*cos(a) -
sin(a))*cos(b*x + a) - b*d^2*x*(cos(a) + I*sin(a))*sin(b*x + a) + (b*d^2*x*
(-I*cos(a) + sin(a)) + I*b*d^2*x*cos(2*b*x + 3*a) - b*d^2*x*sin(2*b*x + 3*a
))*cos(3*b*x + 3*a) + (-I*b*d^2*x*cos(b*x + a) + b*d^2*x*sin(b*x + a))*cos(
2*b*x + 3*a) + (b*d^2*x*(cos(a) + I*sin(a)) - b*d^2*x*cos(2*b*x + 3*a) - I*
b*d^2*x*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) + (b*d^2*x*cos(b*x + a) + I*b*d^
2*x*sin(b*x + a))*sin(2*b*x + 3*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1
) + ((I*b^2*d^2*x^2 + I*b^2*c^2 - 2*b*c*d - 2*I*d^2 - 2*(-I*b^2*c*d + b*d^2
)*x)*cos(3*b*x + 3*a) + (-I*b^2*d^2*x^2 - I*b^2*c^2 + 2*b*c*d + 2*I*d^2 - 2
*(I*b^2*c*d - b*d^2)*x)*cos(b*x + a) - (b^2*d^2*x^2 + b^2*c^2 + 2*I*b*c*d -
 2*d^2 + 2*(b^2*c*d + I*b*d^2)*x)*sin(3*b*x + 3*a) + (b^2*d^2*x^2 + b^2*c^2
 + 2*I*b*c*d - 2*d^2 + 2*(b^2*c*d + I*b*d^2)*x)*sin(b*x + a))*cos(3*b*x + 4
a) - 2((3*I*b^2*d^2*x^2 + 6*I*b^2*c*d*x + 3*I*b^2*c^2 - 2*I*d^2)*cos(b*x
+ 2*a) - (3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*sin(b*x + 2*a))*
cos(3*b*x + 3*a) + (I*b^2*d^2*x^2 + I*b^2*c^2 + 2*b*c*d - 2*I*d^2 - 2*(-I*b
^2*c*d - b*d^2)*x)*cos(2*b*x + 3*a) - 2*((-3*I*b^2*d^2*x^2 - 6*I*b^2*c*d*x
- 3*I*b^2*c^2 + 2*I*d^2)*cos(b*x + a) + (3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^
2*c^2 - 2*d^2)*sin(b*x + a))*cos(b*x + 2*a) - 4*(d^2*(-I*cos(a) + sin(a))*c
os(b*x + a) + d^2*(cos(a) + I*sin(a))*sin(b*x + a) + (d^2*(I*cos(a) - sin(a
)) - I*d^2*cos(2*b*x + 3*a) + d^2*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + (I*d
^2*cos(b*x + a) - d^2*sin(b*x + a))*cos(2*b*x + 3*a) - (d^2*(cos(a) + I*sin
(a)) - d^2*cos(2*b*x + 3*a) - I*d^2*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) - (d
^2*cos(b*x + a) + I*d^2*sin(b*x + a))*sin(2*b*x + 3*a))*dilog(-e^(I*b*x + I
a)) - 4(d^2*(I*cos(a) - sin(a))*cos(b*x + a) - d^2*(cos(a) + I*sin(a))*si
n(b*x + a) + (d^2*(-I*cos(a) + sin(a)) + I*d^2*cos(2*b*x + 3*a) - d^2*sin(2
*b*x + 3*a))*cos(3*b*x + 3*a) + (-I*d^2*cos(b*x + a) + d^2*sin(b*x + a))*co
s(2*b*x + 3*a) + (d^2*(cos(a) + I*sin(a)) - d^2*cos(2*b*x + 3*a) - I*d^2*si
n(2*b*x + 3*a))*sin(3*b*x + 3*a) + (d^2*cos(b*x + a) + I*d^2*sin(b*x + a))*
sin(2*b*x + 3*a))*dilog(e^(I*b*x + I*a)) + 2*((b*d^2*x*(cos(a) + I*sin(a))
+ b*c*d*(cos(a) + I*sin(a)) - (b*d^2*x + b*c*d)*cos(2*b*x + 3*a) - (I*b*d^2
*x + I*b*c*d)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + ((b*d^2*x + b*c*d)*cos(b
*x + a) - (-I*b*d^2*x - I*b*c*d)*sin(b*x + a))*cos(2*b*x + 3*a) - (b*d^2*x*

```
(cos(a) + I*sin(a)) + b*c*d*(cos(a) + I*sin(a)))*cos(b*x + a) - (b*d^2*x*(-
I*cos(a) + sin(a)) + b*c*d*(-I*cos(a) + sin(a)) + (I*b*d^2*x + I*b*c*d)*cos
(2*b*x + 3*a) - (b*d^2*x + b*c*d)*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) - ((-I
*b*d^2*x - I*b*c*d)*cos(b*x + a) + (b*d^2*x + b*c*d)*sin(b*x + a))*sin(2*b*
x + 3*a) - (b*d^2*x*(I*cos(a) - sin(a)) + b*c*d*(I*cos(a) - sin(a)))*sin(b*
x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - 2*((b*d
^2*x*(cos(a) + I*sin(a)) + b*c*d*(cos(a) + I*sin(a)) - (b*d^2*x + b*c*d)*co
s(2*b*x + 3*a) + (-I*b*d^2*x - I*b*c*d)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a)
+ ((b*d^2*x + b*c*d)*cos(b*x + a) + (I*b*d^2*x + I*b*c*d)*sin(b*x + a))*cos
(2*b*x + 3*a) - (b*d^2*x*(cos(a) + I*sin(a)) + b*c*d*(cos(a) + I*sin(a)))*c
os(b*x + a) + (b*d^2*x*(I*cos(a) - sin(a)) + b*c*d*(I*cos(a) - sin(a)) + (-
I*b*d^2*x - I*b*c*d)*cos(2*b*x + 3*a) + (b*d^2*x + b*c*d)*sin(2*b*x + 3*a))
*sin(3*b*x + 3*a) + ((I*b*d^2*x + I*b*c*d)*cos(b*x + a) - (b*d^2*x + b*c*d)
*sin(b*x + a))*sin(2*b*x + 3*a) + (b*d^2*x*(-I*cos(a) + sin(a)) + b*c*d*(-I
*cos(a) + sin(a)))*sin(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*co
s(b*x + a) + 1) - ((b^2*d^2*x^2 + b^2*c^2 + 2*I*b*c*d - 2*d^2 + 2*(b^2*c*d
+ I*b*d^2)*x)*cos(3*b*x + 3*a) - (b^2*d^2*x^2 + b^2*c^2 + 2*I*b*c*d - 2*d^2
 + 2*(b^2*c*d + I*b*d^2)*x)*cos(b*x + a) - (-I*...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(127) = 254$.
time = 3.30, size = 448, normalized size = 3.22

$$-\frac{2b^2d^2x^2+4b^2cdx+2b^2c^2+id^2\mathrm{Li}_2(\cos(bx+a)+i\sin(bx+a))\sin(bx+a)-id^2\mathrm{Li}_2(\cos(bx+a)-i\sin(bx+a))\sin(bx+a)+id^2\mathrm{Li}_2(-\cos(bx+a)+i\sin(bx+a))\sin(bx+a)-id^2\mathrm{Li}_2(-\cos(bx+a)-i\sin(bx+a))\sin(bx+a)-(b^2d^2x^2+2b^2cdx+b^2c^2-2d^2)\cos(bx+a)^2+2(bd^2x+bcd)\cos(bx+a)\sin(bx+a)+(bd^2x+bcd)\log(\cos(bx+a)+i\sin(bx+a)+1)\sin(bx+a)+(bd^2x+bcd)\log(\cos(bx+a)-i\sin(bx+a)+1)\sin(bx+a)-(bcd-ad^2)\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}i\sin(bx+a)+\frac{1}{2}\right)\sin(bx+a)-(bcd-ad^2)\log\left(-\frac{1}{2}\cos(bx+a)-\frac{1}{2}i\sin(bx+a)+\frac{1}{2}\right)\sin(bx+a)-(bd^2x+ad^2)\log(-\cos(bx+a)+i\sin(bx+a)+1)\sin(bx+a)-(bd^2x+ad^2)\log(-\cos(bx+a)-i\sin(bx+a)+1)\sin(bx+a)-2d^2}{b^3\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + I*d^2*dilog(cos(b*x + a) + I*si
n(b*x + a))*sin(b*x + a) - I*d^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b
*x + a) + I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - I*d^2*
dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)*s
in(b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(
b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x
 + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*s
in(b*x + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) +
1/2)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) +
1)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)
*sin(b*x + a) - 2*d^2)/(b^3*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \cos(a+bx) \cot^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)*cot(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)*cot(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)\,{\mathrm{cot}\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^2, x)
```

3.174 $\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=58

$$-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{d\cos(a+bx)}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}-\frac{(c+dx)\sin(a+bx)}{b}$$

```
[Out] -d*arctanh(cos(b*x+a))/b^2-d*cos(b*x+a)/b^2-(d*x+c)*csc(b*x+a)/b-(d*x+c)*si
n(b*x+a)/b
```

Rubi [A]
time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4493, 3377, 2718, 4495, 3855}

$$-\frac{d\cos(a+bx)}{b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}-\frac{(c+dx)\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] -((d*ArcTanh[Cos[a + b*x]])/b^2) - (d*Cos[a + b*x])/b^2 - ((c + d*x)*Csc[a
+ b*x])/b - ((c + d*x)*Sin[a + b*x])/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx = -\int (c+dx)\cos(a+bx)\,dx + \int (c+dx)\cot(a+bx)\csc(a+bx)$$

$$= -\frac{(c+dx)\csc(a+bx)}{b} - \frac{(c+dx)\sin(a+bx)}{b} + \frac{d\int \csc(a+bx)\,d}{b}$$

$$= -\frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{d\cos(a+bx)}{b^2} - \frac{(c+dx)\csc(a+bx)}{b}$$

Mathematica [A]
time = 0.76, size = 104, normalized size = 1.79

$$-\frac{2d\cos(a+bx)+bdx\cot\left(\frac{1}{2}(a+bx)\right)+2bc\csc(a+bx)+2d\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-2d\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)+2bc\sin(a+bx)+2bdx\sin(a+bx)+bdx\tan\left(\frac{1}{2}(a+bx)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] -1/2*(2*d*Cos[a + b*x] + b*d*x*Cot[(a + b*x)/2] + 2*b*c*Csc[a + b*x] + 2*d*
Log[Cos[(a + b*x)/2]] - 2*d*Log[Sin[(a + b*x)/2]] + 2*b*c*Sin[a + b*x] + 2*
b*d*x*Sin[a + b*x] + b*d*x*Tan[(a + b*x)/2])/b^2
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 124, normalized size = 2.14

method	result	size
risch	$\frac{i(dxb+cb+id)e^{i(bx+a)}}{2b^2} - \frac{i(dxb+cb-id)e^{-i(bx+a)}}{2b^2} - \frac{2i(dx+c)e^{i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)} - \frac{d\ln\left(e^{i(bx+a)}+1\right)}{b^2} + \frac{d\ln\left(e^{i(bx+a)}-1\right)}{b^2}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*(d*x*b+c*b+I*d)/b^2*exp(I*(b*x+a))-1/2*I*(d*x*b+c*b-I*d)/b^2*exp(-I*(
b*x+a))-2*I*(d*x+c)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)-d/b^2*ln(exp(I*(b
*x+a))+1)+d/b^2*ln(exp(I*(b*x+a))-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2110 vs. $2(58) = 116$.
time = 0.30, size = 2110, normalized size = 36.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*(1/sin(b*x + a) + sin(b*x + a)) - 2*a*d*(1/sin(b*x + a) + sin(b*x
 + a))/b - (((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*cos(3*b*x +
 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin(2*b*x + 2*a))*sin(3*b
*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2*(4*(b*x + a)*cos(b*x + a)*sin(
2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a) +
 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a)^2 - ((b*x + a)*s
in(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x + a)*cos(2*b*x + 2
*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*cos(
3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 2*a)
 - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a)^2 - ((b*x + a)*sin(b*x + a) +
 cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x + a)*cos(b*x + a)*sin(b*x + a)
 - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^2 + sin(b*x + a)^
2 + 2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a)^2 + cos(b*x + a)^2 + (13*(b*x +
a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a) + sin(2*b*x
+ 2*a)^2 + sin(b*x + a)^2 + 1)*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*sin(b*x +
a)^3 + (3*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x + a) + cos(b*x + a))*
cos(2*b*x + 2*a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x
+ 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*
x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)
+ 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a
)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2
- 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos
(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*
b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x +
 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(c
os(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + ((cos(2*b*x + 2*a)^2
 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b
*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)
^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a
) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos
(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x
 + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x +
 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(
3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(
b*x + a) + 1) + ((b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin(2*b*x + 2*a))*
```

```
cos(3*b*x + 3*a)^2 + (b*x + a)*cos(2*b*x + 2*a)^2 + (b*x + a)*cos(b*x + a)^
2 + (b*x + a)*sin(2*b*x + 2*a)^2 + 13*(b*x + a)*sin(b*x + a)^2 + b*x + 2*((
(b*x + a)*cos(b*x + a) + sin(b*x + a))*cos(2*b*x + 2*a) - (b*x + a)*cos(b*x
 + a) - ((b*x + a)*sin(b*x + a) - cos(b*x + a))*sin(2*b*x + 2*a) - sin(b*x
+ a))*cos(3*b*x + 3*a) - ((b*x + a)*cos(b*x + a)^2 + 13*(b*x + a)*sin(b*x +
 a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x
+ a) - cos(b*x + a)^2 - sin(b*x + a)^2)*sin(2*b*x + 2*a) + a)*sin(3*b*x + 3
*a) - 6*((b*x + a)*cos(b*x + a)^3 + (b*x + a)*cos(b*x + a)*sin(b*x + a)^2)*
sin(2*b*x + 2*a) - (6*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x + a) - co
s(b*x + a))*d/(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*
a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x +
2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2
- 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*
cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x
 + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x
+ 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a
)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*b))/b
```

Fricas [**A**]
time = 2.77, size = 95, normalized size = 1.64

$$-\frac{4\,bdx - 2\,(bdx + bc)\cos\left(bx + a\right)^{2} + 2\,d\cos\left(bx + a\right)\sin\left(bx + a\right) + d\log\left(\frac{1}{2}\,\cos\left(bx + a\right) + \frac{1}{2}\right)\sin\left(bx + a\right) - d\log\left(-\frac{1}{2}\,\cos\left(bx + a\right) + \frac{1}{2}\right)\sin\left(bx + a\right) + 4\,bc}{2\,b^{2}\sin\left(bx + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + 2*d*cos(b*x + a)*sin(b*x +
 a) + d*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - d*log(-1/2*cos(b*x + a)
+ 1/2)*sin(b*x + a) + 4*b*c)/(b^2*sin(b*x + a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)\cos\left(a + bx\right)\cot^{2}\left(a + bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)*cot(a + b*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1967 vs. $2(58) = 116$.
time = 1.07, size = 1967, normalized size = 33.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")

[Out] 1/2*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 + 6*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^3 - d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^3 + d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^3 + 6*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a)^4 + d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a)^4 + 6*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*b*c*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 6*b*c*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + b*d*x*tan(1/2*b*x)^4 - 8*b*d*x*tan(1/2*b*x)^3*tan(1/2*a) - d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a) + d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a) - 12*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - 8*b*d*x*tan(1/2*b*x)*tan(1/2*a)^3 + b*d*x*tan(1/2*a)^4 - d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^4 + d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4 - 8*b*c*tan(1/2*b*x)^3*tan(1/2*a) + 2*d*tan(1/2*b*x)^4*tan(1/2*a) - 12*b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 + 12*d*tan(1/2*b*x)^3*tan(1/2*a)^2 - 8*b*c*tan(1/2*b*x)*tan(1/2*a)^3 + 12*d*tan(1/2*b*x)^2*tan(1/2*a)^3 + b*c*tan(1/2*a)^4 + 2*d*tan(1/2*b*x)*tan(1/2*a)^4 + 6*b*d*x*tan(1/2*b*x)^2 + d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3 - d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3 + 8*b*d*x*tan(1/2*b*x)*tan(1/2*a) + 6*b*d*x*tan(1/2*a)^2 + d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*a)^3 - d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 +

```
tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 +
1))*tan(1/2*a)^3 + 6*b*c*tan(1/2*b*x)^2 - 2*d*tan(1/2*b*x)^3 + 8*b*c*tan(1/
2*b*x)*tan(1/2*a) - 12*d*tan(1/2*b*x)^2*tan(1/2*a) + 6*b*c*tan(1/2*a)^2 - 1
2*d*tan(1/2*b*x)*tan(1/2*a)^2 - 2*d*tan(1/2*a)^3 + b*d*x + d*log(4*(tan(1/2
*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2
*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))
*tan(1/2*b*x) - d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan
(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan
(1/2*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x) + d*log(4*(tan(1/2*b*x)^4*tan(1
/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1
/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*a) -
 d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan
(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan
(1/2*a)^2 + 1))*tan(1/2*a) + b*c + 2*d*tan(1/2*b*x) + 2*d*tan(1/2*a))/(b^2*
tan(1/2*b*x)^4*tan(1/2*a)^3 + b^2*tan(1/2*b*x)^3*tan(1/2*a)^4 + b^2*tan(1/2
*b*x)^4*tan(1/2*a) + b^2*tan(1/2*b*x)*tan(1/2*a)^4 - b^2*tan(1/2*b*x)^3 - b
^2*tan(1/2*a)^3 - b^2*tan(1/2*b*x) - b^2*tan(1/2*a))
```

Mupad [B]
time = 2.31, size = 162, normalized size = 2.79

$$\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\frac{\left(b\,c+d\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b^2}+\frac{d\,x\,1\mathrm{i}}{2\,b}\right)+\mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}\,\left(\frac{\left(-b\,c+d\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b^2}-\frac{d\,x\,1\mathrm{i}}{2\,b}\right)-\frac{d\,\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{b^2}+\frac{d\,\ln\left(d\,2\mathrm{i}-d\,\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}\,2\mathrm{i}\right)}{b^2}+\frac{2\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(c+d\,x\right)}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x),x)
```

```
[Out] exp(a*1i + b*x*1i)*(((d*1i + b*c)*1i)/(2*b^2) + (d*x*1i)/(2*b)) + exp(- a*1
i - b*x*1i)*(((d*1i - b*c)*1i)/(2*b^2) - (d*x*1i)/(2*b)) - (d*log(exp(a*1i
+ b*x*1i)*1i + 1i))/b^2 + (d*log(d*2i - d*exp(a*1i)*exp(b*x*1i)*2i))/b^2 +
(2*exp(a*1i + b*x*1i)*(c + d*x))/(b*(exp(a*2i + b*x*2i)*1i - 1i))
```

3.175 $\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=75

$$-\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\text{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(cot(b*x+a)*csc(b*x+a)/(d*x+c),x)-Ci(b*c/d+b*x)*cos(a-b*c/d)
/d+Si(b*c/d+b*x)*sin(a-b*c/d)/d
```

Rubi [A]
time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x),x]
```

```
[Out] -((Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d) + (Sin[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d + Defer[Int][(Cot[a + b*x]*Csc[a + b*x])/(c + d*x
), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx &= -\int \frac{\cos(a+bx)}{c+dx}\,dx+\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx \\
&= -\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\right)+\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx \\
&= -\frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\int \frac{\cot(a+bx)\csc(}{c+dx}
\end{aligned}$$

Mathematica [A]
time = 4.05, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x),x]
```

```
[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x), x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\cos \left(bx+a\right) \left(\cot^{2}\left(bx+a\right)\right)}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(1, (I*b*d*x +
I*b*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d)
+ (b*c*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*
x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(1, (I*b*d*x + I*
b*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) +
(b*d*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x
+ I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e(1, (I*b*d*x + I*b*
c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*
cos(2*b*x + 2*a)^2 - 4*d*cos(b*x + a)*sin(2*b*x + 2*a) + (b*c*(exp_integral
_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-
(b*c - a*d)/d) - b*c*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_inte
gral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(exp_integral_e
(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b
*c - a*d)/d) - b*d*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integr
al_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*sin(2*b*x + 2*a)^2 +
 (b*d*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x
 + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e(1, (I*b*d*x + I*b
*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x
- 2*(b*c*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(1, (I*b*d*x +
I*b*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d)
+ (b*d*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*
x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e(1, (I*b*d*x + I*
b*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x
```

```
 - 2*d*sin(b*x + a))*cos(2*b*x + 2*a) - 2*(b*d^3*x + b*c*d^2 + (b*d^3*x + b
*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x + b*c*d^2)*sin(2*b*x + 2*a)^2 - 2*(b*
d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^2*x^2 + 2*b*
c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 + (b*d^2*x^2
 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*co
s(b*x + a)), x) - 2*(b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*
a)^2 + (b*d^3*x + b*c*d^2)*sin(2*b*x + 2*a)^2 - 2*(b*d^3*x + b*c*d^2)*cos(2
*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2
*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*
sin(b*x + a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) - 4*d*
sin(b*x + a))/(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*
d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="fricas")
[Out] integral(cos(b*x + a)*cot(b*x + a)^2/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)**2/(d*x+c),x)
[Out] Integral(cos(a + b*x)*cot(a + b*x)**2/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(cos(b*x + a)*cot(b*x + a)^2/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,{\mathrm{cot}\left(a+b\,x\right)}^2}{c+d\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x),x)
[Out] int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x), x)
```

3.176 $\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=93

$$\frac{\cos(a+bx)}{d(c+dx)}+\frac{b\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2}+\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\text{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2},x\right.$$

```
[Out] CannotIntegrate(cot(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)+cos(b*x+a)/d/(d*x+c)+b*c
os(a-b*c/d)*Si(b*c/d+b*x)/d^2+b*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^2
```

Rubi [A]
time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Cos[a + b*x]/(d*(c + d*x)) + (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d]
)/d^2 + (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int][(C
ot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx &= -\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx+\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{b\int \frac{\sin(a+bx)}{c+dx}\,dx}{d}+\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right.}{c+dx}}{d}\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{b\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2}+\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\int
\end{aligned}$$

Mathematica [A]
time = 4.43, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2, x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\cos (bx + a) (\cot^2 (bx + a))}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(2, (I*b*d*x +
I*b*c)/d) - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d)
+ (b*c*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*
x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(2, (I*b*d*x + I*
b*c)/d) - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) +
(b*d*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x
+ I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e(2, (I*b*d*x + I*b*
c)/d) - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*
cos(2*b*x + 2*a)^2 - 4*d*cos(b*x + a)*sin(2*b*x + 2*a) + (b*c*(exp_integral
_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*cos(-
(b*c - a*d)/d) - b*c*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_inte
gral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(exp_integral_e
(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*cos(-(b
*c - a*d)/d) - b*d*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_integr
al_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*sin(2*b*x + 2*a)^2 +
 (b*d*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x
 + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e(2, (I*b*d*x + I*b
*c)/d) - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x
- 2*(b*c*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(2, (I*b*d*x +
```

```
I*b*c)/d) - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d)
+ (b*d*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*
x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e(2, (I*b*d*x + I*
b*c)/d) - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x
 - 2*d*sin(b*x + a))*cos(2*b*x + 2*a) - 4*(b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*
d^2 + (b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*cos(2*b*x + 2*a)^2 + (b*d^4*x^2
 + 2*b*c*d^3*x + b*c^2*d^2)*sin(2*b*x + 2*a)^2 - 2*(b*d^4*x^2 + 2*b*c*d^3*x
 + b*c^2*d^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^3*x^3 + 3*b*c*d
^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b
*c^3)*cos(b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*si
n(b*x + a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x
+ a)), x) - 4*(b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2 + (b*d^4*x^2 + 2*b*c*d^3
*x + b*c^2*d^2)*cos(2*b*x + 2*a)^2 + (b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*
sin(2*b*x + 2*a)^2 - 2*(b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*cos(2*b*x + 2*
a))*integrate(sin(b*x + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3
 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)^2 + (b*d^
3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(b*x + a)^2 - 2*(b*d^3*x^3
+ 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)), x) - 4*d*sin(b*x + a)
)/(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*
cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^2
 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(cos(b*x + a)*cot(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a+bx\right)\cot^{2}\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)**2/(d*x+c)**2,x)
[Out] Integral(cos(a + b*x)*cot(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*cot(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,{\mathrm{cot}\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x)^2, x)
```

3.177 $\int (c+dx)^m \cot^3(a+bx)\,dx$

Optimal. Leaf size=19

$$\text{Int}\left((c+dx)^m\cot^3(a+bx),x\right)$$

```
[Out] Unintegrable((d*x+c)^m*cot(b*x+a)^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m\cot^3(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x]^3,x]
[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m\cot^3(a+bx)\,dx=\int (c+dx)^m\cot^3(a+bx)\,dx$$

Mathematica [A]
time = 10.15, size = 0, normalized size = 0.00

$$\int (c+dx)^m\cot^3(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x]^3,x]
[Out] Integrate[(c + d*x)^m*Cot[a + b*x]^3, x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int (dx+c)^m\left(\cot^3(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cot(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*cot(b*x+a)^3,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cot(b*x + a)^3, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="fricas")
[Out] integral((d*x + c)^m*cot(b*x + a)^3, x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cot(b*x+a)**3,x)
[Out] Integral((c + d*x)**m*cot(a + b*x)**3, x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*cot(b*x + a)^3, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.05

$$\int \cot(a+b\,x)^3\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3*(c + d*x)^m,x)
[Out] int(cot(a + b*x)^3*(c + d*x)^m, x)
```

3.178 $\int (c+dx)^4 \cot^3(a+bx)\,dx$

Optimal. Leaf size=302

$$-\frac{2id(c+dx)^3}{b^2}-\frac{(c+dx)^4}{2b}+\frac{i(c+dx)^5}{5d}-\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{6d^2(c+dx)^2\log(}{b^3}$$

```
[Out] -2*I*d*(d*x+c)^3/b^2-1/2*(d*x+c)^4/b+1/5*I*(d*x+c)^5/d-2*d*(d*x+c)^3*cot(b*
x+a)/b^2-1/2*(d*x+c)^4*cot(b*x+a)^2/b+6*d^2*(d*x+c)^2*ln(1-exp(2*I*(b*x+a))
)/b^3-(d*x+c)^4*ln(1-exp(2*I*(b*x+a)))/b-6*I*d^3*(d*x+c)*polylog(2,exp(2*I*
(b*x+a)))/b^4+2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^2+3*d^4*polylog
(3,exp(2*I*(b*x+a)))/b^5-3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(b*x+a)))/b^3-3*
I*d^3*(d*x+c)*polylog(4,exp(2*I*(b*x+a)))/b^4+3/2*d^4*polylog(5,exp(2*I*(b*
x+a)))/b^5
```

Rubi [A]
time = 0.32, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3801, 3798, 2221, 2611, 2320, 6724, 32, 6744}

$$\frac{3d^4\text{Li}_3\left(e^{2i(a+bx)}\right)}{b^5}+\frac{3d^4\text{Li}_5\left(e^{2i(a+bx)}\right)}{2b^5}-\frac{6id^3(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^4}-\frac{3id^3(c+dx)\text{Li}_4\left(e^{2i(a+bx)}\right)}{b^4}-\frac{3d^2(c+dx)^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^3}+\frac{2id(c+dx)^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}-\frac{2id(c+dx)^3}{b^2}-\frac{(c+dx)^4}{2b}+\frac{i(c+dx)^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]^3,x]
```

```
[Out] ((-2*I)*d*(c + d*x)^3)/b^2 - (c + d*x)^4/(2*b) + ((I/5)*(c + d*x)^5)/d - (2
*d*(c + d*x)^3*Cot[a + b*x])/b^2 - ((c + d*x)^4*Cot[a + b*x]^2)/(2*b) + (6*
d^2*(c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))])/b^3 - ((c + d*x)^4*Log[1 - E^
((2*I)*(a + b*x))])/b - ((6*I)*d^3*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
])/b^4 + ((2*I)*d*(c + d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^4
*PolyLog[3, E^((2*I)*(a + b*x))])/b^5 - (3*d^2*(c + d*x)^2*PolyLog[3, E^((2
*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*PolyLog[4, E^((2*I)*(a + b*x))]
)/b^4 + (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \cot^3(a+bx)\,dx = -\frac{(c+dx)^4 \cot^2(a+bx)}{2b} + \frac{(2d)\int (c+dx)^3 \cot^2(a+bx)\,dx}{b} - \int (c+dx)^4 \cot($$

$$= \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+dx)^4 \cot^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+ba}}{1-}$$

$$= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+da}{}$$

$$= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+da}{}$$

$$= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+da}{}$$

$$= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+da}{}$$

$$= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+da}{}$$

$$= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+da}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1101 vs. $2(302) = 604$.
time = 6.99, size = 1101, normalized size = 3.65

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]^3,x]
```

```
[Out] -1/5*(x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4)*Cot[a
]) - ((c + d*x)^4*Csc[a + b*x]^2)/(2*b) + (c^2*d^2*Csc[a]*(2*b^2*x^2*(2*b*E
^((2*I)*a)*x + (3*I)*(-1 + E^((2*I)*a))*Log[1 - E^((2*I)*(a + b*x))]) + 6*b
*(-1 + E^((2*I)*a))*x*PolyLog[2, E^((2*I)*(a + b*x))] + (3*I)*(-1 + E^((2*I
)*a))*PolyLog[3, E^((2*I)*(a + b*x))]))/(2*b^3*E^(I*a)) - (d^4*Csc[a]*(2*b^
2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(-1 + E^((2*I)*a))*Log[1 - E^((2*I)*(a + b
*x))]) + 6*b*(-1 + E^((2*I)*a))*x*PolyLog[2, E^((2*I)*(a + b*x))] + (3*I)*(
-1 + E^((2*I)*a))*PolyLog[3, E^((2*I)*(a + b*x))]))/(2*b^5*E^(I*a)) + c*d^3
*E^(I*a)*Csc[a]*(x^4 + (-1 + E^((-2*I)*a))*x^4 + ((-1 + E^((2*I)*a))*(2*b^4
*x^4 + (4*I)*b^3*x^3*Log[1 - E^((2*I)*(a + b*x))] + 6*b^2*x^2*PolyLog[2, E^
((2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[3, E^((2*I)*(a + b*x))] - 3*PolyLog[
4, E^((2*I)*(a + b*x))]))/(2*b^4*E^((2*I)*a))) + (d^4*E^(I*a)*Csc[a]*(x^5 +
 (-1 + E^((-2*I)*a))*x^5 + ((-1 + E^((2*I)*a))*(4*b^5*x^5 + (10*I)*b^4*x^4*
```

```
Log[1 - E^((2*I)*(a + b*x))] + 20*b^3*x^3*PolyLog[2, E^((2*I)*(a + b*x))] +
 (30*I)*b^2*x^2*PolyLog[3, E^((2*I)*(a + b*x))] - 30*b*x*PolyLog[4, E^((2*I
)*(a + b*x))] - (15*I)*PolyLog[5, E^((2*I)*(a + b*x))]))/(4*b^5*E^((2*I)*a)
)))/5 - (c^4*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]
*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (6*c^2*d^2*Csc[a]*(-(b*x*Cos[a]) + Lo
g[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) +
 (2*Csc[a]*Csc[a + b*x]*(c^3*d*Sin[b*x] + 3*c^2*d^2*x*Sin[b*x] + 3*c*d^3*x^
2*Sin[b*x] + d^4*x^3*Sin[b*x]))/b^2 + (2*c^3*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcT
an[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b
*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] +
 Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*Pol
yLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^
2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (6*c*d^3*Csc[a]*Sec[a]*(b^2*E^(I*
ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*
I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]])
)] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I
*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))
/(b^4*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1876 vs. $2(273) = 546$.
time = 0.22, size = 1877, normalized size = 6.22

method	result	size
risch	Expression too large to display	1877

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -4/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3-4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3-4/
b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+12*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a))
)*x+12*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+4*I/b^2*d^4*polylog(2,exp
(I*(b*x+a)))*x^3-24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-8*I/b^3*a^3*c^2*d
^2-24*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))+4*I/b^2*c^3*d*polylog(2,exp(I*(
b*x+a)))+4*I/b^2*a^2*c^3*d+6*I/b^4*c*d^3*a^4-2*I/b^4*d^4*a^4*x+12/b^3*c^2*d
^2*a^2*ln(exp(I*(b*x+a)))+4/b^2*c^3*d*a*ln(exp(I*(b*x+a))-1)-8/b^2*c^3*d*a*
ln(exp(I*(b*x+a)))+4/b^4*c*d^3*a^3*ln(exp(I*(b*x+a))-1)+I*d^3*c*x^4+12*d^4*
polylog(3,-exp(I*(b*x+a)))/b^5+12*d^4*polylog(3,exp(I*(b*x+a)))/b^5+1/5*I*d
^4*x^5+12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2+8*I/b^3*c*d^3*a^3*x+12*
I/b^2*polylog(2,exp(I*(b*x+a)))*c^2*d^2*x-12*I/b^2*a^2*c^2*d^2*x+8*I/b*a*c^
3*d*x-I*c^4*x-1/5*I/d*c^5+2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2
*I*(b*x+a))+6*b*c^2*d^2*x^2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2
*I*d^4*x^3*exp(2*I*(b*x+a))+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b
*x+a))-6*I*c^2*d^2*x*exp(2*I*(b*x+a))+2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a)
)+6*I*c*d^3*x^2+6*I*c^2*d^2*x+2*I*c^3*d)/b^2/(exp(2*I*(b*x+a))-1)^2+24*d^4*
```

```
polylog(5,-exp(I*(b*x+a)))/b^5+24*d^4*polylog(5,exp(I*(b*x+a)))/b^5-1/b*c^4
*ln(exp(I*(b*x+a))-1)-1/b*c^4*ln(exp(I*(b*x+a))+1)+2/b*c^4*ln(exp(I*(b*x+a)
))+2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))-12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a))
)-12/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a)))-1/b^5*d^4*a^4*ln(exp(I*(b*x+a))
-1)+1/b^5*d^4*a^4*ln(1-exp(I*(b*x+a)))-12/b^3*d^4*polylog(3,-exp(I*(b*x+a))
)*x^2-12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^2+6*d^2/b^3*c^2*ln(exp(I*(b*x+
a))-1)+6*d^2/b^3*c^2*ln(exp(I*(b*x+a))+1)-12*d^2/b^3*c^2*ln(exp(I*(b*x+a)))
+6*d^4/b^5*a^2*ln(exp(I*(b*x+a))-1)-12*d^4/b^5*a^2*ln(exp(I*(b*x+a)))+6*d^4
/b^3*ln(1-exp(I*(b*x+a)))*x^2-6*d^4/b^5*ln(1-exp(I*(b*x+a)))*a^2+6*d^4/b^3*
ln(exp(I*(b*x+a))+1)*x^2-4*I*d^4/b^2*x^3+8*I*d^4/b^5*a^3+24*d^3/b^4*c*a*ln(
exp(I*(b*x+a)))-12*d^3/b^4*c*a*ln(exp(I*(b*x+a))-1)+12*d^3/b^3*c*ln(exp(I*(
b*x+a))+1)*x+12*d^3/b^3*c*ln(1-exp(I*(b*x+a)))*x+12*d^3/b^4*c*ln(1-exp(I*(b
*x+a)))*a-12*I*d^4/b^4*polylog(2,-exp(I*(b*x+a)))*x+12*I*d^4/b^4*a^2*x-12*I
*d^3/b^4*c*polylog(2,-exp(I*(b*x+a)))-12*I*d^3/b^4*c*polylog(2,exp(I*(b*x+a
)))-12*I*d^3/b^2*c*x^2-12*I*d^3/b^4*c*a^2-12*I*d^4/b^4*polylog(2,exp(I*(b*x
+a)))*x+2*I*d^2*c^2*x^3+2*I*d*c^3*x^2-8/5*I/b^5*d^4*a^5-24*I*d^3/b^3*c*a*x+
4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3-24*I/b^4*d^4*polylog(4,-exp(I*(b
*x+a)))*x-24*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))+4*I/b^2*c^3*d*polylog(2
,-exp(I*(b*x+a)))-4/b*c^3*d*ln(1-exp(I*(b*x+a)))*x-4/b^2*c^3*d*ln(1-exp(I*(
b*x+a)))*a-4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x-1/b*d^4*ln(1-exp(I*(b*x+a)))*x^
4-1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4-8/b^4*c*d^3*a^3*ln(exp(I*(b*x+a)))-6/b^3
*c^2*d^2*a^2*ln(exp(I*(b*x+a))-1)-6/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2-6/b*
c^2*d^2*ln(1-exp(I*(b*x+a)))*x^2+6/b^3*c^2*d^2*ln(1-exp(I*(b*x+a)))*a^2-24/
b^3*c*d^3*polylog(3,-exp(I*(b*x+a)))*x-24/b^3*c*d^3*polylog(3,exp(I*(b*x+a)
))*x
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7158 vs. $2(266) = 532$.
time = 4.28, size = 7158, normalized size = 23.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^4*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 4*a*c^3*d*(1/sin(b*x +
 a)^2 + log(sin(b*x + a)^2))/b + 6*a^2*c^2*d^2*(1/sin(b*x + a)^2 + log(sin(
b*x + a)^2))/b^2 - 4*a^3*c*d^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b^3
 + a^4*d^4*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b^4 - 2*(2*(b*x + a)^5*
d^4 + 40*b^3*c^3*d - 120*a*b^2*c^2*d^2 + 120*a^2*b*c*d^3 - 40*a^3*d^4 + 10*
(b*c*d^3 - a*d^4)*(b*x + a)^4 + 20*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b
*x + a)^3 + 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x
 + a)^2 - 10*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^2*d^4 +
4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*
d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (
```

```
a^3 - 3*a)*d^4)*(b*x + a) + ((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3
 - 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d
^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 -
 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 2*((b*x + a)^4
*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x
+ a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3
*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a)
)*cos(2*b*x + 2*a) - (-I*(b*x + a)^4*d^4 + 6*I*b^2*c^2*d^2 - 12*I*a*b*c*d^3
 + 6*I*a^2*d^4 + 4*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c^2*d^2 +
 2*I*a*b*c*d^3 + (-I*a^2 + I)*d^4)*(b*x + a)^2 + 4*(-I*b^3*c^3*d + 3*I*a*b^
2*c^2*d^2 + 3*(-I*a^2 + I)*b*c*d^3 + (I*a^3 - 3*I*a)*d^4)*(b*x + a))*sin(4*
b*x + 4*a) - 2*(I*(b*x + a)^4*d^4 - 6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 - 6*I*
a^2*d^4 + 4*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^3 + 6*(I*b^2*c^2*d^2 - 2*I*a*b*
c*d^3 + (I*a^2 - I)*d^4)*(b*x + a)^2 + 4*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 +
 3*(I*a^2 - I)*b*c*d^3 + (-I*a^3 + 3*I*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))
*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(4*b*x + 4*a) - 2*(b^2*c
^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(2*b*x + 2*a) + (I*b^2*c^2*d^2 - 2*I*a*b
*c*d^3 + I*a^2*d^4)*sin(4*b*x + 4*a) + 2*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 -
I*a^2*d^4)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 10*(
(b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*
c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^
2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a) + ((b*x + a)^4*d^4 + 4*(b*c*d^3
 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x
+ a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)
*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 2*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4
)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 +
 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b
*x + a))*cos(2*b*x + 2*a) + (I*(b*x + a)^4*d^4 + 4*(I*b*c*d^3 - I*a*d^4)*(b
*x + a)^3 + 6*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + (I*a^2 - I)*d^4)*(b*x + a)^2
 + 4*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*(I*a^2 - I)*b*c*d^3 + (-I*a^3 + 3
*I*a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) + 2*(-I*(b*x + a)^4*d^4 + 4*(-I*b*c*
d^3 + I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 + (-I*a^2 +
I)*d^4)*(b*x + a)^2 + 4*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 + 3*(-I*a^2 + I)*
b*c*d^3 + (I*a^3 - 3*I*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x
 + a), -cos(b*x + a) + 1) + 2*((b*x + a)^5*d^4 + 5*(b*c*d^3 - a*d^4)*(b*x +
 a)^4 + 10*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^3 + 10*(b^
3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a
)^2 - 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(4*b*x + 4*a)
- 4*((b*x + a)^5*d^4 + 10*b^3*c^3*d - 30*a*b^2*c^2*d^2 + 30*a^2*b*c*d^3 - 1
0*a^3*d^4 + 5*(b*c*d^3 - (a - I)*d^4)*(b*x + a)^4 + 10*(b^2*c^2*d^2 - 2*(a
- I)*b*c*d^3 + (a^2 - 2*I*a - 1)*d^4)*(b*x + a)^3 + 10*(b^3*c^3*d - 3*(a -
I)*b^2*c^2*d^2 + 3*(a^2 - 2*I*a - 1)*b*c*d^3 - (a^3 - 3*I*a^2 - 3*a)*d^4)*(
b*x + a)^2 - 10*(-2*I*b^3*c^3*d + 3*(2*I*a + 1)*b^2*c^2*d^2 + 6*(-I*a^2 - a
)*b*c*d^3 + (2*I*a^3 + 3*a^2)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 40*(b^3*c^
```

```
3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)
*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^
2 - 1)*d^4)*(b*x + a) + (b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*
(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(
b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 2*
(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*(a^2 - 1)*b*c*d^3 - (a^3
 - 3*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^
3 + (a^2 - 1)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (I*b^3*c^3*d - 3*I*a*b^2*c
^2*d^2 + I*(b*x + a)^3*d^4 + 3*(I*a^2 - I)*b*c*d^3 + (-I*a^3 + 3*I*a)*d^4 +
 3*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + (
I*a^2 - I)*d^4)*(b*x + a))*sin(4*b*x + 4*a) + 2*(-I*b^3*c^3*d + 3*I*a*b^2*c
^2*d^2 - I*(b*x + a)^3*d^4 + 3*(-I*a^2 + I)*b*c...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1751 vs. $2(266) = 532$.
time = 2.91, size = 1751, normalized size = 5.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^4*d^4*x^4 + 16*b^4*c*d^3*x^3 + 24*b^4*c^2*d^2*x^2 + 16*b^4*c^3*d*x
 + 4*b^4*c^4 - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + I*b^3*c^3*d - 3*I*b*c
*d^3 + 3*I*(b^3*c^2*d^2 - b*d^4)*x + (-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 -
I*b^3*c^3*d + 3*I*b*c*d^3 - 3*I*(b^3*c^2*d^2 - b*d^4)*x)*cos(2*b*x + 2*a))*
dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) - 4*(-I*b^3*d^4*x^3 - 3*I*b^3*
c*d^3*x^2 - I*b^3*c^3*d + 3*I*b*c*d^3 - 3*I*(b^3*c^2*d^2 - b*d^4)*x + (I*b^
3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + I*b^3*c^3*d - 3*I*b*c*d^3 + 3*I*(b^3*c^2*d^
2 - b*d^4)*x)*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)
) + 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^2*d^2 - 4*(a^3 - 3*a)*b*
c*d^3 + (a^4 - 6*a^2)*d^4 - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^2*
d^2 - 4*(a^3 - 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4)*cos(2*b*x + 2*a))*log(-1/2
*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^4*c^4 - 4*a*b^3*c^
3*d + 6*(a^2 - 1)*b^2*c^2*d^2 - 4*(a^3 - 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4 -
 (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^2*d^2 - 4*(a^3 - 3*a)*b*c*d^3
 + (a^4 - 6*a^2)*d^4)*cos(2*b*x + 2*a))*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*s
in(2*b*x + 2*a) + 1/2) + 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d -
 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 3*a)*b*c*d^3 - (a^4 - 6*a^2)*d^4 + 6*(b^4*c^2
*d^2 - b^2*d^4)*x^2 + 4*(b^4*c^3*d - 3*b^2*c*d^3)*x - (b^4*d^4*x^4 + 4*b^4*
c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 3*a)*b*c*d^3 - (a^
4 - 6*a^2)*d^4 + 6*(b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(b^4*c^3*d - 3*b^2*c*d^3
)*x)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) + 2*
(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3
 - 3*a)*b*c*d^3 - (a^4 - 6*a^2)*d^4 + 6*(b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(b^
```

```
4*c^3*d - 3*b^2*c*d^3)*x - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d -
 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 3*a)*b*c*d^3 - (a^4 - 6*a^2)*d^4 + 6*(b^4*c^2
*d^2 - b^2*d^4)*x^2 + 4*(b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(2*b*x + 2*a))*log(
-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) + 3*(d^4*cos(2*b*x + 2*a) - d^4
)*polylog(5, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + 3*(d^4*cos(2*b*x + 2*
a) - d^4)*polylog(5, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) - 6*(-I*b*d^4*x
 - I*b*c*d^3 + (I*b*d^4*x + I*b*c*d^3)*cos(2*b*x + 2*a))*polylog(4, cos(2*b
*x + 2*a) + I*sin(2*b*x + 2*a)) - 6*(I*b*d^4*x + I*b*c*d^3 + (-I*b*d^4*x -
I*b*c*d^3)*cos(2*b*x + 2*a))*polylog(4, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*
a)) + 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - d^4 - (b^2*d^4*x^2 + 2
*b^2*c*d^3*x + b^2*c^2*d^2 - d^4)*cos(2*b*x + 2*a))*polylog(3, cos(2*b*x +
2*a) + I*sin(2*b*x + 2*a)) + 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 -
 d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - d^4)*cos(2*b*x + 2*a))*
polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 8*(b^3*d^4*x^3 + 3*b^3*
c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*sin(2*b*x + 2*a))/(b^5*cos(2*b*x +
 2*a) - b^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \cot^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cot(b*x+a)**3,x)
[Out] Integral((c + d*x)**4*cot(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^4*cot(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(a+b\,x)^3\,(c+d\,x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3*(c + d*x)^4,x)
[Out] int(cot(a + b*x)^3*(c + d*x)^4, x)
```

3.179 $\int (c+dx)^3 \cot^3(a+bx)\,dx$

Optimal. Leaf size=256

$$-\frac{3id(c+dx)^2}{2b^2}-\frac{(c+dx)^3}{2b}+\frac{i(c+dx)^4}{4d}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{3d^2(c+dx)\log\left(1\right.}{b^3}$$

```
[Out] -3/2*I*d*(d*x+c)^2/b^2-1/2*(d*x+c)^3/b+1/4*I*(d*x+c)^4/d-3/2*d*(d*x+c)^2*co
t(b*x+a)/b^2-1/2*(d*x+c)^3*cot(b*x+a)^2/b+3*d^2*(d*x+c)*ln(1-exp(2*I*(b*x+a
)))/b^3-(d*x+c)^3*ln(1-exp(2*I*(b*x+a)))/b-3/2*I*d^3*polylog(2,exp(2*I*(b*x
+a)))/b^4+3/2*I*d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)
*polylog(3,exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4
```

Rubi [A]

time = 0.24, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3801, 3798, 2221, 2317, 2438, 32, 2611, 6744, 2320, 6724}

$$-\frac{3id^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}-\frac{3id(c+dx)^2}{2b^2}-\frac{(c+dx)^3}{2b}+\frac{i(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]^3,x]
```

```
[Out] (((-3*I)/2)*d*(c + d*x)^2)/b^2 - (c + d*x)^3/(2*b) + ((I/4)*(c + d*x)^4)/d
- (3*d*(c + d*x)^2*Cot[a + b*x])/(2*b^2) - ((c + d*x)^3*Cot[a + b*x]^2)/(2*
b) + (3*d^2*(c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b^3 - ((c + d*x)^3*Log[
1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))
])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3
*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) - (((3*I)/4)*d^3*Po
lyLog[4, E^((2*I)*(a + b*x))])/b^4
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \cot^3(a+bx)\,dx = -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(3d)\int (c+dx)^2\cot^2(a+bx)\,dx}{2b} - \int (c+dx)^3\cot($$

$$= \frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx}}{1-}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} + \frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx}{}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} + \frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx}{}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} + \frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx}{}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} + \frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx}{}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} + \frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 788 vs. $2(256) = 512$.
time = 6.84, size = 788, normalized size = 3.08

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]^3,x]
```

```
[Out] -1/4*(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cot[a]) - ((c + d*x)^3*
Csc[a + b*x]^2)/(2*b) + (c*d^2*Csc[a]*(2*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)
*(-1 + E^((2*I)*a))*Log[1 - E^((2*I)*(a + b*x))]) + 6*b*(-1 + E^((2*I)*a))*
x*PolyLog[2, E^((2*I)*(a + b*x))] + (3*I)*(-1 + E^((2*I)*a))*PolyLog[3, E^(
(2*I)*(a + b*x))]))/(4*b^3*E^(I*a)) + (d^3*E^(I*a)*Csc[a]*(x^4 + (-1 + E^((
-2*I)*a))*x^4 + ((-1 + E^((2*I)*a))*(2*b^4*x^4 + (4*I)*b^3*x^3*Log[1 - E^((
```

```
2*I)*(a + b*x))] + 6*b^2*x^2*PolyLog[2, E^((2*I)*(a + b*x))] + (6*I)*b*x*Po
lyLog[3, E^((2*I)*(a + b*x))] - 3*PolyLog[4, E^((2*I)*(a + b*x))]))/(2*b^4*
E^((2*I)*a))))/4 - (c^3*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a
]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (3*c*d^2*Csc[a]*(-(b*x*Cos
[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin
[a]^2)) + (3*Csc[a]*Csc[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3
*x^2*Sin[b*x]))/(2*b^2) + (3*c^2*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*
x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*
x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[
b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((
2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec
[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*d^3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a
]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2
*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[
Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2,
E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^4*Sqrt
[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1202 vs. $2(221) = 442$.
time = 0.15, size = 1203, normalized size = 4.70

method	result	size
risch	Expression too large to display	1203

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)-2/b^4*d^3*a^3*ln(exp(I*(b*x+a)))-6/b^3*c
*d^2*polylog(3,exp(I*(b*x+a)))-6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))-6/b^3
*d^3*polylog(3,-exp(I*(b*x+a)))*x-6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-1/b
*c^3*ln(exp(I*(b*x+a))+1)+2/b*c^3*ln(exp(I*(b*x+a)))-1/b*c^3*ln(exp(I*(b*x+
a))-1)-I*c^3*x-1/4*I/d*c^4+3/2*I*d*c^2*x^2-6*I*d^3/b^3*a*x+I*d^2*c*x^3+3*d^
2/b^3*c*ln(exp(I*(b*x+a))-1)+3*d^2/b^3*c*ln(exp(I*(b*x+a))+1)-6*d^2/b^3*c*l
n(exp(I*(b*x+a)))+3*d^3/b^3*ln(1-exp(I*(b*x+a)))*x+3*d^3/b^4*ln(1-exp(I*(b*
x+a)))*a+3*d^3/b^3*ln(exp(I*(b*x+a))+1)*x-3*d^3/b^4*a*ln(exp(I*(b*x+a))-1)+
6*d^3/b^4*a*ln(exp(I*(b*x+a)))-3*I*d^3/b^2*x^2-3*I*d^3/b^4*a^2-3*I*d^3/b^4*
polylog(2,-exp(I*(b*x+a)))+3*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))+3*I/b^2*
a^2*c^2*d+1/4*I*d^3*x^4-1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3-1/b*d^3*ln(1-exp(I
*(b*x+a)))*x^3-1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3-6/b^2*c^2*d*a*ln(exp(I*(b
*x+a)))-3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)+3/2*I/b^4*d^3*a^4-6*I/b^4*d^3*
polylog(4,exp(I*(b*x+a)))+(2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I
*(b*x+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c
^2*d*x*exp(2*I*(b*x+a))-3*I*c^2*d*exp(2*I*(b*x+a))+3*I*d^3*x^2+2*b*c^3*exp(
2*I*(b*x+a))+6*I*c*d^2*x+3*I*c^2*d)/b^2/(exp(2*I*(b*x+a))-1)^2-3/b*c^2*d*ln
```

```
(exp(I*(b*x+a))+1)*x-3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x-3/b^2*c^2*d*ln(1-exp(
I*(b*x+a)))*a-3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-3/b*c*d^2*ln(1-exp(I*(b*x+
a)))*x^2+3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2+6/b^3*c*d^2*a^2*ln(exp(I*(b*x
+a)))+3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+6*I/b^2*polylog(2,exp(I*(b*x+a)))*
c*d^2*x-6*I/b^2*a^2*c*d^2*x+6*I/b*a*c^2*d*x+3*I/b^2*c^2*d*polylog(2,-exp(I*
(b*x+a)))+3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+6*I/b^2*c*d^2*polylog(
2,-exp(I*(b*x+a)))*x+3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2+2*I/b^3*d^3*
a^3*x-4*I/b^3*a^3*c*d^2-6*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4-3*I*d^3*poly
log(2,exp(I*(b*x+a)))/b^4
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3958 vs. $2(214) = 428$.
time = 1.38, size = 3958, normalized size = 15.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 3*a*c^2*d*(1/sin(b*x +
 a)^2 + log(sin(b*x + a)^2))/b + 3*a^2*c*d^2*(1/sin(b*x + a)^2 + log(sin(b*
x + a)^2))/b^2 - a^3*d^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b^3 - 2*(
(b*x + a)^4*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^3 + 4*(b*c*d^2 - a
*d^3)*(b*x + a)^3 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a)^2 - 4*(
(b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3
*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a) + ((b*x + a)^3*d^3 - 3
*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b
*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 -
3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*
b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^3*d^3
+ 3*I*b*c*d^2 - 3*I*a*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^
2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 + I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2
*(I*(b*x + a)^3*d^3 - 3*I*b*c*d^2 + 3*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*
x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 - I)*d^3)*(b*x + a))*sin
(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 12*(b*c*d^2 - a*d^
3 + (b*c*d^2 - a*d^3)*cos(4*b*x + 4*a) - 2*(b*c*d^2 - a*d^3)*cos(2*b*x + 2*
a) + (I*b*c*d^2 - I*a*d^3)*sin(4*b*x + 4*a) + 2*(-I*b*c*d^2 + I*a*d^3)*sin(
2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 4*((b*x + a)^3*d^3
+ 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*
d^3)*(b*x + a) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^
2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 2*((b*
x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2
 + (a^2 - 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (I*(b*x + a)^3*d^3 + 3*(I*b
*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 - I
)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*(-I*(b*x + a)^3*d^3 + 3*(-I*b*c*d^2
```

```
+ I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 + I)*d^3
)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (
(b*x + a)^4*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a)^3 + 6*(b^2*c^2*d - 2*a*b*c*
d^2 + (a^2 - 2)*d^3)*(b*x + a)^2 - 24*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*
x + 4*a) - 2*((b*x + a)^4*d^3 + 6*b^2*c^2*d - 12*a*b*c*d^2 + 6*a^2*d^3 + 4*
(b*c*d^2 - (a - I)*d^3)*(b*x + a)^3 + 6*(b^2*c^2*d - 2*(a - I)*b*c*d^2 + (a
^2 - 2*I*a - 1)*d^3)*(b*x + a)^2 - 12*(-I*b^2*c^2*d + (2*I*a + 1)*b*c*d^2 +
 (-I*a^2 - a)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 12*(b^2*c^2*d - 2*a*b*c*d^
2 + (b*x + a)^2*d^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) + (b^2*
c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a*d^3)
*(b*x + a))*cos(4*b*x + 4*a) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3
 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (I*b^2
*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + (I*a^2 - I)*d^3 + 2*(I*b*c*d^2
 - I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 -
 I*(b*x + a)^2*d^3 + (-I*a^2 + I)*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))
*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + 12*(b^2*c^2*d - 2*a*b*c*d^2 +
(b*x + a)^2*d^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) + (b^2*c^2*
d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*
x + a))*cos(4*b*x + 4*a) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (
a^2 - 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (I*b^2*c^2
*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + (I*a^2 - I)*d^3 + 2*(I*b*c*d^2 - I
*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(
b*x + a)^2*d^3 + (-I*a^2 + I)*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin
(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + 2*(I*(b*x + a)^3*d^3 - 3*I*b*c*d^2
+ 3*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*
b*c*d^2 + (I*a^2 - I)*d^3)*(b*x + a) + (I*(b*x + a)^3*d^3 - 3*I*b*c*d^2 + 3
*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c
*d^2 + (I*a^2 - I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 2*(-I*(b*x + a)^3*d^3
 + 3*I*b*c*d^2 - 3*I*a*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b
^2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 + I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) -
((b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 +
3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2
*((b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 +
 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*l
og(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 2*(I*(b*x + a)^3
*d^3 - 3*I*b*c*d^2 + 3*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I
*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 - I)*d^3)*(b*x + a) + (I*(b*x + a)^3*d^
3 - 3*I*b*c*d^2 + 3*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^
2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 - I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 2*
(-I*(b*x + a)^3*d^3 + 3*I*b*c*d^2 - 3*I*a*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b
*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1139 vs. $2(214) = 428$.

time = 2.84, size = 1139, normalized size = 4.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 6*(I*b
^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - I*d^3 + (-I*b^2*d^3*x^2 - 2*I*
b^2*c*d^2*x - I*b^2*c^2*d + I*d^3)*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2*a)
 + I*sin(2*b*x + 2*a)) - 6*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d
+ I*d^3 + (I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - I*d^3)*cos(2*b*x
 + 2*a))*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 4*(b^3*c^3 - 3*a*b^
2*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3)*cos(2*b*x + 2*a))*log(-1/2*cos(2*b
*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*
(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 1
)*b*c*d^2 - (a^3 - 3*a)*d^3)*cos(2*b*x + 2*a))*log(-1/2*cos(2*b*x + 2*a) -
1/2*I*sin(2*b*x + 2*a) + 1/2) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2 + (a^3 - 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3)*x - (b^3*d^
3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 3*a)*d^3 +
 3*(b^3*c^2*d - b*d^3)*x)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) + I*sin(2
*b*x + 2*a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2 + (a^3 - 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3)*x - (b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 3*a)*d^3 + 3*(b^3*c^2*d
 - b*d^3)*x)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) +
 1) - 3*(I*d^3*cos(2*b*x + 2*a) - I*d^3)*polylog(4, cos(2*b*x + 2*a) + I*si
n(2*b*x + 2*a)) - 3*(-I*d^3*cos(2*b*x + 2*a) + I*d^3)*polylog(4, cos(2*b*x
+ 2*a) - I*sin(2*b*x + 2*a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*c
os(2*b*x + 2*a))*polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + 6*(b*d
^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*polylog(3, cos(2*b*x
 + 2*a) - I*sin(2*b*x + 2*a)) + 12*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d
)*sin(2*b*x + 2*a))/(b^4*cos(2*b*x + 2*a) - b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cot^3(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cot(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**3*cot(a + b*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^3*cot(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(a+b\,x)^3\,(c+d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3*(c + d*x)^3,x)
[Out] int(cot(a + b*x)^3*(c + d*x)^3, x)
```

3.180 $\int (c+dx)^2 \cot^3(a+bx)\,dx$

Optimal. Leaf size=168

$$-\frac{cdx}{b}-\frac{d^2x^2}{2b}+\frac{i(c+dx)^3}{3d}-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}-\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}+\frac{d^2\,\mathrm{lo}}{}$$

```
[Out] -c*d*x/b-1/2*d^2*x^2/b+1/3*I*(d*x+c)^3/d-d*(d*x+c)*cot(b*x+a)/b^2-1/2*(d*x+
c)^2*cot(b*x+a)^2/b-(d*x+c)^2*ln(1-exp(2*I*(b*x+a)))/b+d^2*ln(sin(b*x+a))/b
^3+I*d*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,exp(2*I*(b
*x+a)))/b^3
```

Rubi [A]
time = 0.15, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3801, 3556, 3798, 2221, 2611, 2320, 6724}

$$-\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\log(\sin(a+bx))}{b^3}+\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}-\frac{cdx}{b}-\frac{d^2x^2}{2b}+\frac{i(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]^3,x]
```

```
[Out] -((c*d*x)/b) - (d^2*x^2)/(2*b) + ((I/3)*(c + d*x)^3)/d - (d*(c + d*x)*Cot[a
 + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]^2)/(2*b) - ((c + d*x)^2*Log[1 - E^
((2*I)*(a + b*x))])/b + (d^2*Log[Sin[a + b*x]])/b^3 + (I*d*(c + d*x)*PolyLo
g[2, E^((2*I)*(a + b*x))])/b^2 - (d^2*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b
^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2 \cot^3(a+bx)\,dx &= -\frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{d\int (c+dx)\cot^2(a+bx)\,dx}{b} - \int (c+dx)^2\cot(a+ \\ &= \frac{i(c+dx)^3}{3d} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}(}{1-e^{2} \\ &= -\frac{cdx}{b} - \frac{d^2x^2}{2b} + \frac{i(c+dx)^3}{3d} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} \\ &= -\frac{cdx}{b} - \frac{d^2x^2}{2b} + \frac{i(c+dx)^3}{3d} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} \\ &= -\frac{cdx}{b} - \frac{d^2x^2}{2b} + \frac{i(c+dx)^3}{3d} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} \\ &= -\frac{cdx}{b} - \frac{d^2x^2}{2b} + \frac{i(c+dx)^3}{3d} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 446 vs. $2(168) = 336$.
time $= 6.67$, size $= 446$, normalized size $= 2.65$

$$-\frac{1}{3}x(3c^2+3cdx+d^2x^2)\cot(a) - \frac{(c+dx)^2\csc^2(a+bx)}{2b} + \frac{d^2e^{-ia}\csc(a)\left(2b^2x^2\left(2be^{2ia}x+3i(-1+e^{2ia})\log\left(1-e^{2i(a+bx)}\right)\right)+6b(-1+e^{2ia})x\,\text{PolyLog}\left(2,e^{2i(a+bx)}\right)+3i(-1+e^{2ia})\,\text{PolyLog}\left(3,e^{2i(a+bx)}\right)\right)}{12b^3} - \frac{c^2\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b(\cos^2(a)+\sin^2(a))} + \frac{d^2\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b^3(\cos^2(a)+\sin^2(a))} + \frac{\csc(a)\csc(a+bx)(cd\sin(bx)+d^2x\sin(bx))}{b^2} + \frac{cd\csc(a)\sec(a)\left(b^2e^{i\text{ArcTan}(\tan(a))}x^2+\frac{\left(ibx(-\pi+2\text{ArcTan}(\tan(a)))-\pi\log(1+e^{-2ibx})-2(bx+\text{ArcTan}(\tan(a)))\log\left(1-e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)+\pi\log(\cos(bx))+2\text{ArcTan}(\tan(a))\log(\sin(bx+\text{ArcTan}(\tan(a))))+i\,\text{PolyLog}\left(2,e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)\right)\tan(a)}{\sqrt{1+\tan^2(a)}}\right)}{b^2\sqrt{\sec^2(a)(\cos^2(a)+\sin^2(a))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]^3,x]
```

```
[Out] -1/3*(x*(3*c^2 + 3*c*d*x + d^2*x^2)*Cot[a]) - ((c + d*x)^2*Csc[a + b*x]^2)/
(2*b) + (d^2*Csc[a]*(2*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(-1 + E^((2*I)*a)
)*Log[1 - E^((2*I)*(a + b*x))]) + 6*b*(-1 + E^((2*I)*a))*x*PolyLog[2, E^((2
*I)*(a + b*x))] + (3*I)*(-1 + E^((2*I)*a))*PolyLog[3, E^((2*I)*(a + b*x))])
)/(12*b^3*E^(I*a)) - (c^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos
[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (d^2*Csc[a]*(-(b*x*Cos[a
]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a
]^2)) + (Csc[a]*Csc[a + b*x]*(c*d*Sin[b*x] + d^2*x*Sin[b*x]))/b^2 + (c*d*Cs
c[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]
) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*
(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x
+ ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a]
)/Sqrt[1 + Tan[a]^2]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(152) = 304$.
time $= 0.12$, size $= 644$, normalized size $= 3.83$

method	result
risch	$\frac{4icdax}{b} - \frac{4cda\ln(e^{i(bx+a)})}{b^2} + \frac{2cda\ln(e^{i(bx+a)}-1)}{b^2} - \frac{2id^2a^2x}{b^2} + \frac{2icd\,a^2}{b^2} + \frac{id^2x^3}{3} + \frac{2id^2\operatorname{polylog}(2,e^{i(bx+a)})x}{b^2} - \frac{2cd\ln(1$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4*I/b*a*c*d*x+2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))-1/b^3*d^2*a^2*ln(exp(I*(b*x+
a))-1)-1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2+1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2-
1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2-2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a-2/b*c*d*l
n(exp(I*(b*x+a))+1)*x-2/b*c*d*ln(1-exp(I*(b*x+a)))*x-4/b^2*c*d*a*ln(exp(I*(
b*x+a)))+2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)-1/b*c^2*ln(exp(I*(b*x+a))-1)-1/b*
c^2*ln(exp(I*(b*x+a))+1)+2/b*c^2*ln(exp(I*(b*x+a)))-2*d^2*polylog(3,-exp(I*
(b*x+a)))/b^3-2*d^2*polylog(3,exp(I*(b*x+a)))/b^3+2*I/b^2*c*d*polylog(2,exp
(I*(b*x+a)))-2*I/b^2*a^2*d^2*x+2*I/b^2*a^2*c*d+2*I/b^2*polylog(2,exp(I*(b*x
+a)))*d^2*x-I*c^2*x-1/3*I/d*c^3-2/b^3*d^2*ln(exp(I*(b*x+a)))+1/b^3*d^2*ln(e
xp(I*(b*x+a))-1)+1/b^3*d^2*ln(exp(I*(b*x+a))+1)+1/3*I*d^2*x^3+2*(b*d^2*x^2*
exp(2*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x*
exp(2*I*(b*x+a))-I*c*d*exp(2*I*(b*x+a))+I*d^2*x+I*d*c)/b^2/(exp(2*I*(b*x+a)
)-1)^2+I*d*c*x^2-4/3*I/b^3*a^3*d^2+2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x
+2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1966 vs. $2(149) = 298$.
time = 0.54, size = 1966, normalized size = 11.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 2*a*c*d*(1/sin(b*x + a
)^2 + log(sin(b*x + a)^2))/b + a^2*d^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)
^2))/b^2 - 2*(2*(b*x + a)^3*d^2 + 6*(b*c*d - a*d^2)*(b*x + a)^2 + 12*b*c*d
- 12*a*d^2 - 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2 + ((b*x
 + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(4*b*x + 4*a) - 2*((b*x
 + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(2*b*x + 2*a) - (-I*(b*
x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a) + I*d^2)*sin(4*b*x + 4*a) -
 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - I*d^2)*sin(2*b*x
+ 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 6*(d^2*cos(4*b*x + 4*a) -
 2*d^2*cos(2*b*x + 2*a) + I*d^2*sin(4*b*x + 4*a) - 2*I*d^2*sin(2*b*x + 2*a)
 + d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 6*((b*x + a)^2*d^2 + 2*(b
*c*d - a*d^2)*(b*x + a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*c
os(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b
```

```
*x + 2*a) + (I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(4*b*x
 + 4*a) + 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*sin(2*b
*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 2*((b*x + a)^3*d^2 +
3*(b*c*d - a*d^2)*(b*x + a)^2 - 6*(b*x + a)*d^2)*cos(4*b*x + 4*a) - 4*((b*x
 + a)^3*d^2 + 3*(b*c*d - (a - I)*d^2)*(b*x + a)^2 + 3*b*c*d - 3*a*d^2 - 3*(
-2*I*b*c*d + (2*I*a + 1)*d^2)*(b*x + a))*cos(2*b*x + 2*a) + 12*(b*c*d + (b*
x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 2*(
b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (I*b*c*d + I*(b*x + a)*d^
2 - I*a*d^2)*sin(4*b*x + 4*a) + 2*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*si
n(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + 12*(b*c*d + (b*x + a)*d^2 - a*d^2
 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 2*(b*c*d + (b*x + a)*
d^2 - a*d^2)*cos(2*b*x + 2*a) + (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(4
*b*x + 4*a) + 2*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(2*b*x + 2*a))*di
log(e^(I*b*x + I*a)) + 3*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x +
a) - I*d^2 + (I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - I*d^2)*
cos(4*b*x + 4*a) + 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a)
 + I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a)
 - d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a)
 - d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x +
 a) + 1) + 3*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - I*d^2 +
 (I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - I*d^2)*cos(4*b*x +
4*a) + 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a) + I*d^2)*co
s(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(
4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(
2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 1
2*(I*d^2*cos(4*b*x + 4*a) - 2*I*d^2*cos(2*b*x + 2*a) - d^2*sin(4*b*x + 4*a)
 + 2*d^2*sin(2*b*x + 2*a) + I*d^2)*polylog(3, -e^(I*b*x + I*a)) + 12*(I*d^2
*cos(4*b*x + 4*a) - 2*I*d^2*cos(2*b*x + 2*a) - d^2*sin(4*b*x + 4*a) + 2*d^2
*sin(2*b*x + 2*a) + I*d^2)*polylog(3, e^(I*b*x + I*a)) + 2*(I*(b*x + a)^3*d
^2 + 3*(I*b*c*d - I*a*d^2)*(b*x + a)^2 - 6*I*(b*x + a)*d^2)*sin(4*b*x + 4*a
) + 4*(-I*(b*x + a)^3*d^2 + 3*(-I*b*c*d + (I*a + 1)*d^2)*(b*x + a)^2 - 3*I*
b*c*d + 3*I*a*d^2 + 3*(2*b*c*d - (2*a - I)*d^2)*(b*x + a))*sin(2*b*x + 2*a)
)/(-6*I*b^2*cos(4*b*x + 4*a) + 12*I*b^2*cos(2*b*x + 2*a) + 6*b^2*sin(4*b*x
+ 4*a) - 12*b^2*sin(2*b*x + 2*a) - 6*I*b^2))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 657 vs. $2(149) = 298$.
time = 3.43, size = 657, normalized size = 3.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*b^2*c^2 - 2*(I*b*d^2*x + I*b*c*d + (-I
*b*d^2*x - I*b*c*d)*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x
```

```
+ 2*a)) - 2*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(2*b*x + 2*a))
*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 2*(b^2*c^2 - 2*a*b*c*d + (a
^2 - 1)*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 1)*d^2)*cos(2*b*x + 2*a))*log(-
1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^2*c^2 - 2*a*b*c
*d + (a^2 - 1)*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 1)*d^2)*cos(2*b*x + 2*a)
)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*
c*d - a^2*d^2)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)
 + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 +
 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a)
 - I*sin(2*b*x + 2*a) + 1) - (d^2*cos(2*b*x + 2*a) - d^2)*polylog(3, cos(2*
b*x + 2*a) + I*sin(2*b*x + 2*a)) - (d^2*cos(2*b*x + 2*a) - d^2)*polylog(3,
cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 4*(b*d^2*x + b*c*d)*sin(2*b*x + 2*
a))/(b^3*cos(2*b*x + 2*a) - b^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \cot^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cot(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*cot(a + b*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cot(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(a+b\,x)^3\,(c+d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] int(cot(a + b*x)^3*(c + d*x)^2, x)
```

3.181 $\int (c+dx)\cot^3(a+bx)\,dx$

Optimal. Leaf size=109

$$-\frac{dx}{2b}+\frac{i(c+dx)^2}{2d}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}-\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}+\frac{idPolyLog\left(2,e^{2i(a+bx)}\right)}{2b^2}$$

```
[Out] -1/2*d*x/b+1/2*I*(d*x+c)^2/d-1/2*d*cot(b*x+a)/b^2-1/2*(d*x+c)*cot(b*x+a)^2/
b-(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b+1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^2
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3801, 3554, 8, 3798, 2221, 2317, 2438}

$$\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\cot^2(a+bx)}{2b}-\frac{dx}{2b}+\frac{i(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]^3,x]
```

```
[Out] -1/2*(d*x)/b + ((I/2)*(c + d*x)^2)/d - (d*Cot[a + b*x])/(2*b^2) - ((c + d*x
)*Cot[a + b*x]^2)/(2*b) - ((c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b + ((I/
2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)\cot^3(a+bx)\,dx &= -\frac{(c+dx)\cot^2(a+bx)}{2b} + \frac{d\int \cot^2(a+bx)\,dx}{2b} - \int (c+dx)\cot(a+bx)\,dx \\
&= \frac{i(c+dx)^2}{2d} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1-e^{2i(a+bx)}}\,d \\
&= -\frac{dx}{2b} + \frac{i(c+dx)^2}{2d} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} - \frac{(c+dx)\log\left(1\right.}{b} \\
&= -\frac{dx}{2b} + \frac{i(c+dx)^2}{2d} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} - \frac{(c+dx)\log\left(1\right.}{b} \\
&= -\frac{dx}{2b} + \frac{i(c+dx)^2}{2d} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} - \frac{(c+dx)\log\left(1\right.}{b}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 240 vs. $2(109) = 218$.
time = 6.19, size = 240, normalized size = 2.20

$$-\frac{1}{2}dx^2\cot(a) - \frac{dx\csc^2(a+bx)}{2b} - \frac{c(\cot^2(a+bx) + 2\log(\cos(a+bx)) + 2\log(\tan(a+bx)))}{2b} + \frac{d\csc(a)\csc(a+bx)\sin(bx)}{2b^2} + \frac{d\csc(a)\sec(a)\left(b^2e^{i\text{ArcTan}(\tan(a))}x^2 + \frac{\left(ibx(-\pi+2\text{ArcTan}(\tan(a))) - \pi\log\left(1+e^{-2ibx}\right) - 2(bx+\text{ArcTan}(\tan(a)))\log\left(1-e^{2i(bx+\text{ArcTan}(\tan(a)))}\right) + \pi\log(\cos(bx)) + 2\text{ArcTan}(\tan(a))\log(\sin(bx+\text{ArcTan}(\tan(a)))) + i\text{PolyLog}\left(2,e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)\right)\tan(a)}{\sqrt{1+\tan^2(a)}}\right)}{2b^2\sqrt{\sec^2(a)\left(\cos^2(a)+\sin^2(a)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]^3,x]
```

```
[Out] -1/2*(d*x^2*Cot[a]) - (d*x*Csc[a + b*x]^2)/(2*b) - (c*(Cot[a + b*x]^2 + 2*L
og[Cos[a + b*x]] + 2*Log[Tan[a + b*x]]))/(2*b) + (d*Csc[a]*Csc[a + b*x]*Sin
[b*x])/(2*b^2) + (d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(
-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[
a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTa
n[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + Arc
Tan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2
 + Sin[a]^2)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(93) = 186$.
time = 0.09, size = 281, normalized size = 2.58

method	result
risch	$\frac{id\,a^2}{b^2}+\frac{id\,x^2}{2}+\frac{2bdx\,\mathrm{e}^{2i(bx+a)}-id\,\mathrm{e}^{2i(bx+a)}+2bc\,\mathrm{e}^{2i(bx+a)}+id}{b^2\left(\mathrm{e}^{2i(bx+a)}-1\right)^2}-\frac{c\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b}-\frac{c\ln\left(\mathrm{e}^{i(bx+a)}+1\right)}{b}+\frac{2c\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b}-i$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] I/b^2*d*a^2+1/2*I*d*x^2+(2*b*d*x*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))+2*b*
c*exp(2*I*(b*x+a))+I*d)/b^2/(exp(2*I*(b*x+a))-1)^2-1/b*c*ln(exp(I*(b*x+a))-
1)-1/b*c*ln(exp(I*(b*x+a))+1)+2/b*c*ln(exp(I*(b*x+a)))-I*c*x+2*I/b*d*a*x+I*
d*polylog(2,-exp(I*(b*x+a)))/b^2-1/b*d*ln(1-exp(I*(b*x+a)))*x-1/b^2*d*ln(1-
exp(I*(b*x+a)))*a+I/b^2*d*polylog(2,exp(I*(b*x+a)))-1/b*d*ln(exp(I*(b*x+a))
+1)*x+1/b^2*d*a*ln(exp(I*(b*x+a))-1)-2/b^2*d*a*ln(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(90) = 180$.
time = 0.42, size = 830, normalized size = 7.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] (b^2*d*x^2 + 2*b^2*c*x - 2*(b*d*x + b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) -
2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (-I*b*d*x - I*b*c)*sin(4*b*x + 4*a) - 2*
(I*b*d*x + I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1)
 - 2*(b*c*cos(4*b*x + 4*a) - 2*b*c*cos(2*b*x + 2*a) + I*b*c*sin(4*b*x + 4*a
) - 2*I*b*c*sin(2*b*x + 2*a) + b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1)
 + 2*(b*d*x*cos(4*b*x + 4*a) - 2*b*d*x*cos(2*b*x + 2*a) + I*b*d*x*sin(4*b*x
 + 4*a) - 2*I*b*d*x*sin(2*b*x + 2*a) + b*d*x)*arctan2(sin(b*x + a), -cos(b*
x + a) + 1) + (b^2*d*x^2 + 2*b^2*c*x)*cos(4*b*x + 4*a) - 2*(b^2*d*x^2 + 2*I
```

```
*b*c + 2*(b^2*c + I*b*d)*x + d)*cos(2*b*x + 2*a) + 2*(d*cos(4*b*x + 4*a) -
2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x + 2*a) + d)*d
ilog(-e^(I*b*x + I*a)) + 2*(d*cos(4*b*x + 4*a) - 2*d*cos(2*b*x + 2*a) + I*d
*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x + 2*a) + d)*dilog(e^(I*b*x + I*a)) - (-
I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(4*b*x + 4*a) - 2*(-I*b*d*x - I*b*c
)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-
I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(4*b*x + 4*a) - 2*(-I*b*d*x - I*b*c
)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-
I*b^2*d*x^2 - 2*I*b^2*c*x)*sin(4*b*x + 4*a) + 2*(-I*b^2*d*x^2 + 2*b*c + 2*(
-I*b^2*c + b*d)*x - I*d)*sin(2*b*x + 2*a) + 2*d)/(-2*I*b^2*cos(4*b*x + 4*a)
 + 4*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b*x + 2*
a) - 2*I*b^2)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(90) = 180$.
time = 3.12, size = 339, normalized size = 3.11

$$\frac{4\,bdx + 4\,bc + (i\,d\cos\left(2\,bx + 2\,a\right) - i\,d){\rm Li}_2(\cos\left(2\,bx + 2\,a\right) + i\,\sin\left(2\,bx + 2\,a\right)) + (-i\,d\cos\left(2\,bx + 2\,a\right) + i\,d){\rm Li}_2(\cos\left(2\,bx + 2\,a\right) - i\,\sin\left(2\,bx + 2\,a\right)) + 2\,(bc - ad - (bc - ad)\cos\left(2\,bx + 2\,a\right))\log\left(-\frac{1}{2}\,\cos\left(2\,bx + 2\,a\right) + \frac{1}{2}i\,\sin\left(2\,bx + 2\,a\right) + \frac{1}{2}\right) + 2\,(bc - ad - (bc - ad)\cos\left(2\,bx + 2\,a\right))\log\left(-\frac{1}{2}\,\cos\left(2\,bx + 2\,a\right) - \frac{1}{2}i\,\sin\left(2\,bx + 2\,a\right) + \frac{1}{2}\right) + 2\,(bdx + ad - (bdx + ad)\cos\left(2\,bx + 2\,a\right))\log\left(-\cos\left(2\,bx + 2\,a\right) + i\,\sin\left(2\,bx + 2\,a\right) + 1\right) + 2\,(bdx + ad - (bdx + ad)\cos\left(2\,bx + 2\,a\right))\log\left(-\cos\left(2\,bx + 2\,a\right) - i\,\sin\left(2\,bx + 2\,a\right) + 1\right) + 2\,d\sin\left(2\,bx + 2\,a\right)}{4\,(b^2\cos\left(2\,bx + 2\,a\right) - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*d*x + 4*b*c + (I*d*cos(2*b*x + 2*a) - I*d)*dilog(cos(2*b*x + 2*a)
+ I*sin(2*b*x + 2*a)) + (-I*d*cos(2*b*x + 2*a) + I*d)*dilog(cos(2*b*x + 2*a
) - I*sin(2*b*x + 2*a)) + 2*(b*c - a*d - (b*c - a*d)*cos(2*b*x + 2*a))*log(
-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b*c - a*d - (b*c
 - a*d)*cos(2*b*x + 2*a))*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a
) + 1/2) + 2*(b*d*x + a*d - (b*d*x + a*d)*cos(2*b*x + 2*a))*log(-cos(2*b*x
+ 2*a) + I*sin(2*b*x + 2*a) + 1) + 2*(b*d*x + a*d - (b*d*x + a*d)*cos(2*b*x
 + 2*a))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) + 2*d*sin(2*b*x +
2*a))/(b^2*cos(2*b*x + 2*a) - b^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)\cot^3{(a + bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*cot(a + b*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*cot(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(a+b\,x)^3\,(c+d\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3*(c + d*x),x)
```

```
[Out] int(cot(a + b*x)^3*(c + d*x), x)
```

3.182 $\int \frac{\cot^3(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\cot^3(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^3/(d*x+c),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^3(a+bx)}{c+dx} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[a + b*x]^3/(c + d*x),x]
```

```
[Out] Defer[Int][Cot[a + b*x]^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{\cot^3(a+bx)}{c+dx} \, dx = \int \frac{\cot^3(a+bx)}{c+dx} \, dx$$

Mathematica [A]
time = 6.57, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(a+bx)}{c+dx} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^3/(c + d*x),x]
```

```
[Out] Integrate[Cot[a + b*x]^3/(c + d*x), x]
```

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(bx+a)}{dx+c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^3/(d*x+c),x)
[Out] int(cot(b*x+a)^3/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="maxima")
[Out] -(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 -
 (2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) -
 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 +
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 +
 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*
b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2
*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 +
 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate((b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2 - d^2)*sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^
2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*cos(b*x + a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*sin(b*x + a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^
3)*cos(b*x + a)), x) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4
*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*si
n(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2
 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*cos(2*b*x + 2*a))*integrate((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
 - d^2)*sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c
^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)
^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x + a)
^2 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x +
a)), x) - (d*cos(2*b*x + 2*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a) - d)*sin(4
*b*x + 4*a) - d*sin(2*b*x + 2*a))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*
x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b
*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="fricas")
[Out] integral(cot(b*x + a)^3/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{3}{\left(a + bx \right)}}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**3/(d*x+c),x)
[Out] Integral(cot(a + b*x)**3/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="giac")
[Out] integrate(cot(b*x + a)^3/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{{\mathrm{cot}\left(a+b\,x\right)}^3}{c+d\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3/(c + d*x),x)
[Out] int(cot(a + b*x)^3/(c + d*x), x)
```

3.183 $\int \frac{\cot^3(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\cot^3(a+bx)}{(c+dx)^2}, x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^3/(d*x+c)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Cot[a + b*x]^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2} dx = \int \frac{\cot^3(a+bx)}{(c+dx)^2} dx$$

Mathematica [A]
time = 9.98, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] Integrate[Cot[a + b*x]^3/(c + d*x)^2, x]
```

Maple [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(cot(b*x+a)^3/(d*x+c)^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 -
 2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) -
 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*
c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^
3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*
x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*
c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b
^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2
*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2
*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*int
egrate((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 3*d^2)*sin(b*x + a)/(b^2*d^4*
x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*
d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*co
s(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c
^3*d*x + b^2*c^4)*sin(b*x + a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2
*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) + (b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
+ 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^
2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*
c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a
) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x +
 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b
^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*c
os(4*b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^
3)*cos(2*b*x + 2*a))*integrate((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 3*d^2
)*sin(b*x + a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c
^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b
^2*c^3*d*x + b^2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b
^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2 - 2*(b^2*d^4*x^4 +
 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a
)), x) - 2*(d*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(2*b*x + 2*a) - d)*sin(4*
```

```
b*x + 4*a) - 2*d*sin(2*b*x + 2*a))/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
 + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^
2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(cot(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**3/(d*x+c)**2,x)
[Out] Integral(cot(a + b*x)**3/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(cot(b*x + a)^3/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cot(a+b\,x)^3}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3/(c + d*x)^2,x)
[Out] int(cot(a + b*x)^3/(c + d*x)^2, x)
```

3.184 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=407

$$\frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c+dx}\,\cos(4a+4bx)}{2048b^3}-\frac{(c+dx)^{5/2}\cos(4a+$$

```
[Out] -1/8*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(5/2)*cos(4*b*x+4*a)/b+5/3
2*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2+5/256*d*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b
^2-15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*
x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/8192*d^(5/2)*FresnelS(2*b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1
/2)/b^(7/2)-15/256*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+15/256*d^(5/2)*FresnelS(2*b^(1/2)*(d*x
+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/128*d^2*co
s(2*b*x+2*a)*(d*x+c)^(1/2)/b^3+15/2048*d^2*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A]

time = 0.62, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15\sqrt{\pi}\,d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}+\frac{15d^2\sqrt{c+dx}\,\cos(4a+4bx)}{2048b^3}+\frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2}+\frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*
a + 2*b*x])/(8*b) + (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) - ((
c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a -
(4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^
(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4
096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*
Sin[2*a + 2*b*x])/(32*b^2) + (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^
2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right)\,dx$$

$$= \frac{1}{8}\int (c+dx)^{5/2}\sin(4a+4bx)\,dx+\frac{1}{4}\int (c+dx)^{5/2}\sin(2a+2bx)$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}+\frac{(5d)\int}{}$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}+\frac{5d(c}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2}{}$$

Mathematica [A]
time = 14.61, size = 550, normalized size = 1.35

$$\frac{-1024b^3c^2\sqrt{c+dx}\,\cos(2(a+bx))+960bd^2\sqrt{c+dx}\,\cos(2(a+bx))-2048b^3cdx\sqrt{c+dx}\,\cos(2(a+bx))-1024b^3d^2x^2\sqrt{c+dx}\,\cos(2(a+bx))-256b^3c^2\sqrt{c+dx}\,\cos(4(a+bx))+60bd^2\sqrt{c+dx}\,\cos(4(a+bx))-512b^3cdx\sqrt{c+dx}\,\cos(4(a+bx))-256b^3d^2x^2\sqrt{c+dx}\,\cos(4(a+bx))-15\sqrt{\frac{b}{d}}d^3\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)-480\sqrt{\frac{b}{d}}d^3\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+15\sqrt{\frac{b}{d}}d^3\sqrt{2\pi}\,S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(4a-\frac{4bc}{d}\right)+480\sqrt{\frac{b}{d}}d^3\sqrt{\pi}\,S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)+1280b^2cd\sqrt{c+dx}\,\sin(2(a+bx))+1280b^2d^2x\sqrt{c+dx}\,\sin(2(a+bx))+160b^2cd\sqrt{c+dx}\,\sin(4(a+bx))+160b^2d^2x\sqrt{c+dx}\,\sin(4(a+bx))}{8192b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos
[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^
2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a +
 b*x)] + 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 512*b^3*c*d*x*Sqrt[c + d
*x]*Cos[4*(a + b*x)] - 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 15*
Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresnel
C[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Fresn
elS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b
/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (
2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sq
```

rt[c + d*x]*Sin[2*(a + b*x)] + 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] +
 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(8192*b^4)

Maple [A]
time = 0.06, size = 470, normalized size = 1.15

method	result
derivativedivides	$\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2}{d}\right.}{4b}\right.}{}\right.}{}}{}$
default	$\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2}{d}\right.}{4b}\right.}{}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+5/16/b*d*(1/4
/b*d*(d*x+c)^(3/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos
(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*(a
*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/64/b*d*(
d*x+c)^(5/2)*cos(4/d*b*(d*x+c)+4*(a*d-b*c)/d)+5/64/b*d*(1/8/b*d*(d*x+c)^(3/
2)*sin(4/d*b*(d*x+c)+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d
```

```
*b*(d*x+c)+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d
-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(
a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 551, normalized size = 1.35

$$\frac{\left(640\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)+5120\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-16\left(\frac{64\,(dx+c)^{\frac{5}{2}}b^4}{d}-15\sqrt{dx+c}\,b^2d\right)\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)-256\left(\frac{16\,(dx+c)^{\frac{5}{2}}b^4}{d}-15\sqrt{dx+c}\,b^2d\right)\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-240\left(-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)-(i+1)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2\,(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{2i\,b}{d}}\right)-15\left(-(i-1)\sqrt{2}\sqrt{\pi}bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)-(i+1)\sqrt{2}\sqrt{\pi}bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4\,(bc-ad)}{d}\right)\right)\mathrm{erf}\left(2\sqrt{dx+c}\sqrt{\frac{i\,b}{d}}\right)-15\left((i+1)\sqrt{2}\sqrt{\pi}bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)+(i-1)\sqrt{2}\sqrt{\pi}bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4\,(bc-ad)}{d}\right)\right)\mathrm{erf}\left(2\sqrt{dx+c}\sqrt{-\frac{i\,b}{d}}\right)-240\left((i+1)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)+(i-1)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2\,(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{2i\,b}{d}}\right)\right)d}{32768\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/32768*(640*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) + 5120*
(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 16*(64*(d*x + c)^(
5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d) - 2
56*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)*b -
 b*c + a*d)/d) - 240*(-(I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/
4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 15*(-(I -
1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I + 1)*s
qrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x
 + c)*sqrt(I*b/d)) - 15*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos
(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*
(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 240*((I + 1)*4^(1/4)*sq
rt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*4^(1/4
)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*
x + c)*sqrt(-2*I*b/d)))*d/b^5
```

Fricas [A]
time = 3.33, size = 376, normalized size = 0.92

$$-\frac{15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)+480\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-480\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\left(192\,b^3d^2x^2+384\,b^3cdx+192\,b^3c^2+360\,bd^2\cos\left(bx+a\right)^2-8\left(64\,b^3d^2x^2+128\,b^3cdx+64\,b^3c^2-15\,bd^2\right)\cos\left(bx+a\right)^4-225\,bd^2+160\left(2\left(b^2d^2x+b^2cd\right)\cos\left(bx+a\right)^3+3\left(b^2d^2x+b^2cd\right)\cos\left(bx+a\right)\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{8192\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))
*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d)
+ 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x +
c)*sqrt(b/(pi*d))) - 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(192*b^3*d^2*x^2 + 384*b^3*c*d*x
+ 192*b^3*c^2 + 360*b*d^2*cos(b*x + a)^2 - 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*
x + 64*b^3*c^2 - 15*b*d^2)*cos(b*x + a)^4 - 225*b*d^2 + 160*(2*(b^2*d^2*x +
 b^2*c*d)*cos(b*x + a)^3 + 3*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x +
a))*sqrt(d*x + c))/b^4
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [C] Result contains complex when optimal does not.
time = 1.25, size = 2446, normalized size = 6.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16384*(512*(-I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)) + I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*
d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))
*c^3 + 24*c*d^2*((-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*e
rf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*
b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(8*I*(d*x
+ c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-4*(-I*
(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (I*sqrt(2)*sqrt(pi)*(64*b^2*c^2
+ 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
1)*b^2) - 4*I*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(
d*x + c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 16*(-I*sq
rt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d
 - 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2
+ 16*(I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x +
 c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b
^2)/d^2) + d^3*((I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*I*b^2*c^2*d - 72*b*c
*d^2 + 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
```

```
2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b
^3) - 4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 192*I*(d*x + c)^(3/2)*b^2*c*d + 192
*I*sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(3/2)*b*d^2 + 72*sqrt(d*x + c)*b*
c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b
^3)/d^3 + (-I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 + 192*I*b^2*c^2*d - 72*b*c*d^2
- 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) -
4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 192*I*(d*x + c)^(3/2)*b^2*c*d + 192*I*sqr
t(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)*b*c*d^2
- 15*I*sqrt(d*x + c)*d^3)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3
 + 32*(I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d^3)*d*e
rf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*
a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5
/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d - 2
0*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3
)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 32*(-I*sqrt(pi)*(64*
b^3*c^3 + 48*I*b^2*c^2*d - 36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x +
 c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2
 - 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-2*(I*(d*x + c)*b
- I*b*c + I*a*d)/d)/b^3)/d^3) + 192*(I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf
(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*
c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(2)*sqrt(pi)
*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) +
 8*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)*b) - 8*I*sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b) + 16*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b +
 4*sqrt(d*x + c)*d*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 16*sqrt(d*x
 + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(
-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c^2)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2),x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2), x)
```

3.185 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=351

$$-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}$$

```
[Out] -1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b-3/1
024*d^(3/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-3/1024*d^(3/2)*FresnelC(2*b^(1/2)*2^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5
/2)-3/64*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/
Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/64*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^
(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+3/32*d*sin(2*b*x+2*a)*(d*
x+c)^(1/2)/b^2+3/256*d*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.41, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2}+\frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -1/8*((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/b - ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(64*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) + (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right)$$

$$= \frac{1}{8}\int (c+dx)^{3/2}\sin(4a+4bx)\,dx+\frac{1}{4}\int (c+dx)^{3/2}\sin(2a+2bx$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{(3d)}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d\sqrt{}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d\sqrt{}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d\sqrt{}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}-\frac{3d^{3/}}{}$$

Mathematica [A]
time = 1.42, size = 393, normalized size = 1.12

$$\frac{-128bc\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(2(a+bx))-128b\sqrt{\frac{b}{d}}dx\sqrt{c+dx}\cos(2(a+bx))-32bc\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(4(a+bx))-32b\sqrt{\frac{b}{d}}dx\sqrt{c+dx}\cos(4(a+bx))-3d\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)-48d\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-3d\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(4a-\frac{4bc}{d}\right)-48d\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)+96\sqrt{\frac{b}{d}}d\sqrt{c+dx}\sin(2(a+bx))+12\sqrt{\frac{b}{d}}d\sqrt{c+dx}\sin(4(a+bx))}{1024b^2\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] - 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*
x)] - 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 3*d*Sqrt[2*Pi]*Co
s[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sq
rt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
- 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (
4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Si
n[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 12*Sqr
t[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(1024*b^2*Sqrt[b/d])
```

Maple [A]
time = 0.06, size = 376, normalized size = 1.07

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+}}{\sqrt{\pi}\sqrt{\frac{b}{d}}}\right.\right.}{}\right.}{8b}$
default	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+}}{\sqrt{\pi}\sqrt{\frac{b}{d}}}\right.\right.}{}\right.}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+3/16/b*d*(1/4
/b*d*(d*x+c)^(1/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d
)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))-1
/64/b*d*(d*x+c)^(3/2)*cos(4/d*b*(d*x+c)+4*(a*d-b*c)/d)+3/64/b*d*(1/8/b*d*(d
*x+c)^(1/2)*sin(4/d*b*(d*x+c)+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+
c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 503, normalized size = 1.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4096*(128*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 512
*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 48*sqrt(d*x + c
)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 384*sqrt(d*x + c)*b^2*sin(2*((d*
x + c)*b - b*c + a*d)/d) + 24*((I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^
2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/
d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + 3*((I
```

```
+ 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I - 1)*s
qrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x +
 c)*sqrt(I*b/d)) + 3*(-(I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*
(b*c - a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c -
 a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + 24*(-(I - 1)*4^(1/4)*sqrt(2)*
sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(2
)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqr
t(-2*I*b/d)))*d/b^4
```

Fricas [A]
time = 2.91, size = 294, normalized size = 0.84

$$-\frac{3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+48\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+48\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+16\left(16\left(b^2dx+b^2c\right)\cos\left(bx+a\right)^4-6\,b^2dx-6\,b^2c-3\left(2\,bd\cos\left(bx+a\right)^3+3\,bd\cos\left(bx+a\right)\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{1024\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*f
resnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d))) + 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4
 - 6*b^2*d*x - 6*b^2*c - 3*(2*b*d*cos(b*x + a)^3 + 3*b*d*cos(b*x + a))*sin(
b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin\left(a+bx\right)\cos^3\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x)**3, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.94, size = 1523, normalized size = 4.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2048*(64*(-I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt
```

```
(b^2*d^2) + 1)) + I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^
2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
 + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c
^2 + d^2*((-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqr
t(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(8*I*(d*x + c)^(3
/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-4*(-I*(d*x +
c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*
b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2)
 - 4*I*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c
)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 16*(-I*sqrt(pi)*
(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d - 3*sq
rt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 16*(I
*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2
) + 16*(I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d
)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*d)*d
*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c -
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sqrt(pi)*(4*b*c +
I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*
b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 16*sqrt(d*x + c)*
d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(I*(
d*x + c)*b - I*b*c + I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)*b
 + I*b*c - I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b + I*b*c -
I*a*d)/d)/b)*c)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\,\sin(a+b\,x)\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2), x)
```

3.186 $\int \sqrt{c+dx} \cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=299

$$-\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}$$

```
[Out] 1/128*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/128*FresnelS(2*b^(1/2)*2^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(
3/2)+1/16*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2
))*d^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/P
i^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/8*cos(2*b*x+2*a)*(d*x+
c)^(1/2)/b-1/32*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.32, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -1/8*(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/b - (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\,\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\,\sin(2a+2bx)+\frac{1}{8}\sqrt{c+dx}\,\sin(4a+4bx)\right)\,dx$$

$$= \frac{1}{8}\int \sqrt{c+dx}\,\sin(4a+4bx)\,dx+\frac{1}{4}\int \sqrt{c+dx}\,\sin(2a+2bx)\,d\ldots$$

$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(4a+4bx)}{32b}+\frac{d\int\frac{\cos(4\ldots}{\sqrt{c\ldots}}}{6\ldots}$$

$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(4a+4bx)}{32b}+\frac{(d\cos(\ldots}{\ldots}$$

$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(4a+4bx)}{32b}+\frac{\cos(4a\ldots}{\ldots}$$

$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(4a+4bx)}{32b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\ldots}{\ldots}$$

Mathematica [A]
time = 0.60, size = 264, normalized size = 0.88

$$\frac{-16\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(2(a+bx))-4\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(4(a+bx))+\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+8\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-\sqrt{2\pi}\,S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(4a-\frac{4bc}{d}\right)-8\sqrt{\pi}\,S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{128b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 4*Sqrt[b/d]*Sqrt[c + d*x]*C
os[4*(a + b*x)] + Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/
d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sq
rt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 8*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
 + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(128*b*Sqrt[b/d])
```

Maple [A]
time = 0.05, size = 286, normalized size = 0.96

method	result

derivativedivides	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx-}}{\sqrt{\pi}\sqrt{}}\right.\right.}{16b\sqrt{\frac{b}{d}}}}{}$
default	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx-}}{\sqrt{\pi}\sqrt{}}\right.\right.}{16b\sqrt{\frac{b}{d}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+1/32/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x
+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d))-1/64/b*d*(d*x+c)^(1/2)*cos(4/d*b*(d*x+c)+4*(a*d-b*c)/d)+1/256/b*d*
2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(
1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 429, normalized size = 1.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/512*(16*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 64*sqrt
(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 4*(-(I - 1)*4^(1/4)*sq
rt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sq
rt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(2*I*b/d)) - (-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b*c -
a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))
*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - ((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(
1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin
(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 4*((I + 1)*4^(1/4)*
sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*4^(1/4)*
sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(-2*I*b/d)))*d/b^3
```

Fricas [A]
time = 3.10, size = 233, normalized size = 0.78

$$\frac{\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)+8\,\pi d\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-8\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\,\left(8\,b\cos\left(bx+a\right)^4-3\,b\right)\sqrt{dx+c}}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 8*pi*d*sqr
t(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)
)) - 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(
-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 3*b)*sqrt(d*x + c))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin\left(a+bx\right)\cos^3\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**3, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.71, size = 830, normalized size = 2.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d
)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*(-I*sqrt(2)*sqrt(pi)*d*erf
(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*
c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(2)*sqrt(pi)*d
*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-
I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*er
f(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*
```

```
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c + 8*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sqrt(pi)*(4*b*c + I*d)*d*
erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 16*sqrt(d*x + c)*d*e^(-2
*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(I*(d*x + c
)*b - I*b*c + I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*
c - I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/
d)/b)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\,\sin(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2), x)
```

3.187 $\int \sqrt{c+dx} \cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=299

$$-\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}$$

```
[Out] 1/128*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/128*FresnelS(2*b^(1/2)*2^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(
3/2)+1/16*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2
))*d^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/P
i^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/8*cos(2*b*x+2*a)*(d*x+
c)^(1/2)/b-1/32*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.30, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -1/8*(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/b - (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\sin(2a+2bx)+\frac{1}{8}\sqrt{c+dx}\sin(4a+4bx)\right)dx$$

$$= \frac{1}{8}\int \sqrt{c+dx}\sin(4a+4bx)\,dx+\frac{1}{4}\int \sqrt{c+dx}\sin(2a+2bx)\,d$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{d\int \frac{\cos(4}{\sqrt{c}}}{6}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{(d\cos(}{}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{\cos(4a}{}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}}{}$$

Mathematica [A]
time = 0.06, size = 264, normalized size = 0.88

$$\frac{-16\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(2(a+bx))-4\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(4(a+bx))+\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+8\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-\sqrt{2\pi}\,S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(4a-\frac{4bc}{d}\right)-8\sqrt{\pi}\,S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{128b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 4*Sqrt[b/d]*Sqrt[c + d*x]*C
os[4*(a + b*x)] + Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/
d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sq
rt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 8*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
 + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(128*b*Sqrt[b/d])
```

Maple [A]
time = 0.00, size = 286, normalized size = 0.96

method	result

derivativedivides	$$-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx-}}{\sqrt{\pi}\sqrt{}}\right.\right.}{16b\sqrt{\frac{b}{d}}}$$
default	$$-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx-}}{\sqrt{\pi}\sqrt{}}\right.\right.}{16b\sqrt{\frac{b}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+1/32/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x
+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d))-1/64/b*d*(d*x+c)^(1/2)*cos(4/d*b*(d*x+c)+4*(a*d-b*c)/d)+1/256/b*d*
2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(
1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 429, normalized size = 1.43

$$-\frac{\left(\frac{16\sqrt{dx+c}\,b^2\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{64\sqrt{dx+c}\,b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d}-4\left(-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)-(i+1)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{2ib}{d}}\right)-\left(-(i-1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)-(i+1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4(bc-ad)}{d}\right)\right)\mathrm{erf}\left(2\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)-\left((i+1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)+(i-1)\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4(bc-ad)}{d}\right)\right)\mathrm{erf}\left(2\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)-4\left((i+1)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(i-1)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{2ib}{d}}\right)\right)d}{512\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/512*(16*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 64*sqrt
(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 4*(-(I - 1)*4^(1/4)*sq
rt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sq
rt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(2*I*b/d)) - (-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b*c -
a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))
*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - ((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(
1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin
(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 4*((I + 1)*4^(1/4)*
sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*4^(1/4)*
sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(-2*I*b/d)))*d/b^3
```

Fricas [A]
time = 3.10, size = 233, normalized size = 0.78

$$\frac{\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)C\left(2\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+8\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)C\left(2\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-8\pi d\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-4\left(8b\cos\left(bx+a\right)^{4}-3b\right)\sqrt{dx+c}}{128\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 8*pi*d*sqr
t(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)
)) - 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(
-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 3*b)*sqrt(d*x + c))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin\left(a+bx\right)\cos^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**3, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.69, size = 830, normalized size = 2.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d
)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*(-I*sqrt(2)*sqrt(pi)*d*erf
(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*
c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(2)*sqrt(pi)*d
*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-
I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*er
f(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*
```

```
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c + 8*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sqrt(pi)*(4*b*c + I*d)*d*
erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 16*sqrt(d*x + c)*d*e^(-2
*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(I*(d*x + c
)*b - I*b*c + I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*
c - I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/
d)/b)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)^3 \sin(a+bx) \sqrt{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2),x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2), x)
```

3.188 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=351

$$-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}$$

```
[Out] -1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b-3/1
024*d^(3/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-3/1024*d^(3/2)*FresnelC(2*b^(1/2)*2^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5
/2)-3/64*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/
Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/64*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^
(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+3/32*d*sin(2*b*x+2*a)*(d*
x+c)^(1/2)/b^2+3/256*d*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.38, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2}+\frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -1/8*((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/b - ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]])/
(64*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) + (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right)$$

$$= \frac{1}{8}\int (c+dx)^{3/2}\sin(4a+4bx)\,dx+\frac{1}{4}\int (c+dx)^{3/2}\sin(2a+2bx$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{(3d)}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d\sqrt{}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d\sqrt{}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d\sqrt{}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}-\frac{3d^{3/}}{}$$

Mathematica [A]
time = 0.14, size = 393, normalized size = 1.12

$$\frac{-128bc\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(2(a+bx))-128b\sqrt{\frac{b}{d}}dx\sqrt{c+dx}\cos(2(a+bx))-32bc\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(4(a+bx))-32b\sqrt{\frac{b}{d}}dx\sqrt{c+dx}\cos(4(a+bx))-3d\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)-48d\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-3d\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\sin\left(4a-\frac{4bc}{d}\right)-48d\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)+96\sqrt{\frac{b}{d}}d\sqrt{c+dx}\sin(2(a+bx))+12\sqrt{\frac{b}{d}}d\sqrt{c+dx}\sin(4(a+bx))}{1024b^2\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] - 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*
x)] - 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 3*d*Sqrt[2*Pi]*Co
s[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sq
rt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
- 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (
4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Si
n[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 12*Sqr
t[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(1024*b^2*Sqrt[b/d])
```

Maple [A]
time = 0.00, size = 376, normalized size = 1.07

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)S\left(\frac{2b\sqrt{dx+}}{\sqrt{\pi}\sqrt{\frac{b}{d}}}\right)\right)}{}\right)}{8b}$
default	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)S\left(\frac{2b\sqrt{dx+}}{\sqrt{\pi}\sqrt{\frac{b}{d}}}\right)\right)}{}\right)}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+3/16/b*d*(1/4
/b*d*(d*x+c)^(1/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d
)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))-1
/64/b*d*(d*x+c)^(3/2)*cos(4/d*b*(d*x+c)+4*(a*d-b*c)/d)+3/64/b*d*(1/8/b*d*(d
*x+c)^(1/2)*sin(4/d*b*(d*x+c)+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+
c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 503, normalized size = 1.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4096*(128*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 512
*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 48*sqrt(d*x + c
)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 384*sqrt(d*x + c)*b^2*sin(2*((d*
x + c)*b - b*c + a*d)/d) + 24*((I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^
2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/
d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + 3*((I
```

```
+ 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I - 1)*s
qrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x +
 c)*sqrt(I*b/d)) + 3*(-(I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*
(b*c - a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c -
 a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + 24*(-(I - 1)*4^(1/4)*sqrt(2)*
sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(2
)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqr
t(-2*I*b/d)))*d/b^4
```

Fricas [A]
time = 3.27, size = 294, normalized size = 0.84

$$-\frac{3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+48\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+48\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+16\left(16\left(b^2dx+b^2c\right)\cos\left(bx+a\right)^4-6\,b^2dx-6\,b^2c-3\left(2\,bd\cos\left(bx+a\right)^3+3\,bd\cos\left(bx+a\right)\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{1024\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*f
resnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d))) + 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4
 - 6*b^2*d*x - 6*b^2*c - 3*(2*b*d*cos(b*x + a)^3 + 3*b*d*cos(b*x + a))*sin(
b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin(a+bx)\cos^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x)**3, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.95, size = 1523, normalized size = 4.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2048*(64*(-I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt
```

```
(b^2*d^2) + 1)) + I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^
2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
 + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c
^2 + d^2*((-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqr
t(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(8*I*(d*x + c)^(3
/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-4*(-I*(d*x +
c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*
b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2)
 - 4*I*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c
)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 16*(-I*sqrt(pi)*
(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d - 3*sq
rt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 16*(I
*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2
) + 16*(I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d
)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*d)*d
*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c -
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sqrt(pi)*(4*b*c +
I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*
b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 16*sqrt(d*x + c)*
d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(I*(
d*x + c)*b - I*b*c + I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)*b
 + I*b*c - I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b + I*b*c -
I*a*d)/d)/b)*c)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos{(a+b\,x)}^3\,\sin{(a+b\,x)}\,{(c+d\,x)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2), x)
```

3.189 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=407

$$\frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c+dx}\,\cos(4a+4bx)}{2048b^3}-\frac{(c+dx)^{5/2}\cos(4a$$

```
[Out] -1/8*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(5/2)*cos(4*b*x+4*a)/b+5/3
2*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2+5/256*d*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b
^2-15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*
x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/8192*d^(5/2)*FresnelS(2*b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1
/2)/b^(7/2)-15/256*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+15/256*d^(5/2)*FresnelS(2*b^(1/2)*(d*x
+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/128*d^2*co
s(2*b*x+2*a)*(d*x+c)^(1/2)/b^3+15/2048*d^2*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A]

time = 0.47, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15\sqrt{\pi}\,d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}+\frac{15d^2\sqrt{c+dx}\,\cos(4a+4bx)}{2048b^3}+\frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2}+\frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*
a + 2*b*x])/(8*b) + (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) - ((
c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a -
(4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^
(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4
096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*
Sin[2*a + 2*b*x])/(32*b^2) + (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^
2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right)$$

$$= \frac{1}{8}\int (c+dx)^{5/2}\sin(4a+4bx)\,dx+\frac{1}{4}\int (c+dx)^{5/2}\sin(2a+2bx$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}+\frac{(5d)}{}$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}+\frac{5d(c}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d}{}$$

Mathematica [A]
time = 10.66, size = 550, normalized size = 1.35

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos
[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^
2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a +
 b*x)] + 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 512*b^3*c*d*x*Sqrt[c + d
*x]*Cos[4*(a + b*x)] - 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 15*
Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresnel
C[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Fresn
elS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b
/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (
2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sq
```

```
rt[c + d*x]*Sin[2*(a + b*x)] + 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] +
 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(8192*b^4)
```

Maple [A]
time = 0.00, size = 470, normalized size = 1.15

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.}{}\right.}{}$
default	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+5/16/b*d*(1/4
/b*d*(d*x+c)^(3/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos
(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*(a
*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/64/b*d*(
d*x+c)^(5/2)*cos(4/d*b*(d*x+c)+4*(a*d-b*c)/d)+5/64/b*d*(1/8/b*d*(d*x+c)^(3/
2)*sin(4/d*b*(d*x+c)+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d
```

```
*b*(d*x+c)+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d
-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(
a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 551, normalized size = 1.35

$$\frac{\left(640\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)+5120\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-16\left(\frac{64\,(dx+c)^{\frac{5}{2}}b^4}{d}-15\,\sqrt{dx+c}\,b^2d\right)\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)-256\left(\frac{16\,(dx+c)^{\frac{5}{2}}b^4}{d}-15\,\sqrt{dx+c}\,b^2d\right)\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-240\left(-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)-(i+1)\cdot 4^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2\,(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{2i\,b}{d}}\right)-15\left(-(i-1)\,\sqrt{2}\,\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)-(i+1)\,\sqrt{2}\,\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4\,(bc-ad)}{d}\right)\right)\operatorname{erf}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{i\,b}{d}}\right)-15\left((i+1)\,\sqrt{2}\,\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)+(i-1)\,\sqrt{2}\,\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{4\,(bc-ad)}{d}\right)\right)\operatorname{erf}\left(2\,\sqrt{dx+c}\,\sqrt{-\frac{i\,b}{d}}\right)-240\left((i+1)\cdot 4^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)+(i-1)\cdot 4^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2\,(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{2i\,b}{d}}\right)\right)d}{32768\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/32768*(640*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) + 5120*
(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 16*(64*(d*x + c)^(
5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d) - 2
56*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)*b -
 b*c + a*d)/d) - 240*(-(I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/
4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 15*(-(I -
1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I + 1)*s
qrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x
 + c)*sqrt(I*b/d)) - 15*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos
(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*
(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 240*((I + 1)*4^(1/4)*sq
rt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*4^(1/4
)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*
x + c)*sqrt(-2*I*b/d)))*d/b^5
```

Fricas [A]
time = 3.39, size = 376, normalized size = 0.92

$$-\frac{15\,\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-15\,\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)+480\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-480\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\left(192\,b^3d^2x^2+384\,b^3cdx+192\,b^3c^2+360\,bd^2\cos\left(bx+a\right)^2-8\left(64\,b^3d^2x^2+128\,b^3cdx+64\,b^3c^2-15\,bd^2\right)\cos\left(bx+a\right)^4-225\,bd^2+160\left(2\left(b^2d^2x+b^2cd\right)\cos\left(bx+a\right)^3+3\left(b^2d^2x+b^2cd\right)\cos\left(bx+a\right)\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{8192\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))
*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d)
+ 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x +
c)*sqrt(b/(pi*d))) - 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(192*b^3*d^2*x^2 + 384*b^3*c*d*x
+ 192*b^3*c^2 + 360*b*d^2*cos(b*x + a)^2 - 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*
x + 64*b^3*c^2 - 15*b*d^2)*cos(b*x + a)^4 - 225*b*d^2 + 160*(2*(b^2*d^2*x +
 b^2*c*d)*cos(b*x + a)^3 + 3*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x +
a))*sqrt(d*x + c))/b^4
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [C] Result contains complex when optimal does not.
time = 1.18, size = 2446, normalized size = 6.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16384*(512*(-I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)) + I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*
d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))
*c^3 + 24*c*d^2*((-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*e
rf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*
b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(8*I*(d*x
+ c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-4*(-I*
(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (I*sqrt(2)*sqrt(pi)*(64*b^2*c^2
+ 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
1)*b^2) - 4*I*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(
d*x + c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 16*(-I*sq
rt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d
 - 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2
+ 16*(I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x +
 c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b
^2)/d^2) + d^3*((I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*I*b^2*c^2*d - 72*b*c
*d^2 + 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
```

```
2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b
^3) - 4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 192*I*(d*x + c)^(3/2)*b^2*c*d + 192
*I*sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(3/2)*b*d^2 + 72*sqrt(d*x + c)*b*
c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b
^3)/d^3 + (-I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 + 192*I*b^2*c^2*d - 72*b*c*d^2
- 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) -
4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 192*I*(d*x + c)^(3/2)*b^2*c*d + 192*I*sqr
t(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)*b*c*d^2
- 15*I*sqrt(d*x + c)*d^3)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3
 + 32*(I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d^3)*d*e
rf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*
a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5
/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d - 2
0*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3
)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 32*(-I*sqrt(pi)*(64*
b^3*c^3 + 48*I*b^2*c^2*d - 36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x +
 c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2
 - 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-2*(I*(d*x + c)*b
- I*b*c + I*a*d)/d)/b^3)/d^3) + 192*(I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf
(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*
c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(2)*sqrt(pi)
*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) +
 8*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)*b) - 8*I*sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b) + 16*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b +
 4*sqrt(d*x + c)*d*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 16*sqrt(d*x
 + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(
-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c^2)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\,\sin(a+b\,x)\,\left(c+d\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2), x)
```

3.190 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=615

$$\frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2}-\frac{d(c+dx)^{3/2}\cos(5a+5bx)}{160b^2}+\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\cdots}{3\cdots}$$

```
[Out] 5/16*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/288*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b^
2-1/160*d*(d*x+c)^(3/2)*cos(5*b*x+5*a)/b^2+1/8*(d*x+c)^(5/2)*sin(b*x+a)/b-1
/48*(d*x+c)^(5/2)*sin(3*b*x+3*a)/b-1/80*(d*x+c)^(5/2)*sin(5*b*x+5*a)/b-3/16
000*d^(5/2)*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/
2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(7/2)-3/16000*d^(5/2)*FresnelC(b^(1/2)*10^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(
7/2)-5/3456*d^(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x
+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)-5/3456*d^(5/2)*FresnelC(b^(1/2)
*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/
b^(7/2)+15/64*d^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^(5/2)*FresnelC(b^(1/2)*2^
(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)
-15/32*d^2*sin(b*x+a)*(d*x+c)^(1/2)/b^3+5/576*d^2*sin(3*b*x+3*a)*(d*x+c)^(1
/2)/b^3+3/1600*d^2*sin(5*b*x+5*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.89, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(16*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a
+ 3*b*x])/(288*b^2) - (d*(c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(160*b^2) + (15*
d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d
*x])/Sqrt[d]])/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fr
esnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) - (3*d^(5
/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*
Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) -
(5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*
Sin[3*a - (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[(Sqrt
```

```
[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) - (15
*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(32*b^3) + ((c + d*x)^(5/2)*Sin[a + b*x])/
(8*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5/2)
*Sin[3*a + 3*b*x])/(48*b) + (3*d^2*Sqrt[c + d*x]*Sin[5*a + 5*b*x])/(1600*b^
3) - ((c + d*x)^(5/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2}\cos(a+bx) - \frac{1}{16}(c+dx)^{5/2}\cos(3a+3bx) - \right.$$

$$= -\left(\frac{1}{16}\int (c+dx)^{5/2}\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^{5/2}\cos($$

$$= \frac{(c+dx)^{5/2}\sin(a+bx)}{8b} - \frac{(c+dx)^{5/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)^{5}}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 23.85, size = 1795, normalized size = 2.92

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*c^2*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d]
)/Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x)
)/d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (c*d*(Sqrt[b/d]
*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c
)/d] + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sq
rt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2
*b*Sqrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(16*b^3) + ((b/d)^
(3/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^
2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d])) - Sqrt[2*Pi]
```

```
*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (b*c)/d] +
 (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2*b
*(c - 5*d*x)*Cos[a + b*x] + d*(-15 + 4*b^2*x^2)*Sin[a + b*x])))/(64*b^5) -
(c^2*(-(Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[
c + d*x]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*
a - (3*b*c)/d] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(96*S
qrt[3]*b*Sqrt[b/d]) - (c*d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/
Pi]*Sqrt[c + d*x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d])
 + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c
*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]
*(Cos[3*(a + b*x)] + 2*b*x*Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) - ((b/d)^(3
/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2
*c^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d])) - Sqrt
[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3
*b*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]
*d*Sqrt[c + d*x]*(-2*b*(c - 5*d*x)*Cos[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*S
in[3*(a + b*x)])))/(1152*Sqrt[3]*b^5) - (c^2*(-(Sqrt[2*Pi]*Cos[5*a - (5*b*c
)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*FresnelC[S
qrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d] + 2*Sqrt[5]*Sqrt[b
/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (c*d*(Sqrt
[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[5*
a - (5*b*c)/d] + 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*Fresne
lS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*
Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*b*Sqrt[c + d*x]*(3*Cos[5*(a + b*x)] + 10*
b*x*Sin[5*(a + b*x)])))/(800*Sqrt[5]*b^3) - ((b/d)^(3/2)*d^2*(-(Sqrt[2*Pi]*
FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*((20*b^2*c^2 - 3*d^2)*Cos[5*a
 - (5*b*c)/d] + 12*b*c*d*Sin[5*a - (5*b*c)/d])) - Sqrt[2*Pi]*FresnelC[Sqrt[
b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[5*a - (5*b*c)/d] + (20*b^2*c
^2 - 3*d^2)*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2
*b*(c - 5*d*x)*Cos[5*(a + b*x)] + d*(-3 + 20*b^2*x^2)*Sin[5*(a + b*x)])))/(
3200*Sqrt[5]*b^5)
```

Maple [A]
time = 0.09, size = 716, normalized size = 1.16

method	result

derivativedivides	$\frac{\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}}{\cdots}$
default	$\frac{\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(5/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-5/16/b*d*(-1/2/b
*d*(d*x+c)^(3/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1
/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(co
s((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin
((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1
/96/b*d*(d*x+c)^(5/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+5/96/b*d*(-1/6/b*d*(
d*x+c)^(3/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2
)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(
d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*b*(d*x+c)^(1/2)/d))))-1/160/b*d*(d*x+c)^(5/2)*sin(5/d*b*(d*x+c)+5*(a*
d-b*c)/d)+1/32/b*d*(-1/10/b*d*(d*x+c)^(3/2)*cos(5/d*b*(d*x+c)+5*(a*d-b*c)/d
)+3/10/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5/d*b*(d*x+c)+5*(a*d-b*c)/d)-1/100/b
```

```
*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2
)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(5*(a*d-b*c)/d)*Fresne
lC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.54, size = 826, normalized size = 1.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/1728000*sqrt(2)*(5400*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b - b
*c + a*d)/d)/d + 15000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b*c
 + a*d)/d)/d - 270000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c +
a*d)/d)/d - 81*(-(I + 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*
c - a*d)/d) + (I - 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c -
 a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 625*(-(I + 1)*9^(1/4)*sqrt(pi)
*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(pi)*b^2
*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d))
- 101250*((I + 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I -
 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(I*b/d)) - 101250*(-(I - 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a
*d)/d) + (I + 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(-I*b/d)) - 625*((I - 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(
1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)
*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - 81*((I - 1)*25^
(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (I + 1)*25^(1/
4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*
sqrt(-5*I*b/d)) + 540*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 3*sqrt(2)*sqrt(
d*x + c)*b^3)*sin(5*((d*x + c)*b - b*c + a*d)/d) + 1500*(12*sqrt(2)*(d*x +
c)^(5/2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*sin(3*((d*x + c)*b - b*c +
a*d)/d) - 27000*(4*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 15*sqrt(2)*sqrt(d*x +
c)*b^3)*sin(((d*x + c)*b - b*c + a*d)/d))*d^2/b^6
```

Fricas [A]
time = 3.72, size = 548, normalized size = 0.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_
sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 625*sqrt(6)*pi*d^3*sqrt(b/(pi*
d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))
```

```
 - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*f
resnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 625*
sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*
d)))*sin(-3*(b*c - a*d)/d) + 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(
sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(90*(b^2
*d^2*x + b^2*c*d)*cos(b*x + a)^5 - 50*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3
 - 300*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (120*b^3*d^2*x^2 + 240*b^3*c*d*x
 + 120*b^3*c^2 - 9*(20*b^3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*c
os(b*x + a)^4 - 428*b*d^2 + (60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 +
11*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep
```

Giac [C] Result contains complex when optimal does not.
time = 1.70, size = 3705, normalized size = 6.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/864000*(1800*(30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)) - 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 5*sqrt(6)*sqrt(pi)*d*erf(-1/2
*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c
 + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*
erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2
*(2250*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)
/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d +
```

4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c -
I*a*d)/d)/b^2)/d^2 - 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*
d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(-2*I
*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3*(
-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 - 9*(sqrt(10)*sqrt(pi)*(100*b^2
*c^2 - 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b^2) - 10*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^(-5*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^
2 + 2250*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*
d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*
c + I*a*d)/d)/b^2)/d^2 - 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 + 4*I*b*c*d - d^
2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(2*
I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3*
(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 - 9*(sqrt(10)*sqrt(pi)*(100*b^2
*c^2 + 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b^2) - 10*(10*I*(d*x + c)^(3/2)*b*d - 20*I*sqrt(d*x + c)*b*c*d
+ 3*sqrt(d*x + c)*d^2)*e^(-5*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2)
- d^3*(6750*(sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15
*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(-
4*I*(d*x + c)^(5/2)*b^2*d + 12*I*(d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d*x +
c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 15*I*s
qrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 - 125*(sq
rt(6)*sqrt(pi)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-
1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*
b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 6*(-12*I*(d*x
+ c)^(5/2)*b^2*d + 36*I*(d*x + c)^(3/2)*b^2*c*d - 36*I*sqrt(d*x + c)*b^2*c^
2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x +
c)*d^3)*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 - 27*(sqrt(10)*s
qrt(pi)*(200*b^3*c^3 - 60*I*b^2*c^2*d - 18*b*c*d^2 + 3*I*d^3)*d*erf(-1/2*sq
rt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c -
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 10*(-20*I*(d*x + c)
^(5/2)*b^2*d + 60*I*(d*x + c)^(3/2)*b^2*c*d - 60*I*sqrt(d*x + c)*b^2*c^2*d
+ 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c)*d
^3)*e^(-5*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 6750*(sqrt(2)*sqrt
(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(4*I*(d*x + c)^(5/2)*b^2
*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x

```
+ c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I
*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 - 125*(sqrt(6)*sqrt(pi)*(72*b^3*c
^3 + 36*I*b^2*c^2*d - 18*b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 6*(12*I*(d*x + c)^(5/2)*b^2*d - 36*I*(d
*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2\,{\left(c+d\,x\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

3.191 $\int (c+dx)^{3/2} \cos^3(a+bx) \sin^2(a+bx) \, dx$

Optimal. Leaf size=534

$$\frac{3d\sqrt{c+dx}\ \cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\ \cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\ \cos(5a+5bx)}{800b^2} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\ \cos\left(a-\frac{bc}{d}\right) \text{Fr}}{1}$$

```
[Out] 1/8*(d*x+c)^(3/2)*sin(b*x+a)/b-1/48*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b-1/80*(d*
x+c)^(3/2)*sin(5*b*x+5*a)/b+3/8000*d^(3/2)*cos(5*a-5*b*c/d)*FresnelC(b^(1/2
)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(5/2)-3/8000
*d^(3/2)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-
5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(5/2)+1/576*d^(3/2)*cos(3*a-3*b*c/d)*FresnelC(
b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/
576*d^(3/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*
a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*cos(a-b*c/d)*FresnelC(b^(1
/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/32*d
^(3/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d
)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d*cos(b*x+a)*(d*x+c)^(1/2)/b^2-1/96*d*cos(3
*b*x+3*a)*(d*x+c)^(1/2)/b^2-3/800*d*cos(5*b*x+5*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.63, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{3\sqrt{\frac{\pi}{2}}\, d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}} + \frac{\sqrt{\frac{\pi}{6}}\, d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} + \frac{3\sqrt{\frac{\pi}{10}}\, d^{3/2}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}} - \frac{3\sqrt{\frac{\pi}{10}}\, d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}} - \frac{\sqrt{\frac{\pi}{6}}\, d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} + \frac{3\sqrt{\frac{\pi}{2}}\, d^{3/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}} + \frac{3d\sqrt{c+dx}\ \cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\ \cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\ \cos(5a+5bx)}{800b^2} + \frac{(c+dx)^{3/2}\sin(a+bx)}{8b} - \frac{(c+dx)^{3/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)^{3/2}\sin(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*
x])/(96*b^2) - (3*d*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(800*b^2) - (3*d^(3/2)*
Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]])/(16*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sq
rt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi
/10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt
[d]])/(800*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt
[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*
c)/d])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + ((c + d*x)^(3/2)*Sin
[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(48*b) - ((c + d*x)^(
3/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2}\cos(a+bx) - \frac{1}{16}(c+dx)^{3/2}\cos(3a+3bx) -\right.$$

$$= -\left(\frac{1}{16}\int (c+dx)^{3/2}\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^{3/2}\cos$$

$$= \frac{(c+dx)^{3/2}\sin(a+bx)}{8b} - \frac{(c+dx)^{3/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)}{}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c}}{}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c}}{}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c}}{}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.32, size = 1043, normalized size = 1.95

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*c*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/
Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/
d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (d*(Sqrt[b/d]*Sqr
t[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/d]
 + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2
/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b*S
qrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(32*b^3) - (c*(-(Sqrt[
2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) -
Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d
] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(96*Sqrt[3]*b*Sqrt
[b/d]) - (d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*
x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[b/d]*Sq
rt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b
```

```
*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(Cos[3*(a + b*
x)] + 2*b*x*Sin[3*(a + b*x)])))/(192*Sqrt[3]*b^3) - (c*(-(Sqrt[2*Pi]*Cos[5*
a - (5*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*
FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d] + 2*Sqrt
[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)]))/(160*Sqrt[5]*b*Sqrt[b/d]) -
(d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-3*
d*Cos[5*a - (5*b*c)/d] + 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi
]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d
] + 3*d*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*b*Sqrt[c + d*x]*(3*Cos[5*(a + b*x
)] + 10*b*x*Sin[5*(a + b*x)])))/(1600*Sqrt[5]*b^3)
```

Maple [A]
time = 0.09, size = 583, normalized size = 1.09

method	result
derivativedivides	$\frac{\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b}{\sqrt{\pi}}\right.\right.}{}\right)}{8b}}{}$
default	$\frac{\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b}{\sqrt{\pi}}\right.\right.}{}\right)}{8b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(3/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-3/16/b*d*(-1/2/b
*d*(d*x+c)^(1/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b
/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)
^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)))-1/96/b*d*(d*x+c)^(3/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/32/b*d
*(-1/6/b*d*(d*x+c)^(1/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*
3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))-1/160/b*d*(d*x+c)^(3/2)*s
in(5/d*b*(d*x+c)+5*(a*d-b*c)/d)+3/160/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5/d*
b*(d*x+c)+5*(a*d-b*c)/d)+1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(co
```

```
s(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(
d*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 760, normalized size = 1.42

$$-\frac{\sqrt{2}\left(\frac{1800\sqrt{2}(dx+c)^{\frac{3}{2}}b^4\sin\left(\frac{5((dx+c)b-bc+ad)}{d}\right)}{d^2}+\frac{3000\sqrt{2}(dx+c)^{\frac{3}{2}}b^4\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d^2}-\frac{18000\sqrt{2}(dx+c)^{\frac{3}{2}}b^4\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d^2}+\frac{540\sqrt{2}\sqrt{dx+c}\,b^3\cos\left(\frac{5((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{1500\sqrt{2}\sqrt{dx+c}\,b^3\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{27000\sqrt{2}\sqrt{dx+c}\,b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}-27\left(-(i-1)\cdot 25^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{5(bc-ad)}{d}\right)-(i+1)\cdot 25^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{5(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{5ib}{d}}\right)-125\left(-(i-1)\cdot 9^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{3ib}{d}}\right)-6750\left((i-1)\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i+1)\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)-6750\left(-(i+1)\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)-(i-1)\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)-125\left((i+1)\cdot 9^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)+(i-1)\cdot 9^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{3ib}{d}}\right)-27\left((i+1)\cdot 25^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{5(bc-ad)}{d}\right)+(i-1)\cdot 25^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{5(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{5ib}{d}}\right)\right)d^2}{288000\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/288000*sqrt(2)*(1800*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b - b*
c + a*d)/d)/d^2 + 3000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b*c
 + a*d)/d)/d^2 - 18000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*c +
 a*d)/d)/d^2 + 540*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c + a*d
)/d)/d + 1500*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d)/
d - 27000*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d - 27
*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (I
 + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(5*I*b/d)) - 125*(-(I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/
4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin
(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 6750*((I - 1)*sqrt(p
i)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(pi)*b^2*(b^2/d^2)
^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 6750*(-(I + 1)
*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - 125*((
I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)
*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x +
 c)*sqrt(-3*I*b/d)) - 27*((I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos
(-5*(b*c - a*d)/d) + (I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(
b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^5
```

Fricas [A]
time = 2.90, size = 446, normalized size = 0.84

$$\frac{27\sqrt{10}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)C\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+125\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-6750\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+6750\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-125\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)-27\sqrt{10}\,\pi d^2\sqrt{\frac{b}{\pi d}}S\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{5(bc-ad)}{d}\right)-480\left(9bd\cos(bx+a)^5-5bd\cos(bx+a)^3-30bd\cos(bx+a)+10\left(3(b^2dx+b^2c)\cos(bx+a)^4-2b^2dx-2b^2c-(b^2dx+b^2c)\cos(bx+a)^2\right)\sin(bx+a)\right)\sqrt{dx+c}}{72000\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_co
s(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d)
)*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) -
 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d))) + 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel
```

```
_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6
)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-3*(b*c - a*d)/d) - 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(1
0)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(9*b*d*cos(b*x
 + a)^5 - 5*b*d*cos(b*x + a)^3 - 30*b*d*cos(b*x + a) + 10*(3*(b^2*d*x + b^2
*c)*cos(b*x + a)^4 - 2*b^2*d*x - 2*b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2
)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x)**3, x)
```

Giac [C] Result contains complex when optimal does not.
time = 1.28, size = 2313, normalized size = 4.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/144000*(300*(30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) - 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*
d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c
+ I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*e
rf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5
*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(2250
*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sq
rt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d
)/d)/b^2)/d^2 - 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(
-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I
```

```
*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(-2*I*(d*x
+ c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*
x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 - 9*(sqrt(10)*sqrt(pi)*(100*b^2*c^2 -
 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^
2) + 1)*b^2) - 10*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d + 3
*sqrt(d*x + c)*d^2)*e^(-5*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 22
50*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(
sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*d - 4*
I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*
a*d)/d)/b^2)/d^2 - 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 + 4*I*b*c*d - d^2)*d*e
rf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*
(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(2*I*(d*x
 + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3*(I*(d*
x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 - 9*(sqrt(10)*sqrt(pi)*(100*b^2*c^2 +
 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b^2) - 10*(10*I*(d*x + c)^(3/2)*b*d - 20*I*sqrt(d*x + c)*b*c*d + 3*s
qrt(d*x + c)*d^2)*e^(-5*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) - 20*(
450*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(10)*sqrt(pi)*(10*b*c -
 I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 45
0*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(10)*sqrt(pi)*(10*b*c +
 I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 900
*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 150*I*sqrt(d*x
 + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 90*I*sqrt(d*x + c)*d*e
^(-5*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 900*I*sqrt(d*x + c)*d*e^((-I*(d
*x + c)*b + I*b*c - I*a*d)/d)/b + 150*I*sqrt(d*x + c)*d*e^(-3*(-I*(d*x + c)
*b + I*b*c - I*a*d)/d)/b + 90*I*sqrt(d*x + c)*d*e^(-5*(-I*(d*x + c)*b + I*b
*c - I*a*d)/d)/b)*c)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\,\sin(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

3.192 $\int \sqrt{c+dx} \cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=459

$$-\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}+\frac{\sqrt{d}}{}$$

```
[Out] 1/800*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(3/2)+1/800*FresnelC(b^(1/2)*10^(1/2)/Pi^
(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(
3/2)+1/288*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)
/d^(1/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1/288*FresnelC(b^(1/2)*6^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^
(3/2)-1/16*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/8
*sin(b*x+a)*(d*x+c)^(1/2)/b-1/48*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b-1/80*sin(5*
b*x+5*a)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.49, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}+\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{c+dx}\sin(a+bx)}{8b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{48b}-\frac{\sqrt{c+dx}\sin(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -1/8*(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/b^(3/2) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x
])/Sqrt[d]])/(80*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(80*b^(3/2)) + (Sqrt[d]*S
qrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3
*b*c)/d])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*S
qrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(3/2)) + (Sqrt[c + d*x]*Sin[a
 + b*x])/(8*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(48*b) - (Sqrt[c + d*x]*S
in[5*a + 5*b*x])/(80*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\,\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx}\,\cos(a+bx) - \frac{1}{16}\sqrt{c+dx}\,\cos(3a+3bx) - \frac{1}{16}\right.$$

$$= -\left(\frac{1}{16}\int \sqrt{c+dx}\,\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int \sqrt{c+dx}\,\cos(5a$$

$$= \frac{\sqrt{c+dx}\,\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\,\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\,\sin}{80}$$

$$= \frac{\sqrt{c+dx}\,\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\,\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\,\sin}{80}$$

$$= \frac{\sqrt{c+dx}\,\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\,\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\,\sin}{80}$$

$$= -\frac{\sqrt{d}\,\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{bc}{d}\right) S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} + \frac{\sqrt{d}\,\sqrt{\frac{\pi}{6}}\,\mathrm{c}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.22, size = 435, normalized size = 0.95

$$-\frac{ie^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b}-\frac{-\sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)-\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\sin\left(3a-\frac{3bc}{d}\right)+2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin(3(a+bx))}{96\sqrt{3}\,b\sqrt{\frac{b}{d}}}-\frac{-\sqrt{2\pi}\cos\left(5a-\frac{5bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right)-\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right)\sin\left(5a-\frac{5bc}{d}\right)+2\sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin(5(a+bx))}{160\sqrt{5}\,b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sq
rt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d]
)/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) - (-(Sqrt[2*Pi]*Cos[3
*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*
FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] + 2*Sqrt[
3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)])/(96*Sqrt[3]*b*Sqrt[b/d]) - (-(
Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x
]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5
*b*c)/d] + 2*Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)])/(160*Sqrt[5]
*b*Sqrt[b/d])
```

Maple [A]
time = 0.09, size = 444, normalized size = 0.97

method	result
derivativedivides	$\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{}}{\sqrt{\pi}}\right.\right.}{16b\sqrt{\frac{b}{d}}}}{}$
default	$\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{}}{\sqrt{\pi}}\right.\right.}{16b\sqrt{\frac{b}{d}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(1/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-1/32/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d))-1/96/b*d*(d*x+c)^(1/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/
d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)
/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))-1/160
/b*d*(d*x+c)^(1/2)*sin(5/d*b*(d*x+c)+5*(a*d-b*c)/d)+1/1600/b*d*2^(1/2)*Pi^(
1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1
/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(
1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 680, normalized size = 1.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/28800*sqrt(2)*(180*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c +
a*d)/d)/d^2 + 300*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*d)
/d)/d^2 - 1800*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d
^2 - 9*((I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)
/d - (I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d)
*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 25*((I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d
^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d - (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^
(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 450*(-(I
```

```
+ 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d + (I - 1)*sqrt(pi)*
b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(I*b/d)) -
 450*((I - 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d - (I + 1)*
sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(
-I*b/d)) - 25*(-(I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-3*(b*c -
a*d)/d)/d + (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)
/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - 9*(-(I - 1)*25^(1/4)*sqrt(pi)*b^
2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d + (I + 1)*25^(1/4)*sqrt(pi)*b^2*(
b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*
d^2/b^4
```

Fricas [A]
time = 3.51, size = 365, normalized size = 0.80

$$\frac{9\sqrt{10}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+25\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-450\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-450\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+25\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)+9\sqrt{10}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{5(bc-ad)}{d}\right)-480\left(3b\cos(bx+a)^4-b\cos(bx+a)^2-2b\right)\sqrt{dx+c}\,\sin(bx+a)}{7200\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_sin(sq
rt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-
3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*sq
rt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x
+ c)*sqrt(b/(pi*d))) - 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*
sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b/
(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d
)/d) + 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(10)*sqrt(d*x + c)*sq
rt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^4 - b*cos(b*x +
 a)^2 - 2*b)*sqrt(d*x + c)*sin(b*x + a))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^2(a+bx)\cos^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**3, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.90, size = 1270, normalized size = 2.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/14400*(450*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c
- I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(10)*sqrt(pi)*
(10*b*c - I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)*b) + 450*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2
*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c
 + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(10)*sqrt(pi)*
(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b) - 30*(30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)) - 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6
)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*erf(-1/
2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b
*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c - 900*I*sqrt(d*x +
c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 150*I*sqrt(d*x + c)*d*e^(-3*
(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 90*I*sqrt(d*x + c)*d*e^(-5*(I*(d*x +
 c)*b - I*b*c + I*a*d)/d)/b + 900*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*
b*c - I*a*d)/d)/b + 150*I*sqrt(d*x + c)*d*e^(-3*(-I*(d*x + c)*b + I*b*c - I
*a*d)/d)/b + 90*I*sqrt(d*x + c)*d*e^(-5*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)
/b)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\sin(a+b\,x)^2\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

3.193 $\int \sqrt{c+dx} \cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=459

$$-\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}+\frac{\sqrt{d}\cdots}{\cdots}$$

```
[Out] 1/800*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(3/2)+1/800*FresnelC(b^(1/2)*10^(1/2)/Pi^
(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(
3/2)+1/288*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)
/d^(1/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1/288*FresnelC(b^(1/2)*6^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^
(3/2)-1/16*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/8
*sin(b*x+a)*(d*x+c)^(1/2)/b-1/48*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b-1/80*sin(5*
b*x+5*a)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.48, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}+\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{c+dx}\sin(a+bx)}{8b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{48b}-\frac{\sqrt{c+dx}\sin(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -1/8*(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/b^(3/2) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x
])/Sqrt[d]])/(80*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(80*b^(3/2)) + (Sqrt[d]*S
qrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3
*b*c)/d])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*S
qrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(3/2)) + (Sqrt[c + d*x]*Sin[a
 + b*x])/(8*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(48*b) - (Sqrt[c + d*x]*S
in[5*a + 5*b*x])/(80*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\,\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx}\,\cos(a+bx) - \frac{1}{16}\sqrt{c+dx}\,\cos(3a+3bx) - \frac{1}{16}\right.$$

$$= -\left(\frac{1}{16}\int \sqrt{c+dx}\,\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int \sqrt{c+dx}\,\cos(5a$$

$$= \frac{\sqrt{c+dx}\,\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\,\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\,\mathrm{si}}{80}$$

$$= \frac{\sqrt{c+dx}\,\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\,\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\,\mathrm{si}}{80}$$

$$= \frac{\sqrt{c+dx}\,\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\,\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\,\mathrm{si}}{80}$$

$$= -\frac{\sqrt{d}\,\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{bc}{d}\right) S\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} + \frac{\sqrt{d}\,\sqrt{\frac{\pi}{6}}\,\mathrm{c}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.72, size = 403, normalized size = 0.88

$$\frac{-450ie^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\mathrm{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\mathrm{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)+\frac{25\left(\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+\sqrt{6\pi}\,\mathrm{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)\sin\left(3a-\frac{3bc}{d}\right)-6\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin(3(a+bx))\right)}{\sqrt{\frac{b}{d}}}+\frac{9\left(\sqrt{10\pi}\cos\left(5a-\frac{5bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right)+\sqrt{10\pi}\,\mathrm{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right)\sin\left(5a-\frac{5bc}{d}\right)-10\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin(5(a+bx))\right)}{\sqrt{\frac{b}{d}}}}{7200b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (((-450*I)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sq
rt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d]
)/Sqrt[(I*b*(c + d*x))/d]))/E^((I*(b*c + a*d))/d) + (25*(Sqrt[6*Pi]*Cos[3*a
 - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[6*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] - 6*Sqrt[b/d
]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/Sqrt[b/d] + (9*(Sqrt[10*Pi]*Cos[5*a - (5
*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[10*Pi]*Fresne
lC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d] - 10*Sqrt[b/d]
*Sqrt[c + d*x]*Sin[5*(a + b*x)]))/Sqrt[b/d])/(7200*b)
```

Maple [A]
time = 0.00, size = 444, normalized size = 0.97

method	result

derivativedivides	$\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{\cdots}}{\sqrt{\pi}\cdots}\right)\right)}{16b\sqrt{\frac{b}{d}}}}{\cdots}$
default	$\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{\cdots}}{\sqrt{\pi}\cdots}\right)\right)}{16b\sqrt{\frac{b}{d}}}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(1/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-1/32/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d))-1/96/b*d*(d*x+c)^(1/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/
d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)
/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))-1/160
/b*d*(d*x+c)^(1/2)*sin(5/d*b*(d*x+c)+5*(a*d-b*c)/d)+1/1600/b*d*2^(1/2)*Pi^(
1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1
/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(
1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 680, normalized size = 1.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/28800*sqrt(2)*(180*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c +
a*d)/d)/d^2 + 300*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*d)
/d)/d^2 - 1800*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d
^2 - 9*((I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)
/d - (I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d)
*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 25*((I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d
^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d - (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^
(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 450*(-(I
+ 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d + (I - 1)*sqrt(pi)*
```

```
b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(I*b/d)) -
 450*((I - 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d - (I + 1)*
sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(
-I*b/d)) - 25*(-(I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-3*(b*c -
a*d)/d)/d + (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)
/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - 9*(-(I - 1)*25^(1/4)*sqrt(pi)*b^
2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d + (I + 1)*25^(1/4)*sqrt(pi)*b^2*(
b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*
d^2/b^4
```

Fricas [**A**]
time = 2.86, size = 365, normalized size = 0.80

$$\frac{9\,\sqrt{10}\,\pi d\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+25\,\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-450\,\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-450\,\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)+25\,\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)+9\,\sqrt{10}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{5\,(bc-ad)}{d}\right)-480\,\left(3\,b\cos\left(bx+a\right)^4-b\cos\left(bx+a\right)^2-2\,b\right)\sqrt{dx+c}\,\sin\left(bx+a\right)}{7200\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_sin(sq
rt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-
3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*sq
rt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x
+ c)*sqrt(b/(pi*d))) - 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*
sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b/
(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d
)/d) + 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(10)*sqrt(d*x + c)*sq
rt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^4 - b*cos(b*x +
 a)^2 - 2*b)*sqrt(d*x + c)*sin(b*x + a))/b^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^2\left(a+bx\right)\cos^3\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**3, x)
```

Giac [**C**] Result contains complex when optimal does not.
time = 0.89, size = 1270, normalized size = 2.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/14400*(450*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c
- I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(10)*sqrt(pi)*
(10*b*c - I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)*b) + 450*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2
*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c
 + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(10)*sqrt(pi)*
(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b) - 30*(30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)) - 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6
)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*erf(-1/
2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b
*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c - 900*I*sqrt(d*x +
c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 150*I*sqrt(d*x + c)*d*e^(-3*
(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 90*I*sqrt(d*x + c)*d*e^(-5*(I*(d*x +
 c)*b - I*b*c + I*a*d)/d)/b + 900*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*
b*c - I*a*d)/d)/b + 150*I*sqrt(d*x + c)*d*e^(-3*(-I*(d*x + c)*b + I*b*c - I
*a*d)/d)/b + 90*I*sqrt(d*x + c)*d*e^(-5*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)
/b)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\sin(a+b\,x)^2\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

3.194 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=534

$$\frac{3d\sqrt{c+dx}\,\cos(a+bx)}{16b^2}-\frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{96b^2}-\frac{3d\sqrt{c+dx}\,\cos(5a+5bx)}{800b^2}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\cos\left(a-\frac{bc}{d}\right)\,\mathrm{Fr}\ldots}{1\ldots}$$

```
[Out] 1/8*(d*x+c)^(3/2)*sin(b*x+a)/b-1/48*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b-1/80*(d*
x+c)^(3/2)*sin(5*b*x+5*a)/b+3/8000*d^(3/2)*cos(5*a-5*b*c/d)*FresnelC(b^(1/2
)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(5/2)-3/8000
*d^(3/2)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-
5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(5/2)+1/576*d^(3/2)*cos(3*a-3*b*c/d)*FresnelC(
b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/
576*d^(3/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*
a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*cos(a-b*c/d)*FresnelC(b^(1
/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/32*d
^(3/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d
)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d*cos(b*x+a)*(d*x+c)^(1/2)/b^2-1/96*d*cos(3
*b*x+3*a)*(d*x+c)^(1/2)/b^2-3/800*d*cos(5*b*x+5*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.60, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}\,d^{3/2}\cos\left(5a-\frac{5bc}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{3\sqrt{\frac{\pi}{10}}\,d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{3d\sqrt{c+dx}\,\cos(a+bx)}{16b^2}-\frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{96b^2}-\frac{3d\sqrt{c+dx}\,\cos(5a+5bx)}{800b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)}{8b}-\frac{(c+dx)^{3/2}\sin(3a+3bx)}{48b}-\frac{(c+dx)^{3/2}\sin(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*
x])/(96*b^2) - (3*d*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(800*b^2) - (3*d^(3/2)*
Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]])/(16*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sq
rt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi
/10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt
[d]])/(800*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt
[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*
c)/d])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + ((c + d*x)^(3/2)*Sin
[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(48*b) - ((c + d*x)^(
3/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2}\cos(a+bx) - \frac{1}{16}(c+dx)^{3/2}\cos(3a+3bx) - \right.$$

$$= -\left(\frac{1}{16}\int (c+dx)^{3/2}\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^{3/2}\cos$$

$$= \frac{(c+dx)^{3/2}\sin(a+bx)}{8b} - \frac{(c+dx)^{3/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)}{}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c}}{}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c}}{}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c}}{}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.14, size = 1043, normalized size = 1.95

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*c*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/
Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/
d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (d*(Sqrt[b/d]*Sqr
t[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/d]
 + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2
/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b*S
qrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(32*b^3) - (c*(-(Sqrt[
2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) -
Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d
] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(96*Sqrt[3]*b*Sqrt
[b/d]) - (d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*
x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[b/d]*Sq
rt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b
```

```
*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(Cos[3*(a + b*
x)] + 2*b*x*Sin[3*(a + b*x)]))))/(192*Sqrt[3]*b^3) - (c*(-(Sqrt[2*Pi]*Cos[5*
a - (5*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*
FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d] + 2*Sqrt
[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)]))/(160*Sqrt[5]*b*Sqrt[b/d]) -
(d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-3*
d*Cos[5*a - (5*b*c)/d] + 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi
]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d
] + 3*d*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*b*Sqrt[c + d*x]*(3*Cos[5*(a + b*x
)] + 10*b*x*Sin[5*(a + b*x)]))))/(1600*Sqrt[5]*b^3)
```

Maple [A]
time = 0.00, size = 583, normalized size = 1.09

method	result
derivativedivides	$\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b}{\sqrt{\pi}}\right.\right.}{}\right.}{8b}$
default	$\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b}{\sqrt{\pi}}\right.\right.}{}\right.}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(3/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-3/16/b*d*(-1/2/b
*d*(d*x+c)^(1/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b
/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)
^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)))-1/96/b*d*(d*x+c)^(3/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/32/b*d
*(-1/6/b*d*(d*x+c)^(1/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*
3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))-1/160/b*d*(d*x+c)^(3/2)*s
in(5/d*b*(d*x+c)+5*(a*d-b*c)/d)+3/160/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5/d*
b*(d*x+c)+5*(a*d-b*c)/d)+1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(co
```

```
s(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(
d*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.54, size = 760, normalized size = 1.42

$$-\frac{\sqrt{2}\left(\frac{1800\sqrt{2}(dx+c)^{\frac{3}{2}}b^4\sin\left(\frac{5((dx+c)b-bc+ad)}{d}\right)}{d^2}+\frac{3000\sqrt{2}(dx+c)^{\frac{3}{2}}b^4\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d^2}-\frac{18000\sqrt{2}(dx+c)^{\frac{3}{2}}b^4\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d^2}+\frac{540\sqrt{2}\sqrt{dx+c}\,b^3\cos\left(\frac{5((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{1500\sqrt{2}\sqrt{dx+c}\,b^3\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{27000\sqrt{2}\sqrt{dx+c}\,b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}-27\left(-(i-1)\cdot 25^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{5(bc-ad)}{d}\right)-(i+1)\cdot 25^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{5(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{5ib}{d}}\right)-125\left(-(i-1)\cdot 9^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{3ib}{d}}\right)-6750\left((i-1)\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i+1)\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)-6750\left(-(i+1)\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)-(i-1)\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{ib}{d}}\right)-125\left((i+1)\cdot 9^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)+(i-1)\cdot 9^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{3ib}{d}}\right)-27\left((i+1)\cdot 25^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{5(bc-ad)}{d}\right)+(i-1)\cdot 25^{\frac{1}{4}}\sqrt{\pi}b^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{5(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{5ib}{d}}\right)\right)d^2}{288000\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/288000*sqrt(2)*(1800*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b - b*
c + a*d)/d)/d^2 + 3000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b*c
 + a*d)/d)/d^2 - 18000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*c +
 a*d)/d)/d^2 + 540*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c + a*d
)/d)/d + 1500*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d)/
d - 27000*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d - 27
*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (I
 + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(5*I*b/d)) - 125*(-(I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/
4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin
(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 6750*((I - 1)*sqrt(p
i)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(pi)*b^2*(b^2/d^2)
^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 6750*(-(I + 1)
*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - 125*((
I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)
*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x +
 c)*sqrt(-3*I*b/d)) - 27*((I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos
(-5*(b*c - a*d)/d) + (I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(
b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^5
```

Fricas [A]
time = 2.79, size = 446, normalized size = 0.84

$$\frac{27\sqrt{10}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+125\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-6750\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+6750\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-125\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{3(bc-ad)}{d}\right)-27\sqrt{10}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{5(bc-ad)}{d}\right)-480\left(9bd\cos(bx+a)^5-5bd\cos(bx+a)^3-30bd\cos(bx+a)+10\left(3\left(b^2dx+b^2c\right)\cos(bx+a)^4-2b^2dx-2b^2c-\left(b^2dx+b^2c\right)\cos(bx+a)^2\right)\sin(bx+a)\right)\sqrt{dx+c}}{72000\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_co
s(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d)
)*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) -
 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d))) + 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel
```

```
_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6
)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-3*(b*c - a*d)/d) - 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(1
0)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(9*b*d*cos(b*x
 + a)^5 - 5*b*d*cos(b*x + a)^3 - 30*b*d*cos(b*x + a) + 10*(3*(b^2*d*x + b^2
*c)*cos(b*x + a)^4 - 2*b^2*d*x - 2*b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2
)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x)**3, x)
```

Giac [C] Result contains complex when optimal does not.
time = 1.26, size = 2313, normalized size = 4.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/144000*(300*(30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) - 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*
d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c
+ I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*e
rf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5
*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(2250
*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sq
rt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d
)/d)/b^2)/d^2 - 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(
-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I
```

```
*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(-2*I*(d*x
+ c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*
x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 - 9*(sqrt(10)*sqrt(pi)*(100*b^2*c^2 -
 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^
2) + 1)*b^2) - 10*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d + 3
*sqrt(d*x + c)*d^2)*e^(-5*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 22
50*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(
sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*d - 4*
I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*
a*d)/d)/b^2)/d^2 - 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 + 4*I*b*c*d - d^2)*d*e
rf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*
(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(2*I*(d*x
 + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3*(I*(d*
x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 - 9*(sqrt(10)*sqrt(pi)*(100*b^2*c^2 +
 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b^2) - 10*(10*I*(d*x + c)^(3/2)*b*d - 20*I*sqrt(d*x + c)*b*c*d + 3*s
qrt(d*x + c)*d^2)*e^(-5*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) - 20*(
450*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(10)*sqrt(pi)*(10*b*c -
 I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 45
0*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(10)*sqrt(pi)*(10*b*c +
 I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 900
*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 150*I*sqrt(d*x
 + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 90*I*sqrt(d*x + c)*d*e
^(-5*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 900*I*sqrt(d*x + c)*d*e^((-I*(d
*x + c)*b + I*b*c - I*a*d)/d)/b + 150*I*sqrt(d*x + c)*d*e^(-3*(-I*(d*x + c)
*b + I*b*c - I*a*d)/d)/b + 90*I*sqrt(d*x + c)*d*e^(-5*(-I*(d*x + c)*b + I*b
*c - I*a*d)/d)/b)*c)/d
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2\,{\left(c+d\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

3.195 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=615

$$\frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2}-\frac{d(c+dx)^{3/2}\cos(5a+5bx)}{160b^2}+\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\right.}{}$$

[Out] 5/16*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/288*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b^2-1/160*d*(d*x+c)^(3/2)*cos(5*b*x+5*a)/b^2+1/8*(d*x+c)^(5/2)*sin(b*x+a)/b-1/48*(d*x+c)^(5/2)*sin(3*b*x+3*a)/b-1/80*(d*x+c)^(5/2)*sin(5*b*x+5*a)/b-3/16000*d^(5/2)*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(7/2)-3/16000*d^(5/2)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(7/2)-5/3456*d^(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)-5/3456*d^(5/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^(5/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)-15/32*d^2*sin(b*x+a)*(d*x+c)^(1/2)/b^3+5/576*d^2*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^3+3/1600*d^2*sin(5*b*x+5*a)*(d*x+c)^(1/2)/b^3

Rubi [A]
time = 0.72, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}-\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\cos\left(5a-\frac{5bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{15d^2\sqrt{c+dx}\sin(a+bx)}{32b^3}+\frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{576b^3}+\frac{3d^2\sqrt{c+dx}\sin(5a+5bx)}{1600b^3}+\frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2}-\frac{d(c+dx)^{3/2}\cos(5a+5bx)}{160b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)}{8b}-\frac{(c+dx)^{5/2}\sin(3a+3bx)}{48b}-\frac{(c+dx)^{5/2}\sin(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]

[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(16*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a + 3*b*x])/(288*b^2) - (d*(c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(160*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[(Sqrt

```
[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) - (15
*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(32*b^3) + ((c + d*x)^(5/2)*Sin[a + b*x])/
(8*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5/2)
*Sin[3*a + 3*b*x])/(48*b) + (3*d^2*Sqrt[c + d*x]*Sin[5*a + 5*b*x])/(1600*b^
3) - ((c + d*x)^(5/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2}\cos(a+bx) - \frac{1}{16}(c+dx)^{5/2}\cos(3a+3bx) -\right.$$

$$= -\left(\frac{1}{16}\int (c+dx)^{5/2}\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^{5/2}\cos$$

$$= \frac{(c+dx)^{5/2}\sin(a+bx)}{8b} - \frac{(c+dx)^{5/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 21.97, size = 1795, normalized size = 2.92

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*c^2*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d]
)/Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x)
)/d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (c*d*(Sqrt[b/d]
*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c
)/d] + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sq
rt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2
*b*Sqrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(16*b^3) + ((b/d)^
(3/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^
2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d])) - Sqrt[2*Pi]
```

```
*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (b*c)/d] +
 (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2*b
*(c - 5*d*x)*Cos[a + b*x] + d*(-15 + 4*b^2*x^2)*Sin[a + b*x])))/(64*b^5) -
(c^2*(-(Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[
c + d*x]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*
a - (3*b*c)/d] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(96*S
qrt[3]*b*Sqrt[b/d]) - (c*d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/
Pi]*Sqrt[c + d*x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d])
 + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c
*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]
*(Cos[3*(a + b*x)] + 2*b*x*Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) - ((b/d)^(3
/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2
*c^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d])) - Sqrt
[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3
*b*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]
*d*Sqrt[c + d*x]*(-2*b*(c - 5*d*x)*Cos[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*S
in[3*(a + b*x)])))/(1152*Sqrt[3]*b^5) - (c^2*(-(Sqrt[2*Pi]*Cos[5*a - (5*b*c
)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*FresnelC[S
qrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d] + 2*Sqrt[5]*Sqrt[b
/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (c*d*(Sqrt
[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[5*
a - (5*b*c)/d] + 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*Fresne
lS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*
Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*b*Sqrt[c + d*x]*(3*Cos[5*(a + b*x)] + 10*
b*x*Sin[5*(a + b*x)])))/(800*Sqrt[5]*b^3) - ((b/d)^(3/2)*d^2*(-(Sqrt[2*Pi]*
FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*((20*b^2*c^2 - 3*d^2)*Cos[5*a
 - (5*b*c)/d] + 12*b*c*d*Sin[5*a - (5*b*c)/d])) - Sqrt[2*Pi]*FresnelC[Sqrt[
b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[5*a - (5*b*c)/d] + (20*b^2*c
^2 - 3*d^2)*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2
*b*(c - 5*d*x)*Cos[5*(a + b*x)] + d*(-3 + 20*b^2*x^2)*Sin[5*(a + b*x)])))/(
3200*Sqrt[5]*b^5)
```

Maple [A]
time = 0.00, size = 716, normalized size = 1.16

method	result

derivativedivides	$\frac{\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}}{\cdots}$
default	$\frac{\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(5/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-5/16/b*d*(-1/2/b
*d*(d*x+c)^(3/2)*cos(1/d*b*(d*x+c)+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1
/2)*sin(1/d*b*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(co
s((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin
((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1
/96/b*d*(d*x+c)^(5/2)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+5/96/b*d*(-1/6/b*d*(
d*x+c)^(3/2)*cos(3/d*b*(d*x+c)+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2
)*sin(3/d*b*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(
d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*b*(d*x+c)^(1/2)/d))))-1/160/b*d*(d*x+c)^(5/2)*sin(5/d*b*(d*x+c)+5*(a*
d-b*c)/d)+1/32/b*d*(-1/10/b*d*(d*x+c)^(3/2)*cos(5/d*b*(d*x+c)+5*(a*d-b*c)/d
)+3/10/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5/d*b*(d*x+c)+5*(a*d-b*c)/d)-1/100/b
```

*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2
)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(5*(a*d-b*c)/d)*Fresne
lC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))

Maxima [C] Result contains complex when optimal does not.
time = 0.56, size = 826, normalized size = 1.34

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")

[Out] -1/1728000*sqrt(2)*(5400*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b - b
*c + a*d)/d)/d + 15000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b*c
 + a*d)/d)/d - 270000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c +
a*d)/d)/d - 81*(-(I + 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*
c - a*d)/d) + (I - 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c -
 a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 625*(-(I + 1)*9^(1/4)*sqrt(pi)
*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(pi)*b^2
d(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d))
- 101250*((I + 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I -
 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(I*b/d)) - 101250*(-(I - 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a
*d)/d) + (I + 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(-I*b/d)) - 625*((I - 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(
1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)
sin(-3(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - 81*((I - 1)*25^
(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (I + 1)*25^(1/
4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*
sqrt(-5*I*b/d)) + 540*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 3*sqrt(2)*sqrt(
d*x + c)*b^3)*sin(5*((d*x + c)*b - b*c + a*d)/d) + 1500*(12*sqrt(2)*(d*x +
c)^(5/2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*sin(3*((d*x + c)*b - b*c +
a*d)/d) - 27000*(4*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 15*sqrt(2)*sqrt(d*x +
c)*b^3)*sin(((d*x + c)*b - b*c + a*d)/d))*d^2/b^6

Fricas [A]
time = 2.51, size = 548, normalized size = 0.89

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")

[Out] -1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_
sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 625*sqrt(6)*pi*d^3*sqrt(b/(pi*
d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))

```
 - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*f
resnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 625*
sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*
d)))*sin(-3*(b*c - a*d)/d) + 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(
sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(90*(b^2
*d^2*x + b^2*c*d)*cos(b*x + a)^5 - 50*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3
- 300*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (120*b^3*d^2*x^2 + 240*b^3*c*d*x
 + 120*b^3*c^2 - 9*(20*b^3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*c
os(b*x + a)^4 - 428*b*d^2 + (60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 +
11*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep
```

Giac [C] Result contains complex when optimal does not.
time = 1.69, size = 3705, normalized size = 6.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/864000*(1800*(30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)) - 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 5*sqrt(6)*sqrt(pi)*d*erf(-1/2
*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c
 + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*
erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2
*(2250*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)
/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d +
```

```
4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c -
 I*a*d)/d)/b^2)/d^2 - 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*
d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(-2*I
*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3*(
-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 - 9*(sqrt(10)*sqrt(pi)*(100*b^2
*c^2 - 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b^2) - 10*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^(-5*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^
2 + 2250*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*
d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*
c + I*a*d)/d)/b^2)/d^2 - 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 + 4*I*b*c*d - d^
2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(2*
I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3*
(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 - 9*(sqrt(10)*sqrt(pi)*(100*b^2
*c^2 + 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b^2) - 10*(10*I*(d*x + c)^(3/2)*b*d - 20*I*sqrt(d*x + c)*b*c*d
 + 3*sqrt(d*x + c)*d^2)*e^(-5*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2)
- d^3*(6750*(sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15
*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(-
4*I*(d*x + c)^(5/2)*b^2*d + 12*I*(d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d*x +
c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 15*I*s
qrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 - 125*(sq
rt(6)*sqrt(pi)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-
1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*
b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 6*(-12*I*(d*x
+ c)^(5/2)*b^2*d + 36*I*(d*x + c)^(3/2)*b^2*c*d - 36*I*sqrt(d*x + c)*b^2*c^
2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x +
c)*d^3)*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 - 27*(sqrt(10)*s
qrt(pi)*(200*b^3*c^3 - 60*I*b^2*c^2*d - 18*b*c*d^2 + 3*I*d^3)*d*erf(-1/2*sq
rt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 10*(-20*I*(d*x + c)
^(5/2)*b^2*d + 60*I*(d*x + c)^(3/2)*b^2*c*d - 60*I*sqrt(d*x + c)*b^2*c^2*d
+ 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c)*d
^3)*e^(-5*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 6750*(sqrt(2)*sqrt
(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(4*I*(d*x + c)^(5/2)*b^2
*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x
```

```
+ c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I
*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 - 125*(sqrt(6)*sqrt(pi)*(72*b^3*c
^3 + 36*I*b^2*c^2*d - 18*b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 6*(12*I*(d*x + c)^(5/2)*b^2*d - 36*I*(d
*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*...
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2\,{\left(c+d\,x\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

3.196 $\int (c+dx)^{5/2} \cos^3(a+bx) \sin^3(a+bx) \, dx$

Optimal. Leaf size=407

$$\frac{45d^2\sqrt{c+dx}\,\cos(2a+2bx)}{1024b^3}-\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b}-\frac{5d^2\sqrt{c+dx}\,\cos(6a+6bx)}{9216b^3}+\frac{(c+dx)^{5/2}\cos(6a+}{192b}$$

```
[Out] -3/64*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(5/2)*cos(6*b*x+6*a)/b+1
5/256*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/2304*d*(d*x+c)^(3/2)*sin(6*b*x+6
*a)/b^2+5/55296*d^(5/2)*cos(6*a-6*b*c/d)*FresnelC(2*b^(1/2)*3^(1/2)/Pi^(1/2
)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)-5/55296*d^(5/2)*FresnelS(
2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*3^(1/2)*
Pi^(1/2)/b^(7/2)-45/2048*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c
)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+45/2048*d^(5/2)*FresnelS(2*b^(1/
2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+45/102
4*d^2*cos(2*b*x+2*a)*(d*x+c)^(1/2)/b^3-5/9216*d^2*cos(6*b*x+6*a)*(d*x+c)^(1
/2)/b^3
```

Rubi [A]

time = 0.64, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}-\frac{45\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{2048b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}\sin\left(6a-\frac{6bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}+\frac{45\sqrt{\pi}\,d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2048b^{7/2}}+\frac{45d^2\sqrt{c+dx}\,\cos(2a+2bx)}{1024b^3}-\frac{5d^2\sqrt{c+dx}\,\cos(6a+6bx)}{9216b^3}+\frac{15d(c+dx)^{3/2}\sin(2a+2bx)}{256b^2}-\frac{5d(c+dx)^{3/2}\sin(6a+6bx)}{2304b^2}-\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b}+\frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (45*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(1024*b^3) - (3*(c + d*x)^(5/2)*Cos
[2*a + 2*b*x])/(64*b) - (5*d^2*Sqrt[c + d*x]*Cos[6*a + 6*b*x])/(9216*b^3) +
 ((c + d*x)^(5/2)*Cos[6*a + 6*b*x])/(192*b) + (5*d^(5/2)*Sqrt[Pi/3]*Cos[6*a
 - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1843
2*b^(7/2)) - (45*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*
Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]])/(2048*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/3]*
FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d]
)/(18432*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2048*b^(7/2)) + (15*d*(c + d*x)
^(3/2)*Sin[2*a + 2*b*x])/(256*b^2) - (5*d*(c + d*x)^(3/2)*Sin[6*a + 6*b*x])
/(2304*b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^{5/2}\sin(2a+2bx) - \frac{1}{32}(c+dx)^{5/2}\sin(6a+6bx)\right.$$

$$= -\left(\frac{1}{32}\int (c+dx)^{5/2}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^{5/2}\sin(2$$

$$= -\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b} - \frac{(5d$$

$$= -\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b} + \frac{15d$$

$$= \frac{45d^2\sqrt{c+dx}\,\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d$$

$$= \frac{45d^2\sqrt{c+dx}\,\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d$$

$$= \frac{45d^2\sqrt{c+dx}\,\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d$$

$$= \frac{45d^2\sqrt{c+dx}\,\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d$$

Mathematica [A]
time = 5.18, size = 550, normalized size = 1.35

$$\frac{-2592b^3c^2\sqrt{c+dx}\cos(2(a+bx)) + 2430bd^2\sqrt{c+dx}\cos(2(a+bx)) - 5184b^3cdx\sqrt{c+dx}\cos(2(a+bx)) - 2592b^3d^2x^2\sqrt{c+dx}\cos(2(a+bx)) + 288b^3c^2\sqrt{c+dx}\cos(6(a+bx)) - 30bd^2\sqrt{c+dx}\cos(6(a+bx)) + 576b^3cdx\sqrt{c+dx}\cos(6(a+bx)) + 288b^3d^2x^2\sqrt{c+dx}\cos(6(a+bx)) + 5\sqrt{\frac{b}{d}}d^3\sqrt{3\pi}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right) - 1215\sqrt{\frac{b}{d}}d^3\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - 5\sqrt{\frac{b}{d}}d^3\sqrt{3\pi}\,S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right)\sin\left(6a-\frac{6bc}{d}\right) + 1215\sqrt{\frac{b}{d}}d^3\sqrt{\pi}\,S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right) + 3240b^2cd\sqrt{c+dx}\sin(2(a+bx)) + 3240b^2d^2x\sqrt{c+dx}\sin(2(a+bx)) - 120b^2cd\sqrt{c+dx}\sin(6(a+bx)) - 120b^2d^2x\sqrt{c+dx}\sin(6(a+bx))}{55296b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-2592*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 2430*b*d^2*Sqrt[c + d*x]*Co
s[2*(a + b*x)] - 5184*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 2592*b^3*d
^2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 288*b^3*c^2*Sqrt[c + d*x]*Cos[6*(a
+ b*x)] - 30*b*d^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 576*b^3*c*d*x*Sqrt[c +
d*x]*Cos[6*(a + b*x)] + 288*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 5*
Sqrt[b/d]*d^3*Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[3/P
i]*Sqrt[c + d*x]] - 1215*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresne
lC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 5*Sqrt[b/d]*d^3*Sqrt[3*Pi]*Fresn
elS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*c)/d] + 1215*Sqrt[
b/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a -
```

```
(2*b*c)/d] + 3240*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 3240*b^2*d^2*x*S
qrt[c + d*x]*Sin[2*(a + b*x)] - 120*b^2*c*d*Sqrt[c + d*x]*Sin[6*(a + b*x)]
- 120*b^2*d^2*x*Sqrt[c + d*x]*Sin[6*(a + b*x)])/(55296*b^4)
```

Maple [A]

time = 0.07, size = 477, normalized size = 1.17

method	result
derivativedivides	$-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad}{}\right.}{4b}\right.}{}\right.}{}$
default	$-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad}{}\right.}{4b}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(5/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+15/128/b*d*(
1/4/b*d*(d*x+c)^(3/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d
*x+c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2
*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))+1/384/b
*d*(d*x+c)^(5/2)*cos(6/d*b*(d*x+c)+6*(a*d-b*c)/d)-5/384/b*d*(1/12/b*d*(d*x+
```

c)^(3/2)*sin(6/d*b*(d*x+c)+6*(a*d-b*c)/d)-1/4/b*d*(-1/12/b*d*(d*x+c)^(1/2)*
cos(6/d*b*(d*x+c)+6*(a*d-b*c)/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*b*(d
x+c)^(1/2)/d)-sin(6(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*b*(d*x+c)^(1/2)/d)))))

Maxima [C] Result contains complex when optimal does not.
time = 0.54, size = 561, normalized size = 1.38

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")

[Out] -1/442368*(960*(d*x + c)^(3/2)*b^3*sin(6*((d*x + c)*b - b*c + a*d)/d) - 259
20*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 48*(48*(d*x + c
)^(5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(6*((d*x + c)*b - b*c + a*d)/d) +
1296*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)*
b - b*c + a*d)/d) - 5*(-(I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(
1/4)*cos(-6*(b*c - a*d)/d) - (I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d
^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) - 1215*((
I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d)
+ (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d
)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 1215*(-(I + 1)*4^(1/4)*sqrt(2)*sqr
t(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(2)
*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(-2*I*b/d)) - 5*((I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*
cos(-6*(b*c - a*d)/d) + (I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(
1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*d/b^5

Fricas [A]
time = 3.36, size = 445, normalized size = 1.09

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")

[Out] 1/55296*(5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*f
resnel_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) -
1215*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c
)*sqrt(b/(pi*d))) + 1215*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 96*(24*b^3*d^2*x^2 + 2*(48*b^3*d^2*
x^2 + 96*b^3*c*d*x + 48*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^6 + 48*b^3*c*d*x +
24*b^3*c^2 + 45*b*d^2*cos(b*x + a)^2 - 3*(48*b^3*d^2*x^2 + 96*b^3*c*d*x + 4

```
8*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^4 - 25*b*d^2 - 20*(2*(b^2*d^2*x + b^2*c*d
)*cos(b*x + a)^5 - 2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3 - 3*(b^2*d^2*x +
b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4845 deep
```

Giac [**C**] Result contains complex when optimal does not.
time = 2.52, size = 2445, normalized size = 6.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/110592*(576*(I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)) - I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)) - 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*
d^2) + 1)) + 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))
*c^3 + 36*c*d^2*((I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 - 8*I*b*c*d - d^2)*d*erf(-
sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c
- I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-4*I*(d*x + c
)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-6*(-I*(d*x +
 c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (-I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 + 8*I
*b*c*d - d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2)
- 6*I*(-4*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d
^2)*e^(-6*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 27*(-I*sqrt(pi)*(16
*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(
d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 27*(I*sq
rt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d
```

```
+ 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) +
 d^3*((-I*sqrt(3)*sqrt(pi)*(576*b^3*c^3 - 144*I*b^2*c^2*d - 36*b*c*d^2 + 5*
I*d^3)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*
(-48*I*(d*x + c)^(5/2)*b^2*d + 144*I*(d*x + c)^(3/2)*b^2*c*d - 144*I*sqrt(d
*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x + c)*b*c*d^2 + 5
*I*sqrt(d*x + c)*d^3)*e^(-6*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 +
(I*sqrt(3)*sqrt(pi)*(576*b^3*c^3 + 144*I*b^2*c^2*d - 36*b*c*d^2 - 5*I*d^3)*
d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(
-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(-48*I*(
d*x + c)^(5/2)*b^2*d + 144*I*(d*x + c)^(3/2)*b^2*c*d - 144*I*sqrt(d*x + c)*
b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(
d*x + c)*d^3)*e^(-6*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 + 81*(I*sqr
t(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(b*d
)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d -
48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^
(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-2*(-I*
(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 81*(-I*sqrt(pi)*(64*b^3*c^3 + 48
*I*b^2*c^2*d - 36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^
2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d
*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-2*(I*(d*x + c)*b - I*b*c + I*
a*d)/d)/b^3)/d^3) + 144*(-I*sqrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-sqrt(3)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(3)*sqrt(pi)*(12*b*c +
 I*d)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 27*I*sqr
t(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 2
7*I*sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*
b) + 54*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*sqrt
(d*x + c)*d*e^(-6*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 54*sqrt(d*x + c)*d
*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^(-6*(-I*
(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)^3 \sin(a+bx)^3 (c+dx)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

3.197 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=351

$$-\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b}+\frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b}+\frac{d^{3/2}\sqrt{\frac{\pi}{3}}\cos\left(6a-\frac{6bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-$$

```
[Out] -3/64*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(3/2)*cos(6*b*x+6*a)/b+1
/4608*d^(3/2)*cos(6*a-6*b*c/d)*FresnelS(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)+1/4608*d^(3/2)*FresnelC(2*b^(1/2)*3
^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*3^(1/2)*Pi^(1/2)/b^
(5/2)-9/512*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/
2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-9/512*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+9/256*d*sin(2*b*x+2*a
)*(d*x+c)^(1/2)/b^2-1/768*d*sin(6*b*x+6*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.46, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{3}}d^{3/2}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{512b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}d^{3/2}\cos\left(6a-\frac{6bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{512b^{5/2}}+\frac{9d\sqrt{c+dx}\sin(2a+2bx)}{256b^2}-\frac{d\sqrt{c+dx}\sin(6a+6bx)}{768b^2}-\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b}+\frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*(c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)^(3/2)*Cos[6*a + 6
*b*x])/(192*b) + (d^(3/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])
])/(512*b^(5/2)) + (d^(3/2)*Sqrt[Pi/3]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[
c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[P
i]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)
/d])/(512*b^(5/2)) + (9*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(256*b^2) - (d*Sq
rt[c + d*x]*Sin[6*a + 6*b*x])/(768*b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^{3/2}\sin(2a+2bx) - \frac{1}{32}(c+dx)^{3/2}\sin(6a+6bx)\right.$$

$$= -\left(\frac{1}{32}\int (c+dx)^{3/2}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^{3/2}\sin(2$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} - \frac{d\int}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{}}{}$$

$$d^{3/2}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{}{}$$

Mathematica [**A**]
time = 2.78, size = 391, normalized size = 1.11

$$\frac{-216bc\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(2(a+bx)) - 216b\sqrt{\frac{b}{d}}dx\sqrt{c+dx}\cos(2(a+bx)) + 24bc\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(6(a+bx)) + 24b\sqrt{\frac{b}{d}}dx\sqrt{c+dx}\cos(6(a+bx)) + d\sqrt{3\pi}\cos\left(6a-\frac{6bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right) - 81d\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + d\sqrt{3\pi}\,\text{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right)\sin\left(6a-\frac{6bc}{d}\right) - 81d\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right) + 162\sqrt{\frac{b}{d}}d\sqrt{c+dx}\sin(2(a+bx)) - 6\sqrt{\frac{b}{d}}d\sqrt{c+dx}\sin(6(a+bx))}{4608b^2\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-216*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 216*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] + 24*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[6*(a + b*
x)] + 24*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[6*(a + b*x)] + d*Sqrt[3*Pi]*Cos[
6*a - (6*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]] - 81*d*Sqrt
[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] +
d*Sqrt[3*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*
c)/d] - 81*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*
a - (2*b*c)/d] + 162*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 6*Sqrt[b/
d]*d*Sqrt[c + d*x]*Sin[6*(a + b*x)])/(4608*b^2*Sqrt[b/d])
```

Maple [**A**]
time = 0.07, size = 383, normalized size = 1.09

method	result
derivativedivides	$-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx}}{\sqrt{\pi}\sqrt{}}\right.\right.}{}\right.}{64b}$
default	$-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx}}{\sqrt{\pi}\sqrt{}}\right.\right.}{}\right.}{64b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(3/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+9/128/b*d*(1
/4/b*d*(d*x+c)^(1/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d
)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)
/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
+1/384/b*d*(d*x+c)^(3/2)*cos(6/d*b*(d*x+c)+6*(a*d-b*c)/d)-1/128/b*d*(1/12/b
*d*(d*x+c)^(1/2)*sin(6/d*b*(d*x+c)+6*(a*d-b*c)/d)-1/144/b*d*2^(1/2)*Pi^(1/2
)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)
/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2
)*6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 511, normalized size = 1.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36864*(192*(d*x + c)^(3/2)*b^3*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 172
8*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 48*sqrt(d*x +
c)*b^2*sin(6*((d*x + c)*b - b*c + a*d)/d) + 1296*sqrt(d*x + c)*b^2*sin(2*((
d*x + c)*b - b*c + a*d)/d) + ((I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^
2)^(1/4)*cos(-6*(b*c - a*d)/d) - (I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2
/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) + 81*(-
```

```
(I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d)
+ (I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + 81*((I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*
b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi
)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/
d)) + (-(I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-6*(b*c -
 a*d)/d) + (I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-6*(b*
c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*d/b^4
```

Fricas [**A**]
time = 4.18, size = 326, normalized size = 0.93

$$\frac{\sqrt{3}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{3}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6(bc-ad)}{d}\right)-81\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-81\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+96\left(8\left(b^2dx+b^2c\right)\cos\left(bx+a\right)^6-12\left(b^2dx+b^2c\right)\cos\left(bx+a\right)^4+2\,b^2dx+2\,b^2c-\left(2\,bd\cos\left(bx+a\right)^5-2\,bd\cos\left(bx+a\right)^3-3\,bd\cos\left(bx+a\right)\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{4608\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4608*(sqrt(3)*pi*d^2*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_sin(2*s
qrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(3)*pi*d^2*sqrt(b/(pi*d))*fresne
l_cos(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 81*pi
*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt(
b/(pi*d))) - 81*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-2*(b*c - a*d)/d) + 96*(8*(b^2*d*x + b^2*c)*cos(b*x + a)^6 - 12*
(b^2*d*x + b^2*c)*cos(b*x + a)^4 + 2*b^2*d*x + 2*b^2*c - (2*b*d*cos(b*x + a
)^5 - 2*b*d*cos(b*x + a)^3 - 3*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c
))/b^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**C**] Result contains complex when optimal does not.
time = 1.77, size = 1522, normalized size = 4.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/9216*(48*(I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
```

```
b^2*d^2) + 1)) - I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) - 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)) + 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^
2 + d^2*((I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 - 8*I*b*c*d - d^2)*d*erf(-sqrt(3)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-4*I*(d*x + c)^(3/2)*
b*d + 8*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-6*(-I*(d*x + c)*b +
I*b*c - I*a*d)/d)/b^2)/d^2 + (-I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 + 8*I*b*c*d -
 d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-
4*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^(-
6*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 27*(-I*sqrt(pi)*(16*b^2*c^2
 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2)
 - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)
*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 27*(I*sqrt(pi)*(
16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2)
+ 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt
(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) + 8*(-I*s
qrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b
^2*d^2) + 1)*b) + I*sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(-sqrt(3)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 27*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 27*I*sqrt(pi)*(4*b*c + I*d)*d*
erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 54*sqrt(d*x + c)*d*e^(-2
*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^(-6*(I*(d*x + c
)*b - I*b*c + I*a*d)/d)/b + 54*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*
c - I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^(-6*(-I*(d*x + c)*b + I*b*c - I*a*d)/
d)/b)*c)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

3.198 $\int \sqrt{c+dx} \cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=299

$$-\frac{3\sqrt{c+dx}\,\cos(2a+2bx)}{64b}+\frac{\sqrt{c+dx}\,\cos(6a+6bx)}{192b}-\frac{\sqrt{d}\,\sqrt{\frac{\pi}{3}}\,\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{\frac{3}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}$$

```
[Out] -1/1152*cos(6*a-6*b*c/d)*FresnelC(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/
d^(1/2))*d^(1/2)*3^(1/2)*Pi^(1/2)/b^(3/2)+1/1152*FresnelS(2*b^(1/2)*3^(1/2)
/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*d^(1/2)*3^(1/2)*Pi^(1/2)/
b^(3/2)+3/128*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^
(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)-3/128*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1
/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-3/64*cos(2*b*x+2*a)
*(d*x+c)^(1/2)/b+1/192*cos(6*b*x+6*a)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.33, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}+\frac{3\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\sin\left(6a-\frac{6bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}-\frac{3\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{3/2}}-\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b}+\frac{\sqrt{c+dx}\cos(6a+6bx)}{192b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b) + (Sqrt[c + d*x]*Cos[6*a + 6*b*x
])/(192*b) - (Sqrt[d]*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*S
qrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(384*b^(3/2)) + (3*Sqrt[d]*Sqrt[Pi]*Cos[
2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(1
28*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d
*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(384*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi]*Fre
snelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(
128*b^(3/2))
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\,\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}\sqrt{c+dx}\,\sin(2a+2bx) - \frac{1}{32}\sqrt{c+dx}\,\sin(6a+6bx)\right) dx$$

$$= -\left(\frac{1}{32}\int \sqrt{c+dx}\,\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int \sqrt{c+dx}\,\sin(2a+$$

$$= -\frac{3\sqrt{c+dx}\,\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\,\cos(6a+6bx)}{192b} - \frac{d\int \frac{\cos(6}{\sqrt{c}}}{3}$$

$$= -\frac{3\sqrt{c+dx}\,\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\,\cos(6a+6bx)}{192b} - \frac{(d\cos(}{}$$

$$= -\frac{3\sqrt{c+dx}\,\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\,\cos(6a+6bx)}{192b} - \frac{\cos(6a}{}$$

$$= -\frac{3\sqrt{c+dx}\,\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\,\cos(6a+6bx)}{192b} - \frac{\sqrt{d}\sqrt{\frac{}{}}}{}$$

Mathematica [A]
time = 1.19, size = 264, normalized size = 0.88

$$\frac{-54\sqrt{\frac{b}{d}}\sqrt{c+dx}\,\cos(2(a+bx))+6\sqrt{\frac{b}{d}}\sqrt{c+dx}\,\cos(6(a+bx))-\sqrt{3\pi}\,\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right)+27\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+\sqrt{3\pi}\,S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right)\sin\left(6a-\frac{6bc}{d}\right)-27\sqrt{\pi}\,S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{1152b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-54*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 6*Sqrt[b/d]*Sqrt[c + d*x]*C
os[6*(a + b*x)] - Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[3/Pi]*Sqrt[c + d*x]] + 27*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b
/d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[3*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*S
qrt[c + d*x]]*Sin[6*a - (6*b*c)/d] - 27*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt
[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(1152*b*Sqrt[b/d])
```

Maple [A]
time = 0.07, size = 293, normalized size = 0.98

method	result

derivativedivides	$\frac{-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{3d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\operatorname{S}\left(\frac{2b\sqrt{d}}{\sqrt{\pi}}\right.\right.}{128b\sqrt{\frac{b}{d}}}}{}$
default	$\frac{-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{3d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\operatorname{S}\left(\frac{2b\sqrt{d}}{\sqrt{\pi}}\right.\right.}{128b\sqrt{\frac{b}{d}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+3/256/b*d*Pi
^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)
^(1/2)/d))+1/384/b*d*(d*x+c)^(1/2)*cos(6/d*b*(d*x+c)+6*(a*d-b*c)/d)-1/4608/
b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(6*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.50, size = 439, normalized size = 1.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/9216*(48*sqrt(d*x + c)*b^2*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 432*sqr
t(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - (-(I - 1)*36^(1/4)*sq
rt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (I + 1)*36^(1/4)*s
qrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*
sqrt(6*I*b/d)) - 27*((I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos
(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin
(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 27*(-(I + 1)*4^(1/4)
*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)
*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(-2*I*b/d)) - ((I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*co
s(-6*(b*c - a*d)/d) + (I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*s
in(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*d/b^3
```

Fricas [A]
time = 2.93, size = 242, normalized size = 0.81

$$-\frac{\sqrt{3}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{3}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-\sqrt{3}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{3}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)-27\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-48\,\left(4\,b\cos\left(bx+a\right)^{6}-6\,b\cos\left(bx+a\right)^{4}+b\right)\sqrt{dx+c}}{1152\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/1152*(sqrt(3)*pi*d*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(2*sq
rt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(3)*pi*d*sqrt(b/(pi*d))*fresnel_s
in(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 27*pi*d*
sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi
*d))) + 27*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*
sin(-2*(b*c - a*d)/d) - 48*(4*b*cos(b*x + a)^6 - 6*b*cos(b*x + a)^4 + b)*sq
rt(d*x + c))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^{3}\left(a+bx\right)\cos^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x)**3, x)
```

Giac [C] Result contains complex when optimal does not.
time = 1.18, size = 830, normalized size = 2.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2304*(-I*sqrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(-sqr
t(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 12*(I*sqrt(3)*sqrt(pi)*d
*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(
I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(3)*sqrt(p
i)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 9*I*sqrt(pi)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c -
```

```
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 9*I*sqrt(pi)*d*erf(-sqr
t(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)
/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c + 27*I*sqrt(pi)*(4*b*c - I*d)*d*e
rf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*
a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 27*I*sqrt(pi)*(4*b*c + I
*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b
*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 54*sqrt(d*x + c)*d
*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^(-6*(I*(d
*x + c)*b - I*b*c + I*a*d)/d)/b + 54*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)*b
+ I*b*c - I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^(-6*(-I*(d*x + c)*b + I*b*c - I
*a*d)/d)/b)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^3\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2),x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

3.199 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=299

$$-\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b}+\frac{\sqrt{c+dx}\cos(6a+6bx)}{192b}-\frac{\sqrt{d}\sqrt{\frac{\pi}{3}}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}$$

```
[Out] -1/1152*cos(6*a-6*b*c/d)*FresnelC(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/
d^(1/2))*d^(1/2)*3^(1/2)*Pi^(1/2)/b^(3/2)+1/1152*FresnelS(2*b^(1/2)*3^(1/2)
/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*d^(1/2)*3^(1/2)*Pi^(1/2)/
b^(3/2)+3/128*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^
(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)-3/128*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1
/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-3/64*cos(2*b*x+2*a)
*(d*x+c)^(1/2)/b+1/192*cos(6*b*x+6*a)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.33, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}+\frac{3\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\sin\left(6a-\frac{6bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}-\frac{3\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{3/2}}-\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b}+\frac{\sqrt{c+dx}\cos(6a+6bx)}{192b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b) + (Sqrt[c + d*x]*Cos[6*a + 6*b*x
])/(192*b) - (Sqrt[d]*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*S
qrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(384*b^(3/2)) + (3*Sqrt[d]*Sqrt[Pi]*Cos[
2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(1
28*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d
*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(384*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi]*Fre
snelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(
128*b^(3/2))
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\,\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}\sqrt{c+dx}\,\sin(2a+2bx) - \frac{1}{32}\sqrt{c+dx}\,\sin(6a+6bx)\right) dx$$

$$= -\left(\frac{1}{32}\int \sqrt{c+dx}\,\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int \sqrt{c+dx}\,\sin(2a+$$

$$= -\frac{3\sqrt{c+dx}\,\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\,\cos(6a+6bx)}{192b} - \frac{d\int \frac{\cos(6}{\sqrt{c}}}{3}$$

$$= -\frac{3\sqrt{c+dx}\,\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\,\cos(6a+6bx)}{192b} - \frac{(d\cos(}{}$$

$$= -\frac{3\sqrt{c+dx}\,\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\,\cos(6a+6bx)}{192b} - \frac{\cos(6a}{}$$

$$= -\frac{3\sqrt{c+dx}\,\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\,\cos(6a+6bx)}{192b} - \frac{\sqrt{d}\,\sqrt{\frac{3}{3}}}{}$$

Mathematica [**A**]
time = 0.04, size = 264, normalized size = 0.88

$$\frac{-54\sqrt{\frac{b}{d}}\sqrt{c+dx}\,\cos(2(a+bx)) + 6\sqrt{\frac{b}{d}}\sqrt{c+dx}\,\cos(6(a+bx)) - \sqrt{3\pi}\cos\left(6a-\frac{6bc}{d}\right)\mathrm{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right) + 27\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + \sqrt{3\pi}\,S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right)\sin\left(6a-\frac{6bc}{d}\right) - 27\sqrt{\pi}\,S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{1152b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-54*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 6*Sqrt[b/d]*Sqrt[c + d*x]*C
os[6*(a + b*x)] - Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[3/Pi]*Sqrt[c + d*x]] + 27*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b
/d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[3*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*S
qrt[c + d*x]]*Sin[6*a - (6*b*c)/d] - 27*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt
[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(1152*b*Sqrt[b/d])
```

Maple [**A**]
time = 0.00, size = 293, normalized size = 0.98

method	result

derivativedivides	$\frac{-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{3d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\operatorname{S}\left(\frac{2b\sqrt{d}}{\sqrt{\pi}}\right.\right.}{128b\sqrt{\frac{b}{d}}}}{}$
default	$\frac{-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{3d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\operatorname{S}\left(\frac{2b\sqrt{d}}{\sqrt{\pi}}\right.\right.}{128b\sqrt{\frac{b}{d}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+3/256/b*d*Pi
^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)
^(1/2)/d))+1/384/b*d*(d*x+c)^(1/2)*cos(6/d*b*(d*x+c)+6*(a*d-b*c)/d)-1/4608/
b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(6*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 439, normalized size = 1.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/9216*(48*sqrt(d*x + c)*b^2*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 432*sqr
t(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - (-(I - 1)*36^(1/4)*sq
rt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (I + 1)*36^(1/4)*s
qrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*
sqrt(6*I*b/d)) - 27*((I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos
(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin
(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 27*(-(I + 1)*4^(1/4)
*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)
*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(-2*I*b/d)) - ((I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*co
s(-6*(b*c - a*d)/d) + (I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*s
in(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*d/b^3
```

Fricas [A]
time = 2.96, size = 242, normalized size = 0.81

$$-\frac{\sqrt{3}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{3}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-\sqrt{3}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{3}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)-27\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-48\,\left(4\,b\cos\left(bx+a\right)^{6}-6\,b\cos\left(bx+a\right)^{4}+b\right)\sqrt{dx+c}}{1152\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/1152*(sqrt(3)*pi*d*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(2*sq
rt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(3)*pi*d*sqrt(b/(pi*d))*fresnel_s
in(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 27*pi*d*
sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi
*d))) + 27*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*
sin(-2*(b*c - a*d)/d) - 48*(4*b*cos(b*x + a)^6 - 6*b*cos(b*x + a)^4 + b)*sq
rt(d*x + c))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^{3}\left(a+bx\right)\cos^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x)**3, x)
```

Giac [C] Result contains complex when optimal does not.
time = 1.21, size = 830, normalized size = 2.78

$$-\frac{-\frac{i\sqrt{3}\sqrt{\pi}(12\,bc-i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{3}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{6\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{i\sqrt{3}\sqrt{\pi}(12\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{3}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{6\,(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+12\left(\frac{i\sqrt{3}\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{3}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{6\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}-\frac{i\sqrt{3}\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{3}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{6\,(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}-\frac{9i\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+\frac{9i\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}\right)c+\frac{27i\sqrt{\pi}(4\,bc-i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{27i\sqrt{\pi}(4\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{54\sqrt{dx+c}\,de^{\left(-\frac{2\,(i\,(dx+c)b-i\,bc+i\,ad)}{d}\right)}}{b}-\frac{6\sqrt{dx+c}\,de^{\left(-\frac{6\,(i\,(dx+c)b-i\,bc+i\,ad)}{d}\right)}}{b}+\frac{54\sqrt{dx+c}\,de^{\left(-\frac{2\,(-i\,(dx+c)b+i\,bc-i\,ad)}{d}\right)}}{b}-\frac{6\sqrt{dx+c}\,de^{\left(-\frac{6\,(-i\,(dx+c)b+i\,bc-i\,ad)}{d}\right)}}{b}}{2304\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2304*(-I*sqrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(-sqr
t(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 12*(I*sqrt(3)*sqrt(pi)*d
*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(
I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(3)*sqrt(p
i)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 9*I*sqrt(pi)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c -
```

```
 I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 9*I*sqrt(pi)*d*erf(-sqr
t(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)
/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c + 27*I*sqrt(pi)*(4*b*c - I*d)*d*e
rf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*
a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 27*I*sqrt(pi)*(4*b*c + I
*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b
*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 54*sqrt(d*x + c)*d
*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^(-6*(I*(d
*x + c)*b - I*b*c + I*a*d)/d)/b + 54*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)*b
+ I*b*c - I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^(-6*(-I*(d*x + c)*b + I*b*c - I
*a*d)/d)/b)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)^3 \sin(a+bx)^3 \sqrt{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2),x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

3.200 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=351

$$-\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b}+\frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b}+\frac{d^{3/2}\sqrt{\frac{\pi}{3}}\,\cos\left(6a-\frac{6bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-$$

```
[Out] -3/64*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(3/2)*cos(6*b*x+6*a)/b+1
/4608*d^(3/2)*cos(6*a-6*b*c/d)*FresnelS(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)+1/4608*d^(3/2)*FresnelC(2*b^(1/2)*3
^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*3^(1/2)*Pi^(1/2)/b^
(5/2)-9/512*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/
2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-9/512*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+9/256*d*sin(2*b*x+2*a
)*(d*x+c)^(1/2)/b^2-1/768*d*sin(6*b*x+6*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.40, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$,
Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{3}}\,d^{3/2}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{512b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}\,d^{3/2}\cos\left(6a-\frac{6bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}\,d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{512b^{5/2}}+\frac{9d\sqrt{c+dx}\sin(2a+2bx)}{256b^2}-\frac{d\sqrt{c+dx}\sin(6a+6bx)}{768b^2}-\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b}+\frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*(c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)^(3/2)*Cos[6*a + 6
*b*x])/(192*b) + (d^(3/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])
])/(512*b^(5/2)) + (d^(3/2)*Sqrt[Pi/3]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[
c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[P
i]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)
/d])/(512*b^(5/2)) + (9*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(256*b^2) - (d*Sq
rt[c + d*x]*Sin[6*a + 6*b*x])/(768*b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^{3/2}\sin(2a+2bx) - \frac{1}{32}(c+dx)^{3/2}\sin(6a+6bx)\right.$$

$$= -\left(\frac{1}{32}\int (c+dx)^{3/2}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^{3/2}\sin(2$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} - \frac{d\int}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{}}{}$$

$$d^{3/2}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{}{}$$

Mathematica [A]
time = 0.09, size = 391, normalized size = 1.11

$$\frac{-216bc\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(2(a+bx)) - 216b\sqrt{\frac{b}{d}}dx\sqrt{c+dx}\cos(2(a+bx)) + 24bc\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(6(a+bx)) + 24b\sqrt{\frac{b}{d}}dx\sqrt{c+dx}\cos(6(a+bx)) + d\sqrt{3\pi}\cos\left(6a-\frac{6bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right) - 81d\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + d\sqrt{3\pi}\,\text{FresnelC}\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right)\sin\left(6a-\frac{6bc}{d}\right) - 81d\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right) + 162\sqrt{\frac{b}{d}}d\sqrt{c+dx}\sin(2(a+bx)) - 6\sqrt{\frac{b}{d}}d\sqrt{c+dx}\sin(6(a+bx))}{4608b^2\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-216*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 216*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] + 24*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[6*(a + b*
x)] + 24*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[6*(a + b*x)] + d*Sqrt[3*Pi]*Cos[
6*a - (6*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]] - 81*d*Sqrt
[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] +
d*Sqrt[3*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*
c)/d] - 81*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*
a - (2*b*c)/d] + 162*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 6*Sqrt[b/
d]*d*Sqrt[c + d*x]*Sin[6*(a + b*x)])/(4608*b^2*Sqrt[b/d])
```

Maple [A]
time = 0.00, size = 383, normalized size = 1.09

method	result
derivativedivides	$-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx}}{\sqrt{\pi}\,\sqrt{}}\right.\right.}{}\right.}{64b}$
default	$-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx}}{\sqrt{\pi}\,\sqrt{}}\right.\right.}{}\right.}{64b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(3/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+9/128/b*d*(1
/4/b*d*(d*x+c)^(1/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d
)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)
/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
+1/384/b*d*(d*x+c)^(3/2)*cos(6/d*b*(d*x+c)+6*(a*d-b*c)/d)-1/128/b*d*(1/12/b
*d*(d*x+c)^(1/2)*sin(6/d*b*(d*x+c)+6*(a*d-b*c)/d)-1/144/b*d*2^(1/2)*Pi^(1/2
)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)
/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2
)*6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 511, normalized size = 1.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36864*(192*(d*x + c)^(3/2)*b^3*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 172
8*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 48*sqrt(d*x +
c)*b^2*sin(6*((d*x + c)*b - b*c + a*d)/d) + 1296*sqrt(d*x + c)*b^2*sin(2*((
d*x + c)*b - b*c + a*d)/d) + ((I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^
2)^(1/4)*cos(-6*(b*c - a*d)/d) - (I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2
/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) + 81*(-
```

```
(I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d)
+ (I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + 81*((I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*
b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi
)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/
d)) + (-(I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-6*(b*c -
 a*d)/d) + (I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-6*(b*
c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*d/b^4
```

Fricas [**A**]
time = 3.08, size = 326, normalized size = 0.93

$$\frac{\sqrt{3}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{3}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+\sqrt{3}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{3}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6(bc-ad)}{d}\right)-81\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-81\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+96\left(8\left(b^2dx+b^2c\right)\cos\left(bx+a\right)^6-12\left(b^2dx+b^2c\right)\cos\left(bx+a\right)^4+2\,b^2dx+2\,b^2c-\left(2\,bd\cos\left(bx+a\right)^5-2\,bd\cos\left(bx+a\right)^3-3\,bd\cos\left(bx+a\right)\right)\sin\left(bx+a\right)\right)\sqrt{dx+c}}{4608\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4608*(sqrt(3)*pi*d^2*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_sin(2*s
qrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(3)*pi*d^2*sqrt(b/(pi*d))*fresne
l_cos(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 81*pi
*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt(
b/(pi*d))) - 81*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-2*(b*c - a*d)/d) + 96*(8*(b^2*d*x + b^2*c)*cos(b*x + a)^6 - 12*
(b^2*d*x + b^2*c)*cos(b*x + a)^4 + 2*b^2*d*x + 2*b^2*c - (2*b*d*cos(b*x + a
)^5 - 2*b*d*cos(b*x + a)^3 - 3*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c
))/b^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**C**] Result contains complex when optimal does not.
time = 1.78, size = 1522, normalized size = 4.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/9216*(48*(I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
```

```
b^2*d^2) + 1)) - I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) - 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)) + 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c^
2 + d^2*((I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 - 8*I*b*c*d - d^2)*d*erf(-sqrt(3)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-4*I*(d*x + c)^(3/2)*
b*d + 8*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-6*(-I*(d*x + c)*b +
I*b*c - I*a*d)/d)/b^2)/d^2 + (-I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 + 8*I*b*c*d -
 d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-
4*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^(-
6*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 27*(-I*sqrt(pi)*(16*b^2*c^2
 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2)
 - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)
*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 27*(I*sqrt(pi)*(
16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2)
+ 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt
(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) + 8*(-I*s
qrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b
^2*d^2) + 1)*b) + I*sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(-sqrt(3)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 27*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 27*I*sqrt(pi)*(4*b*c + I*d)*d*
erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 54*sqrt(d*x + c)*d*e^(-2
*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^(-6*(I*(d*x + c
)*b - I*b*c + I*a*d)/d)/b + 54*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*
c - I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^(-6*(-I*(d*x + c)*b + I*b*c - I*a*d)/
d)/b)*c)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\sin(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

3.201 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=407

$$\frac{45d^2\sqrt{c+dx}\,\cos(2a+2bx)}{1024b^3}-\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b}-\frac{5d^2\sqrt{c+dx}\,\cos(6a+6bx)}{9216b^3}+\frac{(c+dx)^{5/2}\cos(6a+}{192b}$$

```
[Out] -3/64*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(5/2)*cos(6*b*x+6*a)/b+1
5/256*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/2304*d*(d*x+c)^(3/2)*sin(6*b*x+6
*a)/b^2+5/55296*d^(5/2)*cos(6*a-6*b*c/d)*FresnelC(2*b^(1/2)*3^(1/2)/Pi^(1/2
)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)-5/55296*d^(5/2)*FresnelS(
2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*3^(1/2)*
Pi^(1/2)/b^(7/2)-45/2048*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c
)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+45/2048*d^(5/2)*FresnelS(2*b^(1/
2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+45/102
4*d^2*cos(2*b*x+2*a)*(d*x+c)^(1/2)/b^3-5/9216*d^2*cos(6*b*x+6*a)*(d*x+c)^(1
/2)/b^3
```

Rubi [A]

time = 0.47, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4491, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}-\frac{45\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{2048b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}\sin\left(6a-\frac{6bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}+\frac{45\sqrt{\pi}\,d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2048b^{7/2}}+\frac{45d^2\sqrt{c+dx}\,\cos(2a+2bx)}{1024b^3}-\frac{5d^2\sqrt{c+dx}\,\cos(6a+6bx)}{9216b^3}+\frac{15d(c+dx)^{3/2}\sin(2a+2bx)}{256b^2}-\frac{5d(c+dx)^{3/2}\sin(6a+6bx)}{2304b^2}-\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b}+\frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (45*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(1024*b^3) - (3*(c + d*x)^(5/2)*Cos
[2*a + 2*b*x])/(64*b) - (5*d^2*Sqrt[c + d*x]*Cos[6*a + 6*b*x])/(9216*b^3) +
 ((c + d*x)^(5/2)*Cos[6*a + 6*b*x])/(192*b) + (5*d^(5/2)*Sqrt[Pi/3]*Cos[6*a
 - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1843
2*b^(7/2)) - (45*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*
Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2048*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/3]*
FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d]
)/(18432*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2048*b^(7/2)) + (15*d*(c + d*x)
^(3/2)*Sin[2*a + 2*b*x])/(256*b^2) - (5*d*(c + d*x)^(3/2)*Sin[6*a + 6*b*x])
/(2304*b^2)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^{5/2}\sin(2a+2bx) - \frac{1}{32}(c+dx)^{5/2}\sin(6a+6bx)\right.$$

$$= -\left(\frac{1}{32}\int (c+dx)^{5/2}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^{5/2}\sin(2$$

$$= -\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b} - \frac{(5d}{}$$

$$= -\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b} + \frac{15d}{}$$

$$= \frac{45d^2\sqrt{c+dx}\,\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d}{}$$

$$= \frac{45d^2\sqrt{c+dx}\,\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d}{}$$

$$= \frac{45d^2\sqrt{c+dx}\,\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d}{}$$

$$= \frac{45d^2\sqrt{c+dx}\,\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d}{}$$

Mathematica [A]
time = 2.61, size = 550, normalized size = 1.35

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-2592*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 2430*b*d^2*Sqrt[c + d*x]*Co
s[2*(a + b*x)] - 5184*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 2592*b^3*d
^2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 288*b^3*c^2*Sqrt[c + d*x]*Cos[6*(a
+ b*x)] - 30*b*d^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 576*b^3*c*d*x*Sqrt[c +
d*x]*Cos[6*(a + b*x)] + 288*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 5*
Sqrt[b/d]*d^3*Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[3/P
i]*Sqrt[c + d*x]] - 1215*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresne
lC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 5*Sqrt[b/d]*d^3*Sqrt[3*Pi]*Fresn
elS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*c)/d] + 1215*Sqrt[
b/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a -
```

```
(2*b*c)/d] + 3240*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 3240*b^2*d^2*x*S
qrt[c + d*x]*Sin[2*(a + b*x)] - 120*b^2*c*d*Sqrt[c + d*x]*Sin[6*(a + b*x)]
- 120*b^2*d^2*x*Sqrt[c + d*x]*Sin[6*(a + b*x)])/(55296*b^4)
```

Maple [A]
time = 0.00, size = 477, normalized size = 1.17

method	result
derivativedivides	$-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad}{}\right.}{4b}\right.}{}\right.}{}$
default	$-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad}{}\right.}{4b}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(5/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+15/128/b*d*(
1/4/b*d*(d*x+c)^(3/2)*sin(2/d*b*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d
*x+c)^(1/2)*cos(2/d*b*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2
*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/384/b
*d*(d*x+c)^(5/2)*cos(6/d*b*(d*x+c)+6*(a*d-b*c)/d)-5/384/b*d*(1/12/b*d*(d*x+
```

```
c)^(3/2)*sin(6/d*b*(d*x+c)+6*(a*d-b*c)/d)-1/4/b*d*(-1/12/b*d*(d*x+c)^(1/2)*
cos(6/d*b*(d*x+c)+6*(a*d-b*c)/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)-sin(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*b*(d*x+c)^(1/2)/d)))))
```

Maxima [**C**] Result contains complex when optimal does not.
time = 0.53, size = 561, normalized size = 1.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/442368*(960*(d*x + c)^(3/2)*b^3*sin(6*((d*x + c)*b - b*c + a*d)/d) - 259
20*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 48*(48*(d*x + c
)^(5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(6*((d*x + c)*b - b*c + a*d)/d) +
 1296*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)*
b - b*c + a*d)/d) - 5*(-(I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(
1/4)*cos(-6*(b*c - a*d)/d) - (I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d
^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) - 1215*((
I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d)
 + (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d
)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 1215*(-(I + 1)*4^(1/4)*sqrt(2)*sqr
t(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(2)
*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(-2*I*b/d)) - 5*((I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*
cos(-6*(b*c - a*d)/d) + (I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(
1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*d/b^5
```

Fricas [**A**]
time = 4.25, size = 445, normalized size = 1.09

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/55296*(5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*f
resnel_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) -
1215*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c
)*sqrt(b/(pi*d))) + 1215*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 96*(24*b^3*d^2*x^2 + 2*(48*b^3*d^2*
x^2 + 96*b^3*c*d*x + 48*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^6 + 48*b^3*c*d*x +
24*b^3*c^2 + 45*b*d^2*cos(b*x + a)^2 - 3*(48*b^3*d^2*x^2 + 96*b^3*c*d*x + 4
```

```
8*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^4 - 25*b*d^2 - 20*(2*(b^2*d^2*x + b^2*c*d
)*cos(b*x + a)^5 - 2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3 - 3*(b^2*d^2*x +
b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4845 deep
```

Giac [**C**] Result contains complex when optimal does not.
time = 2.54, size = 2445, normalized size = 6.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
[Out] -1/110592*(576*(I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)) - I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)) - 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*
d^2) + 1)) + 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))
*c^3 + 36*c*d^2*((I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 - 8*I*b*c*d - d^2)*d*erf(-
sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c
- I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-4*I*(d*x + c
)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-6*(-I*(d*x +
 c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (-I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 + 8*I
*b*c*d - d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2)
- 6*I*(-4*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d
^2)*e^(-6*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 27*(-I*sqrt(pi)*(16
*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(
d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 27*(I*sq
rt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d
```

```
+ 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) +
 d^3*((-I*sqrt(3)*sqrt(pi)*(576*b^3*c^3 - 144*I*b^2*c^2*d - 36*b*c*d^2 + 5*
I*d^3)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*
(-48*I*(d*x + c)^(5/2)*b^2*d + 144*I*(d*x + c)^(3/2)*b^2*c*d - 144*I*sqrt(d
*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x + c)*b*c*d^2 + 5
*I*sqrt(d*x + c)*d^3)*e^(-6*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 +
(I*sqrt(3)*sqrt(pi)*(576*b^3*c^3 + 144*I*b^2*c^2*d - 36*b*c*d^2 - 5*I*d^3)*
d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(
-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(-48*I*(
d*x + c)^(5/2)*b^2*d + 144*I*(d*x + c)^(3/2)*b^2*c*d - 144*I*sqrt(d*x + c)*
b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(
d*x + c)*d^3)*e^(-6*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 + 81*(I*sqr
t(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(b*d
)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d -
48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^
(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-2*(-I*
(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 81*(-I*sqrt(pi)*(64*b^3*c^3 + 48
*I*b^2*c^2*d - 36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^
2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d
*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-2*(I*(d*x + c)*b - I*b*c + I*
a*d)/d)/b^3)/d^3) + 144*(-I*sqrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-sqrt(3)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(3)*sqrt(pi)*(12*b*c +
 I*d)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 27*I*sqr
t(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 2
7*I*sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*
b) + 54*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*sqrt
(d*x + c)*d*e^(-6*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 54*sqrt(d*x + c)*d
*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^(-6*(-I*
(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c^2)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\sin(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

3.202 $\int x^3 \cos^2(x) \cot^2(x)\, dx$

Optimal. Leaf size=112

$$\frac{3x^2}{8}-ix^3-\frac{3x^4}{8}+\frac{3\cos^2(x)}{8}-\frac{3}{4}x^2\cos^2(x)-x^3\cot(x)+3x^2\log\left(1-e^{2ix}\right)-3ix\text{PolyLog}\left(2,e^{2ix}\right)+\frac{3}{2}\text{PolyLog}\left(3,e^{2i\right.}$$

```
[Out] 3/8*x^2-I*x^3-3/8*x^4+3/8*cos(x)^2-3/4*x^2*cos(x)^2-x^3*cot(x)+3*x^2*ln(1-e
xp(2*I*x))-3*I*x*polylog(2,exp(2*I*x))+3/2*polylog(3,exp(2*I*x))+3/4*x*cos(
x)*sin(x)-1/2*x^3*cos(x)*sin(x)
```

Rubi [A]
time = 0.12, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {4493, 3392, 30, 3391, 3801, 3798, 2221, 2611, 2320, 6724}

$$-3ix\text{Li}_2\left(e^{2ix}\right)+\frac{3}{2}\text{Li}_3\left(e^{2ix}\right)-\frac{3x^4}{8}-ix^3-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)+\frac{3x^2}{8}+3x^2\log\left(1-e^{2ix}\right)-\frac{3}{4}x^2\cos^2(x)+\frac{3\cos^2(x)}{8}+\frac{3}{4}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] (3*x^2)/8 - I*x^3 - (3*x^4)/8 + (3*Cos[x]^2)/8 - (3*x^2*Cos[x]^2)/4 - x^3*C
ot[x] + 3*x^2*Log[1 - E^((2*I)*x)] - (3*I)*x*PolyLog[2, E^((2*I)*x)] + (3*P
olyLog[3, E^((2*I)*x)])/2 + (3*x*Cos[x]*Sin[x])/4 - (x^3*Cos[x]*Sin[x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \cos^2(x) \cot^2(x) \, dx &= -\int x^3 \cos^2(x) \, dx + \int x^3 \cot^2(x) \, dx \\
&= -\frac{3}{4} x^2 \cos^2(x) - x^3 \cot(x) - \frac{1}{2} x^3 \cos(x) \sin(x) - \frac{\int x^3 \, dx}{2} + \frac{3}{2} \int x \cos^2(x) \, dx + 3 \int \\
&= -ix^3 - \frac{3x^4}{8} + \frac{3\cos^2(x)}{8} - \frac{3}{4} x^2 \cos^2(x) - x^3 \cot(x) + \frac{3}{4} x \cos(x) \sin(x) - \frac{1}{2} x^3 \cos(x \\
&= \frac{3x^2}{8} - ix^3 - \frac{3x^4}{8} + \frac{3\cos^2(x)}{8} - \frac{3}{4} x^2 \cos^2(x) - x^3 \cot(x) + 3x^2 \log\left(1 - e^{2ix}\right) + \frac{3}{4} x \\
&= \frac{3x^2}{8} - ix^3 - \frac{3x^4}{8} + \frac{3\cos^2(x)}{8} - \frac{3}{4} x^2 \cos^2(x) - x^3 \cot(x) + 3x^2 \log\left(1 - e^{2ix}\right) - 3ix \\
&= \frac{3x^2}{8} - ix^3 - \frac{3x^4}{8} + \frac{3\cos^2(x)}{8} - \frac{3}{4} x^2 \cos^2(x) - x^3 \cot(x) + 3x^2 \log\left(1 - e^{2ix}\right) - 3ix \\
&= \frac{3x^2}{8} - ix^3 - \frac{3x^4}{8} + \frac{3\cos^2(x)}{8} - \frac{3}{4} x^2 \cos^2(x) - x^3 \cot(x) + 3x^2 \log\left(1 - e^{2ix}\right) - 3ix
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 104, normalized size = 0.93

$$\frac{1}{16}\left(-2i\pi^3 + 16ix^3 - 6x^4 + 3\cos(2x) - 6x^2\cos(2x) - 16x^3\cot(x) + 48x^2\log\left(1 - e^{-2ix}\right) + 48ix\text{PolyLog}\left(2, e^{-2ix}\right) + 24\text{PolyLog}\left(3, e^{-2ix}\right) + 6x\sin(2x) - 4x^3\sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] ((-2*I)*Pi^3 + (16*I)*x^3 - 6*x^4 + 3*Cos[2*x] - 6*x^2*Cos[2*x] - 16*x^3*Co
t[x] + 48*x^2*Log[1 - E^((-2*I)*x)] + (48*I)*x*PolyLog[2, E^((-2*I)*x)] + 2
4*PolyLog[3, E^((-2*I)*x)] + 6*x*Sin[2*x] - 4*x^3*Sin[2*x])/16
```

Maple [A]
time = 0.12, size = 150, normalized size = 1.34

method	result
risch	$-\frac{3x^4}{8} + \frac{i(4x^3+6ix^2-6x-3i)\mathrm{e}^{2ix}}{32} - \frac{i(4x^3-6ix^2-6x+3i)\mathrm{e}^{-2ix}}{32} - \frac{2ix^3}{\mathrm{e}^{2ix}-1} - 2ix^3 + 3x^2\ln\left(\mathrm{e}^{ix}+1\right) - 6ix\,\mathrm{polylog}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x)^2*cot(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*x^4+1/32*I*(6*I*x^2+4*x^3-3*I-6*x)*exp(2*I*x)-1/32*I*(-6*I*x^2+4*x^3+3
*I-6*x)*exp(-2*I*x)-2*I*x^3/(exp(2*I*x)-1)-2*I*x^3+3*x^2*ln(exp(I*x)+1)-6*I
*x*polylog(2,-exp(I*x))+6*polylog(3,-exp(I*x))+3*x^2*ln(1-exp(I*x))-6*I*x*p
olylog(2,exp(I*x))+6*polylog(3,exp(I*x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(84) = 168$.
time = 3.38, size = 244, normalized size = 2.18

$$\frac{4\,(2\,x^3-3\,x)\cos(x)^3+24\,x^2\log(\cos(x)+i\,\sin(x)+1)\sin(x)+24\,x^2\log(\cos(x)-i\,\sin(x)+1)\sin(x)+24\,x^2\log(-\cos(x)+i\,\sin(x)+1)\sin(x)+24\,x^2\log(-\cos(x)-i\,\sin(x)+1)\sin(x)-48i\,x\mathrm{Li}_2(\cos(x)+i\,\sin(x))\sin(x)+48i\,x\mathrm{Li}_2(\cos(x)-i\,\sin(x))\sin(x)+48i\,x\mathrm{Li}_2(-\cos(x)+i\,\sin(x))\sin(x)-48i\,x\mathrm{Li}_2(-\cos(x)-i\,\sin(x))\sin(x)-12\,(2\,x^3-x)\cos(x)-3\,(2\,x^4+2\,(2\,x^2-1)\cos(x)^2-2\,x^2+1)\sin(x)+48\,\mathrm{polylog}(3,\cos(x)+i\,\sin(x))\sin(x)+48\,\mathrm{polylog}(3,\cos(x)-i\,\sin(x))\sin(x)+48\,\mathrm{polylog}(3,-\cos(x)+i\,\sin(x))\sin(x)+48\,\mathrm{polylog}(3,-\cos(x)-i\,\sin(x))\sin(x)}{16\,\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(2*x^3 - 3*x)*cos(x)^3 + 24*x^2*log(cos(x) + I*sin(x) + 1)*sin(x) +
 24*x^2*log(cos(x) - I*sin(x) + 1)*sin(x) + 24*x^2*log(-cos(x) + I*sin(x) +
 1)*sin(x) + 24*x^2*log(-cos(x) - I*sin(x) + 1)*sin(x) - 48*I*x*dilog(cos(x
) + I*sin(x))*sin(x) + 48*I*x*dilog(cos(x) - I*sin(x))*sin(x) + 48*I*x*dilo
g(-cos(x) + I*sin(x))*sin(x) - 48*I*x*dilog(-cos(x) - I*sin(x))*sin(x) - 12
*(2*x^3 - x)*cos(x) - 3*(2*x^4 + 2*(2*x^2 - 1)*cos(x)^2 - 2*x^2 + 1)*sin(x)
 + 48*polylog(3, cos(x) + I*sin(x))*sin(x) + 48*polylog(3, cos(x) - I*sin(x
))*sin(x) + 48*polylog(3, -cos(x) + I*sin(x))*sin(x) + 48*polylog(3, -cos(x
) - I*sin(x))*sin(x))/sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\cos^2(x)\cot^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(x)**2*cot(x)**2,x)
```

```
[Out] Integral(x**3*cos(x)**2*cot(x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*cos(x)^2*cot(x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \cos(x)^2 \cot(x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x)^2*cot(x)^2,x)
```

```
[Out] int(x^3*cos(x)^2*cot(x)^2, x)
```

3.203 $\int x^2 \cos^2(x) \cot^2(x)\,dx$

Optimal. Leaf size=83

$$\frac{x}{4}-ix^2-\frac{x^3}{2}-\frac{1}{2}x\cos^2(x)-x^2\cot(x)+2x\log\left(1-e^{2ix}\right)-i\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}x^2\cos(x)\sin(x)$$

```
[Out] 1/4*x-I*x^2-1/2*x^3-1/2*x*cos(x)^2-x^2*cot(x)+2*x*ln(1-exp(2*I*x))-I*polylo
g(2,exp(2*I*x))+1/4*cos(x)*sin(x)-1/2*x^2*cos(x)*sin(x)
```

Rubi [A]
time = 0.11, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {4493, 3392, 30, 2715, 8, 3801, 3798, 2221, 2317, 2438}

$$-i\text{Li}_2\left(e^{2ix}\right)-\frac{x^3}{2}-ix^2-x^2\cot(x)-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{x}{4}+2x\log\left(1-e^{2ix}\right)-\frac{1}{2}x\cos^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] x/4 - I*x^2 - x^3/2 - (x*Cos[x]^2)/2 - x^2*Cot[x] + 2*x*Log[1 - E^((2*I)*x)
] - I*PolyLog[2, E^((2*I)*x)] + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2\cos^2(x)\cot^2(x)\,dx = -\int x^2\cos^2(x)\,dx + \int x^2\cot^2(x)\,dx$$
$$= -\frac{1}{2}x\cos^2(x) - x^2\cot(x) - \frac{1}{2}x^2\cos(x)\sin(x) - \frac{\int x^2\,dx}{2} + \frac{1}{2}\int\cos^2(x)\,dx + 2\int$$
$$= -ix^2 - \frac{x^3}{2} - \frac{1}{2}x\cos^2(x) - x^2\cot(x) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2\cos(x)\sin(x) - 4i\int$$
$$= \frac{x}{4} - ix^2 - \frac{x^3}{2} - \frac{1}{2}x\cos^2(x) - x^2\cot(x) + 2x\log\left(1-e^{2ix}\right) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}$$
$$= \frac{x}{4} - ix^2 - \frac{x^3}{2} - \frac{1}{2}x\cos^2(x) - x^2\cot(x) + 2x\log\left(1-e^{2ix}\right) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}$$
$$= \frac{x}{4} - ix^2 - \frac{x^3}{2} - \frac{1}{2}x\cos^2(x) - x^2\cot(x) + 2x\log\left(1-e^{2ix}\right) - i\text{Li}_2\left(e^{2ix}\right) + \frac{1}{4}\cos(x$$

Mathematica [A]
time = 0.12, size = 72, normalized size = 0.87

$$\frac{1}{8}\left(-8ix^2 - 4x^3 - 2x\cos(2x) - 8x^2\cot(x) + 16x\log\left(1-e^{2ix}\right) - 8i\text{PolyLog}\left(2,e^{2ix}\right) + \sin(2x) - 2x^2\sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] ((-8*I)*x^2 - 4*x^3 - 2*x*Cos[2*x] - 8*x^2*Cot[x] + 16*x*Log[1 - E^((2*I)*x
)] - (8*I)*PolyLog[2, E^((2*I)*x)] + Sin[2*x] - 2*x^2*Sin[2*x])/8
```

Maple [A]
time = 0.11, size = 112, normalized size = 1.35

method	result
risch	$-\frac{x^3}{2} + \frac{i(2x^2+2ix-1)e^{2ix}}{16} - \frac{i(2x^2-2ix-1)e^{-2ix}}{16} - \frac{2ix^2}{e^{2ix}-1} + 2x\ln\left(e^{ix}+1\right) + 2x\ln\left(1-e^{ix}\right) - 2ix^2 - 2i\,\text{p}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^2*cot(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^3+1/16*I*(2*I*x+2*x^2-1)*exp(2*I*x)-1/16*I*(-2*I*x+2*x^2-1)*exp(-2*I
*x)-2*I*x^2/(exp(2*I*x)-1)+2*x*ln(exp(I*x)+1)+2*x*ln(1-exp(I*x))-2*I*x^2-2*
I*polylog(2,-exp(I*x))-2*I*polylog(2,exp(I*x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(62) = 124.
time = 3.15, size = 162, normalized size = 1.95

$$\frac{(2x^2-1)\cos(x)^3+4x\log(\cos(x)+i\sin(x)+1)\sin(x)+4x\log(\cos(x)-i\sin(x)+1)\sin(x)+4x\log(-\cos(x)+i\sin(x)+1)\sin(x)+4x\log(-\cos(x)-i\sin(x)+1)\sin(x)-(6x^2-1)\cos(x)-(2x^3+2x\cos(x)^2-x)\sin(x)-4i\,\mathrm{Li}_2(\cos(x)+i\sin(x))\sin(x)+4i\,\mathrm{Li}_2(\cos(x)-i\sin(x))\sin(x)+4i\,\mathrm{Li}_2(-\cos(x)+i\sin(x))\sin(x)-4i\,\mathrm{Li}_2(-\cos(x)-i\sin(x))\sin(x)}{4\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*x^2 - 1)*cos(x)^3 + 4*x*log(cos(x) + I*sin(x) + 1)*sin(x) + 4*x*log
(cos(x) - I*sin(x) + 1)*sin(x) + 4*x*log(-cos(x) + I*sin(x) + 1)*sin(x) + 4
*x*log(-cos(x) - I*sin(x) + 1)*sin(x) - (6*x^2 - 1)*cos(x) - (2*x^3 + 2*x*c
os(x)^2 - x)*sin(x) - 4*I*dilog(cos(x) + I*sin(x))*sin(x) + 4*I*dilog(cos(x
) - I*sin(x))*sin(x) + 4*I*dilog(-cos(x) + I*sin(x))*sin(x) - 4*I*dilog(-co
s(x) - I*sin(x))*sin(x))/sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cos^2(x)\cot^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(x)**2*cot(x)**2,x)
```

```
[Out] Integral(x**2*cos(x)**2*cot(x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*cos(x)^2*cot(x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\cos(x)^2\cot(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^2*cot(x)^2,x)
```

```
[Out] int(x^2*cos(x)^2*cot(x)^2, x)
```

3.204 $\int x \cos^2(x) \cot^2(x) \, dx$

Optimal. Leaf size=33

$$-\frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x\cot(x) + \log(\sin(x)) - \frac{1}{2}x\cos(x)\sin(x)$$

[Out] -3/4*x^2-1/4*cos(x)^2-x*cot(x)+ln(sin(x))-1/2*x*cos(x)*sin(x)

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4493, 3391, 30, 3801, 3556}

$$-\frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x\cot(x) + \log(\sin(x)) - \frac{1}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

[In] Int[x*Cos[x]^2*Cot[x]^2,x]

[Out] (-3*x^2)/4 - Cos[x]^2/4 - x*Cot[x] + Log[Sin[x]] - (x*Cos[x]*Sin[x])/2

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\cos^2(x)\cot^2(x)\,dx &= -\int x\cos^2(x)\,dx + \int x\cot^2(x)\,dx\\ &= -\frac{1}{4}\cos^2(x) - x\cot(x) - \frac{1}{2}x\cos(x)\sin(x) - \frac{\int x\,dx}{2} - \int x\,dx + \int \cot(x)\,dx\\ &= -\frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x\cot(x) + \log(\sin(x)) - \frac{1}{2}x\cos(x)\sin(x)\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 33, normalized size = 1.00

$$-\frac{3x^2}{4} - \frac{1}{8}\cos(2x) - x\cot(x) + \log(\sin(x)) - \frac{1}{4}x\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] (-3*x^2)/4 - Cos[2*x]/8 - x*Cot[x] + Log[Sin[x]] - (x*Sin[2*x])/4
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.11, size = 60, normalized size = 1.82

method	result	size
risch	$-\frac{3x^2}{4} + \frac{i(2x+i)e^{2ix}}{16} - \frac{i(-i+2x)e^{-2ix}}{16} - 2ix - \frac{2ix}{e^{2ix}-1} + \ln\left(e^{2ix}-1\right)$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^2*cot(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*x^2+1/16*I*(2*x+I)*exp(2*I*x)-1/16*I*(-I+2*x)*exp(-2*I*x)-2*I*x-2*I*x/
(exp(2*I*x)-1)+ln(exp(2*I*x)-1)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 2.73, size = 45, normalized size = 1.36

$$\frac{4\,x\cos\left(x\right)^{3}-12\,x\cos\left(x\right)-\left(6\,x^{2}+2\,\cos\left(x\right)^{2}-1\right)\sin\left(x\right)+8\,\log\left(\frac{1}{2}\,\sin\left(x\right)\right)\sin\left(x\right)}{8\,\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*x*cos(x)^3 - 12*x*cos(x) - (6*x^2 + 2*cos(x)^2 - 1)*sin(x) + 8*log(1
/2*sin(x))*sin(x))/sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos^{2}\left(x\right)\cot^{2}\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)**2*cot(x)**2,x)
```

```
[Out] Integral(x*cos(x)**2*cot(x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(27) = 54.
time = 0.45, size = 206, normalized size = 6.24

$$-\frac{6\,x^{2}\tan\left(\frac{1}{2}\,x\right)^{5}-4\,x\tan\left(\frac{1}{2}\,x\right)^{6}-4\,\log\left(\frac{16\,\tan\left(\frac{1}{2}\,x\right)^{2}}{\tan\left(\frac{1}{2}\,x\right)^{4}+2\,\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)\tan\left(\frac{1}{2}\,x\right)^{5}+12\,x^{2}\tan\left(\frac{1}{2}\,x\right)^{3}-12\,x\tan\left(\frac{1}{2}\,x\right)^{4}+\tan\left(\frac{1}{2}\,x\right)^{5}-8\,\log\left(\frac{16\,\tan\left(\frac{1}{2}\,x\right)^{2}}{\tan\left(\frac{1}{2}\,x\right)^{4}+2\,\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)\tan\left(\frac{1}{2}\,x\right)^{3}+6\,x^{2}\tan\left(\frac{1}{2}\,x\right)+12\,x\tan\left(\frac{1}{2}\,x\right)^{2}-6\,\tan\left(\frac{1}{2}\,x\right)^{3}-4\,\log\left(\frac{16\,\tan\left(\frac{1}{2}\,x\right)^{2}}{\tan\left(\frac{1}{2}\,x\right)^{4}+2\,\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)\tan\left(\frac{1}{2}\,x\right)+4\,x+\tan\left(\frac{1}{2}\,x\right)}{8\,\left(\tan\left(\frac{1}{2}\,x\right)^{5}+2\,\tan\left(\frac{1}{2}\,x\right)^{3}+\tan\left(\frac{1}{2}\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^2,x, algorithm="giac")
```

```
[Out] -1/8*(6*x^2*tan(1/2*x)^5 - 4*x*tan(1/2*x)^6 - 4*log(16*tan(1/2*x)^2/(tan(1/
2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^5 + 12*x^2*tan(1/2*x)^3 - 12*x*tan
(1/2*x)^4 + tan(1/2*x)^5 - 8*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*
x)^2 + 1))*tan(1/2*x)^3 + 6*x^2*tan(1/2*x) + 12*x*tan(1/2*x)^2 - 6*tan(1/2*
x)^3 - 4*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x
) + 4*x + tan(1/2*x))/(tan(1/2*x)^5 + 2*tan(1/2*x)^3 + tan(1/2*x))
```

Mupad [B]
time = 1.23, size = 56, normalized size = 1.70

$$\ln\left(e^{x\,2i}-1\right)-e^{-x\,2i}\left(\frac{1}{16}+\frac{x\,1i}{8}\right)+e^{x\,2i}\left(-\frac{1}{16}+\frac{x\,1i}{8}\right)-\frac{3\,x^2}{4}-x\,2i-\frac{x\,2i}{e^{x\,2i}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^2*cot(x)^2,x)
```

```
[Out] log(exp(x*2i) - 1) - x*2i - exp(-x*2i)*((x*1i)/8 + 1/16) + exp(x*2i)*((x*1i
)/8 - 1/16) - (x*2i)/(exp(x*2i) - 1) - (3*x^2)/4
```

3.205 $\int x^3 \cos^2(x) \cot^3(x) \, dx$

Optimal. Leaf size=180

$$\frac{3x}{8}-\frac{3ix^2}{2}-\frac{3x^3}{4}+\frac{ix^4}{2}-\frac{3}{2}x^2\cot(x)-\frac{1}{2}x^3\cot^2(x)+3x\log\left(1-e^{2ix}\right)-2x^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\text{PolyLog}\left(2,e^{2ix}\right)+3i$$

```
[Out] 3/8*x-3/2*I*polylog(2,exp(2*I*x))-3/4*x^3+1/2*I*x^4-3/2*x^2*cot(x)-1/2*x^3*
cot(x)^2+3*x*ln(1-exp(2*I*x))-2*x^3*ln(1-exp(2*I*x))-3/2*I*polylog(4,exp(2*
I*x))+3*I*x^2*polylog(2,exp(2*I*x))-3*x*polylog(3,exp(2*I*x))-3/2*I*x^2-3/8
*cos(x)*sin(x)+3/4*x^2*cos(x)*sin(x)-3/4*x*sin(x)^2+1/2*x^3*sin(x)^2
```

Rubi [A]
time = 0.26, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {4493, 3524, 3392, 30, 2715, 8, 3798, 2221, 2611, 6744, 2320, 6724, 3801, 2317, 2438}

$$3ix^2\text{Li}_2\left(e^{2ix}\right)-3x\text{Li}_3\left(e^{2ix}\right)-\frac{3}{2}i\text{Li}_2\left(e^{2ix}\right)-\frac{3}{2}i\text{Li}_4\left(e^{2ix}\right)+\frac{ix^4}{2}-\frac{3x^3}{4}-2x^3\log\left(1-e^{2ix}\right)+\frac{1}{2}x^3\sin^2(x)-\frac{1}{2}x^3\cot^2(x)-\frac{3ix^2}{2}-\frac{3}{2}x^2\cot(x)+\frac{3}{4}x^2\sin(x)\cos(x)+\frac{3x}{8}+3x\log\left(1-e^{2ix}\right)-\frac{3}{4}x\sin^2(x)-\frac{3}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (3*x)/8 - ((3*I)/2)*x^2 - (3*x^3)/4 + (I/2)*x^4 - (3*x^2*Cot[x])/2 - (x^3*C
ot[x]^2)/2 + 3*x*Log[1 - E^((2*I)*x)] - 2*x^3*Log[1 - E^((2*I)*x)] - ((3*I)
/2)*PolyLog[2, E^((2*I)*x)] + (3*I)*x^2*PolyLog[2, E^((2*I)*x)] - 3*x*PolyL
og[3, E^((2*I)*x)] - ((3*I)/2)*PolyLog[4, E^((2*I)*x)] - (3*Cos[x]*Sin[x])/
8 + (3*x^2*Cos[x]*Sin[x])/4 - (3*x*Sin[x]^2)/4 + (x^3*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
```

```
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3\cos^2(x)\cot^3(x)\,dx = -\int x^3\cos^2(x)\cot(x)\,dx + \int x^3\cot^3(x)\,dx$$
$$= -\frac{1}{2}x^3\cot^2(x) + \frac{3}{2}\int x^2\cot^2(x)\,dx - 2\int x^3\cot(x)\,dx + \int x^3\cos(x)\sin(x)\,dx$$
$$= -\frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + \frac{1}{2}x^3\sin^2(x) - 2\left(-\frac{ix^4}{4} - 2i\int \frac{e^{2ix}x^3}{1-e^{2ix}}\,dx\right) - \frac{3\int}{}$$
$$= -\frac{3ix^2}{2} - \frac{x^3}{2} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + \frac{3}{4}x^2\cos(x)\sin(x) - \frac{3}{4}x\sin^2(x) + \frac{1}{2}x^3$$
$$= -\frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + 3x\log\left(1-e^{2ix}\right) - \frac{3}{8}\cos(x)\sin(x) +$$
$$= \frac{3x}{8} - \frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + 3x\log\left(1-e^{2ix}\right) - \frac{3}{8}\cos(x)\sin($$
$$= \frac{3x}{8} - \frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + 3x\log\left(1-e^{2ix}\right) - \frac{3}{2}i\text{Li}_2\left(e^{2ix}\right) -$$
$$= \frac{3x}{8} - \frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + 3x\log\left(1-e^{2ix}\right) - \frac{3}{2}i\text{Li}_2\left(e^{2ix}\right) -$$

Mathematica [A]
time = 0.45, size = 159, normalized size = 0.88

$$\frac{1}{32}\left(i\pi^4 - 48ix^2 - 16ix^4 + 12x\cos(2x) - 8x^3\cos(2x) - 48x^2\cot(x) - 16x^3\csc^2(x) - 64x^3\log\left(1-e^{-2ix}\right) + 96x\log\left(1-e^{2ix}\right) - 96ix^2\text{PolyLog}\left(2,e^{-2ix}\right) - 48i\text{PolyLog}\left(2,e^{2ix}\right) - 96x\text{PolyLog}\left(3,e^{-2ix}\right) + 48i\text{PolyLog}\left(4,e^{-2ix}\right) - 6\sin(2x) + 12x^2\sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (I*Pi^4 - (48*I)*x^2 - (16*I)*x^4 + 12*x*Cos[2*x] - 8*x^3*Cos[2*x] - 48*x^2
*Cot[x] - 16*x^3*Csc[x]^2 - 64*x^3*Log[1 - E^((-2*I)*x)] + 96*x*Log[1 - E^(
(2*I)*x)] - (96*I)*x^2*PolyLog[2, E^((-2*I)*x)] - (48*I)*PolyLog[2, E^((2*I
)*x)] - 96*x*PolyLog[3, E^((-2*I)*x)] + (48*I)*PolyLog[4, E^((-2*I)*x)] - 6
*Sin[2*x] + 12*x^2*Sin[2*x])/32
```

Maple [A]
time = 0.16, size = 240, normalized size = 1.33

method	result
risch	$6ix^2\,\text{polylog}\left(2,-e^{ix}\right) - \frac{\left(4x^3+6ix^2-6x-3i\right)e^{2ix}}{32} - \frac{\left(4x^3-6ix^2-6x+3i\right)e^{-2ix}}{32} + \frac{x^2\left(2x\,e^{2ix}-3ie^{2ix}+3i\right)}{\left(e^{2ix}-1\right)^2} - 2x^3\ln(1$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x)^2*cot(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 6*I*x^2*polylog(2,-exp(I*x))-1/32*(6*I*x^2+4*x^3-3*I-6*x)*exp(2*I*x)-1/32*(
-6*I*x^2+4*x^3+3*I-6*x)*exp(-2*I*x)+x^2*(2*x*exp(2*I*x)-3*I*exp(2*I*x)+3*I)
/(exp(2*I*x)-1)^2-2*x^3*ln(1-exp(I*x))-2*x^3*ln(exp(I*x)+1)-3*I*polylog(2,-
exp(I*x))-12*I*polylog(4,exp(I*x))+3*x*ln(1-exp(I*x))+6*I*x^2*polylog(2,exp
(I*x))-12*x*polylog(3,-exp(I*x))-12*x*polylog(3,exp(I*x))+3*x*ln(exp(I*x)+1
)-12*I*polylog(4,-exp(I*x))-3*I*x^2+1/2*I*x^4-3*I*polylog(2,exp(I*x))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3740 vs. 2(126) = 252.
time = 0.67, size = 3740, normalized size = 20.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(4*x^3 + (4*x^3 + 6*I*x^2 - 6*x - 3*I)*cos(6*x)^2 + 4*(8*I*x^4 + 4*x^
3 - 42*I*x^2 - 6*x - 3*I)*cos(4*x)^2 + 4*(8*I*x^4 - 14*x^3 - 24*I*x^2 - 3*x
)*cos(2*x)^2 - (4*x^3 + 6*I*x^2 - 6*x - 3*I)*sin(6*x)^2 + 4*(-8*I*x^4 - 4*x
^3 + 42*I*x^2 + 6*x + 3*I)*sin(4*x)^2 + 4*(-8*I*x^4 + 14*x^3 + 24*I*x^2 + 3
*x)*sin(2*x)^2 - 6*I*x^2 + 32*(2*(-2*I*x^3 + 3*I*x)*cos(4*x)^2 + 2*(-2*I*x^
3 + 3*I*x)*cos(2*x)^2 + 2*(2*I*x^3 - 3*I*x)*sin(4*x)^2 + 2*(2*I*x^3 - 3*I*x
)*sin(2*x)^2 + (2*I*x^3 + (2*I*x^3 - 3*I*x)*cos(4*x) + 2*(-2*I*x^3 + 3*I*x)
*cos(2*x) - (2*x^3 - 3*x)*sin(4*x) + 2*(2*x^3 - 3*x)*sin(2*x) - 3*I*x)*cos(
6*x) + (-4*I*x^3 + 5*(2*I*x^3 - 3*I*x)*cos(2*x) - 5*(2*x^3 - 3*x)*sin(2*x)
+ 6*I*x)*cos(4*x) + (2*I*x^3 - 3*I*x)*cos(2*x) - (2*x^3 + (2*x^3 - 3*x)*cos
(4*x) - 2*(2*x^3 - 3*x)*cos(2*x) - (-2*I*x^3 + 3*I*x)*sin(4*x) - 2*(2*I*x^3
 - 3*I*x)*sin(2*x) - 3*x)*sin(6*x) + (4*x^3 + 4*(2*x^3 - 3*x)*cos(4*x) - 5*
(2*x^3 - 3*x)*cos(2*x) + 5*(-2*I*x^3 + 3*I*x)*sin(2*x) - 6*x)*sin(4*x) - (2
*x^3 - 4*(2*x^3 - 3*x)*cos(2*x) - 3*x)*sin(2*x))*arctan2(sin(x), cos(x) + 1
) + 32*(2*(2*I*x^3 - 3*I*x)*cos(4*x)^2 + 2*(2*I*x^3 - 3*I*x)*cos(2*x)^2 + 2
*(-2*I*x^3 + 3*I*x)*sin(4*x)^2 + 2*(-2*I*x^3 + 3*I*x)*sin(2*x)^2 + (-2*I*x^
3 + (-2*I*x^3 + 3*I*x)*cos(4*x) + 2*(2*I*x^3 - 3*I*x)*cos(2*x) + (2*x^3 - 3
*x)*sin(4*x) - 2*(2*x^3 - 3*x)*sin(2*x) + 3*I*x)*cos(6*x) + (4*I*x^3 + 5*(-
2*I*x^3 + 3*I*x)*cos(2*x) + 5*(2*x^3 - 3*x)*sin(2*x) - 6*I*x)*cos(4*x) + (-
2*I*x^3 + 3*I*x)*cos(2*x) + (2*x^3 + (2*x^3 - 3*x)*cos(4*x) - 2*(2*x^3 - 3*
x)*cos(2*x) + (2*I*x^3 - 3*I*x)*sin(4*x) + 2*(-2*I*x^3 + 3*I*x)*sin(2*x) -
3*x)*sin(6*x) - (4*x^3 + 4*(2*x^3 - 3*x)*cos(4*x) - 5*(2*x^3 - 3*x)*cos(2*x
) - 5*(2*I*x^3 - 3*I*x)*sin(2*x) - 6*x)*sin(4*x) + (2*x^3 - 4*(2*x^3 - 3*x)
*cos(2*x) - 3*x)*sin(2*x))*arctan2(sin(x), -cos(x) + 1) - (16*I*x^4 + 8*x^3
 - 12*I*x^2 - 4*(-4*I*x^4 - 4*x^3 + 18*I*x^2 + 6*x + 3*I)*cos(4*x) + (-32*I
*x^4 + 52*x^3 + 90*I*x^2 + 18*x + 3*I)*cos(2*x) - 4*(4*x^4 - 4*I*x^3 - 18*x
^2 + 6*I*x - 3)*sin(4*x) + (32*x^4 + 52*I*x^3 - 90*x^2 + 18*I*x - 3)*sin(2*
x) - 12*x + 6*I)*cos(6*x) - (-32*I*x^4 - 20*x^3 + 30*I*x^2 - 2*(-40*I*x^4 +
 52*x^3 + 138*I*x^2 + 18*x + 3*I)*cos(2*x) - 2*(40*x^4 + 52*I*x^3 - 138*x^2
```

```
 + 18*I*x - 3)*sin(2*x) + 30*x - 15*I)*cos(4*x) + 4*(-4*I*x^4 - 4*x^3 + 6*I
*x^2 + 6*x - 3*I)*cos(2*x) + 96*(2*(2*I*x^2 - I)*cos(4*x)^2 + 2*(2*I*x^2 -
I)*cos(2*x)^2 + 2*(-2*I*x^2 + I)*sin(4*x)^2 + 2*(-2*I*x^2 + I)*sin(2*x)^2 +
 (-2*I*x^2 + (-2*I*x^2 + I)*cos(4*x) + 2*(2*I*x^2 - I)*cos(2*x) + (2*x^2 -
1)*sin(4*x) - 2*(2*x^2 - 1)*sin(2*x) + I)*cos(6*x) + (4*I*x^2 + 5*(-2*I*x^2
 + I)*cos(2*x) + 5*(2*x^2 - 1)*sin(2*x) - 2*I)*cos(4*x) + (-2*I*x^2 + I)*co
s(2*x) + (2*x^2 + (2*x^2 - 1)*cos(4*x) - 2*(2*x^2 - 1)*cos(2*x) + (2*I*x^2
- I)*sin(4*x) + 2*(-2*I*x^2 + I)*sin(2*x) - 1)*sin(6*x) - (4*x^2 + 4*(2*x^2
 - 1)*cos(4*x) - 5*(2*x^2 - 1)*cos(2*x) - 5*(2*I*x^2 - I)*sin(2*x) - 2)*sin
(4*x) + (2*x^2 - 4*(2*x^2 - 1)*cos(2*x) - 1)*sin(2*x))*dilog(-e^(I*x)) + 96
*(2*(2*I*x^2 - I)*cos(4*x)^2 + 2*(2*I*x^2 - I)*cos(2*x)^2 + 2*(-2*I*x^2 + I
)*sin(4*x)^2 + 2*(-2*I*x^2 + I)*sin(2*x)^2 + (-2*I*x^2 + (-2*I*x^2 + I)*cos
(4*x) + 2*(2*I*x^2 - I)*cos(2*x) + (2*x^2 - 1)*sin(4*x) - 2*(2*x^2 - 1)*sin
(2*x) + I)*cos(6*x) + (4*I*x^2 + 5*(-2*I*x^2 + I)*cos(2*x) + 5*(2*x^2 - 1)*
sin(2*x) - 2*I)*cos(4*x) + (-2*I*x^2 + I)*cos(2*x) + (2*x^2 + (2*x^2 - 1)*c
os(4*x) - 2*(2*x^2 - 1)*cos(2*x) + (2*I*x^2 - I)*sin(4*x) + 2*(-2*I*x^2 + I
)*sin(2*x) - 1)*sin(6*x) - (4*x^2 + 4*(2*x^2 - 1)*cos(4*x) - 5*(2*x^2 - 1)*
cos(2*x) - 5*(2*I*x^2 - I)*sin(2*x) - 2)*sin(4*x) + (2*x^2 - 4*(2*x^2 - 1)*
cos(2*x) - 1)*sin(2*x))*dilog(e^(I*x)) - 16*(2*(2*x^3 - 3*x)*cos(4*x)^2 + 2
*(2*x^3 - 3*x)*cos(2*x)^2 - 2*(2*x^3 - 3*x)*sin(4*x)^2 - 2*(2*x^3 - 3*x)*si
n(2*x)^2 - (2*x^3 + (2*x^3 - 3*x)*cos(4*x) - 2*(2*x^3 - 3*x)*cos(2*x) + (2*
I*x^3 - 3*I*x)*sin(4*x) + 2*(-2*I*x^3 + 3*I*x)*sin(2*x) - 3*x)*cos(6*x) + (
4*x^3 - 5*(2*x^3 - 3*x)*cos(2*x) - 5*(2*I*x^3 - 3*I*x)*sin(2*x) - 6*x)*cos(
4*x) - (2*x^3 - 3*x)*cos(2*x) - (2*I*x^3 + (2*I*x^3 - 3*I*x)*cos(4*x) + 2*(
-2*I*x^3 + 3*I*x)*cos(2*x) - (2*x^3 - 3*x)*sin(4*x) + 2*(2*x^3 - 3*x)*sin(2
*x) - 3*I*x)*sin(6*x) - (-4*I*x^3 + 4*(-2*I*x^3 + 3*I*x)*cos(4*x) + 5*(2*I*
x^3 - 3*I*x)*cos(2*x) - 5*(2*x^3 - 3*x)*sin(2*x) + 6*I*x)*sin(4*x) - (2*I*x
^3 + 4*(-2*I*x^3 + 3*I*x)*cos(2*x) - 3*I*x)*sin(2*x))*log(cos(x)^2 + sin(x)
^2 + 2*cos(x) + 1) - 16*(2*(2*x^3 - 3*x)*cos(4*x)^2 + 2*(2*x^3 - 3*x)*cos(2
*x)^2 - 2*(2*x^3 - 3*x)*sin(4*x)^2 - 2*(2*x^3 - 3*x)*sin(2*x)^2 - (2*x^3 +
(2*x^3 - 3*x)*cos(4*x) - 2*(2*x^3 - 3*x)*cos(2*x) + (2*I*x^3 - 3*I*x)*sin(4
*x) + 2*(-2*I*x^3 + 3*I*x)*sin(2*x) - 3*x)*cos(6*x) + (4*x^3 - 5*(2*x^3 - 3
*x)*cos(2*x) - 5*(2*I*x^3 - 3*I*x)*sin(2*x) - 6*x)*cos(4*x) - (2*x^3 - 3*x)
*cos(2*x) - (2*I*x^3 + (2*I*x^3 - 3*I*x)*cos(4*x) + 2*(-2*I*x^3 + 3*I*x)*co
s(2*x) - (2*x^3 - 3*x)*sin(4*x) + 2*(2*x^3 - 3*x)*sin(2*x) - 3*I*x)*sin(6*x
) - (-4*I*x^3 + 4*(-2*I*x^3 + 3*I*x)*cos(4*x) + 5*(2*I*x^3 - 3*I*x)*cos(2*x
) - 5*(2*x^3 - 3*x)*sin(2*x) + 6*I*x)*sin(4*x) - (2*I*x^3 + 4*(-2*I*x^3 + 3
*I*x)*cos(2*x) - 3*I*x)*sin(2*x))*log(cos(x)^2 ...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(126) = 252$.
time = 2.90, size = 508, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(2*x^3 - 3*x)*cos(x)^4 - 2*x^3 - 3*(2*x^3 - 3*x)*cos(x)^2 + 12*((-2
*I*x^2 + I)*cos(x)^2 + 2*I*x^2 - I)*dilog(cos(x) + I*sin(x)) + 12*((2*I*x^2
 - I)*cos(x)^2 - 2*I*x^2 + I)*dilog(cos(x) - I*sin(x)) + 12*((2*I*x^2 - I)*
cos(x)^2 - 2*I*x^2 + I)*dilog(-cos(x) + I*sin(x)) + 12*((-2*I*x^2 + I)*cos(
x)^2 + 2*I*x^2 - I)*dilog(-cos(x) - I*sin(x)) - 4*(2*x^3 - (2*x^3 - 3*x)*co
s(x)^2 - 3*x)*log(cos(x) + I*sin(x) + 1) - 4*(2*x^3 - (2*x^3 - 3*x)*cos(x)^
2 - 3*x)*log(cos(x) - I*sin(x) + 1) - 4*(2*x^3 - (2*x^3 - 3*x)*cos(x)^2 - 3
*x)*log(-cos(x) + I*sin(x) + 1) - 4*(2*x^3 - (2*x^3 - 3*x)*cos(x)^2 - 3*x)*
log(-cos(x) - I*sin(x) + 1) + 48*(I*cos(x)^2 - I)*polylog(4, cos(x) + I*sin
(x)) + 48*(-I*cos(x)^2 + I)*polylog(4, cos(x) - I*sin(x)) + 48*(-I*cos(x)^2
 + I)*polylog(4, -cos(x) + I*sin(x)) + 48*(I*cos(x)^2 - I)*polylog(4, -cos(
x) - I*sin(x)) + 48*(x*cos(x)^2 - x)*polylog(3, cos(x) + I*sin(x)) + 48*(x*
cos(x)^2 - x)*polylog(3, cos(x) - I*sin(x)) + 48*(x*cos(x)^2 - x)*polylog(3
, -cos(x) + I*sin(x)) + 48*(x*cos(x)^2 - x)*polylog(3, -cos(x) - I*sin(x))
- 3*((2*x^2 - 1)*cos(x)^3 + (2*x^2 + 1)*cos(x))*sin(x) - 3*x)/(cos(x)^2 - 1
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cos^2(x) \cot^3(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(x)**2*cot(x)**3,x)
```

```
[Out] Integral(x**3*cos(x)**2*cot(x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cos(x)^2*cot(x)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,{\cos\left(x\right)}^2\,{\mathrm{cot}\left(x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x)^2*cot(x)^3,x)
```

```
[Out] int(x^3*cos(x)^2*cot(x)^3, x)
```

3.206 $\int x^2 \cos^2(x) \cot^3(x)\,dx$

Optimal. Leaf size=106

$$-\frac{3x^2}{4}+\frac{2ix^3}{3}-x\cot(x)-\frac{1}{2}x^2\cot^2(x)-2x^2\log\left(1-e^{2ix}\right)+\log(\sin(x))+2ix\text{PolyLog}\left(2,e^{2ix}\right)-\text{PolyLog}\left(3,e^{2ix}\right)$$

```
[Out] -3/4*x^2+2/3*I*x^3-x*cot(x)-1/2*x^2*cot(x)^2-2*x^2*ln(1-exp(2*I*x))+ln(sin(
x))+2*I*x*polylog(2,exp(2*I*x))-polylog(3,exp(2*I*x))+1/2*x*cos(x)*sin(x)-1
/4*sin(x)^2+1/2*x^2*sin(x)^2
```

Rubi [A]
time = 0.19, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {4493, 3524, 3391, 30, 3798, 2221, 2611, 2320, 6724, 3801, 3556}

$$2ix\text{Li}_2\left(e^{2ix}\right)-\text{Li}_3\left(e^{2ix}\right)+\frac{2ix^3}{3}-\frac{3x^2}{4}-2x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\cot(x)+\log(\sin(x))+\frac{1}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (-3*x^2)/4 + ((2*I)/3)*x^3 - x*Cot[x] - (x^2*Cot[x]^2)/2 - 2*x^2*Log[1 - E^
((2*I)*x)] + Log[Sin[x]] + (2*I)*x*PolyLog[2, E^((2*I)*x)] - PolyLog[3, E^(
(2*I)*x)] + (x*Cos[x]*Sin[x])/2 - Sin[x]^2/4 + (x^2*Sin[x]^2)/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
```

```
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^2\cos^2(x)\cot^3(x)\,dx &= -\int x^2\cos^2(x)\cot(x)\,dx+\int x^2\cot^3(x)\,dx\\
&= -\frac{1}{2}x^2\cot^2(x)-2\int x^2\cot(x)\,dx+\int x\cot^2(x)\,dx+\int x^2\cos(x)\sin(x)\,dx\\
&= -x\cot(x)-\frac{1}{2}x^2\cot^2(x)+\frac{1}{2}x^2\sin^2(x)-2\left(-\frac{ix^3}{3}-2i\int\frac{e^{2ix}x^2}{1-e^{2ix}}\,dx\right)-\int x\,dx\\
&= -\frac{x^2}{2}-x\cot(x)-\frac{1}{2}x^2\cot^2(x)+\log(\sin(x))+\frac{1}{2}x\cos(x)\sin(x)-\frac{\sin^2(x)}{4}+\frac{1}{2}x^2\\
&= -\frac{3x^2}{4}-x\cot(x)-\frac{1}{2}x^2\cot^2(x)+\log(\sin(x))+\frac{1}{2}x\cos(x)\sin(x)-\frac{\sin^2(x)}{4}+\frac{1}{2}x\\
&= -\frac{3x^2}{4}-x\cot(x)-\frac{1}{2}x^2\cot^2(x)+\log(\sin(x))+\frac{1}{2}x\cos(x)\sin(x)-\frac{\sin^2(x)}{4}+\frac{1}{2}x\\
&= -\frac{3x^2}{4}-x\cot(x)-\frac{1}{2}x^2\cot^2(x)+\log(\sin(x))-2\left(-\frac{ix^3}{3}+x^2\log\left(1-e^{2ix}\right)-ix\right.
\end{aligned}$$

Mathematica [A]
time = 0.36, size = 108, normalized size = 1.02

$$\frac{i\pi^3}{12}-\frac{2ix^3}{3}+\frac{1}{8}\cos(2x)-\frac{1}{4}x^2\cos(2x)-x\cot(x)-\frac{1}{2}x^2\csc^2(x)-2x^2\log\left(1-e^{-2ix}\right)+\log(\sin(x))-2ix\text{PolyLog}\left(2,e^{-2ix}\right)-\text{PolyLog}\left(3,e^{-2ix}\right)+\frac{1}{4}x\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (I/12)*Pi^3 - ((2*I)/3)*x^3 + Cos[2*x]/8 - (x^2*Cos[2*x])/4 - x*Cot[x] - (x
^2*Csc[x]^2)/2 - 2*x^2*Log[1 - E^((-2*I)*x)] + Log[Sin[x]] - (2*I)*x*PolyLo
g[2, E^((-2*I)*x)] - PolyLog[3, E^((-2*I)*x)] + (x*Sin[2*x])/4
```

Maple [A]
time = 0.16, size = 170, normalized size = 1.60

method	result
risch	$\frac{2ix^3}{3} - \frac{(2x^2+2ix-1)\,\mathrm{e}^{2ix}}{16} - \frac{(2x^2-2ix-1)\,\mathrm{e}^{-2ix}}{16} + \frac{2x(x\,\mathrm{e}^{2ix}-i\mathrm{e}^{2ix}+i)}{(\mathrm{e}^{2ix}-1)^2} + \ln(\mathrm{e}^{ix}+1) - 2\ln(\mathrm{e}^{ix}) + \ln(\mathrm{e}^{ix}-1) -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^2*cot(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*x^3-1/16*(2*I*x+2*x^2-1)*exp(2*I*x)-1/16*(-2*I*x+2*x^2-1)*exp(-2*I*x)
+2*x*(x*exp(2*I*x)-I*exp(2*I*x)+I)/(exp(2*I*x)-1)^2+ln(exp(I*x)+1)-2*ln(exp
(I*x))+ln(exp(I*x)-1)-2*x^2*ln(exp(I*x)+1)+4*I*x*polylog(2,-exp(I*x))-4*pol
ylog(3,-exp(I*x))-2*x^2*ln(1-exp(I*x))+4*I*x*polylog(2,exp(I*x))-4*polylog(
3,exp(I*x))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2842 vs. 2(80) = 160.
time = 0.55, size = 2842, normalized size = 26.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(3*(2*x^2 + 2*I*x - 1)*cos(6*x)^2 + 4*(16*I*x^3 + 6*x^2 - 42*I*x - 3)
*cos(4*x)^2 + 2*(32*I*x^3 - 42*x^2 - 48*I*x - 3)*cos(2*x)^2 - 3*(2*x^2 + 2*
I*x - 1)*sin(6*x)^2 + 4*(-16*I*x^3 - 6*x^2 + 42*I*x + 3)*sin(4*x)^2 + 2*(-3
2*I*x^3 + 42*x^2 + 48*I*x + 3)*sin(2*x)^2 + 6*x^2 + 48*(2*(-2*I*x^2 + I)*co
s(4*x)^2 + 2*(-2*I*x^2 + I)*cos(2*x)^2 + 2*(2*I*x^2 - I)*sin(4*x)^2 + 2*(2*
I*x^2 - I)*sin(2*x)^2 + (2*I*x^2 + (2*I*x^2 - I)*cos(4*x) + 2*(-2*I*x^2 + I
)*cos(2*x) - (2*x^2 - 1)*sin(4*x) + 2*(2*x^2 - 1)*sin(2*x) - I)*cos(6*x) +
(-4*I*x^2 + 5*(2*I*x^2 - I)*cos(2*x) - 5*(2*x^2 - 1)*sin(2*x) + 2*I)*cos(4*
x) + (2*I*x^2 - I)*cos(2*x) - (2*x^2 + (2*x^2 - 1)*cos(4*x) - 2*(2*x^2 - 1)
*cos(2*x) - (-2*I*x^2 + I)*sin(4*x) - 2*(2*I*x^2 - I)*sin(2*x) - 1)*sin(6*x
) + (4*x^2 + 4*(2*x^2 - 1)*cos(4*x) - 5*(2*x^2 - 1)*cos(2*x) + 5*(-2*I*x^2
+ I)*sin(2*x) - 2)*sin(4*x) - (2*x^2 - 4*(2*x^2 - 1)*cos(2*x) - 1)*sin(2*x)
)*arctan2(sin(x), cos(x) + 1) + 48*((-I*cos(4*x) + 2*I*cos(2*x) + sin(4*x)
- 2*sin(2*x) - I)*cos(6*x) + (-5*I*cos(2*x) + 5*sin(2*x) + 2*I)*cos(4*x) +
2*I*cos(4*x)^2 + 2*I*cos(2*x)^2 + (cos(4*x) - 2*cos(2*x) + I*sin(4*x) - 2*I
*sin(2*x) + 1)*sin(6*x) - (4*cos(4*x) - 5*cos(2*x) - 5*I*sin(2*x) + 2)*sin(
4*x) - 2*I*sin(4*x)^2 - (4*cos(2*x) - 1)*sin(2*x) - 2*I*sin(2*x)^2 - I*cos(
2*x))*arctan2(sin(x), cos(x) - 1) + 96*(2*I*x^2*cos(4*x)^2 + 2*I*x^2*cos(2*
x)^2 - 2*I*x^2*sin(4*x)^2 - 2*I*x^2*sin(2*x)^2 - I*x^2*cos(2*x) + (-I*x^2*c
os(4*x) + 2*I*x^2*cos(2*x) + x^2*sin(4*x) - 2*x^2*sin(2*x) - I*x^2)*cos(6*x
) + (-5*I*x^2*cos(2*x) + 5*x^2*sin(2*x) + 2*I*x^2)*cos(4*x) + (x^2*cos(4*x)
 - 2*x^2*cos(2*x) + I*x^2*sin(4*x) - 2*I*x^2*sin(2*x) + x^2)*sin(6*x) - (4*
```

x^2*cos(4*x) - 5*x^2*cos(2*x) - 5*I*x^2*sin(2*x) + 2*x^2)*sin(4*x) - (4*x^2
*cos(2*x) - x^2)*sin(2*x))*arctan2(sin(x), -cos(x) + 1) - (32*I*x^3 + 12*x^
2 - 4*(-8*I*x^3 - 6*x^2 + 18*I*x + 3)*cos(4*x) + (-64*I*x^3 + 78*x^2 + 90*I
*x + 9)*cos(2*x) - 4*(8*x^3 - 6*I*x^2 - 18*x + 3*I)*sin(4*x) + (64*x^3 + 78
*I*x^2 - 90*x + 9*I)*sin(2*x) - 12*I*x - 6)*cos(6*x) - (-64*I*x^3 - 30*x^2
- 2*(-80*I*x^3 + 78*x^2 + 138*I*x + 9)*cos(2*x) - 2*(80*x^3 + 78*I*x^2 - 13
8*x + 9*I)*sin(2*x) + 30*I*x + 15)*cos(4*x) + 4*(-8*I*x^3 - 6*x^2 + 6*I*x +
3)*cos(2*x) + 192*(2*I*x*cos(4*x)^2 + 2*I*x*cos(2*x)^2 - 2*I*x*sin(4*x)^2
- 2*I*x*sin(2*x)^2 + (-I*x*cos(4*x) + 2*I*x*cos(2*x) + x*sin(4*x) - 2*x*sin
(2*x) - I*x)*cos(6*x) + (-5*I*x*cos(2*x) + 5*x*sin(2*x) + 2*I*x)*cos(4*x) -
I*x*cos(2*x) + (x*cos(4*x) - 2*x*cos(2*x) + I*x*sin(4*x) - 2*I*x*sin(2*x)
+ x)*sin(6*x) - (4*x*cos(4*x) - 5*x*cos(2*x) - 5*I*x*sin(2*x) + 2*x)*sin(4*
x) - (4*x*cos(2*x) - x)*sin(2*x))*dilog(-e^(I*x)) + 192*(2*I*x*cos(4*x)^2 +
2*I*x*cos(2*x)^2 - 2*I*x*sin(4*x)^2 - 2*I*x*sin(2*x)^2 + (-I*x*cos(4*x) +
2*I*x*cos(2*x) + x*sin(4*x) - 2*x*sin(2*x) - I*x)*cos(6*x) + (-5*I*x*cos(2*
x) + 5*x*sin(2*x) + 2*I*x)*cos(4*x) - I*x*cos(2*x) + (x*cos(4*x) - 2*x*cos(
2*x) + I*x*sin(4*x) - 2*I*x*sin(2*x) + x)*sin(6*x) - (4*x*cos(4*x) - 5*x*co
s(2*x) - 5*I*x*sin(2*x) + 2*x)*sin(4*x) - (4*x*cos(2*x) - x)*sin(2*x))*dilo
g(e^(I*x)) - 24*(2*(2*x^2 - 1)*cos(4*x)^2 + 2*(2*x^2 - 1)*cos(2*x)^2 - 2*(2
*x^2 - 1)*sin(4*x)^2 - 2*(2*x^2 - 1)*sin(2*x)^2 - (2*x^2 + (2*x^2 - 1)*cos(
4*x) - 2*(2*x^2 - 1)*cos(2*x) + (2*I*x^2 - I)*sin(4*x) + 2*(-2*I*x^2 + I)*s
in(2*x) - 1)*cos(6*x) + (4*x^2 - 5*(2*x^2 - 1)*cos(2*x) - 5*(2*I*x^2 - I)*s
in(2*x) - 2)*cos(4*x) - (2*x^2 - 1)*cos(2*x) - (2*I*x^2 + (2*I*x^2 - I)*cos
(4*x) + 2*(-2*I*x^2 + I)*cos(2*x) - (2*x^2 - 1)*sin(4*x) + 2*(2*x^2 - 1)*si
n(2*x) - I)*sin(6*x) - (-4*I*x^2 + 4*(-2*I*x^2 + I)*cos(4*x) + 5*(2*I*x^2 -
I)*cos(2*x) - 5*(2*x^2 - 1)*sin(2*x) + 2*I)*sin(4*x) - (2*I*x^2 + 4*(-2*I*
x^2 + I)*cos(2*x) - I)*sin(2*x))*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) -
24*(2*(2*x^2 - 1)*cos(4*x)^2 + 2*(2*x^2 - 1)*cos(2*x)^2 - 2*(2*x^2 - 1)*sin
(4*x)^2 - 2*(2*x^2 - 1)*sin(2*x)^2 - (2*x^2 + (2*x^2 - 1)*cos(4*x) - 2*(2*x
^2 - 1)*cos(2*x) + (2*I*x^2 - I)*sin(4*x) + 2*(-2*I*x^2 + I)*sin(2*x) - 1)*
cos(6*x) + (4*x^2 - 5*(2*x^2 - 1)*cos(2*x) - 5*(2*I*x^2 - I)*sin(2*x) - 2)*
cos(4*x) - (2*x^2 - 1)*cos(2*x) - (2*I*x^2 + (2*I*x^2 - I)*cos(4*x) + 2*(-2
*I*x^2 + I)*cos(2*x) - (2*x^2 - 1)*sin(4*x) + 2*(2*x^2 - 1)*sin(2*x) - I)*s
in(6*x) - (-4*I*x^2 + 4*(-2*I*x^2 + I)*cos(4*x) + 5*(2*I*x^2 - I)*cos(2*x)
- 5*(2*x^2 - 1)*sin(2*x) + 2*I)*sin(4*x) - (2*I*x^2 + 4*(-2*I*x^2 + I)*cos(
2*x) - I)*sin(2*x))*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 192*((cos(4*x
) - 2*cos(2*x) + I*sin(4*x) - 2*I*sin(2*x) + 1)*cos(6*x) + (5*cos(2*x) + 5*
I*sin(2*x) - 2)*cos(4*x) - 2*cos(4*x)^2 - 2*cos(2*x)^2 + (I*cos(4*x) - 2*I*
cos(2*x) - sin(4*x) + 2*sin(2*x) + I)*sin(6*x) + (-4*I*cos(4*x) + 5*I*cos(2
*x) - 5*sin(2*x) - 2*I)*sin(4*x) + 2*sin(4*x)^2 + (-4*I*cos(2*x) + I)*sin(2
*x) + 2*sin(2*x)^2 + cos(2*x))*polylog(3, -e^(I*x)) + 192*((cos(4*x) - 2*co
s(2*x) + I*sin(4*x) - 2*I*sin(2*x) + 1)*cos(6*x) + (5*cos(2*x) + 5*I*sin(2*
x) - 2)*cos(4*x) - 2*cos(4*x)^2 - 2*cos(2*x)^2 + (I*cos(4*x) - 2*I*cos(2*x)
- sin(4*x) + 2*sin(2*x) + I)*sin(6*x) + (-4*I*cos(4*x) + 5*I*cos(2*x) - 5*
sin(2*x) - 2*I)*sin(4*x) + 2*sin(4*x)^2 + (-4*I...

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(80) = 160.
time = 3.75, size = 370, normalized size = 3.49

$$-\frac{2\,(2\,x^2-1)\cos(x)^4-3\,(2\,x^2-1)\cos(x)^2-2\,x^2+16\,(-i\,x\cos(x)^2+i\,x){\rm Li}_2(\cos(x)+i\,\sin(x))+16\,(i\,x\cos(x)^2-i\,x){\rm Li}_2(\cos(x)-i\,\sin(x))+16\,(i\,x\cos(x)^2-i\,x){\rm Li}_2(-\cos(x)+i\,\sin(x))+16\,(-i\,x\cos(x)^2+i\,x){\rm Li}_2(-\cos(x)-i\,\sin(x))+4\,((2\,x^2-1)\cos(x)^2-2\,x^2+1)\log(\cos(x)+i\,\sin(x)+1)+4\,((2\,x^2-1)\cos(x)^2-2\,x^2+1)\log(\cos(x)-i\,\sin(x)+1)-4\,(\cos(x)^2-1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}i\,\sin(x)+\frac{1}{2}\right)-4\,(\cos(x)^2-1)\log\left(-\frac{1}{2}\cos(x)-\frac{1}{2}i\,\sin(x)+\frac{1}{2}\right)+8\,(x^2\cos(x)^2-x^2)\log(-\cos(x)+i\,\sin(x)+1)+8\,(x^2\cos(x)^2-x^2)\log(-\cos(x)-i\,\sin(x)+1)+16\,(\cos(x)^2-1){\rm polylog}(3,\cos(x)+i\,\sin(x))+16\,(\cos(x)^2-1){\rm polylog}(3,\cos(x)-i\,\sin(x))+16\,(\cos(x)^2-1){\rm polylog}(3,-\cos(x)+i\,\sin(x))+16\,(\cos(x)^2-1){\rm polylog}(3,-\cos(x)-i\,\sin(x))-4\,(x\cos(x)^3+x\cos(x))\sin(x)-1}{8\,(\cos(x)^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(2*x^2 - 1)*cos(x)^4 - 3*(2*x^2 - 1)*cos(x)^2 - 2*x^2 + 16*(-I*x*co
s(x)^2 + I*x)*dilog(cos(x) + I*sin(x)) + 16*(I*x*cos(x)^2 - I*x)*dilog(cos(
x) - I*sin(x)) + 16*(I*x*cos(x)^2 - I*x)*dilog(-cos(x) + I*sin(x)) + 16*(-I
*x*cos(x)^2 + I*x)*dilog(-cos(x) - I*sin(x)) + 4*((2*x^2 - 1)*cos(x)^2 - 2*
x^2 + 1)*log(cos(x) + I*sin(x) + 1) + 4*((2*x^2 - 1)*cos(x)^2 - 2*x^2 + 1)*
log(cos(x) - I*sin(x) + 1) - 4*(cos(x)^2 - 1)*log(-1/2*cos(x) + 1/2*I*sin(x
) + 1/2) - 4*(cos(x)^2 - 1)*log(-1/2*cos(x) - 1/2*I*sin(x) + 1/2) + 8*(x^2*
cos(x)^2 - x^2)*log(-cos(x) + I*sin(x) + 1) + 8*(x^2*cos(x)^2 - x^2)*log(-c
os(x) - I*sin(x) + 1) + 16*(cos(x)^2 - 1)*polylog(3, cos(x) + I*sin(x)) + 1
6*(cos(x)^2 - 1)*polylog(3, cos(x) - I*sin(x)) + 16*(cos(x)^2 - 1)*polylog(
3, -cos(x) + I*sin(x)) + 16*(cos(x)^2 - 1)*polylog(3, -cos(x) - I*sin(x)) -
 4*(x*cos(x)^3 + x*cos(x))*sin(x) - 1)/(cos(x)^2 - 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cos^2(x)\cot^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(x)**2*cot(x)**3,x)
```

```
[Out] Integral(x**2*cos(x)**2*cot(x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*cos(x)^2*cot(x)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\cos(x)^2\cot(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^2*cot(x)^3,x)
```

```
[Out] int(x^2*cos(x)^2*cot(x)^3, x)
```

3.207 $\int x \cos^2(x) \cot^3(x)\, dx$

Optimal. Leaf size=73

$$-\frac{3x}{4}+ix^2-\frac{\cot(x)}{2}-\frac{1}{2}x\cot^2(x)-2x\log\left(1-e^{2ix}\right)+i\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{4}\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)$$

```
[Out] -3/4*x+I*x^2-1/2*cot(x)-1/2*x*cot(x)^2-2*x*ln(1-exp(2*I*x))+I*polylog(2,exp
(2*I*x))+1/4*cos(x)*sin(x)+1/2*x*sin(x)^2
```

Rubi [A]
time = 0.12, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {4493, 3524, 2715, 8, 3798, 2221, 2317, 2438, 3801, 3554}

$$i\text{Li}_2\left(e^{2ix}\right)+ix^2-\frac{3x}{4}-2x\log\left(1-e^{2ix}\right)+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)-\frac{\cot(x)}{2}+\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (-3*x)/4 + I*x^2 - Cot[x]/2 - (x*Cot[x]^2)/2 - 2*x*Log[1 - E^((2*I)*x)] + I
*PolyLog[2, E^((2*I)*x)] + (Cos[x]*Sin[x])/4 + (x*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\cos^2(x)\cot^3(x)\,dx = -\int x\cos^2(x)\cot(x)\,dx + \int x\cot^3(x)\,dx$$
$$= -\frac{1}{2}x\cot^2(x) + \frac{1}{2}\int \cot^2(x)\,dx - 2\int x\cot(x)\,dx + \int x\cos(x)\sin(x)\,dx$$
$$= -\frac{\cot(x)}{2} - \frac{1}{2}x\cot^2(x) + \frac{1}{2}x\sin^2(x) - 2\left(-\frac{ix^2}{2} - 2i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx\right) - \frac{\int 1\,dx}{2} - \frac{1}{2}$$
$$= -\frac{x}{2} - \frac{\cot(x)}{2} - \frac{1}{2}x\cot^2(x) + \frac{1}{4}\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x) - \frac{\int 1\,dx}{4} - 2\left(-\frac{ix^2}{2} + x\right.$$
$$= -\frac{3x}{4} - \frac{\cot(x)}{2} - \frac{1}{2}x\cot^2(x) + \frac{1}{4}\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x) - 2\left(-\frac{ix^2}{2} + x\log\left(1-\right.\right.$$
$$= -\frac{3x}{4} - \frac{\cot(x)}{2} - \frac{1}{2}x\cot^2(x) - 2\left(-\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right) - \frac{1}{2}i\operatorname{Li}_2\left(e^{2ix}\right)\right) + \frac{1}{4}\cos($$

Mathematica [A]
time = 0.13, size = 62, normalized size = 0.85

$$\frac{1}{8}\left(8ix^2 - 2x\cos(2x) - 4\cot(x) - 4x\csc^2(x) - 16x\log\left(1-e^{2ix}\right) + 8i\text{PolyLog}\left(2,e^{2ix}\right) + \sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] ((8*I)*x^2 - 2*x*Cos[2*x] - 4*Cot[x] - 4*x*Csc[x]^2 - 16*x*Log[1 - E^((2*I)
*x)] + (8*I)*PolyLog[2, E^((2*I)*x)] + Sin[2*x])/8
```

Maple [A]
time = 0.14, size = 109, normalized size = 1.49

method	result
risch	$ix^2 - \frac{(2x+i)e^{2ix}}{16} - \frac{(-i+2x)e^{-2ix}}{16} + \frac{2x\,e^{2ix}-ie^{2ix}+i}{(e^{2ix}-1)^2} - 2x\ln\left(1-e^{ix}\right) - 2x\ln\left(e^{ix}+1\right) + 2i\,\text{polylog}\left(2,-e^{i}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^2*cot(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] I*x^2-1/16*(2*x+I)*exp(2*I*x)-1/16*(-I+2*x)*exp(-2*I*x)+(2*x*exp(2*I*x)-I*e
xp(2*I*x)+I)/(exp(2*I*x)-1)^2-2*x*ln(1-exp(I*x))-2*x*ln(exp(I*x)+1)+2*I*pol
ylog(2,-exp(I*x))+2*I*polylog(2,exp(I*x))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1718 vs. 2(52) = 104.
time = 0.44, size = 1718, normalized size = 23.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((2*x + I)*cos(6*x)^2 + 4*(8*I*x^2 + 2*x + I)*cos(4*x)^2 + 4*(8*I*x^2
 - 7*x + 4*I)*cos(2*x)^2 - (2*x + I)*sin(6*x)^2 + 4*(-8*I*x^2 - 2*x - I)*si
n(4*x)^2 + 4*(-8*I*x^2 + 7*x - 4*I)*sin(2*x)^2 + 32*(-2*I*x*cos(4*x)^2 - 2*
I*x*cos(2*x)^2 + 2*I*x*sin(4*x)^2 + 2*I*x*sin(2*x)^2 + (I*x*cos(4*x) - 2*I*
x*cos(2*x) - x*sin(4*x) + 2*x*sin(2*x) + I*x)*cos(6*x) + (5*I*x*cos(2*x) -
5*x*sin(2*x) - 2*I*x)*cos(4*x) + I*x*cos(2*x) - (x*cos(4*x) - 2*x*cos(2*x)
+ I*x*sin(4*x) - 2*I*x*sin(2*x) + x)*sin(6*x) + (4*x*cos(4*x) - 5*x*cos(2*x
) - 5*I*x*sin(2*x) + 2*x)*sin(4*x) + (4*x*cos(2*x) - x)*sin(2*x))*arctan2(s
in(x), cos(x) + 1) + 32*(2*I*x*cos(4*x)^2 + 2*I*x*cos(2*x)^2 - 2*I*x*sin(4*
x)^2 - 2*I*x*sin(2*x)^2 + (-I*x*cos(4*x) + 2*I*x*cos(2*x) + x*sin(4*x) - 2*
x*sin(2*x) - I*x)*cos(6*x) + (-5*I*x*cos(2*x) + 5*x*sin(2*x) + 2*I*x)*cos(4
*x) - I*x*cos(2*x) + (x*cos(4*x) - 2*x*cos(2*x) + I*x*sin(4*x) - 2*I*x*sin(
2*x) + x)*sin(6*x) - (4*x*cos(4*x) - 5*x*cos(2*x) - 5*I*x*sin(2*x) + 2*x)*s
in(4*x) - (4*x*cos(2*x) - x)*sin(2*x))*arctan2(sin(x), -cos(x) + 1) - (16*I
*x^2 - 4*(-4*I*x^2 - 2*x - I)*cos(4*x) + (-32*I*x^2 + 26*x - 17*I)*cos(2*x)
 - 4*(4*x^2 - 2*I*x + 1)*sin(4*x) + (32*x^2 + 26*I*x + 17)*sin(2*x) + 4*x +
 14*I)*cos(6*x) - (-32*I*x^2 - 2*(-40*I*x^2 + 26*x - 17*I)*cos(2*x) - 2*(40
*x^2 + 26*I*x + 17)*sin(2*x) - 10*x - 27*I)*cos(4*x) + 4*(-4*I*x^2 - 2*x -
3*I)*cos(2*x) + 32*((-I*cos(4*x) + 2*I*cos(2*x) + sin(4*x) - 2*sin(2*x) - I
)*cos(6*x) + (-5*I*cos(2*x) + 5*sin(2*x) + 2*I)*cos(4*x) + 2*I*cos(4*x)^2 +
 2*I*cos(2*x)^2 + (cos(4*x) - 2*cos(2*x) + I*sin(4*x) - 2*I*sin(2*x) + 1)*s
in(6*x) - (4*cos(4*x) - 5*cos(2*x) - 5*I*sin(2*x) + 2)*sin(4*x) - 2*I*sin(4
*x)^2 - (4*cos(2*x) - 1)*sin(2*x) - 2*I*sin(2*x)^2 - I*cos(2*x))*dilog(-e^(
I*x)) + 32*((-I*cos(4*x) + 2*I*cos(2*x) + sin(4*x) - 2*sin(2*x) - I)*cos(6*
x) + (-5*I*cos(2*x) + 5*sin(2*x) + 2*I)*cos(4*x) + 2*I*cos(4*x)^2 + 2*I*cos
(2*x)^2 + (cos(4*x) - 2*cos(2*x) + I*sin(4*x) - 2*I*sin(2*x) + 1)*sin(6*x)
- (4*cos(4*x) - 5*cos(2*x) - 5*I*sin(2*x) + 2)*sin(4*x) - 2*I*sin(4*x)^2 -
(4*cos(2*x) - 1)*sin(2*x) - 2*I*sin(2*x)^2 - I*cos(2*x))*dilog(e^(I*x)) - 1
6*(2*x*cos(4*x)^2 + 2*x*cos(2*x)^2 - 2*x*sin(4*x)^2 - 2*x*sin(2*x)^2 - (x*c
os(4*x) - 2*x*cos(2*x) + I*x*sin(4*x) - 2*I*x*sin(2*x) + x)*cos(6*x) - (5*x
*cos(2*x) + 5*I*x*sin(2*x) - 2*x)*cos(4*x) - x*cos(2*x) - (I*x*cos(4*x) - 2
*I*x*cos(2*x) - x*sin(4*x) + 2*x*sin(2*x) + I*x)*sin(6*x) - (-4*I*x*cos(4*x
) + 5*I*x*cos(2*x) - 5*x*sin(2*x) - 2*I*x)*sin(4*x) - (-4*I*x*cos(2*x) + I*
x)*sin(2*x))*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 16*(2*x*cos(4*x)^2 +
 2*x*cos(2*x)^2 - 2*x*sin(4*x)^2 - 2*x*sin(2*x)^2 - (x*cos(4*x) - 2*x*cos(2
*x) + I*x*sin(4*x) - 2*I*x*sin(2*x) + x)*cos(6*x) - (5*x*cos(2*x) + 5*I*x*s
in(2*x) - 2*x)*cos(4*x) - x*cos(2*x) - (I*x*cos(4*x) - 2*I*x*cos(2*x) - x*s
in(4*x) + 2*x*sin(2*x) + I*x)*sin(6*x) - (-4*I*x*cos(4*x) + 5*I*x*cos(2*x)
- 5*x*sin(2*x) - 2*I*x)*sin(4*x) - (-4*I*x*cos(2*x) + I*x)*sin(2*x))*log(co
s(x)^2 + sin(x)^2 - 2*cos(x) + 1) + (16*x^2 + 2*(2*I*x - 1)*cos(6*x) + 4*(4
*x^2 - 2*I*x + 1)*cos(4*x) - (32*x^2 + 26*I*x + 17)*cos(2*x) + 4*(4*I*x^2 +
```

```
 2*x + I)*sin(4*x) - (32*I*x^2 - 26*x + 17*I)*sin(2*x) - 4*I*x + 14)*sin(6*
x) - (32*x^2 + 8*(8*x^2 - 2*I*x + 1)*cos(4*x) - 2*(40*x^2 + 26*I*x + 17)*co
s(2*x) - 2*(40*I*x^2 - 26*x + 17*I)*sin(2*x) - 10*I*x + 27)*sin(4*x) + 4*(4
*x^2 - 2*(8*x^2 + 7*I*x + 4)*cos(2*x) - 2*I*x + 3)*sin(2*x) + 2*x - I)/((co
s(4*x) - 2*cos(2*x) + I*sin(4*x) - 2*I*sin(2*x) + 1)*cos(6*x) + (5*cos(2*x)
 + 5*I*sin(2*x) - 2)*cos(4*x) - 2*cos(4*x)^2 - 2*cos(2*x)^2 + (I*cos(4*x) -
 2*I*cos(2*x) - sin(4*x) + 2*sin(2*x) + I)*sin(6*x) + (-4*I*cos(4*x) + 5*I*
cos(2*x) - 5*sin(2*x) - 2*I)*sin(4*x) + 2*sin(4*x)^2 + (-4*I*cos(2*x) + I)*
sin(2*x) + 2*sin(2*x)^2 + cos(2*x))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(52) = 104$.
time = 2.94, size = 203, normalized size = 2.78

$$-\frac{2\,x\cos\left(x\right)^{4}-3\,x\cos\left(x\right)^{2}+4\,\left(-i\,\cos\left(x\right)^{2}+i\right){\rm Li}_2(\cos\left(x\right)+i\,\sin\left(x\right))+4\,\left(i\,\cos\left(x\right)^{2}-i\right){\rm Li}_2(\cos\left(x\right)-i\,\sin\left(x\right))+4\,\left(i\,\cos\left(x\right)^{2}-i\right){\rm Li}_2(-\cos\left(x\right)+i\,\sin\left(x\right))+4\,\left(-i\,\cos\left(x\right)^{2}+i\right){\rm Li}_2(-\cos\left(x\right)-i\,\sin\left(x\right))+4\,\left(x\cos\left(x\right)^{2}-x\right)\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right)+4\,\left(x\cos\left(x\right)^{2}-x\right)\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right)+4\,\left(x\cos\left(x\right)^{2}-x\right)\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+1\right)+4\,\left(x\cos\left(x\right)^{2}-x\right)\log\left(-\cos\left(x\right)-i\,\sin\left(x\right)+1\right)-\left(\cos\left(x\right)^{3}+\cos\left(x\right)\right)\sin\left(x\right)-x}{4\,\left(\cos\left(x\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*x*cos(x)^4 - 3*x*cos(x)^2 + 4*(-I*cos(x)^2 + I)*dilog(cos(x) + I*si
n(x)) + 4*(I*cos(x)^2 - I)*dilog(cos(x) - I*sin(x)) + 4*(I*cos(x)^2 - I)*di
log(-cos(x) + I*sin(x)) + 4*(-I*cos(x)^2 + I)*dilog(-cos(x) - I*sin(x)) + 4
*(x*cos(x)^2 - x)*log(cos(x) + I*sin(x) + 1) + 4*(x*cos(x)^2 - x)*log(cos(x
) - I*sin(x) + 1) + 4*(x*cos(x)^2 - x)*log(-cos(x) + I*sin(x) + 1) + 4*(x*c
os(x)^2 - x)*log(-cos(x) - I*sin(x) + 1) - (cos(x)^3 + cos(x))*sin(x) - x)/
(cos(x)^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos^{2}\left(x\right)\cot^{3}\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)**2*cot(x)**3,x)
```

```
[Out] Integral(x*cos(x)**2*cot(x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cos(x)^2*cot(x)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\cos\left(x\right)^2\cot(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^2*cot(x)^3,x)
```

```
[Out] int(x*cos(x)^2*cot(x)^3, x)
```

3.208 $\int (c+dx)^m \tan(a+bx)\, dx$

Optimal. Leaf size=17

$$\mathrm{Int}((c+dx)^m \tan(a+bx), x)$$

```
[Out] Unintegrable((d*x+c)^m*tan(b*x+a),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \tan(a+bx)\, dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Tan[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Tan[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \tan(a+bx)\, dx = \int (c+dx)^m \tan(a+bx)\, dx$$

Mathematica [A]
time = 2.66, size = 0, normalized size = 0.00

$$\int (c+dx)^m \tan(a+bx)\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Tan[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Tan[a + b*x], x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a) \sin(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin (a+bx) \sec (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*sin(b*x+a),x)
[Out] Integral((c + d*x)**m*sin(a + b*x)*sec(a + b*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\sin (a+b\,x)\,(c+d\,x)^m}{\cos (a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*(c + d*x)^m)/cos(a + b*x),x)
[Out] int((sin(a + b*x)*(c + d*x)^m)/cos(a + b*x), x)
```

3.209 $\int (c+dx)^4 \tan(a+bx)\, dx$

Optimal. Leaf size=158

$$\frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4 \log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3 \mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} - \frac{3d^2(c+dx)^2 \mathrm{PolyLog}\left(3,-e^{2i\ldots}\right)}{b^3}$$

```
[Out] 1/5*I*(d*x+c)^5/d-(d*x+c)^4*ln(1+exp(2*I*(b*x+a)))/b+2*I*d*(d*x+c)^3*polylo
g(2,-exp(2*I*(b*x+a)))/b^2-3*d^2*(d*x+c)^2*polylog(3,-exp(2*I*(b*x+a)))/b^3
-3*I*d^3*(d*x+c)*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/2*d^4*polylog(5,-exp(2*
I*(b*x+a)))/b^5
```

Rubi [A]
time = 0.15, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3800, 2221, 2611, 6744, 2320, 6724}

$$\frac{3d^4 \mathrm{Li}_5\left(-e^{2i(a+bx)}\right)}{2b^5} - \frac{3id^3(c+dx)\mathrm{Li}_4\left(-e^{2i(a+bx)}\right)}{b^4} - \frac{3d^2(c+dx)^2\mathrm{Li}_3\left(-e^{2i(a+bx)}\right)}{b^3} + \frac{2id(c+dx)^3\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{(c+dx)^4 \log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{i(c+dx)^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Tan[a + b*x],x]
```

```
[Out] ((I/5)*(c + d*x)^5)/d - ((c + d*x)^4*Log[1 + E^((2*I)*(a + b*x))])/b + ((2*
I)*d*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)^2
*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*PolyLog[4, -E
^((2*I)*(a + b*x))])/b^4 + (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \tan(a+bx)\,dx = \frac{i(c+dx)^5}{5d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^4}{1+e^{2i(a+bx)}}\,dx$$

$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{(4d)\int (c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}$$

$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{(6}{}$$

$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{3a}{}$$

$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{3a}{}$$

$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{3a}{}$$

$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{3a}{}$$

Mathematica [A]
time = 0.07, size = 157, normalized size = 0.99

$$\frac{i(c+dx)^5}{5d}-\frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{2id(c+dx)^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{3d^2\left(2b^2(c+dx)^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)+d\left(2ib(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)-d\text{PolyLog}\left(5,-e^{2i(a+bx)}\right)\right)\right)}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Tan[a + b*x],x]
```

```
[Out] ((I/5)*(c + d*x)^5)/d - ((c + d*x)^4*Log[1 + E^((2*I)*(a + b*x))])/b + ((2*
I)*d*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(2*b^2*(c +
 d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))] + d*((2*I)*b*(c + d*x)*PolyLog[4,
-E^((2*I)*(a + b*x))] - d*PolyLog[5, -E^((2*I)*(a + b*x))])))/(2*b^5)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(141) = 282.
time = 0.13, size = 625, normalized size = 3.96

method	result
risch	$\frac{12c^2d^2a^2\ln\left(e^{i(bx+a)}\right)}{b^3}-\frac{8c^3da\ln\left(e^{i(bx+a)}\right)}{b^2}-\frac{2id^4a^4x}{b^4}+\frac{4ic^3da^2}{b^2}+\frac{6icd^3a^4}{b^4}-\frac{8id^2c^2a^3}{b^3}-\frac{8cd^3a^3\ln\left(e^{i(bx+a)}\right)}{b^4}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/b^2*d^4*polylog(2,-exp(2*I*(b*x+a)))*x^3-3*I/b^4*d^4*polylog(4,-exp(2*I
*(b*x+a)))*x+2*I/b^2*c^3*d*polylog(2,-exp(2*I*(b*x+a)))-3*I/b^4*c*d^3*polyl
og(4,-exp(2*I*(b*x+a)))-4/b*c*d^3*ln(1+exp(2*I*(b*x+a)))*x^3+6*I/b^2*c^2*d^
2*polylog(2,-exp(2*I*(b*x+a)))*x+3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5-8
*I/b^3*a^3*c^2*d^2+4*I/b^2*a^2*c^3*d+6*I/b^4*c*d^3*a^4-2*I/b^4*d^4*a^4*x+12
/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)))-8/b^2*c^3*d*a*ln(exp(I*(b*x+a)))+I*d^3*
c*x^4+1/5*I*d^4*x^5+8*I/b^3*c*d^3*a^3*x-12*I/b^2*a^2*c^2*d^2*x+8*I/b*a*c^3*
d*x-I*c^4*x-1/5*I/d*c^5+2/b*c^4*ln(exp(I*(b*x+a)))+2/b^5*d^4*a^4*ln(exp(I*(
b*x+a)))-1/b*c^4*ln(1+exp(2*I*(b*x+a)))-3/b^3*c^2*d^2*polylog(3,-exp(2*I*(b
*x+a)))-3/b^3*d^4*polylog(3,-exp(2*I*(b*x+a)))*x^2-4/b*c^3*d*ln(1+exp(2*I*(
b*x+a)))*x-6/b*c^2*d^2*ln(1+exp(2*I*(b*x+a)))*x^2-1/b*d^4*ln(1+exp(2*I*(b*x
+a)))*x^4-6/b^3*c*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+6*I/b^2*c*d^3*polylog(
2,-exp(2*I*(b*x+a)))*x^2+2*I*d^2*c^2*x^3+2*I*d*c^3*x^2-8/5*I/b^5*d^4*a^5-8/
b^4*c*d^3*a^3*ln(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 803 vs. 2(137) = 274.
time = 0.57, size = 803, normalized size = 5.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/30*(15*c^4*log(-sin(b*x + a)^2 + 1) - 60*a*c^3*d*log(-sin(b*x + a)^2 + 1
)/b + 90*a^2*c^2*d^2*log(-sin(b*x + a)^2 + 1)/b^2 - 60*a^3*c*d^3*log(-sin(b
*x + a)^2 + 1)/b^3 + 15*a^4*d^4*log(-sin(b*x + a)^2 + 1)/b^4 + 2*(-3*I*(b*x
 + a)^5*d^4 - 15*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^4 - 45*d^4*polylog(5, -e^(
2*I*b*x + 2*I*a)) - 30*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a
)^3 - 30*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 - I*a^3*d^4)*(b
*x + a)^2 - 10*(-3*I*(b*x + a)^4*d^4 + 8*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3
 + 9*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a)^2 + 6*(-I*b^3*c
^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b*c*d^3 + I*a^3*d^4)*(b*x + a))*arctan2(
sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 30*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d
^2 + 3*I*a^2*b*c*d^3 + 2*I*(b*x + a)^3*d^4 - I*a^3*d^4 + 4*(I*b*c*d^3 - I*a
*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a)
)*dilog(-e^(2*I*b*x + 2*I*a)) + 5*(3*(b*x + a)^4*d^4 + 8*(b*c*d^3 - a*d^4)*
(b*x + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 6*(b^3*
c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(2*b*x
 + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 30*(-2*I*b*c*d^3
 - 3*I*(b*x + a)*d^4 + 2*I*a*d^4)*polylog(4, -e^(2*I*b*x + 2*I*a)) + 15*(3*
b^2*c^2*d^2 - 6*a*b*c*d^3 + 6*(b*x + a)^2*d^4 + 3*a^2*d^4 + 8*(b*c*d^3 - a*
d^4)*(b*x + a))*polylog(3, -e^(2*I*b*x + 2*I*a)))/b^4)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1410 vs. $2(137) = 274$.
time = 4.16, size = 1410, normalized size = 8.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(24*d^4*polylog(5, I*cos(b*x + a) + sin(b*x + a)) + 24*d^4*polylog(5, I
*cos(b*x + a) - sin(b*x + a)) + 24*d^4*polylog(5, -I*cos(b*x + a) + sin(b*x
 + a)) + 24*d^4*polylog(5, -I*cos(b*x + a) - sin(b*x + a)) - 4*(I*b^3*d^4*x
^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(I*cos(b*x +
 a) + sin(b*x + a)) - 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d
^2*x - I*b^3*c^3*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 4*(-I*b^3*d^4*x^
3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog(-I*cos(b*x +
 a) + sin(b*x + a)) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^
2*x + I*b^3*c^3*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^4*c^4 - 4*a*b
^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) +
I*sin(b*x + a) + I) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^4*d^4*x^4 +
4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2
*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(b*x + a) + sin(b*x + a) +
 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x +
```

```
4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(b*x
+ a) - sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x
^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^
4*d^4)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3
*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^
2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b^4
*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-co
s(b*x + a) + I*sin(b*x + a) + I) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 2
4*(-I*b*d^4*x - I*b*c*d^3)*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 24*(
I*b*d^4*x + I*b*c*d^3)*polylog(4, I*cos(b*x + a) - sin(b*x + a)) - 24*(I*b*
d^4*x + I*b*c*d^3)*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) - 24*(-I*b*d^
4*x - I*b*c*d^3)*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 12*(b^2*d^4*x
^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a))
 - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, I*cos(b*x + a)
 - sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3
, -I*cos(b*x + a) + sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c
^2*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)))/b^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^4 \sin(a + bx) \sec(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sec(b*x+a)*sin(b*x+a),x)
[Out] Integral((c + d*x)**4*sin(a + b*x)*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^4*sec(b*x + a)*sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a + b\,x)\,\left(c + d\,x\right)^4}{\cos(a + b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*(c + d*x)^4)/cos(a + b*x),x)
[Out] int((sin(a + b*x)*(c + d*x)^4)/cos(a + b*x), x)
```

3.210 $\int (c+dx)^3 \tan(a+bx)\,dx$

Optimal. Leaf size=132

$$\frac{i(c+dx)^4}{4d}-\frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}$$

```
[Out] 1/4*I*(d*x+c)^4/d-(d*x+c)^3*ln(1+exp(2*I*(b*x+a)))/b+3/2*I*d*(d*x+c)^2*poly
log(2,-exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))/b
^3-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4
```

Rubi [A]

time = 0.13, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3800, 2221, 2611, 6744, 2320, 6724}

$$-\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Tan[a + b*x],x]
```

```
[Out] ((I/4)*(c + d*x)^4)/d - ((c + d*x)^3*Log[1 + E^((2*I)*(a + b*x))])/b + (((3
*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*
x)*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E
^((2*I)*(a + b*x))])/b^4
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\tan(a+bx)\,dx &= \frac{i(c+dx)^4}{4d}-2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1+e^{2i(a+bx)}}\,dx\\ &= \frac{i(c+dx)^4}{4d}-\frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{(3d)\int (c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}\\ &= \frac{i(c+dx)^4}{4d}-\frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{(3id}{}\\ &= \frac{i(c+dx)^4}{4d}-\frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{3d^2}{}\\ &= \frac{i(c+dx)^4}{4d}-\frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{3d^2}{}\\ &= \frac{i(c+dx)^4}{4d}-\frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{3d^2}{}\end{aligned}$$

Mathematica [A]

time = 0.09, size = 126, normalized size = 0.95

$$\frac{1}{4}i\left(\frac{(c+dx)^4}{d}+\frac{4i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{3d\left(2b^2(c+dx)^2\text{PolyLog}(2,-e^{2i(a+bx)})+d\left(2ib(c+dx)\text{PolyLog}(3,-e^{2i(a+bx)})-d\text{PolyLog}(4,-e^{2i(a+bx)})\right)\right)}{b^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Tan[a + b*x],x]
```

```
[Out] (I/4)*((c + d*x)^4/d + ((4*I)*(c + d*x)^3*Log[1 + E^((2*I)*(a + b*x))])/b +
 (3*d*(2*b^2*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))] + d*((2*I)*b*(c +
 d*x)*PolyLog[3, -E^((2*I)*(a + b*x))] - d*PolyLog[4, -E^((2*I)*(a + b*x))]
)))/b^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(113) = 226.
time = 0.09, size = 432, normalized size = 3.27

method	result
risch	$\frac{i d^3 x^4}{4}+\frac{2c^3\ln\left(e^{i(bx+a)}\right)}{b}-ic^3x-\frac{ic^4}{4d}-\frac{6icd^2a^2x}{b^2}+\frac{6ic^2dax}{b}+\frac{3id^3a^4}{2b^4}+id^2cx^3+\frac{3idc^2x^2}{2}-\frac{2d^3a^3\ln\left(e^{i(bx+a)}\right)}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^4*d^3*a^3*ln(exp(I*(b*x+a)))+2/b*c^3*ln(exp(I*(b*x+a)))-I*c^3*x-1/4*I/
d*c^4+3/2*I*d*c^2*x^2+I*d^2*c*x^3+3*I/b^2*a^2*c^2*d+1/4*I*d^3*x^4-6/b^2*c^2
*d*a*ln(exp(I*(b*x+a)))-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/2*I/b^
4*d^3*a^4-1/b*c^3*ln(1+exp(2*I*(b*x+a)))-3/2/b^3*c*d^2*polylog(3,-exp(2*I*(
b*x+a)))-3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+6/b^3*c*d^2*a^2*ln(exp(
I*(b*x+a)))-6*I/b^2*a^2*c*d^2*x+6*I/b*a*c^2*d*x+3/2*I/b^2*d^3*polylog(2,-ex
p(2*I*(b*x+a)))*x^2+3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+a)))-3/b*c^2*d*
ln(1+exp(2*I*(b*x+a)))*x-3/b*c*d^2*ln(1+exp(2*I*(b*x+a)))*x^2-1/b*d^3*ln(1+
exp(2*I*(b*x+a)))*x^3+2*I/b^3*d^3*a^3*x-4*I/b^3*a^3*c*d^2+3*I/b^2*c*d^2*pol
ylog(2,-exp(2*I*(b*x+a)))*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 497 vs. 2(109) = 218.
time = 0.54, size = 497, normalized size = 3.77

$$-\frac{6c^3\log\left(-\sin(bx+a)^2+1\right)-\frac{18ac^2d\log\left(-\sin(bx+a)^2+1\right)}{b}+\frac{18a^2cd^2\log\left(-\sin(bx+a)^2+1\right)}{b^2}-\frac{6a^3d^3\log\left(-\sin(bx+a)^2+1\right)}{b^3}+\frac{-3i(bx+a)^4d^3-12(ibcd^2-iad^3)(bx+a)^3+12id^3\mathrm{Li}_4(-e^{(2ibx+2ia)})-18(ib^2c^2d-2iabcd^2+ia^2d^3)(bx+a)^2-4\left(-4i(bx+a)^3d^3+9(-ibcd^2+iad^3)(bx+a)^2+9(-ib^2c^2d+2iabcd^2-ia^2d^3)(bx+a)\ldots\right)\ldots}{b^3}}{12b}$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/12*(6*c^3*log(-sin(b*x + a)^2 + 1) - 18*a*c^2*d*log(-sin(b*x + a)^2 + 1)
/b + 18*a^2*c*d^2*log(-sin(b*x + a)^2 + 1)/b^2 - 6*a^3*d^3*log(-sin(b*x + a
```

```
)^2 + 1)/b^3 + (-3*I*(b*x + a)^4*d^3 - 12*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^3
 + 12*I*d^3*polylog(4, -e^(2*I*b*x + 2*I*a)) - 18*(I*b^2*c^2*d - 2*I*a*b*c*
d^2 + I*a^2*d^3)*(b*x + a)^2 - 4*(-4*I*(b*x + a)^3*d^3 + 9*(-I*b*c*d^2 + I*
a*d^3)*(b*x + a)^2 + 9*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a)
)*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(3*I*b^2*c^2*d - 6*I*
a*b*c*d^2 + 4*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + 6*(I*b*c*d^2 - I*a*d^3)*(b*
x + a))*dilog(-e^(2*I*b*x + 2*I*a)) + 2*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a
*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(co
s(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 6*(3*b*c*
d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*polylog(3, -e^(2*I*b*x + 2*I*a)))/b^3)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(109) = 218$.
time = 4.43, size = 974, normalized size = 7.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*polylog(4,
 I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) + sin(
b*x + a)) + 6*I*d^3*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*d
^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(b*x + a)
) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(I*cos(b*x + a)
 - sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog
(-I*cos(b*x + a) + sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b
^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*x +
 a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^
3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*
x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b
*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) - sin(b*x
+ a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*
x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b*d^3*x + b*c*d^2)*pol
ylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, I
*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x
 + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) -
sin(b*x + a)))/b^4
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \sin (a+bx) \sec (a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*sin(b*x+a),x)
[Out] Integral((c + d*x)**3*sin(a + b*x)*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (a+b\,x)\,\left(c+d\,x\right)^3}{\cos (a+b\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*(c + d*x)^3)/cos(a + b*x),x)
[Out] int((sin(a + b*x)*(c + d*x)^3)/cos(a + b*x), x)
```

3.211 $\int (c+dx)^2 \tan(a+bx)\,dx$

Optimal. Leaf size=96

$$\frac{i(c+dx)^3}{3d}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}$$

```
[Out] 1/3*I*(d*x+c)^3/d-(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b+I*d*(d*x+c)*polylog(2,
-exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3
```

Rubi [A]
time = 0.12, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3800, 2221, 2611, 2320, 6724}

$$-\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Tan[a + b*x],x]
```

```
[Out] ((I/3)*(c + d*x)^3)/d - ((c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b + (I*d
*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (d^2*PolyLog[3, -E^((2*I
)*(a + b*x))])/(2*b^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\tan(a+bx)\,dx &= \frac{i(c+dx)^3}{3d}-2i\int\frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}\,dx\\ &= \frac{i(c+dx)^3}{3d}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{(2d)\int(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}\\ &= \frac{i(c+dx)^3}{3d}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{id(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{(id^2)}{}\\ &= \frac{i(c+dx)^3}{3d}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{id(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\mathrm{Su}}{}\\ &= \frac{i(c+dx)^3}{3d}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{id(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\mathrm{Li}}{}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 100, normalized size = 1.04

$$\frac{2ib^2(c+dx)^2\left(b(c+dx)+3id\log\left(1+e^{2i(a+bx)}\right)\right)+6ibd^2(c+dx)\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)-3d^3\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{6b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Tan[a + b*x],x]
```

```
[Out] ((2*I)*b^2*(c + d*x)^2*(b*(c + d*x) + (3*I)*d*Log[1 + E^((2*I)*(a + b*x))])
 + (6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))] - 3*d^3*PolyLog[3
, -E^((2*I)*(a + b*x))])/(6*b^3*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(84) = 168.
time = 0.07, size = 266, normalized size = 2.77

method	result
risch	$\frac{id^2x^3}{3}-\frac{ic^3}{3d}-\frac{4id^2a^3}{3b^3}+\frac{2icd\,a^2}{b^2}-\frac{c^2\ln(1+e^{2i(bx+a)})}{b}+\frac{2c^2\ln(e^{i(bx+a)})}{b}+\frac{2d^2a^2\ln(e^{i(bx+a)})}{b^3}+idc\,x^2+\frac{id^2\,\mathrm{polylog}(2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I*d^2*x^3-1/3*I/d*c^3-4/3*I/b^3*a^3*d^2+2*I/b^2*a^2*c*d-1/b*c^2*ln(1+ex
p(2*I*(b*x+a)))+2/b*c^2*ln(exp(I*(b*x+a)))+2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))
+I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))+I*d*c*x^2-2/b*c*d*ln(1+exp(2*I*(b*x
+a)))*x-2*I/b^2*a^2*d^2*x-1/b*d^2*ln(1+exp(2*I*(b*x+a)))*x^2+I/b^2*d^2*poly
log(2,-exp(2*I*(b*x+a)))*x-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-4/b^2*c
*d*a*ln(exp(I*(b*x+a)))-I*c^2*x+4*I/b*a*c*d*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(81) = 162.
time = 0.53, size = 284, normalized size = 2.96

$$-\frac{3\,c^2\log\left(-\sin\left(bx+a\right)^2+1\right)-\frac{6\,acd\log\left(-\sin(bx+a)^2+1\right)}{b}+\frac{3\,a^2d^2\log\left(-\sin(bx+a)^2+1\right)}{b^2}+\frac{-2i\,(bx+a)^3d^2-6\,(i\,bcd-i\,ad^2)(bx+a)^2+3\,d^2\mathrm{Li}_3(-e^{(2i\,bx+2i\,a)})-6\left(-i\,(bx+a)^2d^2+2\,(-i\,bcd+i\,ad^2)(bx+a)\right)\arctan(\sin(2\,bx+2\,a),\cos(2\,bx+2\,a)+1)-6\,(i\,bcd+i\,(bx+a)d^2-i\,ad^2)\mathrm{Li}_2(-e^{(2i\,bx+2i\,a)})+3\left((bx+a)^2d^2+2\,(bcd-ad^2)(bx+a)\right)\log\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)}{b^2}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/6*(3*c^2*log(-sin(b*x + a)^2 + 1) - 6*a*c*d*log(-sin(b*x + a)^2 + 1)/b +
 3*a^2*d^2*log(-sin(b*x + a)^2 + 1)/b^2 + (-2*I*(b*x + a)^3*d^2 - 6*(I*b*c*
d - I*a*d^2)*(b*x + a)^2 + 3*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) - 6*(-I*(
b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2(sin(2*b*x + 2*a)
, cos(2*b*x + 2*a) + 1) - 6*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*dilog(-e^
(2*I*b*x + 2*I*a)) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1))/b^2)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 594 vs. 2(81) = 162.
time = 2.81, size = 594, normalized size = 6.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 2*d^2*polylog(3, I*
cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) + sin(b*x +
 a)) + 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 2*(I*b*d^2*x + I*
b*c*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) + 2*(-I*b*d^2*x - I*b*c*d)*dilo
```

```
g(I*cos(b*x + a) - sin(b*x + a)) + 2*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*
x + a) + sin(b*x + a)) + 2*(I*b*d^2*x + I*b*c*d)*dilog(-I*cos(b*x + a) - si
n(b*x + a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*x
+ a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*sin(b*x +
a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x +
 a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)
*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*
b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2
*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) +
 (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I) +
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/b^
3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \sin (a+bx) \sec (a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*sin(b*x+a),x)
[Out] Integral((c + d*x)**2*sin(a + b*x)*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^2*sec(b*x + a)*sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (a+b\,x)\,{\left(c+d\,x\right)}^2}{\cos (a+b\,x)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*(c + d*x)^2)/cos(a + b*x),x)
[Out] int((sin(a + b*x)*(c + d*x)^2)/cos(a + b*x), x)
```

3.212 $\int (c+dx)\tan(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{i(c+dx)^2}{2d}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}$$

```
[Out] 1/2*I*(d*x+c)^2/d-(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b+1/2*I*d*polylog(2,-exp(2
*I*(b*x+a)))/b^2
```

Rubi [A]
time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3800, 2221, 2317, 2438}

$$\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Tan[a + b*x],x]
```

```
[Out] ((I/2)*(c + d*x)^2)/d - ((c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b + ((I/2)
*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
```

```
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)\tan(a+bx)\,dx &= \frac{i(c+dx)^2}{2d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}\,dx \\
&= \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{d\int \log\left(1+e^{2i(a+bx)}\right)\,dx}{b} \\
&= \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{(id)\mathrm{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i(a+bx)}\right)}{2b^2} \\
&= \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}
\end{aligned}
$$

Mathematica [**A**]
time = 0.02, size = 70, normalized size = 1.06

$$
\frac{1}{2}idx^2 - \frac{dx\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{c\log(\cos(a+bx))}{b} + \frac{id\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Tan[a + b*x],x]
```

```
[Out] (I/2)*d*x^2 - (d*x*Log[1 + E^((2*I)*(a + b*x))])/b - (c*Log[Cos[a + b*x]])/
b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(56) = 112.
time = 0.07, size = 123, normalized size = 1.86

method	result
risch	$\frac{id\,x^2}{2} - icx - \frac{c\ln\left(1+\mathrm{e}^{2i(bx+a)}\right)}{b} + \frac{2c\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b} + \frac{2idax}{b} + \frac{id\,a^2}{b^2} - \frac{d\ln\left(1+\mathrm{e}^{2i(bx+a)}\right)x}{b} + \frac{id\,\mathrm{polylog}\left(2,-\mathrm{e}^{2i(bx+a)}\right)}{2b^2} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*d*x^2-I*c*x-1/b*c*ln(1+exp(2*I*(b*x+a)))+2/b*c*ln(exp(I*(b*x+a)))+2*I
/b*d*a*x+I/b^2*d*a^2-1/b*d*ln(1+exp(2*I*(b*x+a)))*x+1/2*I*d*polylog(2,-exp(
2*I*(b*x+a)))/b^2-2/b^2*d*a*ln(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(53) = 106.
time = 0.51, size = 115, normalized size = 1.74

$$-\frac{-i\,b^2dx^2 - 2i\,b^2cx - 2\,(-i\,bdx - i\,bc)\arctan\left(\sin\left(2\,bx + 2\,a\right), \cos\left(2\,bx + 2\,a\right) + 1\right) - i\,d\mathrm{Li}_2\left(-e^{(2i\,bx + 2i\,a)}\right) + (bdx + bc)\log\left(\cos\left(2\,bx + 2\,a\right)^2 + \sin\left(2\,bx + 2\,a\right)^2 + 2\,\cos\left(2\,bx + 2\,a\right) + 1\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(-I*b^2*d*x^2 - 2*I*b^2*c*x - 2*(-I*b*d*x - I*b*c)*arctan2(sin(2*b*x +
 2*a), cos(2*b*x + 2*a) + 1) - I*d*dilog(-e^(2*I*b*x + 2*I*a)) + (b*d*x + b
*c)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1))/
b^2
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(53) = 106.
time = 2.58, size = 310, normalized size = 4.70

$$\frac{-i\,d\mathrm{Li}_2(i\,\cos\left(bx + a\right) + \sin\left(bx + a\right)) + i\,d\mathrm{Li}_2(i\,\cos\left(bx + a\right) - \sin\left(bx + a\right)) + i\,d\mathrm{Li}_2(-i\,\cos\left(bx + a\right) + \sin\left(bx + a\right)) - i\,d\mathrm{Li}_2(-i\,\cos\left(bx + a\right) - \sin\left(bx + a\right)) - (bc - ad)\log\left(\cos\left(bx + a\right) + i\,\sin\left(bx + a\right) + i\right) - (bc - ad)\log\left(\cos\left(bx + a\right) - i\,\sin\left(bx + a\right) + i\right) - (bdx + ad)\log\left(i\,\cos\left(bx + a\right) + \sin\left(bx + a\right) + 1\right) - (bdx + ad)\log\left(i\,\cos\left(bx + a\right) - \sin\left(bx + a\right) + 1\right) - (bdx + ad)\log\left(-i\,\cos\left(bx + a\right) + \sin\left(bx + a\right) + 1\right) - (bdx + ad)\log\left(-i\,\cos\left(bx + a\right) - \sin\left(bx + a\right) + 1\right) - (bc - ad)\log\left(-\cos\left(bx + a\right) + i\,\sin\left(bx + a\right) + i\right) - (bc - ad)\log\left(-\cos\left(bx + a\right) - i\,\sin\left(bx + a\right) + i\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(I*cos(b*x + a) -
 sin(b*x + a)) + I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d*dilog(-I*c
os(b*x + a) - sin(b*x + a)) - (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a)
 + I) - (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d*x + a*d)*
log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*log(I*cos(b*x + a) -
 sin(b*x + a) + 1) - (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)
- (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b*c - a*d)*log(-
cos(b*x + a) + I*sin(b*x + a) + I) - (b*c - a*d)*log(-cos(b*x + a) - I*sin(
b*x + a) + I))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)\sin\left(a + bx\right)\sec\left(a + bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)*sin(a + b*x)*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*sin(b*x + a), x)
```

Mupad [B]
time = 1.57, size = 148, normalized size = 2.24

$$\frac{c\,\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b}-\frac{d\,\left(\pi\,\ln\left(\cos\left(b\,x\right)\right)+\mathrm{polylog}\left(2,-\mathrm{e}^{-a\,2\mathrm{i}}\,\mathrm{e}^{-b\,x\,2\mathrm{i}}\right)\,1\mathrm{i}-\pi\,\ln\left(\mathrm{e}^{-a\,2\mathrm{i}}\,\mathrm{e}^{-b\,x\,2\mathrm{i}}+1\right)+2\,a\,\ln\left(\mathrm{e}^{-a\,2\mathrm{i}}\,\mathrm{e}^{-b\,x\,2\mathrm{i}}+1\right)-\pi\,\ln\left(\mathrm{e}^{b\,x\,2\mathrm{i}}+1\right)+b^2\,x^2\,1\mathrm{i}-\ln\left(\cos\left(a+b\,x\right)\right)\,\left(2\,a-\pi\right)+2\,b\,x\,\ln\left(\mathrm{e}^{-a\,2\mathrm{i}}\,\mathrm{e}^{-b\,x\,2\mathrm{i}}+1\right)+a\,b\,x\,2\mathrm{i}\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*(c + d*x))/cos(a + b*x),x)
```

```
[Out] (c*log(tan(a + b*x)^2 + 1))/(2*b) - (d*(polylog(2, -exp(-a*2i)*exp(-b*x*2i)
)*1i - pi*log(exp(b*x*2i) + 1) - pi*log(exp(-a*2i)*exp(-b*x*2i) + 1) + 2*a*
log(exp(-a*2i)*exp(-b*x*2i) + 1) + pi*log(cos(b*x)) + b^2*x^2*1i - log(cos(
a + b*x))*(2*a - pi) + 2*b*x*log(exp(-a*2i)*exp(-b*x*2i) + 1) + a*b*x*2i))/
(2*b^2)
```

3.213 $\int \frac{\tan(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{\tan(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(tan(b*x+a)/(d*x+c),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tan(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Tan[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][Tan[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\tan(a+bx)}{c+dx}\,dx = \int \frac{\tan(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 3.70, size = 0, normalized size = 0.00

$$\int \frac{\tan(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[a + b*x]/(c + d*x),x]
```

```
[Out] Integrate[Tan[a + b*x]/(c + d*x), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)/(d*x+c),x)
[Out] int(sec(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x)
[Out] Integral(sin(a + b*x)*sec(a + b*x)/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\sin\left(a+b\,x\right)}{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(cos(a + b*x)*(c + d*x)),x)
[Out] int(sin(a + b*x)/(cos(a + b*x)*(c + d*x)), x)
```

3.214 $\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{\tan(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(tan(b*x+a)/(d*x+c)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Tan[a + b*x]/(c + d*x)^2,x]
[Out] Defer[Int][Tan[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 2.61, size = 0, normalized size = 0.00

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[a + b*x]/(c + d*x)^2,x]
[Out] Integrate[Tan[a + b*x]/(c + d*x)^2, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x)
[Out] int(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(sec(b*x + a)*sin(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)**2,x)
[Out] Integral(sin(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\sin\left(a+b\,x\right)}{\cos\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(cos(a + b*x)*(c + d*x)^2),x)
[Out] int(sin(a + b*x)/(cos(a + b*x)*(c + d*x)^2), x)
```

3.215 $\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=148

$$\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1\right.}{2b}$$

```
[Out] 1/2*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)
/d)^m)-1/2*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d
*x+c)/d)^m)+Unintegrable((d*x+c)^m*sec(b*x+a),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((I/2)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/
(b*(((-I)*b*(c + d*x))/d)^m) - ((I/2)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d*
x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Defer[Int][(c + d*x
)^m*Sec[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx &= -\int (c+dx)^m\cos(a+bx)\,dx+\int (c+dx)^m\sec(a+bx)\,dx\\ &= -\left(\frac{1}{2}\int e^{-i(a+bx)}(c+dx)^m\,dx\right)-\frac{1}{2}\int e^{i(a+bx)}(c+dx)^m\,dx+\int (c\\ &= \frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{-i\left(a-\frac{bc}{d}\right.}}{}\end{aligned}$$

Mathematica [A]
time = 6.47, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x], x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int (dx+c)^{m} \sec \left(bx+a\right) \left(\sin^{2}\left(bx+a\right)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x)
[Out] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
[Out] integral(-(cos(b*x + a)^2 - 1)*(d*x + c)^m*sec(b*x + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+dx\right)^{m} \sin^{2}\left(a+bx\right) \sec\left(a+bx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*sin(b*x+a)**2,x)
[Out] Integral((c + d*x)**m*sin(a + b*x)**2*sec(a + b*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^2\,{\left(c+d\,x\right)}^m}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^2*(c + d*x)^m)/cos(a + b*x),x)
```

```
[Out] int((sin(a + b*x)^2*(c + d*x)^m)/cos(a + b*x), x)
```

3.216 $\int (c+dx)^3 \sin(a+bx) \tan(a+bx)\, dx$

Optimal. Leaf size=275

$$-\frac{2i(c+dx)^3\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{6d^3\cos(a+bx)}{b^4}-\frac{3d(c+dx)^2\cos(a+bx)}{b^2}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+}\right.}{b^2}$$

```
[Out] -2*I*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b+6*d^3*cos(b*x+a)/b^4-3*d*(d*x+c)^2*
cos(b*x+a)/b^2+3*I*d*(d*x+c)^2*polylog(2,-I*exp(I*(b*x+a)))/b^2-3*I*d*(d*x+
c)^2*polylog(2,I*exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*polylog(3,-I*exp(I*(b*x+
a)))/b^3+6*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^3-6*I*d^3*polylog(4,-I
*exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4+6*d^2*(d*x+c)*
sin(b*x+a)/b^3-(d*x+c)^3*sin(b*x+a)/b
```

Rubi [A]
time = 0.17, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4492, 3377, 2718, 4266, 2611, 6744, 2320, 6724}

$$-\frac{2i(c+dx)^3\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}-\frac{6id^3\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^4}+\frac{6id^3\text{Li}_4\left(ie^{i(a+bx)}\right)}{b^4}+\frac{6d^3\cos(a+bx)}{b^4}-\frac{6d^2(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\sin(a+bx)}{b^3}+\frac{3id(c+dx)^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{3d(c+dx)^2\cos(a+bx)}{b^2}-\frac{(c+dx)^3\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + (6*d^3*Cos[a + b*x])/b^4 -
 (3*d*(c + d*x)^2*Cos[a + b*x])/b^2 + ((3*I)*d*(c + d*x)^2*PolyLog[2, (-I)*
E^(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])
/b^2 - (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 + (6*d^2*(c +
 d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4, (-I)*E^(I*
(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4 + (6*d^2*(
c + d*x)*Sin[a + b*x])/b^3 - ((c + d*x)^3*Sin[a + b*x])/b
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4492

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \sin(a+bx) \tan(a+bx)\, dx = -\int (c+dx)^3 \cos(a+bx)\, dx + \int (c+dx)^3 \sec(a+bx)\, dx$$

$$= -\frac{2i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(c+dx)^3 \sin(a+bx)}{b} - \frac{(3d)\int (c$$

$$= -\frac{2i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2 \cos(a+bx)}{b^2} + \frac{3id(c$$

$$= -\frac{2i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2 \cos(a+bx)}{b^2} + \frac{3id(c$$

$$= -\frac{2i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{6d^3 \cos(a+bx)}{b^4} - \frac{3d(c+dx)^2 \cos}{b^2}$$

$$= -\frac{2i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{6d^3 \cos(a+bx)}{b^4} - \frac{3d(c+dx)^2 \cos}{b^2}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 557 vs. 2(275) = 550.
time = 1.35, size = 557, normalized size = 2.03

$$-\frac{2ib^3c^3\text{ArcTan}\left(e^{i(a+bx)}\right)+3b^2c^2d\cos(a+bx)-6d^3\cos(a+bx)+6b^2cd^2x\cos(a+bx)+3b^2d^3x^2\cos(a+bx)-3b^3c^2dx\log\left(1-ie^{i(a+bx)}\right)-3b^3cd^2x^2\log\left(1-ie^{i(a+bx)}\right)-b^3d^3x^3\log\left(1-ie^{i(a+bx)}\right)+3b^3c^2dx\log\left(1+ie^{i(a+bx)}\right)+3b^3cd^2x^2\log\left(1+ie^{i(a+bx)}\right)+b^3d^3x^3\log\left(1+ie^{i(a+bx)}\right)-3ib^2d(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)+3ib^2d(c+dx)^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)+6bcd^2\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)+6bd^3x\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)-6bcd^2\text{PolyLog}\left(3,ie^{i(a+bx)}\right)-6bd^3x\text{PolyLog}\left(3,ie^{i(a+bx)}\right)+6id^3\text{PolyLog}\left(4,-ie^{i(a+bx)}\right)-6id^3\text{PolyLog}\left(4,ie^{i(a+bx)}\right)+b^3c^3\sin(a+bx)-6bcd^2\sin(a+bx)+3b^3c^2dx\sin(a+bx)-6bd^3x\sin(a+bx)+3b^3cd^2x^2\sin(a+bx)+b^3d^3x^3\sin(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] -((((2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^2*c^2*d*Cos[a + b*x] - 6*d^3
*Cos[a + b*x] + 6*b^2*c*d^2*x*Cos[a + b*x] + 3*b^2*d^3*x^2*Cos[a + b*x] - 3
*b^3*c^2*d*x*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a
 + b*x))] - b^3*d^3*x^3*Log[1 - I*E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 +
I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + b^3*d^3*x
^3*Log[1 + I*E^(I*(a + b*x))] - (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(
I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] + 6*b
*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a
 + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3,
I*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] - (6*I)*d^3
*PolyLog[4, I*E^(I*(a + b*x))] + b^3*c^3*Sin[a + b*x] - 6*b*c*d^2*Sin[a + b
*x] + 3*b^3*c^2*d*x*Sin[a + b*x] - 6*b*d^3*x*Sin[a + b*x] + 3*b^3*c*d^2*x^2
*Sin[a + b*x] + b^3*d^3*x^3*Sin[a + b*x])/b^4)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 900 vs. 2(250) = 500.
time = 0.43, size = 901, normalized size = 3.28

method	result
risch	$-\frac{3ic^2d\,\mathrm{polylog}(2,ie^{i(bx+a)})}{b^2}-\frac{3id^3\,\mathrm{polylog}(2,ie^{i(bx+a)})x^2}{b^2}+\frac{3id^3\,\mathrm{polylog}(2,-ie^{i(bx+a)})x^2}{b^2}+\frac{3ic^2d\,\mathrm{polylog}(2,-ie^{i(bx+a)})}{b^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -6*I/b^3*c*d^2*a^2*arctan(exp(I*(b*x+a)))+6*I/b^2*c^2*d*a*arctan(exp(I*(b*x
+a)))+6*I/b^2*d^2*c*polylog(2,-I*exp(I*(b*x+a)))*x-6*I*d^3*polylog(4,-I*exp
(I*(b*x+a)))/b^4-1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3
*I*b^2*d^3*x^2-6*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b
^4*exp(-I*(b*x+a))-6/b^3*d^3*polylog(3,-I*exp(I*(b*x+a)))*x+6/b^3*d^3*polyl
og(3,I*exp(I*(b*x+a)))*x-2*I/b*c^3*arctan(exp(I*(b*x+a)))+3/b^3*a^2*d^2*c*l
n(1+I*exp(I*(b*x+a)))+3/b*c^2*d*ln(1-I*exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-I
*exp(I*(b*x+a)))*a-3/b^3*a^2*d^2*c*ln(1-I*exp(I*(b*x+a)))-3/b*d^2*c*ln(1+I*
exp(I*(b*x+a)))*x^2+3/b*d^2*c*ln(1-I*exp(I*(b*x+a)))*x^2-3/b*c^2*d*ln(1+I*e
xp(I*(b*x+a)))*x-3/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a-3*I/b^2*c^2*d*polylog
(2,I*exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,I*exp(I*(b*x+a)))*x^2+3*I/b^2*d^
3*polylog(2,-I*exp(I*(b*x+a)))*x^2+2*I/b^4*d^3*a^3*arctan(exp(I*(b*x+a)))+3
*I/b^2*c^2*d*polylog(2,-I*exp(I*(b*x+a)))+6*I*d^3*polylog(4,I*exp(I*(b*x+a)
))/b^4+1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3
*x^2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(
b*x+a))+1/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3-1/b*d^3*ln(1+I*exp(I*(b*x+a)))*x
^3+1/b^4*a^3*d^3*ln(1-I*exp(I*(b*x+a)))-1/b^4*a^3*d^3*ln(1+I*exp(I*(b*x+a))
)-6/b^3*d^2*c*polylog(3,-I*exp(I*(b*x+a)))+6/b^3*d^2*c*polylog(3,I*exp(I*(b
*x+a)))-6*I/b^2*d^2*c*polylog(2,I*exp(I*(b*x+a)))*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 934 vs. $2(237) = 474$.
time = 0.61, size = 934, normalized size = 3.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^3*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a)) -
 3*a*c^2*d*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))
/b + 3*a^2*c*d^2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x
 + a))/b^2 - a^3*d^3*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin
(b*x + a))/b^3 + (12*I*d^3*polylog(4, I*e^(I*b*x + I*a)) - 12*I*d^3*polylog
(4, -I*e^(I*b*x + I*a)) - 2*(I*(b*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b
*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b*x + a))*arctan2(
cos(b*x + a), sin(b*x + a) + 1) - 2*(I*(b*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a
```

```
*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b*x + a))*
arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*
x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x + a)
- 6*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c
*d^2 - I*a*d^3)*(b*x + a))*dilog(I*e^(I*b*x + I*a)) - 6*(-I*b^2*c^2*d + 2*I
*a*b*c*d^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x
+ a))*dilog(-I*e^(I*b*x + I*a)) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b
*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x +
a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - ((b*x + a)^3*d^3 + 3*(b*c*d^2
 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*lo
g(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + 12*(b*c*d^2 + (b*
x + a)*d^3 - a*d^3)*polylog(3, I*e^(I*b*x + I*a)) - 12*(b*c*d^2 + (b*x + a)
*d^3 - a*d^3)*polylog(3, -I*e^(I*b*x + I*a)) - 2*((b*x + a)^3*d^3 - 6*b*c*d
^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2
 + (a^2 - 2)*d^3)*(b*x + a))*sin(b*x + a))/b^3)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1075 vs. $2(237) = 474$.
time = 5.48, size = 1075, normalized size = 3.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*polylog(4,
 I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) + sin(
b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 6*(b^2*d^3
*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a) - 3*(I*b^2*d^3*x^2 +
 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) - 3*(I
*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(b*x + a) - sin(b*
x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-I*cos(b
*x + a) + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d
)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*x + a) + I)
 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3
*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*l
og(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*
b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a)
+ sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) - sin(b*x + a) + 1
) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) +
 I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
```

```
log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b*d^3*x + b*c*d^2)*polylog(3,
I*cos(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x
 + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) +
sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x
+ a)) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2
*d - 2*b*d^3)*x)*sin(b*x + a))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \sin^2(a + bx) \sec(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*sin(b*x+a)**2,x)
[Out] Integral((c + d*x)**3*sin(a + b*x)**2*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(a + b\,x)^2\,(c + d\,x)^3}{\cos(a + b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^2*(c + d*x)^3)/cos(a + b*x),x)
[Out] int((sin(a + b*x)^2*(c + d*x)^3)/cos(a + b*x), x)
```

3.217 $\int (c+dx)^2 \sin(a+bx) \tan(a+bx) \, dx$

Optimal. Leaf size=186

$$-\frac{2i(c+dx)^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}-\frac{2d(c+dx)\cos(a+bx)}{b^2}+\frac{2id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{Po}}{}$$

```
[Out] -2*I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b-2*d*(d*x+c)*cos(b*x+a)/b^2+2*I*d*(d
*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylog(2,I*exp(I*(b*x
+a)))/b^2-2*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,I*exp(I*(b
*x+a)))/b^3+2*d^2*sin(b*x+a)/b^3-(d*x+c)^2*sin(b*x+a)/b
```

Rubi [A]
time = 0.10, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {4492, 3377, 2717, 4266, 2611, 2320, 6724}

$$-\frac{2i(c+dx)^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}-\frac{2d^2\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{2d^2\sin(a+bx)}{b^3}+\frac{2id(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{2d(c+dx)\cos(a+bx)}{b^2}-\frac{(c+dx)^2\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b - (2*d*(c + d*x)*Cos[a + b*x
])/b^2 + ((2*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((2*I)*
d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (2*d^2*PolyLog[3, (-I)*E^(
I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + (2*d^2*Sin
[a + b*x])/b^3 - ((c + d*x)^2*Sin[a + b*x])/b
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4492

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \sin(a+bx)\tan(a+bx)\,dx = -\int (c+dx)^2\cos(a+bx)\,dx + \int (c+dx)^2\sec(a+bx)\,dx$$
$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(c+dx)^2\sin(a+bx)}{b} - \frac{(2d)\int(c+$$
$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)\cos(a+bx)}{b^2} + \frac{2id(c+$$
$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)\cos(a+bx)}{b^2} + \frac{2id(c+$$
$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)\cos(a+bx)}{b^2} + \frac{2id(c+$$

Mathematica [A]
time = 0.79, size = 315, normalized size = 1.69

$$-\frac{2ib^2c^2\text{ArcTan}\left(e^{i(a+bx)}\right)+2bcd\cos(a+bx)+2bd^2x\cos(a+bx)-2b^2cdx\log\left(1-ie^{i(a+bx)}\right)-b^2d^2x^2\log\left(1-ie^{i(a+bx)}\right)+2b^2cdx\log\left(1+ie^{i(a+bx)}\right)+b^2d^2x^2\log\left(1+ie^{i(a+bx)}\right)-2ibd(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)+2ibd(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)+2d^2\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)-2d^2\text{PolyLog}\left(3,ie^{i(a+bx)}\right)+b^2c^2\sin(a+bx)-2d^2\sin(a+bx)+2b^2cdx\sin(a+bx)+b^2d^2x^2\sin(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] -(((2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b*c*d*Cos[a + b*x] + 2*b*d^2*x
*Cos[a + b*x] - 2*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1
- I*E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] + b^2*d^2*x^2
*Log[1 + I*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a +
 b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] + 2*d^2*PolyLog
[3, (-I)*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, I*E^(I*(a + b*x))] + b^2*c^2*S
in[a + b*x] - 2*d^2*Sin[a + b*x] + 2*b^2*c*d*x*Sin[a + b*x] + b^2*d^2*x^2*S
in[a + b*x])/b^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(169) = 338.
time = 0.31, size = 512, normalized size = 2.75

method	result
risch	$\frac{i\left(x^2d^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2\right)\mathrm{e}^{i(bx+a)}}{2b^3}-\frac{2id^2\,\mathrm{polylog}\left(2,i\mathrm{e}^{i(bx+a)}\right)x}{b^2}-\frac{a^2d^2\ln\left(1-i\mathrm{e}^{i(bx+a)}\right)}{b^3}-\frac{d^2\ln\left(1+i\mathrm{e}^{i(bx+}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp
(I*(b*x+a))-2*I/b^2*c*d*polylog(2,I*exp(I*(b*x+a)))-1/b^3*a^2*d^2*ln(1-I*ex
p(I*(b*x+a)))-1/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^2+2*I/b^2*d^2*polylog(2,-I*e
xp(I*(b*x+a)))*x-2*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3-2/b^2*c*d*ln(1+I*ex
p(I*(b*x+a)))*a+2/b*c*d*ln(1-I*exp(I*(b*x+a)))*x-2/b*c*d*ln(1+I*exp(I*(b*x+
a)))*x+2*I/b^2*c*d*polylog(2,-I*exp(I*(b*x+a)))-2*I/b^2*d^2*polylog(2,I*exp
(I*(b*x+a)))*x-2*I/b*c^2*arctan(exp(I*(b*x+a)))+1/b^3*a^2*d^2*ln(1+I*exp(I*
(b*x+a)))+1/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2+2/b^2*c*d*ln(1-I*exp(I*(b*x+a)
))*a-2*I/b^3*d^2*a^2*arctan(exp(I*(b*x+a)))+2*d^2*polylog(3,I*exp(I*(b*x+a)
))/b^3-1/2*I*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)/
b^3*exp(-I*(b*x+a))+4*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 516 vs. 2(160) = 320.
time = 0.58, size = 516, normalized size = 2.77

$$\frac{c^2\left(\log\left(\sin\left(bx+a\right)+1\right)-\log\left(\sin\left(bx+a\right)-1\right)-2\sin\left(bx+a\right)\right)-\frac{2acd\left(\log(\sin(bx+a)+1)-\log(\sin(bx+a)-1)-2\sin(bx+a)\right)}{b}+\frac{a^2d^2\left(\log(\sin(bx+a)+1)-\log(\sin(bx+a)-1)-2\sin(bx+a)\right)}{b^2}+\frac{4d^2\mathrm{Li}_3\left(ie^{(ibx+ia)}\right)-4d^2\mathrm{Li}_3\left(-ie^{(ibx+ia)}\right)-2\left(i(bx+a)^2d^2+2\left(ibcd-iad^2\right)(bx+a)\right)\arctan\left(\cos(bx+a),\sin(bx+a)+1\right)-2\left(i(bx+a)^2d^2+2\left(ibcd-iad^2\right)(bx+a)\right)\arctan\left(\cos(bx+a),-\sin(bx+a)+1\right)-4\left(bcd+(bx+a)d^2-ad^2\right)\cos(bx+a)-4\left(ibcd+i(bx+a)d^2-iad^2\right)\mathrm{Li}_2\left(ie^{(ibx+ia)}\right)-4\left(-ibcd-i(bx+a)d^2+iad^2\right)\mathrm{Li}_2\left(-ie^{(ibx+ia)}\right)+\left((bx+a)^2d^2+2\left(bcd-ad^2\right)(bx+a)\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2+2\sin(bx+a)+1\right)-\left((bx+a)^2d^2+2\left(bcd-ad^2\right)(bx+a)\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2-2\sin(bx+a)+1\right)-2\left((bx+a)^2d^2+2\left(bcd-ad^2\right)(bx+a)-2d^2\right)\sin(bx+a)}{b^2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a)) -
 2*a*c*d*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))/b
 + a^2*d^2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))
/b^2 + (4*d^2*polylog(3, I*e^(I*b*x + I*a)) - 4*d^2*polylog(3, -I*e^(I*b*x
+ I*a)) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*arctan2(c
os(b*x + a), sin(b*x + a) + 1) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^
2)*(b*x + a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 4*(b*c*d + (b*x +
a)*d^2 - a*d^2)*cos(b*x + a) - 4*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*dilo
g(I*e^(I*b*x + I*a)) - 4*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*dilog(-I*e^
(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*
x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - ((b*x + a)^2*d^2 + 2*(b*c
*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a)
 + 1) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*sin(b*x +
 a))/b^2)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(160) = 320$.
time = 1.63, size = 656, normalized size = 3.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*polylog(3, I*
cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) + sin(b*x +
 a)) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 4*(b*d^2*x + b*c*
d)*cos(b*x + a) + 2*(I*b*d^2*x + I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*x +
a)) + 2*(I*b*d^2*x + I*b*c*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + 2*(-I*
b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 2*(-I*b*d^2*x -
I*b*c*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2
*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 +
 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1)
- (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + s
in(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I
*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-co
s(b*x + a) + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos
(b*x + a) - I*sin(b*x + a) + I) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 -
2*d^2)*sin(b*x + a))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \sin^2(a+bx) \sec(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*sin(b*x+a)**2,x)
[Out] Integral((c + d*x)**2*sin(a + b*x)**2*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^2*sec(b*x + a)*sin(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^2\,(c+d\,x)^2}{\cos(a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^2*(c + d*x)^2)/cos(a + b*x),x)
[Out] int((sin(a + b*x)^2*(c + d*x)^2)/cos(a + b*x), x)
```

3.218 $\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=103

$$-\frac{2i(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}-\frac{d\cos(a+bx)}{b^2}+\frac{id\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)\,\text{s}}{}$$

```
[Out] -2*I*(d*x+c)*arctan(exp(I*(b*x+a)))/b-d*cos(b*x+a)/b^2+I*d*polylog(2,-I*exp
(I*(b*x+a)))/b^2-I*d*polylog(2,I*exp(I*(b*x+a)))/b^2-(d*x+c)*sin(b*x+a)/b
```

Rubi [A]
time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4492, 3377, 2718, 4266, 2317, 2438}

$$-\frac{2i(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{d\cos(a+bx)}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b - (d*Cos[a + b*x])/b^2 + (I*d*
PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])
/b^2 - ((c + d*x)*Sin[a + b*x])/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4492

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx &= -\int (c+dx)\cos(a+bx)\,dx + \int (c+dx)\sec(a+bx)\,dx\\ &= -\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(c+dx)\sin(a+bx)}{b} - \frac{d\int \log\left(1-\right.}{}\\ &= -\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)}{b^2} - \frac{(c+dx)\sin(a+bx}{b}\\ &= -\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)}{b^2} + \frac{id\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2} -\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 213 vs. 2(103) = 206.
time = 0.46, size = 213, normalized size = 2.07

$$\frac{c\tanh^{-1}(\sin(a+bx))}{b}+\frac{d\left(\left(-a+\frac{\pi}{2}-bx\right)\left(\log\left(1-e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)-\log\left(1+e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)\right)-\left(-a+\frac{\pi}{2}\right)\log\left(\tan\left(\frac{1}{2}\left(-a+\frac{\pi}{2}-bx\right)\right)\right)+i\left(\mathrm{PolyLog}\left(2,-e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)-\mathrm{PolyLog}\left(2,e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)\right)\right)}{b^2}-\frac{d\cos(bx)(\cos(a)+bx\sin(a))}{b^2}-\frac{d(bx\cos(a)-\sin(a))\sin(bx)}{b^2}-\frac{c\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] (c*ArcTanh[Sin[a + b*x]])/b + (d*((-a + Pi/2 - b*x)*(Log[1 - E^(I*(-a + Pi/
2 - b*x))] - Log[1 + E^(I*(-a + Pi/2 - b*x))]) - (-a + Pi/2)*Log[Tan[(-a +
Pi/2 - b*x)/2]] + I*(PolyLog[2, -E^(I*(-a + Pi/2 - b*x))] - PolyLog[2, E^(I
*(-a + Pi/2 - b*x))])))/b^2 - (d*Cos[b*x]*(Cos[a] + b*x*Sin[a]))/b^2 - (d*(
b*x*Cos[a] - Sin[a])*Sin[b*x])/b^2 - (c*Sin[a + b*x])/b
```

Maple [A]
time = 0.11, size = 180, normalized size = 1.75

method	result
default	$\frac{\frac{da\sin(bx+a)}{b}-c\sin(bx+a)-\frac{d(\cos(bx+a)+(bx+a)\sin(bx+a))}{b}}{b}+\frac{-\frac{da\ln(\sec(bx+a)+\tan(bx+a))}{b}+c\ln(\sec(bx+a)+\tan(bx+a))+\frac{d\left(-(bx+a)\ln\right.}{}}{}$
risch	$\frac{i(dxb+cb+id)e^{i(bx+a)}}{2b^2}-\frac{i(dxb+cb-id)e^{-i(bx+a)}}{2b^2}-\frac{2ic\arctan\left(e^{i(bx+a)}\right)}{b}-\frac{d\ln\left(1+ie^{i(bx+a)}\right)x}{b}-\frac{d\ln\left(1+ie^{i(bx+a)}\right)a}{b^2}+\frac{d\ln(1}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/b*d*a*sin(b*x+a)-c*sin(b*x+a)-1/b*d*(cos(b*x+a)+(b*x+a)*sin(b*x+a)))
+1/b*(-1/b*d*a*ln(sec(b*x+a)+tan(b*x+a))+c*ln(sec(b*x+a)+tan(b*x+a))+1/b*d*
(-(b*x+a)*ln(1+I*exp(I*(b*x+a)))+(b*x+a)*ln(1-I*exp(I*(b*x+a)))+I*dilog(1+I
*exp(I*(b*x+a)))-I*dilog(1-I*exp(I*(b*x+a)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*b^2*d*integrate((x*cos(2*b*x + 2*a)*cos(b*x + a) + x*sin(2*b*x + 2*a
)*sin(b*x + a) + x*cos(b*x + a))/(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 +
 2*cos(2*b*x + 2*a) + 1), x) + b*c*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*
sin(b*x + a) + 1) - b*c*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a
) + 1) - 2*d*cos(b*x + a) - 2*(b*d*x + b*c)*sin(b*x + a))/b^2
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(88) = 176$.
time = 1.94, size = 331, normalized size = 3.21

$$-\frac{2\,d\cos(bx+a)+i\,d\,\mathrm{Li}_2(i\cos(bx+a)+\sin(bx+a))+i\,d\,\mathrm{Li}_2(i\cos(bx+a)-\sin(bx+a))-i\,d\,\mathrm{Li}_2(-i\cos(bx+a)+\sin(bx+a))-i\,d\,\mathrm{Li}_2(-i\cos(bx+a)-\sin(bx+a))-(bc-ad)\log(\cos(bx+a)+i\sin(bx+a)+i)+(bc-ad)\log(\cos(bx+a)-i\sin(bx+a)+i)-(bdx+ad)\log(i\cos(bx+a)+\sin(bx+a)+1)+(bdx+ad)\log(i\cos(bx+a)-\sin(bx+a)+1)-(bdx+ad)\log(-i\cos(bx+a)+\sin(bx+a)+1)+(bdx+ad)\log(-i\cos(bx+a)-\sin(bx+a)+1)-(bc-ad)\log(-\cos(bx+a)+i\sin(bx+a)+i)+(bc-ad)\log(-\cos(bx+a)-i\sin(bx+a)+i)+2\,(bdx+bc)\sin(bx+a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*cos(b*x + a) + I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*dil
og(I*cos(b*x + a) - sin(b*x + a)) - I*d*dilog(-I*cos(b*x + a) + sin(b*x + a
)) - I*d*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b*c - a*d)*log(cos(b*x +
a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) +
I) - (b*d*x + a*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a*d)*l
og(I*cos(b*x + a) - sin(b*x + a) + 1) - (b*d*x + a*d)*log(-I*cos(b*x + a) +
 sin(b*x + a) + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)
```

```
- (b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(-co
s(b*x + a) - I*sin(b*x + a) + I) + 2*(b*d*x + b*c)*sin(b*x + a))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\sin^2(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)**2,x)
[Out] Integral((c + d*x)*sin(a + b*x)**2*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)*sec(b*x + a)*sin(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^2\,(c+d\,x)}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^2*(c + d*x))/cos(a + b*x),x)
[Out] int((sin(a + b*x)^2*(c + d*x))/cos(a + b*x), x)
```

3.219 $\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=69

$$-\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\text{Int}\left(\frac{\sec(a+bx)}{c+dx},x\right)$$

```
[Out] -Ci(b*c/d+b*x)*cos(a-b*c/d)/d+Si(b*c/d+b*x)*sin(a-b*c/d)/d+Unintegrable(sec
(b*x+a)/(d*x+c),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] -((Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d) + (Sin[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d + Defer[Int][Sec[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx &= -\int \frac{\cos(a+bx)}{c+dx}\,dx+\int \frac{\sec(a+bx)}{c+dx}\,dx\\ &= -\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\right)+\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\\ &= -\frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\int \frac{\sec(a+bx)}{c+dx}\,dx\end{aligned}$$

Mathematica [A]
time = 2.99, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x), x]
```

Maple [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\left(\sin^2(bx+a)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*((exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x
+ I*b*c)/d))*cos(-(b*c - a*d)/d) + 4*d*integrate((cos(2*b*x + 2*a)*cos(b*x
+ a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/((d*x + c)*cos(2*b*x +
 2*a)^2 + (d*x + c)*sin(2*b*x + 2*a)^2 + d*x + 2*(d*x + c)*cos(2*b*x + 2*a)
 + c), x) + (-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(1
, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))/d
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**2*sec(a + b*x)/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(sec(b*x + a)*sin(b*x + a)^2/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^2}{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(cos(a + b*x)*(c + d*x)),x)
[Out] int(sin(a + b*x)^2/(cos(a + b*x)*(c + d*x)), x)
```

3.220 $\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=87

$$\frac{\cos(a+bx)}{d(c+dx)}+\frac{b\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2}+\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\text{Int}\left(\frac{\sec(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] cos(b*x+a)/d/(d*x+c)+b*cos(a-b*c/d)*Si(b*c/d+b*x)/d^2+b*Ci(b*c/d+b*x)*sin(a
-b*c/d)/d^2+Unintegrable(sec(b*x+a)/(d*x+c)^2,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Cos[a + b*x]/(d*(c + d*x)) + (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d]
)/d^2 + (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int][Se
c[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx &= -\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx+\int \frac{\sec(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{b\int \frac{\sin(a+bx)}{c+dx}\,dx}{d}+\int \frac{\sec(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{b\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2}+\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\int
\end{aligned}$$

Mathematica [A]
time = 7.26, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

Maple [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\sec \left(bx + a\right) \left(\sin^{2}\left(bx + a\right)\right)}{\left(dx + c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x
+ I*b*c)/d))*cos(-(b*c - a*d)/d) + 4*(d^2*x + c*d)*integrate((cos(2*b*x + 2
*a)*cos(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(d^2*x^2 +
 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x + 2*a)^2 + (d^2*x^2 + 2*c*d*
x + c^2)*sin(2*b*x + 2*a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x +
 2*a)), x) + (-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(
2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))/(d^2*x + c*d)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}\left(a + bx\right) \sec \left(a + bx\right)}{\left(c + dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^2}{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(cos(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(sin(a + b*x)^2/(cos(a + b*x)*(c + d*x)^2), x)
```

3.221 $\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=152

$$\frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-3-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-}}{b}$$

```
[Out] 2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*b*
(d*x+c)/d)^m)+2^(-3-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a-b*
c/d))/((I*b*(d*x+c)/d)^m)+Unintegrable((d*x+c)^m*tan(b*x+a),x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c +
 d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (2^(-3 - m)*(c + d*x)^m*Gamma[1 +
 m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^
m) + Defer[Int][(c + d*x)^m*Tan[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx &= -\int (c+dx)^m\cos(a+bx)\sin(a+bx)\,dx+\int (c+dx)^m\tan(a+\\ &= -\int \frac{1}{2}(c+dx)^m\sin(2a+2bx)\,dx+\int (c+dx)^m\tan(a+bx)\,dx\\ &= -\left(\frac{1}{2}\int (c+dx)^m\sin(2a+2bx)\,dx\right)+\int (c+dx)^m\tan(a+bx)\,d\\ &= -\left(\frac{1}{4}i\int e^{-i(2a+2bx)}(c+dx)^m\,dx\right)+\frac{1}{4}i\int e^{i(2a+2bx)}(c+dx)^m\,dx\\ &= \frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}+\end{aligned}$$

Mathematica [A]
time = 7.70, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Sin[a + b*x]^2*Tan[a + b*x], x]
```

Maple [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a) \left(\sin^3(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*(d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^m}{\cos\left(a+b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^3*(c + d*x)^m)/cos(a + b*x),x)
[Out] int((sin(a + b*x)^3*(c + d*x)^m)/cos(a + b*x), x)
```

3.222 $\int (c+dx)^3 \sin^2(a+bx) \tan(a+bx) \, dx$

Optimal. Leaf size=251

$$-\frac{3d^3x}{8b^3}+\frac{(c+dx)^3}{4b}+\frac{i(c+dx)^4}{4d}-\frac{(c+dx)^3 \log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2 \text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3d^2(c+$$

```
[Out] -3/8*d^3*x/b^3+1/4*(d*x+c)^3/b+1/4*I*(d*x+c)^4/d-(d*x+c)^3*ln(1+exp(2*I*(b*
x+a)))/b+3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)
*polylog(3,-exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^
4+3/8*d^3*cos(b*x+a)*sin(b*x+a)/b^4-3/4*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b
^2+3/4*d^2*(d*x+c)*sin(b*x+a)^2/b^3-1/2*(d*x+c)^3*sin(b*x+a)^2/b
```

Rubi [A]

time = 0.20, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {4492, 4489, 3392, 32, 2715, 8, 3800, 2221, 2611, 6744, 2320, 6724}

$$-\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^3\sin(a+bx)\cos(a+bx)}{8b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3}+\frac{3id(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{3d(c+dx)^2\sin(a+bx)\cos(a+bx)}{4b^2}-\frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{3d^3x}{8b^3}+\frac{(c+dx)^3}{4b}+\frac{i(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (-3*d^3*x)/(8*b^3) + (c + d*x)^3/(4*b) + ((I/4)*(c + d*x)^4)/d - ((c + d*x)
^3*Log[1 + E^((2*I)*(a + b*x))])/b + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E
^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))
])/(2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (3*d^3*
Cos[a + b*x]*Sin[a + b*x])/(8*b^4) - (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a +
b*x])/(4*b^2) + (3*d^2*(c + d*x)*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^3*Sin
[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
```

```
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4492

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \sin^2(a+bx)\tan(a+bx)\,dx = -\int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx + \int (c+dx)^3 \tan(a+b$$

$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1+e^{2i(a+bx)}}\,dx$$

$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\cos(a+}{4b^2}$$

$$= \frac{(c+dx)^3}{4b} + \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+}{}$$

$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} + \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}$$

$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} + \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}$$

$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} + \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1734 vs. $2(251) = 502$.

time = 6.43, size = 1734, normalized size = 6.91

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (c*d^2*((2*I)*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(1 + E^((2*I)*a))*Log[1 +
E^((2*I)*(a + b*x))]) + (6*I)*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((2*I)*(a
 + b*x))] - 3*(1 + E^((2*I)*a))*PolyLog[3, -E^((2*I)*(a + b*x))])*Sec[a])/(
4*b^3*E^(I*a)) - (I/4)*d^3*E^(I*a)*(-x^4 + (1 + E^((-2*I)*a))*x^4 - ((1 + E
^((2*I)*a))*(2*b^4*x^4 + (4*I)*b^3*x^3*Log[1 + E^((2*I)*(a + b*x))] + 6*b^2
*x^2*PolyLog[2, -E^((2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[3, -E^((2*I)*(a +
 b*x))] - 3*PolyLog[4, -E^((2*I)*(a + b*x))]))/(2*b^4*E^((2*I)*a)))*Sec[a]
- (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))
/(b*(Cos[a]^2 + Sin[a]^2)) - (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]
) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] -
2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log
[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2,
 E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqr
t[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + Sec[a]*(Cos[2*a + 2*b*x]/(64*b^4) - ((
I/64)*Sin[2*a + 2*b*x])/b^4)*(8*b^3*c^3*Cos[a] - (12*I)*b^2*c^2*d*Cos[a] -
12*b*c*d^2*Cos[a] + (6*I)*d^3*Cos[a] + 24*b^3*c^2*d*x*Cos[a] - (24*I)*b^2*c
*d^2*x*Cos[a] - 12*b*d^3*x*Cos[a] + 24*b^3*c*d^2*x^2*Cos[a] - (12*I)*b^2*d^
3*x^2*Cos[a] + 8*b^3*d^3*x^3*Cos[a] + (32*I)*b^4*c^3*x*Cos[a + 2*b*x] + (48
*I)*b^4*c^2*d*x^2*Cos[a + 2*b*x] + (32*I)*b^4*c*d^2*x^3*Cos[a + 2*b*x] + (8
*I)*b^4*d^3*x^4*Cos[a + 2*b*x] - (32*I)*b^4*c^3*x*Cos[3*a + 2*b*x] - (48*I)
*b^4*c^2*d*x^2*Cos[3*a + 2*b*x] - (32*I)*b^4*c*d^2*x^3*Cos[3*a + 2*b*x] - (
8*I)*b^4*d^3*x^4*Cos[3*a + 2*b*x] + 4*b^3*c^3*Cos[3*a + 4*b*x] + (6*I)*b^2*
c^2*d*Cos[3*a + 4*b*x] - 6*b*c*d^2*Cos[3*a + 4*b*x] - (3*I)*d^3*Cos[3*a + 4
*b*x] + 12*b^3*c^2*d*x*Cos[3*a + 4*b*x] + (12*I)*b^2*c*d^2*x*Cos[3*a + 4*b*
x] - 6*b*d^3*x*Cos[3*a + 4*b*x] + 12*b^3*c*d^2*x^2*Cos[3*a + 4*b*x] + (6*I)
*b^2*d^3*x^2*Cos[3*a + 4*b*x] + 4*b^3*d^3*x^3*Cos[3*a + 4*b*x] + 4*b^3*c^3*
Cos[5*a + 4*b*x] + (6*I)*b^2*c^2*d*Cos[5*a + 4*b*x] - 6*b*c*d^2*Cos[5*a + 4
*b*x] - (3*I)*d^3*Cos[5*a + 4*b*x] + 12*b^3*c^2*d*x*Cos[5*a + 4*b*x] + (12*
I)*b^2*c*d^2*x*Cos[5*a + 4*b*x] - 6*b*d^3*x*Cos[5*a + 4*b*x] + 12*b^3*c*d^2
*x^2*Cos[5*a + 4*b*x] + (6*I)*b^2*d^3*x^2*Cos[5*a + 4*b*x] + 4*b^3*d^3*x^3*
Cos[5*a + 4*b*x] - 32*b^4*c^3*x*Sin[a + 2*b*x] - 48*b^4*c^2*d*x^2*Sin[a + 2
*b*x] - 32*b^4*c*d^2*x^3*Sin[a + 2*b*x] - 8*b^4*d^3*x^4*Sin[a + 2*b*x] + 32
*b^4*c^3*x*Sin[3*a + 2*b*x] + 48*b^4*c^2*d*x^2*Sin[3*a + 2*b*x] + 32*b^4*c*
d^2*x^3*Sin[3*a + 2*b*x] + 8*b^4*d^3*x^4*Sin[3*a + 2*b*x] + (4*I)*b^3*c^3*S
in[3*a + 4*b*x] - 6*b^2*c^2*d*Sin[3*a + 4*b*x] - (6*I)*b*c*d^2*Sin[3*a + 4*
b*x] + 3*d^3*Sin[3*a + 4*b*x] + (12*I)*b^3*c^2*d*x*Sin[3*a + 4*b*x] - 12*b^
2*c*d^2*x*Sin[3*a + 4*b*x] - (6*I)*b*d^3*x*Sin[3*a + 4*b*x] + (12*I)*b^3*c*
d^2*x^2*Sin[3*a + 4*b*x] - 6*b^2*d^3*x^2*Sin[3*a + 4*b*x] + (4*I)*b^3*d^3*x
```

```
^3*Sin[3*a + 4*b*x] + (4*I)*b^3*c^3*Sin[5*a + 4*b*x] - 6*b^2*c^2*d*Sin[5*a
+ 4*b*x] - (6*I)*b*c*d^2*Sin[5*a + 4*b*x] + 3*d^3*Sin[5*a + 4*b*x] + (12*I)
*b^3*c^2*d*x*Sin[5*a + 4*b*x] - 12*b^2*c*d^2*x*Sin[5*a + 4*b*x] - (6*I)*b*d
^3*x*Sin[5*a + 4*b*x] + (12*I)*b^3*c*d^2*x^2*Sin[5*a + 4*b*x] - 6*b^2*d^3*x
^2*Sin[5*a + 4*b*x] + (4*I)*b^3*d^3*x^3*Sin[5*a + 4*b*x])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 649 vs. 2(220) = 440.
time = 0.23, size = 650, normalized size = 2.59

method	result
risch	$\frac{i d^{3} x^{4}}{4}+\frac{2 c^{3} \ln \left({\mathrm e}^{i \left(b x +a \right)}\right)}{b}-i c^{3} x -\frac{i c^{4}}{4 d}-\frac{6 i c \,d^{2} a^{2} x}{b^{2}}+\frac{6 i c^{2} d a x}{b}+\frac{3 i d^{3} a^{4}}{2 b^{4}}+i d^{2} c \,x^{3}+\frac{3 i d \,c^{2} x^{2}}{2}-\frac{2 d^{3} a^{3} \ln \left({\mathrm e}^{i \left(b x +a \right)}\right)}{b^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] -2/b^4*d^3*a^3*ln(exp(I*(b*x+a)))+1/32*(4*d^3*x^3*b^3+6*I*b^2*d^3*x^2+12*b^
3*c*d^2*x^2+12*I*b^2*c*d^2*x+12*b^3*c^2*d*x+6*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3
*x-3*I*d^3-6*c*d^2*b)/b^4*exp(2*I*(b*x+a))+1/32*(4*d^3*x^3*b^3-6*I*b^2*d^3*
x^2+12*b^3*c*d^2*x^2-12*I*b^2*c*d^2*x+12*b^3*c^2*d*x-6*I*b^2*c^2*d+4*b^3*c^
3-6*b*d^3*x+3*I*d^3-6*c*d^2*b)/b^4*exp(-2*I*(b*x+a))+2/b*c^3*ln(exp(I*(b*x+
a)))-I*c^3*x-1/4*I/d*c^4+3/2*I*d*c^2*x^2+I*d^2*c*x^3+3*I/b^2*a^2*c^2*d+1/4*
I*d^3*x^4-6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))-3/4*I*d^3*polylog(4,-exp(2*I*(b*
x+a)))/b^4+3/2*I/b^4*d^3*a^4-1/b*c^3*ln(1+exp(2*I*(b*x+a)))-3/2/b^3*c*d^2*p
olylog(3,-exp(2*I*(b*x+a)))-3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+6/b^
3*c*d^2*a^2*ln(exp(I*(b*x+a)))-6*I/b^2*a^2*c*d^2*x+6*I/b*a*c^2*d*x+3/2*I/b^
2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2+3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(
b*x+a)))-3/b*c^2*d*ln(1+exp(2*I*(b*x+a)))*x-3/b*c*d^2*ln(1+exp(2*I*(b*x+a))
)*x^2-1/b*d^3*ln(1+exp(2*I*(b*x+a)))*x^3+2*I/b^3*d^3*a^3*x-4*I/b^3*a^3*c*d^
2+3*I/b^2*c*d^2*polylog(2,-exp(2*I*(b*x+a)))*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 692 vs. 2(216) = 432.
time = 0.58, size = 692, normalized size = 2.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")

[Out] -1/48*(24*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))*c^3 - 72*(sin(b*x + a)
^2 + log(sin(b*x + a)^2 - 1))*a*c^2*d/b + 72*(sin(b*x + a)^2 + log(sin(b*x
+ a)^2 - 1))*a^2*c*d^2/b^2 - 24*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))*
a^3*d^3/b^3 + (-12*I*(b*x + a)^4*d^3 - 48*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^3
 + 48*I*d^3*polylog(4, -e^(2*I*b*x + 2*I*a)) - 72*(I*b^2*c^2*d - 2*I*a*b*c*
```

```
d^2 + I*a^2*d^3)*(b*x + a)^2 - 16*(-4*I*(b*x + a)^3*d^3 + 9*(-I*b*c*d^2 + I
*a*d^3)*(b*x + a)^2 + 9*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a
))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(2*(b*x + a)^3*d^3 -
 3*b*c*d^2 + 3*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4
*a*b*c*d^2 + (2*a^2 - 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 24*(3*I*b^2*c^2
*d - 6*I*a*b*c*d^2 + 4*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + 6*(I*b*c*d^2 - I*a
*d^3)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) + 8*(4*(b*x + a)^3*d^3 + 9*(b*
c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a
))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) +
24*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*polylog(3, -e^(2*I*b*x + 2*I*a))
 + 9*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a^2 - 1)*d^3 + 4*(
b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))/b^3)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(216) = 432$.
time = 2.09, size = 1134, normalized size = 4.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 24*I*d^3*polylog(4, I*cos(b*x + a)
+ sin(b*x + a)) + 24*I*d^3*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 24*I
*d^3*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) - 24*I*d^3*polylog(4, -I*co
s(b*x + a) - sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3
 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b^2*d^3*x^2
 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 3*(2*b^3*
c^2*d - b*d^3)*x + 12*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog
(I*cos(b*x + a) + sin(b*x + a)) + 12*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*
b^2*c^2*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + 12*(-I*b^2*d^3*x^2 - 2*I*
b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 12*(I*b^
2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x
+ a)) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x +
 a) + I*sin(b*x + a) + I) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*log(cos(b*x + a) - I*sin(b*x + a) + I) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^
2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(
b*x + a) + sin(b*x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2
*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*
x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 4*
(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*
d^2 + a^3*d^3)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + 4*(b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + I)
 + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a)
```

```
- I*sin(b*x + a) + I) + 24*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) +
sin(b*x + a)) + 24*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) - sin(b*x
+ a)) + 24*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) +
 24*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)))/b^4
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*sin(b*x+a)**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(a+b\,x)^3\,(c+d\,x)^3}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^3*(c + d*x)^3)/cos(a + b*x),x)
[Out] int((sin(a + b*x)^3*(c + d*x)^3)/cos(a + b*x), x)
```

3.223 $\int (c+dx)^2 \sin^2(a+bx) \tan(a+bx) \, dx$

Optimal. Leaf size=184

$$\frac{cdx}{2b}+\frac{d^2x^2}{4b}+\frac{i(c+dx)^3}{3d}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i\right.}}{2b^3}$$

```
[Out] 1/2*c*d*x/b+1/4*d^2*x^2/b+1/3*I*(d*x+c)^3/d-(d*x+c)^2*ln(1+exp(2*I*(b*x+a))
)/b+I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,-exp(2*I
*(b*x+a)))/b^3-1/2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2+1/4*d^2*sin(b*x+a)^2
/b^3-1/2*(d*x+c)^2*sin(b*x+a)^2/b
```

Rubi [A]
time = 0.15, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4492, 4489, 3391, 3800, 2221, 2611, 2320, 6724}

$$-\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\sin^2(a+bx)}{4b^3}+\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}+\frac{cdx}{2b}+\frac{d^2x^2}{4b}+\frac{i(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (c*d*x)/(2*b) + (d^2*x^2)/(4*b) + ((I/3)*(c + d*x)^3)/d - ((c + d*x)^2*Log[
1 + E^((2*I)*(a + b*x))])/b + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x)
)])/b^2 - (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) - (d*(c + d*x)*Cos
[a + b*x]*Sin[a + b*x])/(2*b^2) + (d^2*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)
^2*Sin[a + b*x]^2)/(2*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4492

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \sin^2(a+bx)\tan(a+bx)\,dx = -\int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx + \int (c+dx)^2\tan(a+bx$$

$$= \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}\,dx +$$

$$= \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cos(a+bx)}{2b^2}$$

$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} + \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id(c+}{}$$

$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} + \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id(c+}{}$$

$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} + \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id(c+}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 525 vs. $2(184) = 368$.
time = 6.41, size = 525, normalized size = 2.85

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (d^2*((2*I)*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(1 + E^((2*I)*a))*Log[1 + E^
((2*I)*(a + b*x))]) + (6*I)*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((2*I)*(a +
 b*x))] - 3*(1 + E^((2*I)*a))*PolyLog[3, -E^((2*I)*(a + b*x))])*Sec[a])/(12
*b^3*E^(I*a)) - (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]]
+ b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcT
an[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I
)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))
] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*
PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/
(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Cos[2*b*x]*(2*b^2*c^2*Cos[2*a
] - d^2*Cos[2*a] + 4*b^2*c*d*x*Cos[2*a] + 2*b^2*d^2*x^2*Cos[2*a] - 2*b*c*d*
Sin[2*a] - 2*b*d^2*x*Sin[2*a]))/(8*b^3) - ((2*b*c*d*Cos[2*a] + 2*b*d^2*x*Co
s[2*a] + 2*b^2*c^2*Sin[2*a] - d^2*Sin[2*a] + 4*b^2*c*d*x*Sin[2*a] + 2*b^2*d
^2*x^2*Sin[2*a])*Sin[2*b*x])/(8*b^3) + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tan[a
])/3
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(162) = 324$.
time = 0.32, size = 388, normalized size = 2.11

method	result
risch	$\frac{id^2x^3}{3} - \frac{ic^3}{3d} + \frac{icd\,\mathrm{polylog}(2,-e^{2i(bx+a)})}{b^2} - \frac{2id^2a^2x}{b^2} + \frac{(2x^2d^2b^2+4b^2cdx+2ib\,d^2x+2b^2c^2+2ibcd-d^2)\,e^{2i(bx+a)}}{16b^3} + \frac{(2x^2d^2b^2-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I*d^2*x^3-1/3*I/d*c^3-2*I/b^2*a^2*d^2*x+4*I/b*a*c*d*x+1/16*(2*x^2*d^2*b
^2+2*I*b*d^2*x+4*b^2*c*d*x+2*I*b*c*d+2*b^2*c^2-d^2)/b^3*exp(2*I*(b*x+a))+1/
16*(2*x^2*d^2*b^2-2*I*b*d^2*x+4*b^2*c*d*x-2*I*b*c*d+2*b^2*c^2-d^2)/b^3*exp(
-2*I*(b*x+a))-1/b*c^2*ln(1+exp(2*I*(b*x+a)))+2/b*c^2*ln(exp(I*(b*x+a)))+2/b
^3*d^2*a^2*ln(exp(I*(b*x+a)))-4/3*I/b^3*a^3*d^2+I*d*c*x^2+I/b^2*c*d*polylog
(2,-exp(2*I*(b*x+a)))-I*c^2*x-1/b*d^2*ln(1+exp(2*I*(b*x+a)))*x^2+2*I/b^2*a^
2*c*d-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-4/b^2*c*d*a*ln(exp(I*(b*x+a)
))+I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-2/b*c*d*ln(1+exp(2*I*(b*x+a)))*
x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(159) = 318$.
time = 0.54, size = 383, normalized size = 2.08

$$-\frac{12\left(\sin(bx+a)^2+\log\left(\sin(bx+a)^2-1\right)\right)c^2-\frac{24\left(\sin(bx+a)^2+\log\left(\sin(bx+a)^2-1\right)\right)acd}{b}+\frac{12\left(\sin(bx+a)^2+\log\left(\sin(bx+a)^2-1\right)\right)a^2d^2}{b^2}+\frac{-8i(bx+a)^3d^2-24(ibcd-iad^2)(bx+a)^2+12d^2\mathrm{Li}_3(-e^{(2ibx+2ia)})-24\left(-i(bx+a)^2d^2+2(-ibcd+iad^2)(bx+a)\right)\arctan(\sin(2bx+2a),\cos(2bx+2a)+1)-3\left(2(bx+a)^2d^2+4(bcd-ad^2)(bx+a)-d^2\right)\cos(2bx+2a)-24(ibcd+i(bx+a)d^2-iad^2)\mathrm{Li}_2(-e^{(2ibx+2ia)})+12\left((bx+a)^2d^2+2(bcd-ad^2)(bx+a)\right)\log\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2+2\cos(2bx+2a)+1\right)+6(bcd+(bx+a)d^2-ad^2)\sin(2bx+2a)}{b^2}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(12*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))*c^2 - 24*(sin(b*x + a)
^2 + log(sin(b*x + a)^2 - 1))*a*c*d/b + 12*(sin(b*x + a)^2 + log(sin(b*x +
a)^2 - 1))*a^2*d^2/b^2 + (-8*I*(b*x + a)^3*d^2 - 24*(I*b*c*d - I*a*d^2)*(b*
x + a)^2 + 12*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) - 24*(-I*(b*x + a)^2*d^2
 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x +
2*a) + 1) - 3*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(2
*b*x + 2*a) - 24*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*dilog(-e^(2*I*b*x +
2*I*a)) + 12*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(2*b*x
+ 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 6*(b*c*d + (b*x +
 a)*d^2 - a*d^2)*sin(2*b*x + 2*a))/b^2)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(159) = 318$.
time = 1.95, size = 688, normalized size = 3.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2
- d^2)*cos(b*x + a)^2 + 4*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 4
*d^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 4*d^2*polylog(3, -I*cos(b*
x + a) + sin(b*x + a)) + 4*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) +
 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) + 4*(I*b*d^2*x + I*b*c*d)*di
log(I*cos(b*x + a) + sin(b*x + a)) + 4*(-I*b*d^2*x - I*b*c*d)*dilog(I*cos(b
*x + a) - sin(b*x + a)) + 4*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x + a) +
sin(b*x + a)) + 4*(I*b*d^2*x + I*b*c*d)*dilog(-I*cos(b*x + a) - sin(b*x + a
)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*x + a) +
I) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*sin(b*x + a) +
I) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a)
 + sin(b*x + a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*
log(I*cos(b*x + a) - sin(b*x + a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a
*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b^2*d^2*x^2
+ 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1
) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) +
I) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) +
 I))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \sin^3(a+bx) \sec(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*sin(a + b*x)**3*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*sin(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^3\,(c+d\,x)^2}{\cos(a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^3*(c + d*x)^2)/cos(a + b*x),x)
[Out] int((sin(a + b*x)^3*(c + d*x)^2)/cos(a + b*x), x)
```

3.224 $\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=115

$$\frac{dx}{4b}+\frac{i(c+dx)^2}{2d}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\cos(a+bx)\sin(a+bx)}{4b^2}-\frac{(c+dx)}{}$$

```
[Out] 1/4*d*x/b+1/2*I*(d*x+c)^2/d-(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b+1/2*I*d*polylo
g(2,-exp(2*I*(b*x+a)))/b^2-1/4*d*cos(b*x+a)*sin(b*x+a)/b^2-1/2*(d*x+c)*sin(
b*x+a)^2/b
```

Rubi [A]
time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4492, 4489, 2715, 8, 3800, 2221, 2317, 2438}

$$\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{dx}{4b}+\frac{i(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (d*x)/(4*b) + ((I/2)*(c + d*x)^2)/d - ((c + d*x)*Log[1 + E^((2*I)*(a + b*x)
)])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (d*Cos[a + b*x]*Si
n[a + b*x])/(4*b^2) - ((c + d*x)*Sin[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4492

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx &= -\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx+\int (c+dx)\tan(a+bx)\\ &= \frac{i(c+dx)^2}{2d}-\frac{(c+dx)\sin^2(a+bx)}{2b}-2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}\,dx+\\ &= \frac{i(c+dx)^2}{2d}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{d\cos(a+bx)\sin(a+b}{4b^2}\\ &= \frac{dx}{4b}+\frac{i(c+dx)^2}{2d}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{d\cos(a+bx)\sin}{4b^2}\\ &= \frac{dx}{4b}+\frac{i(c+dx)^2}{2d}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{id\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}\end{aligned}$$

Mathematica [**A**]

time = 0.33, size = 134, normalized size = 1.17

$$\frac{dx\cos(2(a+bx))}{4b}+\frac{ad\log(\cos(a+bx))}{b^2}-\frac{c\left(-\frac{1}{2}\cos^2(a+bx)+\log(\cos(a+bx))\right)}{b}+\frac{d\left(\frac{1}{2}i(a+bx)^2-(a+bx)\log\left(1+e^{2i(a+bx)}\right)+\frac{1}{2}i\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)\right)}{b^2}-\frac{d\sin(2(a+bx))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (d*x*Cos[2*(a + b*x)])/(4*b) + (a*d*Log[Cos[a + b*x]])/b^2 - (c*(-1/2*Cos[a
 + b*x]^2 + Log[Cos[a + b*x]]))/b + (d*((I/2)*(a + b*x)^2 - (a + b*x)*Log[1
 + E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^((2*I)*(a + b*x))]))/b^2 - (d
*Sin[2*(a + b*x)])/(8*b^2)
```

Maple [A]
time = 0.41, size = 179, normalized size = 1.56

method	result
risch	$\frac{id\,x^2}{2}-icx+\frac{(2dxb+2cb+id)\mathrm{e}^{2i(bx+a)}}{16b^2}+\frac{(2dxb+2cb-id)\mathrm{e}^{-2i(bx+a)}}{16b^2}-\frac{c\ln\left(1+\mathrm{e}^{2i(bx+a)}\right)}{b}+\frac{2c\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b}+\frac{2idax}{b}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*d*x^2-I*c*x+1/16*(2*d*x*b+I*d+2*c*b)/b^2*exp(2*I*(b*x+a))+1/16*(2*d*x
*b-I*d+2*c*b)/b^2*exp(-2*I*(b*x+a))-1/b*c*ln(1+exp(2*I*(b*x+a)))+2/b*c*ln(e
xp(I*(b*x+a)))+2*I/b*d*a*x+I/b^2*d*a^2-1/b*d*ln(1+exp(2*I*(b*x+a)))*x+1/2*I
*d*polylog(2,-exp(2*I*(b*x+a)))/b^2-2/b^2*d*a*ln(exp(I*(b*x+a)))
```

Maxima [A]
time = 0.53, size = 146, normalized size = 1.27

$$-\frac{-4i\,b^2dx^2-8i\,b^2cx-8\,(-i\,bdx-i\,bc)\arctan\left(\sin\left(2\,bx+2\,a\right),\cos\left(2\,bx+2\,a\right)+1\right)-2\,(bdx+bc)\cos\left(2\,bx+2\,a\right)-4i\,d\mathrm{Li}_2\left(-e^{(2i\,bx+2i\,a)}\right)+4\,(bdx+bc)\log\left(\cos\left(2\,bx+2\,a\right)^2+\sin\left(2\,bx+2\,a\right)^2+2\,\cos\left(2\,bx+2\,a\right)+1\right)+d\sin\left(2\,bx+2\,a\right)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(-4*I*b^2*d*x^2 - 8*I*b^2*c*x - 8*(-I*b*d*x - I*b*c)*arctan2(sin(2*b*x
 + 2*a), cos(2*b*x + 2*a) + 1) - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - 4*I*d*d
ilog(-e^(2*I*b*x + 2*I*a)) + 4*(b*d*x + b*c)*log(cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + d*sin(2*b*x + 2*a))/b^2
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(96) = 192.
time = 1.32, size = 346, normalized size = 3.01

$$-\frac{bdx-2\,(bdx+bc)\cos\left(bx+a\right)^2+d\cos\left(bx+a\right)\sin\left(bx+a\right)+2i\,d\mathrm{Li}_2(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right))-2i\,d\mathrm{Li}_2(i\,\cos\left(bx+a\right)-\sin\left(bx+a\right))-2i\,d\mathrm{Li}_2(-i\,\cos\left(bx+a\right)+\sin\left(bx+a\right))+2i\,d\mathrm{Li}_2(-i\,\cos\left(bx+a\right)-\sin\left(bx+a\right))+2\,(bc-ad)\log\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)+i\right)+2\,(bc-ad)\log\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)+i\right)+2\,(bdx+ad)\log\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)+1\right)+2\,(bdx+ad)\log\left(i\,\cos\left(bx+a\right)-\sin\left(bx+a\right)+1\right)+2\,(bdx+ad)\log\left(-i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)+1\right)+2\,(bdx+ad)\log\left(-i\,\cos\left(bx+a\right)-\sin\left(bx+a\right)+1\right)+2\,(bc-ad)\log\left(-\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)+i\right)+2\,(bc-ad)\log\left(-\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)+i\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a)
+ 2*I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) - 2*I*d*dilog(I*cos(b*x + a) -
 sin(b*x + a)) - 2*I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 2*I*d*dilog(
-I*cos(b*x + a) - sin(b*x + a)) + 2*(b*c - a*d)*log(cos(b*x + a) + I*sin(b*
x + a) + I) + 2*(b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) + 2*(b*d
*x + a*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b*d*x + a*d)*log(I*co
s(b*x + a) - sin(b*x + a) + 1) + 2*(b*d*x + a*d)*log(-I*cos(b*x + a) + sin(
b*x + a) + 1) + 2*(b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + 2
*(b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 2*(b*c - a*d)*log(-c
os(b*x + a) - I*sin(b*x + a) + I))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\sin^3(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*sin(a + b*x)**3*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*sin(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^3\,(c+d\,x)}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^3*(c + d*x))/cos(a + b*x),x)
```

```
[Out] int((sin(a + b*x)^3*(c + d*x))/cos(a + b*x), x)
```

3.225 $\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=82

$$-\frac{\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d}-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\text{Int}\left(\frac{\tan(a+bx)}{c+dx},x\right)$$

```
[Out] -1/2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d-1/2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b
*c/d)/d+Unintegrable(tan(b*x+a)/(d*x+c),x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] -1/2*(CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/d - (Cos[2*a - (
2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Defer[Int][Tan[a + b*x]/(
c + d*x), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx &= -\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx+\int \frac{\tan(a+bx)}{c+dx}\,dx\\ &= -\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx+\int \frac{\tan(a+bx)}{c+dx}\,dx\\ &= -\left(\frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx\right)+\int \frac{\tan(a+bx)}{c+dx}\,dx\\ &= -\left(\frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx\right)-\frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx+\int \frac{\tan(a+bx)}{c+dx}\,dx\\ &= -\frac{\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d}-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\int \frac{\tan(a+bx)}{c+dx}\,dx\end{aligned}$$

Mathematica [A]
time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]
```

Maple [A]
time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\sec \left(bx+a\right) \left(\sin^{3}\left(bx+a\right)\right)}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*((-I*exp_integral_e(1, 2*(-I*b*d*x - I*b*c)/d) + I*exp_integral_e(1, -2
*(-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 8*d*integrate(sin(2*b*x + 2
*a)/((d*x + c)*cos(2*b*x + 2*a)^2 + (d*x + c)*sin(2*b*x + 2*a)^2 + d*x + 2*
(d*x + c)*cos(2*b*x + 2*a) + c), x) + (exp_integral_e(1, 2*(-I*b*d*x - I*b*
c)/d) + exp_integral_e(1, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c - a*d)/d))/
d
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**3/(d*x+c),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
[Out] integrate(sec(b*x + a)*sin(b*x + a)^3/(d*x + c), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^3}{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(cos(a + b*x)*(c + d*x)),x)
[Out] int(sin(a + b*x)^3/(cos(a + b*x)*(c + d*x)), x)
```

3.226 $\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=102

$$-\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin(2a+2bx)}{2d(c+dx)}+\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\text{Int}\left(\frac{\tan(a+bx)}{(c+dx)^2}\right.$$

```
[Out] -b*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^2+b*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c
/d)/d^2+1/2*sin(2*b*x+2*a)/d/(d*x+c)+Unintegrable(tan(b*x+a)/(d*x+c)^2,x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -((b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2) + Sin[2*a +
2*b*x]/(2*d*(c + d*x)) + (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*
b*x])/d^2 + Defer[Int][Tan[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx &= -\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx+\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx\\
&= -\int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx+\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx\\
&= -\left(\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx\right)+\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(2a+2bx)}{2d(c+dx)}-\frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d}+\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(2a+2bx)}{2d(c+dx)}-\frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(2a-\frac{2bc}{d}\right)\right)\int}{d}\\
&= -\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin(2a+2bx)}{2d(c+dx)}+\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}\right.}{d^2}
\end{aligned}$$

Mathematica [A]
time = 2.69, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]
```

Maple [A]
time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\left(\sin^3(bx+a)\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((-I*exp_integral_e(2, 2*(-I*b*d*x - I*b*c)/d) + I*exp_integral_e(2, -2
*(-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 8*(d^2*x + c*d)*integrate(s
in(2*b*x + 2*a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x +
2*a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(2*b*x + 2*a)^2 + c^2 + 2*(d^2*x^2 +
2*c*d*x + c^2)*cos(2*b*x + 2*a)), x) + (exp_integral_e(2, 2*(-I*b*d*x - I*b
*c)/d) + exp_integral_e(2, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c - a*d)/d))
/(d^2*x + c*d)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)*sin(b*x + a)/(d^2*x^2 + 2*c*d*x
 + c^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)^3/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^3}{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(cos(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(sin(a + b*x)^3/(cos(a + b*x)*(c + d*x)^2), x)
```

3.227 $\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}((c+dx)^m \csc(a+bx)\sec(a+bx), x)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx = \int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$$

Mathematica [A]
time = 5.70, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x], x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x)
[Out] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc (a+bx) \sec (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a),x)
[Out] Integral((c + d*x)**m*csc(a + b*x)*sec(a + b*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos (a+b\,x)\,\sin (a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)),x)
[Out] int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)), x)
```

3.228 $\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=247

$$-\frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}+\frac{2id(c+dx)^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{3d}{}$$

```
[Out] -2*(d*x+c)^4*arctanh(exp(2*I*(b*x+a)))/b+2*I*d*(d*x+c)^3*polylog(2,-exp(2*I
*(b*x+a)))/b^2-2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^2-3*d^2*(d*x+c
)^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(b*x
+a)))/b^3-3*I*d^3*(d*x+c)*polylog(4,-exp(2*I*(b*x+a)))/b^4+3*I*d^3*(d*x+c)*
polylog(4,exp(2*I*(b*x+a)))/b^4+3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5-3/
2*d^4*polylog(5,exp(2*I*(b*x+a)))/b^5
```

Rubi [A]
time = 0.16, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4504, 4268, 2611, 6744, 2320, 6724}

$$\frac{3d^4\text{Li}_5\left(-e^{2i(a+bx)}\right)}{2b^5}-\frac{3d^4\text{Li}_5\left(e^{2i(a+bx)}\right)}{2b^5}-\frac{3id^3(c+dx)\text{Li}_4\left(-e^{2i(a+bx)}\right)}{b^4}+\frac{3id^3(c+dx)\text{Li}_4\left(e^{2i(a+bx)}\right)}{b^4}-\frac{3d^2(c+dx)^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{3d^2(c+dx)^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{b^3}+\frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^4*ArcTanh[E^((2*I)*(a + b*x))])/b + ((2*I)*d*(c + d*x)^3*Poly
Log[2, -E^((2*I)*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)^3*PolyLog[2, E^((2*I
)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b
^3 + (3*d^2*(c + d*x)^2*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(
c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + ((3*I)*d^3*(c + d*x)*PolyL
og[4, E^((2*I)*(a + b*x))])/b^4 + (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/
(2*b^5) - (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx = 2\int (c+dx)^4 \csc(2a+2bx)\,dx$$

$$= -\frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{(4d)\int (c+dx)^3 \log\left(1-e^{i(2a+2bx}\right.}{b}$$

$$= -\frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2}{}$$

$$= -\frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2}{}$$

$$= -\frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2}{}$$

$$= -\frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2}{}$$

$$= -\frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2}{}$$

<u>Mathematica</u> [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 578 vs. $2(247) = 494$.
time = 0.84, size = 578, normalized size = 2.34

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-4*b^4*c^4*ArcTanh[E^((2*I)*(a + b*x))] + 8*b^4*c^3*d*x*Log[1 - E^((2*I)*(
a + b*x))] + 12*b^4*c^2*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] + 8*b^4*c*d^3*
x^3*Log[1 - E^((2*I)*(a + b*x))] + 2*b^4*d^4*x^4*Log[1 - E^((2*I)*(a + b*x)
)] - 8*b^4*c^3*d*x*Log[1 + E^((2*I)*(a + b*x))] - 12*b^4*c^2*d^2*x^2*Log[1
+ E^((2*I)*(a + b*x))] - 8*b^4*c*d^3*x^3*Log[1 + E^((2*I)*(a + b*x))] - 2*b
^4*d^4*x^4*Log[1 + E^((2*I)*(a + b*x))] + (4*I)*b^3*d*(c + d*x)^3*PolyLog[2
, -E^((2*I)*(a + b*x))] - (4*I)*b^3*d*(c + d*x)^3*PolyLog[2, E^((2*I)*(a +
b*x))] - 6*b^2*c^2*d^2*PolyLog[3, -E^((2*I)*(a + b*x))] - 12*b^2*c*d^3*x*Po
lyLog[3, -E^((2*I)*(a + b*x))] - 6*b^2*d^4*x^2*PolyLog[3, -E^((2*I)*(a + b*
x))] + 6*b^2*c^2*d^2*PolyLog[3, E^((2*I)*(a + b*x))] + 12*b^2*c*d^3*x*PolyL
og[3, E^((2*I)*(a + b*x))] + 6*b^2*d^4*x^2*PolyLog[3, E^((2*I)*(a + b*x))]
- (6*I)*b*c*d^3*PolyLog[4, -E^((2*I)*(a + b*x))] - (6*I)*b*d^4*x*PolyLog[4,
 -E^((2*I)*(a + b*x))] + (6*I)*b*c*d^3*PolyLog[4, E^((2*I)*(a + b*x))] + (6
*I)*b*d^4*x*PolyLog[4, E^((2*I)*(a + b*x))] + 3*d^4*PolyLog[5, -E^((2*I)*(a
 + b*x))] - 3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1241 vs. $2(221) = 442$.
time = 0.18, size = 1242, normalized size = 5.03

method	result	size
risch	Expression too large to display	1242

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3+4/b
*c*d^3*ln(exp(I*(b*x+a))+1)*x^3-12*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*
x-12*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-12*I/b^2*c*d^3*polylog(2,-e
xp(I*(b*x+a)))*x^2-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2+2*I/b^2*d^4
*polylog(2,-exp(2*I*(b*x+a)))*x^3-3*I/b^4*d^4*polylog(4,-exp(2*I*(b*x+a)))*
x+2*I/b^2*c^3*d*polylog(2,-exp(2*I*(b*x+a)))-3*I/b^4*c*d^3*polylog(4,-exp(2
*I*(b*x+a)))-4/b*c*d^3*ln(1+exp(2*I*(b*x+a)))*x^3+6*I/b^2*c^2*d^2*polylog(2
,-exp(2*I*(b*x+a)))*x+3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5-4/b^2*c^3*d*
a*ln(exp(I*(b*x+a))-1)-4/b^4*c*d^3*a^3*ln(exp(I*(b*x+a))-1)-4*I/b^2*d^4*pol
ylog(2,exp(I*(b*x+a)))*x^3-4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+24*I/
b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))
*x+24*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))-4*I/b^2*c^3*d*polylog(2,exp(I*
(b*x+a)))+24*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))-4*I/b^2*c^3*d*polylog(2,
-exp(I*(b*x+a)))-24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*polylog(5,exp
(I*(b*x+a)))/b^5+1/b*c^4*ln(exp(I*(b*x+a))-1)+1/b*c^4*ln(exp(I*(b*x+a))+1)+
12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))+12/b^3*c^2*d^2*polylog(3,-exp(I*(b
*x+a)))+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)-1/b*c^4*ln(1+exp(2*I*(b*x+a)))-1
/b^5*d^4*a^4*ln(1-exp(I*(b*x+a)))+12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2
+12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^2-3/b^3*c^2*d^2*polylog(3,-exp(2*I*
(b*x+a)))-3/b^3*d^4*polylog(3,-exp(2*I*(b*x+a)))*x^2-4/b*c^3*d*ln(1+exp(2*I
*(b*x+a)))*x-6/b*c^2*d^2*ln(1+exp(2*I*(b*x+a)))*x^2-1/b*d^4*ln(1+exp(2*I*(b
*x+a)))*x^4-6/b^3*c*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+6*I/b^2*c*d^3*polylo
g(2,-exp(2*I*(b*x+a)))*x^2+4/b*c^3*d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c^3*d*ln(
1-exp(I*(b*x+a)))*a+4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x+1/b*d^4*ln(1-exp(I*(b*
x+a)))*x^4+1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4+6/b^3*c^2*d^2*a^2*ln(exp(I*(b*x
+a))-1)+6/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2+6/b*c^2*d^2*ln(1-exp(I*(b*x+a)
))*x^2-6/b^3*c^2*d^2*ln(1-exp(I*(b*x+a)))*a^2+24/b^3*c*d^3*polylog(3,-exp(I
*(b*x+a)))*x+24/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1794 vs. $2(215) = 430$.
time = 0.66, size = 1794, normalized size = 7.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")

[Out] -1/6*(3*c^4*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 12*a*c^3*d*(l
og(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + 18*a^2*c^2*d^2*(log(sin(b
*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 12*a^3*c*d^3*(log(sin(b*x + a)^
2 - 1) - log(sin(b*x + a)^2))/b^3 + 3*a^4*d^4*(log(sin(b*x + a)^2 - 1) - lo
g(sin(b*x + a)^2))/b^4 - (18*d^4*polylog(5, -e^(2*I*b*x + 2*I*a)) - 144*d^4
*polylog(5, -e^(I*b*x + I*a)) - 144*d^4*polylog(5, e^(I*b*x + I*a)) + 4*(-3
*I*(b*x + a)^4*d^4 + 8*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 9*(-I*b^2*c^2*d
^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a)^2 + 6*(-I*b^3*c^3*d + 3*I*a*b^2*c
^2*d^2 - 3*I*a^2*b*c*d^3 + I*a^3*d^4)*(b*x + a))*arctan2(sin(2*b*x + 2*a),
cos(2*b*x + 2*a) + 1) + 6*(I*(b*x + a)^4*d^4 + 4*(I*b*c*d^3 - I*a*d^4)*(b*x
 + a)^3 + 6*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a)^2 + 4*(I*
b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 - I*a^3*d^4)*(b*x + a))*arc
tan2(sin(b*x + a), cos(b*x + a) + 1) + 6*(-I*(b*x + a)^4*d^4 + 4*(-I*b*c*d^
3 + I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(
b*x + a)^2 + 4*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b*c*d^3 + I*a^3*
d^4)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 12*(I*b^3*c^3*d
 - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 + 2*I*(b*x + a)^3*d^4 - I*a^3*d^4 + 4
*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a
^2*d^4)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) + 24*(-I*b^3*c^3*d + 3*I*a*b
^2*c^2*d^2 - 3*I*a^2*b*c*d^3 - I*(b*x + a)^3*d^4 + I*a^3*d^4 + 3*(-I*b*c*d^
3 + I*a*d^4)*(b*x + a)^2 + 3*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(
b*x + a))*dilog(-e^(I*b*x + I*a)) + 24*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 -
3*I*a^2*b*c*d^3 - I*(b*x + a)^3*d^4 + I*a^3*d^4 + 3*(-I*b*c*d^3 + I*a*d^4)*
(b*x + a)^2 + 3*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a))*dil
og(e^(I*b*x + I*a)) - 2*(3*(b*x + a)^4*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^
3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*
a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(2*b*x + 2*a)^2
+ sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 3*((b*x + a)^4*d^4 + 4*(b*
c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x +
 a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a)
)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 3*((b*x + a)^
4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^
2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d
^4)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) +
12*(-2*I*b*c*d^3 - 3*I*(b*x + a)*d^4 + 2*I*a*d^4)*polylog(4, -e^(2*I*b*x +
2*I*a)) + 144*(I*b*c*d^3 + I*(b*x + a)*d^4 - I*a*d^4)*polylog(4, -e^(I*b*x
+ I*a)) + 144*(I*b*c*d^3 + I*(b*x + a)*d^4 - I*a*d^4)*polylog(4, e^(I*b*x +
 I*a)) - 6*(3*b^2*c^2*d^2 - 6*a*b*c*d^3 + 6*(b*x + a)^2*d^4 + 3*a^2*d^4 + 8
*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, -e^(2*I*b*x + 2*I*a)) + 72*(b^2*c^
2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x
+ a))*polylog(3, -e^(I*b*x + I*a)) + 72*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x +
 a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, e^(I*b*x +
I*a)))/b^4)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2600 vs. $2(215) = 430$.
time = 2.03, size = 2600, normalized size = 10.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(24*d^4*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*polylog(5,
cos(b*x + a) - I*sin(b*x + a)) - 24*d^4*polylog(5, I*cos(b*x + a) + sin(b*x
 + a)) - 24*d^4*polylog(5, I*cos(b*x + a) - sin(b*x + a)) - 24*d^4*polylog(
5, -I*cos(b*x + a) + sin(b*x + a)) - 24*d^4*polylog(5, -I*cos(b*x + a) - si
n(b*x + a)) + 24*d^4*polylog(5, -cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*po
lylog(5, -cos(b*x + a) - I*sin(b*x + a)) + 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3
*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(cos(b*x + a) + I*sin(b*x + a)
) + 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d
)*dilog(cos(b*x + a) - I*sin(b*x + a)) + 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x
^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(I*cos(b*x + a) + sin(b*x + a))
+ 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*
dilog(I*cos(b*x + a) - sin(b*x + a)) + 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^
2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog(-I*cos(b*x + a) + sin(b*x + a))
+ 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*d
ilog(-I*cos(b*x + a) - sin(b*x + a)) + 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^
2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog(-cos(b*x + a) + I*sin(b*x + a))
+ 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*d
ilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b
^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x + a) + I*sin(b*x + a)
 + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*
d^4)*log(cos(b*x + a) + I*sin(b*x + a) + I) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^
3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x + a) - I*sin(b
*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b^4*d^4*x^4 + 4*b^4*c
*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^
2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (
b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3
*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(b*x + a) -
sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*
b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*
log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 +
6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a
^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b^4*c^4 -
4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/2*cos(b
*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x +
```

```
a) + 1/2) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*
d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-cos
(b*x + a) + I*sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b
^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*
c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) - I*
sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*
c*d^3 + a^4*d^4)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 24*(-I*b*d^4*x -
 I*b*c*d^3)*polylog(4, cos(b*x + a) + I*sin(b*x + a)) + 24*(I*b*d^4*x + I*b
*c*d^3)*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 24*(-I*b*d^4*x - I*b*c*
d^3)*polylog(4, I*cos(b*x + a) + sin(b*x + a)) + 24*(I*b*d^4*x + I*b*c*d^3)
*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 24*(I*b*d^4*x + I*b*c*d^3)*pol
ylog(4, -I*cos(b*x + a) + sin(b*x + a)) + 24*(-I*b*d^4*x - I*b*c*d^3)*polyl
og(4, -I*cos(b*x + a) - sin(b*x + a)) + 24*(I*b*d^4*x + I*b*c*d^3)*polylog(
4, -cos(b*x + a) + I*sin(b*x + a)) + 24*(-I*b*d^4*x - I*b*c*d^3)*polylog(4,
 -cos(b*x + a) - I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^
2*d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*
c*d^3*x + b^2*c^2*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) + 12*(b^2*
d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, I*cos(b*x + a) + sin(b*x
+ a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, I*cos(b*x
 + a) - sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*poly
log(3, -I*cos(b*x + a) + sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x +
b^2*c^2*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 12*(b^2*d^4*x^2 +
 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) -
12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) - I
*sin(b*x + a)))/b^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \csc(a+bx) \sec(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**4*csc(a + b*x)*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*csc(b*x + a)*sec(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^4}{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^4/(cos(a + b*x)*sin(a + b*x)),x)
[Out] int((c + d*x)^4/(cos(a + b*x)*sin(a + b*x)), x)
```

3.229 $\int (c+dx)^3 \csc(a+bx) \sec(a+bx)\, dx$

Optimal. Leaf size=197

$$-\frac{2(c+dx)^3 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2 \text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3id(c+dx)^2 \text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{3\cdots}{}$$

```
[Out] -2*(d*x+c)^3*arctanh(exp(2*I*(b*x+a)))/b+3/2*I*d*(d*x+c)^2*polylog(2,-exp(2
*I*(b*x+a)))/b^2-3/2*I*d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3/2*d^2*
(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))/b^3+3/2*d^2*(d*x+c)*polylog(3,exp(2*I*
(b*x+a)))/b^3-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/4*I*d^3*polylog(
4,exp(2*I*(b*x+a)))/b^4
```

Rubi [A]
time = 0.12, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4504, 4268, 2611, 6744, 2320, 6724}

$$-\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}+\frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{3id(c+dx)^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^3*ArcTanh[E^((2*I)*(a + b*x))])/b + (((3*I)/2)*d*(c + d*x)^2*
PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2,
 E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x
))])/(2*b^3) + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) -
(((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/4)*d^3*PolyL
og[4, E^((2*I)*(a + b*x))])/b^4
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\csc(a+bx)\sec(a+bx)\,dx = 2\int (c+dx)^3\csc(2a+2bx)\,dx$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{(3d)\int (c+dx)^2\log\left(1-e^{i(2a+2bx)}\right)}{b}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3ic}{}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3ic}{}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3ic}{}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3ic}{}$$

Mathematica [A]
time = 0.64, size = 350, normalized size = 1.78

$$\frac{-8b^3c^3\tanh^{-1}\left(e^{2i(a+bx)}\right)+12b^3c^2dx\log\left(1-e^{2i(a+bx)}\right)+12b^3cd^2x^2\log\left(1-e^{2i(a+bx)}\right)+4b^3d^3x^3\log\left(1-e^{2i(a+bx)}\right)-12b^3c^2dx\log\left(1+e^{2i(a+bx)}\right)-12b^3cd^2x^2\log\left(1+e^{2i(a+bx)}\right)-4b^3d^3x^3\log\left(1+e^{2i(a+bx)}\right)+6ib^2d(c+dx)^2\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)-6ib^2d(c+dx)^2\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)-6bd^2(c+dx)\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)+6bcd^2\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)+6bd^3x\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)-3id^3\mathrm{PolyLog}\left(4,-e^{2i(a+bx)}\right)+3id^3\mathrm{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-8*b^3*c^3*ArcTanh[E^((2*I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 - E^((2*I)*
(a + b*x))] + 12*b^3*c*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] + 4*b^3*d^3*x^3
*Log[1 - E^((2*I)*(a + b*x))] - 12*b^3*c^2*d*x*Log[1 + E^((2*I)*(a + b*x))]
 - 12*b^3*c*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] - 4*b^3*d^3*x^3*Log[1 + E^
((2*I)*(a + b*x))] + (6*I)*b^2*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x)
)] - (6*I)*b^2*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))] - 6*b*d^2*(c +
 d*x)*PolyLog[3, -E^((2*I)*(a + b*x))] + 6*b*c*d^2*PolyLog[3, E^((2*I)*(a +
 b*x))] + 6*b*d^3*x*PolyLog[3, E^((2*I)*(a + b*x))] - (3*I)*d^3*PolyLog[4,
-E^((2*I)*(a + b*x))] + (3*I)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/(4*b^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 815 vs. 2(167) = 334.
time = 0.12, size = 816, normalized size = 4.14

method	result
risch	$\frac{c^3\ln\left(e^{i(bx+a)}+1\right)}{b}+\frac{c^3\ln\left(e^{i(bx+a)}-1\right)}{b}-\frac{6icd^2\,\mathrm{polylog}\left(2,e^{i(bx+a)}\right)x}{b^2}-\frac{6icd^2\,\mathrm{polylog}\left(2,-e^{i(bx+a)}\right)x}{b^2}+\frac{6id^3\,\mathrm{polylog}\left(4,-e^{i(b}\right.}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 6*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)))-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+6
/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))
+6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))
*x+1/b*c^3*ln(exp(I*(b*x+a))+1)+1/b*c^3*ln(exp(I*(b*x+a))-1)-6*I/b^2*c*d^2*
polylog(2,exp(I*(b*x+a)))*x+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+1/b*d^3*l
n(exp(I*(b*x+a))+1)*x^3+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1-exp
(I*(b*x+a)))*a^3+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)-3/4*I*d^3*polylog(4,-
exp(2*I*(b*x+a)))/b^4-1/b*c^3*ln(1+exp(2*I*(b*x+a)))-3/2/b^3*c*d^2*polylog(
3,-exp(2*I*(b*x+a)))-3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+3/b*c^2*d*l
n(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-exp
(I*(b*x+a)))*a+3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b*c*d^2*ln(1-exp(I*(b*x
+a)))*x^2-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-3/b^2*c^2*d*a*ln(exp(I*(b*x+
a))-1)-3*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,-exp
(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog
(2,exp(I*(b*x+a)))*x^2+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2+3/2*I
/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+a)))-3/b*c^2*d*ln(1+exp(2*I*(b*x+a)))*x-
```

```
3/b*c*d^2*ln(1+exp(2*I*(b*x+a)))*x^2-1/b*d^3*ln(1+exp(2*I*(b*x+a)))*x^3-6*I
/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x+3*I/b^2*c*d^2*polylog(2,-exp(2*I*(b
*x+a)))*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1078 vs. 2(161) = 322.
time = 0.58, size = 1078, normalized size = 5.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/6*(3*c^3*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 9*a*c^2*d*(lo
g(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + 9*a^2*c*d^2*(log(sin(b*x +
 a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 3*a^3*d^3*(log(sin(b*x + a)^2 - 1)
- log(sin(b*x + a)^2))/b^3 + (6*I*d^3*polylog(4, -e^(2*I*b*x + 2*I*a)) - 36
*I*d^3*polylog(4, -e^(I*b*x + I*a)) - 36*I*d^3*polylog(4, e^(I*b*x + I*a))
- 2*(-4*I*(b*x + a)^3*d^3 + 9*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 9*(-I*b^
2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a))*arctan2(sin(2*b*x + 2*a), c
os(2*b*x + 2*a) + 1) - 6*(I*(b*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x
+ a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b*x + a))*arctan2(sin
(b*x + a), cos(b*x + a) + 1) - 6*(-I*(b*x + a)^3*d^3 + 3*(-I*b*c*d^2 + I*a*
d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a))*
arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 3*(3*I*b^2*c^2*d - 6*I*a*b*c*d^2
 + 4*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + 6*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*d
ilog(-e^(2*I*b*x + 2*I*a)) - 18*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)
^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*dilog(-e^(I*b*x +
I*a)) - 18*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 +
2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*dilog(e^(I*b*x + I*a)) + (4*(b*x + a)^3
*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d
^3)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
2*a) + 1) - 3*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c
^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^
2 + 2*cos(b*x + a) + 1) - 3*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a
)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 +
 sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 3*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*
a*d^3)*polylog(3, -e^(2*I*b*x + 2*I*a)) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d
^3)*polylog(3, -e^(I*b*x + I*a)) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*pol
ylog(3, e^(I*b*x + I*a)))/b^3)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1786 vs. 2(161) = 322.
time = 2.16, size = 1786, normalized size = 9.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")

[Out] 1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*polylog(4, I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) + 6*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog

```
(3, -I*cos(b*x + a) - sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos
(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a
) - I*sin(b*x + a)))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \csc (a+bx) \sec (a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*csc(a + b*x)*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^3}{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)),x)
```

```
[Out] int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)), x)
```

3.230 $\int (c+dx)^2 \csc(a+bx) \sec(a+bx)\, dx$

Optimal. Leaf size=127

$$-\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}+\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{Pol}}{}$$

```
[Out] -2*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b+I*d*(d*x+c)*polylog(2,-exp(2*I*(b*
x+a)))/b^2-I*d*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,-e
xp(2*I*(b*x+a)))/b^3+1/2*d^2*polylog(3,exp(2*I*(b*x+a)))/b^3
```

Rubi [A]
time = 0.08, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4504, 4268, 2611, 2320, 6724}

$$-\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/b + (I*d*(c + d*x)*PolyLog[2,
 -E^((2*I)*(a + b*x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
])/b^2 - (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E
^((2*I)*(a + b*x))])/(2*b^3)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
```

```
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\csc(a+bx)\sec(a+bx)\,dx &= 2\int (c+dx)^2\csc(2a+2bx)\,dx\\ &= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{(2d)\int (c+dx)\log\left(1-e^{i(2a+2bx)}\right)}{b}\\ &= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}+\frac{id(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c\cdot}{}\\ &= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}+\frac{id(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c\cdot}{}\\ &= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}+\frac{id(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c\cdot}{}\end{aligned}$$

Mathematica [A]
time = 0.38, size = 213, normalized size = 1.68

$$\frac{-4b^2c^2\tanh^{-1}\left(e^{2i(a+bx)}\right)+4b^2cdx\log\left(1-e^{2i(a+bx)}\right)+2b^2d^2x^2\log\left(1-e^{2i(a+bx)}\right)-4b^2cdx\log\left(1+e^{2i(a+bx)}\right)-2b^2d^2x^2\log\left(1+e^{2i(a+bx)}\right)+2ibd(c+dx)\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)-2ibd(c+dx)\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)-d^2\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)+d^2\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-4*b^2*c^2*ArcTanh[E^((2*I)*(a + b*x))] + 4*b^2*c*d*x*Log[1 - E^((2*I)*(a
+ b*x))] + 2*b^2*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] - 4*b^2*c*d*x*Log[1 +
 E^((2*I)*(a + b*x))] - 2*b^2*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] + (2*I)*
b*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLo
```

```
g[2, E^((2*I)*(a + b*x))] - d^2*PolyLog[3, -E^((2*I)*(a + b*x))] + d^2*Poly
Log[3, E^((2*I)*(a + b*x))])/(2*b^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(111) = 222$.
time = 0.10, size = 469, normalized size = 3.69

method	result
risch	$\frac{d^2 a^2 \ln(\mathrm{e}^{i(bx+a)}-1)}{b^3} + \frac{d^2 \ln(1-\mathrm{e}^{i(bx+a)}) x^2}{b} - \frac{d^2 \ln(1-\mathrm{e}^{i(bx+a)}) a^2}{b^3} + \frac{d^2 \ln(\mathrm{e}^{i(bx+a)}+1) x^2}{b} - \frac{d^2 \ln(1+\mathrm{e}^{2i(bx+a)}) x^2}{b} - \frac{c^2 \ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d
^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2-1/b*d^2*ln(1+e
xp(2*I*(b*x+a)))*x^2-1/b*c^2*ln(1+exp(2*I*(b*x+a)))+2/b^2*c*d*ln(1-exp(I*(b
*x+a)))*a+2/b*c*d*ln(exp(I*(b*x+a))+1)*x+2/b*c*d*ln(1-exp(I*(b*x+a)))*x-2/b
^2*c*d*a*ln(exp(I*(b*x+a))-1)-2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-2*I/b^
2*c*d*polylog(2,exp(I*(b*x+a)))-2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+I/
b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))-2/b*c*d*ln(1+exp(2*I*(b*x+a)))*x+1/b*c
^2*ln(exp(I*(b*x+a))-1)+1/b*c^2*ln(exp(I*(b*x+a))+1)-1/2*d^2*polylog(3,-exp
(2*I*(b*x+a)))/b^3+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp
(I*(b*x+a)))/b^3-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x+I/b^2*d^2*polylog(
2,-exp(2*I*(b*x+a)))*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(107) = 214$.
time = 0.58, size = 599, normalized size = 4.72

$$-\frac{c^2 \left(\log\left(\sin(bx+a)^2 - 1\right) - \log\left(\sin(bx+a)^2\right)\right) - \frac{2acd\left(\log\left(\sin(bx+a)^2-1\right)-\log\left(\sin(bx+a)^2\right)\right)}{b} + \frac{a^2d^2\left(\log\left(\sin(bx+a)^2-1\right)-\log\left(\sin(bx+a)^2\right)\right)}{b^2} + \ldots}{2b}$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 2*a*c*d*(log(si
n(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + a^2*d^2*(log(sin(b*x + a)^2 -
1) - log(sin(b*x + a)^2))/b^2 + (d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) - 4*d
^2*polylog(3, -e^(I*b*x + I*a)) - 4*d^2*polylog(3, e^(I*b*x + I*a)) - 2*(-I
*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2(sin(2*b*x + 2*
a), cos(2*b*x + 2*a) + 1) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b
*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(-I*(b*x + a)^2*d^2 +
2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1)
- 2*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*dilog(-e^(2*I*b*x + 2*I*a)) - 4*(
-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*dilog(-e^(I*b*x + I*a)) - 4*(-I*b*c*d
 - I*(b*x + a)*d^2 + I*a*d^2)*dilog(e^(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2
```

```
*(b*c*d - a*d^2)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2
*cos(2*b*x + 2*a) + 1) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*lo
g(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - ((b*x + a)^2*d^2
+ 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(
b*x + a) + 1))/b^2)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(107) = 214$.
time = 2.07, size = 1098, normalized size = 8.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, cos
(b*x + a) - I*sin(b*x + a)) - 2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a
)) - 2*d^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 2*d^2*polylog(3, -I*
cos(b*x + a) + sin(b*x + a)) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x +
 a)) + 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3,
-cos(b*x + a) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(cos(b*x + a
) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(cos(b*x + a) - I*sin(b
*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) - 2
*(-I*b*d^2*x - I*b*c*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 2*(-I*b*d^2*
x - I*b*c*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d
)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(-c
os(b*x + a) + I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(-cos(b*x + a)
 - I*sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a)
 + I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) +
 I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x +
a) - I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a)
 - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*
log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b
*c*d - a^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b
^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (
b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(
b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a) + 1/2
*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x +
 a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d -
a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*
x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/b^3
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \csc (a + bx) \sec (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a),x)
[Out] Integral((c + d*x)**2*csc(a + b*x)*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^2*csc(b*x + a)*sec(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d\,x)^2}{\cos (a + b\,x)\, \sin (a + b\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)),x)
[Out] int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)), x)
```

3.231 $\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=71

$$-\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}+\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}$$

```
[Out] -2*(d*x+c)*arctanh(exp(2*I*(b*x+a)))/b+1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))
/b^2-1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4504, 4268, 2317, 2438}

$$\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-2*(c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b + ((I/2)*d*PolyLog[2, -E^((2*
I)*(a + b*x))])/b^2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
```

```
x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx = 2\int (c+dx)\csc(2a+2bx)\,dx$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\int \log\left(1-e^{i(2a+2bx)}\right)\,dx}{b} + \frac{d\int \mathrm{lo}$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{(id)\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{i(2a+2b}}{2b^2}$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{id\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{id\mathrm{Li}_2\left(e^{2i(a+bx}}{2b^2}$$

Mathematica [A]
time = 0.13, size = 141, normalized size = 1.99

$$-\frac{c\log(\cos(a+bx))}{b}+\frac{c\log(\sin(a+bx))}{b}+\frac{d\left((2a+2bx)\left(\log\left(1-e^{i(2a+2bx)}\right)-\log\left(1+e^{i(2a+2bx)}\right)\right)-2a\log\left(\tan\left(\frac{1}{2}(2a+2bx)\right)\right)+i\left(\mathrm{PolyLog}\left(2,-e^{i(2a+2bx)}\right)-\mathrm{PolyLog}\left(2,e^{i(2a+2bx)}\right)\right)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] -((c*Log[Cos[a + b*x]])/b) + (c*Log[Sin[a + b*x]])/b + (d*((2*a + 2*b*x)*(L
og[1 - E^(I*(2*a + 2*b*x))] - Log[1 + E^(I*(2*a + 2*b*x))]) - 2*a*Log[Tan[(
2*a + 2*b*x)/2]] + I*(PolyLog[2, -E^(I*(2*a + 2*b*x))] - PolyLog[2, E^(I*(2
*a + 2*b*x))])))/(2*b^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(59) = 118.
time = 0.08, size = 208, normalized size = 2.93

method	result
risch	$\frac{c\ln\left(e^{i(bx+a)}-1\right)}{b}+\frac{c\ln\left(e^{i(bx+a)}+1\right)}{b}-\frac{c\ln\left(1+e^{2i(bx+a)}\right)}{b}+\frac{d\ln\left(1-e^{i(bx+a)}\right)x}{b}+\frac{d\ln\left(1-e^{i(bx+a)}\right)a}{b^2}-\frac{id\,\mathrm{polylog}\left(2,e^{i(bx+a}}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*c*ln(exp(I*(b*x+a))-1)+1/b*c*ln(exp(I*(b*x+a))+1)-1/b*c*ln(1+exp(2*I*(b
*x+a)))+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b*x+a)))*a-I*d*pol
ylog(2,exp(I*(b*x+a)))/b^2-1/b*d*ln(1+exp(2*I*(b*x+a)))*x+1/2*I*d*polylog(2
,-exp(2*I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*polylog(2,-exp(I*(
b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(55) = 110$.
time = 0.56, size = 269, normalized size = 3.79

$$-\frac{2 i \, b d x \arctan\left(\sin\left(b x + a\right), -\cos\left(b x + a\right) + 1\right) - 2 i \, b c \arctan\left(\sin\left(b x + a\right), \cos\left(b x + a\right) - 1\right) - 2 \, {\left(-i \, b d x - i \, b c\right)} \arctan\left(\sin\left(2 \, b x + 2 \, a\right), \cos\left(2 \, b x + 2 \, a\right) + 1\right) - 2 \, {\left(i \, b d x + i \, b c\right)} \arctan\left(\sin\left(b x + a\right), \cos\left(b x + a\right) + 1\right) - i \, d {\rm Li}_2\left(-e^{\left(2 i \, b x + 2 i \, a\right)}\right) + 2 i \, d {\rm Li}_2\left(-e^{\left(i \, b x + i \, a\right)}\right) + 2 i \, d {\rm Li}_2\left(e^{\left(i \, b x + i \, a\right)}\right) + {\left(b d x + b c\right)} \log\left(\cos\left(2 \, b x + 2 \, a\right)^{2} + \sin\left(2 \, b x + 2 \, a\right)^{2} + 2 \, \cos\left(2 \, b x + 2 \, a\right) + 1\right) - {\left(b d x + b c\right)} \log\left(\cos\left(b x + a\right)^{2} + \sin\left(b x + a\right)^{2} + 2 \, \cos\left(b x + a\right) + 1\right) - {\left(b d x + b c\right)} \log\left(\cos\left(b x + a\right)^{2} + \sin\left(b x + a\right)^{2} - 2 \, \cos\left(b x + a\right) + 1\right)}{2 \, b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*I*b*c*arctan2(
sin(b*x + a), cos(b*x + a) - 1) - 2*(-I*b*d*x - I*b*c)*arctan2(sin(2*b*x +
2*a), cos(2*b*x + 2*a) + 1) - 2*(I*b*d*x + I*b*c)*arctan2(sin(b*x + a), cos
(b*x + a) + 1) - I*d*dilog(-e^(2*I*b*x + 2*I*a)) + 2*I*d*dilog(-e^(I*b*x +
I*a)) + 2*I*d*dilog(e^(I*b*x + I*a)) + (b*d*x + b*c)*log(cos(2*b*x + 2*a)^2
 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (b*d*x + b*c)*log(cos(b*x
 + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (b*d*x + b*c)*log(cos(b*x
+ a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1))/b^2
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(55) = 110$.
time = 1.68, size = 554, normalized size = 7.80

$$\frac{-i \, d {\rm Li}_2\left(\cos\left(b x + a\right) + i \, \sin\left(b x + a\right)\right) + i \, d {\rm Li}_2\left(\cos\left(b x + a\right) - i \, \sin\left(b x + a\right)\right) - i \, d {\rm Li}_2\left(i \, \cos\left(b x + a\right) + \sin\left(b x + a\right)\right) + i \, d {\rm Li}_2\left(i \, \cos\left(b x + a\right) - \sin\left(b x + a\right)\right) + i \, d {\rm Li}_2\left(-i \, \cos\left(b x + a\right) + \sin\left(b x + a\right)\right) - i \, d {\rm Li}_2\left(-i \, \cos\left(b x + a\right) - \sin\left(b x + a\right)\right) + i \, d {\rm Li}_2\left(-\cos\left(b x + a\right) + i \, \sin\left(b x + a\right)\right) - i \, d {\rm Li}_2\left(-\cos\left(b x + a\right) - i \, \sin\left(b x + a\right)\right) + {\left(b d x + b c\right)} \log\left(\cos\left(b x + a\right) + i \, \sin\left(b x + a\right) + 1\right) - {\left(b c - a d\right)} \log\left(\cos\left(b x + a\right) + i \, \sin\left(b x + a\right) + i\right) + {\left(b d x + b c\right)} \log\left(\cos\left(b x + a\right) - i \, \sin\left(b x + a\right) + 1\right) - {\left(b c - a d\right)} \log\left(\cos\left(b x + a\right) - i \, \sin\left(b x + a\right) + i\right) - {\left(b d x + a d\right)} \log\left(i \, \cos\left(b x + a\right) + \sin\left(b x + a\right) + 1\right) - {\left(b d x + a d\right)} \log\left(i \, \cos\left(b x + a\right) - \sin\left(b x + a\right) + 1\right) - {\left(b d x + a d\right)} \log\left(-i \, \cos\left(b x + a\right) + \sin\left(b x + a\right) + 1\right) - {\left(b d x + a d\right)} \log\left(-i \, \cos\left(b x + a\right) - \sin\left(b x + a\right) + 1\right) + {\left(b c - a d\right)} \log\left(-\frac{1}{2} \, \cos\left(b x + a\right) + \frac{1}{2} i \, \sin\left(b x + a\right) + \frac{1}{2}\right) + {\left(b c - a d\right)} \log\left(-\frac{1}{2} \, \cos\left(b x + a\right) - \frac{1}{2} i \, \sin\left(b x + a\right) + \frac{1}{2}\right) + {\left(b d x + a d\right)} \log\left(-\cos\left(b x + a\right) + i \, \sin\left(b x + a\right) + 1\right) - {\left(b c - a d\right)} \log\left(-\cos\left(b x + a\right) + i \, \sin\left(b x + a\right) + i\right) + {\left(b d x + a d\right)} \log\left(-\cos\left(b x + a\right) - i \, \sin\left(b x + a\right) + 1\right) - {\left(b c - a d\right)} \log\left(-\cos\left(b x + a\right) - i \, \sin\left(b x + a\right) + i\right)}{2 \, b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*dilog(cos(b*x + a) - I
*sin(b*x + a)) - I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(I*cos
(b*x + a) - sin(b*x + a)) + I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d
*dilog(-I*cos(b*x + a) - sin(b*x + a)) + I*d*dilog(-cos(b*x + a) + I*sin(b*
x + a)) - I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b*d*x + b*c)*log(cos
(b*x + a) + I*sin(b*x + a) + 1) - (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x
+ a) + I) + (b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b*c - a
*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d*x + a*d)*log(I*cos(b*x +
a) + sin(b*x + a) + 1) - (b*d*x + a*d)*log(I*cos(b*x + a) - sin(b*x + a) +
1) - (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*
log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*log(-1/2*cos(b*x + a)
 + 1/2*I*sin(b*x + a) + 1/2) + (b*c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*si
n(b*x + a) + 1/2) + (b*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) -
 (b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*d*x + a*d)*log(-c
os(b*x + a) - I*sin(b*x + a) + 1) - (b*c - a*d)*log(-cos(b*x + a) - I*sin(b
*x + a) + I))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + d x\right) \csc{\left(a + b x \right)} \sec{\left(a + b x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)*sec(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\,x}{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)*sin(a + b*x)),x)
```

```
[Out] int((c + d*x)/(cos(a + b*x)*sin(a + b*x)), x)
```

3.232 $\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=22

$$2\text{Int}\left(\frac{\csc(2a+2bx)}{c+dx},x\right)$$

```
[Out] 2*Unintegrable(csc(2*b*x+2*a)/(d*x+c),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x),x]
```

```
[Out] 2*Defer[Int][Csc[2*a + 2*b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx = 2\int \frac{\csc(2a+2bx)}{c+dx}\,dx$$

Mathematica [A]
time = 4.48, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x), x]
```

Maple [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)\sec\left(bx+a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\cos(a+b\,x)\,\sin(a+b\,x)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)),x)
[Out] int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)), x)
```

3.233 $\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=22

$$2\text{Int}\left(\frac{\csc(2a+2bx)}{(c+dx)^2},x\right)$$

```
[Out] 2*Unintegrable(csc(2*b*x+2*a)/(d*x+c)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2,x]
[Out] 2*Defer[Int][Csc[2*a + 2*b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = 2\int \frac{\csc(2a+2bx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 5.88, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2,x]
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2, x]
```

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)\sec(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x)
[Out] int(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)**2,x)
[Out] Integral(csc(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\cos(a+b\,x)\,\sin(a+b\,x)\,{(c+d\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)^2),x)
[Out] int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)^2), x)
```

3.234 $\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=25

$$\text{Int}\left((c+dx)^m \csc^2(a+bx)\sec(a+bx),x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx = \int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$$

Mathematica [A]
time = 15.62, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x], x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\csc^2(bx+a)\right)\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x)
[Out] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \csc^2 (a + bx) \sec (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a),x)
[Out] Integral((c + d*x)**m*csc(a + b*x)**2*sec(a + b*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c + d\,x)^m}{\cos (a + b\,x)\, \sin (a + b\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^2),x)
[Out] int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^2), x)
```

3.235 $\int (c+dx)^3 \csc^2(a+bx) \sec(a+bx)\, dx$

Optimal. Leaf size=350

$$-\frac{2i(c+dx)^3 \text{ArcTan}\left(e^{i(a+bx)}\right)}{b}-\frac{6d(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3 \csc(a+bx)}{b}+\frac{6id^2(c+dx)\text{PolyLog}}{b^3}$$

```
[Out] -2*I*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b-6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a)
))/b^2-(d*x+c)^3*csc(b*x+a)/b+6*I*d^2*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^
3+3*I*d*(d*x+c)^2*polylog(2,-I*exp(I*(b*x+a)))/b^2-3*I*d*(d*x+c)^2*polylog(
2,I*exp(I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^3-6*d^3
*polylog(3,-exp(I*(b*x+a)))/b^4-6*d^2*(d*x+c)*polylog(3,-I*exp(I*(b*x+a)))/
b^3+6*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^3+6*d^3*polylog(3,exp(I*(b*
x+a)))/b^4-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,I*exp
(I*(b*x+a)))/b^4
```

Rubi [A]
time = 0.43, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {2701, 327, 213, 4505, 6873, 12, 6874, 6408, 4266, 2611, 6744, 2320, 6724, 4268}

$$-\frac{2i(c+dx)^3 \text{ArcTan}\left(e^{i(a+bx)}\right)}{b}-\frac{6d^3\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{Li}_3\left(e^{i(a+bx)}\right)}{b^4}-\frac{6id^3\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^4}+\frac{6id^3\text{Li}_4\left(ie^{i(a+bx)}\right)}{b^4}+\frac{6id^2(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}-\frac{6d^2(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b - (6*d*(c + d*x)^2*ArcTanh[E
^(I*(a + b*x))])/b^2 - ((c + d*x)^3*Csc[a + b*x])/b + ((6*I)*d^2*(c + d*x)*
PolyLog[2, -E^(I*(a + b*x))])/b^3 + ((3*I)*d*(c + d*x)^2*PolyLog[2, (-I)*E^
(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b
^2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog
[3, -E^(I*(a + b*x))])/b^4 - (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x
))])/b^3 + (6*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + (6*d^3*Pol
yLog[3, E^(I*(a + b*x))])/b^4 - ((6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))]
)/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
```

```
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx = \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - (3d)\int (c$$

$$= \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - (3d)\int \frac{(c}{}$$

$$= \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - \frac{(3d)\int (c}{}$$

$$= \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - \frac{(3d)\int ((}{}$$

$$= \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - \frac{(3d)\int (c}{}$$

$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^3\csc(a+bx)}{b} + \frac{\int b(c+}{}$$

$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^3\csc(a+bx)}{b} + \frac{6id^2(c+}{}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c}{}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c}{}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c}{}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c}{}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 760 vs. $2(350) = 700$.
time = 3.12, size = 760, normalized size = 2.17

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] -((((2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 6*b^2*c^2*d*ArcTanh[E^(I*(a + b*
x))] + b^3*c^3*Csc[a + b*x] + 3*b^3*c^2*d*x*Csc[a + b*x] + 3*b^3*c*d^2*x^2*
Csc[a + b*x] + b^3*d^3*x^3*Csc[a + b*x] - 6*b^2*c*d^2*x*Log[1 - E^(I*(a + b
*x))] - 3*b^2*d^3*x^2*Log[1 - E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 - I*E^
```

```
(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - b^3*d^3*x^3*L
og[1 - I*E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] + 3*b^
3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*
x))] + 6*b^2*c*d^2*x*Log[1 + E^(I*(a + b*x))] + 3*b^2*d^3*x^2*Log[1 + E^(I*
(a + b*x))] - (6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (3*I)*b^
2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))] + (3*I)*b^2*c^2*d*PolyLog[
2, I*E^(I*(a + b*x))] + (6*I)*b^2*c*d^2*x*PolyLog[2, I*E^(I*(a + b*x))] + (
3*I)*b^2*d^3*x^2*PolyLog[2, I*E^(I*(a + b*x))] + (6*I)*b*c*d^2*PolyLog[2, E
^(I*(a + b*x))] + (6*I)*b*d^3*x*PolyLog[2, E^(I*(a + b*x))] + 6*d^3*PolyLog
[3, -E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*d^
3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(a + b*x
))] - 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] - 6*d^3*PolyLog[3, E^(I*(a +
b*x))] + (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4,
I*E^(I*(a + b*x))])/b^4)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1157 vs. $2(313) = 626$.
time = 0.55, size = 1158, normalized size = 3.31

method	result	size
risch	Expression too large to display	1158

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -6*I/b^3*c*d^2*a^2*arctan(exp(I*(b*x+a)))+6*I/b^2*c^2*d*a*arctan(exp(I*(b*x
+a)))+6*I/b^2*d^2*c*polylog(2,-I*exp(I*(b*x+a)))*x+6/b^3*d^3*ln(1-exp(I*(b*
x+a)))*a*x-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4-6*d^2/b^2*c*ln(exp(I*(b
*x+a))+1)*x+6*I*d^3/b^3*polylog(2,-exp(I*(b*x+a)))*x-6*I*d^3/b^3*polylog(2,
exp(I*(b*x+a)))*x-6*I/b^4*d^3*a*dilog(exp(I*(b*x+a)))+6*I/b^3*c*d^2*dilog(e
xp(I*(b*x+a)))+6*I/b^4*d^3*polylog(2,-exp(I*(b*x+a)))*a-6*I/b^4*d^3*polylog
(2,exp(I*(b*x+a)))*a+6*I/b^3*c*d^2*dilog(exp(I*(b*x+a))+1)-6*I/b^4*a*d^3*di
log(exp(I*(b*x+a))+1)+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4-6*d^2/b^3*c*a*ln(
exp(I*(b*x+a))-1)-6/b^3*d^3*polylog(3,-I*exp(I*(b*x+a)))*x+6/b^3*d^3*polylo
g(3,I*exp(I*(b*x+a)))*x-2*I/b*c^3*arctan(exp(I*(b*x+a)))+3/b^3*a^2*d^2*c*ln
(1+I*exp(I*(b*x+a)))+3/b*c^2*d*ln(1-I*exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-I*
exp(I*(b*x+a)))*a-3/b^3*a^2*d^2*c*ln(1-I*exp(I*(b*x+a)))-3/b*d^2*c*ln(1+I*e
xp(I*(b*x+a)))*x^2+3/b*d^2*c*ln(1-I*exp(I*(b*x+a)))*x^2-3/b*c^2*d*ln(1+I*ex
p(I*(b*x+a)))*x-3/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a-3*I/b^2*c^2*d*polylog(
2,I*exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,I*exp(I*(b*x+a)))*x^2+3*I/b^2*d^3
*polylog(2,-I*exp(I*(b*x+a)))*x^2+2*I/b^4*d^3*a^3*arctan(exp(I*(b*x+a)))+3*
I/b^2*c^2*d*polylog(2,-I*exp(I*(b*x+a)))+6*I*d^3*polylog(4,I*exp(I*(b*x+a))
)/b^4-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(b*
x+a))-1)+3*d/b^2*c^2*ln(exp(I*(b*x+a))-1)-3*d/b^2*c^2*ln(exp(I*(b*x+a))+1)+
3*d^3/b^4*a^2*ln(exp(I*(b*x+a))-1)+3*d^3/b^2*ln(1-exp(I*(b*x+a)))*x^2+3*d^3
```

```
/b^4*ln(1-exp(I*(b*x+a)))*a^2-3*d^3/b^2*ln(exp(I*(b*x+a))+1)*x^2+1/b*d^3*ln
(1-I*exp(I*(b*x+a)))*x^3-1/b*d^3*ln(1+I*exp(I*(b*x+a)))*x^3+1/b^4*a^3*d^3*l
n(1-I*exp(I*(b*x+a)))-1/b^4*a^3*d^3*ln(1+I*exp(I*(b*x+a)))-6/b^3*d^2*c*poly
log(3,-I*exp(I*(b*x+a)))+6/b^3*d^2*c*polylog(3,I*exp(I*(b*x+a)))-6*d^3*poly
log(3,-exp(I*(b*x+a)))/b^4-6*I/b^2*d^2*c*polylog(2,I*exp(I*(b*x+a)))*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3257 vs. $2(296) = 592$.
time = 1.04, size = 3257, normalized size = 9.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))
- 3*a*c^2*d*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1)
)/b + 3*a^2*c*d^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a
) - 1))/b^2 - a^3*d^3*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x
 + a) - 1))/b^3 - 2*(2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 +
 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) - ((b*x + a)^3*d^3 + 3*(b*
c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a
))*cos(2*b*x + 2*a) - (I*(b*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a
)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b*x + a))*sin(2*b*x + 2*
a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + 2*((b*x + a)^3*d^3 + 3*(b*c*d
^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) -
 ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (I*(b*x + a)^3*d^3 + 3*(I*b*
c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*
(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) + 6*(
b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(
b*x + a) - (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^
2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(
b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(2*b*x +
 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 6*(b^2*c^2*d - 2*a*b*c*d^2
 + a^2*d^3 - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(2*b*x + 2*a) + (-I*b^2
*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a),
 cos(b*x + a) - 1) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) - (
(b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (I*(b*x
 + a)^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(
sin(b*x + a), -cos(b*x + a) + 1) - 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(b*x + a
) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a
*d^3)*(b*x + a) - (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*
(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (I*b^2*c^2*d - 2*I*a*b*c*d^
```

```
2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(
2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x
+ a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) - (b^2*c^2*d - 2*a*b*c
*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x
 + 2*a) + (-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2
*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*
a)) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3 - (b*c*d^2 + (b*x + a)*d^3 - a*d^
3)*cos(2*b*x + 2*a) + (-I*b*c*d^2 - I*(b*x + a)*d^3 + I*a*d^3)*sin(2*b*x +
2*a))*dilog(-e^(I*b*x + I*a)) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3 - (b*c*
d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (I*b*c*d^2 + I*(b*x + a)*d^
3 - I*a*d^3)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - 3*(I*b^2*c^2*d - 2*
I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x
+ a) + (-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-
I*b*c*d^2 + I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (b^2*c^2*d - 2*a*b*c*d^2
 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2
*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - 3*(-I*b^2*
c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a
*d^3)*(b*x + a) + (I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*
d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (b^2*c^2*d - 2*
a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(
2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (
I*(b*x + a)^3*d^3 - 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 - 3*(-I*b^2*c^2*d
+ 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a) + (-I*(b*x + a)^3*d^3 - 3*(I*b*c*d^2
 - I*a*d^3)*(b*x + a)^2 - 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b*x
+ a))*cos(2*b*x + 2*a) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2
 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(c
os(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + (-I*(b*x + a)^3*d^3
- 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 - 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*
a^2*d^3)*(b*x + a) + (I*(b*x + a)^3*d^3 - 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a
)^2 - 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a))*cos(2*b*x + 2
*a) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2
*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin
(b*x + a)^2 - 2*sin(b*x + a) + 1) + 12*(d^3*cos(2*b*x + 2*a) + I*d^3*sin(2*
b*x + 2*a) - d^3)*polylog(4, I*e^(I*b*x + I*a)) - 12*(d^3*cos(2*b*x + 2*a)
+ I*d^3*sin(2*b*x + 2*a) - d^3)*polylog(4, -I*e^(I*b*x + I*a)) - 12*(-I*b*c
*d^2 - I*(b*x + a)*d^3 + I*a*d^3 + (I*b*c*d^2 + I*(b*x + a)*d^3 - I*a*d^3)*
cos(2*b*x + 2*a) - (b*c*d^2 + (b*x + a)*d^3 - a...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1753 vs. $2(296) = 592$.
time = 3.61, size = 1753, normalized size = 5.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 6*I*d^3
*polylog(4, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 6*I*d^3*polylog(4
, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + 6*I*d^3*polylog(4, -I*cos(b
*x + a) + sin(b*x + a))*sin(b*x + a) + 6*I*d^3*polylog(4, -I*cos(b*x + a) -
 sin(b*x + a))*sin(b*x + a) - 6*d^3*polylog(3, cos(b*x + a) + I*sin(b*x + a
))*sin(b*x + a) - 6*d^3*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x +
 a) + 6*d^3*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*d^3
*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*(I*b*d^3*x + I
*b*c*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(-I*b*d^3*x
 - I*b*c*d^2)*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 3*(I*b^2*
d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(b*x + a
))*sin(b*x + a) + 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I
*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*
d^2*x - I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + 3
*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - s
in(b*x + a))*sin(b*x + a) + 6*(I*b*d^3*x + I*b*c*d^2)*dilog(-cos(b*x + a) +
 I*sin(b*x + a))*sin(b*x + a) + 6*(-I*b*d^3*x - I*b*c*d^2)*dilog(-cos(b*x +
 a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c
^2*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - (b^3*c^3 - 3*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I)
*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a
) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) - (b^3*
d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
 a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b^3*d^3*x^
3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d
^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) - (b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*lo
g(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b^3*d^3*x^3 + 3*b^3*c
*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*
cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2
+ a^2*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) -
 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*
x + a) + 1/2)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 -
 a^2*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - (b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a
) + I)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^
3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b^3*c^3 - 3*a*b^
2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + I)*
sin(b*x + a) + 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) + sin(b*x +
a))*sin(b*x + a) - 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) - sin(b*
x + a))*sin(b*x + a) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) + s
in(b*x + a))*sin(b*x + a) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a
```

```
) - sin(b*x + a))*sin(b*x + a))/(b^4*sin(b*x + a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \csc^2(a+bx) \sec(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a),x)
[Out] Integral((c + d*x)**3*csc(a + b*x)**2*sec(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^3*csc(b*x + a)^2*sec(b*x + a), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)^2),x)
[Out] \text{Hanged}
```

3.236 $\int (c+dx)^2 \csc^2(a+bx) \sec(a+bx) \, dx$

Optimal. Leaf size=226

$$-\frac{2i(c+dx)^2 \text{ArcTan}\left(e^{i(a+bx)}\right)}{b}-\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{2id^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}$$

[Out] -2*I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b-4*d*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^2-(d*x+c)^2*csc(b*x+a)/b+2*I*d^2*polylog(2,-exp(I*(b*x+a)))/b^3+2*I*d*(d*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^2-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3-2*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3

Rubi [A]
time = 0.26, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {2701, 327, 213, 4505, 6873, 12, 6874, 6408, 4266, 2611, 2320, 6724, 4268, 2317, 2438}

$$-\frac{2i(c+dx)^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{2id^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}-\frac{2d^2\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{2id(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x],x]

[Out] ((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b - (4*d*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^2*Csc[a + b*x])/b + ((2*I)*d^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 + ((2*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^2 \csc^2(a+bx)\sec(a+bx)\,dx = \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - (2d)\int($$

$$= \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - (2d)\int$$

$$= \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - \frac{(2d)\int(}{}$$

$$= \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - \frac{(2d)\int(}{}$$

$$= \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - \frac{(2d)\int(}{}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2\csc(a+bx)}{b} + \frac{\int b(c+}{}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2\csc(a+bx)}{b} + \frac{(2id^2)\,\mathrm{S}}{}$$

$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c}{}$$

$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c}{}$$

$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c}{}$$

$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 593 vs. $2(226) = 452$.
time = 6.22, size = 593, normalized size = 2.62

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] -(((c + d*x)^2*Csc[a])/b) + ((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2
*c*d*x*Log[1 - I*E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))]
- 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a +
b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(
c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*
x))] + 2*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + ((4*I)*c*d*ArcTan[(I*Cos[
a] - I*Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/(b^2*Sqrt[Cos[a]^2
```

```
+ Sin[a]^2]) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c^2*Sin[(b*x)/2]) - 2*c*d*x*
Sin[(b*x)/2] - d^2*x^2*Sin[(b*x)/2]))/(2*b) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*
(c^2*Sin[(b*x)/2] + 2*c*d*x*Sin[(b*x)/2] + d^2*x^2*Sin[(b*x)/2]))/(2*b) + (
2*d^2*((-2*ArcTan[Tan[a]]*ArcTanh[(-Cos[a] + Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[
a]^2 + Sin[a]^2]])/Sqrt[Cos[a]^2 + Sin[a]^2] + (((b*x + ArcTan[Tan[a]])*(Lo
g[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(I*(b*x + ArcTan[Tan[a]]))]
) + I*(PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))] - PolyLog[2, E^(I*(b*x + A
rcTan[Tan[a]]))]))*Sec[a])/Sqrt[1 + Tan[a]^2]))/b^3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(201) = 402.
time = 0.30, size = 556, normalized size = 2.46

method	result
risch	$-\frac{2i(x^2d^2+2cdx+c^2)e^{i(bx+a)}}{b(e^{2i(bx+a)}-1)}-\frac{d^2\ln(1+ie^{i(bx+a)})x^2}{b}-\frac{2d^2\,\mathrm{polylog}(3,-ie^{i(bx+a)})}{b^3}-\frac{2d^2\ln(e^{i(bx+a)}+1)x}{b^2}-\frac{2cd\ln(1+ie^{i(bx+}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*(d^2*x^2+2*c*d*x+c^2)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)-1/b*d^2*ln
(1+I*exp(I*(b*x+a)))*x^2-2*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3-2*d^2/b^2*l
n(exp(I*(b*x+a))+1)*x-2/b*c*d*ln(1+I*exp(I*(b*x+a)))*x-2*d/b^2*c*ln(exp(I*(
b*x+a))+1)+2*d/b^2*c*ln(exp(I*(b*x+a))-1)+2*d^2*polylog(3,I*exp(I*(b*x+a)))
/b^3+2/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a+2*I/b^2*d^2*polylog(2,-I*exp(I*(b*x
+a)))*x-2*d^2/b^3*a*ln(exp(I*(b*x+a))-1)+2/b*c*d*ln(1-I*exp(I*(b*x+a)))*x-2
*I/b*c^2*arctan(exp(I*(b*x+a)))+2*I/b^2*c*d*polylog(2,-I*exp(I*(b*x+a)))-2*
I/b^2*d^2*polylog(2,I*exp(I*(b*x+a)))*x-2*I/b^3*d^2*a^2*arctan(exp(I*(b*x+a
)))+4*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))-1/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)
))+1/b^3*a^2*d^2*ln(1+I*exp(I*(b*x+a)))+2*I/b^3*d^2*dilog(exp(I*(b*x+a)))-2
/b^2*c*d*ln(1+I*exp(I*(b*x+a)))*a-2*I/b^2*c*d*polylog(2,I*exp(I*(b*x+a)))+1
/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2+2*I/b^3*d^2*dilog(exp(I*(b*x+a))+1)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1638 vs. 2(188) = 376.
time = 0.63, size = 1638, normalized size = 7.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))
- 2*a*c*d*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/
b + a^2*d^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1)
)/b^2 - 2*(2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - ((b*x + a)^2*
```

```
d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (I*(b*x + a)^2*d^2 +
2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), si
n(b*x + a) + 1) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - ((b*x
+ a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (I*(b*x + a)^2
*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x +
 a), -sin(b*x + a) + 1) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2 - (b*c*d + (b*x
+ a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*
sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 4*(b*c*d - a*d^
2 - (b*c*d - a*d^2)*cos(2*b*x + 2*a) + (-I*b*c*d + I*a*d^2)*sin(2*b*x + 2*a
))*arctan2(sin(b*x + a), cos(b*x + a) - 1) - 4*((b*x + a)*d^2*cos(2*b*x + 2
*a) + I*(b*x + a)*d^2*sin(2*b*x + 2*a) - (b*x + a)*d^2)*arctan2(sin(b*x + a
), -cos(b*x + a) + 1) - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*c
os(b*x + a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2 - (b*c*d + (b*x + a)*d^2 - a
*d^2)*cos(2*b*x + 2*a) - (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(2*b*x +
2*a))*dilog(I*e^(I*b*x + I*a)) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2 - (b*c*d
+ (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (-I*b*c*d - I*(b*x + a)*d^2 + I
*a*d^2)*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) + 4*(d^2*cos(2*b*x + 2*
a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(-e^(I*b*x + I*a)) - 4*(d^2*cos(2*b
*x + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(e^(I*b*x + I*a)) - 2*(I*b*c
*d + I*(b*x + a)*d^2 - I*a*d^2 + (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*cos
(2*b*x + 2*a) + (b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b
*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - 2*(-I*b*c*d - I*(b*x + a
)*d^2 + I*a*d^2 + (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*cos(2*b*x + 2*a) -
(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(
b*x + a)^2 - 2*cos(b*x + a) + 1) + (I*(b*x + a)^2*d^2 - 2*(-I*b*c*d + I*a*d
^2)*(b*x + a) + (-I*(b*x + a)^2*d^2 - 2*(I*b*c*d - I*a*d^2)*(b*x + a))*cos(
2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(2*b*x +
2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + (-I*(b*x
+ a)^2*d^2 - 2*(I*b*c*d - I*a*d^2)*(b*x + a) + (I*(b*x + a)^2*d^2 - 2*(-I*b
*c*d + I*a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d -
 a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 -
2*sin(b*x + a) + 1) - 4*(I*d^2*cos(2*b*x + 2*a) - d^2*sin(2*b*x + 2*a) - I*
d^2)*polylog(3, I*e^(I*b*x + I*a)) - 4*(-I*d^2*cos(2*b*x + 2*a) + d^2*sin(2
*b*x + 2*a) + I*d^2)*polylog(3, -I*e^(I*b*x + I*a)) - 4*(I*(b*x + a)^2*d^2
+ 2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(b*x + a))/(-2*I*b^2*cos(2*b*x + 2*a)
 + 2*b^2*sin(2*b*x + 2*a) + 2*I*b^2))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1067 vs. $2(188) = 376$.
time = 2.62, size = 1067, normalized size = 4.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*I*d^2*dilog(cos(b*x + a)
+ I*sin(b*x + a))*sin(b*x + a) - 2*I*d^2*dilog(cos(b*x + a) - I*sin(b*x + a
))*sin(b*x + a) + 2*I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a
) - 2*I*d^2*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 2*d^2*poly
log(3, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 2*d^2*polylog(3, I*cos
(b*x + a) - sin(b*x + a))*sin(b*x + a) + 2*d^2*polylog(3, -I*cos(b*x + a) +
 sin(b*x + a))*sin(b*x + a) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x +
a))*sin(b*x + a) + 2*(I*b*d^2*x + I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*x +
 a))*sin(b*x + a) + 2*(I*b*d^2*x + I*b*c*d)*dilog(I*cos(b*x + a) - sin(b*x
+ a))*sin(b*x + a) + 2*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x + a) + sin(b
*x + a))*sin(b*x + a) + 2*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x + a) - si
n(b*x + a))*sin(b*x + a) + 2*(b*d^2*x + b*c*d)*log(cos(b*x + a) + I*sin(b*x
 + a) + 1)*sin(b*x + a) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a)
+ I*sin(b*x + a) + I)*sin(b*x + a) + 2*(b*d^2*x + b*c*d)*log(cos(b*x + a) -
 I*sin(b*x + a) + 1)*sin(b*x + a) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos
(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x +
 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) +
 (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) - sin
(b*x + a) + 1)*sin(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*
d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b^2*d^2*x^2 +
2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)*
sin(b*x + a) - 2*(b*c*d - a*d^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a)
 + 1/2)*sin(b*x + a) - 2*(b*c*d - a*d^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(
b*x + a) + 1/2)*sin(b*x + a) - 2*(b*d^2*x + a*d^2)*log(-cos(b*x + a) + I*si
n(b*x + a) + 1)*sin(b*x + a) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x
 + a) + I*sin(b*x + a) + I)*sin(b*x + a) - 2*(b*d^2*x + a*d^2)*log(-cos(b*x
 + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*
log(-cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a))/(b^3*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \csc^2(a + bx) \sec(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*csc(a + b*x)**2*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^2*sec(b*x + a), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)^2),x)
```

```
[Out] \text{Hanged}
```

3.237 $\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=131

$$-\frac{2idx\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{dx\tanh^{-1}(\sin(a+bx))}{b}+\frac{(c+dx)\tanh^{-1}(\sin(a+bx))}{b}-\frac{(c$$

```
[Out] -2*I*d*x*arctan(exp(I*(b*x+a)))/b-d*arctanh(cos(b*x+a))/b^2-d*x*arctanh(sin
(b*x+a))/b+(d*x+c)*arctanh(sin(b*x+a))/b-(d*x+c)*csc(b*x+a)/b+I*d*polylog(2
,-I*exp(I*(b*x+a)))/b^2-I*d*polylog(2,I*exp(I*(b*x+a)))/b^2
```

Rubi [A]
time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2701, 327, 213, 4505, 6406, 12, 4266, 2317, 2438, 3855}

$$-\frac{2idx\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}+\frac{(c+dx)\tanh^{-1}(\sin(a+bx))}{b}-\frac{dx\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*d*x*ArcTan[E^(I*(a + b*x))])/b - (d*ArcTanh[Cos[a + b*x]])/b^2 - (d
*x*ArcTanh[Sin[a + b*x]])/b + ((c + d*x)*ArcTanh[Sin[a + b*x]])/b - ((c + d
*x)*Csc[a + b*x])/b + (I*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*Pol
yLog[2, I*E^(I*(a + b*x))])/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6406

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx = \frac{(c+dx)\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)\csc(a+bx)}{b} - d\int \left(\frac{\tanh^{-1}}{}\right.$$

$$= \frac{(c+dx)\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)\csc(a+bx)}{b} - \frac{d\int \tanh^{-1}(}{}$$

$$= -\frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a+bx))}{b} + \frac{(c+dx)\tanh}{}$$

$$= -\frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a+bx))}{b} + \frac{(c+dx)\tanh}{}$$

$$= -\frac{2idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a}{b}$$

$$= -\frac{2idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a}{b}$$

$$= -\frac{2idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a}{b}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.24, size = 508, normalized size = 3.88

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] (d*(a*Cos[(a + b*x)/2] - (a + b*x)*Cos[(a + b*x)/2])*Csc[(a + b*x)/2])/(2*b
^2) - (c*Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/b -
(d*Log[Cos[(a + b*x)/2]])/b^2 + (d*Log[Sin[(a + b*x)/2]])/b^2 - (d*x*(a*(Lo
g[1 - Tan[(a + b*x)/2]] - Log[1 + Tan[(a + b*x)/2]]) + I*(Log[1 + I*Tan[(a
+ b*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(a + b*x)/2])] - Log[1 - I*Tan[(a + b
*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(a + b*x)/2])] - Log[1 + I*Tan[(a + b*x)
/2]]*Log[(1/2 - I/2)*(1 + Tan[(a + b*x)/2])] + Log[1 - I*Tan[(a + b*x)/2]]*
Log[(1/2 + I/2)*(1 + Tan[(a + b*x)/2])] - PolyLog[2, ((1 + I) - (1 - I)*Tan
[(a + b*x)/2])/2] + PolyLog[2, (-1/2 - I/2)*(I + Tan[(a + b*x)/2])] - PolyL
og[2, ((1 + I) + (1 - I)*Tan[(a + b*x)/2])/2] + PolyLog[2, ((1 - I) + (1 +
I)*Tan[(a + b*x)/2])/2])))/(b*(a - I*Log[1 - I*Tan[(a + b*x)/2]] + I*Log[1
+ I*Tan[(a + b*x)/2]])) + (d*Sec[(a + b*x)/2]*(a*Sin[(a + b*x)/2] - (a + b*
x)*Sin[(a + b*x)/2]))/(2*b^2)
```

Maple [**A**]
time = 0.14, size = 235, normalized size = 1.79

method	result
risch	$-\frac{2i(dx+c)\,\mathrm{e}^{i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)}+\frac{2ida\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b^2}+\frac{d\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b^2}-\frac{d\ln\left(\mathrm{e}^{i(bx+a)}+1\right)}{b^2}-\frac{2ic\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b}-\frac{id\,\mathrm{dilog}\left(1-i\right.}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*(d*x+c)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)+2*I/b^2*d*a*arctan(exp(I
*(b*x+a)))+d/b^2*ln(exp(I*(b*x+a))-1)-d/b^2*ln(exp(I*(b*x+a))+1)-2*I/b*c*ar
ctan(exp(I*(b*x+a)))-I/b^2*d*dilog(1-I*exp(I*(b*x+a)))+I/b^2*d*dilog(1+I*ex
p(I*(b*x+a)))+1/b*d*ln(1-I*exp(I*(b*x+a)))*x+1/b^2*d*ln(1-I*exp(I*(b*x+a)))
*a-1/b*d*ln(1+I*exp(I*(b*x+a)))*x-1/b^2*d*ln(1+I*exp(I*(b*x+a)))*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(4*(b*d*x + b*c)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*d*x + b*c)*cos(2
*b*x + 2*a)*sin(b*x + a) - 4*(b^2*d*cos(2*b*x + 2*a)^2 + b^2*d*sin(2*b*x +
2*a)^2 - 2*b^2*d*cos(2*b*x + 2*a) + b^2*d)*integrate((x*cos(2*b*x + 2*a)*co
s(b*x + a) + x*sin(2*b*x + 2*a)*sin(b*x + a) + x*cos(b*x + a))/(cos(2*b*x +
 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1), x) - (b*c*cos(2*b*x
 + 2*a)^2 + b*c*sin(2*b*x + 2*a)^2 - 2*b*c*cos(2*b*x + 2*a) + b*c)*log(cos(
b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + (b*c*cos(2*b*x + 2*a)^2
 + b*c*sin(2*b*x + 2*a)^2 - 2*b*c*cos(2*b*x + 2*a) + b*c)*log(cos(b*x + a)^
2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (d*cos(2*b*x + 2*a)^2 + d*sin(2*
b*x + 2*a)^2 - 2*d*cos(2*b*x + 2*a) + d)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a)
 + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (d*cos(2*b*x + 2
*a)^2 + d*sin(2*b*x + 2*a)^2 - 2*d*cos(2*b*x + 2*a) + d)*log(cos(b*x)^2 - 2
*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) +
4*(b*d*x + b*c)*sin(b*x + a))/(b^2*cos(2*b*x + 2*a)^2 + b^2*sin(2*b*x + 2*a
)^2 - 2*b^2*cos(2*b*x + 2*a) + b^2)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(113) = 226$.
time = 3.41, size = 434, normalized size = 3.31

$$-\frac{2\,bdx+i\,d\mathrm{Li}_2(i\cos(bx+a)+\sin(bx+a))\sin(bx+a)+i\,d\mathrm{Li}_2(i\cos(bx+a)-\sin(bx+a))\sin(bx+a)-i\,d\mathrm{Li}_2(-i\cos(bx+a)+\sin(bx+a))\sin(bx+a)-i\,d\mathrm{Li}_2(-i\cos(bx+a)-\sin(bx+a))\sin(bx+a)-(bc-ad)\log(\cos(bx+a)+i\sin(bx+a)+i)\sin(bx+a)+(bc-ad)\log(\cos(bx+a)-i\sin(bx+a)+i)\sin(bx+a)+d\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)\sin(bx+a)-(bdx+ad)\log(i\cos(bx+a)+\sin(bx+a)+1)\sin(bx+a)+(bdx+ad)\log(i\cos(bx+a)-\sin(bx+a)+1)\sin(bx+a)-(bdx+ad)\log(-i\cos(bx+a)+\sin(bx+a)+1)\sin(bx+a)+(bdx+ad)\log(-i\cos(bx+a)-\sin(bx+a)+1)\sin(bx+a)-d\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)\sin(bx+a)-(bc-ad)\log(-\cos(bx+a)+i\sin(bx+a)+i)\sin(bx+a)+(bc-ad)\log(-\cos(bx+a)-i\sin(bx+a)+i)\sin(bx+a)+2\,bc}{2\,b^2\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + I*d*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + I*d
*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - I*d*dilog(-I*cos(b*x +
 a) + sin(b*x + a))*sin(b*x + a) - I*d*dilog(-I*cos(b*x + a) - sin(b*x + a)
)*sin(b*x + a) - (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x
 + a) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + d
*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - (b*d*x + a*d)*log(I*cos(b*x + a
) + sin(b*x + a) + 1)*sin(b*x + a) + (b*d*x + a*d)*log(I*cos(b*x + a) - sin
(b*x + a) + 1)*sin(b*x + a) - (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x +
 a) + 1)*sin(b*x + a) + (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) +
1)*sin(b*x + a) - d*log(-1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - (b*c - a*d)
*log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + (b*c - a*d)*log(-co
s(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 2*b*c)/(b^2*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)**2*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^2*sec(b*x + a), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)*sin(a + b*x)^2),x)
```

```
[Out] \text{Hanged}
```

3.238 $\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{\csc^2(a+bx)\sec(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x),x]
[Out] Defer[Int][(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 10.57, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x),x]
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x), x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^2\left(bx+a\right)\right)\sec\left(bx+a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] (2*(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*
a)^2 + b*c - 2*(b*d*x + b*c)*cos(2*b*x + 2*a))*integrate((cos(2*b*x + 2*a)*
cos(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/((d*x + c)*cos
(2*b*x + 2*a)^2 + (d*x + c)*sin(2*b*x + 2*a)^2 + d*x + 2*(d*x + c)*cos(2*b*
x + 2*a) + c), x) - (b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2
 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2
*a))*integrate(sin(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2
*b*c*d*x + b*c^2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x
+ a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) - (b*d^2*x + b
*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x +
 2*a)^2 - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d
^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2
 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*
x + b*c^2)*cos(b*x + a)), x) - 2*cos(b*x + a)*sin(2*b*x + 2*a) + 2*cos(2*b*
x + 2*a)*sin(b*x + a) - 2*sin(b*x + a))/(b*d*x + (b*d*x + b*c)*cos(2*b*x +
2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)^2 + b*c - 2*(b*d*x + b*c)*cos(2*b*x
 + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sec(b*x + a)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sec(b*x + a)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^2\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)),x)
```

```
[Out] int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)), x)
```

3.239 $\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x)
```

Rubi [A]
time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2,x]
[Out] Defer[Int][(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 10.04, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2,x]
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2, x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^2\left(bx+a\right)\right)\sec\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 2*((b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b
*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)^2 - 2*(b*d^2
*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a))*integrate((cos(2*b*x + 2*a)*cos
(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(d^2*x^2 + 2*c*d*
x + (d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x + 2*a)^2 + (d^2*x^2 + 2*c*d*x + c^2
)*sin(2*b*x + 2*a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x + 2*a)),
 x) - (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2
*d)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(2*b*x + 2*
a)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a))*integrate(si
n(b*x + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 +
3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^
2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(b*x + a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2
+ 3*b*c^2*d*x + b*c^3)*cos(b*x + a)), x) - (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2
*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 +
2*b*c*d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*
c^2*d)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2
+ 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*c
os(b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(b*x +
 a)^2 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)),
x) - cos(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a)*sin(b*x + a) - sin(b*
x + a))/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*co
s(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)^2 - 2*(
b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+b\,x)\,\sin(a+b\,x)^2\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^2),x)
```

```
[Out] int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^2), x)
```

3.240 $\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=25

$$\text{Int}\left((c+dx)^m \csc^3(a+bx)\sec(a+bx),x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x)
```

Rubi [A]
time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx = \int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$$

Mathematica [A]
time = 15.80, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x], x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\csc^3(bx+a)\right)\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x)
[Out] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^3),x)
[Out] int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^3), x)
```

3.241 $\int (c+dx)^3 \csc^3(a+bx) \sec(a+bx) \, dx$

Optimal. Leaf size=325

$$-\frac{3id(c+dx)^2}{2b^2}-\frac{(c+dx)^3}{2b}-\frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}$$

```
[Out] -3/2*I*d*(d*x+c)^2/b^2-1/2*(d*x+c)^3/b-2*(d*x+c)^3*arctanh(exp(2*I*(b*x+a))
)/b-3/2*d*(d*x+c)^2*cot(b*x+a)/b^2-1/2*(d*x+c)^3*cot(b*x+a)^2/b+3*d^2*(d*x+
c)*ln(1-exp(2*I*(b*x+a)))/b^3+3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a))
)/b^2-3/2*I*d^3*polylog(2,exp(2*I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polylog(2
,exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))/b^3+3/2
*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*polylog(4,-exp(2*I*(
b*x+a)))/b^4+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4
```

Rubi [A]
time = 0.55, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {2700, 14, 4505, 6873, 12, 6874, 3801, 3798, 2221, 2317, 2438, 32, 2631, 4268, 2611, 6744, 2320, 6724}

$$-\frac{3id^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}+\frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{3id(c+dx)^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}-\frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{3id(c+dx)^2}{2b^2}-\frac{(c+dx)^3}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] (((-3*I)/2)*d*(c + d*x)^2)/b^2 - (c + d*x)^3/(2*b) - (2*(c + d*x)^3*ArcTanh
[E^((2*I)*(a + b*x))])/b - (3*d*(c + d*x)^2*Cot[a + b*x])/(2*b^2) - ((c + d
*x)^3*Cot[a + b*x]^2)/(2*b) + (3*d^2*(c + d*x)*Log[1 - E^((2*I)*(a + b*x))]
)/b^3 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((
(3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^4 - (((3*I)/2)*d*(c + d*x)^
2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2
*I)*(a + b*x))])/(2*b^3) + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))]
)/(2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/
4)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
```

```
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^3 \csc^3(a+bx)\sec(a+bx)\,dx = -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} - (3d)\int (c$$

$$= -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} - (3d)\int \frac{(c}{}$$

$$= -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} - \frac{(3d)\int (c-}{}$$

$$= -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} - \frac{(3d)\int (-}{}$$

$$= -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} + \frac{(3d)\int (c-}{}$$

$$= -\frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{\int 2b(c+dx}{}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3}{}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c}{}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c}{}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c}{}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c}{}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1285 vs. $2(325) = 650$.
time = 6.83, size = 1285, normalized size = 3.95

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -1/2*((c + d*x)^3*Csc[a + b*x]^2)/b - (c*d^2*Csc[a]*(2*b^2*x^2*(2*b*E^((2*I
)*a)*x + (3*I)*(-1 + E^((2*I)*a))*Log[1 - E^((2*I)*(a + b*x))]) + 6*b*(-1 +
 E^((2*I)*a))*x*PolyLog[2, E^((2*I)*(a + b*x))] + (3*I)*(-1 + E^((2*I)*a))*
PolyLog[3, E^((2*I)*(a + b*x))]))/(4*b^3*E^(I*a)) - (d^3*E^(I*a)*Csc[a]*(x^
4 + (-1 + E^((-2*I)*a))*x^4 + ((-1 + E^((2*I)*a))*(2*b^4*x^4 + (4*I)*b^3*x^
3*Log[1 - E^((2*I)*(a + b*x))] + 6*b^2*x^2*PolyLog[2, E^((2*I)*(a + b*x))]
+ (6*I)*b*x*PolyLog[3, E^((2*I)*(a + b*x))] - 3*PolyLog[4, E^((2*I)*(a + b*
x))]))/(2*b^4*E^((2*I)*a))))/4 + (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*
x^3)*Csc[a]*Sec[a])/4 + (c*d^2*((2*I)*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(1
 + E^((2*I)*a))*Log[1 + E^((2*I)*(a + b*x))]) + (6*I)*b*(1 + E^((2*I)*a))*x
*PolyLog[2, -E^((2*I)*(a + b*x))] - 3*(1 + E^((2*I)*a))*PolyLog[3, -E^((2*I
)*(a + b*x))])*Sec[a])/(4*b^3*E^(I*a)) - (I/4)*d^3*E^(I*a)*(-x^4 + (1 + E^(
(-2*I)*a))*x^4 - ((1 + E^((2*I)*a))*(2*b^4*x^4 + (4*I)*b^3*x^3*Log[1 + E^((
2*I)*(a + b*x))] + 6*b^2*x^2*PolyLog[2, -E^((2*I)*(a + b*x))] + (6*I)*b*x*P
olyLog[3, -E^((2*I)*(a + b*x))] - 3*PolyLog[4, -E^((2*I)*(a + b*x))]))/(2*b
^4*E^((2*I)*a)))*Sec[a] - (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*
Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (c^3*Csc[a]*(-(b*x*Cos
[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a
]^2)) + (3*c*d^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b
*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) - (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I
*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^(
(-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[
a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]]
 + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec
[a])/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (3*Csc[a]*Csc[a + b*x]*
(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[b*x]))/(2*b^2) - (3*c^2*
d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan
[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2
*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[
b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Ta
n[a])/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (
3*d^3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan
[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E
^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[
Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))]
)*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^4*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1222 vs. 2(279) = 558.
time = 0.20, size = 1223, normalized size = 3.76

method	result	size
risch	Expression too large to display	1223

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 6*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)))-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+6
/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))
+6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))
*x+1/b*c^3*ln(exp(I*(b*x+a))+1)+1/b*c^3*ln(exp(I*(b*x+a))-1)-6*I/b^2*c*d^2*
polylog(2,exp(I*(b*x+a)))*x+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4-6*I*d^3/b
^3*a*x+3*d^2/b^3*c*ln(exp(I*(b*x+a))-1)+3*d^2/b^3*c*ln(exp(I*(b*x+a))+1)-6*
d^2/b^3*c*ln(exp(I*(b*x+a)))+3*d^3/b^3*ln(1-exp(I*(b*x+a)))*x+3*d^3/b^4*ln(
1-exp(I*(b*x+a)))*a+3*d^3/b^3*ln(exp(I*(b*x+a))+1)*x-3*d^3/b^4*a*ln(exp(I*(
b*x+a))-1)+6*d^3/b^4*a*ln(exp(I*(b*x+a)))-3*I*d^3/b^2*x^2-3*I*d^3/b^4*a^2-3
*I*d^3/b^4*polylog(2,-exp(I*(b*x+a)))+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+1/b*
d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+3/b^3*c*d^2
*a^2*ln(exp(I*(b*x+a))-1)-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4-1/b*c^
3*ln(1+exp(2*I*(b*x+a)))-3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))-3/2/b^3
*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+(2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x
^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x
+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*I*c^2*d*exp(2*I*(b*x+a))+3*I*d^3*x^2+2*
b*c^3*exp(2*I*(b*x+a))+6*I*c*d^2*x+3*I*c^2*d)/b^2/(exp(2*I*(b*x+a))-1)^2+3/
b*c^2*d*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*c^2*d
*ln(1-exp(I*(b*x+a)))*a+3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b*c*d^2*ln(1-e
xp(I*(b*x+a)))*x^2-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-3/b^2*c^2*d*a*ln(ex
p(I*(b*x+a))-1)-3*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-3*I/b^2*c^2*d*polyl
og(2,-exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3*I/b^2*d^
3*polylog(2,exp(I*(b*x+a)))*x^2+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*
x^2+3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+a)))-3/b*c^2*d*ln(1+exp(2*I*(b*
x+a)))*x-3/b*c*d^2*ln(1+exp(2*I*(b*x+a)))*x^2-1/b*d^3*ln(1+exp(2*I*(b*x+a))
)*x^3-6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x+3*I/b^2*c*d^2*polylog(2,-e
xp(2*I*(b*x+a)))*x-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5165 vs. 2(270) = 540.
time = 1.80, size = 5165, normalized size = 15.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")

[Out] -1/2*(c^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 3*a*c^2*d*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + 3*a^2*c*d^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - a^3*d^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^3 - 2*(18*b^2*c^2*d - 36*a*b*c*d^2 + 18*a^2*d^3 - 2*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) + (4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 2*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-4*I*(b*x + a)^3*d^3 + 9*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 9*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(4*I*(b*x + a)^3*d^3 + 9*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 9*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + 6*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a) + ((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (I*(b*x + a)^3*d^3 + 3*I*b*c*d^2 - 3*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 + I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*(-I*(b*x + a)^3*d^3 - 3*I*b*c*d^2 + 3*I*a*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 - I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 18*(b*c*d^2 - a*d^3 + (b*c*d^2 - a*d^3)*cos(4*b*x + 4*a) - 2*(b*c*d^2 - a*d^3)*cos(2*b*x + 2*a) + (I*b*c*d^2 - I*a*d^3)*sin(4*b*x + 4*a) + 2*(-I*b*c*d^2 + I*a*d^3)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) - 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^3*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 - I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(I*(b*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 + I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 18*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 6*(-2*I*(b*x + a)^3*d^3 - 3*b^2*c^2*d + 6*a*b*c*d^2 - 3*a^2*d^3 + 3*(-2*I*b*c*d^2 + (2*I*a + 1)*d^3)*(b*x + a)^2 + 6*(-I*b^2*c^2*d + (2*I*a + 1)*b*c*d^2 + (-I*a^2 - a)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 3*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*a^2*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a) + (3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*a^2*d^3 + 6*(b*c*d^2 - a*d

```
^3)*(b*x + a))*cos(4*b*x + 4*a) - 2*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a
)^2*d^3 + 3*a^2*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (3*
I*b^2*c^2*d - 6*I*a*b*c*d^2 + 4*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + 6*(I*b*c*
d^2 - I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*(-3*I*b^2*c^2*d + 6*I*a*b*c*
d^2 - 4*I*(b*x + a)^2*d^3 - 3*I*a^2*d^3 + 6*(-I*b*c*d^2 + I*a*d^3)*(b*x + a
))*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) - 18*(b^2*c^2*d - 2*a*b*c*
d^2 + (b*x + a)^2*d^3 + (a^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) + (b^
2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^
3)*(b*x + a))*cos(4*b*x + 4*a) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d
^3 + (a^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-I*
b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 + (-I*a^2 - I)*d^3 + 2*(-I*b*
c*d^2 + I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(I*b^2*c^2*d - 2*I*a*b*c*d
^2 + I*(b*x + a)^2*d^3 + (I*a^2 + I)*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a
))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) - 18*(b^2*c^2*d - 2*a*b*c*d^2
+ (b*x + a)^2*d^3 + (a^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) + (b^2*c^
2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(
b*x + a))*cos(4*b*x + 4*a) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 +
 (a^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-I*b^2*
c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 + (-I*a^2 - I)*d^3 + 2*(-I*b*c*d^
2 + I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(I*b^2*c^2*d - 2*I*a*b*c*d^2 +
 I*(b*x + a)^2*d^3 + (I*a^2 + I)*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*s
in(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-4*I*(b*x + a)^3*d^3 - 9*(I*b*c*
d^2 - I*a*d^3)*(b*x + a)^2 - 9*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b
*x + a) + (-4*I*(b*x + a)^3*d^3 - 9*(I*b*c*d^2 ...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3475 vs. $2(270) = 540$.
time = 3.32, size = 3475, normalized size = 10.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 + 3*(b^2*d^3*x
^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) - 3*(-I*b^2*d^3*x
^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d - I*d^3 + (I*b^2*d^3*x^2 + 2*I*b^2*c*d^2
*x + I*b^2*c^2*d + I*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) + I*sin(b*x +
a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d + I*d^3 + (-I*b^2*d^
3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d - I*d^3)*cos(b*x + a)^2)*dilog(cos(b*
x + a) - I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*
d + (I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*cos(b*x + a)^2)*dilog(I
*cos(b*x + a) + sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*
c^2*d + (-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*cos(b*x + a)^2)*di
log(I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I
```

*b^2*c^2*d + (-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*cos(b*x + a)^
2)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^
2*x - I*b^2*c^2*d + (I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*cos(b*x
 + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2
*c*d^2*x + I*b^2*c^2*d + I*d^3 + (-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*
c^2*d - I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 3*(-
I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d - I*d^3 + (I*b^2*d^3*x^2 + 2*
I*b^2*c*d^2*x + I*b^2*c^2*d + I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) -
I*sin(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2 - (b
^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2 + 3*(b^3*c^2*d + b*d^3)*
x)*cos(b*x + a)^2 + 3*(b^3*c^2*d + b*d^3)*x)*log(cos(b*x + a) + I*sin(b*x +
 a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2)*log(cos(b*x + a) +
 I*sin(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2
 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2 + 3*(b^3*c^2*d + b*
d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d + b*d^3)*x)*log(cos(b*x + a) - I*sin(
b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2)*log(cos(b*x +
 a) - I*sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x
+ 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
+ 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)^2)*
log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3
*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - (b^3*d^3*x^3 + 3*b
^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cos
(b*x + a)^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*
c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - (b^3*
d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
 a^3*d^3)*cos(b*x + a)^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^3*d^
3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a
^3*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)^2)*log(-I*cos(b*x + a) - sin(b*x + a)
+ 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3 - (
b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x +
a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^3*c^3 - 3*a*b^
2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x +
 a) - 1/2*I*sin(b*x + a) + 1/2) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*
cos(b*x + a)^2 + 3*(b^3*c^2*d + b*d^3)*x)*log(-cos(b*x + a) + I*sin(b*x + a
) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*
a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2)*log(-cos(b*x + a) +
I*sin(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^
2*b*c*d^2 + (a^3 + 3*a)*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*
d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x + a)

```
^2 + 3*(b^3*c^2*d + b*d^3)*x)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + (b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x +
a) + I) - 6*(-I*d^3*cos(b*x + a)^2 + I*d^3)*polylog(4, cos(b*x + a) + I*sin
(b*x + a)) - 6*(I*d^3*cos(b*x + a)^2 - I*d^3)*polylog(4, cos(b*x + a) - I*s
in(b*x + a)) - 6*(-I*d^3*cos(b*x + a)^2 + I*d^3)*polylog(4, I*cos(b*x + a)
+ sin(b*x + a)) - 6*(I*d^3*cos(b*x + a)^2 - I*d^3)*polylog(4, I*cos(b*x + a
) - sin(b*x + a)) - 6*(I*d^3*cos(b*x + a)^2 - I*d^3)*polylog(4, -I*cos(b*x
+ a) + sin(b*x + a)) - 6*(-I*d^3*cos(b*x + a)^2 + I*d^3)*polylog(4, -I*cos(
b*x + a) - sin(b*x + a)) - 6*(I*d^3*cos(b*x + a)^2 - I*d^3)*polylog(4, -cos
(b*x + a) + I*sin(b*x + a)) - 6*(-I*d^3*cos(b*x...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \csc^3 (a + bx) \sec (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a),x)
[Out] Integral((c + d*x)**3*csc(a + b*x)**3*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^3*csc(b*x + a)^3*sec(b*x + a), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)^3),x)
[Out] \text{Hanged}
```

3.242 $\int (c+dx)^2 \csc^3(a+bx) \sec(a+bx) \, dx$

Optimal. Leaf size=201

$$-\frac{cdx}{b}-\frac{d^2x^2}{2b}-\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{d^2\log(\sin(a+}{b^3}$$

```
[Out] -c*d*x/b-1/2*d^2*x^2/b-2*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b-d*(d*x+c)*co
t(b*x+a)/b^2-1/2*(d*x+c)^2*cot(b*x+a)^2/b+d^2*ln(sin(b*x+a))/b^3+I*d*(d*x+c
)*polylog(2,-exp(2*I*(b*x+a)))/b^2-I*d*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/
b^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+1/2*d^2*polylog(3,exp(2*I*(b*x
+a)))/b^3
```

Rubi [A]
time = 0.31, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {2700, 14, 4505, 6873, 12, 6874, 3801, 3556, 2631, 4268, 2611, 2320, 6724}

$$-\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\log(\sin(a+bx))}{b^3}+\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}-\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{cdx}{b}-\frac{d^2x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -((c*d*x)/b) - (d^2*x^2)/(2*b) - (2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))
])/b - (d*(c + d*x)*Cot[a + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]^2)/(2*b)
+ (d^2*Log[Sin[a + b*x]])/b^3 + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*
x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d^2*Poly
Log[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E^((2*I)*(a + b*x))
])/(2*b^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx = -\frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{(c+dx)^2\log(\tan(a+bx))}{b} - (2d)\int (c$$

$$= -\frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{(c+dx)^2\log(\tan(a+bx))}{b} - (2d)\int \frac{(c}{}$$

$$= -\frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{(c+dx)^2\log(\tan(a+bx))}{b} - \frac{d\int (c+d}{}$$

$$= -\frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{(c+dx)^2\log(\tan(a+bx))}{b} - \frac{d\int (-(c}{}$$

$$= -\frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{d\int (c+d}{}$$

$$= -\frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{\int 2b(c+dx)^2}{}$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} +$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cot(a+}{b^2}$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cot(a+}{b^2}$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cot(a+}{b^2}$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cot(a+}{b^2}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 785 vs. $2(201) = 402$.
time = 6.70, size = 785, normalized size = 3.91

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -1/2*((c + d*x)^2*Csc[a + b*x]^2)/b - (d^2*Csc[a]*(2*b^2*x^2*(2*b*E^((2*I)*
a)*x + (3*I)*(-1 + E^((2*I)*a))*Log[1 - E^((2*I)*(a + b*x))]) + 6*b*(-1 + E
^((2*I)*a))*x*PolyLog[2, E^((2*I)*(a + b*x))] + (3*I)*(-1 + E^((2*I)*a))*Po
lyLog[3, E^((2*I)*(a + b*x))]))/(12*b^3*E^(I*a)) + (x*(3*c^2 + 3*c*d*x + d^
2*x^2)*Csc[a]*Sec[a])/3 + (d^2*((2*I)*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(1
 + E^((2*I)*a))*Log[1 + E^((2*I)*(a + b*x))]) + (6*I)*b*(1 + E^((2*I)*a))*x
*PolyLog[2, -E^((2*I)*(a + b*x))] - 3*(1 + E^((2*I)*a))*PolyLog[3, -E^((2*I
)*(a + b*x))])*Sec[a])/(12*b^3*E^(I*a)) - (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*Co
```

```
s[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (c^2*C
sc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(
Cos[a]^2 + Sin[a]^2)) + (d^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] +
Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) - (c*d*Csc[a]*((b^2*x
^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log
[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - Arc
Tan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[C
ot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]
^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Csc[a]*Csc[a + b
*x]*(c*d*Sin[b*x] + d^2*x*Sin[b*x]))/b^2 - (c*d*Csc[a]*Sec[a]*(b^2*E^(I*Arc
Tan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*
b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))]
+ Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*Po
lyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b
^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(181) = 362$.
time = 0.17, size = 632, normalized size = 3.14

method	result
risch	$-\frac{d^2 \ln\left(1+e^{2i(bx+a)}\right) x^2}{b} - \frac{2cda \ln\left(e^{i(bx+a)}-1\right)}{b^2} - \frac{c^2 \ln\left(1+e^{2i(bx+a)}\right)}{b} - \frac{d^2 \operatorname{polylog}\left(3,-e^{2i(bx+a)}\right)}{2b^3} - \frac{2id^2 \operatorname{polylog}\left(2,e^{i(bx+a)}\right)}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d
^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2-1/b*d^2*ln(1+e
xp(2*I*(b*x+a)))*x^2-1/b*c^2*ln(1+exp(2*I*(b*x+a)))+2/b^2*c*d*ln(1-exp(I*(b
*x+a)))*a+2/b*c*d*ln(exp(I*(b*x+a))+1)*x+2/b*c*d*ln(1-exp(I*(b*x+a)))*x-2/b
^2*c*d*a*ln(exp(I*(b*x+a))-1)-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b
^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-
2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))-2/b*c*d*ln(1+exp(2*I*(b*x+a)))*x+1/b*
c^2*ln(exp(I*(b*x+a))-1)+1/b*c^2*ln(exp(I*(b*x+a))+1)-1/2*d^2*polylog(3,-ex
p(2*I*(b*x+a)))/b^3+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,ex
p(I*(b*x+a)))/b^3-2/b^3*d^2*ln(exp(I*(b*x+a)))+1/b^3*d^2*ln(exp(I*(b*x+a))-
1)+1/b^3*d^2*ln(exp(I*(b*x+a))+1)+2*(b*d^2*x^2*exp(2*I*(b*x+a))+2*b*c*d*x*e
xp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))-I*c*d*exp(2
*I*(b*x+a))+I*d^2*x+I*d*c)/b^2/(exp(2*I*(b*x+a))-1)^2+I/b^2*d^2*polylog(2,-
exp(2*I*(b*x+a)))*x+I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2528 vs. $2(177) = 354$.
time = 0.75, size = 2528, normalized size = 12.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)
) - 2*a*c*d*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^
2))/b + a^2*d^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x +
 a)^2))/b^2 + 2*(4*(b*x + a)*d^2*cos(4*b*x + 4*a) + 4*I*(b*x + a)*d^2*sin(4
*b*x + 4*a) - 4*b*c*d + 4*a*d^2 + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b
*x + a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a)
- 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (-I*
(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a) - 2*(I
*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arcta
n2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 2*((b*x + a)^2*d^2 + 2*(b*c*d
- a*d^2)*(b*x + a) + d^2 + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) +
 d^2)*cos(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) +
 d^2)*cos(2*b*x + 2*a) + (I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x +
a) + I*d^2)*sin(4*b*x + 4*a) + 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^
2)*(b*x + a) - I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a)
+ 1) - 2*(d^2*cos(4*b*x + 4*a) - 2*d^2*cos(2*b*x + 2*a) + I*d^2*sin(4*b*x +
 4*a) - 2*I*d^2*sin(2*b*x + 2*a) + d^2)*arctan2(sin(b*x + a), cos(b*x + a)
- 1) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + ((b*x + a)^2*d^2
+ 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b
*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^2 + 2*(-I*b*c
*d + I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c
*d - I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x +
 a) + 1) - 4*(-I*(b*x + a)^2*d^2 - b*c*d + a*d^2 + (-2*I*b*c*d + (2*I*a + 1
)*d^2)*(b*x + a))*cos(2*b*x + 2*a) - 2*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*
c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 2*(b*c*d + (b*x + a)*d^2 -
a*d^2)*cos(2*b*x + 2*a) + (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(4*b*x +
 4*a) + 2*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e
^(2*I*b*x + 2*I*a)) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)
*d^2 - a*d^2)*cos(4*b*x + 4*a) - 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*
x + 2*a) - (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(4*b*x + 4*a) - 2*(I*b
*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a))
 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4
*b*x + 4*a) - 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-I*b*c*
d - I*(b*x + a)*d^2 + I*a*d^2)*sin(4*b*x + 4*a) - 2*(I*b*c*d + I*(b*x + a)*
d^2 - I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b*x + a)^2*d
^2 - 2*(I*b*c*d - I*a*d^2)*(b*x + a) + (-I*(b*x + a)^2*d^2 - 2*(I*b*c*d - I
*a*d^2)*(b*x + a))*cos(4*b*x + 4*a) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d +
 I*a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2
)*(b*x + a))*sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
 + a))*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*co
s(2*b*x + 2*a) + 1) + (I*(b*x + a)^2*d^2 - 2*(-I*b*c*d + I*a*d^2)*(b*x + a)
```

```
 + I*d^2 + (I*(b*x + a)^2*d^2 - 2*(-I*b*c*d + I*a*d^2)*(b*x + a) + I*d^2)*c
os(4*b*x + 4*a) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) +
I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) +
d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) +
d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a)
 + 1) + (I*(b*x + a)^2*d^2 - 2*(-I*b*c*d + I*a*d^2)*(b*x + a) + I*d^2 + (I*
(b*x + a)^2*d^2 - 2*(-I*b*c*d + I*a*d^2)*(b*x + a) + I*d^2)*cos(4*b*x + 4*a
) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) + I*d^2)*cos(2*b
*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(4*b*x
 + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(2*b*x
 + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (-I*d^
2*cos(4*b*x + 4*a) + 2*I*d^2*cos(2*b*x + 2*a) + d^2*sin(4*b*x + 4*a) - 2*d^
2*sin(2*b*x + 2*a) - I*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) - 4*(-I*d^2*co
s(4*b*x + 4*a) + 2*I*d^2*cos(2*b*x + 2*a) + d^2*sin(4*b*x + 4*a) - 2*d^2*si
n(2*b*x + 2*a) - I*d^2)*polylog(3, -e^(I*b*x + I*a)) - 4*(-I*d^2*cos(4*b*x
+ 4*a) + 2*I*d^2*cos(2*b*x + 2*a) + d^2*sin(4*b*x + 4*a) - 2*d^2*sin(2*b*x
+ 2*a) - I*d^2)*polylog(3, e^(I*b*x + I*a)) - 4*((b*x + a)^2*d^2 - I*b*c*d
+ I*a*d^2 + (2*b*c*d - (2*a - I)*d^2)*(b*x + a))*sin(2*b*x + 2*a))/(-2*I*b^
2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4*b*x + 4*a) - 4*
b^2*sin(2*b*x + 2*a) - 2*I*b^2))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1995 vs. $2(177) = 354$.
time = 3.13, size = 1995, normalized size = 9.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)
*sin(b*x + a) - 2*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x + a
)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d + (-I*b*
d^2*x - I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 2*(
-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(I*cos(b*
x + a) + sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*co
s(b*x + a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d
 + (-I*b*d^2*x - I*b*c*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + sin(b*x +
 a)) - 2*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilo
g(-I*cos(b*x + a) - sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d + (-I*b*d^2*x -
I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 2*(-I*b*d^
2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) -
 I*sin(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2 + d^2)*cos(b*x + a)^2 + d^2)*log(cos(b*x + a) + I*sin(b
*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*
```

```
d^2)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + I) - (b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + d^2)*cos(b
*x + a)^2 + d^2)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*
c*d + a^2*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2)*log(cos(b*x
 + a) - I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*
d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log
(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*
d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a
)^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x +
2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos
(b*x + a)^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2
*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2
*d^2)*cos(b*x + a)^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*c^2 -
2*a*b*c*d + (a^2 + 1)*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(b*x +
 a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^2*c^2 - 2*a*b
*c*d + (a^2 + 1)*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(b*x + a)^2
)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - (b^2*d^2*x^2 + 2*b^2*
c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*
d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*c^2 - 2
*a*b*c*d + a^2*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2)*log(-c
os(b*x + a) + I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d
- a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^
2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2
 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*si
n(b*x + a) + I) + 2*(d^2*cos(b*x + a)^2 - d^2)*polylog(3, cos(b*x + a) + I*
sin(b*x + a)) + 2*(d^2*cos(b*x + a)^2 - d^2)*polylog(3, cos(b*x + a) - I*si
n(b*x + a)) - 2*(d^2*cos(b*x + a)^2 - d^2)*polylog(3, I*cos(b*x + a) + sin(
b*x + a)) - 2*(d^2*cos(b*x + a)^2 - d^2)*polylog(3, I*cos(b*x + a) - sin(b*
x + a)) - 2*(d^2*cos(b*x + a)^2 - d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x
 + a)) - 2*(d^2*cos(b*x + a)^2 - d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x
+ a)) + 2*(d^2*cos(b*x + a)^2 - d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x +
 a)) + 2*(d^2*cos(b*x + a)^2 - d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x +
a)))/(b^3*cos(b*x + a)^2 - b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*csc(a + b*x)**3*sec(a + b*x), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)^3),x)
[Out] \text{Hanged}
```

3.243 $\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=141

$$-\frac{dx}{2b}-\frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}-\frac{dx\log(\tan(a+bx))}{b}+\frac{(c+dx)\log(\tan(}{b}$$

```
[Out] -1/2*d*x/b-2*d*x*arctanh(exp(2*I*(b*x+a)))/b-1/2*d*cot(b*x+a)/b^2-1/2*(d*x+
c)*cot(b*x+a)^2/b-d*x*ln(tan(b*x+a))/b+(d*x+c)*ln(tan(b*x+a))/b+1/2*I*d*pol
ylog(2,-exp(2*I*(b*x+a)))/b^2-1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^2
```

Rubi [A]

time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2700, 14, 4505, 3554, 8, 2628, 12, 4268, 2317, 2438}

$$\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+bx))}{b}-\frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{dx\log(\tan(a+bx))}{b}-\frac{dx}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -1/2*(d*x)/b - (2*d*x*ArcTanh[E^((2*I)*(a + b*x))])/b - (d*Cot[a + b*x])/(2
*b^2) - ((c + d*x)*Cot[a + b*x]^2)/(2*b) - (d*x*Log[Tan[a + b*x]])/b + ((c
+ d*x)*Log[Tan[a + b*x]])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^
2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx &= -\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+bx))}{b}-d\int\Bigl(-\frac{\cot^2}{} \\
&= -\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+bx))}{b}+\frac{d\int\cot^2(a+}{2b} \\
&= -\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}-\frac{dx\log(\tan(a+bx))}{b}+\frac{(}{} \\
&= -\frac{dx}{2b}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}-\frac{dx\log(\tan(a+bx)}{b} \\
&= -\frac{dx}{2b}-\frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+}{2b} \\
&= -\frac{dx}{2b}-\frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+}{2b} \\
&= -\frac{dx}{2b}-\frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+}{2b}
\end{aligned}$$

Mathematica [**A**]
time = 0.99, size = 210, normalized size = 1.49

$$-\frac{d\cot(a+bx)}{2b^2}-\frac{dx\csc^2(a+bx)}{2b}+\frac{ad\log(\cos(a+bx))}{b^2}-\frac{c(\csc^2(a+bx)+2\log(\cos(a+bx))-2\log(\sin(a+bx)))}{2b}-\frac{ad\log(\sin(a+bx))}{b^2}+\frac{d\left(\frac{1}{2}i(a+bx)^2-(a+bx)\log\left(1+e^{2i(a+bx)}\right)+\frac{1}{2}i\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)\right)}{b^2}+\frac{d\left((a+bx)\log\left(1-e^{2i(a+bx)}\right)-\frac{1}{2}i\left((a+bx)^2+\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -1/2*(d*Cot[a + b*x])/b^2 - (d*x*Csc[a + b*x]^2)/(2*b) + (a*d*Log[Cos[a + b
*x]])/b^2 - (c*(Csc[a + b*x]^2 + 2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]])
)/(2*b) - (a*d*Log[Sin[a + b*x]])/b^2 + (d*((I/2)*(a + b*x)^2 - (a + b*x)*L
og[1 + E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^((2*I)*(a + b*x))]))/b^2
+ (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x))] - (I/2)*((a + b*x)^2 + PolyLog
[2, E^((2*I)*(a + b*x))])))/b^2
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(123) = 246.
time = 0.13, size = 270, normalized size = 1.91

method	result
risch	$\frac{2bdx\,\mathrm{e}^{2i(bx+a)}-id\,\mathrm{e}^{2i(bx+a)}+2bc\,\mathrm{e}^{2i(bx+a)}+id}{b^2\left(\mathrm{e}^{2i(bx+a)}-1\right)^2}+\frac{c\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b}+\frac{c\ln\left(\mathrm{e}^{i(bx+a)}+1\right)}{b}-\frac{c\ln\left(1+\mathrm{e}^{2i(bx+a)}\right)}{b}+\frac{d\ln\left(1-\mathrm{e}^{i(bx+a)}\right)}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] (2*b*d*x*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))+I*d)/
b^2/(exp(2*I*(b*x+a))-1)^2+1/b*c*ln(exp(I*(b*x+a))-1)+1/b*c*ln(exp(I*(b*x+a
))+1)-1/b*c*ln(1+exp(2*I*(b*x+a)))+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(
1-exp(I*(b*x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/b*d*ln(1+exp(2*I*(b
*x+a)))*x+1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+
1)*x-I*d*polylog(2,-exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1028 vs. 2(119) = 238.
time = 0.61, size = 1028, normalized size = 7.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -(2*(b*d*x + b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) - 2*(b*d*x + b*c)*cos(2*b
*x + 2*a) - (-I*b*d*x - I*b*c)*sin(4*b*x + 4*a) - 2*(I*b*d*x + I*b*c)*sin(2
*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 2*(b*d*x + b
*c + (b*d*x + b*c)*cos(4*b*x + 4*a) - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (I
*b*d*x + I*b*c)*sin(4*b*x + 4*a) + 2*(-I*b*d*x - I*b*c)*sin(2*b*x + 2*a))*a
rctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(b*c*cos(4*b*x + 4*a) - 2*b*c*co
s(2*b*x + 2*a) + I*b*c*sin(4*b*x + 4*a) - 2*I*b*c*sin(2*b*x + 2*a) + b*c)*a
rctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*(b*d*x*cos(4*b*x + 4*a) - 2*b*d*
x*cos(2*b*x + 2*a) + I*b*d*x*sin(4*b*x + 4*a) - 2*I*b*d*x*sin(2*b*x + 2*a)
 + b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*(-2*I*b*d*x - 2*I*b*c
 - d)*cos(2*b*x + 2*a) - (d*cos(4*b*x + 4*a) - 2*d*cos(2*b*x + 2*a) + I*d*s
in(4*b*x + 4*a) - 2*I*d*sin(2*b*x + 2*a) + d)*dilog(-e^(2*I*b*x + 2*I*a)) +
 2*(d*cos(4*b*x + 4*a) - 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*
d*sin(2*b*x + 2*a) + d)*dilog(-e^(I*b*x + I*a)) + 2*(d*cos(4*b*x + 4*a) - 2
*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x + 2*a) + d)*di
log(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(4*b*x + 4
*a) - 2*(-I*b*d*x - I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*b*x + 4*a
) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x +
2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos
(4*b*x + 4*a) - 2*(I*b*d*x + I*b*c)*cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*
b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x
 + a)^2 + 2*cos(b*x + a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(4*
b*x + 4*a) - 2*(I*b*d*x + I*b*c)*cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*b*x
 + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
a)^2 - 2*cos(b*x + a) + 1) - 2*(2*b*d*x + 2*b*c - I*d)*sin(2*b*x + 2*a) - 2
*d)/(-2*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4*b*x
 + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 2*I*b^2)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 942 vs. 2(119) = 238.

time = 2.70, size = 942, normalized size = 6.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x + d*cos(b*x + a)*sin(b*x + a) + b*c + (-I*d*cos(b*x + a)^2 + I*d
)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(c
os(b*x + a) - I*sin(b*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(I*cos(b*x
 + a) + sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(I*cos(b*x + a) - s
in(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(-I*cos(b*x + a) + sin(b*x +
 a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) +
(I*d*cos(b*x + a)^2 - I*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-I*d*co
s(b*x + a)^2 + I*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b*d*x - (b*d*x
 + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - ((b*
c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(cos(b*x + a) + I*sin(b*x + a) + I)
 - (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a) - I*sin(b*
x + a) + 1) - ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(cos(b*x + a) - I
*sin(b*x + a) + I) + (b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*log(I*cos
(b*x + a) + sin(b*x + a) + 1) + (b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d
)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b*d*x - (b*d*x + a*d)*cos(b*x +
 a)^2 + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x - (b*d*x + a*
d)*cos(b*x + a)^2 + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + ((b*c -
a*d)*cos(b*x + a)^2 - b*c + a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a)
 + 1/2) + ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(-1/2*cos(b*x + a) -
1/2*I*sin(b*x + a) + 1/2) - (b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*lo
g(-cos(b*x + a) + I*sin(b*x + a) + 1) - ((b*c - a*d)*cos(b*x + a)^2 - b*c +
 a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b*d*x - (b*d*x + a*d)*cos(
b*x + a)^2 + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - ((b*c - a*d)*co
s(b*x + a)^2 - b*c + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/(b^2*cos
(b*x + a)^2 - b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \csc^3 (a + bx) \sec (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)**3*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

3.244 $\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{\csc^3(a+bx)\sec(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x),x]
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 10.79, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x),x]
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(\csc^3(bx+a))\sec(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 -
 (2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) -
 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 +
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 +
 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*
b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2
*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 +
 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate((b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2 + d^2)*sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^
2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*cos(b*x + a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*sin(b*x + a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^
3)*cos(b*x + a)), x) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4
*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*si
n(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2
 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*cos(2*b*x + 2*a))*integrate((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
 + d^2)*sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c
^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)
^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x + a)
^2 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x +
a)), x) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*
cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)
^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2
*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*co
s(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*
cos(2*b*x + 2*a))*integrate(sin(2*b*x + 2*a)/((d*x + c)*cos(2*b*x + 2*a)^2
```

```
+ (d*x + c)*sin(2*b*x + 2*a)^2 + d*x + 2*(d*x + c)*cos(2*b*x + 2*a) + c), x
) - (d*cos(2*b*x + 2*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a) - d)*sin(4*b*x +
 4*a) - d*sin(2*b*x + 2*a))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*
sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*
a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2
*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(csc(b*x + a)^3*sec(b*x + a)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)/(d*x+c),x)
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+b\,x)\,{\sin(a+b\,x)}^3\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)),x)
[Out] int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)), x)
```

3.245 $\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2} dx = \int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2} dx$$

Mathematica [A]
time = 12.94, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2, x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(\csc^3(bx+a))\sec(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x)

[Out] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")

[Out] -(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 - 2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 3*d^2)*sin(b*x + a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 3*d^2)*sin(b*x + a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2 - 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*

```
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4
*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)
^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x +
4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*
b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*
d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^
2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^
2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(sin(2*b*x + 2*a)/(
d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x + 2*a)^2 + (d^2*x^2
 + 2*c*d*x + c^2)*sin(2*b*x + 2*a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*co
s(2*b*x + 2*a)), x) - 2*(d*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(2*b*x + 2*a
) - d)*sin(4*b*x + 4*a) - 2*d*sin(2*b*x + 2*a))/(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*
d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d
^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*
(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 +
 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4
*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x
 + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)^3*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)/(d*x+c)**2,x)
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)/(c + d*x)**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^3\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^2),x)
[Out] int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^2), x)
```

3.246 $\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=23

$$\text{Int}((c+dx)^m \sec(a+bx)\tan(a+bx),x)$$

```
[Out] CannotIntegrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] Defer[Int][(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx$$

Mathematica [A]
time = 2.60, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x], x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)\tan(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)
[Out] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \tan (a+bx) \sec (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*tan(b*x+a),x)
[Out] Integral((c + d*x)**m*tan(a + b*x)*sec(a + b*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(a+b\,x)\ (c+d\,x)^m}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x),x)
[Out] int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x), x)
```

3.247 $\int (c+dx)^4 \sec(a+bx) \tan(a+bx) \, dx$

Optimal. Leaf size=227

$$\frac{8id(c+dx)^3\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}-\frac{12id^2(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{12id^2(c+dx)^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+$$

```
[Out] 8*I*d*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b^2-12*I*d^2*(d*x+c)^2*polylog(2,-I*
exp(I*(b*x+a)))/b^3+12*I*d^2*(d*x+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^3+24*d
^3*(d*x+c)*polylog(3,-I*exp(I*(b*x+a)))/b^4-24*d^3*(d*x+c)*polylog(3,I*exp(
I*(b*x+a)))/b^4+24*I*d^4*polylog(4,-I*exp(I*(b*x+a)))/b^5-24*I*d^4*polylog(
4,I*exp(I*(b*x+a)))/b^5+(d*x+c)^4*sec(b*x+a)/b
```

Rubi [A]
time = 0.13, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4494, 4266, 2611, 6744, 2320, 6724}

$$\frac{8id(c+dx)^3\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}+\frac{24id^4\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^5}-\frac{24id^4\text{Li}_4\left(ie^{i(a+bx)}\right)}{b^5}+\frac{24d^3(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{24d^3(c+dx)\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}-\frac{12id^2(c+dx)^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{12id^2(c+dx)^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}+\frac{(c+dx)^4\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((8*I)*d*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b^2 - ((12*I)*d^2*(c + d*x)^2
*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((12*I)*d^2*(c + d*x)^2*PolyLog[2,
 I*E^(I*(a + b*x))])/b^3 + (24*d^3*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x)
)])/b^4 - (24*d^3*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + ((24*I)*d^
4*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^5 - ((24*I)*d^4*PolyLog[4, I*E^(I*(a
+ b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4\sec(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)^4\sec(a+bx)}{b} - \frac{(4d)\int (c+dx)^3\sec(a+bx)\,dx}{b}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^4\sec(a+bx)}{b} + \frac{(12d^2)\int (\cdots}{}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \cdots$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \cdots$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \cdots$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \cdots$$

Mathematica [**A**]
time = 1.10, size = 428, normalized size = 1.89

$$-\frac{4d\left(-2ib^3c^3\mathrm{ArcTan}\left(e^{i(a+bx)}\right)+3b^3c^2dx\log\left(1-ie^{i(a+bx)}\right)+3b^3cd^2x^2\log\left(1-ie^{i(a+bx)}\right)+b^3d^3x^3\log\left(1-ie^{i(a+bx)}\right)-3b^3c^2dx\log\left(1+ie^{i(a+bx)}\right)-3b^3cd^2x^2\log\left(1+ie^{i(a+bx)}\right)-b^3d^3x^3\log\left(1+ie^{i(a+bx)}\right)+3ib^2d(c+dx)^2\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)-3ib^2d(c+dx)^2\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)-6bcd^2\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)-6bd^3x\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)+6bcd^2\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)+6bd^3x\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)-6id^3\mathrm{PolyLog}\left(4,-ie^{i(a+bx)}\right)+6id^3\mathrm{PolyLog}\left(4,ie^{i(a+bx)}\right)\right)}{b^5}+\frac{(c+dx)^4\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] (-4*d*((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - I*E^(
I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] + b^3*d^3*x^3*Lo
g[1 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 3*b^3
*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x
))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))] - (3*I)*b^2*
d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, (-I)*E^(
I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*c*d^2*Poly
Log[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] - (6*I)
*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*
x))]))/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 766 vs. 2(202) = 404.
time = 0.19, size = 767, normalized size = 3.38

method	result
risch	$-\frac{24id^2c^2a\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b^3}+\frac{24id^3c\,\mathrm{polylog}\left(2,i\mathrm{e}^{i(bx+a)}\right)x}{b^3}-\frac{24id^3c\,\mathrm{polylog}\left(2,-i\mathrm{e}^{i(bx+a)}\right)x}{b^3}+\frac{2\left(d^4x^4+4c\,d^3x^3+6c^2d^2x^2+4c^3\right.}{b\left(1+\mathrm{e}^{2i(bx+a)}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x^2+4*c^3*d*x+c^4)*exp(I*(b*x+a))/b/(1+exp
(2*I*(b*x+a)))+4/b^2*d^4*ln(1+I*exp(I*(b*x+a)))*x^3-4/b^2*d^4*ln(1-I*exp(I*
(b*x+a)))*x^3+24/b^4*d^3*c*polylog(3,-I*exp(I*(b*x+a)))-4/b^5*d^4*a^3*ln(1-
I*exp(I*(b*x+a)))+4/b^5*d^4*a^3*ln(1+I*exp(I*(b*x+a)))+24/b^4*d^4*polylog(3
,-I*exp(I*(b*x+a)))*x-24/b^4*d^3*c*polylog(3,I*exp(I*(b*x+a)))-24/b^4*d^4*p
olylog(3,I*exp(I*(b*x+a)))*x-12/b^2*d^3*c*ln(1-I*exp(I*(b*x+a)))*x^2+12/b^2
*d^3*c*ln(1+I*exp(I*(b*x+a)))*x^2+12/b^2*d^2*c^2*ln(1+I*exp(I*(b*x+a)))*x+1
2/b^3*d^2*c^2*ln(1+I*exp(I*(b*x+a)))*a+12/b^4*d^3*a^2*c*ln(1-I*exp(I*(b*x+a
)))-12/b^4*d^3*a^2*c*ln(1+I*exp(I*(b*x+a)))-12/b^2*d^2*c^2*ln(1-I*exp(I*(b*
x+a)))*x-12/b^3*d^2*c^2*ln(1-I*exp(I*(b*x+a)))*a-12*I/b^3*d^2*c^2*polylog(2
,-I*exp(I*(b*x+a)))-8*I/b^5*d^4*a^3*arctan(exp(I*(b*x+a)))-12*I/b^3*d^4*pol
ylog(2,-I*exp(I*(b*x+a)))*x^2+12*I/b^3*d^4*polylog(2,I*exp(I*(b*x+a)))*x^2+
8*I/b^2*d*c^3*arctan(exp(I*(b*x+a)))+12*I/b^3*d^2*c^2*polylog(2,I*exp(I*(b*
x+a)))+24*I*d^4*polylog(4,-I*exp(I*(b*x+a)))/b^5+24*I/b^3*d^3*c*polylog(2,I
*exp(I*(b*x+a)))*x-24*I/b^3*d^3*c*polylog(2,-I*exp(I*(b*x+a)))*x+24*I/b^4*d
```

```
^3*c*a^2*arctan(exp(I*(b*x+a)))-24*I/b^3*d^2*c^2*a*arctan(exp(I*(b*x+a)))-2
4*I*d^4*polylog(4,I*exp(I*(b*x+a)))/b^5
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2944 vs. $2(189) = 378$.
time = 0.72, size = 2944, normalized size = 12.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] (2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x + 2*a
)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2
*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
 + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))*c^3
*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b) -
 6*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x + 2*a
)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2
*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
 + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))*a*c
^2*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*
b^2) + 6*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x
 + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin
(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)
^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos
(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1
))*a^2*c*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)
 + 1)*b^3) - 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin
(2*b*x + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2
 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 +
 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x +
a) + 1))*a^3*d^4/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
2*a) + 1)*b^4) + c^4/cos(b*x + a) - 4*a*c^3*d/(b*cos(b*x + a)) + 6*a^2*c^2*
d^2/(b^2*cos(b*x + a)) - 4*a^3*c*d^3/(b^3*cos(b*x + a)) + a^4*d^4/(b^4*cos(
b*x + a)) + 2*(2*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^
2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + ((b*x + a)^3*d^4 + 3*(b*c*d^
3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))
*cos(2*b*x + 2*a) + (I*(b*x + a)^3*d^4 + 3*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^
2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*
a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + 2*((b*x + a)^3*d^4 + 3*(b*c*d
^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)
```

```
 + ((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*
a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (I*(b*x + a)^3*d^4 + 3*(
I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2
*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1)
 + (-I*(b*x + a)^4*d^4 + 4*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c
^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a)^2)*cos(b*x + a) + 6*(b^2*c^2*
d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x +
a) + (b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 -
a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*(b*
x + a)^2*d^4 + I*a^2*d^4 + 2*(I*b*c*d^3 - I*a*d^4)*(b*x + a))*sin(2*b*x + 2
*a))*dilog(I*e^(I*b*x + I*a)) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*
d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a) + (b^2*c^2*d^2 - 2*a*b*c*d^3
+ (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*
a) - (-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*(b*x + a)^2*d^4 - I*a^2*d^4 + 2*(-
I*b*c*d^3 + I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a))
 + (I*(b*x + a)^3*d^4 + 3*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*
d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a) + (I*(b*x + a)^3*d^4 + 3*(I*b*c*
d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*
(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x +
 a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a)
)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + (-I*(b*x + a)
^3*d^4 + 3*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^2 + 3*(-I*b^2*c^2*d^2 + 2*I*a*b
*c*d^3 - I*a^2*d^4)*(b*x + a) + (-I*(b*x + a)^3*d^4 + 3*(-I*b*c*d^3 + I*a*d
^4)*(b*x + a)^2 + 3*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a))
*cos(2*b*x + 2*a) + ((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*
(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(
b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - 12*(d^4*cos(2*b*x + 2*a
) + I*d^4*sin(2*b*x + 2*a) + d^4)*polylog(4, I*e^(I*b*x + I*a)) + 12*(d^4*c
os(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a) + d^4)*polylog(4, -I*e^(I*b*x + I*
a)) + 12*(I*b*c*d^3 + I*(b*x + a)*d^4 - I*a*d^4 + (I*b*c*d^3 + I*(b*x + a)*
d^4 - I*a*d^4)*cos(2*b*x + 2*a) - (b*c*d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*b
*x + 2*a))*polylog(3, I*e^(I*b*x + I*a)) + 12*(-I*b*c*d^3 - I*(b*x + a)*d^4
 + I*a*d^4 + (-I*b*c*d^3 - I*(b*x + a)*d^4 + I*a*d^4)*cos(2*b*x + 2*a) + (b
*c*d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2*a...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1190 vs. $2(189) = 378$.
time = 1.23, size = 1190, normalized size = 5.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^
4 - 12*I*d^4*cos(b*x + a)*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 12*I*
```

```
d^4*cos(b*x + a)*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 12*I*d^4*cos(b
*x + a)*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) + 12*I*d^4*cos(b*x + a)*
polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 6*(-I*b^2*d^4*x^2 - 2*I*b^2*c*
d^3*x - I*b^2*c^2*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) -
6*(-I*b^2*d^4*x^2 - 2*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*cos(b*x + a)*dilog(I*c
os(b*x + a) - sin(b*x + a)) - 6*(I*b^2*d^4*x^2 + 2*I*b^2*c*d^3*x + I*b^2*c^
2*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 6*(I*b^2*d^4*x^
2 + 2*I*b^2*c*d^3*x + I*b^2*c^2*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - s
in(b*x + a)) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*co
s(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) + 2*(b^3*c^3*d - 3*a*b^2*
c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*
x + a) + I) - 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*
c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*
x + a) + 1) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*
c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*
x + a) + 1) - 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*
c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b
*x + a) + 1) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2
*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(
b*x + a) + 1) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*c
os(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 2*(b^3*c^3*d - 3*a*b^
2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(-cos(b*x + a) - I*sin
(b*x + a) + I) + 12*(b*d^4*x + b*c*d^3)*cos(b*x + a)*polylog(3, I*cos(b*x +
 a) + sin(b*x + a)) - 12*(b*d^4*x + b*c*d^3)*cos(b*x + a)*polylog(3, I*cos(
b*x + a) - sin(b*x + a)) + 12*(b*d^4*x + b*c*d^3)*cos(b*x + a)*polylog(3, -
I*cos(b*x + a) + sin(b*x + a)) - 12*(b*d^4*x + b*c*d^3)*cos(b*x + a)*polylo
g(3, -I*cos(b*x + a) - sin(b*x + a)))/(b^5*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \tan(a+bx) \sec(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sec(b*x+a)*tan(b*x+a),x)
[Out] Integral((c + d*x)**4*tan(a + b*x)*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*sec(b*x + a)*tan(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(a+b\,x)\,\left(c+d\,x\right)^4}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x),x)
[Out] int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x), x)
```

3.248 $\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=159

$$\frac{6id(c+dx)^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{6d}{}$$

```
[Out] 6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)*polylog(2,-I*exp
(I*(b*x+a)))/b^3+6*I*d^2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^3+6*d^3*poly
log(3,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4+(d*x+c)^
3*sec(b*x+a)/b
```

Rubi [A]
time = 0.08, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4494, 4266, 2611, 2320, 6724}

$$\frac{6id(c+dx)^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}+\frac{6d^3\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}-\frac{6id^2(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}+\frac{(c+dx)^3\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*Po
lyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(
I*(a + b*x))])/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*
PolyLog[3, I*E^(I*(a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x])/b
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
```

```
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)^3\sec(a+bx)}{b} - \frac{(3d)\int (c+dx)^2\sec(a+bx)\,dx}{b}$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^3\sec(a+bx)}{b} + \frac{(6d^2)\int(c+$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6id^2(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6id^2(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6id^2(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2$$

Mathematica [A]
time = 0.78, size = 256, normalized size = 1.61

$$-\frac{3d\left(-2ib^2c^2\text{ArcTan}\left(e^{i(a+bx)}\right)+2b^2cdx\log\left(1-ie^{i(a+bx)}\right)+b^2d^2x^2\log\left(1-ie^{i(a+bx)}\right)-2b^2cdx\log\left(1+ie^{i(a+bx)}\right)-b^2d^2x^2\log\left(1+ie^{i(a+bx)}\right)+2ibd(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)-2ibd(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)-2d^2\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)+2d^2\text{PolyLog}\left(3,ie^{i(a+bx)}\right)\right)}{b^4}+\frac{(c+dx)^3\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] (-3*d*((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - I*E^(I*
(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1 +
I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + (2*I)*b*d*(c
```

```
+ d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, I*
E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 2*d^2*PolyLog[3
, I*E^(I*(a + b*x))]))/b^4 + ((c + d*x)^3*Sec[a + b*x])/b
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 462 vs. 2(142) = 284.
time = 0.13, size = 463, normalized size = 2.91

method	result
risch	$\frac{2\,\mathrm{e}^{i(bx+a)}\left(d^3x^3+3c\,d^2x^2+3c^2dx+c^3\right)}{b\left(1+\mathrm{e}^{2i(bx+a)}\right)}-\frac{12id^2ca\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b^3}+\frac{6id^3a^2\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b^4}+\frac{6id^3x\,\mathrm{polylog}\left(2,i\mathrm{e}^{i(bx+a)}\right)}{b^3}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*exp(I*(b*x+a))*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(1+exp(2*I*(b*x+a)))
+6*I*c*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3-12*I/b^3*d^2*c*a*arctan(exp(I*(b
*x+a)))+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b^3+6*I/b^4*d^3*a^2*arctan(ex
p(I*(b*x+a)))+6/b^3*d^2*c*ln(1+I*exp(I*(b*x+a)))*a+6*d^3*polylog(3,-I*exp(I
*(b*x+a)))/b^4+6*I/b^2*d*c^2*arctan(exp(I*(b*x+a)))+3/b^2*d^3*ln(1+I*exp(I*
(b*x+a)))*x^2+6/b^2*d^2*c*ln(1+I*exp(I*(b*x+a)))*x-6*I*d^3*x*polylog(2,-I*e
xp(I*(b*x+a)))/b^3-6*I*c*d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3-6/b^3*d^2*c*l
n(1-I*exp(I*(b*x+a)))*a-3/b^2*d^3*ln(1-I*exp(I*(b*x+a)))*x^2+3/b^4*d^3*a^2*
ln(1-I*exp(I*(b*x+a)))-3/b^4*d^3*a^2*ln(1+I*exp(I*(b*x+a)))-6/b^2*d^2*c*ln(
1-I*exp(I*(b*x+a)))*x-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1770 vs. 2(133) = 266.
time = 0.58, size = 1770, normalized size = 11.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(3*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x +
 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2
 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))
*c^2*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*
b) - 6*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x +
 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2
 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))
```

```
*a*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1
)*b^2) + 3*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b
*x + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + s
in(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x +
a)^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*c
os(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) +
 1))*a^2*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)
 + 1)*b^3) + 2*c^3/cos(b*x + a) - 6*a*c^2*d/(b*cos(b*x + a)) + 6*a^2*c*d^2/
(b^2*cos(b*x + a)) - 2*a^3*d^3/(b^3*cos(b*x + a)) + 2*(6*((b*x + a)^2*d^3 +
 2*(b*c*d^2 - a*d^3)*(b*x + a) + ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*
x + a))*cos(2*b*x + 2*a) + (I*(b*x + a)^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*
x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + 6*((b*x
 + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) + ((b*x + a)^2*d^3 + 2*(b*c*d^2
 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (I*(b*x + a)^2*d^3 + 2*(I*b*c*d^2 -
 I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a)
+ 1) + 4*(-I*(b*x + a)^3*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2)*cos(b*
x + a) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3 + (b*c*d^2 + (b*x + a)*d^3 - a
*d^3)*cos(2*b*x + 2*a) + (I*b*c*d^2 + I*(b*x + a)*d^3 - I*a*d^3)*sin(2*b*x
+ 2*a))*dilog(I*e^(I*b*x + I*a)) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3 + (b
*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (-I*b*c*d^2 - I*(b*x + a
)*d^3 + I*a*d^3)*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) + 3*(I*(b*x +
a)^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a) + (I*(b*x + a)^2*d^3 + 2*(I*b*
c*d^2 - I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^2*d^3 + 2*(b*c*d^
2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2
 + 2*sin(b*x + a) + 1) + 3*(-I*(b*x + a)^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(
b*x + a) + (-I*(b*x + a)^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*cos(2*
b*x + 2*a) + ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x +
2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + 12*(I*d^3
*cos(2*b*x + 2*a) - d^3*sin(2*b*x + 2*a) + I*d^3)*polylog(3, I*e^(I*b*x + I
*a)) + 12*(-I*d^3*cos(2*b*x + 2*a) + d^3*sin(2*b*x + 2*a) - I*d^3)*polylog(
3, -I*e^(I*b*x + I*a)) + 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)
^2)*sin(b*x + a))/(-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x + 2*a) - 2*I
*b^3))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 783 vs. $2(133) = 266$.
time = 2.04, size = 783, normalized size = 4.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*co
s(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*p
```

```
olylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, -I
*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x +
a) - sin(b*x + a)) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a)*dilog(I*cos(b*
x + a) + sin(b*x + a)) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a)*dilog(I*co
s(b*x + a) - sin(b*x + a)) - 6*(I*b*d^3*x + I*b*c*d^2)*cos(b*x + a)*dilog(-
I*cos(b*x + a) + sin(b*x + a)) - 6*(I*b*d^3*x + I*b*c*d^2)*cos(b*x + a)*dil
og(-I*cos(b*x + a) - sin(b*x + a)) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*
cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I) - 3*(b
^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(
b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2
 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 3*(b^2*d^
3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x
+ a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a
^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - 3*(b^2*c^2*d
 - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) +
 I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a)
- I*sin(b*x + a) + I))/(b^4*cos(b*x + a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3\tan(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*tan(b*x+a),x)
[Out] Integral((c + d*x)**3*tan(a + b*x)*sec(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^3*sec(b*x + a)*tan(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(a+b\,x)\,{(c+d\,x)}^3}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x),x)
[Out] int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x), x)
```

3.249 $\int (c+dx)^2 \sec(a+bx) \tan(a+bx) \, dx$

Optimal. Leaf size=97

$$\frac{4id(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

```
[Out] 4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2-2*I*d^2*polylog(2,-I*exp(I*(b*x+a)
))/b^3+2*I*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3+(d*x+c)^2*sec(b*x+a)/b
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4494, 4266, 2317, 2438}

$$\frac{4id(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2id^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*PolyLog[2, (-I
)*E^(I*(a + b*x))])/b^3 + ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 + (
(c + d*x)^2*Sec[a + b*x])/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
```

```
Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2\sec(a+bx)\tan(a+bx)\,dx &= \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{(2d)\int (c+dx)\sec(a+bx)\,dx}{b} \\
&= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\sec(a+bx)}{b} + \frac{(2d^2)\int \log
\\
&= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{(2id^2)\,\mathrm{Sub}
\\
&= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2id^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{2id^2\mathrm{Li}_2\left(ie^{i(a
\end{aligned}$$

Mathematica [A]
time = 1.76, size = 174, normalized size = 1.79

$$\frac{-4bcd\tanh^{-1}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right) - 4d^2\mathrm{ArcTan}(\cot(a))\tanh^{-1}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right) + \frac{2d^2\csc(a)\left((bx-\mathrm{ArcTan}(\cot(a)))\left(\log\left(1-e^{i(bx-\mathrm{ArcTan}(\cot(a)))}\right)-\log\left(1+e^{i(bx-\mathrm{ArcTan}(\cot(a)))}\right)\right)+i\mathrm{PolyLog}\left(2,-e^{i(bx-\mathrm{ArcTan}(\cot(a)))}\right)-i\mathrm{PolyLog}\left(2,e^{i(bx-\mathrm{ArcTan}(\cot(a)))}\right)\right)}{\sqrt{\csc^2(a)}} + b^2(c+dx)^2\sec(a+bx)}{b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] (-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*ArcT
anh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + (2*d^2*Csc[a]*((b*x - ArcTan[Cot[a]])*(
Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a]])
)]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2, E^(I*(b*x
- ArcTan[Cot[a]]))]))/Sqrt[Csc[a]^2] + b^2*(c + d*x)^2*Sec[a + b*x])/b^3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(86) = 172.
time = 0.07, size = 234, normalized size = 2.41

method	result
risch	$\frac{2\,\mathrm{e}^{i(bx+a)}\left(x^2d^2+2cdx+c^2\right)}{b\left(1+\mathrm{e}^{2i(bx+a)}\right)}+\frac{4idc\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b^2}+\frac{2d^2\ln\left(1+i\mathrm{e}^{i(bx+a)}\right)x}{b^2}+\frac{2d^2\ln\left(1+i\mathrm{e}^{i(bx+a)}\right)a}{b^3}-\frac{2d^2\ln(1$
derivativedivides	$\frac{\frac{a^2d^2}{b^2\cos(bx+a)}-\frac{2acd}{b\cos(bx+a)}-\frac{2a\,d^2\left(\frac{bx+a}{\cos(bx+a)}-\ln(\sec(bx+a)+\tan(bx+a))\right)}{b^2}+\frac{c^2}{\cos(bx+a)}+\frac{2cd\left(\frac{bx+a}{\cos(bx+a)}-\ln(\sec(bx+a)+\tan(bx+\right.}{b}}{b}$
default	$\frac{\frac{a^2d^2}{b^2\cos(bx+a)}-\frac{2acd}{b\cos(bx+a)}-\frac{2a\,d^2\left(\frac{bx+a}{\cos(bx+a)}-\ln(\sec(bx+a)+\tan(bx+a))\right)}{b^2}+\frac{c^2}{\cos(bx+a)}+\frac{2cd\left(\frac{bx+a}{\cos(bx+a)}-\ln(\sec(bx+a)+\tan(bx+\right.}{b}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/b^2*a^2*d^2/cos(b*x+a)-2/b*a*c*d/cos(b*x+a)-2/b^2*a*d^2*((b*x+a)/cos
(b*x+a)-ln(sec(b*x+a)+tan(b*x+a)))+1/cos(b*x+a)*c^2+2/b*c*d*((b*x+a)/cos(b*
x+a)-ln(sec(b*x+a)+tan(b*x+a)))+1/b^2*d^2*((b*x+a)^2/cos(b*x+a)+2*(b*x+a)*l
n(1+I*exp(I*(b*x+a)))-2*(b*x+a)*ln(1-I*exp(I*(b*x+a)))-2*I*dilog(1+I*exp(I*
(b*x+a)))+2*I*dilog(1-I*exp(I*(b*x+a)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] (2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)*cos(b*x + a) + 2*(b*d^2
*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)*sin(b*x + a) + 2*(b*d^2*x^2 + 2*
b*c*d*x + b*c^2)*cos(b*x + a) - 4*(b^2*d^2*cos(2*b*x + 2*a)^2 + b^2*d^2*sin
(2*b*x + 2*a)^2 + 2*b^2*d^2*cos(2*b*x + 2*a) + b^2*d^2)*integrate((x*cos(2*
b*x + 2*a)*cos(b*x + a) + x*sin(2*b*x + 2*a)*sin(b*x + a) + x*cos(b*x + a))
/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 + 2*b*cos(2*b*x + 2*a) + b),
x) - (c*d*cos(2*b*x + 2*a)^2 + c*d*sin(2*b*x + 2*a)^2 + 2*c*d*cos(2*b*x + 2
*a) + c*d)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + (c*d
*cos(2*b*x + 2*a)^2 + c*d*sin(2*b*x + 2*a)^2 + 2*c*d*cos(2*b*x + 2*a) + c*d
)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))/(b^2*cos(2*b*x
 + 2*a)^2 + b^2*sin(2*b*x + 2*a)^2 + 2*b^2*cos(2*b*x + 2*a) + b^2)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 446 vs. 2(79) = 158.
time = 1.40, size = 446, normalized size = 4.60

$$\frac{b^2d^2x^2 + 2b^2cdx + b^2c^2 + i d^2\cos(bx+a)\,\mathrm{Li}_2(i\cos(bx+a)+\sin(bx+a)) + i d^2\cos(bx+a)\,\mathrm{Li}_2(i\cos(bx+a)-\sin(bx+a)) - i d^2\cos(bx+a)\,\mathrm{Li}_2(-i\cos(bx+a)+\sin(bx+a)) - i d^2\cos(bx+a)\,\mathrm{Li}_2(-i\cos(bx+a)-\sin(bx+a)) - (bcd-ad^2)\cos(bx+a)\log(\cos(bx+a)+i\sin(bx+a)+i) + (bcd-ad^2)\cos(bx+a)\log(\cos(bx+a)-i\sin(bx+a)+i) - (bd^2x+ad^2)\cos(bx+a)\log(i\cos(bx+a)+\sin(bx+a)+1) + (bd^2x+ad^2)\cos(bx+a)\log(i\cos(bx+a)-\sin(bx+a)+1) - (bd^2x+ad^2)\cos(bx+a)\log(-i\cos(bx+a)+\sin(bx+a)+1) + (bd^2x+ad^2)\cos(bx+a)\log(-i\cos(bx+a)-\sin(bx+a)+1) - (bcd-ad^2)\cos(bx+a)\log(-\cos(bx+a)+i\sin(bx+a)+i) + (bcd-ad^2)\cos(bx+a)\log(-\cos(bx+a)-i\sin(bx+a)+i)}{b^3\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*cos(b*x + a)*dilog(I*cos(b*x +
 a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a
)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d^2*cos(b
*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b*c*d - a*d^2)*cos(b*x + a
)*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log
```

```
(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*
cos(b*x + a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos
(b*x + a) - sin(b*x + a) + 1) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b
*x + a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x
 + a) - sin(b*x + a) + 1) - (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a)
+ I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*
sin(b*x + a) + I))/(b^3*cos(b*x + a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \tan{(a + bx)} \sec{(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*tan(b*x+a),x)
[Out] Integral((c + d*x)**2*tan(a + b*x)*sec(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^2*sec(b*x + a)*tan(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(a+b\,x)\,{\left(c+d\,x\right)}^2}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^2)/cos(a + b*x),x)
[Out] int((tan(a + b*x)*(c + d*x)^2)/cos(a + b*x), x)
```

3.250 $\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=29

$$-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{(c+dx)\sec(a+bx)}{b}$$

```
[Out] -d*arctanh(sin(b*x+a))/b^2+(d*x+c)*sec(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4494, 3855}

$$\frac{(c+dx)\sec(a+bx)}{b}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]*Tan[a + b*x],x]
[Out] -((d*ArcTanh[Sin[a + b*x]])/b^2) + ((c + d*x)*Sec[a + b*x])/b
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx &= \frac{(c+dx)\sec(a+bx)}{b}-\frac{d\int \sec(a+bx)\,dx}{b}\\ &= -\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{(c+dx)\sec(a+bx)}{b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 93 vs. 2(29) = 58.
time = 0.06, size = 93, normalized size = 3.21

$$\frac{d\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}-\frac{d\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}+\frac{c\sec(a+bx)}{b}+\frac{dx\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] (d*Log[Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2]])/b^2 - (d*Log[Cos[a/2 + (b*
x)/2] + Sin[a/2 + (b*x)/2]])/b^2 + (c*Sec[a + b*x])/b + (d*x*Sec[a + b*x])/
b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(29) = 58$.
time = 0.07, size = 67, normalized size = 2.31

method	result	size
derivativedivides	$\frac{-\frac{da}{b\cos(bx+a)}+\frac{c}{\cos(bx+a)}+\frac{d\left(\frac{bx+a}{\cos(bx+a)}-\ln(\sec(bx+a)+\tan(bx+a))\right)}{b}}{b}$	67
default	$\frac{-\frac{da}{b\cos(bx+a)}+\frac{c}{\cos(bx+a)}+\frac{d\left(\frac{bx+a}{\cos(bx+a)}-\ln(\sec(bx+a)+\tan(bx+a))\right)}{b}}{b}$	67
risch	$\frac{2\,\mathrm{e}^{i(bx+a)}(dx+c)}{b\left(1+\mathrm{e}^{2i(bx+a)}\right)}+\frac{d\ln\left(\mathrm{e}^{i(bx+a)}-i\right)}{b^2}-\frac{d\ln\left(\mathrm{e}^{i(bx+a)}+i\right)}{b^2}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b*d*a/cos(b*x+a)+c/cos(b*x+a)+1/b*d*((b*x+a)/cos(b*x+a)-ln(sec(b*x+
a)+tan(b*x+a))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(29) = 58$.
time = 0.47, size = 259, normalized size = 8.93

$$\frac{\frac{\left(4\,(bx+a)\cos(2\,bx+2\,a)\cos(bx+a)+4\,(bx+a)\sin(2\,bx+2\,a)\sin(bx+a)+4\,(bx+a)\cos(bx+a)-\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2+2\,\sin(bx+a)+1\right)+\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2-2\,\sin(bx+a)+1\right)\right)d}{\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)b}+\frac{2\,c}{\cos(bx+a)}-\frac{2\,ad}{b\cos(bx+a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*((4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x + 2
*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b
*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 +
 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b
*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))*d
/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b) + 2
*c/cos(b*x + a) - 2*a*d/(b*cos(b*x + a)))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(29) = 58$.

time = 1.14, size = 60, normalized size = 2.07

$$\frac{2\,bdx - d\cos\left(bx+a\right)\log\left(\sin\left(bx+a\right)+1\right) + d\cos\left(bx+a\right)\log\left(-\sin\left(bx+a\right)+1\right) + 2\,bc}{2\,b^2\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x - d*cos(b*x + a)*log(sin(b*x + a) + 1) + d*cos(b*x + a)*log(-s
in(b*x + a) + 1) + 2*b*c)/(b^2*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + dx\right)\tan\left(a + bx\right)\sec\left(a + bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)*tan(a + b*x)*sec(a + b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1537 vs. $2(29) = 58$.
time = 0.71, size = 1537, normalized size = 53.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2*tan(1/2*a)^
2 + d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*
tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^
2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan
(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*
b*x)^2*tan(1/2*a)^2 - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)
^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*
b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*ta
n(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*a
)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2 + 2*b*d*x*ta
n(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2 - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 +
2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^
4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1
/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) +
 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2 + d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)
```

```
^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*
b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*
tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2
*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2 - 4*d*log(2*(tan(1/2*b*x)^4*tan
(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + t
an(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/
2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*
tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a) + 4*d*log(2*(ta
n(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*
tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b
*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*t
an(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)
 + 2*b*c*tan(1/2*a)^2 - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*
x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/
2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*
tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2
*a)^2 + 1))*tan(1/2*a)^2 + d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2
*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan
(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 +
 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(
1/2*a)^2 + 1))*tan(1/2*a)^2 + 2*b*d*x + 2*b*c + d*log(2*(tan(1/2*b*x)^4*tan
(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + t
an(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/
2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*
tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1)) - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2
- 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x
)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan
(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a)
 + 1)/(tan(1/2*a)^2 + 1)))/(b^2*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*tan(1/2*b
*x)^2 - 4*b^2*tan(1/2*b*x)*tan(1/2*a) - b^2*tan(1/2*a)^2 + b^2)
```

Mupad [B]
time = 2.58, size = 78, normalized size = 2.69

$$\frac{d\,\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}-\mathrm{i}\right)}{b^2}-\frac{d\,\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}+1\mathrm{i}\right)}{b^2}+\frac{2\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(c+d\,x\right)}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x))/cos(a + b*x),x)
```

```
[Out] (d*log(exp(a*1i + b*x*1i) - 1i))/b^2 - (d*log(exp(a*1i + b*x*1i) + 1i))/b^2
 + (2*exp(a*1i + b*x*1i)*(c + d*x))/(b*(exp(a*2i + b*x*2i) + 1))
```

3.251 $\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\sec(a+bx)\tan(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x),x]
[Out] Defer[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 11.03, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x),x]
[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x]
```

Maple [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\tan(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] 2*(cos(2*b*x + 2*a)*cos(b*x + a) + (b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos
(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x + b*c*d
)*cos(2*b*x + 2*a))*integrate((cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b*x +
2*a)*sin(b*x + a) + cos(b*x + a))/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x
^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2
)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)),
 x) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d*x + (b*d*x + b*c)*
cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)^2 + b*c + 2*(b*d*x + b*
c)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*tan(b*x + a)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(tan(a + b*x)*sec(a + b*x)/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*tan(b*x + a)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(a+b\,x)}{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)),x)
```

```
[Out] int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)), x)
```

3.252 $\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
[Out] Defer[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 19.65, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\tan(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 2*(cos(2*b*x + 2*a)*cos(b*x + a) + 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (
b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*c*
d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)
*cos(2*b*x + 2*a))*integrate((cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b*x + 2
*a)*sin(b*x + a) + cos(b*x + a))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x +
 b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(2*b*x + 2*a)
^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(2*b*x + 2*a)^2 +
 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(2*b*x + 2*a)), x)
+ sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d^2*x^2 + 2*b*c*d*x + b*
c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b
*c*d*x + b*c^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(
2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*tan(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan{\left(a + bx \right)} \sec{\left(a + bx \right)}}{\left(c + dx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(tan(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sec(b*x + a)*tan(b*x + a)/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(a+b\,x)}{\cos\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)^2),x)
[Out] int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)^2), x)
```

3.253 $\int (c+dx)^m \tan^2(a+bx)\,dx$

Optimal. Leaf size=19

$$\text{Int}\left((c+dx)^m \tan^2(a+bx),x\right)$$

```
[Out] Unintegrable((d*x+c)^m*tan(b*x+a)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \tan^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Tan[a + b*x]^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Tan[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \tan^2(a+bx)\,dx = \int (c+dx)^m \tan^2(a+bx)\,dx$$

Mathematica [A]
time = 3.20, size = 0, normalized size = 0.00

$$\int (c+dx)^m \tan^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Tan[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Tan[a + b*x]^2, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\tan^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*tan(b*x+a)^2,x)
```

[Out] int((d*x+c)^m*tan(b*x+a)^2,x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*tan(b*x + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*tan(b*x + a)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \tan^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*tan(b*x+a)**2,x)
[Out] Integral((c + d*x)**m*tan(a + b*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*tan(b*x + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \tan(a+b\,x)^2\,(c+d\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2*(c + d*x)^m,x)
[Out] int(tan(a + b*x)^2*(c + d*x)^m, x)
```

3.254 $\int (c+dx)^3 \tan^2(a+bx)\,dx$

Optimal. Leaf size=128

$$-\frac{i(c+dx)^3}{b}-\frac{(c+dx)^4}{4d}+\frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2}-\frac{3id^2(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3}+\frac{3d^3\text{PolyLog}(3}{2b}$$

```
[Out] -I*(d*x+c)^3/b-1/4*(d*x+c)^4/d+3*d*(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b^2-3*I
*d^2*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^3+3/2*d^3*polylog(3,-exp(2*I*(b
*x+a)))/b^4+(d*x+c)^3*tan(b*x+a)/b
```

Rubi [A]
time = 0.14, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3801, 3800, 2221, 2611, 2320, 6724, 32}

$$\frac{3d^3\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^2(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{i(c+dx)^3}{b}-\frac{(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Tan[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^3)/b - (c + d*x)^4/(4*d) + (3*d*(c + d*x)^2*Log[1 + E^((2*I
)*(a + b*x))])/b^2 - ((3*I)*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])
/b^3 + (3*d^3*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^4) + ((c + d*x)^3*Tan[
a + b*x])/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \tan^2(a+bx)\,dx = \frac{(c+dx)^3\tan(a+bx)}{b} - \frac{(3d)\int (c+dx)^2\tan(a+bx)\,dx}{b} - \int (c+dx)^3\,dx$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{(c+dx)^3\tan(a+bx)}{b} + \frac{(6id)\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}\,dx}{b}$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{(c+dx)^3\tan(a}{b}$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\mathrm{Li}_2}{b^3}$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\mathrm{Li}_2}{b^3}$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\mathrm{Li}_2}{b^3}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 431 vs. $2(128) = 256$.
time = 6.44, size = 431, normalized size = 3.37

$$-\frac{1}{4}x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)-\frac{d^3e^{-ia}\left(2ib^2x^2\left(2be^{2ia}x+3i\left(1+e^{2ia}\right)\log\left(1+e^{2i(a+bx)}\right)\right)+6ib\left(1+e^{2ia}\right)x\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)-3\left(1+e^{2ia}\right)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)\right)\sec(a)}{4b^4}+\frac{3c^2d\sec(a)\left(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a)\right)}{b^2\left(\cos^2(a)+\sin^2(a)\right)}+\frac{3cd^2\csc(a)\left(b^2e^{-i\text{ArcTan}(\cot(a))}x^2-\frac{\cot(a)\left(ibx(-\pi-2\text{ArcTan}(\cot(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx-\text{ArcTan}(\cot(a)))\log\left(1-e^{2i(bx-\text{ArcTan}(\cot(a)))}\right)+\pi\log(\cos(bx))-2\text{ArcTan}(\cot(a))\log(\sin(bx-\text{ArcTan}(\cot(a))))+i\,\text{PolyLog}\left(2,e^{2i(bx-\text{ArcTan}(\cot(a)))}\right)\right)}{\sqrt{1+\cot^2(a)}}\right)\sec(a)}{b^3\sqrt{\csc^2(a)\left(\cos^2(a)+\sin^2(a)\right)}}+\frac{\sec(a)\sec(a+bx)\left(c^3\sin(bx)+3c^2dx\sin(bx)+3cd^2x^2\sin(bx)+d^3x^3\sin(bx)\right)}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Tan[a + b*x]^2,x]
```

```
[Out] -1/4*(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)) - (d^3*((2*I)*b^2*x^2*
(2*b*E^((2*I)*a)*x + (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((2*I)*(a + b*x))])
+ (6*I)*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((2*I)*(a + b*x))] - 3*(1 + E^(
(2*I)*a))*PolyLog[3, -E^((2*I)*(a + b*x))])*Sec[a])/(4*b^4*E^(I*a)) + (3*c^
2*d*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b
^2*(Cos[a]^2 + Sin[a]^2)) + (3*c*d^2*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]])
 - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2
*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[
Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2,
E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^3*Sqrt[C
sc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Sec[a]*Sec[a + b*x]*(c^3*Sin[b*x] + 3*c^
2*d*x*Sin[b*x] + 3*c*d^2*x^2*Sin[b*x] + d^3*x^3*Sin[b*x]))/b
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(116) = 232$.
time = 0.11, size = 356, normalized size = 2.78

method	result
risch	$-\frac{d^3x^4}{4}-d^2c\,x^3-\frac{3d\,c^2x^2}{2}-c^3x-\frac{c^4}{4d}+\frac{2i\left(d^3x^3+3c\,d^2x^2+3c^2dx+c^3\right)}{b\left(1+e^{2i(bx+a)}\right)}+\frac{3d\,c^2\ln\left(1+e^{2i(bx+a)}\right)}{b^2}-\frac{6d\,c^2\ln\left(e^{i(bx+a)}\right)}{b^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d^3*x^4-d^2*c*x^3-3/2*d*c^2*x^2-c^3*x-1/4/d*c^4+2*I*(d^3*x^3+3*c*d^2*x
^2+3*c^2*d*x+c^3)/b/(1+exp(2*I*(b*x+a)))+3*d/b^2*c^2*ln(1+exp(2*I*(b*x+a)))
-6*d/b^2*c^2*ln(exp(I*(b*x+a)))-6*d^3/b^4*a^2*ln(exp(I*(b*x+a)))+6*I*d^3/b^
3*a^2*x-3*I*d^2/b^3*c*polylog(2,-exp(2*I*(b*x+a)))-12*I*d^2/b^2*c*a*x-6*I*d
^2/b*c*x^2+3*d^3/b^2*ln(1+exp(2*I*(b*x+a)))*x^2+4*I*d^3/b^4*a^3+3/2*d^3*pol
ylog(3,-exp(2*I*(b*x+a)))/b^4+12*d^2/b^3*c*a*ln(exp(I*(b*x+a)))+6*d^2/b^2*c
*ln(1+exp(2*I*(b*x+a)))*x-2*I*d^3/b*x^3-3*I*d^3/b^3*polylog(2,-exp(2*I*(b*x
+a)))*x-6*I*d^2/b^3*c*a^2
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1363 vs. $2(113) = 226$.
time = 0.56, size = 1363, normalized size = 10.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b*x + a - tan(b*x + a))*c^3 - 6*(b*x + a - tan(b*x + a))*a*c^2*d/b
 + 6*(b*x + a - tan(b*x + a))*a^2*c*d^2/b^2 - 2*(b*x + a - tan(b*x + a))*a^
3*d^3/b^3 + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a
)^2 + 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 +
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*c^2
*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b) -
 6*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 + 2*(b*
x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x +
 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)
^2 + 2*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^2/((cos(
2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b^2) + 3*((b*
x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 + 2*(b*x + a)^
2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*
cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^3/((cos(2*b*x +
 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b^3) - 2*(I*(b*x + a
)^4*d^3 - 4*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^3 + 12*((b*x + a)^2*d^3 + 2*(b
*c*d^2 - a*d^3)*(b*x + a) + ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a
))*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x +
 a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + (I
*(b*x + a)^4*d^3 - 4*(-I*b*c*d^2 + (I*a + 2)*d^3)*(b*x + a)^3 - 24*(b*c*d^2
 - a*d^3)*(b*x + a)^2)*cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d
^3 + (b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (I*b*c*d^2 + I*(b
*x + a)*d^3 - I*a*d^3)*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) - 6*(I
*(b*x + a)^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a) + (I*(b*x + a)^2*d^3 +
 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^2*d^3 + 2
*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + si
n(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 6*(I*d^3*cos(2*b*x + 2*a) - d^
3*sin(2*b*x + 2*a) + I*d^3)*polylog(3, -e^(2*I*b*x + 2*I*a)) - ((b*x + a)^4
*d^3 + 4*(b*c*d^2 - (a - 2*I)*d^3)*(b*x + a)^3 + 24*(I*b*c*d^2 - I*a*d^3)*(
b*x + a)^2)*sin(2*b*x + 2*a))/(-4*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(2*b*x
+ 2*a) - 4*I*b^3))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(113) = 226$.
time = 1.17, size = 373, normalized size = 2.91

$$-\frac{b^4d^3x^4+4\,b^4cd^2x^3+6\,b^4c^2dx^2+4\,b^4c^3x-3\,d^3\mathrm{polylog}\left(3,\frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)-3\,d^3\mathrm{polylog}\left(3,\frac{\tan(bx+a)^2-2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)+6\,(-i\,bd^3x-i\,bcd^2)\mathrm{Li}_2\left(\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)+6\,(i\,bd^3x+i\,bcd^2)\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)-6\,(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d)\log\left(-\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right)-6\,(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d)\log\left(-\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right)-4\,(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\tan(bx+a)}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 6*b^4*c^2*d*x^2 + 4*b^4*c^3*x - 3*d^3
*polylog(3, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) -
 3*d^3*polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 +
 1)) + 6*(-I*b*d^3*x - I*b*c*d^2)*dilog(2*(I*tan(b*x + a) - 1)/(tan(b*x + a
)^2 + 1) + 1) + 6*(I*b*d^3*x + I*b*c*d^2)*dilog(2*(-I*tan(b*x + a) - 1)/(ta
n(b*x + a)^2 + 1) + 1) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(-2
*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*
x + b^2*c^2*d)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 4*(b^3*
d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*tan(b*x + a))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \tan^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*tan(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*tan(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(a+b\,x)^2\,(c+d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] int(tan(a + b*x)^2*(c + d*x)^3, x)
```

3.255 $\int (c+dx)^2 \tan^2(a+bx)\,dx$

Optimal. Leaf size=96

$$-\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}+\frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2}-\frac{id^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3}+\frac{(c+dx)^2\tan(a+bx)}{b}$$

```
[Out] -I*(d*x+c)^2/b-1/3*(d*x+c)^3/d+2*d*(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b^2-I*d^2
*polylog(2,-exp(2*I*(b*x+a)))/b^3+(d*x+c)^2*tan(b*x+a)/b
```

Rubi [A]
time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3801, 3800, 2221, 2317, 2438, 32}

$$-\frac{id^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Tan[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^2)/b - (c + d*x)^3/(3*d) + (2*d*(c + d*x)*Log[1 + E^((2*I)*
(a + b*x))])/b^2 - (I*d^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^3 + ((c + d*x
)^2*Tan[a + b*x])/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2\tan^2(a+bx)\,dx = \frac{(c+dx)^2\tan(a+bx)}{b} - \frac{(2d)\int (c+dx)\tan(a+bx)\,dx}{b} - \int (c+dx)^2\,dx$$

$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} + \frac{(c+dx)^2\tan(a+bx)}{b} + \frac{(4id)\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}\,dx}{b}$$

$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\tan(a+}{b}$$

$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\tan(a+}{b}$$

$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{id^2\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 276 vs. 2(96) = 192.
time = 6.34, size = 276, normalized size = 2.88

$$-\frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right)+\frac{2cd\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))}{b^2\left(\cos^2(a)+\sin^2(a)\right)}+\frac{d^2\csc(a)\left(b^2e^{-i\text{ArcTan}(\cot(a))}x^2-\frac{\cot(a)\left(ibx(-\pi-2\text{ArcTan}(\cot(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx-\text{ArcTan}(\cot(a)))\log\left(1-e^{2i(bx-\text{ArcTan}(\cot(a)))}\right)+\pi\log(\cos(bx))-2\text{ArcTan}(\cot(a))\log(\sin(bx-\text{ArcTan}(\cot(a))))+i\text{PolyLog}\left(2,e^{2i(bx-\text{ArcTan}(\cot(a)))}\right)\right)}{\sqrt{1+\cot^2(a)}}\right)\sec(a)}{b^3\sqrt{\csc^2(a)\left(\cos^2(a)+\sin^2(a)\right)}}+\frac{\sec(a)\sec(a+bx)\left(c^2\sin(bx)+2cdx\sin(bx)+d^2x^2\sin(bx)\right)}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Tan[a + b*x]^2,x]
```

```
[Out] -1/3*(x*(3*c^2 + 3*c*d*x + d^2*x^2)) + (2*c*d*Sec[a]*(Cos[a]*Log[Cos[a]*Cos
[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^2*(Cos[a]^2 + Sin[a]^2)) + (d^2*
Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot
[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2
```

```
*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[
b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/S
qrt[1 + Cot[a]^2])*Sec[a])/(b^3*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Se
c[a]*Sec[a + b*x]*(c^2*Sin[b*x] + 2*c*d*x*Sin[b*x] + d^2*x^2*Sin[b*x]))/b
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(88) = 176.
time = 0.10, size = 199, normalized size = 2.07

method	result
risch	$-\frac{d^2x^3}{3} - cd\,x^2 - c^2x - \frac{c^3}{3d} + \frac{2i(x^2d^2+2cdx+c^2)}{b(1+e^{2i(bx+a)})} + \frac{2dc\ln(1+e^{2i(bx+a)})}{b^2} - \frac{4dc\ln(e^{i(bx+a)})}{b^2} - \frac{2id^2x^2}{b} - \frac{4id^2ax}{b^2} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*d^2*x^3-c*d*x^2-c^2*x-1/3/d*c^3+2*I*(d^2*x^2+2*c*d*x+c^2)/b/(1+exp(2*I
*(b*x+a)))+2*d/b^2*c*ln(1+exp(2*I*(b*x+a)))-4*d/b^2*c*ln(exp(I*(b*x+a)))-2*
I*d^2/b*x^2-4*I*d^2/b^2*a*x-2*I*d^2/b^3*a^2+2*d^2/b^2*ln(1+exp(2*I*(b*x+a))
)*x-I*d^2*polylog(2,-exp(2*I*(b*x+a)))/b^3+4*d^2/b^3*a*ln(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(85) = 170.
time = 0.57, size = 418, normalized size = 4.35

$$\frac{i\,b^3d^2x^3 + 3i\,b^3cdx^2 + 3i\,b^3c^2x + 6\,b^2c^2 + 6\,(bd^2x + bcd + (bd^2x + bcd)\cos(2\,bx + 2\,a) - (-i\,bd^2x - i\,bcd)\sin(2\,bx + 2\,a))\arctan(\sin(2\,bx + 2\,a), \cos(2\,bx + 2\,a) + 1) + (i\,b^3d^2x^3 - 3\,(-i\,b^3cd + 2\,b^2d^2)x^2 - 3\,(-i\,b^3c^2 + 4\,b^2cd)x)\cos(2\,bx + 2\,a) - 3\,(d^2\cos(2\,bx + 2\,a) + i\,d^2\sin(2\,bx + 2\,a) + d^2)\mathrm{Li}_2(-e^{(2i\,bx+2i\,a)}) - 3\,(i\,bd^2x + i\,bcd + (i\,bd^2x + i\,bcd)\cos(2\,bx + 2\,a) - (bd^2x + bcd)\sin(2\,bx + 2\,a))\log(\cos(2\,bx + 2\,a)^2 + \sin(2\,bx + 2\,a)^2 + 2\,\cos(2\,bx + 2\,a) + 1) - (b^3d^2x^3 + 3\,(b^3cd + 2i\,b^2d^2)x^2 + 3\,(b^3c^2 + 4i\,b^2cd)x)\sin(2\,bx + 2\,a)}{-3i\,b^3\cos(2\,bx + 2\,a) + 3\,b^3\sin(2\,bx + 2\,a) - 3i\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (I*b^3*d^2*x^3 + 3*I*b^3*c*d*x^2 + 3*I*b^3*c^2*x + 6*b^2*c^2 + 6*(b*d^2*x +
 b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a) - (-I*b*d^2*x - I*b*c*d)*sin(2*
b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + (I*b^3*d^2*x^
3 - 3*(-I*b^3*c*d + 2*b^2*d^2)*x^2 - 3*(-I*b^3*c^2 + 4*b^2*c*d)*x)*cos(2*b*
x + 2*a) - 3*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) + d^2)*dilog(-e
^(2*I*b*x + 2*I*a)) - 3*(I*b*d^2*x + I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(2*
b*x + 2*a) - (b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + s
in(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (b^3*d^2*x^3 + 3*(b^3*c*d + 2
*I*b^2*d^2)*x^2 + 3*(b^3*c^2 + 4*I*b^2*c*d)*x)*sin(2*b*x + 2*a))/(-3*I*b^3*
cos(2*b*x + 2*a) + 3*b^3*sin(2*b*x + 2*a) - 3*I*b^3)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(85) = 170.
time = 1.11, size = 210, normalized size = 2.19

$$-\frac{2\,b^3d^2x^3 + 6\,b^3cdx^2 + 6\,b^3c^2x - 3i\,d^2\mathrm{Li}_2\left(\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1} + 1\right) + 3i\,d^2\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1} + 1\right) - 6\,(bd^2x + bcd)\log\left(-\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right) - 6\,(bd^2x + bcd)\log\left(-\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right) - 6\,(b^2d^2x^2 + 2\,b^2cdx + b^2c^2)\tan(bx+a)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 + 6*b^3*c^2*x - 3*I*d^2*dilog(2*(I*tan(
b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + 3*I*d^2*dilog(2*(-I*tan(b*x + a)
- 1)/(tan(b*x + a)^2 + 1) + 1) - 6*(b*d^2*x + b*c*d)*log(-2*(I*tan(b*x + a)
 - 1)/(tan(b*x + a)^2 + 1)) - 6*(b*d^2*x + b*c*d)*log(-2*(-I*tan(b*x + a) -
 1)/(tan(b*x + a)^2 + 1)) - 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*tan(b*x
 + a))/b^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \tan^2 (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*tan(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*tan(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(a+b\,x)^2\,(c+d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] int(tan(a + b*x)^2*(c + d*x)^2, x)
```

3.256 $\int (c+dx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=40

$$-cx-\frac{dx^2}{2}+\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}$$

```
[Out] -c*x-1/2*d*x^2+d*ln(cos(b*x+a))/b^2+(d*x+c)*tan(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {3801, 3556}

$$\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}-cx-\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Tan[a + b*x]^2,x]
[Out] -(c*x) - (d*x^2)/2 + (d*Log[Cos[a + b*x]])/b^2 + ((c + d*x)*Tan[a + b*x])/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\tan^2(a+bx)\,dx&=\frac{(c+dx)\tan(a+bx)}{b}-\frac{d\int \tan(a+bx)\,dx}{b}-\int (c+dx)\,dx\\&=-cx-\frac{dx^2}{2}+\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.32, size = 76, normalized size = 1.90

$$-\frac{c\text{ArcTan}(\tan(a+bx))}{b}+\frac{d\log(\cos(a+bx))}{b^2}-\frac{dx\sec(a)(bx\cos(a)-2\sin(a))}{2b}+\frac{dx\sec(a)\sec(a+bx)\sin(bx)}{b}+\frac{c\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Tan[a + b*x]^2,x]
```

```
[Out] -((c*ArcTan[Tan[a + b*x]])/b) + (d*Log[Cos[a + b*x]])/b^2 - (d*x*Sec[a]*(b*
x*Cos[a] - 2*Sin[a]))/(2*b) + (d*x*Sec[a]*Sec[a + b*x]*Sin[b*x])/b + (c*Tan
[a + b*x])/b
```

Maple [A]
time = 0.07, size = 63, normalized size = 1.58

method	result	size
norman	$\frac{c\tan(bx+a)}{b}+\frac{dx\tan(bx+a)}{b}-cx-\frac{dx^2}{2}-\frac{d\ln(1+\tan^2(bx+a))}{2b^2}$	52
default	$-\frac{dx^2}{2}-cx+\frac{-\frac{da\tan(bx+a)}{b}+c\tan(bx+a)+\frac{d((bx+a)\tan(bx+a)+\ln(\cos(bx+a)))}{b}}{b}$	63
risch	$-\frac{dx^2}{2}-cx-\frac{2idx}{b}-\frac{2ida}{b^2}+\frac{2i(dx+c)}{b(1+e^{2i(bx+a)})}+\frac{d\ln(1+e^{2i(bx+a)})}{b^2}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d*x^2-c*x+1/b*(-1/b*d*a*tan(b*x+a)+c*tan(b*x+a)+1/b*d*((b*x+a)*tan(b*x
+a)+ln(cos(b*x+a))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(38) = 76.
time = 0.49, size = 237, normalized size = 5.92

$$-\frac{2\,(bx+a-\tan(bx+a))c-\frac{2\,(bx+a-\tan(bx+a))ad}{b}+\frac{\left((bx+a)^2\cos(2\,bx+2\,a)^2+(bx+a)^2\sin(2\,bx+2\,a)^2+2\,(bx+a)^2\cos(2\,bx+2\,a)+(bx+a)^2-\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)\log\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)-4\,(bx+a)\sin(2\,bx+2\,a)\right)d}{\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b*x + a - tan(b*x + a))*c - 2*(b*x + a - tan(b*x + a))*a*d/b + ((b
*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 + 2*(b*x + a)
^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^
2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2
*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*d/((cos(2*b*x + 2*a)
^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b))/b
```

Fricas [A]
time = 1.42, size = 53, normalized size = 1.32

$$-\frac{b^2dx^2+2\,b^2cx-d\log\left(\frac{1}{\tan(bx+a)^2+1}\right)-2\,(bdx+bc)\tan(bx+a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*d*x^2 + 2*b^2*c*x - d*log(1/(tan(b*x + a)^2 + 1)) - 2*(b*d*x + b*
c)*tan(b*x + a))/b^2
```

Sympy [A]
time = 0.09, size = 65, normalized size = 1.62

$$\begin{cases} -cx - \frac{dx^2}{2} + \frac{c\tan{(a+bx)}}{b} + \frac{dx\tan{(a+bx)}}{b} - \frac{d\log{(\tan^2{(a+bx)}+1)}}{2b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\tan^2{(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)**2,x)
```

```
[Out] Piecewise((-c*x - d*x**2/2 + c*tan(a + b*x)/b + d*x*tan(a + b*x)/b - d*log(
tan(a + b*x)**2 + 1)/(2*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*tan(a)**2, True
))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(38) = 76.
time = 0.59, size = 223, normalized size = 5.58

$$-\frac{b^2dx^2\tan{(bx)}\tan{(a)} + 2b^2cx\tan{(bx)}\tan{(a)} - b^2dx^2 - 2b^2cx + 2bdx\tan{(bx)} + 2bdx\tan{(a)} - d\log\left(\frac{4\left(\tan(bx)^4\tan(a)^2 - 2\tan(bx)^3\tan(a) + \tan(bx)^2\tan(a)^2 + \tan(bx)^2 - 2\tan(bx)\tan(a) + 1\right)}{\tan(a)^2+1}\right)\tan{(bx)}\tan{(a)} + 2bc\tan{(bx)} + 2bc\tan{(a)} + d\log\left(\frac{4\left(\tan(bx)^4\tan(a)^2 - 2\tan(bx)^3\tan(a) + \tan(bx)^2\tan(a)^2 + \tan(bx)^2 - 2\tan(bx)\tan(a) + 1\right)}{\tan(a)^2+1}\right)}{2\left(b^2\tan{(bx)}\tan{(a)} - b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*d*x^2*tan(b*x)*tan(a) + 2*b^2*c*x*tan(b*x)*tan(a) - b^2*d*x^2 - 2
*b^2*c*x + 2*b*d*x*tan(b*x) + 2*b*d*x*tan(a) - d*log(4*(tan(b*x)^4*tan(a)^2
 - 2*tan(b*x)^3*tan(a) + tan(b*x)^2*tan(a)^2 + tan(b*x)^2 - 2*tan(b*x)*tan(
a) + 1)/(tan(a)^2 + 1))*tan(b*x)*tan(a) + 2*b*c*tan(b*x) + 2*b*c*tan(a) + d
*log(4*(tan(b*x)^4*tan(a)^2 - 2*tan(b*x)^3*tan(a) + tan(b*x)^2*tan(a)^2 + t
an(b*x)^2 - 2*tan(b*x)*tan(a) + 1)/(tan(a)^2 + 1)))/(b^2*tan(b*x)*tan(a) -
b^2)
```

Mupad [B]
time = 1.44, size = 52, normalized size = 1.30

$$-c\,x-\frac{d\,x^2}{2}-\frac{\frac{d\,\ln\left({\mathrm{tan}\left(a+b\,x\right)}^2+1\right)}{2}-b\,\left(c\,\mathrm{tan}\left(a+b\,x\right)+d\,x\,\mathrm{tan}\left(a+b\,x\right)\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2*(c + d*x),x)
```

```
[Out] - c*x - (d*x^2)/2 - ((d*log(tan(a + b*x)^2 + 1))/2 - b*(c*tan(a + b*x) + d*
x*tan(a + b*x)))/b^2
```

3.257 $\int \frac{\tan^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{\tan^2(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(tan(b*x+a)^2/(d*x+c),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Tan[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][Tan[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\tan^2(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 3.62, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[a + b*x]^2/(c + d*x),x]
```

```
[Out] Integrate[Tan[a + b*x]^2/(c + d*x), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(tan(b*x+a)^2/(d*x+c),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] (2*(b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x +
 b*c*d^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*inte
grate(sin(2*b*x + 2*a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*
d*x + b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x
 + 2*a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)), x) - (b*d*
x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)^2 + b
*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a))*log(d*x + c) + 2*d*sin(2*b*x + 2*a))
/(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d
)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(tan(b*x + a)^2/(d*x + c), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(tan(a + b*x)**2/(c + d*x), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(tan(b*x + a)^2/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\tan(a+b\,x)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2/(c + d*x),x)
```

```
[Out] int(tan(a + b*x)^2/(c + d*x), x)
```

3.258 $\int \frac{\tan^2(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\tan^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(tan(b*x+a)^2/(d*x+c)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Tan[a + b*x]^2/(c + d*x)^2,x]
[Out] Defer[Int][Tan[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 5.44, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[a + b*x]^2/(c + d*x)^2,x]
[Out] Integrate[Tan[a + b*x]^2/(c + d*x)^2, x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(tan(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)^
2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + 4*(b*d^4*x^2 + 2*b*c*d^3*x + b
*c^2*d^2 + (b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*cos(2*b*x + 2*a)^2 + (b*d^
4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^4*x^2 + 2*b*c*
d^3*x + b*c^2*d^2)*cos(2*b*x + 2*a))*integrate(sin(2*b*x + 2*a)/(b*d^3*x^3
+ 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^
2*d*x + b*c^3)*cos(2*b*x + 2*a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*
x + b*c^3)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x
+ b*c^3)*cos(2*b*x + 2*a)), x) + 2*d*sin(2*b*x + 2*a))/(b*d^3*x^2 + 2*b*c*d
^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a)^2 + (
b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x^2 + 2*b*
c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(tan(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(tan(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\tan(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2/(c + d*x)^2,x)
```

```
[Out] int(tan(a + b*x)^2/(c + d*x)^2, x)
```

3.259 $\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=150

$$\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}(1+$$

```
[Out] CannotIntegrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)+1/2*exp(I*(a-b*c/d))*(d*
x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+1/2*(d*x+c)^m*GAMMA
(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(2*b*(
((-I)*b*(c + d*x))/d)^m) + ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(2
*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Defer[Int][(c + d*x)^m*Sec[
a + b*x]*Tan[a + b*x], x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx &= -\int (c+dx)^m \sin(a+bx)\,dx + \int (c+dx)^m \sec(a+bx)\tan(a+b\\
&= -\left(\frac{1}{2}i\int e^{-i(a+bx)}(c+dx)^m\,dx\right) + \frac{1}{2}i\int e^{i(a+bx)}(c+dx)^m\,dx + \int\\
&= \frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b} + \frac{e^{-i\left(a-\frac{bc}{d}\right.}}{}
\end{aligned}$$

Mathematica [A]
time = 14.42, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(dx+c\right)^{m} \sin \left(bx+a\right) \left(\tan^{2}\left(bx+a\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x)
[Out] int((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+dx\right)^{m} \sin \left(a+bx\right) \tan^{2}\left(a+bx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sin(b*x+a)*tan(b*x+a)**2,x)
[Out] Integral((c + d*x)**m*sin(a + b*x)*tan(a + b*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)\,\tan(a+b\,x)^2\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^m,x)
```

```
[Out] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^m, x)
```

3.260 $\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx) \, dx$

Optimal. Leaf size=228

$$\frac{6id(c+dx)^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}-\frac{6d^2(c+dx)\cos(a+bx)}{b^3}+\frac{(c+dx)^3\cos(a+bx)}{b}-\frac{6id^2(c+dx)\text{PolyLog}(2,-$$

```
[Out] 6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*cos(b*x+a)/b^3+(d*
x+c)^3*cos(b*x+a)/b-6*I*d^2*(d*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^3+6*I*d^
2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^3+6*d^3*polylog(3,-I*exp(I*(b*x+a))
)/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4+(d*x+c)^3*sec(b*x+a)/b+6*d^3*si
n(b*x+a)/b^4-3*d*(d*x+c)^2*sin(b*x+a)/b^2
```

Rubi [A]
time = 0.14, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4492, 3377, 2717, 4494, 4266, 2611, 2320, 6724}

$$\frac{6id(c+dx)^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}+\frac{6d^3\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}+\frac{6d^3\sin(a+bx)}{b^4}-\frac{6id^2(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}-\frac{6d^2(c+dx)\cos(a+bx)}{b^3}-\frac{3d(c+dx)^2\sin(a+bx)}{b^2}+\frac{(c+dx)^3\cos(a+bx)}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] ((6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 - (6*d^2*(c + d*x)*Cos[a
+ b*x])/b^3 + ((c + d*x)^3*Cos[a + b*x])/b - ((6*I)*d^2*(c + d*x)*PolyLog[2
, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(I*(a +
b*x))])/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLog
[3, I*E^(I*(a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x])/b + (6*d^3*Sin[a +
 b*x])/b^4 - (3*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4492

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\, dx = -\int (c+dx)^3 \sin(a+bx)\, dx + \int (c+dx)^3 \sec(a+bx) \tan(a+b$$

$$= \frac{(c+dx)^3 \cos(a+bx)}{b} + \frac{(c+dx)^3 \sec(a+bx)}{b} - \frac{(3d) \int (c+dx)^2}{b}$$

$$= \frac{6id(c+dx)^2 \tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^3 \cos(a+bx)}{b} + \frac{(c+dx)}{}$$

$$= \frac{6id(c+dx)^2 \tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+d}{}$$

$$= \frac{6id(c+dx)^2 \tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+d}{}$$

$$= \frac{6id(c+dx)^2 \tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+d}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 532 vs. 2(228) = 456.
time = 1.45, size = 532, normalized size = 2.33

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (Sec[a + b*x]*(3*b^3*c^3 - 6*b*c*d^2 + 9*b^3*c^2*d*x - 6*b*d^3*x + 9*b^3*c*
d^2*x^2 + 3*b^3*d^3*x^3 + (12*I)*b^2*c^2*d*ArcTan[E^(I*(a + b*x))]*Cos[a +
b*x] + b^3*c^3*Cos[2*(a + b*x)] - 6*b*c*d^2*Cos[2*(a + b*x)] + 3*b^3*c^2*d*
x*Cos[2*(a + b*x)] - 6*b*d^3*x*Cos[2*(a + b*x)] + 3*b^3*c*d^2*x^2*Cos[2*(a
+ b*x)] + b^3*d^3*x^3*Cos[2*(a + b*x)] - 12*b^2*c*d^2*x*Cos[a + b*x]*Log[1
- I*E^(I*(a + b*x))] - 6*b^2*d^3*x^2*Cos[a + b*x]*Log[1 - I*E^(I*(a + b*x))
] + 12*b^2*c*d^2*x*Cos[a + b*x]*Log[1 + I*E^(I*(a + b*x))] + 6*b^2*d^3*x^2*
Cos[a + b*x]*Log[1 + I*E^(I*(a + b*x))] - (12*I)*b*d^2*(c + d*x)*Cos[a + b*
x]*PolyLog[2, (-I)*E^(I*(a + b*x))] + (12*I)*b*d^2*(c + d*x)*Cos[a + b*x]*P
olyLog[2, I*E^(I*(a + b*x))] + 12*d^3*Cos[a + b*x]*PolyLog[3, (-I)*E^(I*(a
+ b*x))] - 12*d^3*Cos[a + b*x]*PolyLog[3, I*E^(I*(a + b*x))] - 3*b^2*c^2*d*
Sin[2*(a + b*x)] + 6*d^3*Sin[2*(a + b*x)] - 6*b^2*c*d^2*x*Sin[2*(a + b*x)]
- 3*b^2*d^3*x^2*Sin[2*(a + b*x)]))/(2*b^4)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 676 vs. 2(211) = 422.
time = 0.13, size = 677, normalized size = 2.97

method	result
risch	$\frac{(d^3x^3b^3+3b^3c\,d^2x^2+3ib^2d^3x^2+3b^3c^2dx+6ib^2c\,d^2x+b^3c^3+3ib^2c^2d-6b\,d^3x-6c\,d^2b-6id^3)\,\mathrm{e}^{i(bx+a)}}{2b^4}+\frac{(d^3x^3b^3+3b^3c\,d^2x^2-3ib^2d^3x^2-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*
d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))+1
/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d
^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))+2
*exp(I*(b*x+a))*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(1+exp(2*I*(b*x+a)))-
3/b^2*d^3*ln(1-I*exp(I*(b*x+a)))*x^2+3/b^2*d^3*ln(1+I*exp(I*(b*x+a)))*x^2+6
*I/b^2*d*c^2*arctan(exp(I*(b*x+a)))-6*I*c*d^2*polylog(2,-I*exp(I*(b*x+a)))/
b^3+6*I/b^4*d^3*a^2*arctan(exp(I*(b*x+a)))+6*d^3*polylog(3,-I*exp(I*(b*x+a)
))/b^4+6/b^3*d^2*c*ln(1+I*exp(I*(b*x+a)))*a-6*I*d^3*x*polylog(2,-I*exp(I*(b
*x+a)))/b^3-6/b^2*d^2*c*ln(1-I*exp(I*(b*x+a)))*x-3/b^4*d^3*a^2*ln(1+I*exp(I
*(b*x+a)))+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b^3+3/b^4*d^3*a^2*ln(1-I*e
xp(I*(b*x+a)))+6*I*c*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3+6/b^2*d^2*c*ln(1+I
*exp(I*(b*x+a)))*x-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4-12*I/b^3*d^2*c*a*a
rctan(exp(I*(b*x+a)))-6/b^3*d^2*c*ln(1-I*exp(I*(b*x+a)))*a
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 11010 vs. $2(202) = 404$.
time = 1.61, size = 11010, normalized size = 48.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^3*(1/cos(b*x + a) + cos(b*x + a)) - 6*a*c^2*d*(1/cos(b*x + a) + co
s(b*x + a))/b + 6*a^2*c*d^2*(1/cos(b*x + a) + cos(b*x + a))/b^2 - 2*a^3*d^3
*(1/cos(b*x + a) + cos(b*x + a))/b^3 + 3*((b*x + (b*x + a)*cos(2*b*x + 2*a)
 + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)^3 +
((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a)^3 + 6*
(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos
(b*x + a) + 4*(b*x + a)*cos(b*x + a) + (3*(b*x + a)*sin(b*x + a) + cos(b*x
+ a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(
b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x + a)*sin(2*b*x + 2*a)*sin(b*x + a) +
 (b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a)
 + 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a) + 6*(b*x +
a)*cos(b*x + a) - 2*sin(b*x + a))*sin(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x +
 a) - sin(b*x + a))*sin(2*b*x + 2*a)^2 + ((b*x + a)*cos(2*b*x + 2*a)^2 + 13
```

```
*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2 + (b*x + a)*sin(b*
x + a)^2 + b*x + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2 +
2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + c
os(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a) + a)*cos(3*b*x + 3*a) + 2*
(3*(b*x + a)*cos(b*x + a)^3 + 3*(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + (b*
x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a) + (b*x + a)*cos(b*x +
a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*co
s(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x
 + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2
*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
+ 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 +
sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*s
in(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x
+ a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2
+ sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + ((cos(2*b*x + 2*a)^2 + sin(2*b*x +
 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + si
n(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x +
 a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x +
a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos
(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(
b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*
x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) +
 sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1)
+ (((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a)^2 +
 12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + 2*(((b*x + a)*sin(b*x + a) - cos(
b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)*cos(b*x + a) + sin(b*x + a))*sin(2*
b*x + 2*a) + (b*x + a)*sin(b*x + a) - cos(b*x + a))*cos(3*b*x + 3*a) + (12*
(b*x + a)*cos(b*x + a)*sin(b*x + a) - cos(b*x + a)^2 - sin(b*x + a)^2 - 2)*
cos(2*b*x + 2*a) - cos(2*b*x + 2*a)^2 - cos(b*x + a)^2 + ((b*x + a)*cos(b*x
 + a)^2 + 13*(b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)^
2 - sin(b*x + a)^2 - 1)*sin(3*b*x + 3*a) + 6*((b*x + a)*cos(b*x + a)^2*sin(
b*x + a) + (b*x + a)*sin(b*x + a)^3)*sin(2*b*x + 2*a) - sin(b*x + a))*c^2*d
/(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3
*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos
(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x +
3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*
x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2
*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin
(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(
b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a
) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*b) - 6*((b*x + (b*x +
a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^3 + 6*(b*x + a
)*cos(b*x + a)^3 + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(
```

```
3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + 2*(4*(b*x + a)*c
os(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*cos(b*x + a) + (3*(b*x + a)*sin(
b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + ((b*x + a)*
cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x + a)*sin(2*b*x +
2*a)*sin(b*x + a) + (b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a
))*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x
 + 2*a) + 6*(b*x + a)*cos(b*x + a) - 2*sin(b*x + a))*sin(3*b*x + 3*a)^2 + (
(b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 2*a)^2 + ((b*x + a)*cos(
2*b*x + 2*a)^2 + 13*(b*x + a)*cos(b*x + a)^2 + ...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(202) = 404$.
time = 2.10, size = 896, normalized size = 3.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*co
s(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*p
olylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, -I
*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x +
a) - sin(b*x + a)) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2
 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 - 6*(-I*b*d^3*x - I*b*c*d^2)*c
os(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) - 6*(-I*b*d^3*x - I*b*c*d^
2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 6*(I*b*d^3*x + I*b*c
*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 6*(I*b*d^3*x + I
*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(b^2*c^2*d
 - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) +
I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) -
I*sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^
3)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2
*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - sin
(b*x + a) + 1) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*co
s(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2
*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*
x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b
*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b
*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b^2*d^3*x^2 + 2*b^2*c*
d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)*sin(b*x + a))/(b^4*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \sin(a + bx) \tan^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sin(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*sin(a + b*x)*tan(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sin(b*x + a)*tan(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,x\right)\,{\mathrm{tan}\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^3, x)
```

3.261 $\int (c+dx)^2 \sin(a+bx) \tan^2(a+bx) \, dx$

Optimal. Leaf size=145

$$\frac{4id(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2d^2\cos(a+bx)}{b^3}+\frac{(c+dx)^2\cos(a+bx)}{b}-\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{Pol}}{}$$

```
[Out] 4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2-2*d^2*cos(b*x+a)/b^3+(d*x+c)^2*cos
(b*x+a)/b-2*I*d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3+2*I*d^2*polylog(2,I*exp(
I*(b*x+a)))/b^3+(d*x+c)^2*sec(b*x+a)/b-2*d*(d*x+c)*sin(b*x+a)/b^2
```

Rubi [A]
time = 0.09, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {4492, 3377, 2718, 4494, 4266, 2317, 2438}

$$\frac{4id(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2id^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}-\frac{2d^2\cos(a+bx)}{b^3}-\frac{2d(c+dx)\sin(a+bx)}{b^2}+\frac{(c+dx)^2\cos(a+bx)}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] ((4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 - (2*d^2*Cos[a + b*x])/b^3
+ ((c + d*x)^2*Cos[a + b*x])/b - ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))
])/b^3 + ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 + ((c + d*x)^2*Sec[a
 + b*x])/b - (2*d*(c + d*x)*Sin[a + b*x])/b^2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4492

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2 \sin(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)^2 \sin(a+bx)\,dx + \int (c+dx)^2 \sec(a+bx)\tan(a+b$$

$$= \frac{(c+dx)^2\cos(a+bx)}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{(2d)\int (c+dx)\,c}{b}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\cos(a+bx)}{b} + \frac{(c+dx)^2}{}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{(c+dx)^2\cos(a}{b}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{(c+dx)^2\cos(a}{b}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 362 vs. 2(145) = 290.
time = 3.23, size = 362, normalized size = 2.50

$$\frac{-4bcd\tanh^{-1}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right) - 4d^2\text{ArcTan}(\cot(a))\tanh^{-1}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right) + \frac{2d^2\csc(a)\left((bx-\text{ArcTan}(\cot(a)))\left(\log\left(1-e^{i(bx-\text{ArcTan}(\cot(a)))}\right)-\log\left(1+e^{i(bx-\text{ArcTan}(\cot(a)))}\right)\right)+i\text{PolyLog}\left(2,-e^{i(bx-\text{ArcTan}(\cot(a)))}\right)-i\text{PolyLog}\left(2,e^{i(bx-\text{ArcTan}(\cot(a)))}\right)\right)}{\sqrt{\csc^2(a)}} + b^2(c+dx)^2\sec(a) + \cos(bx)\left(\left(-2d^2+b^2(c+dx)^2\right)\cos(a)-2bd(c+dx)\sin(a)\right) - \left(2bd(c+dx)\cos(a)+\left(-2d^2+b^2(c+dx)^2\right)\sin(a)\right)\sin(bx) + \frac{b^2(c+dx)^2\sin\left(\frac{bx}{2}\right)}{\left(\cos\left(\frac{a}{2}\right)-\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)} - \frac{b^2(c+dx)^2\sin\left(\frac{bx}{2}\right)}{\left(\cos\left(\frac{a}{2}\right)+\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)}}{b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*ArcT
anh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + (2*d^2*Csc[a]*((b*x - ArcTan[Cot[a]])*(
Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a]])
)]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2, E^(I*(b*x
- ArcTan[Cot[a]]))]))/Sqrt[Csc[a]^2] + b^2*(c + d*x)^2*Sec[a] + Cos[b*x]*((
-2*d^2 + b^2*(c + d*x)^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) - (2*b*d*(c + d*
x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] + (b^2*(c + d*x)^2*
Sin[(b*x)/2])/((Cos[a/2] - Sin[a/2])*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2]))
 - (b^2*(c + d*x)^2*Sin[(b*x)/2])/((Cos[a/2] + Sin[a/2])*(Cos[(a + b*x)/2]
+ Sin[(a + b*x)/2])))/b^3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(134) = 268$.
time = 0.12, size = 345, normalized size = 2.38

method	result
risch	$\frac{(x^2 d^2 b^2+2b^2 cdx+2ib\, d^2 x+b^2 c^2+2ibcd-2d^2)\mathrm{e}^{i(bx+a)}}{2b^3}+\frac{(x^2 d^2 b^2+2b^2 cdx-2ib\, d^2 x+b^2 c^2-2ibcd-2d^2)\mathrm{e}^{-i(bx+a)}}{2b^3}+\frac{2\,\mathrm{e}^{i(bx+a)}(x^2 d^2}{b(1+\mathrm{e}^{2i(b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(I
*(b*x+a))+1/2*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)
/b^3*exp(-I*(b*x+a))+2*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(1+exp(2*I*(b
*x+a)))+4*I*d/b^2*c*arctan(exp(I*(b*x+a)))+2*d^2/b^2*ln(1+I*exp(I*(b*x+a)))
*x+2*d^2/b^3*ln(1+I*exp(I*(b*x+a)))*a-2*d^2/b^2*ln(1-I*exp(I*(b*x+a)))*x-2*
d^2/b^3*ln(1-I*exp(I*(b*x+a)))*a-2*I*d^2/b^3*dilog(1+I*exp(I*(b*x+a)))+2*I*
d^2/b^3*dilog(1-I*exp(I*(b*x+a)))-4*I*d^2/b^3*a*arctan(exp(I*(b*x+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*((3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*cos(2*b*x + 3*a)*
cos(b*x + 2*a) + (3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*sin(2*b*
x + 3*a)*sin(b*x + 2*a) + (3*b^2*d^2*x^2*cos(a) + 6*b^2*c*d*x*cos(a) + 3*b^
2*c^2*cos(a) - 2*d^2*cos(a))*cos(b*x + 2*a) + (3*b^2*d^2*x^2*sin(a) + 6*b^2
```

*c*d*x*sin(a) + 3*b^2*c^2*sin(a) - 2*d^2*sin(a))*sin(b*x + 2*a))*cos(3*b*x
+ 3*a)^2 + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) - 2*
(b*d^2*x + b*c*d)*sin(b*x + a))*cos(2*b*x + 3*a)^2 + 2*((3*b^2*d^2*x^2 + 6*
b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*cos(2*b*x + 3*a)*cos(b*x + 2*a) + (3*b^2*d^2
*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*sin(2*b*x + 3*a)*sin(b*x + 2*a) + (
3*b^2*d^2*x^2*cos(a) + 6*b^2*c*d*x*cos(a) + 3*b^2*c^2*cos(a) - 2*d^2*cos(a)
)*cos(b*x + 2*a) + (3*b^2*d^2*x^2*sin(a) + 6*b^2*c*d*x*sin(a) + 3*b^2*c^2*s
in(a) - 2*d^2*sin(a))*sin(b*x + 2*a))*sin(3*b*x + 3*a)^2 + ((b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) - 2*(b*d^2*x + b*c*d)*sin(b*x +
 a))*sin(2*b*x + 3*a)^2 + ((b^2*d^2*x^2*cos(a) + b^2*c^2*cos(a) + 2*b*c*d*s
in(a) - 2*d^2*cos(a) + 2*(b^2*c*d*cos(a) + b*d^2*sin(a))*x + (b^2*d^2*x^2 +
 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(2*b*x + 3*a) + 2*(b*d^2*x + b*c*d)*sin(
2*b*x + 3*a))*cos(3*b*x + 3*a)^2 + (b^2*d^2*x^2*cos(a) + b^2*c^2*cos(a) + 2
*b*c*d*sin(a) - 2*d^2*cos(a) + 2*(b^2*c*d*cos(a) + b*d^2*sin(a))*x)*cos(b*x
 + a)^2 + (b^2*d^2*x^2*cos(a) + b^2*c^2*cos(a) + 2*b*c*d*sin(a) - 2*d^2*cos
(a) + 2*(b^2*c*d*cos(a) + b*d^2*sin(a))*x + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2 - 2*d^2)*cos(2*b*x + 3*a) + 2*(b*d^2*x + b*c*d)*sin(2*b*x + 3*a))*sin
(3*b*x + 3*a)^2 + (b^2*d^2*x^2*cos(a) + b^2*c^2*cos(a) + 2*b*c*d*sin(a) - 2
*d^2*cos(a) + 2*(b^2*c*d*cos(a) + b*d^2*sin(a))*x)*sin(b*x + a)^2 + 2*((b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(2*b*x + 3*a)*cos(b*x + a) + 2
*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(2*b*x + 3*a) + (b^2*d^2*x^2*cos(a) + b^
2*c^2*cos(a) + 2*b*c*d*sin(a) - 2*d^2*cos(a) + 2*(b^2*c*d*cos(a) + b*d^2*si
n(a))*x)*cos(b*x + a))*cos(3*b*x + 3*a) + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2 - 2*d^2)*cos(b*x + a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2
)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2 - 2*d^2)*cos(2*b*x + 3*a)*sin(b*x + a) + 2*(b*d^2*x + b*c*d)*sin(2*b*x +
3*a)*sin(b*x + a) + (b^2*d^2*x^2*cos(a) + b^2*c^2*cos(a) + 2*b*c*d*sin(a) -
 2*d^2*cos(a) + 2*(b^2*c*d*cos(a) + b*d^2*sin(a))*x)*sin(b*x + a))*sin(3*b*
x + 3*a) + 2*((b*d^2*x + b*c*d)*cos(b*x + a)^2 + (b*d^2*x + b*c*d)*sin(b*x
+ a)^2)*sin(2*b*x + 3*a))*cos(3*b*x + 4*a) + ((cos(a)^2 + sin(a)^2)*b^2*d^2
x^2 + 2(cos(a)^2 + sin(a)^2)*b^2*c*d*x + (cos(a)^2 + sin(a)^2)*b^2*c^2 -
2*(cos(a)^2 + sin(a)^2)*d^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)
*cos(2*b*x + 3*a)^2 + 4*(3*b^2*d^2*x^2*cos(a) + 6*b^2*c*d*x*cos(a) + 3*b^2*
c^2*cos(a) - 2*d^2*cos(a))*cos(b*x + 2*a)*cos(b*x + a) + (b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2 - 2*d^2)*sin(2*b*x + 3*a)^2 + 4*(3*b^2*d^2*x^2*sin(a) +
6*b^2*c*d*x*sin(a) + 3*b^2*c^2*sin(a) - 2*d^2*sin(a))*cos(b*x + a)*sin(b*x
+ 2*a) + 2*(b^2*d^2*x^2*cos(a) + 2*b^2*c*d*x*cos(a) + b^2*c^2*cos(a) + 2*(3
*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*cos(b*x + 2*a)*cos(b*x + a)
 - 2*d^2*cos(a))*cos(2*b*x + 3*a) + 2*(b^2*d^2*x^2*sin(a) + 2*b^2*c*d*x*sin
(a) + b^2*c^2*sin(a) + 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*
cos(b*x + a)*sin(b*x + 2*a) - 2*d^2*sin(a))*sin(2*b*x + 3*a))*cos(3*b*x + 3
a) + 2(((3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*cos(b*x + a)^2
+ (3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*sin(b*x + a)^2)*cos(b*x
 + 2*a) + (b^2*d^2*x^2*cos(a) + 2*b^2*c*d*x*cos(a) + b^2*c^2*cos(a) - 2*d^2
*cos(a))*cos(b*x + a) - 2*(b*d^2*x*cos(a) + b*c*d*cos(a))*sin(b*x + a))*cos

```
(2*b*x + 3*a) + 2*((3*b^2*d^2*x^2*cos(a) + 6*b^2*c*d*x*cos(a) + 3*b^2*c^2*c
os(a) - 2*d^2*cos(a))*cos(b*x + a)^2 + (3*b^2*d^2*x^2*cos(a) + 6*b^2*c*d*x*
cos(a) + 3*b^2*c^2*cos(a) - 2*d^2*cos(a))*sin(b*x + a)^2)*cos(b*x + 2*a) +
((cos(a)^2 + sin(a)^2)*b^2*d^2*x^2 + 2*(cos(a)^2 + sin(a)^2)*b^2*c*d*x + (c
os(a)^2 + sin(a)^2)*b^2*c^2 - 2*(cos(a)^2 + sin(a)^2)*d^2)*cos(b*x + a) - 8
*((cos(a)^2 + sin(a)^2)*b^3*d^2*cos(b*x + a)^2 + (cos(a)^2 + sin(a)^2)*b^3*
d^2*sin(b*x + a)^2 + (b^3*d^2*cos(2*b*x + 3*a)^2 + 2*b^3*d^2*cos(2*b*x + 3*
a)*cos(a) + b^3*d^2*sin(2*b*x + 3*a)^2 + 2*b^3*d^2*sin(2*b*x + 3*a)*sin(a)
+ (cos(a)^2 + sin(a)^2)*b^3*d^2)*cos(3*b*x + 3*a)^2 + (b^3*d^2*cos(b*x + a)
^2 + b^3*d^2*sin(b*x + a)^2)*cos(2*b*x + 3*a)^2 + (b^3*d^2*cos(2*b*x + 3*a)
^2 + 2*b^3*d^2*cos(2*b*x + 3*a)*cos(a) + b^3*d^2*sin(2*b*x + 3*a)^2 + 2*b^3
*d^2*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^3*d^2)*sin(3*b*x + 3
*a)^2 + (b^3*d^2*cos(b*x + a)^2 + b^3*d^2*sin(b*x + a)^2)*sin(2*b*x + 3*a)^
2 + 2*(b^3*d^2*cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*b^3*d^2*cos(2*b*x + 3*a)
*cos(b*x + a)*cos(a) + b^3*d^2*cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*b^3*d^2*
cos(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^3*d^2*cos(b*
x + a))*cos(3*b*x + 3*a) + 2*(b^3*d^2*cos(b*x + a)^2*cos(a) + b^3*d^2*cos(a
)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(b^3*d^2...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(127) = 254$.
time = 1.16, size = 511, normalized size = 3.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*cos(b*x + a)*dilog(I*cos(b*x +
 a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a
)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d^2*cos(b
*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*
x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a
) - I*sin(b*x + a) + I) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a)
 + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) -
sin(b*x + a) + 1) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + si
n(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(
b*x + a) + 1) - (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x
+ a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a)
 + I) - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \sin(a+bx) \tan^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sin(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*sin(a + b*x)*tan(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sin(b*x + a)*tan(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)\,{\mathrm{tan}\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^2, x)
```

3.262 $\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=56

$$-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}+\frac{(c+dx)\sec(a+bx)}{b}-\frac{d\sin(a+bx)}{b^2}$$

```
[Out] -d*arctanh(sin(b*x+a))/b^2+(d*x+c)*cos(b*x+a)/b+(d*x+c)*sec(b*x+a)/b-d*sin(
b*x+a)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4492, 3377, 2717, 4494, 3855}

$$-\frac{d\sin(a+bx)}{b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}+\frac{(c+dx)\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] -((d*ArcTanh[Sin[a + b*x]])/b^2) + ((c + d*x)*Cos[a + b*x])/b + ((c + d*x)*
Sec[a + b*x])/b - (d*Sin[a + b*x])/b^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4492

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)\sin(a+bx)\,dx + \int (c+dx)\sec(a+bx)\tan(a+bx)\,$$
$$= \frac{(c+dx)\cos(a+bx)}{b} + \frac{(c+dx)\sec(a+bx)}{b} - \frac{d\int \cos(a+bx)\,dx}{b}$$
$$= -\frac{d\tanh^{-1}(\sin(a+bx))}{b^2} + \frac{(c+dx)\cos(a+bx)}{b} + \frac{(c+dx)\sec(a}{b}$$

Mathematica [A]
time = 0.34, size = 107, normalized size = 1.91

$$\frac{\sec(a+bx)\left(3bc+3bdx+b(c+dx)\cos(2(a+bx))+2d\cos(a+bx)\left(\log\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)\right)-d\sin(2(a+bx))\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (Sec[a + b*x]*(3*b*c + 3*b*d*x + b*(c + d*x)*Cos[2*(a + b*x)] + 2*d*Cos[a +
 b*x]*(Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] - Log[Cos[(a + b*x)/2] + Si
n[(a + b*x)/2]]) - d*Sin[2*(a + b*x)]))/(2*b^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 123, normalized size = 2.20

method	result	size
risch	$\frac{(dxb+cb+id)e^{i(bx+a)}}{2b^2} + \frac{(dxb+cb-id)e^{-i(bx+a)}}{2b^2} + \frac{2\,e^{i(bx+a)}(dx+c)}{b\left(1+e^{2i(bx+a)}\right)} + \frac{d\ln\left(e^{i(bx+a)}-i\right)}{b^2} - \frac{d\ln\left(e^{i(bx+a)}+i\right)}{b^2}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(d*x*b+c*b+I*d)/b^2*exp(I*(b*x+a))+1/2*(d*x*b+c*b-I*d)/b^2*exp(-I*(b*x+
a))+2*exp(I*(b*x+a))*(d*x+c)/b/(1+exp(2*I*(b*x+a)))+d/b^2*ln(exp(I*(b*x+a))
-I)-d/b^2*ln(exp(I*(b*x+a))+I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2123 vs. $2(56) = 112$.
time = 0.53, size = 2123, normalized size = 37.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*c*(1/cos(b*x + a) + cos(b*x + a)) - 2*a*d*(1/cos(b*x + a) + cos(b*x
+ a))/b + ((b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*
b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)^3 + ((b*x + a)*sin(2*b*x + 2*a) - c
os(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)*sin(b*x
+ a)^2 + 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*cos(b*x
 + a) + (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a))*cos(3*b
*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a)^2 +
(8*(b*x + a)*sin(2*b*x + 2*a)*sin(b*x + a) + (b*x + (b*x + a)*cos(2*b*x + 2
*a) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a)
- sin(b*x + a))*cos(2*b*x + 2*a) + 6*(b*x + a)*cos(b*x + a) - 2*sin(b*x + a
))*sin(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b*x +
 2*a)^2 + ((b*x + a)*cos(2*b*x + 2*a)^2 + 13*(b*x + a)*cos(b*x + a)^2 + (b*
x + a)*sin(2*b*x + 2*a)^2 + (b*x + a)*sin(b*x + a)^2 + b*x + (13*(b*x + a)*
cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) +
 (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^2 + sin(b*x + a)^2)
*sin(2*b*x + 2*a) + a)*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a)^3 + 3
*(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + (b*x + a)*cos(b*x + a) - sin(b*x +
 a))*cos(2*b*x + 2*a) + (b*x + a)*cos(b*x + a) - ((cos(2*b*x + 2*a)^2 + sin
(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a
)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + s
in(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + co
s(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x +
 a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a
) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^
2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x
+ 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x +
a) + 1) + ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) +
1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^
2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(
3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(
cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)
^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a
)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin
(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x +
```

```
a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (((b*x + a)*sin(2*b*x + 2*a)
- cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a)^2 + 12*(b*x + a)*cos(b*x + a)*sin(
b*x + a) + 2*(((b*x + a)*sin(b*x + a) - cos(b*x + a))*cos(2*b*x + 2*a) + ((
b*x + a)*cos(b*x + a) + sin(b*x + a))*sin(2*b*x + 2*a) + (b*x + a)*sin(b*x
+ a) - cos(b*x + a))*cos(3*b*x + 3*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x
+ a) - cos(b*x + a)^2 - sin(b*x + a)^2 - 2)*cos(2*b*x + 2*a) - cos(2*b*x +
2*a)^2 - cos(b*x + a)^2 + ((b*x + a)*cos(b*x + a)^2 + 13*(b*x + a)*sin(b*x
+ a)^2)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)^2 - sin(b*x + a)^2 - 1)*sin(3*b
*x + 3*a) + 6*((b*x + a)*cos(b*x + a)^2*sin(b*x + a) + (b*x + a)*sin(b*x +
a)^3)*sin(2*b*x + 2*a) - sin(b*x + a))*d/(((cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + s
in(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^
2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x
+ a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x +
 a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*co
s(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos
(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b
*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a)
+ sin(b*x + a)^2)*b))/b
```

Fricas [**A**]
time = 2.21, size = 93, normalized size = 1.66

$$\frac{2\,bdx+2\,(bdx+bc)\cos\left(bx+a\right)^{2}-d\cos\left(bx+a\right)\log\left(\sin\left(bx+a\right)+1\right)+d\cos\left(bx+a\right)\log\left(-\sin\left(bx+a\right)+1\right)-2\,d\cos\left(bx+a\right)\sin\left(bx+a\right)+2\,bc}{2\,b^{2}\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x + 2*(b*d*x + b*c)*cos(b*x + a)^2 - d*cos(b*x + a)*log(sin(b*x
+ a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a) + 1) - 2*d*cos(b*x + a)*sin(b*
x + a) + 2*b*c)/(b^2*cos(b*x + a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\sin\left(a+bx\right)\tan^{2}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*sin(a + b*x)*tan(a + b*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2762 vs. $2(56) = 112$.
time = 1.19, size = 2762, normalized size = 49.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b*c*tan(1/2*b*x)^4*tan(1/2*a)^
4 + d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*
tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^
2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan
(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*
b*x)^4*tan(1/2*a)^4 - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)
^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*
b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*ta
n(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*a
)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^4 - 16*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^
3 - 16*b*c*tan(1/2*b*x)^3*tan(1/2*a)^3 - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*
a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/
2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x
)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1
/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a)^3 + 4*d*log(2*(tan
(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*t
an(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*
x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*ta
n(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a
)^3 + 4*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + 4
*b*d*x*tan(1/2*b*x)^4 + 16*b*d*x*tan(1/2*b*x)^3*tan(1/2*a) + 48*b*d*x*tan(1
/2*b*x)^2*tan(1/2*a)^2 + 16*b*d*x*tan(1/2*b*x)*tan(1/2*a)^3 + 4*b*d*x*tan(1
/2*a)^4 + 4*b*c*tan(1/2*b*x)^4 - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*t
an(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 +
 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*
a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)
/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4 + d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2
- 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x
)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan
(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a)
 + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4 + 16*b*c*tan(1/2*b*x)^3*tan(1/2*a)
 - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2
*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)
^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + ta
n(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2
*b*x)^3*tan(1/2*a) + 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x
)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2
*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*t
an(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*
a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a) - 4*d*tan(1/2*b*x)^4*tan(1/2*a) + 48*b
*c*tan(1/2*b*x)^2*tan(1/2*a)^2 - 24*d*tan(1/2*b*x)^3*tan(1/2*a)^2 + 16*b*c*
```

```
tan(1/2*b*x)*tan(1/2*a)^3 - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(
1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*
tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^
2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(t
an(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^3 + 4*d*log(2*(tan(1/2*b*x)^4*tan
(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + t
an(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/
2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*
tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^3 - 24*d*tan(1/
2*b*x)^2*tan(1/2*a)^3 + 4*b*c*tan(1/2*a)^4 - d*log(2*(tan(1/2*b*x)^4*tan(1/
2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(
1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b
*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan
(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*a)^4 + d*log(2*(tan(1/2*b*x)^4*tan
(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + t
an(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/
2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*
tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*a)^4 - 4*d*tan(1/2*b*x)*tan(1/2
*a)^4 - 16*b*d*x*tan(1/2*b*x)*tan(1/2*a) + 4*d*tan(1/2*b*x)^3 - 16*b*c*tan(
1/2*b*x)*tan(1/2*a) - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*
x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/
2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*
tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2
*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a) + 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^
2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b
*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1...
```

Mupad [B]

time = 1.16, size = 151, normalized size = 2.70

$$e^{a\,1i+b\,x\,1i}\left(\frac{b\,c+d\,1i}{2\,b^2}+\frac{d\,x}{2\,b}\right)-e^{-a\,1i-b\,x\,1i}\left(\frac{-b\,c+d\,1i}{2\,b^2}-\frac{d\,x}{2\,b}\right)+\frac{d\,\ln\left(e^{a\,1i+b\,x\,1i}-i\right)}{b^2}-\frac{d\,\ln\left(e^{a\,1i+b\,x\,1i}+1i\right)}{b^2}+\frac{e^{a\,1i+b\,x\,1i}\,\left(c+d\,x\right)\,2i}{b\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x),x)
```

```
[Out] exp(a*1i + b*x*1i)*((d*1i + b*c)/(2*b^2) + (d*x)/(2*b)) - exp(- a*1i - b*x*
1i)*((d*1i - b*c)/(2*b^2) - (d*x)/(2*b)) + (d*log(exp(a*1i + b*x*1i) - 1i))
/b^2 - (d*log(exp(a*1i + b*x*1i) + 1i))/b^2 + (exp(a*1i + b*x*1i)*(c + d*x)
*2i)/(b*(exp(a*2i + b*x*2i)*1i + 1i))
```

3.263 $\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=76

$$-\frac{\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\text{Int}\left(\frac{\sec(a+bx)\tan(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)-cos(a-b*c/d)*Si(b*c/d+b*x)
/d-Ci(b*c/d+b*x)*sin(a-b*c/d)/d
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

```
[Out] -((CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d) - (Cos[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d + Defer[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*x
), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx &= -\int \frac{\sin(a+bx)}{c+dx}\,dx+\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx\\
&= -\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\right)-\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\\
&= -\frac{\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\int \frac{\sec(a+bx)\tan(}{c+dx}
\end{aligned}$$

Mathematica [A]
time = 4.07, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x), x]

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sin \left(bx + a\right) \left(\tan^{2}\left(bx + a\right)\right)}{dx + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(1, -(
I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*(exp_integral_e(1, (I*b*d*x
+ I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d)
+ (b*c*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(1, -(I*
b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*(exp_integral_e(1, (I*b*d*x +
I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) +
(b*d*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(1, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d*(exp_integral_e(1, (I*b*d*x + I*
b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*
cos(2*b*x + 2*a)^2 + (b*c*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp
_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*(exp_integr
al_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin
(-(b*c - a*d)/d) + (b*d*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_i
ntegral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d*(exp_integral
_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-
(b*c - a*d)/d))*x)*sin(2*b*x + 2*a)^2 + 4*d*sin(2*b*x + 2*a)*sin(b*x + a) +
 (b*d*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(1, -(I*b
*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d*(exp_integral_e(1, (I*b*d*x + I
*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x
- 2*(b*c*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(1, -
(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(1, (I*b*d*x
 + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d)
 + (b*d*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(1, -(
I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integral_e(1, (I*b*d*x
+ I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))
```

```
*x - 2*d*cos(b*x + a))*cos(2*b*x + 2*a) + 4*d*cos(b*x + a) + 4*(b*d^3*x + b
*c*d^2 + (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x + b*c*d^2)*sin(2
*b*x + 2*a)^2 + 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*integrate((cos(2*b*
x + 2*a)*cos(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d^
2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a
)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x^2 + 2
*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)), x))/(b*d^2*x + b*c*d + (b*d^2*x + b*c*
d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x +
 b*c*d)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")
[Out] integral(sin(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)**2/(d*x+c),x)
[Out] Integral(sin(a + b*x)*tan(a + b*x)**2/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(sin(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)\,{\tan(a+b\,x)}^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x),x)
[Out] int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x), x)
```

3.264 $\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=94

$$-\frac{b\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\sin(a+bx)}{d(c+dx)}+\frac{b\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\operatorname{Int}\left(\frac{\sec(a+bx)\tan(a+bx}{(c+dx)^2}\right.$$

```
[Out] CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)-b*Ci(b*c/d+b*x)*cos(a-b*
c/d)/d^2+b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2+sin(b*x+a)/d/(d*x+c)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -((b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2) + Sin[a + b*x]/(d*(c
 + d*x)) + (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int]
[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx &= -\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx+\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(a+bx)}{d(c+dx)}-\frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{d}+\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(a+bx)}{d(c+dx)}-\frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+b\right.}{c+dx}}{d}\\
&= -\frac{b\cos\left(a-\frac{bc}{d}\right)\operatorname{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\sin(a+bx)}{d(c+dx)}+\frac{b\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+
\end{aligned}$$

Mathematica [A]
time = 4.40, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2, x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sin \left(bx + a\right) \left(\tan^{2}\left(bx + a\right)\right)}{\left(dx + c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(2, -(
I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*(exp_integral_e(2, (I*b*d*x
+ I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d)
+ (b*c*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(2, -(I*
b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*(exp_integral_e(2, (I*b*d*x +
I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) +
(b*d*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(2, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d*(exp_integral_e(2, (I*b*d*x + I*
b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*
cos(2*b*x + 2*a)^2 + (b*c*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp
_integral_e(2, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*(exp_integr
al_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin
(-(b*c - a*d)/d) + (b*d*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_i
ntegral_e(2, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d*(exp_integral
_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-
(b*c - a*d)/d))*x)*sin(2*b*x + 2*a)^2 + 4*d*sin(2*b*x + 2*a)*sin(b*x + a) +
 (b*d*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(2, -(I*b
*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d*(exp_integral_e(2, (I*b*d*x + I
*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x
- 2*(b*c*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(2, -
(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(2, (I*b*d*x
```

```
 + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d)
 + (b*d*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(2, -(
I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integral_e(2, (I*b*d*x
+ I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))
*x - 2*d*cos(b*x + a))*cos(2*b*x + 2*a) + 4*d*cos(b*x + a) + 8*(b*d^4*x^2 +
 2*b*c*d^3*x + b*c^2*d^2 + (b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*cos(2*b*x
+ 2*a)^2 + (b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*sin(2*b*x + 2*a)^2 + 2*(b*
d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*cos(2*b*x + 2*a))*integrate((cos(2*b*x +
 2*a)*cos(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d^3*x
^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b
*c^2*d*x + b*c^3)*cos(2*b*x + 2*a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2
*d*x + b*c^3)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d
*x + b*c^3)*cos(2*b*x + 2*a)), x))/(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (b*
d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*c*d^
2*x + b*c^2*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*c
os(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(sin(b*x + a)*tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin{\left(a + bx \right)} \tan^{2}{\left(a + bx \right)}}{\left(c + dx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)**2/(d*x+c)**2,x)
[Out] Integral(sin(a + b*x)*tan(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)*tan(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)\,{\tan\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x)^2,x)
[Out] int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x)^2, x)
```

3.265 $\int (c+dx)^m \csc(a+bx) \sec^2(a+bx) \, dx$

Optimal. Leaf size=25

$$\text{Int}\left((c+dx)^m \csc(a+bx) \sec^2(a+bx), x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x)
```

Rubi [A]
time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \csc(a+bx) \sec^2(a+bx) \, dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2,x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \csc(a+bx) \sec^2(a+bx) \, dx = \int (c+dx)^m \csc(a+bx) \sec^2(a+bx) \, dx$$

Mathematica [A]
time = 22.43, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc(a+bx) \sec^2(a+bx) \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a) \left(\sec^2(bx+a)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x)
[Out] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc (a+bx) \sec^2 (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a)**2,x)
[Out] Integral((c + d*x)**m*csc(a + b*x)*sec(a + b*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos (a+b\,x)^2\,\sin (a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)),x)
[Out] int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)), x)
```

3.266 $\int (c+dx)^4 \csc(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=469

$$\frac{8id(c+dx)^3\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{4id(c+dx)^3\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{12id^2(c+\cdots}{}$$

```
[Out] -4*I*d*(d*x+c)^3*polylog(2,exp(I*(b*x+a)))/b^2-2*(d*x+c)^4*arctanh(exp(I*(b
*x+a)))/b+24*I*d^4*polylog(4,-I*exp(I*(b*x+a)))/b^5+12*I*d^2*(d*x+c)^2*poly
log(2,I*exp(I*(b*x+a)))/b^3+8*I*d*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b^2-12*I
*d^2*(d*x+c)^2*polylog(2,-I*exp(I*(b*x+a)))/b^3-12*d^2*(d*x+c)^2*polylog(3,
-exp(I*(b*x+a)))/b^3+24*d^3*(d*x+c)*polylog(3,-I*exp(I*(b*x+a)))/b^4-24*d^3
*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^4+12*d^2*(d*x+c)^2*polylog(3,exp(I*(
b*x+a)))/b^3-24*I*d^4*polylog(4,I*exp(I*(b*x+a)))/b^5+4*I*d*(d*x+c)^3*polyl
og(2,-exp(I*(b*x+a)))/b^2-24*I*d^3*(d*x+c)*polylog(4,-exp(I*(b*x+a)))/b^4+2
4*I*d^3*(d*x+c)*polylog(4,exp(I*(b*x+a)))/b^4+24*d^4*polylog(5,-exp(I*(b*x+
a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5+(d*x+c)^4*sec(b*x+a)/b
```

Rubi [A]
time = 0.52, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {2702, 327, 213, 4505, 6873, 12, 6874, 6408, 4268, 2611, 6744, 2320, 6724, 4266}

$$\frac{8id(c+dx)^3\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}+\frac{24id^4\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^5}-\frac{24id^4\text{Li}_4\left(ie^{i(a+bx)}\right)}{b^5}+\frac{24d^4\text{Li}_5\left(-e^{i(a+bx)}\right)}{b^5}-\frac{24d^4\text{Li}_5\left(e^{i(a+bx)}\right)}{b^5}+\frac{24d^3(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{24d^3(c+dx)\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}-\frac{24id^3(c+dx)\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}+\frac{24id^3(c+dx)\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}-\frac{12id^2(c+dx)^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{12id^2(c+dx)^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}-\frac{12d^2(c+dx)^2\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{12d^2(c+dx)^2\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}+\frac{4id(c+dx)^3\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{4id(c+dx)^3\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}+\frac{(c+dx)^4\sec(a+bx)}{b}-\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] ((8*I)*d*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^4*ArcTanh[
E^(I*(a + b*x))])/b + ((4*I)*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b^
2 - ((12*I)*d^2*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((12*I)
*d^2*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((4*I)*d*(c + d*x)^3*
PolyLog[2, E^(I*(a + b*x))])/b^2 - (12*d^2*(c + d*x)^2*PolyLog[3, -E^(I*(a
+ b*x))])/b^3 + (24*d^3*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (
24*d^3*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + (12*d^2*(c + d*x)^2*P
olyLog[3, E^(I*(a + b*x))])/b^3 - ((24*I)*d^3*(c + d*x)*PolyLog[4, -E^(I*(a
 + b*x))])/b^4 + ((24*I)*d^4*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^5 - ((24*I
)*d^4*PolyLog[4, I*E^(I*(a + b*x))])/b^5 + ((24*I)*d^3*(c + d*x)*PolyLog[4,
 E^(I*(a + b*x))])/b^4 + (24*d^4*PolyLog[5, -E^(I*(a + b*x))])/b^5 - (24*d^
4*PolyLog[5, E^(I*(a + b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^4 \csc(a+bx)\sec^2(a+bx)\,dx = -\frac{(c+dx)^4\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^4\sec(a+bx)}{b} - (4d)$$

$$= -\frac{(c+dx)^4\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^4\sec(a+bx)}{b} - (4d)$$

$$= -\frac{(c+dx)^4\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^4\sec(a+bx)}{b} - \frac{(4d)}{}$$

$$= -\frac{(c+dx)^4\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^4\sec(a+bx)}{b} - \frac{(4d)}{}$$

$$= -\frac{(c+dx)^4\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^4\sec(a+bx)}{b} + \frac{(4d)}{}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^4\sec(a+bx)}{b} + \frac{\int b(c+d}{}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\operatorname{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} +$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{12}{}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4i}{}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4i}{}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4i}{}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4i}{}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4i}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 998 vs. $2(469) = 938$.
time = 2.77, size = 998, normalized size = 2.13

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] (-2*b^4*c^4*ArcTanh[E^(I*(a + b*x))] + 4*b^4*c^3*d*x*Log[1 - E^(I*(a + b*x)
)] + 6*b^4*c^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] + 4*b^4*c*d^3*x^3*Log[1 - E
```

```
^(I*(a + b*x))] + b^4*d^4*x^4*Log[1 - E^(I*(a + b*x))] - 4*b^4*c^3*d*x*Log[
1 + E^(I*(a + b*x))] - 6*b^4*c^2*d^2*x^2*Log[1 + E^(I*(a + b*x))] - 4*b^4*c
*d^3*x^3*Log[1 + E^(I*(a + b*x))] - b^4*d^4*x^4*Log[1 + E^(I*(a + b*x))] +
(4*I)*b^3*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))] - (4*I)*b^3*d*(c + d*x
)^3*PolyLog[2, E^(I*(a + b*x))] - 12*b^2*c^2*d^2*PolyLog[3, -E^(I*(a + b*x)
)] - 24*b^2*c*d^3*x*PolyLog[3, -E^(I*(a + b*x))] - 12*b^2*d^4*x^2*PolyLog[3
, -E^(I*(a + b*x))] + 12*b^2*c^2*d^2*PolyLog[3, E^(I*(a + b*x))] + 24*b^2*c
*d^3*x*PolyLog[3, E^(I*(a + b*x))] + 12*b^2*d^4*x^2*PolyLog[3, E^(I*(a + b*
x))] - (24*I)*b*c*d^3*PolyLog[4, -E^(I*(a + b*x))] - (24*I)*b*d^4*x*PolyLog
[4, -E^(I*(a + b*x))] - 4*d*((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3
*c^2*d*x*Log[1 - I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b
*x))] + b^3*d^3*x^3*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^
(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*L
og[1 + I*E^(I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a
 + b*x))] - (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d
^2*PolyLog[3, (-I)*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b
*x))] + 6*b*c*d^2*PolyLog[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^
(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*Pol
yLog[4, I*E^(I*(a + b*x))]) + (24*I)*b*c*d^3*PolyLog[4, E^(I*(a + b*x))] +
(24*I)*b*d^4*x*PolyLog[4, E^(I*(a + b*x))] + 24*d^4*PolyLog[5, -E^(I*(a + b
*x))] - 24*d^4*PolyLog[5, E^(I*(a + b*x))] + b^4*(c + d*x)^4*Sec[a + b*x])/
b^5
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1865 vs. $2(422) = 844$.
time = 0.64, size = 1866, normalized size = 3.98

method	result	size
risch	Expression too large to display	1866

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3-4/b
*c*d^3*ln(exp(I*(b*x+a))+1)*x^3-12*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*
x-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2+12*I/b^2*c^2*d^2*polylog(2,-
exp(I*(b*x+a)))*x+12*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-24*I/b^4*c*
d^3*polylog(2,-I*exp(I*(b*x+a)))*a+24*I/b^4*c*d^3*polylog(2,I*exp(I*(b*x+a)
))*a+24*I/b^4*a*c*d^3*dilog(1+I*exp(I*(b*x+a)))-24*I/b^4*a*c*d^3*dilog(1-I*
exp(I*(b*x+a)))+2*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x^2+4*c^3*d*x+c^4)*exp(I*(
b*x+a))/b/(1+exp(2*I*(b*x+a)))-4/b^2*c^3*d*a*ln(exp(I*(b*x+a))-1)-4/b^4*c*d
^3*a^3*ln(exp(I*(b*x+a))-1)-4*I/b^2*d^4*polylog(2,exp(I*(b*x+a)))*x^3+24*I/
b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-4*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))
+24*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))+4/b^2*d^4*ln(1+I*exp(I*(b*x+a)))*
x^3-4/b^2*d^4*ln(1-I*exp(I*(b*x+a)))*x^3+24/b^4*d^3*c*polylog(3,-I*exp(I*(b
```

```
*x+a)))-4/b^5*d^4*a^3*ln(1-I*exp(I*(b*x+a)))+4/b^5*d^4*a^3*ln(1+I*exp(I*(b*
x+a)))+24/b^4*d^4*polylog(3,-I*exp(I*(b*x+a)))*x-24/b^4*d^3*c*polylog(3,I*e
xp(I*(b*x+a)))-24/b^4*d^4*polylog(3,I*exp(I*(b*x+a)))*x+24*d^4*polylog(5,-e
xp(I*(b*x+a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5+1/b*c^4*ln(exp(I*(b
*x+a))-1)-1/b*c^4*ln(exp(I*(b*x+a))+1)+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+
a)))-12/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a)))+1/b^5*d^4*a^4*ln(exp(I*(b*x+
a))-1)-12/b^2*d^3*c*ln(1-I*exp(I*(b*x+a)))*x^2+12/b^2*d^3*c*ln(1+I*exp(I*(b
*x+a)))*x^2+12/b^2*d^2*c^2*ln(1+I*exp(I*(b*x+a)))*x+12/b^3*d^2*c^2*ln(1+I*e
xp(I*(b*x+a)))*a+12/b^4*d^3*a^2*c*ln(1-I*exp(I*(b*x+a)))-12/b^4*d^3*a^2*c*l
n(1+I*exp(I*(b*x+a)))-12/b^2*d^2*c^2*ln(1-I*exp(I*(b*x+a)))*x-12/b^3*d^2*c^
2*ln(1-I*exp(I*(b*x+a)))*a-8*I/b^5*d^4*a^3*arctan(exp(I*(b*x+a)))-12*I/b^3*
d^4*polylog(2,-I*exp(I*(b*x+a)))*x^2+12*I/b^3*d^4*polylog(2,I*exp(I*(b*x+a)
))*x^2+8*I/b^2*d*c^3*arctan(exp(I*(b*x+a)))-1/b^5*d^4*a^4*ln(1-exp(I*(b*x+a
)))-12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2+12/b^3*d^4*polylog(3,exp(I*(b
*x+a)))*x^2-12*I/b^5*a^2*d^4*polylog(2,I*exp(I*(b*x+a)))-12*I/b^3*d^2*c^2*d
ilog(1+I*exp(I*(b*x+a)))+12*I/b^3*d^2*c^2*dilog(1-I*exp(I*(b*x+a)))-12*I/b^
5*a^2*d^4*dilog(1+I*exp(I*(b*x+a)))+12*I/b^5*a^2*d^4*dilog(1-I*exp(I*(b*x+a
)))+12*I/b^5*a^2*d^4*polylog(2,-I*exp(I*(b*x+a)))+24*I*d^4*polylog(4,-I*exp
(I*(b*x+a)))/b^5+24*I/b^3*d^3*c*polylog(2,I*exp(I*(b*x+a)))*x-24*I/b^3*d^3*
c*polylog(2,-I*exp(I*(b*x+a)))*x+24*I/b^4*d^3*c*a^2*arctan(exp(I*(b*x+a)))-
24*I/b^3*d^2*c^2*a*arctan(exp(I*(b*x+a)))-24*I*d^4*polylog(4,I*exp(I*(b*x+a
)))/b^5+4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3-24*I/b^4*d^4*polylog(4,-
exp(I*(b*x+a)))*x-24*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))+4*I/b^2*c^3*d*p
olylog(2,-exp(I*(b*x+a)))+4/b*c^3*d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c^3*d*ln(1
-exp(I*(b*x+a)))*a-4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x+1/b*d^4*ln(1-exp(I*(b*x
+a)))*x^4-1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4+6/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+
a))-1)-6/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2+6/b*c^2*d^2*ln(1-exp(I*(b*x+a))
)*x^2-6/b^3*c^2*d^2*ln(1-exp(I*(b*x+a)))*a^2-24/b^3*c*d^3*polylog(3,-exp(I*
(b*x+a)))*x+24/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5709 vs. $2(403) = 806$.
time = 2.17, size = 5709, normalized size = 12.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^4*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 4*a*c^3*d*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + 6*a^2*c^2*d^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x +
a) - 1))/b^2 - 4*a^3*c*d^3*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(co
s(b*x + a) - 1))/b^3 + a^4*d^4*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + lo
g(cos(b*x + a) - 1))/b^4 + 2*(8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^
```

3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^
2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + (b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3
*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2
+ 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (I*
b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 + I*(b*x + a)^3*d^4 - I*a^3
d^4 + 3(I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d
^3 + I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x
+ a) + 1) + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^
4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^
3 + a^2*d^4)*(b*x + a) + (b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*
x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 -
2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (I*b^3*c^3*d - 3*I*a*
b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 + I*(b*x + a)^3*d^4 - I*a^3*d^4 + 3*(I*b*c*d^
3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b
*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 2*((b
*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*
d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3
- a^3*d^4)*(b*x + a) + ((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3
+ 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*
b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b
*x + a)^4*d^4 + 4*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c^2*d^2 +
2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a)^2 + 4*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^
2 - 3*I*a^2*b*c*d^3 + I*a^3*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b
*x + a), cos(b*x + a) + 1) - 2*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x
+ a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d
- 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a) + ((b*x + a)^4*d^4
+ 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b
*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^4*d^4 + 4*(-I*b*c*d^3 + I*a*d^4)*
(b*x + a)^3 + 6*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a)^2 +
4*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b*c*d^3 + I*a^3*d^4)*(b*x + a
))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 4*(-I*(b*x
+ a)^4*d^4 + 4*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c^2*d^2 + 2*I
*a*b*c*d^3 - I*a^2*d^4)*(b*x + a)^2 + 4*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 -
3*I*a^2*b*c*d^3 + I*a^3*d^4)*(b*x + a))*cos(b*x + a) + 24*(b^2*c^2*d^2 - 2
*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a) + (b
^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*
(b*x + a))*cos(2*b*x + 2*a) + (I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*(b*x + a)^
2*d^4 + I*a^2*d^4 + 2*(I*b*c*d^3 - I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*di
log(I*e^(I*b*x + I*a)) - 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 +
a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a) + (b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x
+ a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (
-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*(b*x + a)^2*d^4 - I*a^2*d^4 + 2*(-I*b*c*
d^3 + I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) + 8*(
b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3

```
*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b
*x + a) + (b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 -
a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3
*I*a^2*b*c*d^3 + I*(b*x + a)^3*d^4 - I*a^3*d^4 + 3*(I*b*c*d^3 - I*a*d^4)*(b
*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a))*sin(2*
b*x + 2*a))*dilog(-e^(I*b*x + I*a)) - 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^
2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3
*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + (b^3*c^3*d - 3*a*b^2*c^2
*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x
 + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*
a) - (-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b*c*d^3 - I*(b*x + a)^3*d^
4 + I*a^3*d^4 + 3*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^2 + 3*(-I*b^2*c^2*d^2 +
2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*
a)) - (-I*(b*x + a)^4*d^4 - 4*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^3 - 6*(I*b^2*
c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a)^...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2519 vs. $2(403) = 806$.
time = 2.49, size = 2519, normalized size = 5.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x +
 2*b^4*c^4 - 24*d^4*cos(b*x + a)*polylog(5, cos(b*x + a) + I*sin(b*x + a))
- 24*d^4*cos(b*x + a)*polylog(5, cos(b*x + a) - I*sin(b*x + a)) + 24*d^4*co
s(b*x + a)*polylog(5, -cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*cos(b*x + a)
*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) - 24*I*d^4*cos(b*x + a)*polylog
(4, I*cos(b*x + a) + sin(b*x + a)) - 24*I*d^4*cos(b*x + a)*polylog(4, I*cos
(b*x + a) - sin(b*x + a)) + 24*I*d^4*cos(b*x + a)*polylog(4, -I*cos(b*x + a
) + sin(b*x + a)) + 24*I*d^4*cos(b*x + a)*polylog(4, -I*cos(b*x + a) - sin(
b*x + a)) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^
3*c^3*d)*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 4*(-I*b^3*d^4*
x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*cos(b*x + a)*dil
og(cos(b*x + a) - I*sin(b*x + a)) - 12*(-I*b^2*d^4*x^2 - 2*I*b^2*c*d^3*x -
I*b^2*c^2*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) - 12*(-I*b
^2*d^4*x^2 - 2*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*cos(b*x + a)*dilog(I*cos(b*x
+ a) - sin(b*x + a)) - 12*(I*b^2*d^4*x^2 + 2*I*b^2*c*d^3*x + I*b^2*c^2*d^2)
*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 12*(I*b^2*d^4*x^2 + 2
*I*b^2*c*d^3*x + I*b^2*c^2*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*
x + a)) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*
c^3*d)*cos(b*x + a)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 4*(-I*b^3*d^4*x
```

```
^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*cos(b*x + a)*dilo
g(-cos(b*x + a) - I*sin(b*x + a)) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*
c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*cos(b*x + a)*log(cos(b*x + a) + I*si
n(b*x + a) + 1) - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)
*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) - (b^4*d^4*x^4 + 4*b^4
*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*cos(b*x + a)*log(
cos(b*x + a) - I*sin(b*x + a) + 1) + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2
*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I) - 4
*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2
*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 4
*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2
*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 4
*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2
*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) +
4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^
2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) +
 (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*co
s(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*cos(b*x + a)*lo
g(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^4*d^4*x^4 + 4*b^4*c*d^
3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d
^2 + 4*a^3*b*c*d^3 - a^4*d^4)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x +
a) + 1) - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x
 + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*
x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2
 + 4*a^3*b*c*d^3 - a^4*d^4)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a)
 + 1) + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x +
 a)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 24*(-I*b*d^4*x - I*b*c*d^3)*c
os(b*x + a)*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 24*(I*b*d^4*x + I*b
*c*d^3)*cos(b*x + a)*polylog(4, cos(b*x + a) - I*sin(b*x + a)) - 24*(-I*b*d
^4*x - I*b*c*d^3)*cos(b*x + a)*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) -
 24*(I*b*d^4*x + I*b*c*d^3)*cos(b*x + a)*polylog(4, -cos(b*x + a) - I*sin(b
*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)*poly
log(3, cos(b*x + a) + I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b
^2*c^2*d^2)*cos(b*x + a)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) + 24*(b*
d^4*x + b*c*d^3)*cos(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2
4*(b*d^4*x + b*c*d^3)*cos(b*x + a)*polylog(3, I*cos(b*x + a) - sin(b*x + a)
) + 24*(b*d^4*x + b*c*d^3)*cos(b*x + a)*polylog(3, -I*cos(b*x + a) + sin(b*
x + a)) - 24*(b*d^4*x + b*c*d^3)*cos(b*x + a)*polylog(3, -I*cos(b*x + a) -
sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)
*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3
*x + b^2*c^2*d^2)*cos(b*x + a)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/
(b^5*cos(b*x + a))
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*csc(b*x + a)*sec(b*x + a)^2, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^4/(cos(a + b*x)^2*sin(a + b*x)),x)
```

```
[Out] \text{Hanged}
```

3.267 $\int (c+dx)^3 \csc(a+bx) \sec^2(a+bx) \, dx$

Optimal. Leaf size=343

$$\frac{6id(c+dx)^2 \text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2 \text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{6id^2(c+d$$

```
[Out] 6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2-2*(d*x+c)^3*arctanh(exp(I*(b*x+a
)))/b+3*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)*polylo
g(2,-I*exp(I*(b*x+a)))/b^3+6*I*d^2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^3-
3*I*d*(d*x+c)^2*polylog(2,exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*polylog(3,-exp(
I*(b*x+a)))/b^3+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,I*ex
p(I*(b*x+a)))/b^4+6*d^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-6*I*d^3*polyl
og(4,-exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+(d*x+c)^3*s
ec(b*x+a)/b
```

Rubi [A]
time = 0.37, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {2702, 327, 213, 4505, 6873, 12, 6874, 6408, 4268, 2611, 6744, 2320, 6724, 4266}

$$\frac{6id(c+dx)^2 \text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}+\frac{6d^3 \text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3 \text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}-\frac{6id^3 \text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}+\frac{6id^3 \text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}-\frac{6id^2(c+dx) \text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx) \text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}-\frac{6d^2(c+dx) \text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx) \text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2 \text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)^2 \text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}+\frac{(c+dx)^3 \sec(a+bx)}{b}-\frac{2(c+dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] ((6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^3*ArcTanh[
E^(I*(a + b*x))])/b + ((3*I)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^
2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*d^2
*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d*(c + d*x)^2*PolyLo
g[2, E^(I*(a + b*x))])/b^2 - (6*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])
/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLog[3, I*E
^(I*(a + b*x))])/b^4 + (6*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 -
((6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, E^(I*(
a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
```

```
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^3 \csc(a+bx)\sec^2(a+bx)\,dx = -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^3\sec(a+bx)}{b} - (3d)$$

$$= -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^3\sec(a+bx)}{b} - (3d)$$

$$= -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^3\sec(a+bx)}{b} - \frac{(3d)}{}$$

$$= -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^3\sec(a+bx)}{b} - \frac{(3d)}{}$$

$$= -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^3\sec(a+bx)}{b} + \frac{(3d)}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^3\sec(a+bx)}{b} - \frac{\int b(-c-}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6id^2(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{6i}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6i}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3i}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3i}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3i}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3i}{}$$

Mathematica [A]
time = 1.65, size = 616, normalized size = 1.80

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] (-2*b^3*c^3*ArcTanh[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - E^(I*(a + b*x)
)] + 3*b^3*c*d^2*x^2*Log[1 - E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1 - E^(I*(a
 + b*x))] - 3*b^3*c^2*d*x*Log[1 + E^(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1
+ E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + E^(I*(a + b*x))] + (3*I)*b^2*d*(c
+ d*x)^2*PolyLog[2, -E^(I*(a + b*x))] - (3*I)*b^2*d*(c + d*x)^2*PolyLog[2,
E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, -E^(I*(a + b*x))] - 6*b*d^3*x*PolyL
```

```
og[3, -E^(I*(a + b*x))] - 3*d*((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b
^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))
] - 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a
+ b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d
*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a +
b*x))] + 2*d^2*PolyLog[3, I*E^(I*(a + b*x))]) + 6*b*c*d^2*PolyLog[3, E^(I*(
a + b*x))] + 6*b*d^3*x*PolyLog[3, E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, -
E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, E^(I*(a + b*x))] + b^3*(c + d*x)^3*
Sec[a + b*x])/b^4
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1151 vs. $2(308) = 616$.
time = 0.51, size = 1152, normalized size = 3.36

method	result	size
risch	Expression too large to display	1152

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))-6
/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))-6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*
x+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-1/b*c^3*ln(exp(I*(b*x+a))+1)+1/b*c^
3*ln(exp(I*(b*x+a))-1)-12*I/b^3*d^2*c*a*arctan(exp(I*(b*x+a)))-6*I/b^2*c*d^
2*polylog(2,exp(I*(b*x+a)))*x+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*d^3*
polylog(3,I*exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+3/b^2
*d^3*ln(1+I*exp(I*(b*x+a)))*x^2-3/b^2*d^3*ln(1-I*exp(I*(b*x+a)))*x^2+3/b^4*
d^3*a^2*ln(1-I*exp(I*(b*x+a)))-3/b^4*d^3*a^2*ln(1+I*exp(I*(b*x+a)))-6*I*d^3
*x*polylog(2,-I*exp(I*(b*x+a)))/b^3+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b
^3-1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*
d^3*ln(1-exp(I*(b*x+a)))*a^3+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)+2*exp(I*(
b*x+a))*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(1+exp(2*I*(b*x+a)))-3/b*c^2*
d*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-
exp(I*(b*x+a)))*a-3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b*c*d^2*ln(1-exp(I*(
b*x+a)))*x^2-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-3/b^2*c^2*d*a*ln(exp(I*(b
*x+a))-1)-3*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,exp
(I*(b*x+a)))*x^2+3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))+3*I/b^2*d^3*polyl
og(2,-exp(I*(b*x+a)))*x^2+6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x-6*I*d^
3*polylog(4,-exp(I*(b*x+a)))/b^4+6/b^3*d^2*c*ln(1+I*exp(I*(b*x+a)))*a+6/b^2
*d^2*c*ln(1+I*exp(I*(b*x+a)))*x-6/b^3*d^2*c*ln(1-I*exp(I*(b*x+a)))*a-6/b^2*
d^2*c*ln(1-I*exp(I*(b*x+a)))*x+6*I/b^2*d*c^2*arctan(exp(I*(b*x+a)))+6*I/b^4
*d^3*a^2*arctan(exp(I*(b*x+a)))-6*I/b^3*c*d^2*dilog(1+I*exp(I*(b*x+a)))+6*I
/b^3*c*d^2*dilog(1-I*exp(I*(b*x+a)))-6*I/b^4*d^3*polylog(2,-I*exp(I*(b*x+a)
))*a+6*I/b^4*d^3*polylog(2,I*exp(I*(b*x+a)))*a+6*I/b^4*a*d^3*dilog(1+I*exp(
I*(b*x+a)))-6*I/b^4*a*d^3*dilog(1-I*exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3202 vs. 2(293) = 586.
time = 0.99, size = 3202, normalized size = 9.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^3*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 3*a*c^2*d*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + 3*a^2*c*d^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a)
 - 1))/b^2 - a^3*d^3*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x
+ a) - 1))/b^3 + 2*(6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3
+ 2*(b*c*d^2 - a*d^3)*(b*x + a) + (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^
3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (I*b^2*c^2*
d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)
*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + 6*(
b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(
b*x + a) + (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^
2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(
b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(2*b*x +
 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 2*((b*x + a)^3*d^3 + 3*(b
*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x +
a) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^3*d^3 + 3*
(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^
2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1)
 - 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*(b*x + a) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*
x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a
) - (-I*(b*x + a)^3*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*
c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin
(b*x + a), -cos(b*x + a) + 1) + 4*(-I*(b*x + a)^3*d^3 + 3*(-I*b*c*d^2 + I*a
*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a))
*cos(b*x + a) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3 + (b*c*d^2 + (b*x + a)*
d^3 - a*d^3)*cos(2*b*x + 2*a) + (I*b*c*d^2 + I*(b*x + a)*d^3 - I*a*d^3)*sin
(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d
^3 + (b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (-I*b*c*d^2 - I*(
b*x + a)*d^3 + I*a*d^3)*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) + 6*(b^
2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*
x + a) + (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2
- a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*
x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(2*b*x + 2
*a))*dilog(-e^(I*b*x + I*a)) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3
```

```
 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) + (b^2*c^2*d - 2*a*b*c*d^2 + (b*
x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) -
(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d
^2 + I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-I*(b*
x + a)^3*d^3 - 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 - 3*(I*b^2*c^2*d - 2*I*a
*b*c*d^2 + I*a^2*d^3)*(b*x + a) + (-I*(b*x + a)^3*d^3 - 3*(I*b*c*d^2 - I*a*
d^3)*(b*x + a)^2 - 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b*x + a))*c
os(2*b*x + 2*a) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b
^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x
+ a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (I*(b*x + a)^3*d^3 - 3*(-I*
b*c*d^2 + I*a*d^3)*(b*x + a)^2 - 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^
3)*(b*x + a) + (I*(b*x + a)^3*d^3 - 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 -
3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) -
((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c
*d^2 + a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
 a)^2 - 2*cos(b*x + a) + 1) + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^
2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a) + (I*b^2*c^2*d - 2*I*
a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x +
a))*cos(2*b*x + 2*a) - (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3
 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + si
n(b*x + a)^2 + 2*sin(b*x + a) + 1) + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b
*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a) + (-I*b^2*c^
2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d
^3)*(b*x + a))*cos(2*b*x + 2*a) + (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^
3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x
+ a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - 12*(d^3*cos(2*b*x + 2*a) +
I*d^3*sin(2*b*x + 2*a) + d^3)*polylog(4, -e^(I*b*x + I*a)) + 12*(d^3*cos(2*
b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) + d^3)*polylog(4, e^(I*b*x + I*a)) + 12
*(I*d^3*cos(2*b*x + 2*a) - d^3*sin(2*b*x + 2*a) + I*d^3)*polylog(3, I*e^(I*
b*x + I*a)) + 12*(-I*d^3*cos(2*b*x + 2*a) + d^3*sin(2*b*x + 2*a) - I*d^3)*p
olylog(3, -I*e^(I*b*x + I*a)) + 12*(I*b*c*d^2 +...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1705 vs. $2(293) = 586$.
time = 2.64, size = 1705, normalized size = 4.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*I*d^3*
cos(b*x + a)*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*cos(b*x +
a)*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*cos(b*x + a)*polylog
(4, -cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*cos(b*x + a)*polylog(4, -cos(
```

```
b*x + a) - I*sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) +
 sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) - sin(b*x + a
)) + 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*
cos(b*x + a)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*d^3*x^2
+ 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(b*
x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*cos(b*x + a)*d
ilog(cos(b*x + a) - I*sin(b*x + a)) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos(b*x +
a)*dilog(I*cos(b*x + a) + sin(b*x + a)) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos(b*
x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 6*(I*b*d^3*x + I*b*c*d^2)*cos
(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 6*(I*b*d^3*x + I*b*c*d^2)
*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*
I*b^2*c*d^2*x + I*b^2*c^2*d)*cos(b*x + a)*dilog(-cos(b*x + a) + I*sin(b*x +
 a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*cos(b*x + a)*dilo
g(-cos(b*x + a) - I*sin(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*
c^2*d*x + b^3*c^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b
*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*co
s(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 3*(b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I) - 3*(b^2
*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*
x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 -
 a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 3*(b^2*d^3*
x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x +
a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2
*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)*log(-1/2*cos(b*x + a) +
1/2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^3*
d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
 a^3*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b^2*c^2
*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a)
 + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*
a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)
 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) - I
*sin(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)*polylog(3, cos(b*x
+ a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)*polylog(3, cos(
b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)*polylog(3,
-cos(b*x + a) + I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)*polylo
g(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^4*cos(b*x + a))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \csc(a + bx) \sec^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*csc(a + b*x)*sec(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a)^2, x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)),x)
```

```
[Out] \text{Hanged}
```

3.268 $\int (c+dx)^2 \csc(a+bx) \sec^2(a+bx) \, dx$

Optimal. Leaf size=219

$$\frac{4id(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{2id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{2id^2\text{PolyLo}\ldots}{}$$

```
[Out] 4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2-2*(d*x+c)^2*arctanh(exp(I*(b*x+a))
)/b+2*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*d^2*polylog(2,-I*exp(I
*(b*x+a)))/b^3+2*I*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3-2*I*d*(d*x+c)*polylo
g(2,exp(I*(b*x+a)))/b^2-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(
3,exp(I*(b*x+a)))/b^3+(d*x+c)^2*sec(b*x+a)/b
```

Rubi [A]

time = 0.25, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {2702, 327, 213, 4505, 6873, 12, 6874, 6408, 4268, 2611, 2320, 6724, 4266, 2317, 2438}

$$\frac{4id(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2id^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}-\frac{2d^2\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}+\frac{2id(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\sec(a+bx)}{b}-\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] ((4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^2*ArcTanh[E^
(I*(a + b*x))])/b + ((2*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 -
((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((2*I)*d^2*PolyLog[2, I*
E^(I*(a + b*x))])/b^3 - ((2*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2
 - (2*d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, E^(I*(a + b
*x))])/b^3 + ((c + d*x)^2*Sec[a + b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx = -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} - (2d)\int$$
$$= -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} - (2d)\int$$
$$= -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{(2d)\int(}{}$$
$$= -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{(2d)\int}{}$$
$$= -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} + \frac{(2d)\int(}{}$$
$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\sec(a+bx)}{b} + \frac{\int b(c+dx)}{}$$
$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{(2id^2)\,\text{Subs}}{}$$
$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2id^2\text{I}}{}$$
$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c}{}$$
$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c}{}$$
$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c}{}$$

Mathematica [A]
time = 4.07, size = 379, normalized size = 1.73

$$\frac{-2b^2c^2\tanh^{-1}\left(e^{i(a+bx)}\right) - 4bcd\tanh^{-1}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right) - 4d^2\text{ArcTan}(\cot(a))\tanh^{-1}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right) + 2b^2cdx\log\left(1-e^{i(a+bx)}\right) + b^2d^2x^2\log\left(1-e^{i(a+bx)}\right) - 2b^2cdx\log\left(1+e^{i(a+bx)}\right) - b^2d^2x^2\log\left(1+e^{i(a+bx)}\right) + 2ibd(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right) - 2ibd(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right) + \frac{2d^2\csc(a)\left((bx-\text{ArcTan}(\cot(a)))\left(\log\left(1-e^{i(bx-\text{ArcTan}(\cot(a)))}\right)-\log\left(1+e^{i(bx-\text{ArcTan}(\cot(a)))}\right)\right)+i\text{PolyLog}\left(2,-e^{i(bx-\text{ArcTan}(\cot(a)))}\right)-i\text{PolyLog}\left(2,e^{i(bx-\text{ArcTan}(\cot(a)))}\right)\right)}{\sqrt{\csc^2(a)}} - 2d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right) + 2d^2\text{PolyLog}\left(3,e^{i(a+bx)}\right) + b^2(c+dx)^2\sec(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] (-2*b^2*c^2*ArcTanh[E^(I*(a + b*x))] - 4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[
(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + 2*
b^2*c*d*x*Log[1 - E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] -
 2*b^2*c*d*x*Log[1 + E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + E^(I*(a + b*x))
] + (2*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*
PolyLog[2, E^(I*(a + b*x))] + (2*d^2*Csc[a]*((b*x - ArcTan[Cot[a]])*(Log[1
- E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a]]))]) +
I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2, E^(I*(b*x - ArcT
```

```
an[Cot[a]]))]))/Sqrt[Csc[a]^2] - 2*d^2*PolyLog[3, -E^(I*(a + b*x))] + 2*d^2
*PolyLog[3, E^(I*(a + b*x))] + b^2*(c + d*x)^2*Sec[a + b*x])/b^3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(196) = 392$.
time = 0.28, size = 568, normalized size = 2.59

method	result
risch	$-\frac{2cda\ln(\mathrm{e}^{i(bx+a)}-1)}{b^2}-\frac{2id^2\,\mathrm{polylog}(2,\mathrm{e}^{i(bx+a)})x}{b^2}+\frac{2cd\ln(1-\mathrm{e}^{i(bx+a)})a}{b^2}-\frac{2cd\ln(\mathrm{e}^{i(bx+a)}+1)x}{b}+\frac{2cd\ln(1-\mathrm{e}^{i(bx+a)})x}{b}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d
^2*ln(1-exp(I*(b*x+a)))*a^2-1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+2*exp(I*(b*x+a
))*(d^2*x^2+2*c*d*x+c^2)/b/(1+exp(2*I*(b*x+a)))+2/b^2*c*d*ln(1-exp(I*(b*x+a
)))*a-2/b*c*d*ln(exp(I*(b*x+a))+1)*x+2/b*c*d*ln(1-exp(I*(b*x+a)))*x-2/b^2*c
*d*a*ln(exp(I*(b*x+a))-1)-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*c
*d*polylog(2,exp(I*(b*x+a)))+1/b*c^2*ln(exp(I*(b*x+a))-1)-1/b*c^2*ln(exp(I*
(b*x+a))+1)-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x
+a)))/b^3+2*d^2/b^2*ln(1+I*exp(I*(b*x+a)))*x+2*d^2/b^3*ln(1+I*exp(I*(b*x+a)
))*a-2*d^2/b^2*ln(1-I*exp(I*(b*x+a)))*x-2*d^2/b^3*ln(1-I*exp(I*(b*x+a)))*a-
2*I*d^2/b^3*dilog(1+I*exp(I*(b*x+a)))+2*I*d^2/b^3*dilog(1-I*exp(I*(b*x+a)))
+4*I*d/b^2*c*arctan(exp(I*(b*x+a)))-4*I*d^2/b^3*a*arctan(exp(I*(b*x+a)))+2*
I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)
))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1590 vs. $2(185) = 370$.
time = 0.64, size = 1590, normalized size = 7.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 2*a*c*d*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
 + a^2*d^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b^2 + 2*(4*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2
)*cos(2*b*x + 2*a) + (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(2*b*x + 2*a)
)*arctan2(cos(b*x + a), sin(b*x + a) + 1) + 4*(b*c*d + (b*x + a)*d^2 - a*d^
2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (I*b*c*d + I*(b*x +
a)*d^2 - I*a*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1
) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + ((b*x + a)^2*d^2 + 2
```

```
*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^2 + 2*(-I*
b*c*d + I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x
 + a) + 1) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + ((b*x + a)^
2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^2
 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a)
, -cos(b*x + a) + 1) + 4*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x
+ a))*cos(b*x + a) + 4*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) + d^2
)*dilog(I*e^(I*b*x + I*a)) - 4*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*
a) + d^2)*dilog(-I*e^(I*b*x + I*a)) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b
*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (I*b*c*d + I*(b*x + a)*d^2
 - I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) - 4*(b*c*d + (b*x + a
)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-I*b*c*
d - I*(b*x + a)*d^2 + I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (
-I*(b*x + a)^2*d^2 - 2*(I*b*c*d - I*a*d^2)*(b*x + a) + (-I*(b*x + a)^2*d^2
- 2*(I*b*c*d - I*a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*
(b*c*d - a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
 a)^2 + 2*cos(b*x + a) + 1) - (I*(b*x + a)^2*d^2 - 2*(-I*b*c*d + I*a*d^2)*(
b*x + a) + (I*(b*x + a)^2*d^2 - 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*cos(2*b*x
 + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(2*b*x + 2*a))
*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 2*(I*b*c*d + I
*(b*x + a)*d^2 - I*a*d^2 + (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*cos(2*b*x
+ 2*a) - (b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + 2*(-I*b*c*d - I*(b*x + a)*d^2 +
 I*a*d^2 + (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*cos(2*b*x + 2*a) + (b*c*d
 + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
a)^2 - 2*sin(b*x + a) + 1) + 4*(I*d^2*cos(2*b*x + 2*a) - d^2*sin(2*b*x + 2*
a) + I*d^2)*polylog(3, -e^(I*b*x + I*a)) + 4*(-I*d^2*cos(2*b*x + 2*a) + d^2
*sin(2*b*x + 2*a) - I*d^2)*polylog(3, e^(I*b*x + I*a)) + 4*((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(b*x + a))/(-2*I*b^2*cos(2*b*x + 2*a) +
2*b^2*sin(2*b*x + 2*a) - 2*I*b^2))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1035 vs. $2(185) = 370$.
time = 1.15, size = 1035, normalized size = 4.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*I*d^2*cos(b*x + a)*dilog(I
*cos(b*x + a) + sin(b*x + a)) + 2*I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) -
 sin(b*x + a)) - 2*I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a))
 - 2*I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b
*x + a)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)*poly
```

```
log(3, cos(b*x + a) - I*sin(b*x + a)) - 2*d^2*cos(b*x + a)*polylog(3, -cos(
b*x + a) + I*sin(b*x + a)) - 2*d^2*cos(b*x + a)*polylog(3, -cos(b*x + a) -
I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*cos(b*x + a)*dilog(cos(b*x + a) +
 I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*cos(b*x + a)*dilog(cos(b*x + a)
 - I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*cos(b*x + a)*dilog(-cos(b*x +
a) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*cos(b*x + a)*dilog(-cos(b*x
 + a) - I*sin(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a
)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b*c*d - a*d^2)*cos(b*x + a)*l
og(cos(b*x + a) + I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 2*(b*c*d - a*d^2)*
cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I) - 2*(b*d^2*x + a*d^2)*c
os(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b*d^2*x + a*d^2)*co
s(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 2*(b*d^2*x + a*d^2)*cos
(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b*d^2*x + a*d^2)*cos
(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/2*I*s
in(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(
b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b*c*d - a*d^2)*cos(b*
x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + 1
) + 2*(b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I))
/(b^3*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*csc(a + b*x)*sec(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)*sec(b*x + a)^2, x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)),x)
[Out] \text{Hanged}
```

3.269 $\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=113

$$-\frac{2dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{c\tanh^{-1}(\cos(a+bx))}{b}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyL}}{}$$

```
[Out] -2*d*x*arctanh(exp(I*(b*x+a)))/b-c*arctanh(cos(b*x+a))/b-d*arctanh(sin(b*x+
a))/b^2+I*d*polylog(2,-exp(I*(b*x+a)))/b^2-I*d*polylog(2,exp(I*(b*x+a)))/b^
2+c*sec(b*x+a)/b+d*x*sec(b*x+a)/b
```

Rubi [A]
time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2702, 327, 213, 4505, 6406, 12, 4268, 2317, 2438, 3855}

$$\frac{id\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{(c+dx)\sec(a+bx)}{b}-\frac{(c+dx)\tanh^{-1}(\cos(a+bx))}{b}-\frac{2dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{dx\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] (-2*d*x*ArcTanh[E^(I*(a + b*x))])/b + (d*x*ArcTanh[Cos[a + b*x]])/b - ((c +
 d*x)*ArcTanh[Cos[a + b*x]])/b - (d*ArcTanh[Sin[a + b*x]])/b^2 + (I*d*PolyL
og[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/b^2 + ((c
+ d*x)*Sec[a + b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6406

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx = -\frac{(c+dx)\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)\sec(a+bx)}{b} - d\int \Big(-$$

$$= -\frac{(c+dx)\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)\sec(a+bx)}{b} + \frac{d\int \tanh$$

$$= \frac{dx\tanh^{-1}(\cos(a+bx))}{b} - \frac{(c+dx)\tanh^{-1}(\cos(a+bx))}{b} - \frac{d\tanh$$

$$= \frac{dx\tanh^{-1}(\cos(a+bx))}{b} - \frac{(c+dx)\tanh^{-1}(\cos(a+bx))}{b} - \frac{d\tanh$$

$$= -\frac{2dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{dx\tanh^{-1}(\cos(a+bx))}{b} - \frac{(c+dx)\tanh$$

$$= -\frac{2dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{dx\tanh^{-1}(\cos(a+bx))}{b} - \frac{(c+dx)\tanh$$

$$= -\frac{2dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{dx\tanh^{-1}(\cos(a+bx))}{b} - \frac{(c+dx)\tanh$$

<u>Mathematica</u> [A]

time = 0.55, size = 212, normalized size = 1.88

$$-\frac{c\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{d\log\left(\cos\left(\frac{1}{2}(a+bx)\right) - \sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} + \frac{c\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b} - \frac{d\log\left(\cos\left(\frac{1}{2}(a+bx)\right) + \sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} - \frac{ad\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} + \frac{d\left((a+bx)\left(\log\left(1-e^{i(a+bx)}\right) - \log\left(1+e^{i(a+bx)}\right)\right) + i\left(\text{PolyLog}\left(2,-e^{i(a+bx)}\right) - \text{PolyLog}\left(2,e^{i(a+bx)}\right)\right)\right)}{b^2} + \frac{c\sec(a+bx)}{b} + \frac{dx\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] -((c*Log[Cos[(a + b*x)/2]])/b) + (d*Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]
])/b^2 + (c*Log[Sin[(a + b*x)/2]])/b - (d*Log[Cos[(a + b*x)/2] + Sin[(a + b
*x)/2]])/b^2 - (a*d*Log[Tan[(a + b*x)/2]])/b^2 + (d*((a + b*x)*(Log[1 - E^(
I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a + b*x))]
 - PolyLog[2, E^(I*(a + b*x))])))/b^2 + (c*Sec[a + b*x])/b + (d*x*Sec[a + b
*x])/b
```

<u>Maple</u> [A]

time = 0.11, size = 160, normalized size = 1.42

method	result
risch	$\frac{2\,\mathrm{e}^{i(bx+a)}(dx+c)}{b\left(1+\mathrm{e}^{2i(bx+a)}\right)} + \frac{c\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b} - \frac{c\ln\left(\mathrm{e}^{i(bx+a)}+1\right)}{b} + \frac{2id\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b^2} + \frac{id\,\mathrm{dilog}\left(\mathrm{e}^{i(bx+a)}\right)}{b^2} + \frac{id\,\mathrm{dilog}\left(\mathrm{e}^{i(bx+a)}+1\right.}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*exp(I*(b*x+a))*(d*x+c)/b/(1+exp(2*I*(b*x+a)))+1/b*c*ln(exp(I*(b*x+a))-1)-
1/b*c*ln(exp(I*(b*x+a))+1)+2*I/b^2*d*arctan(exp(I*(b*x+a)))+I/b^2*d*dilog(e
xp(I*(b*x+a)))+I/b^2*d*dilog(exp(I*(b*x+a))+1)-1/b*d*ln(exp(I*(b*x+a))+1)*x
-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(101) = 202$.
time = 0.63, size = 800, normalized size = 7.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x + 2*a) + d)*arctan2(2*(cos(b*x + 2*
a)*cos(a) + sin(b*x + 2*a)*sin(a))/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*
sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2), (c
os(b*x + 2*a)^2 - cos(a)^2 + sin(b*x + 2*a)^2 - sin(a)^2)/(cos(b*x + 2*a)^2
 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)
*sin(a) + sin(a)^2)) + 2*(b*d*x + b*c + (b*d*x + b*c)*cos(2*b*x + 2*a) + (I
*b*d*x + I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) -
 2*(b*c*cos(2*b*x + 2*a) + I*b*c*sin(2*b*x + 2*a) + b*c)*arctan2(sin(b*x +
a), cos(b*x + a) - 1) + 2*(b*d*x*cos(2*b*x + 2*a) + I*b*d*x*sin(2*b*x + 2*a
) + b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 4*(I*b*d*x + I*b*c)*c
os(b*x + a) - 2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x + 2*a) + d)*dilog(-e^(I
*b*x + I*a)) + 2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x + 2*a) + d)*dilog(e^(I
*b*x + I*a)) - (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(2*b*x + 2*a) - (b*d
*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x
 + a) + 1) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(2*b*x + 2*a) + (b*d
*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x
 + a) + 1) - (-I*d*cos(2*b*x + 2*a) + d*sin(2*b*x + 2*a) - I*d)*log((cos(b*
x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(
b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b
*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(b*
d*x + b*c)*sin(b*x + a))/(-2*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(2*b*x + 2*a
) - 2*I*b^2)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(101) = 202$.
time = 1.91, size = 366, normalized size = 3.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x - I*d*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*
cos(b*x + a)*dilog(cos(b*x + a) - I*sin(b*x + a)) - I*d*cos(b*x + a)*dilog(
-cos(b*x + a) + I*sin(b*x + a)) + I*d*cos(b*x + a)*dilog(-cos(b*x + a) - I*
sin(b*x + a)) - (b*d*x + b*c)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a
) + 1) - (b*d*x + b*c)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1)
+ (b*c - a*d)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2
) + (b*c - a*d)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1
/2) + (b*d*x + a*d)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) +
(b*d*x + a*d)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - d*cos(
b*x + a)*log(sin(b*x + a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a) + 1) + 2*
b*c)/(b^2*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \csc (a + bx) \sec^2 (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)*sec(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)*sec(b*x + a)^2, x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)^2*sin(a + b*x)),x)
```

```
[Out] \text{Hanged}
```

3.270 $\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\csc(a+bx)\sec^2(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x),x]
[Out] Defer[Int][(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 9.20, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x),x]
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x), x]
```

Maple [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)\left(\sec^2\left(bx+a\right)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] (2*cos(2*b*x + 2*a)*cos(b*x + a) + 2*(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*c
os(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x + b*c
*d)*cos(2*b*x + 2*a))*integrate((cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b*x
+ 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2
*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c
^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)
), x) + (b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x
 + 2*a)^2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/
((d*x + c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*co
s(b*x + a) + c), x) + (b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x +
b*c)*sin(2*b*x + 2*a)^2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a))*integrate
(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x -
2*(d*x + c)*cos(b*x + a) + c), x) + 2*sin(2*b*x + 2*a)*sin(b*x + a) + 2*cos
(b*x + a))/(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*
b*x + 2*a)^2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)*sec(b*x + a)^2/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)**2/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)*sec(b*x + a)^2/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)),x)
[Out] int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)), x)
```

3.271 $\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 9.42, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)\left(\sec^2\left(bx+a\right)\right)}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (2*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (
b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*c*
d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)
*cos(2*b*x + 2*a))*integrate((cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b*x + 2
*a)*sin(b*x + a) + cos(b*x + a))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x +
 b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(2*b*x + 2*a)
^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(2*b*x + 2*a)^2 +
 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(2*b*x + 2*a)), x)
+ (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*
x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*
x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(d^2*x^2
+ 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x +
 c^2)*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x)
+ (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*
x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*
x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(d^2*x^2
+ 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x +
 c^2)*sin(b*x + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x)
+ 2*sin(2*b*x + 2*a)*sin(b*x + a) + 2*cos(b*x + a))/(b*d^2*x^2 + 2*b*c*d*x
+ b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 +
 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*
cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^2), x)
```

3.272 $\int (c+dx)^m \csc^2(a+bx) \sec^2(a+bx) \, dx$

Optimal. Leaf size=27

$$\text{Int}\left((c+dx)^m \csc^2(a+bx) \sec^2(a+bx), x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

Rubi [A]
time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \csc^2(a+bx) \sec^2(a+bx) \, dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \csc^2(a+bx) \sec^2(a+bx) \, dx = \int (c+dx)^m \csc^2(a+bx) \sec^2(a+bx) \, dx$$

Mathematica [A]
time = 1.88, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^2(a+bx) \sec^2(a+bx) \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\csc^2(bx+a)\right) \left(\sec^2(bx+a)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x)
[Out] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a)**2,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^2),x)
[Out] int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^2), x)
```

3.273 $\int (c+dx)^3 \csc^2(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=118

$$-\frac{2i(c+dx)^3}{b}-\frac{2(c+dx)^3\cot(2a+2bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{4i(a+bx)}\right)}{b^2}-\frac{3id^2(c+dx)\mathrm{PolyLog}\left(2,e^{4i(a+bx)}\right)}{2b^3}$$

```
[Out] -2*I*(d*x+c)^3/b-2*(d*x+c)^3*cot(2*b*x+2*a)/b+3*d*(d*x+c)^2*ln(1-exp(4*I*(b
*x+a)))/b^2-3/2*I*d^2*(d*x+c)*polylog(2,exp(4*I*(b*x+a)))/b^3+3/8*d^3*polyl
og(3,exp(4*I*(b*x+a)))/b^4
```

Rubi [A]
time = 0.18, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4504, 4269, 3798, 2221, 2611, 2320, 6724}

$$\frac{3d^3\mathrm{Li}_3\left(e^{4i(a+bx)}\right)}{8b^4}-\frac{3id^2(c+dx)\mathrm{Li}_2\left(e^{4i(a+bx)}\right)}{2b^3}+\frac{3d(c+dx)^2\log\left(1-e^{4i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^3\cot(2a+2bx)}{b}-\frac{2i(c+dx)^3}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] ((-2*I)*(c + d*x)^3)/b - (2*(c + d*x)^3*Cot[2*a + 2*b*x])/b + (3*d*(c + d*x
)^2*Log[1 - E^((4*I)*(a + b*x))])/b^2 - (((3*I)/2)*d^2*(c + d*x)*PolyLog[2,
 E^((4*I)*(a + b*x))])/b^3 + (3*d^3*PolyLog[3, E^((4*I)*(a + b*x))])/(8*b^4
)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\csc^2(a+bx)\sec^2(a+bx)\,dx &= 4\int (c+dx)^3\csc^2(2a+2bx)\,dx\\ &= -\frac{2(c+dx)^3\cot(2a+2bx)}{b}+\frac{(6d)\int (c+dx)^2\cot(2a+2bx)\,dx}{b}\\ &= -\frac{2i(c+dx)^3}{b}-\frac{2(c+dx)^3\cot(2a+2bx)}{b}-\frac{(12id)\int \frac{e^{2i(2a+2bx)}(c+d}{1-e^{2i(2a+2bx}}}{b}\\ &= -\frac{2i(c+dx)^3}{b}-\frac{2(c+dx)^3\cot(2a+2bx)}{b}+\frac{3d(c+dx)^2\log(1-}{b^2}\\ &= -\frac{2i(c+dx)^3}{b}-\frac{2(c+dx)^3\cot(2a+2bx)}{b}+\frac{3d(c+dx)^2\log(1-}{b^2}\\ &= -\frac{2i(c+dx)^3}{b}-\frac{2(c+dx)^3\cot(2a+2bx)}{b}+\frac{3d(c+dx)^2\log(1-}{b^2}\\ &= -\frac{2i(c+dx)^3}{b}-\frac{2(c+dx)^3\cot(2a+2bx)}{b}+\frac{3d(c+dx)^2\log(1-}{b^2}\end{aligned}$$

Mathematica [A]
time = 1.50, size = 154, normalized size = 1.31

$$\frac{d\left(-\frac{32ib^3e^{4ia}x(3c^2+3cdx+d^2x^2)}{-1+e^{4ia}}+24b^2(c+dx)^2\log\left(1-e^{4i(a+bx)}\right)-12ibd(c+dx)\text{PolyLog}\left(2,e^{4i(a+bx)}\right)+3d^2\text{PolyLog}\left(3,e^{4i(a+bx)}\right)\right)+16b^3(c+dx)^3\csc(2a)\csc(2(a+bx))\sin(2bx)}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] (d*(((-32*I)*b^3*E^((4*I)*a)*x*(3*c^2 + 3*c*d*x + d^2*x^2))/(-1 + E^((4*I)*
a)) + 24*b^2*(c + d*x)^2*Log[1 - E^((4*I)*(a + b*x))] - (12*I)*b*d*(c + d*x
)*PolyLog[2, E^((4*I)*(a + b*x))] + 3*d^2*PolyLog[3, E^((4*I)*(a + b*x))])
+ 16*b^3*(c + d*x)^3*Csc[2*a]*Csc[2*(a + b*x)]*Sin[2*b*x])/(8*b^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 686 vs. 2(106) = 212.
time = 0.14, size = 687, normalized size = 5.82

method	result
risch	$-\frac{3id^3\,\mathrm{polylog}(2,-\mathrm{e}^{2i(bx+a)})x}{b^3}+\frac{6d^2c\ln(1+\mathrm{e}^{2i(bx+a)})x}{b^2}-\frac{3id^2c\,\mathrm{polylog}(2,-\mathrm{e}^{2i(bx+a)})}{b^3}+\frac{12id^3a^2x}{b^3}-\frac{12id^2cx^2}{b}+\frac{24d^2ca\ln}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 12*I*d^3/b^3*a^2*x-12*I*d^2/b*c*x^2-12*I*d^2/b^3*c*a^2-24*I*d^2/b^2*c*a*x-6
*I*d^3/b^3*polylog(2,-exp(I*(b*x+a)))*x-6*I*d^2/b^3*c*polylog(2,-exp(I*(b*x
```

```
+a))+6*d^2/b^3*c*ln(1-exp(I*(b*x+a)))*a+6*d^2/b^2*c*ln(exp(I*(b*x+a))+1)*x
-6*I*d^2/b^3*c*polylog(2,exp(I*(b*x+a)))-6*I*d^3/b^3*polylog(2,exp(I*(b*x+a
)))*x+6*d^2/b^2*c*ln(1-exp(I*(b*x+a)))*x+3/2*d^3*polylog(3,-exp(2*I*(b*x+a)
))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4+24*d^2/b^3*c*a*ln(exp(I*(b*x+a))
)-6*d^2/b^3*c*a*ln(exp(I*(b*x+a))-1)+6*d^2/b^2*c*ln(1+exp(2*I*(b*x+a)))*x-3
*I*d^2/b^3*c*polylog(2,-exp(2*I*(b*x+a)))-3*I*d^3/b^3*polylog(2,-exp(2*I*(b
*x+a)))*x-4*I*d^3/b*x^3+8*I*d^3/b^4*a^3-4*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+
c^3)/b/(1+exp(2*I*(b*x+a)))/(exp(2*I*(b*x+a))-1)+3*d/b^2*c^2*ln(exp(I*(b*x+
a))-1)+3*d/b^2*c^2*ln(exp(I*(b*x+a))+1)-12*d/b^2*c^2*ln(exp(I*(b*x+a)))+3*d
^3/b^4*a^2*ln(exp(I*(b*x+a))-1)-12*d^3/b^4*a^2*ln(exp(I*(b*x+a)))+3*d/b^2*c
^2*ln(1+exp(2*I*(b*x+a)))+3*d^3/b^2*ln(1+exp(2*I*(b*x+a)))*x^2+3*d^3/b^2*ln
(1-exp(I*(b*x+a)))*x^2-3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2+3*d^3/b^2*ln(exp(
I*(b*x+a))+1)*x^2+6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2360 vs. $2(103) = 206$.
time = 0.67, size = 2360, normalized size = 20.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^3*(1/tan(b*x + a) - tan(b*x + a)) - 6*a*c^2*d*(1/tan(b*x + a) - t
an(b*x + a))/b + 6*a^2*c*d^2*(1/tan(b*x + a) - tan(b*x + a))/b^2 - 2*a^3*d^
3*(1/tan(b*x + a) - tan(b*x + a))/b^3 - 3*((cos(4*b*x + 4*a)^2 + sin(4*b*x
+ 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a
)^2 + 2*cos(2*b*x + 2*a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 -
2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a
) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)
*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*si
n(4*b*x + 4*a))*c^2*d/((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b
*x + 4*a) + 1)*b) + 6*((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b
*x + 4*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
2*a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) +
 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(4*b*x
+ 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2
+ sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*sin(4*b*x + 4*a))*a*c*
d^2/((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*b^2
) - 3*((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*l
og(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (cos
(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x
+ a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4
*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2
 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*sin(4*b*x + 4*a))*a^2*d^3/((cos(4*b*x
```

```
+ 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*b^3) + 2*(6*((b*x +
 a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) - ((b*x + a)^2*d^3 + 2*(b*c*d^2 -
 a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-I*(b*x + a)^2*d^3 + 2*(-I*b*c*d^2 +
 I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x
+ 2*a) + 1) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) - ((b*x +
a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-I*(b*x + a)^
2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a))*arctan2(sin(b
*x + a), cos(b*x + a) + 1) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x
+ a) - ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) -
 (I*(b*x + a)^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a))*
arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 8*((b*x + a)^3*d^3 + 3*(b*c*d^2
- a*d^3)*(b*x + a)^2)*cos(4*b*x + 4*a) - 6*(b*c*d^2 + (b*x + a)*d^3 - a*d^3
 - (b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(4*b*x + 4*a) - (I*b*c*d^2 + I*(b*x
 + a)*d^3 - I*a*d^3)*sin(4*b*x + 4*a))*dilog(-e^(2*I*b*x + 2*I*a)) - 12*(b*
c*d^2 + (b*x + a)*d^3 - a*d^3 - (b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(4*b*x
 + 4*a) - (I*b*c*d^2 + I*(b*x + a)*d^3 - I*a*d^3)*sin(4*b*x + 4*a))*dilog(-
e^(I*b*x + I*a)) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3 - (b*c*d^2 + (b*x +
a)*d^3 - a*d^3)*cos(4*b*x + 4*a) - (I*b*c*d^2 + I*(b*x + a)*d^3 - I*a*d^3)*
sin(4*b*x + 4*a))*dilog(e^(I*b*x + I*a)) + 3*(-I*(b*x + a)^2*d^3 + 2*(-I*b*
c*d^2 + I*a*d^3)*(b*x + a) + (I*(b*x + a)^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(
b*x + a))*cos(4*b*x + 4*a) - ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x +
a))*sin(4*b*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a) + 1) + 3*(-I*(b*x + a)^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x +
a) + (I*(b*x + a)^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*cos(4*b*x + 4*
a) - ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(4*b*x + 4*a))*lo
g(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 3*(-I*(b*x + a)^2
*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a) + (I*(b*x + a)^2*d^3 + 2*(I*b*c*d
^2 - I*a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - ((b*x + a)^2*d^3 + 2*(b*c*d^2 -
 a*d^3)*(b*x + a))*sin(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 -
2*cos(b*x + a) + 1) + 3*(I*d^3*cos(4*b*x + 4*a) - d^3*sin(4*b*x + 4*a) - I*
d^3)*polylog(3, -e^(2*I*b*x + 2*I*a)) + 12*(I*d^3*cos(4*b*x + 4*a) - d^3*si
n(4*b*x + 4*a) - I*d^3)*polylog(3, -e^(I*b*x + I*a)) + 12*(I*d^3*cos(4*b*x
+ 4*a) - d^3*sin(4*b*x + 4*a) - I*d^3)*polylog(3, e^(I*b*x + I*a)) + 8*(I*(
b*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2)*sin(4*b*x + 4*a))/(-2
*I*b^3*cos(4*b*x + 4*a) + 2*b^3*sin(4*b*x + 4*a) + 2*I*b^3))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1635 vs. $2(103) = 206$.
time = 1.62, size = 1635, normalized size = 13.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*co
s(b*x + a)*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*d^3*c
os(b*x + a)*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*d^3*
cos(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + 6*d^3
*cos(b*x + a)*polylog(3, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + 6*d^
3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + 6*
d^3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) +
6*d^3*cos(b*x + a)*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a)
+ 6*d^3*cos(b*x + a)*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a
) - 6*(I*b*d^3*x + I*b*c*d^2)*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(b*x +
 a))*sin(b*x + a) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a)*dilog(cos(b*x +
 a) - I*sin(b*x + a))*sin(b*x + a) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a
)*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 6*(I*b*d^3*x + I*b*c*
d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 6*(I*
b*d^3*x + I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a))*sin
(b*x + a) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) -
 sin(b*x + a))*sin(b*x + a) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a)*dilog
(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 6*(I*b*d^3*x + I*b*c*d^2)*c
os(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d^3
*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x
 + a) + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a
)*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*
b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1
)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos
(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2
*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a)
+ 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)
*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*
d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*
x + a) + sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x +
2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)
*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-1/2
*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a
*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a)
 + 1/2)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d
^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(
b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b
*x + a) + I)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 -
a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a)
+ 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) - I*
sin(b*x + a) + I)*sin(b*x + a) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c
^2*d*x + b^3*c^3)*cos(b*x + a)^2)/(b^4*cos(b*x + a)*sin(b*x + a))
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^3}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)^2),x)
```

```
[Out] int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)^2), x)
```

3.274 $\int (c+dx)^2 \csc^2(a+bx) \sec^2(a+bx) \, dx$

Optimal. Leaf size=88

$$-\frac{2i(c+dx)^2}{b}-\frac{2(c+dx)^2\cot(2a+2bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{4i(a+bx)}\right)}{b^2}-\frac{id^2\text{PolyLog}\left(2,e^{4i(a+bx)}\right)}{2b^3}$$

```
[Out] -2*I*(d*x+c)^2/b-2*(d*x+c)^2*cot(2*b*x+2*a)/b+2*d*(d*x+c)*ln(1-exp(4*I*(b*x
+a)))/b^2-1/2*I*d^2*polylog(2,exp(4*I*(b*x+a)))/b^3
```

Rubi [A]
time = 0.12, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4504, 4269, 3798, 2221, 2317, 2438}

$$-\frac{id^2\text{Li}_2\left(e^{4i(a+bx)}\right)}{2b^3}+\frac{2d(c+dx)\log\left(1-e^{4i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^2\cot(2a+2bx)}{b}-\frac{2i(c+dx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] ((-2*I)*(c + d*x)^2)/b - (2*(c + d*x)^2*Cot[2*a + 2*b*x])/b + (2*d*(c + d*x
)*Log[1 - E^((4*I)*(a + b*x))])/b^2 - ((I/2)*d^2*PolyLog[2, E^((4*I)*(a + b
*x))])/b^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
```

```
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\csc^2(a+bx)\sec^2(a+bx)\,dx &= 4\int (c+dx)^2\csc^2(2a+2bx)\,dx\\
&= -\frac{2(c+dx)^2\cot(2a+2bx)}{b}+\frac{(4d)\int (c+dx)\cot(2a+2bx)\,dx}{b}\\
&= -\frac{2i(c+dx)^2}{b}-\frac{2(c+dx)^2\cot(2a+2bx)}{b}-\frac{(8id)\int \frac{e^{2i(2a+2bx)}(c+dx}{1-e^{2i(2a+2bx)}}}{b}\\
&= -\frac{2i(c+dx)^2}{b}-\frac{2(c+dx)^2\cot(2a+2bx)}{b}+\frac{2d(c+dx)\log\left(1-e\right.}{b^2}\\
&= -\frac{2i(c+dx)^2}{b}-\frac{2(c+dx)^2\cot(2a+2bx)}{b}+\frac{2d(c+dx)\log\left(1-e\right.}{b^2}\\
&= -\frac{2i(c+dx)^2}{b}-\frac{2(c+dx)^2\cot(2a+2bx)}{b}+\frac{2d(c+dx)\log\left(1-e\right.}{b^2}\end{aligned}$$

Mathematica [A]
time = 1.98, size = 113, normalized size = 1.28

$$\frac{-id^2\text{PolyLog}\left(2,e^{4i(a+bx)}\right)+4b\left(-\frac{2ibde^{4ia}x(2c+dx)}{-1+e^{4ia}}+d(c+dx)\log\left(1-e^{4i(a+bx)}\right)+b(c+dx)^2\csc(2a)\csc(2(a+bx))\sin(2bx)\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] ((-I)*d^2*PolyLog[2, E^((4*I)*(a + b*x))] + 4*b*(((-2*I)*b*d*E^((4*I)*a)*x*
(2*c + d*x))/(-1 + E^((4*I)*a)) + d*(c + d*x)*Log[1 - E^((4*I)*(a + b*x))]
+ b*(c + d*x)^2*Csc[2*a]*Csc[2*(a + b*x)]*Sin[2*b*x]))/(2*b^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(80) = 160.
time = 0.12, size = 351, normalized size = 3.99

method	result
risch	$-\frac{4i(x^2d^2+2cdx+c^2)}{b(1+e^{2i(bx+a)})(e^{2i(bx+a)}-1)}+\frac{2dc\ln(e^{i(bx+a)}-1)}{b^2}+\frac{2dc\ln(e^{i(bx+a)}+1)}{b^2}+\frac{2dc\ln(1+e^{2i(bx+a)})}{b^2}-\frac{8dc\ln(e^{i(bx+a)})}{b^2}+$ 2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4*I*(d^2*x^2+2*c*d*x+c^2)/b/(1+exp(2*I*(b*x+a)))/(exp(2*I*(b*x+a))-1)+2*d/
b^2*c*ln(exp(I*(b*x+a))-1)+2*d/b^2*c*ln(exp(I*(b*x+a))+1)+2*d/b^2*c*ln(1+ex
p(2*I*(b*x+a)))-8*d/b^2*c*ln(exp(I*(b*x+a)))+2*d^2/b^2*ln(1-exp(I*(b*x+a)))
*x+2*d^2/b^3*ln(1-exp(I*(b*x+a)))*a-I*d^2*polylog(2,-exp(2*I*(b*x+a)))/b^3-
8*I*d^2/b^2*a*x-4*I*d^2/b*x^2+2*d^2/b^2*ln(exp(I*(b*x+a))+1)*x-4*I*d^2/b^3*
a^2-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3+2*d^2/b^2*ln(1+exp(2*I*(b*x+a)))*
x-2*I*d^2/b^3*polylog(2,-exp(I*(b*x+a)))-2*d^2/b^3*a*ln(exp(I*(b*x+a))-1)+8
*d^2/b^3*a*ln(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 772 vs. 2(77) = 154.
time = 0.64, size = 772, normalized size = 8.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(4*b^2*c^2 + 2*(b*d^2*x + b*c*d - (b*d^2*x + b*c*d)*cos(4*b*x + 4*a) + (-I
*b*d^2*x - I*b*c*d)*sin(4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x +
 2*a) + 1) + 2*(b*d^2*x + b*c*d - (b*d^2*x + b*c*d)*cos(4*b*x + 4*a) + (-I*
b*d^2*x - I*b*c*d)*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1
) - 2*(b*c*d*cos(4*b*x + 4*a) + I*b*c*d*sin(4*b*x + 4*a) - b*c*d)*arctan2(s
in(b*x + a), cos(b*x + a) - 1) + 2*(b*d^2*x*cos(4*b*x + 4*a) + I*b*d^2*x*si
n(4*b*x + 4*a) - b*d^2*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 4*(b^2
*d^2*x^2 + 2*b^2*c*d*x)*cos(4*b*x + 4*a) + (d^2*cos(4*b*x + 4*a) + I*d^2*si
n(4*b*x + 4*a) - d^2)*dilog(-e^(2*I*b*x + 2*I*a)) + 2*(d^2*cos(4*b*x + 4*a)
 + I*d^2*sin(4*b*x + 4*a) - d^2)*dilog(-e^(I*b*x + I*a)) + 2*(d^2*cos(4*b*x
 + 4*a) + I*d^2*sin(4*b*x + 4*a) - d^2)*dilog(e^(I*b*x + I*a)) - (I*b*d^2*x
 + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(4*b*x + 4*a) + (b*d^2*x + b*c*d)*si
n(4*b*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
 2*a) + 1) - (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(4*b*x + 4*a)
 + (b*d^2*x + b*c*d)*sin(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2
+ 2*cos(b*x + a) + 1) - (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(4
```

```
*b*x + 4*a) + (b*d^2*x + b*c*d)*sin(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(
b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(I*b^2*d^2*x^2 + 2*I*b^2*c*d*x)*sin(4*
b*x + 4*a))/(-I*b^3*cos(4*b*x + 4*a) + b^3*sin(4*b*x + 4*a) + I*b^3)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 950 vs. 2(77) = 154.
time = 1.68, size = 950, normalized size = 10.80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + 2*b^2*c*d*x - I*d^2*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(
b*x + a))*sin(b*x + a) + I*d^2*cos(b*x + a)*dilog(cos(b*x + a) - I*sin(b*x
+ a))*sin(b*x + a) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)
)*sin(b*x + a) - I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a))*si
n(b*x + a) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a))*sin(b
*x + a) + I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x
+ a) + I*d^2*cos(b*x + a)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a
) - I*d^2*cos(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) +
 b^2*c^2 + (b*d^2*x + b*c*d)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a)
 + 1)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a) + I*sin(
b*x + a) + I)*sin(b*x + a) + (b*d^2*x + b*c*d)*cos(b*x + a)*log(cos(b*x + a
) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos
(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + (b*d^2*x + a*d^2)*cos(b*x +
a)*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b*d^2*x + a*d^2)*
cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) + (b*d^2*x
 + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a)
 + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)*s
in(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*si
n(b*x + a) + 1/2)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(-1/2*cos(
b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + (b*d^2*x + a*d^2)*cos(b
*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + (b*c*d - a*d
^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + (b*
d^2*x + a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x
 + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I
)*sin(b*x + a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a)^2)/(b
^3*cos(b*x + a)*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \csc^2 (a + bx) \sec^2 (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*csc(a + b*x)**2*sec(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^2}{{\cos\left(a+b\,x\right)}^2\,{\sin\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)^2),x)
```

```
[Out] int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)^2), x)
```

3.275 $\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=35

$$-\frac{2(c+dx)\cot(2a+2bx)}{b}+\frac{d\log(\sin(2a+2bx))}{b^2}$$

[Out] -2*(d*x+c)*cot(2*b*x+2*a)/b+d*ln(sin(2*b*x+2*a))/b^2

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4504, 4269, 3556}

$$\frac{d\log(\sin(2a+2bx))}{b^2}-\frac{2(c+dx)\cot(2a+2bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^2,x]

[Out] (-2*(c + d*x)*Cot[2*a + 2*b*x])/b + (d*Log[Sin[2*a + 2*b*x]])/b^2

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx &= 4\int (c+dx)\csc^2(2a+2bx)\,dx\\ &= -\frac{2(c+dx)\cot(2a+2bx)}{b}+\frac{(2d)\int \cot(2a+2bx)\,dx}{b}\\ &= -\frac{2(c+dx)\cot(2a+2bx)}{b}+\frac{d\log(\sin(2a+2bx))}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 32, normalized size = 0.91

$$\frac{-2b(c+dx)\cot(2(a+bx))+d\log(\sin(2(a+bx)))}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] (-2*b*(c + d*x)*Cot[2*(a + b*x)] + d*Log[Sin[2*(a + b*x)]])/b^2
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 72, normalized size = 2.06

method	result
risch	$-\frac{4idx}{b}-\frac{4ida}{b^2}-\frac{4i(dx+c)}{b\left(1+e^{2i(bx+a)}\right)\left(e^{2i(bx+a)}-1\right)}+\frac{d\ln\left(e^{4i(bx+a)}-1\right)}{b^2}$
norman	$\frac{\frac{c}{2b}-\frac{3c\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{c\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}+\frac{dx}{2b}-\frac{3dx\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{dx\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}+\frac{d\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}+\frac{d\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-\right.}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4*I*d/b*x-4*I*d/b^2*a-4*I*(d*x+c)/b/(1+exp(2*I*(b*x+a)))/(exp(2*I*(b*x+a))
-1)+d/b^2*ln(exp(4*I*(b*x+a))-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(35) = 70.
time = 0.50, size = 308, normalized size = 8.80

$$-\frac{2\,c\left(\frac{1}{\tan\left(bx+a\right)}-\tan\left(bx+a\right)\right)-\frac{2\,ad\left(\frac{1}{\tan\left(bx+a\right)}-\tan\left(bx+a\right)\right)}{b}-\frac{\left(\left(\cos\left(4\,bx+4\,a\right)^2+\sin\left(4\,bx+4\,a\right)^2-2\,\cos\left(4\,bx+4\,a\right)+1\right)\log\left(\cos\left(2\,bx+2\,a\right)^2+\sin\left(2\,bx+2\,a\right)^2+2\,\cos\left(2\,bx+2\,a\right)+1\right)+\left(\cos\left(4\,bx+4\,a\right)^2+\sin\left(4\,bx+4\,a\right)^2-2\,\cos\left(4\,bx+4\,a\right)+1\right)\log\left(\cos\left(bx+a\right)^2+\sin\left(bx+a\right)^2+2\,\cos\left(bx+a\right)+1\right)+\left(\cos\left(4\,bx+4\,a\right)^2+\sin\left(4\,bx+4\,a\right)^2-2\,\cos\left(4\,bx+4\,a\right)+1\right)\log\left(\cos\left(bx+a\right)^2+\sin\left(bx+a\right)^2-2\,\cos\left(bx+a\right)+1\right)-8\,\left(bx+a\right)\sin\left(4\,bx+4\,a\right)\right)d}{\left(\cos\left(4\,bx+4\,a\right)^2+\sin\left(4\,bx+4\,a\right)^2-2\,\cos\left(4\,bx+4\,a\right)+1\right)b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*(1/tan(b*x + a) - tan(b*x + a)) - 2*a*d*(1/tan(b*x + a) - tan(b*x
 + a))/b - ((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) +
 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) +
 (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos
(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(4*b*x + 4*a)^2 +
sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin(b*x +
 a)^2 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*sin(4*b*x + 4*a))*d/((cos(4*b*x +
 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*b))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(35) = 70$.
time = 1.76, size = 75, normalized size = 2.14

$$\frac{d\cos\left(bx+a\right)\log\left(-\frac{1}{2}\cos\left(bx+a\right)\sin\left(bx+a\right)\right)\sin\left(bx+a\right)+bdx-2\left(bdx+bc\right)\cos\left(bx+a\right)^2+bc}{b^2\cos\left(bx+a\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (d*cos(b*x + a)*log(-1/2*cos(b*x + a)*sin(b*x + a))*sin(b*x + a) + b*d*x -
2*(b*d*x + b*c)*cos(b*x + a)^2 + b*c)/(b^2*cos(b*x + a)*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+dx\right)\csc^2\left(a+bx\right)\sec^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)**2*sec(a + b*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 13091 vs. $2(35) = 70$.
time = 2.30, size = 13091, normalized size = 374.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 -
6*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 16*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^3
+ d*log(64*(tan(1/2*b*x)^16*tan(1/2*a)^6 + 2*tan(1/2*b*x)^15*tan(1/2*a)^7 +
 tan(1/2*b*x)^14*tan(1/2*a)^8 - 2*tan(1/2*b*x)^16*tan(1/2*a)^4 - 14*tan(1/2
*b*x)^15*tan(1/2*a)^5 - 20*tan(1/2*b*x)^14*tan(1/2*a)^6 - 6*tan(1/2*b*x)^13
*tan(1/2*a)^7 + 2*tan(1/2*b*x)^12*tan(1/2*a)^8 + tan(1/2*b*x)^16*tan(1/2*a)
^2 + 14*tan(1/2*b*x)^15*tan(1/2*a)^3 + 54*tan(1/2*b*x)^14*tan(1/2*a)^4 + 42
*tan(1/2*b*x)^13*tan(1/2*a)^5 - 28*tan(1/2*b*x)^12*tan(1/2*a)^6 - 30*tan(1/
2*b*x)^11*tan(1/2*a)^7 - tan(1/2*b*x)^10*tan(1/2*a)^8 - 2*tan(1/2*b*x)^15*t
an(1/2*a) - 20*tan(1/2*b*x)^14*tan(1/2*a)^2 - 42*tan(1/2*b*x)^13*tan(1/2*a)
^3 + 84*tan(1/2*b*x)^12*tan(1/2*a)^4 + 210*tan(1/2*b*x)^11*tan(1/2*a)^5 + 8
4*tan(1/2*b*x)^10*tan(1/2*a)^6 - 22*tan(1/2*b*x)^9*tan(1/2*a)^7 - 4*tan(1/2
*b*x)^8*tan(1/2*a)^8 + tan(1/2*b*x)^14 + 6*tan(1/2*b*x)^13*tan(1/2*a) - 28*
```

tan(1/2*b*x)^12*tan(1/2*a)^2 - 210*tan(1/2*b*x)^11*tan(1/2*a)^3 - 182*tan(1/2*b*x)^10*tan(1/2*a)^4 + 154*tan(1/2*b*x)^9*tan(1/2*a)^5 + 182*tan(1/2*b*x)^8*tan(1/2*a)^6 + 22*tan(1/2*b*x)^7*tan(1/2*a)^7 - tan(1/2*b*x)^6*tan(1/2*a)^8 + 2*tan(1/2*b*x)^12 + 30*tan(1/2*b*x)^11*tan(1/2*a) + 84*tan(1/2*b*x)^10*tan(1/2*a)^2 - 154*tan(1/2*b*x)^9*tan(1/2*a)^3 - 420*tan(1/2*b*x)^8*tan(1/2*a)^4 - 154*tan(1/2*b*x)^7*tan(1/2*a)^5 + 84*tan(1/2*b*x)^6*tan(1/2*a)^6 + 30*tan(1/2*b*x)^5*tan(1/2*a)^7 + 2*tan(1/2*b*x)^4*tan(1/2*a)^8 - tan(1/2*b*x)^10 + 22*tan(1/2*b*x)^9*tan(1/2*a) + 182*tan(1/2*b*x)^8*tan(1/2*a)^2 + 154*tan(1/2*b*x)^7*tan(1/2*a)^3 - 182*tan(1/2*b*x)^6*tan(1/2*a)^4 - 210*tan(1/2*b*x)^5*tan(1/2*a)^5 - 28*tan(1/2*b*x)^4*tan(1/2*a)^6 + 6*tan(1/2*b*x)^3*tan(1/2*a)^7 + tan(1/2*b*x)^2*tan(1/2*a)^8 - 4*tan(1/2*b*x)^8 - 22*tan(1/2*b*x)^7*tan(1/2*a) + 84*tan(1/2*b*x)^6*tan(1/2*a)^2 + 210*tan(1/2*b*x)^5*tan(1/2*a)^3 + 84*tan(1/2*b*x)^4*tan(1/2*a)^4 - 42*tan(1/2*b*x)^3*tan(1/2*a)^5 - 20*tan(1/2*b*x)^2*tan(1/2*a)^6 - 2*tan(1/2*b*x)*tan(1/2*a)^7 - tan(1/2*b*x)^6 - 30*tan(1/2*b*x)^5*tan(1/2*a) - 28*tan(1/2*b*x)^4*tan(1/2*a)^2 + 42*tan(1/2*b*x)^3*tan(1/2*a)^3 + 54*tan(1/2*b*x)^2*tan(1/2*a)^4 + 14*tan(1/2*b*x)*tan(1/2*a)^5 + tan(1/2*a)^6 + 2*tan(1/2*b*x)^4 - 6*tan(1/2*b*x)^3*tan(1/2*a) - 20*tan(1/2*b*x)^2*tan(1/2*a)^2 - 14*tan(1/2*b*x)*tan(1/2*a)^3 - 2*tan(1/2*a)^4 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^8 + 4*tan(1/2*a)^6 + 6*tan(1/2*a)^4 + 4*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^3 - 6*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 + d*log(64*(tan(1/2*b*x)^16*tan(1/2*a)^6 + 2*tan(1/2*b*x)^15*tan(1/2*a)^7 + tan(1/2*b*x)^14*tan(1/2*a)^8 - 2*tan(1/2*b*x)^16*tan(1/2*a)^4 - 14*tan(1/2*b*x)^15*tan(1/2*a)^5 - 20*tan(1/2*b*x)^14*tan(1/2*a)^6 - 6*tan(1/2*b*x)^13*tan(1/2*a)^7 + 2*tan(1/2*b*x)^12*tan(1/2*a)^8 + tan(1/2*b*x)^16*tan(1/2*a)^2 + 14*tan(1/2*b*x)^15*tan(1/2*a)^3 + 54*tan(1/2*b*x)^14*tan(1/2*a)^4 + 42*tan(1/2*b*x)^13*tan(1/2*a)^5 - 28*tan(1/2*b*x)^12*tan(1/2*a)^6 - 30*tan(1/2*b*x)^11*tan(1/2*a)^7 - tan(1/2*b*x)^10*tan(1/2*a)^8 - 2*tan(1/2*b*x)^15*tan(1/2*a) - 20*tan(1/2*b*x)^14*tan(1/2*a)^2 - 42*tan(1/2*b*x)^13*tan(1/2*a)^3 + 84*tan(1/2*b*x)^12*tan(1/2*a)^4 + 210*tan(1/2*b*x)^11*tan(1/2*a)^5 + 84*tan(1/2*b*x)^10*tan(1/2*a)^6 - 22*tan(1/2*b*x)^9*tan(1/2*a)^7 - 4*tan(1/2*b*x)^8*tan(1/2*a)^8 + tan(1/2*b*x)^14 + 6*tan(1/2*b*x)^13*tan(1/2*a) - 28*tan(1/2*b*x)^12*tan(1/2*a)^2 - 210*tan(1/2*b*x)^11*tan(1/2*a)^3 - 182*tan(1/2*b*x)^10*tan(1/2*a)^4 + 154*tan(1/2*b*x)^9*tan(1/2*a)^5 + 182*tan(1/2*b*x)^8*tan(1/2*a)^6 + 22*tan(1/2*b*x)^7*tan(1/2*a)^7 - tan(1/2*b*x)^6*tan(1/2*a)^8 + 2*tan(1/2*b*x)^12 + 30*tan(1/2*b*x)^11*tan(1/2*a) + 84*tan(1/2*b*x)^10*tan(1/2*a)^2 - 154*tan(1/2*b*x)^9*tan(1/2*a)^3 - 420*tan(1/2*b*x)^8*tan(1/2*a)^4 - 154*tan(1/2*b*x)^7*tan(1/2*a)^5 + 84*tan(1/2*b*x)^6*tan(1/2*a)^6 + 30*tan(1/2*b*x)^5*tan(1/2*a)^7 + 2*tan(1/2*b*x)^4*tan(1/2*a)^8 - tan(1/2*b*x)^10 + 22*tan(1/2*b*x)^9*tan(1/2*a) + 182*tan(1/2*b*x)^8*tan(1/2*a)^2 + 154*tan(1/2*b*x)^7*tan(1/2*a)^3 - 182*tan(1/2*b*x)^6*tan(1/2*a)^4 - 210*tan(1/2*b*x)^5*tan(1/2*a)^5 - 28*tan(1/2*b*x)^4*tan(1/2*a)^6 + 6*tan(1/2*b*x)^3*tan(1/2*a)^7 + tan(1/2*b*x)^2*tan(1/2*a)^8 - 4*tan(1/2*b*x)^8 - 22*tan(1/2*b*x)^7*tan(1/2*a) + 84*tan(1/2*b*x)^6*tan(1/2*a)^2 + 210*tan(1/2*b*x)^5*tan(1/2*a)^3 + 84*tan(1/2*b*x)^4*tan(1/2*a)^4 - 42*tan(1/2*b*x)^3*tan(1/2*a)^5 - 20*tan

```
(1/2*b*x)^2*tan(1/2*a)^6 - 2*tan(1/2*b*x)*tan(1/2*a)^7 - tan(1/2*b*x)^6 - 3
0*tan(1/2*b*x)^5*tan(1/2*a) - 28*tan(1/2*b*x)^4*tan(1/2*a)^2 + 42*tan(1/2*b
*x)^3*tan(1/2*a)^3 + 54*tan(1/2*b*x)^2*tan(1/2*a)^4 + 14*tan(1/2*b*x)*tan(1
/2*a)^5 + tan(1/2*a)^6 + 2*tan(1/2*b*x)^4 - 6*tan(1/2*b*x)^3*tan(1/2*a) - 2
0*tan(1/2*b*x)^2*tan(1/2*a)^2 - 14*tan(1/2*b*x)*tan(1/2*a)^3 - 2*tan(1/2*a)
^4 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)
^8 + 4*tan(1/2*a)^6 + 6*tan(1/2*a)^4 + 4*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*
tan(1/2*a)^4 - 6*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 - 16*b*c*tan(1/2*b*x)^3*ta
n(1/2*a)^3 - 6*b*c*tan(1/2*b*x)^2*tan(1/2*a)^4 ...
```

Mupad [B]
time = 1.66, size = 55, normalized size = 1.57

$$\frac{d\,\ln\left(\mathrm{e}^{a\,4\mathrm{i}}\,\mathrm{e}^{b\,x\,4\mathrm{i}}-1\right)}{b^2}-\frac{\left(c+d\,x\right)\,4\mathrm{i}}{b\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}-1\right)}-\frac{d\,x\,4\mathrm{i}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)^2*sin(a + b*x)^2),x)
```

```
[Out] (d*log(exp(a*4i)*exp(b*x*4i) - 1))/b^2 - ((c + d*x)*4i)/(b*(exp(a*4i + b*x*
4i) - 1)) - (d*x*4i)/b
```

3.276 $\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=24

$$4\text{Int}\left(\frac{\csc^2(2a+2bx)}{c+dx},x\right)$$

```
[Out] 4*Unintegrable(csc(2*b*x+2*a)^2/(d*x+c),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x),x]
```

```
[Out] 4*Defer[Int][Csc[2*a + 2*b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx = 4\int \frac{\csc^2(2a+2bx)}{c+dx}\,dx$$

Mathematica [A]
time = 6.78, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x),x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x), x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^2\left(bx+a\right)\right)\left(\sec^2\left(bx+a\right)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] (2*(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(4*b*x + 4*a)^2 + (b*d^2*x + b*c
*d)*sin(4*b*x + 4*a)^2 - 2*(b*d^2*x + b*c*d)*cos(4*b*x + 4*a))*integrate(si
n(2*b*x + 2*a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*
c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)^
2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)), x) + (b*d^2*x + b*
c*d + (b*d^2*x + b*c*d)*cos(4*b*x + 4*a)^2 + (b*d^2*x + b*c*d)*sin(4*b*x +
4*a)^2 - 2*(b*d^2*x + b*c*d)*cos(4*b*x + 4*a))*integrate(sin(b*x + a)/(b*d^
2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2
+ (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x
 + b*c^2)*cos(b*x + a)), x) - (b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(4*b*
x + 4*a)^2 + (b*d^2*x + b*c*d)*sin(4*b*x + 4*a)^2 - 2*(b*d^2*x + b*c*d)*cos
(4*b*x + 4*a))*integrate(sin(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d
^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2
)*sin(b*x + a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) - 4*
sin(4*b*x + 4*a))/(b*d*x + (b*d*x + b*c)*cos(4*b*x + 4*a)^2 + (b*d*x + b*c)
*sin(4*b*x + 4*a)^2 + b*c - 2*(b*d*x + b*c)*cos(4*b*x + 4*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)**2/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)),x)
[Out] int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)), x)
```

3.277 $\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=24

$$4\text{Int}\left(\frac{\csc^2(2a+2bx)}{(c+dx)^2},x\right)$$

```
[Out] 4*Unintegrable(csc(2*b*x+2*a)^2/(d*x+c)^2,x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] 4*Defer[Int][Csc[2*a + 2*b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = 4\int \frac{\csc^2(2a+2bx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 6.90, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(\csc^2(bx+a))(\sec^2(bx+a))}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 2*(2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*
d)*cos(4*b*x + 4*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(4*b*x + 4*a
)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(4*b*x + 4*a))*integrate(sin
(2*b*x + 2*a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3
 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(2*b*x + 2*a)^2 + (b*d^3*x^3 + 3
*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x^3 + 3*b
*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(2*b*x + 2*a)), x) + (b*d^3*x^2 + 2*b*
c*d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(4*b*x + 4*a)^2
+ (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(4*b*x + 4*a)^2 - 2*(b*d^3*x^2 + 2
*b*c*d^2*x + b*c^2*d)*cos(4*b*x + 4*a))*integrate(sin(b*x + a)/(b*d^3*x^3 +
 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2
*d*x + b*c^3)*cos(b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b
*c^3)*sin(b*x + a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*
cos(b*x + a)), x) - (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c
*d^2*x + b*c^2*d)*cos(4*b*x + 4*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*
sin(4*b*x + 4*a)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(4*b*x + 4*a)
)*integrate(sin(b*x + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 +
 (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)^2 + (b*d^3*
x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(b*x + a)^2 - 2*(b*d^3*x^3 +
3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)), x) - 2*sin(4*b*x + 4*a)
)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(4*b*
x + 4*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(4*b*x + 4*a)^2 - 2*(b*d^2*
x^2 + 2*b*c*d*x + b*c^2)*cos(4*b*x + 4*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+b\,x)^2\,\sin(a+b\,x)^2\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^2),x)
```

```
[Out] int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^2), x)
```

3.278 $\int (c+dx)^m \csc^3(a+bx) \sec^2(a+bx)\, dx$

Optimal. Leaf size=27

$$\text{Int}\left((c+dx)^m \csc^3(a+bx) \sec^2(a+bx),x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

Rubi [A]
time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \csc^3(a+bx) \sec^2(a+bx)\, dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \csc^3(a+bx) \sec^2(a+bx)\, dx = \int (c+dx)^m \csc^3(a+bx) \sec^2(a+bx)\, dx$$

Mathematica [A]
time = 21.06, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^3(a+bx) \sec^2(a+bx)\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\csc^3(bx+a)\right) \left(\sec^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
[Out] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a)**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^3),x)
[Out] int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

3.279 $\int (c+dx)^3 \csc^3(a+bx) \sec^2(a+bx) \, dx$

Optimal. Leaf size=601

$$\frac{12icd^2x\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}+\frac{6id^3x^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}-\frac{6d^3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3}-\frac{3(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-$$

```
[Out] -3*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b
^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4-6*I*c*d^2*polylog(2,-I*exp(I*(b*x+
a)))/b^3-6*I*d^3*x*polylog(2,-I*exp(I*(b*x+a)))/b^3-9/2*I*d*(d*x+c)^2*polyl
og(2,exp(I*(b*x+a)))/b^2-9*d^2*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^3+9*d^2
*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-3*c*d^2*x*csc(b*x+a)/b^2-3*I*d^3*pol
ylog(2,exp(I*(b*x+a)))/b^4+3/2*(d*x+c)^3*sec(b*x+a)/b-6*d^3*x*arctanh(exp(I
*(b*x+a)))/b^3-3*c*d^2*arctanh(cos(b*x+a))/b^3-3*c^2*d*arctanh(sin(b*x+a))/
b^2-3/2*c^2*d*csc(b*x+a)/b^2-3/2*d^3*x^2*csc(b*x+a)/b^2-1/2*(d*x+c)^3*csc(b
*x+a)^2*sec(b*x+a)/b-9*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4+3*I*d^3*polylog
(2,-exp(I*(b*x+a)))/b^4+6*I*d^3*x^2*arctan(exp(I*(b*x+a)))/b^2+6*I*c*d^2*po
lylog(2,I*exp(I*(b*x+a)))/b^3+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b^3+9*I
*d^3*polylog(4,exp(I*(b*x+a)))/b^4+12*I*c*d^2*x*arctan(exp(I*(b*x+a)))/b^2+
9/2*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^2
```

Rubi [A]
time = 1.61, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 64, number of rules used = 24, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {2702, 294, 327, 213, 4505, 6820, 12, 6874, 6408, 4268, 2611, 6744, 2320, 6724, 4218, 464, 212, 4266, 2317, 2438, 2701, 6406, 3855, 14}

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] ((12*I)*c*d^2*x*ArcTan[E^(I*(a + b*x))])/b^2 + ((6*I)*d^3*x^2*ArcTan[E^(I*(
a + b*x))])/b^2 - (6*d^3*x*ArcTanh[E^(I*(a + b*x))])/b^3 - (3*(c + d*x)^3*A
rcTanh[E^(I*(a + b*x))])/b - (3*c*d^2*ArcTanh[Cos[a + b*x]])/b^3 - (3*c^2*d
*ArcTanh[Sin[a + b*x]])/b^2 - (3*c^2*d*Csc[a + b*x])/(2*b^2) - (3*c*d^2*x*C
sc[a + b*x])/b^2 - (3*d^3*x^2*Csc[a + b*x])/(2*b^2) + ((3*I)*d^3*PolyLog[2,
 -E^(I*(a + b*x))])/b^4 + (((9*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*
x))])/b^2 - ((6*I)*c*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 - ((6*I)*d^3
*x*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*c*d^2*PolyLog[2, I*E^(I*(
a + b*x))])/b^3 + ((6*I)*d^3*x*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*
d^3*PolyLog[2, E^(I*(a + b*x))])/b^4 - (((9*I)/2)*d*(c + d*x)^2*PolyLog[2,
E^(I*(a + b*x))])/b^2 - (9*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3
+ (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLog[3, I*E^(I*(
a + b*x))])/b^4 + (9*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((9*I
```

```
)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((9*I)*d^3*PolyLog[4, E^(I*(a + b
*x))])/b^4 + (3*(c + d*x)^3*Sec[a + b*x])/(2*b) - ((c + d*x)^3*Csc[a + b*x]
^2*Sec[a + b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6406

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
```

```
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^3 \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+}{}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+}{}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+}{}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+}{}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+}{}$$

$$= \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+dx)^3\csc^2(a+bx)\sec(a+bx)}{2b} - \frac{3\int}{}$$

$$= \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+dx)^3\csc^2(a+bx)\sec(a+bx)}{2b} - \frac{3}{2}\int$$

$$= -\frac{3(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+dx}{}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3c^2d\csc(a+bx)}{2b^2} + \frac{9id(c+dx)^2}{2}$$

$$= \frac{18icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^3\tan}{}$$

$$= \frac{18icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^3\tan}{}$$

$$= \frac{18icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^3\tan}{}$$

$$= \frac{18icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^3x\tanh^{-1}}{b^3}$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^3x\tanh^{-1}}{b^3}$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^3x\tanh^{-1}}{b^3}$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^3x\tanh^{-1}}{b^3}$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^3x\tanh^{-1}}{b^3}$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^3x\tanh^{-1}}{b^3}$$

Mathematica [**A**]
time = 7.29, size = 857, normalized size = 1.43

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (-3*d*((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - I*E^(I*
(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1 +
I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + (2*I)*b*d*(c
+ d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, I*
E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 2*d^2*PolyLog[3
, I*E^(I*(a + b*x))]))/b^4 - (3*(2*b^3*c^3*ArcTanh[E^(I*(a + b*x))] + 4*b*c
*d^2*ArcTanh[E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 - E^(I*(a + b*x))] - 2*
b*d^3*x*Log[1 - E^(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 - E^(I*(a + b*x))]
 - b^3*d^3*x^3*Log[1 - E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 + E^(I*(a + b
*x))] + 2*b*d^3*x*Log[1 + E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 + E^(I*(
a + b*x))] + b^3*d^3*x^3*Log[1 + E^(I*(a + b*x))] - I*d*(2*d^2 + 3*b^2*(c +
 d*x)^2)*PolyLog[2, -E^(I*(a + b*x))] + I*d*(2*d^2 + 3*b^2*(c + d*x)^2)*Pol
yLog[2, E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog[3, -E^(I*(a + b*x))] + 6*b*d^3
*x*PolyLog[3, -E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, E^(I*(a + b*x))] - 6
*b*d^3*x*PolyLog[3, E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, -E^(I*(a + b*x)
)] - (6*I)*d^3*PolyLog[4, E^(I*(a + b*x))]))/(2*b^4) - (Csc[a + b*x]^2*Sec[
a + b*x]*(-(b*c^3) - 3*b*c^2*d*x - 3*b*c*d^2*x^2 - b*d^3*x^3 + 3*b*c^3*Cos[
2*a + 2*b*x] + 9*b*c^2*d*x*Cos[2*a + 2*b*x] + 9*b*c*d^2*x^2*Cos[2*a + 2*b*x
] + 3*b*d^3*x^3*Cos[2*a + 2*b*x] + 3*c^2*d*Sin[2*a + 2*b*x] + 6*c*d^2*x*Sin
[2*a + 2*b*x] + 3*d^3*x^2*Sin[2*a + 2*b*x]))/(4*b^2)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1612 vs. 2(535) = 1070.
time = 0.59, size = 1613, normalized size = 2.68

method	result	size
risch	Expression too large to display	1613

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/2/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+9/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))
-9/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))-9/b^3*d^3*polylog(3,-exp(I*(b*x+a))
)*x+9/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-3/2/b*c^3*ln(exp(I*(b*x+a))+1)+3/
2/b*c^3*ln(exp(I*(b*x+a))-1)-12*I/b^3*d^2*c*a*arctan(exp(I*(b*x+a)))+1/b^2/
(exp(2*I*(b*x+a))-1)^2/(1+exp(2*I*(b*x+a)))*(3*d^3*x^3*b*exp(5*I*(b*x+a))+9
*c*d^2*x^2*b*exp(5*I*(b*x+a))+9*c^2*d*x*b*exp(5*I*(b*x+a))-2*d^3*x^3*b*exp(
```

```
3*I*(b*x+a))+3*c^3*b*exp(5*I*(b*x+a))-6*c*d^2*x^2*b*exp(3*I*(b*x+a))-3*I*c^
2*d*exp(5*I*(b*x+a))-6*c^2*d*x*b*exp(3*I*(b*x+a))+3*d^3*x^3*b*exp(I*(b*x+a)
)+3*I*c^2*d*exp(I*(b*x+a))-2*c^3*b*exp(3*I*(b*x+a))+9*c*d^2*x^2*b*exp(I*(b*
x+a))+3*I*d^3*x^2*exp(I*(b*x+a))+9*c^2*d*x*b*exp(I*(b*x+a))+3*c^3*b*exp(I*(
b*x+a))+6*I*c*d^2*x*exp(I*(b*x+a))-6*I*c*d^2*x*exp(5*I*(b*x+a))-3*I*d^3*x^2
*exp(5*I*(b*x+a)))+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,I
*exp(I*(b*x+a)))/b^4+3*I*d^3*polylog(2,-exp(I*(b*x+a)))/b^4+3/b^2*d^3*ln(1+
I*exp(I*(b*x+a)))*x^2-3/b^2*d^3*ln(1-I*exp(I*(b*x+a)))*x^2+3/b^4*d^3*a^2*ln
(1-I*exp(I*(b*x+a)))-3/b^4*d^3*a^2*ln(1+I*exp(I*(b*x+a)))-6*I*d^3*x*polylog
(2,-I*exp(I*(b*x+a)))/b^3+3*d^2/b^3*c*ln(exp(I*(b*x+a))-1)-3*d^2/b^3*c*ln(e
xp(I*(b*x+a))+1)+3*d^3/b^3*ln(1-exp(I*(b*x+a)))*x+3*d^3/b^4*ln(1-exp(I*(b*x
+a)))*a-3*d^3/b^3*ln(exp(I*(b*x+a))+1)*x-3*d^3/b^4*a*ln(exp(I*(b*x+a))-1)+9
/2*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-9/2*I/b^2*c^2*d*polylog(2,exp(I*(
b*x+a)))-9/2*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2+9/2*I/b^2*d^3*polylog(
2,-exp(I*(b*x+a)))*x^2+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b^3+9*I*d^3*po
lylog(4,exp(I*(b*x+a)))/b^4-3/2/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+3/2/b*d^3*ln
(1-exp(I*(b*x+a)))*x^3+3/2/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+9/2/b^3*c*d^2*a
^2*ln(exp(I*(b*x+a))-1)-9/2/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+9/2/b*c^2*d*ln(1
-exp(I*(b*x+a)))*x+9/2/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-9/2/b*c*d^2*ln(exp(
I*(b*x+a))+1)*x^2-9*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4+9/2/b*c*d^2*ln(1-e
xp(I*(b*x+a)))*x^2-9/2/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-9/2/b^2*c^2*d*a*l
n(exp(I*(b*x+a))-1)-9*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x+9*I/b^2*c*d^2
*polylog(2,-exp(I*(b*x+a)))*x-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4+6/b^3*d
^2*c*ln(1+I*exp(I*(b*x+a)))*a+6/b^2*d^2*c*ln(1+I*exp(I*(b*x+a)))*x-6/b^3*d^
2*c*ln(1-I*exp(I*(b*x+a)))*a-6/b^2*d^2*c*ln(1-I*exp(I*(b*x+a)))*x+6*I/b^2*d
*c^2*arctan(exp(I*(b*x+a)))+6*I/b^4*d^3*a^2*arctan(exp(I*(b*x+a)))-6*I/b^3*
c*d^2*dilog(1+I*exp(I*(b*x+a)))+6*I/b^3*c*d^2*dilog(1-I*exp(I*(b*x+a)))-6*I
/b^4*d^3*polylog(2,-I*exp(I*(b*x+a)))*a+6*I/b^4*d^3*polylog(2,I*exp(I*(b*x+
a)))*a+6*I/b^4*a*d^3*dilog(1+I*exp(I*(b*x+a)))-6*I/b^4*a*d^3*dilog(1-I*exp(
I*(b*x+a)))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8032 vs. $2(509) = 1018$.
time = 4.17, size = 8032, normalized size = 13.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c^3*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(
cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) - 3*a*c^2*d*(2*(3*cos(b*x + a)
^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(c
os(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3
 - cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b^2 -
```

a^3*d^3*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(
cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b^3 + 4*(12*(b^2*c^2*d - 2*a*b
*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) + (b^2*c
^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x +
 a))*cos(6*b*x + 6*a) - (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^
3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - (b^2*c^2*d - 2*a*b*c*
d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x
+ 2*a) + (I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(
I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a) + (-I*b^2*c^2*d + 2*I*a*b*
c*d^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))
*sin(4*b*x + 4*a) + (-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 - I*a
^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(
b*x + a), sin(b*x + a) + 1) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3
 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) + (b^2*c^2*d - 2*a*b*c*d^2 + (b*
x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6*a) -
(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*
(b*x + a))*cos(4*b*x + 4*a) - (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 +
a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (I*b^2*c^2*d -
2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*
x + a))*sin(6*b*x + 6*a) + (-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^
3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-I*
b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 +
 I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a)
+ 1) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x
+ a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a) + ((b*x +
a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c
^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(6*b*x + 6*a) - ((b*x +
 a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*
c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - ((b*x
+ a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2
*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(
b*x + a)^3*d^3 - 2*I*b*c*d^2 + 2*I*a*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x +
a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 + (-3*I*a^2 - 2*I)*d^3)*(b*x + a))*s
in(6*b*x + 6*a) - (I*(b*x + a)^3*d^3 + 2*I*b*c*d^2 - 2*I*a*d^3 + 3*(I*b*c*d
^2 - I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 + 2*I
)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (I*(b*x + a)^3*d^3 + 2*I*b*c*d^2 - 2*I
*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d
^2 + (3*I*a^2 + 2*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a)
, cos(b*x + a) + 1) + 12*(b*c*d^2 - a*d^3 + (b*c*d^2 - a*d^3)*cos(6*b*x + 6
*a) - (b*c*d^2 - a*d^3)*cos(4*b*x + 4*a) - (b*c*d^2 - a*d^3)*cos(2*b*x + 2*
a) + (I*b*c*d^2 - I*a*d^3)*sin(6*b*x + 6*a) + (-I*b*c*d^2 + I*a*d^3)*sin(4*
b*x + 4*a) + (-I*b*c*d^2 + I*a*d^3)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a),
 cos(b*x + a) - 1) - 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 +
 (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a) + ((b*x + a)^3*d^3
 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 +

```
2)*d^3)*(b*x + a))*cos(6*b*x + 6*a) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3
)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*co
s(4*b*x + 4*a) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^
2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-I*
(b*x + a)^3*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d +
6*I*a*b*c*d^2 + (-3*I*a^2 - 2*I)*d^3)*(b*x + a))*sin(6*b*x + 6*a) - (I*(b*x
 + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*
b*c*d^2 + (3*I*a^2 + 2*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (I*(b*x + a)^3
*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2
 + (3*I*a^2 + 2*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a),
-cos(b*x + a) + 1) + 12*(-I*(b*x + a)^3*d^3 - b^2*c^2*d + 2*a*b*c*d^2 - a^2
*d^3 + (-3*I*b*c*d^2 + (3*I*a - 1)*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 2*(
3*I*a - 1)*b*c*d^2 + (-3*I*a^2 + 2*a)*d^3)*(b*x + a))*cos(5*b*x + 5*a) + 8*
(I*(b*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d -
 2*I*a*b*c*d^2 + I*a^2*d^3)*(b*x + a))*cos(3*b*x + 3*a) + 12*(-I*(b*x + a)^
3*d^3 + b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3 + (-3*I*b*c*d^2 + (3*I*a + 1)*d^3
)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 2*(3*I*a + 1)...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3173 vs. 2(509) = 1018.
time = 2.40, size = 3173, normalized size = 5.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 4*b^3*c^3 - 6*(b^
3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)^2 - 6*(
b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) + 3*((3*
I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^3 +
 (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a
))*dilog(cos(b*x + a) + I*sin(b*x + a)) + 3*((-3*I*b^2*d^3*x^2 - 6*I*b^2*c*
d^2*x - 3*I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^3 + (3*I*b^2*d^3*x^2 + 6*I*b^
2*c*d^2*x + 3*I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a))*dilog(cos(b*x + a) - I*s
in(b*x + a)) + 12*((-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a)^3 + (I*b*d^3*x + I
*b*c*d^2)*cos(b*x + a))*dilog(I*cos(b*x + a) + sin(b*x + a)) + 12*((-I*b*d^
3*x - I*b*c*d^2)*cos(b*x + a)^3 + (I*b*d^3*x + I*b*c*d^2)*cos(b*x + a))*dil
og(I*cos(b*x + a) - sin(b*x + a)) + 12*((I*b*d^3*x + I*b*c*d^2)*cos(b*x + a
)^3 + (-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a))*dilog(-I*cos(b*x + a) + sin(b*
x + a)) + 12*((I*b*d^3*x + I*b*c*d^2)*cos(b*x + a)^3 + (-I*b*d^3*x - I*b*c*
d^2)*cos(b*x + a))*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 3*((3*I*b^2*d^3*
x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^3 + (-3*I*b^2
*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a))*dilog(-
cos(b*x + a) + I*sin(b*x + a)) + 3*((-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3
```

*I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^3 + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x
 + 3*I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x +
 a)) + 3*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 2*b*c*d^2 + (3*b^3*c^2
*d + 2*b*d^3)*x)*cos(b*x + a)^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3
+ 2*b*c*d^2 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a))*log(cos(b*x + a) + I
*sin(b*x + a) + 1) + 6*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)^3
- (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a))*log(cos(b*x + a) + I*si
n(b*x + a) + I) + 3*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 2*b*c*d^2 +
 (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
 + b^3*c^3 + 2*b*c*d^2 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a))*log(cos(b
*x + a) - I*sin(b*x + a) + 1) - 6*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(
b*x + a)^3 - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a))*log(cos(b*x
+ a) - I*sin(b*x + a) + I) + 6*((b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2
- a^2*d^3)*cos(b*x + a)^3 - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^
2*d^3)*cos(b*x + a))*log(I*cos(b*x + a) + sin(b*x + a) + 1) - 6*((b^2*d^3*x
^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)^3 - (b^2*d^3*x^2 +
 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a))*log(I*cos(b*x + a) -
sin(b*x + a) + 1) + 6*((b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3
)*cos(b*x + a)^3 - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*co
s(b*x + a))*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - 6*((b^2*d^3*x^2 + 2*b
^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)^3 - (b^2*d^3*x^2 + 2*b^2*c
*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a))*log(-I*cos(b*x + a) - sin(b*x
 + a) + 1) - 3*((b^3*c^3 - 3*a*b^2*c^2*d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*a
)*d^3)*cos(b*x + a)^3 - (b^3*c^3 - 3*a*b^2*c^2*d + (3*a^2 + 2)*b*c*d^2 - (a
^3 + 2*a)*d^3)*cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1
/2) - 3*((b^3*c^3 - 3*a*b^2*c^2*d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*
cos(b*x + a)^3 - (b^3*c^3 - 3*a*b^2*c^2*d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*
a)*d^3)*cos(b*x + a))*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 3
*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 2
*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^3 - (b^3*d^3*x^3 + 3*b^3*
c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d
+ 2*b*d^3)*x)*cos(b*x + a))*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 6*((b
^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)^3 - (b^2*c^2*d - 2*a*b*c*d^2
 + a^2*d^3)*cos(b*x + a))*log(-cos(b*x + a) + I*sin(b*x + a) + I) - 3*((b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^
3 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*
x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*
d^3)*x)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 6*((b^2*c^2
*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)^3 - (b^2*c^2*d - 2*a*b*c*d^2 + a^2
*d^3)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 18*(-I*d^3*co
s(b*x + a)^3 + I*d^3*cos(b*x + a))*polylog(4, cos(b*x + a) + I*sin(b*x + a)
) + 18*(I*d^3*cos(b*x + a)^3 - I*d^3*cos(b*x + a))*polylog(4, cos(b*x + a)
- I*sin(b*x + a)) + 18*(-I*d^3*cos(b*x + a)^3 + I*d^3*cos(b*x + a))*polylog
(4, -cos(b*x + a) + I*sin(b*x + a)) + 18*(I*d^3*cos(b*x + a)^3 - I*d^3*cos(
b*x + a))*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) - 18*((b*d^3*x + b*c*d

```
^2)*cos(b*x + a)^3 - (b*d^3*x + b*c*d^2)*cos(b*x + a))*polylog(3, cos(b*x +
 a) + I*sin(b*x + a)) - 18*((b*d^3*x + b*c*d^2)*cos(b*x + a)^3 - (b*d^3*x +
 b*c*d^2)*cos(b*x + a))*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 12*(d^3
*cos(b*x + a)^3 - d^3*cos(b*x + a))*polylog(3, ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

3.280 $\int (c+dx)^2 \csc^3(a+bx) \sec^2(a+bx)\,dx$

Optimal. Leaf size=305

$$\frac{4id^2x\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{2cd\tanh^{-1}(\sin(a+bx))}{b^2}-c$$

```
[Out] 4*I*d^2*x*arctan(exp(I*(b*x+a)))/b^2-3*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-
d^2*arctanh(cos(b*x+a))/b^3-2*c*d*arctanh(sin(b*x+a))/b^2-c*d*csc(b*x+a)/b^
2-d^2*x*csc(b*x+a)/b^2+3*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*d^2
*polylog(2,-I*exp(I*(b*x+a)))/b^3+2*I*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3-3
*I*d*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^2-3*d^2*polylog(3,-exp(I*(b*x+a)))
/b^3+3*d^2*polylog(3,exp(I*(b*x+a)))/b^3+3/2*(d*x+c)^2*sec(b*x+a)/b-1/2*(d*
x+c)^2*csc(b*x+a)^2*sec(b*x+a)/b
```

Rubi [A]
time = 0.60, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 22, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {2702, 294, 327, 213, 4505, 6820, 12, 6874, 6408, 4268, 2611, 2320, 6724, 4218, 464, 212, 4266, 2317, 2438, 2701, 6406, 3855}

$$\frac{4id^2x\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2id^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}-\frac{3d^2\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{3d^2\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3}+\frac{3id(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{cd\csc(a+bx)}{b^2}-\frac{2cd\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{d^2x\csc(a+bx)}{b^2}+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] ((4*I)*d^2*x*ArcTan[E^(I*(a + b*x))])/b^2 - (3*(c + d*x)^2*ArcTanh[E^(I*(a
+ b*x))])/b - (d^2*ArcTanh[Cos[a + b*x]])/b^3 - (2*c*d*ArcTanh[Sin[a + b*x]
])/b^2 - (c*d*Csc[a + b*x])/b^2 - (d^2*x*Csc[a + b*x])/b^2 + ((3*I)*d*(c +
d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a
 + b*x))])/b^3 + ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d*(
c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (3*d^2*PolyLog[3, -E^(I*(a + b*
x))])/b^3 + (3*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3 + (3*(c + d*x)^2*Sec[a
+ b*x])/(2*b) - ((c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6406

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^2 \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^2\sec(a+bx)}{2b} - (c$$

$$= -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^2\sec(a+bx)}{2b} - (c$$

$$= -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^2\sec(a+bx)}{2b} - (c$$

$$= -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^2\sec(a+bx)}{2b} - (c$$

$$= -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^2\sec(a+bx)}{2b} - (c$$

$$= \frac{3(c+dx)^2\sec(a+bx)}{2b} - \frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b} + 3$$

$$= \frac{3(c+dx)^2\sec(a+bx)}{2b} - \frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3}{2}$$

$$= -\frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3(c+dx)^2\sec(a+bx)}{2b} - (c+d$$

$$= -\frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{cd\csc(a+bx)}{b^2} + \frac{3id(c+dx)\mathrm{Li}}{b^2}$$

$$= \frac{6id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - 2cd\tanh$$

$$= \frac{6id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - 2cd\tanh$$

$$= \frac{6id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - d^2\tanh^{-}$$

$$= \frac{6id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - d^2\tanh^{-}$$

$$= \frac{4id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - d^2\tanh^{-}$$

$$= \frac{4id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - d^2\tanh^{-}$$

$$= \frac{4id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - d^2\tanh^{-}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 840 vs. $2(305) = 610$.
time = 7.20, size = 840, normalized size = 2.75

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]

[Out] ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a/2 + (b*x)/2]^2)/(8*b) - (6*b^2*c^2*ArcTanh[E^(I*(a + b*x))] + 4*d^2*ArcTanh[E^(I*(a + b*x))] - 6*b^2*c*d*x*Log[1 - E^(I*(a + b*x))] - 3*b^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] + 6*b^2*c*d*x*Log[1 + E^(I*(a + b*x))] + 3*b^2*d^2*x^2*Log[1 + E^(I*(a + b*x))] - (6*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + (6*I)*b*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] + 6*d^2*PolyLog[3, -E^(I*(a + b*x))] - 6*d^2*PolyLog[3, E^(I*(a + b*x))])/(2*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*Sec[a/2 + (b*x)/2]^2)/(8*b) + ((c + d*x)*Csc[a]*Sec[a]*(-(d*Cos[a]) + b*c*Sin[a] + b*d*x*Sin[a]))/b^2 - ((4*I)*c*d*ArcTan[((-I)*Sin[a] - I*Cos[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/(b^2*Sqrt[Cos[a]^2 + Sin[a]^2]) - (2*d^2*(-((Csc[a]*((b*x - ArcTan[Cot[a]])*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a]]))]) + I*(PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - PolyLog[2, E^(I*(b*x - ArcTan[Cot[a]]))])))/Sqrt[1 + Cot[a]^2]) + (2*ArcTan[Cot[a]]*ArcTanh[(Sin[a] + Cos[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/Sqrt[Cos[a]^2 + Sin[a]^2]))/b^3 + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c*d*Sin[(b*x)/2]) - d^2*x*Sin[(b*x)/2]))/(2*b^2) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*(c*d*Sin[(b*x)/2] + d^2*x*Sin[(b*x)/2]))/(2*b^2) + (c^2*Sin[(b*x)/2] + 2*c*d*x*Sin[(b*x)/2] + d^2*x^2*Sin[(b*x)/2])/(b*(Cos[a/2] - Sin[a/2])*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) + (-(c^2*Sin[(b*x)/2]) - 2*c*d*x*Sin[(b*x)/2] - d^2*x^2*Sin[(b*x)/2])/(b*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2]))

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 801 vs. 2(278) = 556.
time = 0.33, size = 802, normalized size = 2.63

method	result
risch	$-\frac{3cda\ln(\mathrm{e}^{i(bx+a)}-1)}{b^2}-\frac{3id^2\,\mathrm{polylog}(2,\mathrm{e}^{i(bx+a)})x}{b^2}+\frac{3cd\ln(1-\mathrm{e}^{i(bx+a)})a}{b^2}-\frac{3cd\ln(\mathrm{e}^{i(bx+a)}+1)x}{b}+\frac{3cd\ln(1-\mathrm{e}^{i(bx+a)})x}{b}-$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] -3*I/b^2*polylog(2,exp(I*(b*x+a)))*d^2*x+3*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-3*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+3/2/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+3/2/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2-3/2/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b^2*c*d*ln(1-exp(I*(b*x+a)))*a-3/b*c*d*ln(exp(I*(b*x+a))+1)*x+3/b*c*d*ln(1-exp(I*(b*x+a)))*x-3/b^2*c*d*a*ln(exp(I*(b*x+a))-1)+1/b^2/(exp(2*I*(b*x+a))-1)^2/(1+exp(2*I*(b*x+a)))*(3*d^2*x^2*b*exp(5*I*(b*x+a))+6*c*d*x*b*exp(5*I*(b*x+a))+3*c^2*b*exp(5*I*(b*x+a))-2*d^2*x^2*b*exp(3*I*(b*x+a))-4*c*d*x*b*exp(3*I*(b*x+a))-2*I*d^2*x*exp(5*I*(b*x+a))-2*c^2*b*exp(3*I*(b*x+a))+3*d^2*x^2*b*exp(I*(b*x+a))-2*I*c*d*exp(5*I*

```
(b*x+a))+6*c*d*x*b*exp(I*(b*x+a))+3*c^2*b*exp(I*(b*x+a))+2*I*d^2*x*exp(I*(b
*x+a))+2*I*d*c*exp(I*(b*x+a)))+3/2/b*c^2*ln(exp(I*(b*x+a))-1)-3/2/b*c^2*ln(
exp(I*(b*x+a))+1)-3*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+3*d^2*polylog(3,exp(
I*(b*x+a)))/b^3+3*I/b^2*d*c*polylog(2,-exp(I*(b*x+a)))+1/b^3*d^2*ln(exp(I*(
b*x+a))-1)-1/b^3*d^2*ln(exp(I*(b*x+a))+1)+2*d^2/b^2*ln(1+I*exp(I*(b*x+a)))*
x+2*d^2/b^3*ln(1+I*exp(I*(b*x+a)))*a-2*d^2/b^2*ln(1-I*exp(I*(b*x+a)))*x-2*d
^2/b^3*ln(1-I*exp(I*(b*x+a)))*a-2*I*d^2/b^3*dilog(1+I*exp(I*(b*x+a)))+2*I*d
^2/b^3*dilog(1-I*exp(I*(b*x+a)))+4*I*d/b^2*c*arctan(exp(I*(b*x+a)))-4*I*d^2
/b^3*a*arctan(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3814 vs. $2(267) = 534$.
time = 1.28, size = 3814, normalized size = 12.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(
cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) - 2*a*c*d*(2*(3*cos(b*x + a)^2
 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(cos
(b*x + a) - 1))/b + a^2*d^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos
(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b^2 + 4*(8*
(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(6*b*x
+ 6*a) - (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - (b*c*d + (b*x +
 a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*s
in(6*b*x + 6*a) + (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(4*b*x + 4*a) +
 (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x +
 a), sin(b*x + a) + 1) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x +
 a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b
*x + 4*a) - (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (I*b*c*d + I
*(b*x + a)*d^2 - I*a*d^2)*sin(6*b*x + 6*a) + (-I*b*c*d - I*(b*x + a)*d^2 +
I*a*d^2)*sin(4*b*x + 4*a) + (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(2*b*
x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 2*(3*(b*x + a)^2*d^2 +
 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^
2)*(b*x + a) + 2*d^2)*cos(6*b*x + 6*a) - (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*
d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) - (3*(b*x + a)^2*d^2 + 6*(b*c*d -
a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) - (-3*I*(b*x + a)^2*d^2 + 6*(-I*
b*c*d + I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(6*b*x + 6*a) - (3*I*(b*x + a)^2*d
^2 + 6*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(4*b*x + 4*a) - (3*I*(b*
x + a)^2*d^2 + 6*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(2*b*x + 2*a))
*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 4*(d^2*cos(6*b*x + 6*a) - d^2*co
s(4*b*x + 4*a) - d^2*cos(2*b*x + 2*a) + I*d^2*sin(6*b*x + 6*a) - I*d^2*sin(
4*b*x + 4*a) - I*d^2*sin(2*b*x + 2*a) + d^2)*arctan2(sin(b*x + a), cos(b*x
```

```
+ a) - 1) - 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + ((b*x + a)^2
*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(6*b*x + 6*a) - ((b*x + a)^2*d^2 + 2
*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a) - ((b*x + a)^2*d^2 + 2*(b*c*d
- a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d +
I*a*d^2)*(b*x + a))*sin(6*b*x + 6*a) - (I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*
a*d^2)*(b*x + a))*sin(4*b*x + 4*a) - (I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*
d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1)
+ 4*(-3*I*(b*x + a)^2*d^2 - 2*b*c*d + 2*a*d^2 + 2*(-3*I*b*c*d + (3*I*a - 1)
*d^2)*(b*x + a))*cos(5*b*x + 5*a) + 8*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a
*d^2)*(b*x + a))*cos(3*b*x + 3*a) + 4*(-3*I*(b*x + a)^2*d^2 + 2*b*c*d - 2*a
*d^2 + 2*(-3*I*b*c*d + (3*I*a + 1)*d^2)*(b*x + a))*cos(b*x + a) + 8*(d^2*co
s(6*b*x + 6*a) - d^2*cos(4*b*x + 4*a) - d^2*cos(2*b*x + 2*a) + I*d^2*sin(6*
b*x + 6*a) - I*d^2*sin(4*b*x + 4*a) - I*d^2*sin(2*b*x + 2*a) + d^2)*dilog(I
*e^(I*b*x + I*a)) - 8*(d^2*cos(6*b*x + 6*a) - d^2*cos(4*b*x + 4*a) - d^2*co
s(2*b*x + 2*a) + I*d^2*sin(6*b*x + 6*a) - I*d^2*sin(4*b*x + 4*a) - I*d^2*si
n(2*b*x + 2*a) + d^2)*dilog(-I*e^(I*b*x + I*a)) + 12*(b*c*d + (b*x + a)*d^2
 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - (b*c*d + (b*x
 + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2
*b*x + 2*a) + (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(6*b*x + 6*a) + (-I*
b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(4*b*x + 4*a) + (-I*b*c*d - I*(b*x +
a)*d^2 + I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) - 12*(b*c*d + (
b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - (
b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - (b*c*d + (b*x + a)*d^2 -
a*d^2)*cos(2*b*x + 2*a) - (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(6*b*x
+ 6*a) - (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(4*b*x + 4*a) - (I*b*c*d
+ I*(b*x + a)*d^2 - I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-3
*I*(b*x + a)^2*d^2 - 6*(I*b*c*d - I*a*d^2)*(b*x + a) - 2*I*d^2 + (-3*I*(b*x
 + a)^2*d^2 - 6*(I*b*c*d - I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(6*b*x + 6*a) +
 (3*I*(b*x + a)^2*d^2 - 6*(-I*b*c*d + I*a*d^2)*(b*x + a) + 2*I*d^2)*cos(4*b
*x + 4*a) + (3*I*(b*x + a)^2*d^2 - 6*(-I*b*c*d + I*a*d^2)*(b*x + a) + 2*I*d
^2)*cos(2*b*x + 2*a) + (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2
*d^2)*sin(6*b*x + 6*a) - (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) +
 2*d^2)*sin(4*b*x + 4*a) - (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a)
 + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x
 + a) + 1) - (3*I*(b*x + a)^2*d^2 - 6*(-I*b*c*d + I*a*d^2)*(b*x + a) + 2*I*
d^2 + (3*I*(b*x + a)^2*d^2 - 6*(-I*b*c*d + I*a*d^2)*(b*x + a) + 2*I*d^2)*co
s(6*b*x + 6*a) + (-3*I*(b*x + a)^2*d^2 - 6*(I*b*c*d - I*a*d^2)*(b*x + a) -
2*I*d^2)*cos(4*b*x + 4*a) + (-3*I*(b*x + a)^2*d^2 - 6*(I*b*c*d - I*a*d^2)*(
b*x + a) - 2*I*d^2)*cos(2*b*x + 2*a) - (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^
2)*(b*x + a) + 2*d^2)*sin(6*b*x + 6*a) + (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*
d^2)*(b*x + a) + 2*d^2)*sin(4*b*x + 4*a) + (3*(...
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1801 vs. $2(267) = 534$.

time = 1.84, size = 1801, normalized size = 5.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*b^2*c^2 - 6*(b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*cos(b*x + a)^2 - 4*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a)
 + 6*((I*b*d^2*x + I*b*c*d)*cos(b*x + a)^3 + (-I*b*d^2*x - I*b*c*d)*cos(b*x
 + a))*dilog(cos(b*x + a) + I*sin(b*x + a)) + 6*((-I*b*d^2*x - I*b*c*d)*cos
(b*x + a)^3 + (I*b*d^2*x + I*b*c*d)*cos(b*x + a))*dilog(cos(b*x + a) - I*si
n(b*x + a)) + 4*(-I*d^2*cos(b*x + a)^3 + I*d^2*cos(b*x + a))*dilog(I*cos(b*
x + a) + sin(b*x + a)) + 4*(-I*d^2*cos(b*x + a)^3 + I*d^2*cos(b*x + a))*dil
og(I*cos(b*x + a) - sin(b*x + a)) + 4*(I*d^2*cos(b*x + a)^3 - I*d^2*cos(b*x
 + a))*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 4*(I*d^2*cos(b*x + a)^3 - I*
d^2*cos(b*x + a))*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 6*((I*b*d^2*x + I
*b*c*d)*cos(b*x + a)^3 + (-I*b*d^2*x - I*b*c*d)*cos(b*x + a))*dilog(-cos(b*
x + a) + I*sin(b*x + a)) + 6*((-I*b*d^2*x - I*b*c*d)*cos(b*x + a)^3 + (I*b*
d^2*x + I*b*c*d)*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) + ((3*
b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(b*x + a)^3 - (3*b^2*d^2*
x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(b*x + a))*log(cos(b*x + a) + I*s
in(b*x + a) + 1) + 4*((b*c*d - a*d^2)*cos(b*x + a)^3 - (b*c*d - a*d^2)*cos(
b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + I) + ((3*b^2*d^2*x^2 + 6*b^2*
c*d*x + 3*b^2*c^2 + 2*d^2)*cos(b*x + a)^3 - (3*b^2*d^2*x^2 + 6*b^2*c*d*x +
3*b^2*c^2 + 2*d^2)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 4
*((b*c*d - a*d^2)*cos(b*x + a)^3 - (b*c*d - a*d^2)*cos(b*x + a))*log(cos(b*
x + a) - I*sin(b*x + a) + I) + 4*((b*d^2*x + a*d^2)*cos(b*x + a)^3 - (b*d^2
*x + a*d^2)*cos(b*x + a))*log(I*cos(b*x + a) + sin(b*x + a) + 1) - 4*((b*d^
2*x + a*d^2)*cos(b*x + a)^3 - (b*d^2*x + a*d^2)*cos(b*x + a))*log(I*cos(b*x
 + a) - sin(b*x + a) + 1) + 4*((b*d^2*x + a*d^2)*cos(b*x + a)^3 - (b*d^2*x
+ a*d^2)*cos(b*x + a))*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - 4*((b*d^2*
x + a*d^2)*cos(b*x + a)^3 - (b*d^2*x + a*d^2)*cos(b*x + a))*log(-I*cos(b*x
+ a) - sin(b*x + a) + 1) - ((3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b
*x + a)^3 - (3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a))*log(-1/
2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - ((3*b^2*c^2 - 6*a*b*c*d + (3*a
^2 + 2)*d^2)*cos(b*x + a)^3 - (3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos
(b*x + a))*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 3*((b^2*d^2*
x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^3 - (b^2*d^2*x^2 + 2*
b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a))*log(-cos(b*x + a) + I*sin(b*
x + a) + 1) + 4*((b*c*d - a*d^2)*cos(b*x + a)^3 - (b*c*d - a*d^2)*cos(b*x +
 a))*log(-cos(b*x + a) + I*sin(b*x + a) + I) - 3*((b^2*d^2*x^2 + 2*b^2*c*d*
x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^3 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*
b*c*d - a^2*d^2)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 4*
((b*c*d - a*d^2)*cos(b*x + a)^3 - (b*c*d - a*d^2)*cos(b*x + a))*log(-cos(b*
```

```
x + a) - I*sin(b*x + a) + I) - 6*(d^2*cos(b*x + a)^3 - d^2*cos(b*x + a))*po
lylog(3, cos(b*x + a) + I*sin(b*x + a)) - 6*(d^2*cos(b*x + a)^3 - d^2*cos(b
*x + a))*polylog(3, cos(b*x + a) - I*sin(b*x + a)) + 6*(d^2*cos(b*x + a)^3
- d^2*cos(b*x + a))*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 6*(d^2*cos
(b*x + a)^3 - d^2*cos(b*x + a))*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))
/(b^3*cos(b*x + a)^3 - b^3*cos(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

3.281 $\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=154

$$-\frac{3dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{3c\tanh^{-1}(\cos(a+bx))}{2b}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{d\csc(a+bx)}{2b^2}+\frac{3id\text{PolyLog}(2,-e$$

```
[Out] -3*d*x*arctanh(exp(I*(b*x+a)))/b-3/2*c*arctanh(cos(b*x+a))/b-d*arctanh(sin(
b*x+a))/b^2-1/2*d*csc(b*x+a)/b^2+3/2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2-3/2
*I*d*polylog(2,exp(I*(b*x+a)))/b^2+3/2*(d*x+c)*sec(b*x+a)/b-1/2*(d*x+c)*csc
(b*x+a)^2*sec(b*x+a)/b
```

Rubi [A]

time = 0.12, antiderivative size = 174, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2702, 294, 327, 213, 4505, 6406, 12, 4268, 2317, 2438, 3855, 2701}

$$\frac{3id\text{Li}_2\left(-e^{i(a+bx)}\right)}{2b^2}-\frac{3id\text{Li}_2\left(e^{i(a+bx)}\right)}{2b^2}-\frac{d\csc(a+bx)}{2b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{3(c+dx)\sec(a+bx)}{2b}-\frac{3(c+dx)\tanh^{-1}(\cos(a+bx))}{2b}-\frac{(c+dx)\csc^2(a+bx)\sec(a+bx)}{2b}-\frac{3dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (-3*d*x*ArcTanh[E^(I*(a + b*x))])/b + (3*d*x*ArcTanh[Cos[a + b*x]])/(2*b) -
 (3*(c + d*x)*ArcTanh[Cos[a + b*x]])/(2*b) - (d*ArcTanh[Sin[a + b*x]])/b^2
- (d*Csc[a + b*x])/(2*b^2) + (((3*I)/2)*d*PolyLog[2, -E^(I*(a + b*x))])/b^2
 - (((3*I)/2)*d*PolyLog[2, E^(I*(a + b*x))])/b^2 + (3*(c + d*x)*Sec[a + b*x
])/(2*b) - ((c + d*x)*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6406

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3(c+dx)\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3(c+dx)\sec(a+bx)}{2b}-\frac{(c+$$

$$= -\frac{3(c+dx)\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3(c+dx)\sec(a+bx)}{2b}-\frac{(c+$$

$$= \frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3d\,t$$

$$= \frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3d\,t$$

$$= -\frac{3dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\,t$$

$$= -\frac{3dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\,t$$

$$= -\frac{3dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\,t$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 520 vs. 2(154) = 308.
time = 5.39, size = 520, normalized size = 3.38

$$\frac{dx}{b}-\frac{d\cot\left(\frac{1}{2}(a+bx)\right)}{4b^2}-\frac{c\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{dx\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{3c\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b}+\frac{d\log\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2}+\frac{3c\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{d\log\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2}-\frac{3ad\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)}{2b^2}+\frac{3d\left((a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)+i\left(\text{PolyLog}\left(2,-e^{i(a+bx)}\right)-\text{PolyLog}\left(2,e^{i(a+bx)}\right)\right)\right)}{2b^2}+\frac{c\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{dx\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{c\sin\left(\frac{1}{2}(a+bx)\right)}{b\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)}-\frac{c\sin\left(\frac{1}{2}(a+bx)\right)}{b\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)}+\frac{d\left(a\sin\left(\frac{1}{2}(a+bx)\right)-(a+bx)\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)}+\frac{d\left(-a\sin\left(\frac{1}{2}(a+bx)\right)+(a+bx)\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)}-\frac{d\tan\left(\frac{1}{2}(a+bx)\right)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (d*x)/b - (d*Cot[(a + b*x)/2])/(4*b^2) - (c*Csc[(a + b*x)/2]^2)/(8*b) - (d*
x*Csc[(a + b*x)/2]^2)/(8*b) - (3*c*Log[Cos[(a + b*x)/2]])/(2*b) + (d*Log[Co
```

```
s[(a + b*x)/2] - Sin[(a + b*x)/2]])/b^2 + (3*c*Log[Sin[(a + b*x)/2]])/(2*b)
 - (d*Log[Cos[(a + b*x)/2] + Sin[(a + b*x)/2]])/b^2 - (3*a*d*Log[Tan[(a + b
*x)/2]])/(2*b^2) + (3*d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I
*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x)
)])))/(2*b^2) + (c*Sec[(a + b*x)/2]^2)/(8*b) + (d*x*Sec[(a + b*x)/2]^2)/(8*
b) + (c*Sin[(a + b*x)/2])/(b*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - (c*Si
n[(a + b*x)/2])/(b*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])) + (d*(a*Sin[(a +
b*x)/2] - (a + b*x)*Sin[(a + b*x)/2]))/(b^2*(Cos[(a + b*x)/2] + Sin[(a + b*
x)/2])) + (d*(-(a*Sin[(a + b*x)/2]) + (a + b*x)*Sin[(a + b*x)/2]))/(b^2*(Co
s[(a + b*x)/2] - Sin[(a + b*x)/2])) - (d*Tan[(a + b*x)/2])/(4*b^2)
```

Maple [A]
time = 0.16, size = 267, normalized size = 1.73

method	result
risch	$\frac{3dxb\,\mathrm{e}^{5i(bx+a)}+3cb\,\mathrm{e}^{5i(bx+a)}-2dxb\,\mathrm{e}^{3i(bx+a)}-2cb\,\mathrm{e}^{3i(bx+a)}-id\,\mathrm{e}^{5i(bx+a)}+3dxb\,\mathrm{e}^{i(bx+a)}+3cb\,\mathrm{e}^{i(bx+a)}+id\,\mathrm{e}^{i(bx+a)}}{b^2\left(\mathrm{e}^{2i(bx+a)}-1\right)^2\left(1+\mathrm{e}^{2i(bx+a)}\right)}+\frac{3c\ln\left(\mathrm{e}^{i(bx+a)}\right.}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2/(1+exp(2*I*(b*x+a)))*(3*d*x*b*exp(5*I*(b*x+a))
+3*c*b*exp(5*I*(b*x+a))-2*d*x*b*exp(3*I*(b*x+a))-2*c*b*exp(3*I*(b*x+a))-I*d
*exp(5*I*(b*x+a))+3*d*x*b*exp(I*(b*x+a))+3*c*b*exp(I*(b*x+a))+I*d*exp(I*(b*
x+a)))+3/2/b*c*ln(exp(I*(b*x+a))-1)-3/2/b*c*ln(exp(I*(b*x+a))+1)-3/2/b^2*d*
a*ln(exp(I*(b*x+a))-1)+2*I/b^2*d*arctan(exp(I*(b*x+a)))+3/2*I/b^2*d*dilog(e
xp(I*(b*x+a)))+3/2*I/b^2*d*dilog(exp(I*(b*x+a))+1)-3/2/b*d*ln(exp(I*(b*x+a)
)+1)*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1481 vs. 2(130) = 260.
time = 0.79, size = 1481, normalized size = 9.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(4*(d*cos(6*b*x + 6*a) - d*cos(4*b*x + 4*a) - d*cos(2*b*x + 2*a) + I*d*sin
(6*b*x + 6*a) - I*d*sin(4*b*x + 4*a) - I*d*sin(2*b*x + 2*a) + d)*arctan2(2*
(cos(b*x + 2*a)*cos(a) + sin(b*x + 2*a)*sin(a))/(cos(b*x + 2*a)^2 + cos(a)^
2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) +
sin(a)^2), (cos(b*x + 2*a)^2 - cos(a)^2 + sin(b*x + 2*a)^2 - sin(a)^2)/(cos
(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*c
os(b*x + 2*a)*sin(a) + sin(a)^2)) + 6*(b*d*x + b*c + (b*d*x + b*c)*cos(6*b*
```

```
x + 6*a) - (b*d*x + b*c)*cos(4*b*x + 4*a) - (b*d*x + b*c)*cos(2*b*x + 2*a)
+ (I*b*d*x + I*b*c)*sin(6*b*x + 6*a) + (-I*b*d*x - I*b*c)*sin(4*b*x + 4*a)
+ (-I*b*d*x - I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) +
 1) - 6*(b*c*cos(6*b*x + 6*a) - b*c*cos(4*b*x + 4*a) - b*c*cos(2*b*x + 2*a)
 + I*b*c*sin(6*b*x + 6*a) - I*b*c*sin(4*b*x + 4*a) - I*b*c*sin(2*b*x + 2*a)
 + b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 6*(b*d*x*cos(6*b*x + 6*a)
 - b*d*x*cos(4*b*x + 4*a) - b*d*x*cos(2*b*x + 2*a) + I*b*d*x*sin(6*b*x + 6*
a) - I*b*d*x*sin(4*b*x + 4*a) - I*b*d*x*sin(2*b*x + 2*a) + b*d*x)*arctan2(s
in(b*x + a), -cos(b*x + a) + 1) + 4*(3*I*b*d*x + 3*I*b*c + d)*cos(5*b*x + 5
*a) + 8*(-I*b*d*x - I*b*c)*cos(3*b*x + 3*a) + 4*(3*I*b*d*x + 3*I*b*c - d)*c
os(b*x + a) - 6*(d*cos(6*b*x + 6*a) - d*cos(4*b*x + 4*a) - d*cos(2*b*x + 2*
a) + I*d*sin(6*b*x + 6*a) - I*d*sin(4*b*x + 4*a) - I*d*sin(2*b*x + 2*a) + d
)*dilog(-e^(I*b*x + I*a)) + 6*(d*cos(6*b*x + 6*a) - d*cos(4*b*x + 4*a) - d*
cos(2*b*x + 2*a) + I*d*sin(6*b*x + 6*a) - I*d*sin(4*b*x + 4*a) - I*d*sin(2*
b*x + 2*a) + d)*dilog(e^(I*b*x + I*a)) + 3*(-I*b*d*x - I*b*c + (-I*b*d*x -
I*b*c)*cos(6*b*x + 6*a) + (I*b*d*x + I*b*c)*cos(4*b*x + 4*a) + (I*b*d*x + I
*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(6*b*x + 6*a) - (b*d*x + b*c)*sin
(4*b*x + 4*a) - (b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*cos(b*x + a) + 1) + 3*(I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos
(6*b*x + 6*a) + (-I*b*d*x - I*b*c)*cos(4*b*x + 4*a) + (-I*b*d*x - I*b*c)*co
s(2*b*x + 2*a) - (b*d*x + b*c)*sin(6*b*x + 6*a) + (b*d*x + b*c)*sin(4*b*x +
 4*a) + (b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2
 - 2*cos(b*x + a) + 1) + 2*(I*d*cos(6*b*x + 6*a) - I*d*cos(4*b*x + 4*a) - I
*d*cos(2*b*x + 2*a) - d*sin(6*b*x + 6*a) + d*sin(4*b*x + 4*a) + d*sin(2*b*x
 + 2*a) + I*d)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) +
 sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 +
 cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*s
in(a) + sin(a)^2)) - 4*(3*b*d*x + 3*b*c - I*d)*sin(5*b*x + 5*a) + 8*(b*d*x
+ b*c)*sin(3*b*x + 3*a) - 4*(3*b*d*x + 3*b*c + I*d)*sin(b*x + a))/(-4*I*b^2
*cos(6*b*x + 6*a) + 4*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 4
*b^2*sin(6*b*x + 6*a) - 4*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 4
*I*b^2)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(130) = 260$.
time = 2.00, size = 621, normalized size = 4.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b*d*x - 6*(b*d*x + b*c)*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x +
 a) + 4*b*c + 3*(I*d*cos(b*x + a)^3 - I*d*cos(b*x + a))*dilog(cos(b*x + a)
+ I*sin(b*x + a)) + 3*(-I*d*cos(b*x + a)^3 + I*d*cos(b*x + a))*dilog(cos(b*
```

```
x + a) - I*sin(b*x + a)) + 3*(I*d*cos(b*x + a)^3 - I*d*cos(b*x + a))*dilog(
-cos(b*x + a) + I*sin(b*x + a)) + 3*(-I*d*cos(b*x + a)^3 + I*d*cos(b*x + a)
)*dilog(-cos(b*x + a) - I*sin(b*x + a)) + 3*((b*d*x + b*c)*cos(b*x + a)^3 -
 (b*d*x + b*c)*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + 1) + 3*((b
*d*x + b*c)*cos(b*x + a)^3 - (b*d*x + b*c)*cos(b*x + a))*log(cos(b*x + a) -
 I*sin(b*x + a) + 1) - 3*((b*c - a*d)*cos(b*x + a)^3 - (b*c - a*d)*cos(b*x
+ a))*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 3*((b*c - a*d)*co
s(b*x + a)^3 - (b*c - a*d)*cos(b*x + a))*log(-1/2*cos(b*x + a) - 1/2*I*sin(
b*x + a) + 1/2) - 3*((b*d*x + a*d)*cos(b*x + a)^3 - (b*d*x + a*d)*cos(b*x +
 a))*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 3*((b*d*x + a*d)*cos(b*x + a
)^3 - (b*d*x + a*d)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + 1) +
 2*(d*cos(b*x + a)^3 - d*cos(b*x + a))*log(sin(b*x + a) + 1) - 2*(d*cos(b*x
 + a)^3 - d*cos(b*x + a))*log(-sin(b*x + a) + 1))/(b^2*cos(b*x + a)^3 - b^2
*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a)**2,x)
[Out] Integral((c + d*x)*csc(a + b*x)**3*sec(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)^2*sin(a + b*x)^3),x)
[Out] \text{Hanged}
```

3.282 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x)
```

Rubi [A]
time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x),x]
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 21.03, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x),x]
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x), x]
```

Maple [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(\csc^3(bx+a))(\sec^2(bx+a))}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] (2*(b*d*x + b*c)*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) + (3*(b*d*x + b*c)*cos(5
*b*x + 5*a) - 2*(b*d*x + b*c)*cos(3*b*x + 3*a) + 3*(b*d*x + b*c)*cos(b*x +
a) - d*sin(5*b*x + 5*a) + d*sin(b*x + a))*cos(6*b*x + 6*a) + (3*b*d*x + 3*b
*c - 3*(b*d*x + b*c)*cos(4*b*x + 4*a) - 3*(b*d*x + b*c)*cos(2*b*x + 2*a) -
d*sin(4*b*x + 4*a) - d*sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + (2*(b*d*x + b*c
)*cos(3*b*x + 3*a) - 3*(b*d*x + b*c)*cos(b*x + a) - d*sin(b*x + a))*cos(4*b
*x + 4*a) - 2*(b*d*x + b*c - (b*d*x + b*c)*cos(2*b*x + 2*a))*cos(3*b*x + 3*
a) - (3*(b*d*x + b*c)*cos(b*x + a) + d*sin(b*x + a))*cos(2*b*x + 2*a) + 3*(
b*d*x + b*c)*cos(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(6*b*x + 6*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*co
s(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(6*b*x + 6*a)^2
 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 + 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a) - (b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(6*b*x + 6*a) - 2*(b^
2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*c
os(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*cos(2*b*x + 2*a) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*
a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a))*sin(6*b*x + 6*
a))*integrate(1/2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*sin(b*x
 + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)^2 + (b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x + a)^2 + 2*(b^2*d^3
*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)), x) + (b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos
(6*b*x + 6*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 +
 2*b^2*c*d*x + b^2*c^2)*sin(6*b*x + 6*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*sin(4*b*x + 4*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*
b*x + 4*a)*sin(2*b*x + 2*a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b
*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*cos(4*b*x + 4*a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*
```

```
cos(2*b*x + 2*a))*cos(6*b*x + 6*a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a)
 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a) - 2*((b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a) + (b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*sin(2*b*x + 2*a))*sin(6*b*x + 6*a))*integrate(1/2*(3*b^2*d^2*x^2
+ 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*
d*x + b^2*c^3)*cos(b*x + a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*
d*x + b^2*c^3)*sin(b*x + a)^2 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^
2*d*x + b^2*c^3)*cos(b*x + a)), x) + 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c
^2*d + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(6*b*x + 6*a)^2 + (b^2*
d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(4*b*x + 4*a)^2 + (b^2*d^3*x^2 + 2*
b^2*c*d^2*x + b^2*c^2*d)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x
+ b^2*c^2*d)*sin(6*b*x + 6*a)^2 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)
*sin(4*b*x + 4*a)^2 + 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(4*b*x
 + 4*a)*sin(2*b*x + 2*a) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(2*
b*x + 2*a)^2 + 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - (b^2*d^3*x^2 +
2*b^2*c*d^2*x + b^2*c^2*d)*cos(4*b*x + 4*a) - (b^2*d^3*x^2 + 2*b^2*c*d^2*x
+ b^2*c^2*d)*cos(2*b*x + 2*a))*cos(6*b*x + 6*a) - 2*(b^2*d^3*x^2 + 2*b^2*c*
d^2*x + b^2*c^2*d - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(2*b*x + 2
*a))*cos(4*b*x + 4*a) - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(2*b
*x + 2*a) - 2*((b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(4*b*x + 4*a) +
 (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(2*b*x + 2*a))*sin(6*b*x + 6*
a))*integrate((cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a
) + cos(b*x + a))/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x +
 b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*
a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)), x) + (d*cos(5*b
*x + 5*a) - d*cos(b*x + a) + 3*(b*d*x + b*c)*sin(5*b*x + 5*a) - 2*(b*d*x +
b*c)*sin(3*b*x + 3*a) + 3*(b*d*x + b*c)*sin(b*x + a))*sin(6*b*x + 6*a) + (d
*cos(4*b*x + 4*a) + d*cos(2*b*x + 2*a) - 3*(b*d*x + b*c)*sin(4*b*x + 4*a) -
 3*(b*d*x + b*c)*sin(2*b*x + 2*a) - d)*sin(5*b*x + 5*a) + (d*cos(b*x + a) +
 2*(b*d*x + b*c)*sin(3*b*x + 3*a) - 3*(b*d*x + b*c)*sin(b*x + a))*sin(4*b*x
 + 4*a) + (d*cos(b*x + a) - 3*(b*d*x + b*c)*sin...
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/(d*x + c), x)
```

Sympy [**A**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/(d*x+c),x)
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**2/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^2/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)),x)
[Out] int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)), x)
```

3.283 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)
```

Rubi [A]
time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2,x]
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx$$

Mathematica [A]
time = 29.15, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2,x]
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

Maple [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^3\left(bx+a\right)\right)\left(\sec^2\left(bx+a\right)\right)}{\left(dx+c\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)
[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] (2*(b*d*x + b*c)*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) + (3*(b*d*x + b*c)*cos(5
*b*x + 5*a) - 2*(b*d*x + b*c)*cos(3*b*x + 3*a) + 3*(b*d*x + b*c)*cos(b*x +
a) - 2*d*sin(5*b*x + 5*a) + 2*d*sin(b*x + a))*cos(6*b*x + 6*a) + (3*b*d*x +
 3*b*c - 3*(b*d*x + b*c)*cos(4*b*x + 4*a) - 3*(b*d*x + b*c)*cos(2*b*x + 2*a
) - 2*d*sin(4*b*x + 4*a) - 2*d*sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + (2*(b*d
*x + b*c)*cos(3*b*x + 3*a) - 3*(b*d*x + b*c)*cos(b*x + a) - 2*d*sin(b*x + a
))*cos(4*b*x + 4*a) - 2*(b*d*x + b*c - (b*d*x + b*c)*cos(2*b*x + 2*a))*cos(
3*b*x + 3*a) - (3*(b*d*x + b*c)*cos(b*x + a) + 2*d*sin(b*x + a))*cos(2*b*x
+ 2*a) + 3*(b*d*x + b*c)*cos(b*x + a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^
2*c^3)*cos(6*b*x + 6*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*cos(4*b*x + 4*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*
d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*
c^2*d*x + b^2*c^3)*sin(6*b*x + 6*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^
2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2
*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - (b^2*d^3*x^3 + 3*b^2
*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a) - (b^2*d^3*x^3 + 3*b
^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(6*b*x + 6*a)
- 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x +
 4*a) - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b
*x + 2*a) - 2*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*si
n(4*b*x + 4*a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*
sin(2*b*x + 2*a))*sin(6*b*x + 6*a))*integrate(3/2*(b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2 + 2*d^2)*sin(b*x + a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^
2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^
2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*
b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2
 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b
^2*c^4)*cos(b*x + a)), x) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(6
*b*x + 6*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*c
```

```
os(4*b*x + 4*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^
3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^
2*c^3)*sin(6*b*x + 6*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*sin(4*b*x + 4*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^
2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b^2*d^3*x^3 + 3*b^2*c
*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3
*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(6*b*x + 6*a) - 2*(b^2*d^3
*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - (b^2*d^3*x^3 + 3*b^2*c*d
^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b
^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a) -
2*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*
a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x +
2*a))*sin(6*b*x + 6*a))*integrate(3/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
+ 2*d^2)*sin(b*x + a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 +
4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^
2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^
3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2 - 2*(b^2*d^
4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(
b*x + a)), x) + 4*(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^
3*d + (b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*cos(6*b
*x + 6*a)^2 + (b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)
*cos(4*b*x + 4*a)^2 + (b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^
2*c^3*d)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^
2*x + b^2*c^3*d)*sin(6*b*x + 6*a)^2 + (b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^
2*c^2*d^2*x + b^2*c^3*d)*sin(4*b*x + 4*a)^2 + 2*(b^2*d^4*x^3 + 3*b^2*c*d^3*
x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b^2
*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*sin(2*b*x + 2*a)^
2 + 2*(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d - (b^2*d
^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*cos(4*b*x + 4*a) -
(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*cos(2*b*x + 2
*a))*cos(6*b*x + 6*a) - 2*(b^2*d^4*x^3 + 3*b^2*...
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [**A**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**2/(c + d*x)**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+b\,x)^2\,\sin(a+b\,x)^3\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^2),x)
```

```
[Out] int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^2), x)
```

3.284 $\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(x^m \csc^3(a+bx)\sec^2(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

Rubi [A]
time = 0.69, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] Defer[Int][x^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

Rubi steps

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx$$

Mathematica [A]
time = 36.04, size = 0, normalized size = 0.00

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] Integrate[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^m \left(\csc^3(bx+a)\right)\left(\sec^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
[Out] int(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csc(b*x+a)**3*sec(b*x+a)**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\cos(a+b\,x)^2\,\sin(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(cos(a + b*x)^2*sin(a + b*x)^3),x)
[Out] int(x^m/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

3.285 $\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=387

$$\frac{6ix^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}-\frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3}-\frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{3x^2\csc(a+bx)}{2b^2}+\frac{3i\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^4}$$

```
[Out] 3*I*polylog(2,-exp(I*(b*x+a)))/b^4-6*x*arctanh(exp(I*(b*x+a)))/b^3-3*x^3*ar
ctanh(exp(I*(b*x+a)))/b-3/2*x^2*csc(b*x+a)/b^2-9/2*I*x^2*polylog(2,exp(I*(b
*x+a)))/b^2+9/2*I*x^2*polylog(2,-exp(I*(b*x+a)))/b^2+6*I*x*polylog(2,I*exp(
I*(b*x+a)))/b^3+9*I*polylog(4,exp(I*(b*x+a)))/b^4-3*I*polylog(2,exp(I*(b*x+
a)))/b^4-9*I*polylog(4,-exp(I*(b*x+a)))/b^4-9*x*polylog(3,-exp(I*(b*x+a)))/
b^3+6*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*polylog(3,I*exp(I*(b*x+a)))/b^4+9*
x*polylog(3,exp(I*(b*x+a)))/b^3-6*I*x*polylog(2,-I*exp(I*(b*x+a)))/b^3+6*I*
x^2*arctan(exp(I*(b*x+a)))/b^2+3/2*x^3*sec(b*x+a)/b-1/2*x^3*csc(b*x+a)^2*se
c(b*x+a)/b
```

Rubi [A]
time = 0.63, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 18, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {2702, 294, 327, 213, 4505, 14, 6408, 12, 4268, 2611, 6744, 2320, 6724, 6874, 4266, 2701, 2317, 2438}

$$\frac{6ix^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}+\frac{3i\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^4}-\frac{3i\text{Li}_2\left(e^{i(a+bx)}\right)}{b^4}+\frac{6\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{6\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}-\frac{9i\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}+\frac{9i\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}-\frac{6ix\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6ix\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}-\frac{9x\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{9x\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3}+\frac{9ix^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{2b^2}-\frac{9ix^2\text{Li}_2\left(e^{i(a+bx)}\right)}{2b^2}-\frac{3x^2\csc(a+bx)}{2b^2}+\frac{3x^3\sec(a+bx)}{2b}-\frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] ((6*I)*x^2*ArcTan[E^(I*(a + b*x))])/b^2 - (6*x*ArcTanh[E^(I*(a + b*x))])/b^
3 - (3*x^3*ArcTanh[E^(I*(a + b*x))])/b - (3*x^2*Csc[a + b*x])/(2*b^2) + ((3
*I)*PolyLog[2, -E^(I*(a + b*x))])/b^4 + (((9*I)/2)*x^2*PolyLog[2, -E^(I*(a
+ b*x))])/b^2 - ((6*I)*x*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*x*P
olyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*PolyLog[2, E^(I*(a + b*x))])/b^4
 - (((9*I)/2)*x^2*PolyLog[2, E^(I*(a + b*x))])/b^2 - (9*x*PolyLog[3, -E^(I*
(a + b*x))])/b^3 + (6*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*PolyLog[3,
 I*E^(I*(a + b*x))])/b^4 + (9*x*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((9*I)*P
olyLog[4, -E^(I*(a + b*x))])/b^4 + ((9*I)*PolyLog[4, E^(I*(a + b*x))])/b^4
+ (3*x^3*Sec[a + b*x])/(2*b) - (x^3*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
```

```
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3x^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+}{2b}$$

$$= -\frac{3x^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+}{2b}$$

$$= -\frac{3x^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+}{2b}$$

$$= \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3\int bx^3\csc(a+bx)\,dx}{2b}$$

$$= \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3}{2}\int x^3\csc(a+bx)\,dx$$

$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3x^2\tanh^{-1}(\sin(a+bx)}{2b^2}$$

$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3x^2\tanh^{-1}(\sin(a+bx)}{2b^2}$$

$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3x^2\tanh^{-1}(\sin(a+bx)}{2b^2}$$

$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} -$$

$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} -$$

$$= \frac{6ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} -$$

$$= \frac{6ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} -$$

$$= \frac{6ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} -$$

$$= \frac{6ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} -$$

Mathematica [A]
time = 7.12, size = 616, normalized size = 1.59

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] -1/8*(x^3*Csc[a/2 + (b*x)/2]^2)/b + (6*(I*b^2*x^2*ArcTan[Cos[a + b*x] + I*S
in[a + b*x]] + I*b*x*PolyLog[2, I*Cos[a + b*x] - Sin[a + b*x]] - I*b*x*Poly
Log[2, (-I)*Cos[a + b*x] + Sin[a + b*x]] - PolyLog[3, I*Cos[a + b*x] - Sin[
a + b*x]] + PolyLog[3, (-I)*Cos[a + b*x] + Sin[a + b*x]]))/b^4 + (3*(2*b*x*
Log[1 - E^(I*(a + b*x))] + b^3*x^3*Log[1 - E^(I*(a + b*x))] - 2*b*x*Log[1 +
 E^(I*(a + b*x))] - b^3*x^3*Log[1 + E^(I*(a + b*x))] + I*(2 + 3*b^2*x^2)*Po
lyLog[2, -E^(I*(a + b*x))] - I*(2 + 3*b^2*x^2)*PolyLog[2, E^(I*(a + b*x))]
- 6*b*x*PolyLog[3, -E^(I*(a + b*x))] + 6*b*x*PolyLog[3, E^(I*(a + b*x))] -
(6*I)*PolyLog[4, -E^(I*(a + b*x))] + (6*I)*PolyLog[4, E^(I*(a + b*x))]))/(2
*b^4) + (x^3*Sec[a/2 + (b*x)/2]^2)/(8*b) + (x^2*Csc[a]*Sec[a]*(-3*Cos[a] +
2*b*x*Sin[a]))/(2*b^2) + (3*x^2*Csc[a/2]*Csc[a/2 + (b*x)/2]*Sin[(b*x)/2])/(
4*b^2) - (3*x^2*Sec[a/2]*Sec[a/2 + (b*x)/2]*Sin[(b*x)/2])/(4*b^2) + (x^3*Si
n[(b*x)/2])/(b*(Cos[a/2] - Sin[a/2])*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/
2])) - (x^3*Sin[(b*x)/2])/(b*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Si
n[a/2 + (b*x)/2]))
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int x^3 \left(\csc^3\left(bx+a\right)\right) \left(\sec^2\left(bx+a\right)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] int(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3940 vs. 2(315) = 630.
time = 0.99, size = 3940, normalized size = 10.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a^3*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log
(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) - 4*(12*((b*x + a)^2 - 2*(b*x
 + a)*a + a^2 + ((b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(6*b*x + 6*a) - ((b*
x + a)^2 - 2*(b*x + a)*a + a^2)*cos(4*b*x + 4*a) - ((b*x + a)^2 - 2*(b*x +
a)*a + a^2)*cos(2*b*x + 2*a) + (I*(b*x + a)^2 - 2*I*(b*x + a)*a + I*a^2)*si
n(6*b*x + 6*a) + (-I*(b*x + a)^2 + 2*I*(b*x + a)*a - I*a^2)*sin(4*b*x + 4*a
) + (-I*(b*x + a)^2 + 2*I*(b*x + a)*a - I*a^2)*sin(2*b*x + 2*a))*arctan2(co
s(b*x + a), sin(b*x + a) + 1) + 12*((b*x + a)^2 - 2*(b*x + a)*a + a^2 + ((b
*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(6*b*x + 6*a) - ((b*x + a)^2 - 2*(b*x +
 a)*a + a^2)*cos(4*b*x + 4*a) - ((b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(2*b
```

*x + 2*a) + (I*(b*x + a)^2 - 2*I*(b*x + a)*a + I*a^2)*sin(6*b*x + 6*a) + (-I*(b*x + a)^2 + 2*I*(b*x + a)*a - I*a^2)*sin(4*b*x + 4*a) + (-I*(b*x + a)^2 + 2*I*(b*x + a)*a - I*a^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 6*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a) + ((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a) - 2*a)*cos(6*b*x + 6*a) - ((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a) - 2*a)*cos(4*b*x + 4*a) - ((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a) - 2*a)*cos(2*b*x + 2*a) - (-I*(b*x + a)^3 + 3*I*(b*x + a)^2*a + (-3*I*a^2 - 2*I)*(b*x + a) + 2*I*a)*sin(6*b*x + 6*a) - (I*(b*x + a)^3 - 3*I*(b*x + a)^2*a + (3*I*a^2 + 2*I)*(b*x + a) - 2*I*a)*sin(4*b*x + 4*a) - (I*(b*x + a)^3 - 3*I*(b*x + a)^2*a + (3*I*a^2 + 2*I)*(b*x + a) - 2*I*a)*sin(2*b*x + 2*a) - 2*a)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 12*(a*cos(6*b*x + 6*a) - a*cos(4*b*x + 4*a) - a*cos(2*b*x + 2*a) + I*a*sin(6*b*x + 6*a) - I*a*sin(4*b*x + 4*a) - I*a*sin(2*b*x + 2*a) + a)*arctan2(sin(b*x + a), cos(b*x + a) - 1) - 6*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a) + ((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a))*cos(6*b*x + 6*a) - ((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a))*cos(4*b*x + 4*a) - ((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^3 + 3*I*(b*x + a)^2*a + (-3*I*a^2 - 2*I)*(b*x + a))*sin(6*b*x + 6*a) - (I*(b*x + a)^3 - 3*I*(b*x + a)^2*a + (3*I*a^2 + 2*I)*(b*x + a))*sin(4*b*x + 4*a) - (I*(b*x + a)^3 - 3*I*(b*x + a)^2*a + (3*I*a^2 + 2*I)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 12*(-I*(b*x + a)^3 + (b*x + a)^2*(3*I*a - 1) + (-3*I*a^2 + 2*a)*(b*x + a) - a^2)*cos(5*b*x + 5*a) + 8*(I*(b*x + a)^3 - 3*I*(b*x + a)^2*a + 3*I*(b*x + a)*a^2)*cos(3*b*x + 3*a) + 12*(-I*(b*x + a)^3 + (b*x + a)^2*(3*I*a + 1) + (-3*I*a^2 - 2*a)*(b*x + a) + a^2)*cos(b*x + a) + 24*(b*x*cos(6*b*x + 6*a) - b*x*cos(4*b*x + 4*a) - b*x*cos(2*b*x + 2*a) + I*b*x*sin(6*b*x + 6*a) - I*b*x*sin(4*b*x + 4*a) - I*b*x*sin(2*b*x + 2*a) + b*x)*dilog(I*e^(I*b*x + I*a)) - 24*(b*x*cos(6*b*x + 6*a) - b*x*cos(4*b*x + 4*a) - b*x*cos(2*b*x + 2*a) + I*b*x*sin(6*b*x + 6*a) - I*b*x*sin(4*b*x + 4*a) - I*b*x*sin(2*b*x + 2*a) + b*x)*dilog(-I*e^(I*b*x + I*a)) + 6*(3*(b*x + a)^2 - 6*(b*x + a)*a + 3*a^2 + (3*(b*x + a)^2 - 6*(b*x + a)*a + 3*a^2 + 2)*cos(6*b*x + 6*a) - (3*(b*x + a)^2 - 6*(b*x + a)*a + 3*a^2 + 2)*cos(4*b*x + 4*a) - (3*(b*x + a)^2 - 6*(b*x + a)*a + 3*a^2 + 2)*cos(2*b*x + 2*a) + (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2 + 2*I)*sin(6*b*x + 6*a) + (-3*I*(b*x + a)^2 + 6*I*(b*x + a)*a - 3*I*a^2 - 2*I)*sin(4*b*x + 4*a) + (-3*I*(b*x + a)^2 + 6*I*(b*x + a)*a - 3*I*a^2 - 2*I)*sin(2*b*x + 2*a) + 2)*dilog(-e^(I*b*x + I*a)) - 6*(3*(b*x + a)^2 - 6*(b*x + a)*a + 3*a^2 + (3*(b*x + a)^2 - 6*(b*x + a)*a + 3*a^2 + 2)*cos(6*b*x + 6*a) - (3*(b*x + a)^2 - 6*(b*x + a)*a + 3*a^2 + 2)*cos(4*b*x + 4*a) - (3*(b*x + a)^2 - 6*(b*x + a)*a + 3*a^2 + 2)*cos(2*b*x + 2*a) - (-3*I*(b*x + a)^2 + 6*I*(b*x + a)*a - 3*I*a^2 - 2*I)*sin(6*b*x + 6*a) - (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2 + 2*I)*sin(4*b*x + 4*a) - (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2 + 2*I)*sin(2*b*x + 2*a) + 2)*dilog(e^(I*b*x + I*a)) + 3*(I*(b*x + a)^3 - 3*I*(b*x + a)^2*a + (3*I*a^2 + 2*I)*(b*x + a) + (I*(b*x + a)^3 - 3*I*(b*x + a)^2*a + (3*I*a^2 + 2*I)*(b*x + a) - 2*I*a)*cos(6*b*x + 6*a) + (-I*(b*x + a

```
)^3 + 3*I*(b*x + a)^2*a + (-3*I*a^2 - 2*I)*(b*x + a) + 2*I*a)*cos(4*b*x + 4
*a) + (-I*(b*x + a)^3 + 3*I*(b*x + a)^2*a + (-3*I*a^2 - 2*I)*(b*x + a) + 2*
I*a)*cos(2*b*x + 2*a) - ((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x +
 a) - 2*a)*sin(6*b*x + 6*a) + ((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*
(b*x + a) - 2*a)*sin(4*b*x + 4*a) + ((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2
 + 2)*(b*x + a) - 2*a)*sin(2*b*x + 2*a) - 2*I*a)*log(cos(b*x + a)^2 + sin(b
*x + a)^2 + 2*cos(b*x + a) + 1) + 3*(-I*(b*x + a)^3 + 3*I*(b*x + a)^2*a + (
-3*I*a^2 - 2*I)*(b*x + a) + (-I*(b*x + a)^3 + 3*I*(b*x + a)^2*a + (-3*I*a^2
 - 2*I)*(b*x + a) + 2*I*a)*cos(6*b*x + 6*a) + (I*(b*x + a)^3 - 3*I*(b*x + a
)^2*a + (3*I*a^2 + 2*I)*(b*x + a) - 2*I*a)*cos(4*b*x + 4*a) + (I*(b*x + a)^
3 - 3*I*(b*x + a)^2*a + (3*I*a^2 + 2*I)*(b*x + a) - 2*I*a)*cos(2*b*x + 2*a)
 + ((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1747 vs. $2(315) = 630$.
time = 4.43, size = 1747, normalized size = 4.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*b^3*x^3*cos(b*x + a)^2 - 4*b^3*x^3 + 6*b^2*x^2*cos(b*x + a)*sin(b*x
+ a) - 3*((3*I*b^2*x^2 + 2*I)*cos(b*x + a)^3 + (-3*I*b^2*x^2 - 2*I)*cos(b*x
 + a))*dilog(cos(b*x + a) + I*sin(b*x + a)) - 3*((-3*I*b^2*x^2 - 2*I)*cos(b
*x + a)^3 + (3*I*b^2*x^2 + 2*I)*cos(b*x + a))*dilog(cos(b*x + a) - I*sin(b*
x + a)) - 12*(-I*b*x*cos(b*x + a)^3 + I*b*x*cos(b*x + a))*dilog(I*cos(b*x +
 a) + sin(b*x + a)) - 12*(-I*b*x*cos(b*x + a)^3 + I*b*x*cos(b*x + a))*dilog
(I*cos(b*x + a) - sin(b*x + a)) - 12*(I*b*x*cos(b*x + a)^3 - I*b*x*cos(b*x
+ a))*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 12*(I*b*x*cos(b*x + a)^3 - I*
b*x*cos(b*x + a))*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*((3*I*b^2*x^2 +
 2*I)*cos(b*x + a)^3 + (-3*I*b^2*x^2 - 2*I)*cos(b*x + a))*dilog(-cos(b*x +
a) + I*sin(b*x + a)) - 3*((-3*I*b^2*x^2 - 2*I)*cos(b*x + a)^3 + (3*I*b^2*x^
2 + 2*I)*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*((b^3*x^3
+ 2*b*x)*cos(b*x + a)^3 - (b^3*x^3 + 2*b*x)*cos(b*x + a))*log(cos(b*x + a)
+ I*sin(b*x + a) + 1) - 6*(a^2*cos(b*x + a)^3 - a^2*cos(b*x + a))*log(cos(b
*x + a) + I*sin(b*x + a) + I) - 3*((b^3*x^3 + 2*b*x)*cos(b*x + a)^3 - (b^3*
x^3 + 2*b*x)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 6*(a^2*
cos(b*x + a)^3 - a^2*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + I) -
 6*((b^2*x^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 - a^2)*cos(b*x + a))*log(I*co
s(b*x + a) + sin(b*x + a) + 1) + 6*((b^2*x^2 - a^2)*cos(b*x + a)^3 - (b^2*x
^2 - a^2)*cos(b*x + a))*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 6*((b^2*x^
2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 - a^2)*cos(b*x + a))*log(-I*cos(b*x + a)
 + sin(b*x + a) + 1) + 6*((b^2*x^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 - a^2)*
cos(b*x + a))*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - 3*((a^3 + 2*a)*cos(
```

```
b*x + a)^3 - (a^3 + 2*a)*cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*
x + a) + 1/2) - 3*((a^3 + 2*a)*cos(b*x + a)^3 - (a^3 + 2*a)*cos(b*x + a))*l
og(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*((b^3*x^3 + a^3 + 2*b*
x + 2*a)*cos(b*x + a)^3 - (b^3*x^3 + a^3 + 2*b*x + 2*a)*cos(b*x + a))*log(-
cos(b*x + a) + I*sin(b*x + a) + 1) - 6*(a^2*cos(b*x + a)^3 - a^2*cos(b*x +
a))*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 3*((b^3*x^3 + a^3 + 2*b*x + 2
*a)*cos(b*x + a)^3 - (b^3*x^3 + a^3 + 2*b*x + 2*a)*cos(b*x + a))*log(-cos(b
*x + a) - I*sin(b*x + a) + 1) + 6*(a^2*cos(b*x + a)^3 - a^2*cos(b*x + a))*l
og(-cos(b*x + a) - I*sin(b*x + a) + I) - 18*(-I*cos(b*x + a)^3 + I*cos(b*x
+ a))*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 18*(I*cos(b*x + a)^3 - I*
cos(b*x + a))*polylog(4, cos(b*x + a) - I*sin(b*x + a)) - 18*(-I*cos(b*x +
a)^3 + I*cos(b*x + a))*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) - 18*(I*c
os(b*x + a)^3 - I*cos(b*x + a))*polylog(4, -cos(b*x + a) - I*sin(b*x + a))
+ 18*(b*x*cos(b*x + a)^3 - b*x*cos(b*x + a))*polylog(3, cos(b*x + a) + I*si
n(b*x + a)) + 18*(b*x*cos(b*x + a)^3 - b*x*cos(b*x + a))*polylog(3, cos(b*x
 + a) - I*sin(b*x + a)) + 12*(cos(b*x + a)^3 - cos(b*x + a))*polylog(3, I*c
os(b*x + a) + sin(b*x + a)) - 12*(cos(b*x + a)^3 - cos(b*x + a))*polylog(3,
 I*cos(b*x + a) - sin(b*x + a)) + 12*(cos(b*x + a)^3 - cos(b*x + a))*polylo
g(3, -I*cos(b*x + a) + sin(b*x + a)) - 12*(cos(b*x + a)^3 - cos(b*x + a))*p
olylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 18*(b*x*cos(b*x + a)^3 - b*x*co
s(b*x + a))*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 18*(b*x*cos(b*x +
a)^3 - b*x*cos(b*x + a))*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^4*c
os(b*x + a)^3 - b^4*cos(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(cos(a + b*x)^2*sin(a + b*x)^3),x)
[Out] int(x^3/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

3.286 $\int x^2 \csc^3(a+bx) \sec^2(a+bx) \, dx$

Optimal. Leaf size=235

$$\frac{4ix\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{x\csc(a+bx)}{b^2}+\frac{3ix\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}$$

```
[Out] 4*I*x*arctan(exp(I*(b*x+a)))/b^2-3*x^2*arctanh(exp(I*(b*x+a)))/b-arctanh(co
s(b*x+a))/b^3-x*csc(b*x+a)/b^2+3*I*x*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*pol
ylog(2,-I*exp(I*(b*x+a)))/b^3+2*I*polylog(2,I*exp(I*(b*x+a)))/b^3-3*I*x*pol
ylog(2,exp(I*(b*x+a)))/b^2-3*polylog(3,-exp(I*(b*x+a)))/b^3+3*polylog(3,exp
(I*(b*x+a)))/b^3+3/2*x^2*sec(b*x+a)/b-1/2*x^2*csc(b*x+a)^2*sec(b*x+a)/b
```

Rubi [A]
time = 0.37, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 19, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.950$, Rules used = {2702, 294, 327, 213, 4505, 14, 6408, 12, 4268, 2611, 2320, 6724, 6874, 4266, 2317, 2438, 2701, 6406, 3855}

$$\frac{4ix\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2i\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2i\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}-\frac{3\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{3\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{\tanh^{-1}(\cos(a+bx))}{b^3}+\frac{3ix\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{3ix\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{x\csc(a+bx)}{b^2}+\frac{3x^2\sec(a+bx)}{2b}-\frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] ((4*I)*x*ArcTan[E^(I*(a + b*x))])/b^2 - (3*x^2*ArcTanh[E^(I*(a + b*x))])/b
- ArcTanh[Cos[a + b*x]]/b^3 - (x*Csc[a + b*x])/b^2 + ((3*I)*x*PolyLog[2, -E
^(I*(a + b*x))])/b^2 - ((2*I)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((2*I
)*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*x*PolyLog[2, E^(I*(a + b*x))]
)/b^2 - (3*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (3*PolyLog[3, E^(I*(a + b*x)
)])/b^3 + (3*x^2*Sec[a + b*x])/(2*b) - (x^2*Csc[a + b*x]^2*Sec[a + b*x])/(2
*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6406

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3x^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x^2\sec(a+bx)}{2b} - \frac{x^2\csc^2(a+bx)\sec(a+}{2b}$$

$$= -\frac{3x^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x^2\sec(a+bx)}{2b} - \frac{x^2\csc^2(a+bx)\sec(a+}{2b}$$

$$= -\frac{3x^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x^2\sec(a+bx)}{2b} - \frac{x^2\csc^2(a+bx)\sec(a+}{2b}$$

$$= \frac{3x^2\sec(a+bx)}{2b} - \frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{\int(-3x\sec(a+bx)+}{}$$

$$= \frac{3x^2\sec(a+bx)}{2b} - \frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3}{2}\int x^2\csc(a+bx)\,dx$$

$$= \frac{6ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{x\tanh^{-1}(\sin(a+bx))}{b^2} -$$

$$= \frac{6ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{x\tanh^{-1}(\sin(a+bx))}{b^2} -$$

$$= \frac{6ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{\tanh^{-1}(\cos(a+bx))}{b^3} -$$

$$= \frac{6ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{\tanh^{-1}(\cos(a+bx))}{b^3} -$$

$$= \frac{4ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{\tanh^{-1}(\cos(a+bx))}{b^3} -$$

$$= \frac{4ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{\tanh^{-1}(\cos(a+bx))}{b^3} -$$

$$= \frac{4ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{\tanh^{-1}(\cos(a+bx))}{b^3} -$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 557 vs. $2(235) = 470$.
time = 6.64, size = 557, normalized size = 2.37

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] -1/8*(x^2*Csc[a/2 + (b*x)/2]^2)/b - (2*((-a + Pi/2 - b*x)*(Log[1 - E^(I*(-a
 + Pi/2 - b*x))] - Log[1 + E^(I*(-a + Pi/2 - b*x))]) - (-a + Pi/2)*Log[Tan[
(-a + Pi/2 - b*x)/2]] + I*(PolyLog[2, -E^(I*(-a + Pi/2 - b*x))] - PolyLog[2
, E^(I*(-a + Pi/2 - b*x))])))/b^3 - (2*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x
```

```
]] + 3*b^2*x^2*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] - (3*I)*b*x*PolyLog[2
, -Cos[a + b*x] - I*Sin[a + b*x]] + (3*I)*b*x*PolyLog[2, Cos[a + b*x] + I*S
in[a + b*x]] + 3*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] - 3*PolyLog[3,
Cos[a + b*x] + I*Sin[a + b*x]])/b^3 + (x^2*Sec[a/2 + (b*x)/2]^2)/(8*b) + (x
*Csc[a]*Sec[a]*(-Cos[a] + b*x*Sin[a]))/b^2 + (x*Csc[a/2]*Csc[a/2 + (b*x)/2]
*Sin[(b*x)/2])/(2*b^2) - (x*Sec[a/2]*Sec[a/2 + (b*x)/2]*Sin[(b*x)/2])/(2*b^
2) + (x^2*Sin[(b*x)/2])/(b*(Cos[a/2] - Sin[a/2])*(Cos[a/2 + (b*x)/2] - Sin[
a/2 + (b*x)/2])) - (x^2*Sin[(b*x)/2])/(b*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (
b*x)/2] + Sin[a/2 + (b*x)/2]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(208) = 416$.
time = 0.24, size = 429, normalized size = 1.83

method	result
risch	$\frac{x\left(3bx\,e^{5i(bx+a)}-2bx\,e^{3i(bx+a)}-2i e^{5i(bx+a)}+3bx\,e^{i(bx+a)}+2i e^{i(bx+a)}\right)}{b^{2}\left(e^{2i(bx+a)}-1\right)^{2}\left(1+e^{2i(bx+a)}\right)}+\frac{3a^{2}\ln\left(e^{i(bx+a)}-1\right)}{2b^{3}}-\frac{3a^{2}\ln\left(1-e^{i(bx+a)}\right)}{2b^{3}}+$ 3ix polylo

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/b^2/(exp(2*I*(b*x+a))-1)^2/(1+exp(2*I*(b*x+a)))*(3*b*x*exp(5*I*(b*x+a))-2
*b*x*exp(3*I*(b*x+a))-2*I*exp(5*I*(b*x+a))+3*b*x*exp(I*(b*x+a))+2*I*exp(I*(
b*x+a)))+3/2/b^3*a^2*ln(exp(I*(b*x+a))-1)-3/2/b^3*a^2*ln(1-exp(I*(b*x+a)))-
3*I*x*polylog(2,exp(I*(b*x+a)))/b^2-4*I/b^3*a*arctan(exp(I*(b*x+a)))+2/b^3*
ln(1+I*exp(I*(b*x+a)))*a-2/b^3*ln(1-I*exp(I*(b*x+a)))*a+3*I*x*polylog(2,-ex
p(I*(b*x+a)))/b^2+1/b^3*ln(exp(I*(b*x+a))-1)-1/b^3*ln(exp(I*(b*x+a))+1)-3*p
olylog(3,-exp(I*(b*x+a)))/b^3+3*polylog(3,exp(I*(b*x+a)))/b^3+2/b^2*ln(1+I*
exp(I*(b*x+a)))*x+3/2/b*ln(1-exp(I*(b*x+a)))*x^2-2/b^2*ln(1-I*exp(I*(b*x+a)
))*x-2*I/b^3*dilog(1+I*exp(I*(b*x+a)))-3/2/b*ln(exp(I*(b*x+a))+1)*x^2+2*I/b
^3*dilog(1-I*exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2205 vs. $2(197) = 394$.
time = 0.70, size = 2205, normalized size = 9.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(
cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) + 4*(8*(b*x*cos(6*b*x + 6*a) -
 b*x*cos(4*b*x + 4*a) - b*x*cos(2*b*x + 2*a) + I*b*x*sin(6*b*x + 6*a) - I*b
*x*sin(4*b*x + 4*a) - I*b*x*sin(2*b*x + 2*a) + b*x)*arctan2(cos(b*x + a), s
```

in(b*x + a) + 1) + 8*(b*x*cos(6*b*x + 6*a) - b*x*cos(4*b*x + 4*a) - b*x*cos
(2*b*x + 2*a) + I*b*x*sin(6*b*x + 6*a) - I*b*x*sin(4*b*x + 4*a) - I*b*x*sin
(2*b*x + 2*a) + b*x)*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 2*(3*(b*x +
 a)^2 - 6*(b*x + a)*a + (3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*cos(6*b*x + 6*a
) - (3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*cos(4*b*x + 4*a) - (3*(b*x + a)^2 -
 6*(b*x + a)*a + 2)*cos(2*b*x + 2*a) - (-3*I*(b*x + a)^2 + 6*I*(b*x + a)*a
- 2*I)*sin(6*b*x + 6*a) - (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 2*I)*sin(4*b
*x + 4*a) - (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 2*I)*sin(2*b*x + 2*a) + 2)
*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 4*(cos(6*b*x + 6*a) - cos(4*b*x
+ 4*a) - cos(2*b*x + 2*a) + I*sin(6*b*x + 6*a) - I*sin(4*b*x + 4*a) - I*sin
(2*b*x + 2*a) + 1)*arctan2(sin(b*x + a), cos(b*x + a) - 1) - 6*((b*x + a)^2
 - 2*(b*x + a)*a + ((b*x + a)^2 - 2*(b*x + a)*a)*cos(6*b*x + 6*a) - ((b*x +
 a)^2 - 2*(b*x + a)*a)*cos(4*b*x + 4*a) - ((b*x + a)^2 - 2*(b*x + a)*a)*cos
(2*b*x + 2*a) - (-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*sin(6*b*x + 6*a) - (I*(b
*x + a)^2 - 2*I*(b*x + a)*a)*sin(4*b*x + 4*a) - (I*(b*x + a)^2 - 2*I*(b*x +
 a)*a)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 4*(-3*I
*(b*x + a)^2 + 2*(b*x + a)*(3*I*a - 1) + 2*a)*cos(5*b*x + 5*a) + 8*(I*(b*x
+ a)^2 - 2*I*(b*x + a)*a)*cos(3*b*x + 3*a) + 4*(-3*I*(b*x + a)^2 + 2*(b*x +
 a)*(3*I*a + 1) - 2*a)*cos(b*x + a) + 8*(cos(6*b*x + 6*a) - cos(4*b*x + 4*a
) - cos(2*b*x + 2*a) + I*sin(6*b*x + 6*a) - I*sin(4*b*x + 4*a) - I*sin(2*b*
x + 2*a) + 1)*dilog(I*e^(I*b*x + I*a)) - 8*(cos(6*b*x + 6*a) - cos(4*b*x +
4*a) - cos(2*b*x + 2*a) + I*sin(6*b*x + 6*a) - I*sin(4*b*x + 4*a) - I*sin(2
*b*x + 2*a) + 1)*dilog(-I*e^(I*b*x + I*a)) + 12*(b*x*cos(6*b*x + 6*a) - b*x
*cos(4*b*x + 4*a) - b*x*cos(2*b*x + 2*a) + I*b*x*sin(6*b*x + 6*a) - I*b*x*s
in(4*b*x + 4*a) - I*b*x*sin(2*b*x + 2*a) + b*x)*dilog(-e^(I*b*x + I*a)) - 1
2*(b*x*cos(6*b*x + 6*a) - b*x*cos(4*b*x + 4*a) - b*x*cos(2*b*x + 2*a) + I*b
*x*sin(6*b*x + 6*a) - I*b*x*sin(4*b*x + 4*a) - I*b*x*sin(2*b*x + 2*a) + b*x
)*dilog(e^(I*b*x + I*a)) - (-3*I*(b*x + a)^2 + 6*I*(b*x + a)*a + (-3*I*(b*x
 + a)^2 + 6*I*(b*x + a)*a - 2*I)*cos(6*b*x + 6*a) + (3*I*(b*x + a)^2 - 6*I*
(b*x + a)*a + 2*I)*cos(4*b*x + 4*a) + (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a +
2*I)*cos(2*b*x + 2*a) + (3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*sin(6*b*x + 6*a
) - (3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*sin(4*b*x + 4*a) - (3*(b*x + a)^2 -
 6*(b*x + a)*a + 2)*sin(2*b*x + 2*a) - 2*I)*log(cos(b*x + a)^2 + sin(b*x +
a)^2 + 2*cos(b*x + a) + 1) - (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a + (3*I*(b*x
 + a)^2 - 6*I*(b*x + a)*a + 2*I)*cos(6*b*x + 6*a) + (-3*I*(b*x + a)^2 + 6*I
*(b*x + a)*a - 2*I)*cos(4*b*x + 4*a) + (-3*I*(b*x + a)^2 + 6*I*(b*x + a)*a
- 2*I)*cos(2*b*x + 2*a) - (3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*sin(6*b*x + 6
a) + (3(b*x + a)^2 - 6*(b*x + a)*a + 2)*sin(4*b*x + 4*a) + (3*(b*x + a)^2
 - 6*(b*x + a)*a + 2)*sin(2*b*x + 2*a) + 2*I)*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 - 2*cos(b*x + a) + 1) + 4*(I*b*x*cos(6*b*x + 6*a) - I*b*x*cos(4*b*x
+ 4*a) - I*b*x*cos(2*b*x + 2*a) - b*x*sin(6*b*x + 6*a) + b*x*sin(4*b*x + 4*
a) + b*x*sin(2*b*x + 2*a) + I*b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*
sin(b*x + a) + 1) + 4*(-I*b*x*cos(6*b*x + 6*a) + I*b*x*cos(4*b*x + 4*a) + I
*b*x*cos(2*b*x + 2*a) + b*x*sin(6*b*x + 6*a) - b*x*sin(4*b*x + 4*a) - b*x*s
in(2*b*x + 2*a) - I*b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x +

```
a) + 1) + 12*(I*cos(6*b*x + 6*a) - I*cos(4*b*x + 4*a) - I*cos(2*b*x + 2*a)
- sin(6*b*x + 6*a) + sin(4*b*x + 4*a) + sin(2*b*x + 2*a) + I)*polylog(3, -e
^(I*b*x + I*a)) + 12*(-I*cos(6*b*x + 6*a) + I*cos(4*b*x + 4*a) + I*cos(2*b*
x + 2*a) + sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - sin(2*b*x + 2*a) - I)*poly
log(3, e^(I*b*x + I*a)) + 4*(3*(b*x + a)^2 - 2*(b*x + a)*(3*a + I) + 2*I*a)
*sin(5*b*x + 5*a) - 8*((b*x + a)^2 - 2*(b*x + a)*a)*sin(3*b*x + 3*a) + 4*(3
*(b*x + a)^2 - 2*(b*x + a)*(3*a - I) - 2*I*a)*sin(b*x + a))/(-4*I*cos(6*b*x
 + 6*a) + 4*I*cos(4*b*x + 4*a) + 4*I*cos(2*b*x + 2*a) + 4*sin(6*b*x + 6*a)
- 4*sin(4*b*x + 4*a) - 4*sin(2*b*x + 2*a) - 4*I))/b^3
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1237 vs. $2(197) = 394$.
time = 3.59, size = 1237, normalized size = 5.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*b^2*x^2*cos(b*x + a)^2 - 4*b^2*x^2 + 4*b*x*cos(b*x + a)*sin(b*x + a)
 - 6*(I*b*x*cos(b*x + a)^3 - I*b*x*cos(b*x + a))*dilog(cos(b*x + a) + I*sin
(b*x + a)) - 6*(-I*b*x*cos(b*x + a)^3 + I*b*x*cos(b*x + a))*dilog(cos(b*x +
 a) - I*sin(b*x + a)) - 4*(-I*cos(b*x + a)^3 + I*cos(b*x + a))*dilog(I*cos(
b*x + a) + sin(b*x + a)) - 4*(-I*cos(b*x + a)^3 + I*cos(b*x + a))*dilog(I*c
os(b*x + a) - sin(b*x + a)) - 4*(I*cos(b*x + a)^3 - I*cos(b*x + a))*dilog(-
I*cos(b*x + a) + sin(b*x + a)) - 4*(I*cos(b*x + a)^3 - I*cos(b*x + a))*dilo
g(-I*cos(b*x + a) - sin(b*x + a)) - 6*(I*b*x*cos(b*x + a)^3 - I*b*x*cos(b*x
 + a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 6*(-I*b*x*cos(b*x + a)^3 + I
*b*x*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) - ((3*b^2*x^2 + 2)
*cos(b*x + a)^3 - (3*b^2*x^2 + 2)*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*
x + a) + 1) + 4*(a*cos(b*x + a)^3 - a*cos(b*x + a))*log(cos(b*x + a) + I*si
n(b*x + a) + I) - ((3*b^2*x^2 + 2)*cos(b*x + a)^3 - (3*b^2*x^2 + 2)*cos(b*x
 + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 4*(a*cos(b*x + a)^3 - a*cos
(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + I) - 4*((b*x + a)*cos(b*x +
a)^3 - (b*x + a)*cos(b*x + a))*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 4*(
(b*x + a)*cos(b*x + a)^3 - (b*x + a)*cos(b*x + a))*log(I*cos(b*x + a) - sin
(b*x + a) + 1) - 4*((b*x + a)*cos(b*x + a)^3 - (b*x + a)*cos(b*x + a))*log(
-I*cos(b*x + a) + sin(b*x + a) + 1) + 4*((b*x + a)*cos(b*x + a)^3 - (b*x +
a)*cos(b*x + a))*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + ((3*a^2 + 2)*cos
(b*x + a)^3 - (3*a^2 + 2)*cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*sin(b
*x + a) + 1/2) + ((3*a^2 + 2)*cos(b*x + a)^3 - (3*a^2 + 2)*cos(b*x + a))*lo
g(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*((b^2*x^2 - a^2)*cos(b*
x + a)^3 - (b^2*x^2 - a^2)*cos(b*x + a))*log(-cos(b*x + a) + I*sin(b*x + a)
 + 1) + 4*(a*cos(b*x + a)^3 - a*cos(b*x + a))*log(-cos(b*x + a) + I*sin(b*x
 + a) + I) + 3*((b^2*x^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 - a^2)*cos(b*x +
```

```
a))*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 4*(a*cos(b*x + a)^3 - a*cos(b
*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 6*(cos(b*x + a)^3 - cos(
b*x + a))*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(cos(b*x + a)^3 - c
os(b*x + a))*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*(cos(b*x + a)^3
- cos(b*x + a))*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 6*(cos(b*x + a
)^3 - cos(b*x + a))*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^3*cos(b*
x + a)^3 - b^3*cos(b*x + a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csc(b*x+a)**3*sec(b*x+a)**2,x)
[Out] Integral(x**2*csc(a + b*x)**3*sec(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^2*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\cos(a+b\,x)^2\,\sin(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cos(a + b*x)^2*sin(a + b*x)^3),x)
[Out] int(x^2/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

3.287 $\int x \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=126

$$-\frac{3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{\csc(a+bx)}{2b^2}+\frac{3i\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{3i\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}$$

```
[Out] -3*x*arctanh(exp(I*(b*x+a)))/b-arctanh(sin(b*x+a))/b^2-1/2*csc(b*x+a)/b^2+3
/2*I*polylog(2,-exp(I*(b*x+a)))/b^2-3/2*I*polylog(2,exp(I*(b*x+a)))/b^2+3/2
*x*sec(b*x+a)/b-1/2*x*csc(b*x+a)^2*sec(b*x+a)/b
```

Rubi [A]
time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2702, 294, 327, 213, 4505, 6406, 12, 4268, 2317, 2438, 3855, 2701}

$$\frac{3i\text{Li}_2\left(-e^{i(a+bx)}\right)}{2b^2}-\frac{3i\text{Li}_2\left(e^{i(a+bx)}\right)}{2b^2}-\frac{\csc(a+bx)}{2b^2}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{3x\sec(a+bx)}{2b}-\frac{3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{x\csc^2(a+bx)\sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (-3*x*ArcTanh[E^(I*(a + b*x))])/b - ArcTanh[Sin[a + b*x]]/b^2 - Csc[a + b*x
]/(2*b^2) + (((3*I)/2)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (((3*I)/2)*PolyL
og[2, E^(I*(a + b*x))])/b^2 + (3*x*Sec[a + b*x])/(2*b) - (x*Csc[a + b*x]^2*
Sec[a + b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6406

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int x\csc^3(a+bx)\sec^2(a+bx)\,dx &= -\frac{3x\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)\sec(a+bx)}{2b} \\
&= -\frac{3x\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)\sec(a+bx)}{2b} \\
&= -\frac{3\tanh^{-1}(\sin(a+bx))}{2b^2}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)\sec(a+bx)}{2b}- \\
&= -\frac{3\tanh^{-1}(\sin(a+bx))}{2b^2}-\frac{\csc(a+bx)}{2b^2}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)}{2b} \\
&= -\frac{3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{\csc(a+bx)}{2b^2}+\frac{3x\sec(a+}{2b} \\
&= -\frac{3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{\csc(a+bx)}{2b^2}+\frac{3x\sec(a+}{2b} \\
&= -\frac{3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{\csc(a+bx)}{2b^2}+\frac{3i\text{Li}_2\left(-e^{i}\right.}{2b^2}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 282 vs. 2(126) = 252.
time = 2.55, size = 282, normalized size = 2.24

$$\frac{8bx-2\cot\left(\frac{1}{2}(a+bx)\right)-bx\csc^2\left(\frac{1}{2}(a+bx)\right)+12(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)+8\log\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)-8\log\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)-12a\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)+12i\left(\text{PolyLog}\left(2,-e^{i(a+bx)}\right)-\text{PolyLog}\left(2,e^{i(a+bx)}\right)\right)+bx\sec^2\left(\frac{1}{2}(a+bx)\right)+\frac{8bx\sin\left(\frac{1}{2}(a+bx)\right)}{\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)}-\frac{8bx\sin\left(\frac{1}{2}(a+bx)\right)}{\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)}-2\tan\left(\frac{1}{2}(a+bx)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (8*b*x - 2*Cot[(a + b*x)/2] - b*x*Csc[(a + b*x)/2]^2 + 12*(a + b*x)*(Log[1
- E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + 8*Log[Cos[(a + b*x)/2] - S
in[(a + b*x)/2]] - 8*Log[Cos[(a + b*x)/2] + Sin[(a + b*x)/2]] - 12*a*Log[Ta
n[(a + b*x)/2]] + (12*I)*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a
```

```
 + b*x))]) + b*x*Sec[(a + b*x)/2]^2 + (8*b*x*Sin[(a + b*x)/2])/(Cos[(a + b*
x)/2] - Sin[(a + b*x)/2]) - (8*b*x*Sin[(a + b*x)/2])/(Cos[(a + b*x)/2] + Si
n[(a + b*x)/2]) - 2*Tan[(a + b*x)/2])/(8*b^2)
```

Maple [A]
time = 0.14, size = 182, normalized size = 1.44

method	result
risch	$\frac{3bx\,\mathrm{e}^{5i(bx+a)}-2bx\,\mathrm{e}^{3i(bx+a)}-i\mathrm{e}^{5i(bx+a)}+3bx\,\mathrm{e}^{i(bx+a)}+i\mathrm{e}^{i(bx+a)}}{b^2\left(\mathrm{e}^{2i(bx+a)}-1\right)^2\left(1+\mathrm{e}^{2i(bx+a)}\right)}-\frac{3a\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{2b^2}+\frac{2i\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b^2}+\frac{3i\,\mathrm{dilog}\left(\mathrm{e}^{i(bx+}\right.}{2b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(b*x+a)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2/(1+exp(2*I*(b*x+a)))*(3*b*x*exp(5*I*(b*x+a))-2
*b*x*exp(3*I*(b*x+a))-I*exp(5*I*(b*x+a))+3*b*x*exp(I*(b*x+a))+I*exp(I*(b*x+
a)))-3/2/b^2*a*ln(exp(I*(b*x+a))-1)+2*I/b^2*arctan(exp(I*(b*x+a)))+3/2*I/b^
2*dilog(exp(I*(b*x+a)))+3/2*I/b^2*dilog(exp(I*(b*x+a))+1)-3/2/b*ln(exp(I*(b
*x+a))+1)*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1173 vs. $2(104) = 208$.
time = 0.69, size = 1173, normalized size = 9.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (8*I*b*x*cos(3*b*x + 3*a) - 8*b*x*sin(3*b*x + 3*a) - 4*(cos(6*b*x + 6*a) -
cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + I*sin(6*b*x + 6*a) - I*sin(4*b*x + 4*
a) - I*sin(2*b*x + 2*a) + 1)*arctan2(2*(cos(b*x + 2*a)*cos(a) + sin(b*x + 2
*a)*sin(a))/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*
x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2), (cos(b*x + 2*a)^2 - cos(a
)^2 + sin(b*x + 2*a)^2 - sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*
sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) -
6*(b*x*cos(6*b*x + 6*a) - b*x*cos(4*b*x + 4*a) - b*x*cos(2*b*x + 2*a) + I*b
*x*sin(6*b*x + 6*a) - I*b*x*sin(4*b*x + 4*a) - I*b*x*sin(2*b*x + 2*a) + b*x
)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 6*(b*x*cos(6*b*x + 6*a) - b*x*c
os(4*b*x + 4*a) - b*x*cos(2*b*x + 2*a) + I*b*x*sin(6*b*x + 6*a) - I*b*x*sin
(4*b*x + 4*a) - I*b*x*sin(2*b*x + 2*a) + b*x)*arctan2(sin(b*x + a), -cos(b*
x + a) + 1) - 4*(3*I*b*x + 1)*cos(5*b*x + 5*a) - 4*(3*I*b*x - 1)*cos(b*x +
a) + 6*(cos(6*b*x + 6*a) - cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + I*sin(6*b*
x + 6*a) - I*sin(4*b*x + 4*a) - I*sin(2*b*x + 2*a) + 1)*dilog(-e^(I*b*x + I
*a)) - 6*(cos(6*b*x + 6*a) - cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + I*sin(6*
```

```
b*x + 6*a) - I*sin(4*b*x + 4*a) - I*sin(2*b*x + 2*a) + 1)*dilog(e^(I*b*x +
I*a)) - 3*(-I*b*x*cos(6*b*x + 6*a) + I*b*x*cos(4*b*x + 4*a) + I*b*x*cos(2*b
*x + 2*a) + b*x*sin(6*b*x + 6*a) - b*x*sin(4*b*x + 4*a) - b*x*sin(2*b*x + 2
*a) - I*b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - 3*
(I*b*x*cos(6*b*x + 6*a) - I*b*x*cos(4*b*x + 4*a) - I*b*x*cos(2*b*x + 2*a) -
 b*x*sin(6*b*x + 6*a) + b*x*sin(4*b*x + 4*a) + b*x*sin(2*b*x + 2*a) + I*b*x
)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 2*(I*cos(6*b*
x + 6*a) - I*cos(4*b*x + 4*a) - I*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + sin
(4*b*x + 4*a) + sin(2*b*x + 2*a) + I)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*
cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)
^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)
^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) + 4*(3*b*x - I)*sin(5*b*x + 5*a)
+ 4*(3*b*x + I)*sin(b*x + a))/(-4*I*b^2*cos(6*b*x + 6*a) + 4*I*b^2*cos(4*b*
x + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(6*b*x + 6*a) - 4*b^2*sin(4*
b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 4*I*b^2)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(104) = 208$.
time = 3.69, size = 531, normalized size = 4.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*b*x*cos(b*x + a)^2 - 4*b*x - 3*(I*cos(b*x + a)^3 - I*cos(b*x + a))*d
ilog(cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*cos(b*x + a)^3 + I*cos(b*x + a)
)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*(I*cos(b*x + a)^3 - I*cos(b*x +
a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*cos(b*x + a)^3 + I*cos(b*
x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b*x*cos(b*x + a)^3 - b*x
*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b*x*cos(b*x + a)
^3 - b*x*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 3*(a*cos(b*
x + a)^3 - a*cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2
) - 3*(a*cos(b*x + a)^3 - a*cos(b*x + a))*log(-1/2*cos(b*x + a) - 1/2*I*sin
(b*x + a) + 1/2) + 3*((b*x + a)*cos(b*x + a)^3 - (b*x + a)*cos(b*x + a))*lo
g(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*((b*x + a)*cos(b*x + a)^3 - (b*x
+ a)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 2*(cos(b*x + a
)^3 - cos(b*x + a))*log(sin(b*x + a) + 1) + 2*(cos(b*x + a)^3 - cos(b*x + a
))*log(-sin(b*x + a) + 1) + 2*cos(b*x + a)*sin(b*x + a))/(b^2*cos(b*x + a)^
3 - b^2*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\csc^3(a+bx)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Integral(x*csc(a + b*x)**3*sec(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

```
[Out] int(x/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

3.288 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x)
```

Rubi [A]
time = 0.37, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x} dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x,x]
```

```
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x]^2)/x, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x} dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x} dx$$

Mathematica [A]
time = 42.35, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x,x]
```

```
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x, x]
```

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(\csc^3(bx+a))(\sec^2(bx+a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^2/x,x)
```

```
[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] (2*b*x*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) + 3*b*x*cos(b*x + a) + (3*b*x*cos(
5*b*x + 5*a) - 2*b*x*cos(3*b*x + 3*a) + 3*b*x*cos(b*x + a) - sin(5*b*x + 5*
a) + sin(b*x + a))*cos(6*b*x + 6*a) - (3*b*x*cos(4*b*x + 4*a) + 3*b*x*cos(2
*b*x + 2*a) - 3*b*x + sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*cos(5*b*x + 5*a)
 + (2*b*x*cos(3*b*x + 3*a) - 3*b*x*cos(b*x + a) - sin(b*x + a))*cos(4*b*x +
 4*a) + 2*(b*x*cos(2*b*x + 2*a) - b*x)*cos(3*b*x + 3*a) - (3*b*x*cos(b*x +
a) + sin(b*x + a))*cos(2*b*x + 2*a) + (b^2*x^2*cos(6*b*x + 6*a)^2 + b^2*x^2
*cos(4*b*x + 4*a)^2 + b^2*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(6*b*x + 6*a)
^2 + b^2*x^2*sin(4*b*x + 4*a)^2 + 2*b^2*x^2*sin(4*b*x + 4*a)*sin(2*b*x + 2*
a) + b^2*x^2*sin(2*b*x + 2*a)^2 - 2*b^2*x^2*cos(2*b*x + 2*a) + b^2*x^2 - 2*
(b^2*x^2*cos(4*b*x + 4*a) + b^2*x^2*cos(2*b*x + 2*a) - b^2*x^2)*cos(6*b*x +
 6*a) + 2*(b^2*x^2*cos(2*b*x + 2*a) - b^2*x^2)*cos(4*b*x + 4*a) - 2*(b^2*x^
2*sin(4*b*x + 4*a) + b^2*x^2*sin(2*b*x + 2*a))*sin(6*b*x + 6*a))*integrate(
1/2*(3*b^2*x^2 + 2)*sin(b*x + a)/(b^2*x^3*cos(b*x + a)^2 + b^2*x^3*sin(b*x
+ a)^2 + 2*b^2*x^3*cos(b*x + a) + b^2*x^3), x) + (b^2*x^2*cos(6*b*x + 6*a)^
2 + b^2*x^2*cos(4*b*x + 4*a)^2 + b^2*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(6
*b*x + 6*a)^2 + b^2*x^2*sin(4*b*x + 4*a)^2 + 2*b^2*x^2*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) + b^2*x^2*sin(2*b*x + 2*a)^2 - 2*b^2*x^2*cos(2*b*x + 2*a) + b
^2*x^2 - 2*(b^2*x^2*cos(4*b*x + 4*a) + b^2*x^2*cos(2*b*x + 2*a) - b^2*x^2)*
cos(6*b*x + 6*a) + 2*(b^2*x^2*cos(2*b*x + 2*a) - b^2*x^2)*cos(4*b*x + 4*a)
- 2*(b^2*x^2*sin(4*b*x + 4*a) + b^2*x^2*sin(2*b*x + 2*a))*sin(6*b*x + 6*a))
*integrate(1/2*(3*b^2*x^2 + 2)*sin(b*x + a)/(b^2*x^3*cos(b*x + a)^2 + b^2*x
^3*sin(b*x + a)^2 - 2*b^2*x^3*cos(b*x + a) + b^2*x^3), x) + 2*(b^2*x^2*cos(
6*b*x + 6*a)^2 + b^2*x^2*cos(4*b*x + 4*a)^2 + b^2*x^2*cos(2*b*x + 2*a)^2 +
b^2*x^2*sin(6*b*x + 6*a)^2 + b^2*x^2*sin(4*b*x + 4*a)^2 + 2*b^2*x^2*sin(4*b
*x + 4*a)*sin(2*b*x + 2*a) + b^2*x^2*sin(2*b*x + 2*a)^2 - 2*b^2*x^2*cos(2*b
*x + 2*a) + b^2*x^2 - 2*(b^2*x^2*cos(4*b*x + 4*a) + b^2*x^2*cos(2*b*x + 2*a
) - b^2*x^2)*cos(6*b*x + 6*a) + 2*(b^2*x^2*cos(2*b*x + 2*a) - b^2*x^2)*cos(
4*b*x + 4*a) - 2*(b^2*x^2*sin(4*b*x + 4*a) + b^2*x^2*sin(2*b*x + 2*a))*sin(
6*b*x + 6*a))*integrate((cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b*x + 2*a)*s
in(b*x + a) + cos(b*x + a))/(b*x^2*cos(2*b*x + 2*a)^2 + b*x^2*sin(2*b*x + 2
*a)^2 + 2*b*x^2*cos(2*b*x + 2*a) + b*x^2), x) + (3*b*x*sin(5*b*x + 5*a) - 2
*b*x*sin(3*b*x + 3*a) + 3*b*x*sin(b*x + a) + cos(5*b*x + 5*a) - cos(b*x + a
))*sin(6*b*x + 6*a) - (3*b*x*sin(4*b*x + 4*a) + 3*b*x*sin(2*b*x + 2*a) - co
```

```
s(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*sin(5*b*x + 5*a) + (2*b*x*sin(3*b*x
+ 3*a) - 3*b*x*sin(b*x + a) + cos(b*x + a))*sin(4*b*x + 4*a) - (3*b*x*sin(b
*x + a) - cos(b*x + a))*sin(2*b*x + 2*a) + sin(b*x + a))/(b^2*x^2*cos(6*b*x
 + 6*a)^2 + b^2*x^2*cos(4*b*x + 4*a)^2 + b^2*x^2*cos(2*b*x + 2*a)^2 + b^2*x
^2*sin(6*b*x + 6*a)^2 + b^2*x^2*sin(4*b*x + 4*a)^2 + 2*b^2*x^2*sin(4*b*x +
4*a)*sin(2*b*x + 2*a) + b^2*x^2*sin(2*b*x + 2*a)^2 - 2*b^2*x^2*cos(2*b*x +
2*a) + b^2*x^2 - 2*(b^2*x^2*cos(4*b*x + 4*a) + b^2*x^2*cos(2*b*x + 2*a) - b
^2*x^2)*cos(6*b*x + 6*a) + 2*(b^2*x^2*cos(2*b*x + 2*a) - b^2*x^2)*cos(4*b*x
 + 4*a) - 2*(b^2*x^2*sin(4*b*x + 4*a) + b^2*x^2*sin(2*b*x + 2*a))*sin(6*b*x
 + 6*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="fricas")
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/x,x)
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**2/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="giac")
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^2/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cos(a + b*x)^2*sin(a + b*x)^3),x)
[Out] int(1/(x*cos(a + b*x)^2*sin(a + b*x)^3), x)
```

3.289 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)
```

Rubi [A]
time = 0.39, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2,x]
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$$

Mathematica [A]
time = 17.81, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2,x]
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2, x]
```

Maple [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^3(bx+a)\right)\left(\sec^2(bx+a)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)
```

```
[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] (2*b*x*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) + 3*b*x*cos(b*x + a) + (3*b*x*cos(
5*b*x + 5*a) - 2*b*x*cos(3*b*x + 3*a) + 3*b*x*cos(b*x + a) - 2*sin(5*b*x +
5*a) + 2*sin(b*x + a))*cos(6*b*x + 6*a) - (3*b*x*cos(4*b*x + 4*a) + 3*b*x*c
os(2*b*x + 2*a) - 3*b*x + 2*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(5*b*
x + 5*a) + (2*b*x*cos(3*b*x + 3*a) - 3*b*x*cos(b*x + a) - 2*sin(b*x + a))*c
os(4*b*x + 4*a) + 2*(b*x*cos(2*b*x + 2*a) - b*x)*cos(3*b*x + 3*a) - (3*b*x*
cos(b*x + a) + 2*sin(b*x + a))*cos(2*b*x + 2*a) + (b^2*x^3*cos(6*b*x + 6*a)
^2 + b^2*x^3*cos(4*b*x + 4*a)^2 + b^2*x^3*cos(2*b*x + 2*a)^2 + b^2*x^3*sin(
6*b*x + 6*a)^2 + b^2*x^3*sin(4*b*x + 4*a)^2 + 2*b^2*x^3*sin(4*b*x + 4*a)*si
n(2*b*x + 2*a) + b^2*x^3*sin(2*b*x + 2*a)^2 - 2*b^2*x^3*cos(2*b*x + 2*a) +
b^2*x^3 - 2*(b^2*x^3*cos(4*b*x + 4*a) + b^2*x^3*cos(2*b*x + 2*a) - b^2*x^3)
*cos(6*b*x + 6*a) + 2*(b^2*x^3*cos(2*b*x + 2*a) - b^2*x^3)*cos(4*b*x + 4*a)
 - 2*(b^2*x^3*sin(4*b*x + 4*a) + b^2*x^3*sin(2*b*x + 2*a))*sin(6*b*x + 6*a)
)*integrate(3/2*(b^2*x^2 + 2)*sin(b*x + a)/(b^2*x^4*cos(b*x + a)^2 + b^2*x^
4*sin(b*x + a)^2 + 2*b^2*x^4*cos(b*x + a) + b^2*x^4), x) + (b^2*x^3*cos(6*b
*x + 6*a)^2 + b^2*x^3*cos(4*b*x + 4*a)^2 + b^2*x^3*cos(2*b*x + 2*a)^2 + b^2
*x^3*sin(6*b*x + 6*a)^2 + b^2*x^3*sin(4*b*x + 4*a)^2 + 2*b^2*x^3*sin(4*b*x
+ 4*a)*sin(2*b*x + 2*a) + b^2*x^3*sin(2*b*x + 2*a)^2 - 2*b^2*x^3*cos(2*b*x
+ 2*a) + b^2*x^3 - 2*(b^2*x^3*cos(4*b*x + 4*a) + b^2*x^3*cos(2*b*x + 2*a) -
 b^2*x^3)*cos(6*b*x + 6*a) + 2*(b^2*x^3*cos(2*b*x + 2*a) - b^2*x^3)*cos(4*b
*x + 4*a) - 2*(b^2*x^3*sin(4*b*x + 4*a) + b^2*x^3*sin(2*b*x + 2*a))*sin(6*b
*x + 6*a))*integrate(3/2*(b^2*x^2 + 2)*sin(b*x + a)/(b^2*x^4*cos(b*x + a)^2
 + b^2*x^4*sin(b*x + a)^2 - 2*b^2*x^4*cos(b*x + a) + b^2*x^4), x) + 4*(b^2*
x^3*cos(6*b*x + 6*a)^2 + b^2*x^3*cos(4*b*x + 4*a)^2 + b^2*x^3*cos(2*b*x + 2
*a)^2 + b^2*x^3*sin(6*b*x + 6*a)^2 + b^2*x^3*sin(4*b*x + 4*a)^2 + 2*b^2*x^3
*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b^2*x^3*sin(2*b*x + 2*a)^2 - 2*b^2*x^3
*cos(2*b*x + 2*a) + b^2*x^3 - 2*(b^2*x^3*cos(4*b*x + 4*a) + b^2*x^3*cos(2*b
*x + 2*a) - b^2*x^3)*cos(6*b*x + 6*a) + 2*(b^2*x^3*cos(2*b*x + 2*a) - b^2*x
^3)*cos(4*b*x + 4*a) - 2*(b^2*x^3*sin(4*b*x + 4*a) + b^2*x^3*sin(2*b*x + 2*
a))*sin(6*b*x + 6*a))*integrate((cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b*x
+ 2*a)*sin(b*x + a) + cos(b*x + a))/(b*x^3*cos(2*b*x + 2*a)^2 + b*x^3*sin(2
*b*x + 2*a)^2 + 2*b*x^3*cos(2*b*x + 2*a) + b*x^3), x) + (3*b*x*sin(5*b*x +
5*a) - 2*b*x*sin(3*b*x + 3*a) + 3*b*x*sin(b*x + a) + 2*cos(5*b*x + 5*a) - 2
*cos(b*x + a))*sin(6*b*x + 6*a) - (3*b*x*sin(4*b*x + 4*a) + 3*b*x*sin(2*b*x
```

```
 + 2*a) - 2*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 2)*sin(5*b*x + 5*a) + (
2*b*x*sin(3*b*x + 3*a) - 3*b*x*sin(b*x + a) + 2*cos(b*x + a))*sin(4*b*x + 4
*a) - (3*b*x*sin(b*x + a) - 2*cos(b*x + a))*sin(2*b*x + 2*a) + 2*sin(b*x +
a))/(b^2*x^3*cos(6*b*x + 6*a)^2 + b^2*x^3*cos(4*b*x + 4*a)^2 + b^2*x^3*cos(
2*b*x + 2*a)^2 + b^2*x^3*sin(6*b*x + 6*a)^2 + b^2*x^3*sin(4*b*x + 4*a)^2 +
2*b^2*x^3*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b^2*x^3*sin(2*b*x + 2*a)^2 -
2*b^2*x^3*cos(2*b*x + 2*a) + b^2*x^3 - 2*(b^2*x^3*cos(4*b*x + 4*a) + b^2*x^
3*cos(2*b*x + 2*a) - b^2*x^3)*cos(6*b*x + 6*a) + 2*(b^2*x^3*cos(2*b*x + 2*a
) - b^2*x^3)*cos(4*b*x + 4*a) - 2*(b^2*x^3*sin(4*b*x + 4*a) + b^2*x^3*sin(2
*b*x + 2*a))*sin(6*b*x + 6*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/x**2,x)
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**2/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="giac")
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^2/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*cos(a + b*x)^2*sin(a + b*x)^3),x)
[Out] int(1/(x^2*cos(a + b*x)^2*sin(a + b*x)^3), x)
```

3.290 $\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=25

$$\text{Int}\left((c+dx)^m \sec^2(a+bx)\tan(a+bx),x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$$

Mathematica [A]
time = 5.06, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x], x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\sec^2(bx+a)\right)\tan(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x)
[Out] int((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \tan(a+bx)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)**2*tan(b*x+a),x)
[Out] Integral((c + d*x)**m*tan(a + b*x)*sec(a + b*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(a+b\,x)\,{(c+d\,x)}^m}{{\cos\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x)^2,x)
```

```
[Out] int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x)^2, x)
```

3.291 $\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=139

$$\frac{2id(c+dx)^3}{b^2}-\frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3}+\frac{6id^3(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^4}-\frac{3d^4\text{PolyLog}\left(3,-e^{2i(a+}\right.}{b^5}$$

```
[Out] 2*I*d*(d*x+c)^3/b^2-6*d^2*(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b^3+6*I*d^3*(d*x
+c)*polylog(2,-exp(2*I*(b*x+a)))/b^4-3*d^4*polylog(3,-exp(2*I*(b*x+a)))/b^5
+1/2*(d*x+c)^4*sec(b*x+a)^2/b-2*d*(d*x+c)^3*tan(b*x+a)/b^2
```

Rubi [A]
time = 0.17, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {4494, 4269, 3800, 2221, 2611, 2320, 6724}

$$-\frac{3d^4\text{Li}_3\left(-e^{2i(a+bx)}\right)}{b^5}+\frac{6id^3(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^4}-\frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)^3\tan(a+bx)}{b^2}+\frac{(c+dx)^4\sec^2(a+bx)}{2b}+\frac{2id(c+dx)^3}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((2*I)*d*(c + d*x)^3)/b^2 - (6*d^2*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))]
)/b^3 + ((6*I)*d^3*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 - (3*d^4
*PolyLog[3, -E^((2*I)*(a + b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x]^2)/(2*b)
 - (2*d*(c + d*x)^3*Tan[a + b*x])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)^4\sec^2(a+bx)}{2b} - \frac{(2d)\int (c+dx)^3\sec^2(a+bx)\,dx}{b}$$
$$= \frac{(c+dx)^4\sec^2(a+bx)}{2b} - \frac{2d(c+dx)^3\tan(a+bx)}{b^2} + \frac{(6d^2)\int (c+$$
$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4\sec^2(a+bx)}{2b} - \frac{2d(c+dx)^3\tan(a+bx}{b^2}$$
$$= \frac{2id(c+dx)^3}{b^2} - \frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4\sec^2(}{2b}$$
$$= \frac{2id(c+dx)^3}{b^2} - \frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{6id^3(c+dx)\mathrm{Li}}{b^4}$$
$$= \frac{2id(c+dx)^3}{b^2} - \frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{6id^3(c+dx)\mathrm{Li}}{b^4}$$
$$= \frac{2id(c+dx)^3}{b^2} - \frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{6id^3(c+dx)\mathrm{Li}}{b^4}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 425 vs. 2(139) = 278.
time = 6.47, size = 425, normalized size = 3.06

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (d^4*((2*I)*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(1 + E^((2*I)*a))*Log[1 + E^
((2*I)*(a + b*x))]) + (6*I)*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((2*I)*(a +
 b*x))] - 3*(1 + E^((2*I)*a))*PolyLog[3, -E^((2*I)*(a + b*x))])*Sec[a])/(2*
b^5*E^(I*a)) + ((c + d*x)^4*Sec[a + b*x]^2)/(2*b) - (6*c^2*d^2*Sec[a]*(Cos[
a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + S
in[a]^2)) - (6*c*d^3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*
x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[C
ot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*Ar
cTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x -
ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^4*Sqrt[Csc[a]^2*(Cos[a]^
2 + Sin[a]^2)]) - (2*Sec[a]*Sec[a + b*x]*(c^3*d*Sin[b*x] + 3*c^2*d^2*x*Sin[
b*x] + 3*c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin[b*x]))/b^2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(129) = 258.
time = 0.11, size = 489, normalized size = 3.52

method	result
risch	$\frac{2bd^4x^4e^{2i(bx+a)}+8bcd^3x^3e^{2i(bx+a)}+12bc^2d^2x^2e^{2i(bx+a)}+8bc^3dx\,e^{2i(bx+a)}-4id^4x^3e^{2i(bx+a)}+2bc^4e^{2i(bx+a)}-12icd^3x^2e^{2i(bx+a)}-1}{b^2\left(1+e^{2i(bx+a)}\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x,method=_RETURNVERBOSE)

[Out] 2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x^2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2*I*d^4*x^3*exp(2*I*(b*x+a))+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b*x+a))-6*I*c^2*d^2*x*exp(2*I*(b*x+a))-2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a))-6*I*c*d^3*x^2-6*I*c^2*d^2*x-2*I*c^3*d)/b^2/(1+exp(2*I*(b*x+a)))^2-6/b^3*d^2*c^2*ln(1+exp(2*I*(b*x+a)))+12*d^2/b^3*c^2*ln(exp(I*(b*x+a)))+12*d^4/b^5*a^2*ln(exp(I*(b*x+a)))-12/b^3*d^3*c*ln(1+exp(2*I*(b*x+a)))*x+4*I/b^2*d^4*x^3+24*I/b^3*d^3*c*a*x-8*I/b^5*d^4*a^3-6/b^3*d^4*ln(1+exp(2*I*(b*x+a)))*x^2+12*I/b^4*d^3*c*a^2-3*d^4*polylog(3,-exp(2*I*(b*x+a)))/b^5-24*d^3/b^4*c*a*ln(exp(I*(b*x+a)))-12*I/b^4*d^4*a^2*x+6*I/b^4*d^4*polylog(2,-exp(2*I*(b*x+a)))*x+6*I/b^4*d^3*c*polylog(2,-exp(2*I*(b*x+a)))+12*I/b^2*d^3*c*x^2

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3446 vs. $2(126) = 252$.
time = 0.64, size = 3446, normalized size = 24.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")

[Out] 1/2*(c^4*tan(b*x + a)^2 - 4*a*c^3*d*tan(b*x + a)^2/b + 6*a^2*c^2*d^2*tan(b*x + a)^2/b^2 - 4*a^3*c*d^3*tan(b*x + a)^2/b^3 + a^4*d^4*tan(b*x + a)^2/b^4 + 8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*c^3*d/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b) - 24*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*a*c^2*d^2/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2) + 24*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + s in(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*a^2*c*d^3/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^3) - 8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*a^3*d^4/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^4) + 6*(8*(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 + 4*(b*x + a)^2*cos(2*b*x + 2*a) + 4*((b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - (2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*((b*x + a)^2*sin(2*b*x + 2*a) - b*x - (b*x + a)*cos(2*b*x + 2*a) - a)*sin(4*b*x + 4*a) - 4*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2) - 12*(8*(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 + 4*(b*x + a)^2*cos(2*b*x + 2*a) + 4*((b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - (2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*((b*x + a)^2*sin(2*b*x + 2*a) - b*x - (b*x + a)*cos(2*b*x + 2*a) - a)*sin(4*b*x + 4*a) - 4*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^3) + 6*(8*(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 + 4*(b*x + a)^2*cos(2*b*x + 2*a) + 4*((b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - (2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*((b*x + a)^2*sin(2*b*x + 2*a) - b*x - (b*x + a)*cos(2*b*x + 2*a) - a)*sin(4*b*x + 4*a) - 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^4/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^4) - 2*(6*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a) + ((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 2*((b*x + a)^2*d^4 + 2*(b*c*d^3 -

```
a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^4 + 2*(-I*b*c*d^3 +
 I*a*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 2*(-I*(b*x + a)^2*d^4 + 2*(-I*b*c*d
^3 + I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*
b*x + 2*a) + 1) - 4*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2)*cos
(4*b*x + 4*a) - 2*(-I*(b*x + a)^4*d^4 + 2*(-2*I*b*c*d^3 + (2*I*a + 1)*d^4)*
(b*x + a)^3 + 6*(b*c*d^3 - a*d^4)*(b*x + a)^2)*...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(126) = 252$.
time = 2.94, size = 892, normalized size = 6.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^
4*c^4 - 12*d^4*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 1
2*d^4*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 12*d^4*cos
(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 12*d^4*cos(b*x + a
)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 12*(I*b*d^4*x + I*b*c*d^3)
*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) - 12*(-I*b*d^4*x - I*b
*c*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - 12*(-I*b*d^4*
x - I*b*c*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 12*(I
*b*d^4*x + I*b*c*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a))
- 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(cos(b*x + a) +
 I*sin(b*x + a) + I) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)
^2*log(cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x
+ 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) +
 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^
2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x +
 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) +
 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^
2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) - 6*(b^2*c^
2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x
 + a) + I) - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d
)*cos(b*x + a)*sin(b*x + a))/(b^5*cos(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^4 \tan (a + bx) \sec^2 (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**4*tan(a + b*x)*sec(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*sec(b*x + a)^2*tan(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(a+b\,x)\ {\left(c+d\,x\right)}^4}{{\cos\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x)^2,x)
```

```
[Out] int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x)^2, x)
```

3.292 $\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=115

$$\frac{3id(c+dx)^2}{2b^2}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}+\frac{3id^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^4}+\frac{(c+dx)^3\sec^2(a+bx)}{2b}-\frac{3d(c+d$$

```
[Out] 3/2*I*d*(d*x+c)^2/b^2-3*d^2*(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b^3+3/2*I*d^3*po
lylog(2,-exp(2*I*(b*x+a)))/b^4+1/2*(d*x+c)^3*sec(b*x+a)^2/b-3/2*d*(d*x+c)^2
*tan(b*x+a)/b^2
```

Rubi [A]
time = 0.11, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4494, 4269, 3800, 2221, 2317, 2438}

$$\frac{3id^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^4}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}-\frac{3d(c+dx)^2\tan(a+bx)}{2b^2}+\frac{(c+dx)^3\sec^2(a+bx)}{2b}+\frac{3id(c+dx)^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (((3*I)/2)*d*(c + d*x)^2)/b^2 - (3*d^2*(c + d*x)*Log[1 + E^((2*I)*(a + b*x)
)])/b^3 + (((3*I)/2)*d^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + ((c + d*x)
^3*Sec[a + b*x]^2)/(2*b) - (3*d*(c + d*x)^2*Tan[a + b*x])/(2*b^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3\sec^2(a+bx)\tan(a+bx)\,dx &= \frac{(c+dx)^3\sec^2(a+bx)}{2b}-\frac{(3d)\int(c+dx)^2\sec^2(a+bx)\,dx}{2b}\\
&= \frac{(c+dx)^3\sec^2(a+bx)}{2b}-\frac{3d(c+dx)^2\tan(a+bx)}{2b^2}+\frac{(3d^2)\int(c+\\
&= \frac{3id(c+dx)^2}{2b^2}+\frac{(c+dx)^3\sec^2(a+bx)}{2b}-\frac{3d(c+dx)^2\tan(a+bx}{2b^2}\\
&= \frac{3id(c+dx)^2}{2b^2}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}+\frac{(c+dx)^3\sec^2(a}{2b}\\
&= \frac{3id(c+dx)^2}{2b^2}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}+\frac{(c+dx)^3\sec^2(a}{2b}\\
&= \frac{3id(c+dx)^2}{2b^2}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}+\frac{3id^3\mathrm{Li}_2\left(-e^{2i(a+}\right.}{2b^4}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 286 vs. 2(115) = 230.
time = 6.35, size = 286, normalized size = 2.49

$$\frac{(c+dx)^3\sec^2(a+bx)}{2b}-\frac{3cd^2\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))}{b^3\left(\cos^2(a)+\sin^2(a)\right)}-\frac{3d^3\csc(a)\left(b^2e^{-i\mathrm{ArcTan}(\cot(a))}x^2-\frac{\cot(a)\left(ibx(-\pi-2\mathrm{ArcTan}(\cot(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx-\mathrm{ArcTan}(\cot(a)))\log\left(1-e^{2i(bx-\mathrm{ArcTan}(\cot(a)))}\right)+\pi\log(\cos(bx))-2\mathrm{ArcTan}(\cot(a))\log(\sin(bx-\mathrm{ArcTan}(\cot(a))))+i\mathrm{PolyLog}\left(2,e^{2i(bx-\mathrm{ArcTan}(\cot(a)))}\right)\right)}{\sqrt{1+\cot^2(a)}}\right)\sec(a)}{2b^4\sqrt{\csc^2(a)\left(\cos^2(a)+\sin^2(a)\right)}}-\frac{3\sec(a)\sec(a+bx)\left(c^2d\sin(bx)+2cd^2x\sin(bx)+d^3x^2\sin(bx)\right)}{2b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((c + d*x)^3*Sec[a + b*x]^2)/(2*b) - (3*c*d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos
[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) - (3*d^
3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[C
ot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^(
(2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Si
n[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))
/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) -
 (3*Sec[a]*Sec[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[
b*x]))/(2*b^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(101) = 202$.
time = 0.10, size = 301, normalized size = 2.62

method	result
risch	$\frac{2b\,d^3x^3\mathrm{e}^{2i(bx+a)}-3id^3x^2\mathrm{e}^{2i(bx+a)}+6bc\,d^2x^2\mathrm{e}^{2i(bx+a)}-6ic\,d^2x\,\mathrm{e}^{2i(bx+a)}+6b\,c^2dx\,\mathrm{e}^{2i(bx+a)}-3ic^2d\,\mathrm{e}^{2i(bx+a)}-3id^3x^2+2b\,c^3\mathrm{e}^{2i(bx+a)}-}{b^2\left(1+\mathrm{e}^{2i(bx+a)}\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] (2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*ex
p(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*
I*c^2*d*exp(2*I*(b*x+a))-3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))-6*I*c*d^2*x-3
*I*c^2*d)/b^2/(1+exp(2*I*(b*x+a)))^2-3/b^3*d^2*c*ln(1+exp(2*I*(b*x+a)))+6*d
^2/b^3*c*ln(exp(I*(b*x+a)))+3*I/b^2*d^3*x^2+6*I/b^3*d^3*a*x+3*I/b^4*d^3*a^2
-3/b^3*d^3*ln(1+exp(2*I*(b*x+a)))*x+3/2*I*d^3*polylog(2,-exp(2*I*(b*x+a)))/
b^4-6*d^3/b^4*a*ln(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(98) = 196$.
time = 0.63, size = 668, normalized size = 5.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] -(6*b^2*c^2*d + 6*(b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cos(4*b*x + 4*a)
 + 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a) - (-I*b*d^3*x - I*b*c*d^2)*sin(4*
b*x + 4*a) - 2*(-I*b*d^3*x - I*b*c*d^2)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x
 + 2*a), cos(2*b*x + 2*a) + 1) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x)*cos(4*b*x
+ 4*a) - 2*(-2*I*b^3*d^3*x^3 - 2*I*b^3*c^3 - 3*b^2*c^2*d + 3*(-2*I*b^3*c*d^
2 + b^2*d^3)*x^2 + 6*(-I*b^3*c^2*d + b^2*c*d^2)*x)*cos(2*b*x + 2*a) - 3*(d^
3*cos(4*b*x + 4*a) + 2*d^3*cos(2*b*x + 2*a) + I*d^3*sin(4*b*x + 4*a) + 2*I*
```

```
d^3*sin(2*b*x + 2*a) + d^3)*dilog(-e^(2*I*b*x + 2*I*a)) - 3*(I*b*d^3*x + I*
b*c*d^2 + (I*b*d^3*x + I*b*c*d^2)*cos(4*b*x + 4*a) + 2*(I*b*d^3*x + I*b*c*d
^2)*cos(2*b*x + 2*a) - (b*d^3*x + b*c*d^2)*sin(4*b*x + 4*a) - 2*(b*d^3*x +
b*c*d^2)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*
cos(2*b*x + 2*a) + 1) - 6*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x)*sin(4*b*x + 4*a
) - 2*(2*b^3*d^3*x^3 + 2*b^3*c^3 - 3*I*b^2*c^2*d + 3*(2*b^3*c*d^2 + I*b^2*d
^3)*x^2 + 6*(b^3*c^2*d + I*b^2*c*d^2)*x)*sin(2*b*x + 2*a))/(-2*I*b^4*cos(4*
b*x + 4*a) - 4*I*b^4*cos(2*b*x + 2*a) + 2*b^4*sin(4*b*x + 4*a) + 4*b^4*sin(
2*b*x + 2*a) - 2*I*b^4)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(98) = 196$.
time = 3.18, size = 540, normalized size = 4.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 3*I*d^3*cos(
b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + 3*I*d^3*cos(b*x + a)^2*di
log(I*cos(b*x + a) - sin(b*x + a)) + 3*I*d^3*cos(b*x + a)^2*dilog(-I*cos(b*
x + a) + sin(b*x + a)) - 3*I*d^3*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin
(b*x + a)) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*
x + a) + I) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b
*x + a) + I) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(
b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin
(b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(-I*cos(b*x + a) + s
in(b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(-I*cos(b*x + a) -
 sin(b*x + a) + 1) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) +
 I*sin(b*x + a) + I) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(-cos(b*x + a)
 - I*sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*
x + a)*sin(b*x + a))/(b^4*cos(b*x + a)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \tan(a + bx) \sec^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*tan(a + b*x)*sec(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)^2*tan(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(a+b\,x)\,\left(c+d\,x\right)^3}{\cos\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x)^2,x)
```

```
[Out] int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x)^2, x)
```

3.293 $\int (c+dx)^2 \sec^2(a+bx) \tan(a+bx) \, dx$

Optimal. Leaf size=55

$$-\frac{d^2 \log(\cos(a+bx))}{b^3}+\frac{(c+dx)^2 \sec^2(a+bx)}{2b}-\frac{d(c+dx)\tan(a+bx)}{b^2}$$

```
[Out] -d^2*ln(cos(b*x+a))/b^3+1/2*(d*x+c)^2*sec(b*x+a)^2/b-d*(d*x+c)*tan(b*x+a)/b
^2
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4494, 4269, 3556}

$$-\frac{d^2 \log(\cos(a+bx))}{b^3}-\frac{d(c+dx)\tan(a+bx)}{b^2}+\frac{(c+dx)^2 \sec^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] -((d^2*Log[Cos[a + b*x]])/b^3) + ((c + d*x)^2*Sec[a + b*x]^2)/(2*b) - (d*(c
 + d*x)*Tan[a + b*x])/b^2
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2 \sec^2(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)^2\sec^2(a+bx)}{2b} - \frac{d\int (c+dx)\sec^2(a+bx)\,dx}{b}$$
$$= \frac{(c+dx)^2\sec^2(a+bx)}{2b} - \frac{d(c+dx)\tan(a+bx)}{b^2} + \frac{d^2\int \tan(a+bx}{b^2}$$
$$= -\frac{d^2\log(\cos(a+bx))}{b^3} + \frac{(c+dx)^2\sec^2(a+bx)}{2b} - \frac{d(c+dx)\tan(a}{b^2}$$

Mathematica [A]
time = 0.53, size = 66, normalized size = 1.20

$$\frac{b^2(c+dx)^2\sec^2(a+bx) - 2bd(c+dx)\sec(a)\sec(a+bx)\sin(bx) - 2d^2(\log(\cos(a+bx)) + bx\tan(a))}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (b^2*(c + d*x)^2*Sec[a + b*x]^2 - 2*b*d*(c + d*x)*Sec[a]*Sec[a + b*x]*Sin[b
*x] - 2*d^2*(Log[Cos[a + b*x]] + b*x*Tan[a]))/(2*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(53) = 106.
time = 0.07, size = 165, normalized size = 3.00

method	result
risch	$\frac{2id^2x}{b^2} + \frac{2id^2a}{b^3} + \frac{2b\,d^2x^2e^{2i(bx+a)}+4bcdx\,e^{2i(bx+a)}+2b\,c^2e^{2i(bx+a)}-2id^2x\,e^{2i(bx+a)}-2icd\,e^{2i(bx+a)}-2id^2x-2idc}{b^2\left(1+e^{2i(bx+a)}\right)^2} -$
derivativedivides	$\frac{\frac{a^2d^2}{2b^2\cos(bx+a)^2} - \frac{acd}{b\cos(bx+a)^2} - \frac{2a\,d^2\left(\frac{bx+a}{2\cos(bx+a)^2} - \frac{\tan(bx+a)}{2}\right)}{b^2} + \frac{c^2}{2\cos(bx+a)^2} + \frac{2cd\left(\frac{bx+a}{2\cos(bx+a)^2} - \frac{\tan(bx+a)}{2}\right)}{b} + \frac{d^2\left(\frac{(bx+}{2\cos(b}\right.}{}}{b}$
default	$\frac{\frac{a^2d^2}{2b^2\cos(bx+a)^2} - \frac{acd}{b\cos(bx+a)^2} - \frac{2a\,d^2\left(\frac{bx+a}{2\cos(bx+a)^2} - \frac{\tan(bx+a)}{2}\right)}{b^2} + \frac{c^2}{2\cos(bx+a)^2} + \frac{2cd\left(\frac{bx+a}{2\cos(bx+a)^2} - \frac{\tan(bx+a)}{2}\right)}{b} + \frac{d^2\left(\frac{(bx+}{2\cos(b}\right.}{}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2/b^2*a^2*d^2/cos(b*x+a)^2-1/b*a*c*d/cos(b*x+a)^2-2/b^2*a*d^2*(1/2*(
b*x+a)/cos(b*x+a)^2-1/2*tan(b*x+a))+1/2*c^2/cos(b*x+a)^2+2/b*c*d*(1/2*(b*x+
a)/cos(b*x+a)^2-1/2*tan(b*x+a))+1/b^2*d^2*(1/2*(b*x+a)^2/cos(b*x+a)^2-(b*x+
a)*tan(b*x+a)-ln(cos(b*x+a))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 988 vs. 2(53) = 106.
time = 0.49, size = 988, normalized size = 17.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*tan(b*x + a)^2 - 2*a*c*d*tan(b*x + a)^2/b + a^2*d^2*tan(b*x + a)^2
/b^2 + 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 +
 (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*
x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
- 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*c*d/((2*(2*cos(2*b*x + 2*a) + 1)*
cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x +
4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos
(2*b*x + 2*a) + 1)*b) - 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin
(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b
*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) -
cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*a*d^2/((2*(2*cos
(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*
a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b
*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2) + (8*(b*x + a)^2*cos(2*b*x + 2*a
)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 + 4*(b*x + a)^2*cos(2*b*x + 2*a) + 4
*((b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*
a) - (2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*
cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*
a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2
+ sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*((b*x + a)^2*sin(2*b*x +
 2*a) - b*x - (b*x + a)*cos(2*b*x + 2*a) - a)*sin(4*b*x + 4*a) - 4*(b*x + a
)*sin(2*b*x + 2*a))*d^2/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos
(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x +
 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2
))/b
```

Fricas [A]
time = 2.49, size = 86, normalized size = 1.56

$$\frac{b^2d^2x^2 + 2\,b^2cdx - 2\,d^2\cos\left(bx + a\right)^2\log\left(-\cos\left(bx + a\right)\right) + b^2c^2 - 2\,\left(bd^2x + bcd\right)\cos\left(bx + a\right)\sin\left(bx + a\right)}{2\,b^3\cos\left(bx + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x - 2*d^2*cos(b*x + a)^2*log(-cos(b*x + a)) +
b^2*c^2 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a)^
2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + dx\right)^2\tan\left(a + bx\right)\sec^2\left(a + bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*tan(a + b*x)*sec(a + b*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 4474 vs. $2(53) = 106$.
time = 1.10, size = 4474, normalized size = 81.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4*t
an(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*d^2*x^2*tan
(1/2*b*x)^2*tan(1/2*a)^4 + b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^2*c*d*
x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 4*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*b
^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 + 4*b*d^2*x*tan(1/2*b*x)^3*tan(1/2*a)^
4 - d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2
- 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2
*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*ta
n(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4
*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/
2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1
/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(t
an(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^4 + b^2*d^2*x^
2*tan(1/2*b*x)^4 + 4*b^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b^2*c^2*ta
n(1/2*b*x)^4*tan(1/2*a)^2 + 4*b*c*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + b^2*d^2*x
^2*tan(1/2*a)^4 + 2*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a)^4 + 4*b*c*d*tan(1/2*b
*x)^3*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4 - 4*b*d^2*x*tan(1/2*b*x)^4*
tan(1/2*a) + 8*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - 24*b*d^2*x*tan(1/2*b
*x)^3*tan(1/2*a)^2 + 2*d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b
*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan
(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*
tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a)
+ 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/
2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8
*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*t
an(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/
2*a)^2 - 24*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^3 + 8*d^2*log(4*(tan(1/2*b*x)
^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*
a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan
(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 +
 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b
```

```
*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*ta
n(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*
tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^
2 + 1))*tan(1/2*b*x)^3*tan(1/2*a)^3 + 2*b^2*c*d*x*tan(1/2*a)^4 - 4*b*d^2*x*
tan(1/2*b*x)*tan(1/2*a)^4 + 2*d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*ta
n(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8
+ 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*
b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(
1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2
*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a
)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a
) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2
*tan(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^2 + b^2*c^2*tan(1/2*b*x)^4 - 4*b
*c*d*tan(1/2*b*x)^4*tan(1/2*a) + 2*b^2*d^2*x^2*tan(1/2*a)^2 + 4*b^2*c^2*tan
(1/2*b*x)^2*tan(1/2*a)^2 - 24*b*c*d*tan(1/2*b*x)^3*tan(1/2*a)^2 - 24*b*c*d*
tan(1/2*b*x)^2*tan(1/2*a)^3 + b^2*c^2*tan(1/2*a)^4 - 4*b*c*d*tan(1/2*b*x)*t
an(1/2*a)^4 + 4*b^2*c*d*x*tan(1/2*b*x)^2 + 4*b*d^2*x*tan(1/2*b*x)^3 - d^2*l
og(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1
/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*
tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x
)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*
a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*
tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 +
 tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)
^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4 + 24*b*d^2*x*tan(1/2*b*x)^2*tan(1/
2*a) - 8*d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*
a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*ta
n(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 -
 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b
*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*t
an(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*
tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 +
1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a) + 4*b^2*c
*d*x*tan(1/2*a)^2 + 24*b*d^2*x*tan(1/2*b*x)*tan(1/2*a)^2 - 20*d^2*log(4*(ta
n(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^
7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x...
```

Mupad [B]

time = 3.06, size = 150, normalized size = 2.73

$$-\frac{\frac{(c+d\,x)^2}{b}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,(c+d\,x)^2}{b}}{2\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1}+\frac{d^2\,x\,2\mathrm{i}}{b^2}+\frac{b\,c^2+2\,b\,c\,d\,x-c\,d\,2\mathrm{i}+b\,d^2\,x^2-d^2\,x\,2\mathrm{i}}{b^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}-\frac{d^2\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^2)/cos(a + b*x)^2,x)
```

```
[Out] (d^2*x*2i)/b^2 - ((c + d*x)^2/b - (exp(a*2i + b*x*2i)*(c + d*x)^2)/b)/(2*ex
p(a*2i + b*x*2i) + exp(a*4i + b*x*4i) + 1) + (b*c^2 - c*d*2i - d^2*x*2i + b
*d^2*x^2 + 2*b*c*d*x)/(b^2*(exp(a*2i + b*x*2i) + 1)) - (d^2*log(exp(a*2i)*e
xp(b*x*2i) + 1))/b^3
```

3.294 $\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{(c+dx)\sec^2(a+bx)}{2b}-\frac{d\tan(a+bx)}{2b^2}$$

```
[Out] 1/2*(d*x+c)*sec(b*x+a)^2/b-1/2*d*tan(b*x+a)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4494, 3852, 8}

$$\frac{(c+dx)\sec^2(a+bx)}{2b}-\frac{d\tan(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((c + d*x)*Sec[a + b*x]^2)/(2*b) - (d*Tan[a + b*x])/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx &= \frac{(c+dx)\sec^2(a+bx)}{2b}-\frac{d\int \sec^2(a+bx)\,dx}{2b}\\ &= \frac{(c+dx)\sec^2(a+bx)}{2b}+\frac{d\text{Subst}\left(\int 1\,dx,x,-\tan(a+bx)\right)}{2b^2}\\ &= \frac{(c+dx)\sec^2(a+bx)}{2b}-\frac{d\tan(a+bx)}{2b^2}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 48, normalized size = 1.37

$$\frac{c\sec^2(a+bx)}{2b}+\frac{dx\sec^2(a+bx)}{2b}-\frac{d\tan(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (c*Sec[a + b*x]^2)/(2*b) + (d*x*Sec[a + b*x]^2)/(2*b) - (d*Tan[a + b*x])/(2
*b^2)
```

Maple [A]
time = 0.08, size = 61, normalized size = 1.74

method	result	size
derivativedivides	$\frac{-\frac{da}{2b\cos(bx+a)^2}+\frac{c}{2\cos(bx+a)^2}+\frac{d\left(\frac{bx+a}{2\cos(bx+a)^2}-\frac{\tan(bx+a)}{2}\right)}{b}}{b}$	61
default	$\frac{-\frac{da}{2b\cos(bx+a)^2}+\frac{c}{2\cos(bx+a)^2}+\frac{d\left(\frac{bx+a}{2\cos(bx+a)^2}-\frac{\tan(bx+a)}{2}\right)}{b}}{b}$	61
risch	$\frac{2bdx\,e^{2i(bx+a)}-id\,e^{2i(bx+a)}+2bc\,e^{2i(bx+a)}-id}{b^2\left(1+e^{2i(bx+a)}\right)^2}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2/b*d*a/cos(b*x+a)^2+1/2*c/cos(b*x+a)^2+1/b*d*(1/2*(b*x+a)/cos(b*x+
a)^2-1/2*tan(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(31) = 62.
time = 0.27, size = 283, normalized size = 8.09

$$\frac{c\tan(bx+a)^2-\frac{ad\tan(bx+a)^2}{b}+\frac{2\left(4\,(bx+a)\cos(2\,bx+2\,a)^2+4\,(bx+a)\sin(2\,bx+2\,a)^2+(2\,(bx+a)\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a))\cos(4\,bx+4\,a)+2\,(bx+a)\cos(2\,bx+2\,a)+(2\,(bx+a)\sin(2\,bx+2\,a)-\cos(2\,bx+2\,a)-1)\sin(4\,bx+4\,a)-\sin(2\,bx+2\,a)\right)d}{\left(2\,(2\cos(2\,bx+2\,a)+1)\cos(4\,bx+4\,a)+\cos(4\,bx+4\,a)^2+4\cos(2\,bx+2\,a)^2+\sin(4\,bx+4\,a)^2+4\sin(4\,bx+4\,a)\sin(2\,bx+2\,a)+4\sin(2\,bx+2\,a)^2+4\cos(2\,bx+2\,a)+1\right)b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c*tan(b*x + a)^2 - a*d*tan(b*x + a)^2/b + 2*(4*(b*x + a)*cos(2*b*x + 2
*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + si
n(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x +
 a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x +
 2*a))*d/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2
```

```
+ 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x
+ 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b))/b
```

Fricas [A]
time = 2.08, size = 36, normalized size = 1.03

$$\frac{bdx - d\cos(bx+a)\sin(bx+a) + bc}{2\,b^2\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x - d*cos(b*x + a)*sin(b*x + a) + b*c)/(b^2*cos(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\tan(a+bx)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)*tan(a + b*x)*sec(a + b*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(31) = 62.
time = 0.49, size = 571, normalized size = 16.31

$$\frac{bdx\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^4 + bc\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^4 + 2bdx\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^2 + 2bdx\tan\left(\frac{1}{2}bx\right)^2\tan\left(\frac{1}{2}a\right)^4 + 2bc\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^2 + 2d\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^3 + 2bc\tan\left(\frac{1}{2}bx\right)^2\tan\left(\frac{1}{2}a\right)^4 + 2d\tan\left(\frac{1}{2}bx\right)^3\tan\left(\frac{1}{2}a\right)^4 + bdx\tan\left(\frac{1}{2}bx\right)^4 + 4bdx\tan\left(\frac{1}{2}bx\right)^2\tan\left(\frac{1}{2}a\right)^2 + bdx\tan\left(\frac{1}{2}a\right)^4 + bc\tan\left(\frac{1}{2}bx\right)^4 - 2d\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right) + 4bc\tan\left(\frac{1}{2}bx\right)^2\tan\left(\frac{1}{2}a\right)^2 - 12d\tan\left(\frac{1}{2}bx\right)^3\tan\left(\frac{1}{2}a\right)^2 - 12d\tan\left(\frac{1}{2}bx\right)^2\tan\left(\frac{1}{2}a\right)^3 + bc\tan\left(\frac{1}{2}a\right)^4 - 2d\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}a\right)^4 + 2bdx\tan\left(\frac{1}{2}bx\right)^2 + 2bdx\tan\left(\frac{1}{2}a\right)^2 + 2bc\tan\left(\frac{1}{2}bx\right)^2 + 2d\tan\left(\frac{1}{2}bx\right)^3 + 12d\tan\left(\frac{1}{2}bx\right)^2\tan\left(\frac{1}{2}a\right) + 2bc\tan\left(\frac{1}{2}a\right)^2 + 12d\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}a\right)^2 + 2d\tan\left(\frac{1}{2}a\right)^3 + bdx + bc - 2d\tan\left(\frac{1}{2}bx\right) - 2d\tan\left(\frac{1}{2}a\right)}{2\left(b^2\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^4 - 2b^2\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^2 - 8b^2\tan\left(\frac{1}{2}bx\right)^3\tan\left(\frac{1}{2}a\right)^3 - 2b^2\tan\left(\frac{1}{2}bx\right)^2\tan\left(\frac{1}{2}a\right)^4 + b^2\tan\left(\frac{1}{2}bx\right)^4 + 8b^2\tan\left(\frac{1}{2}bx\right)^3\tan\left(\frac{1}{2}a\right) + 20b^2\tan\left(\frac{1}{2}bx\right)^2\tan\left(\frac{1}{2}a\right)^2 + 8b^2\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}a\right)^3 + b^2\tan\left(\frac{1}{2}a\right)^4 - 2b^2\tan\left(\frac{1}{2}bx\right)^2 - 8b^2\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}a\right) - 2b^2\tan\left(\frac{1}{2}a\right)^2 + b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 +
2*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 +
 2*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 2*b*
c*tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + b*d*x*tan
(1/2*b*x)^4 + 4*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*d*x*tan(1/2*a)^4 + b*
c*tan(1/2*b*x)^4 - 2*d*tan(1/2*b*x)^4*tan(1/2*a) + 4*b*c*tan(1/2*b*x)^2*tan
(1/2*a)^2 - 12*d*tan(1/2*b*x)^3*tan(1/2*a)^2 - 12*d*tan(1/2*b*x)^2*tan(1/2*
a)^3 + b*c*tan(1/2*a)^4 - 2*d*tan(1/2*b*x)*tan(1/2*a)^4 + 2*b*d*x*tan(1/2*b
*x)^2 + 2*b*d*x*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2 + 2*d*tan(1/2*b*x)^3 +
12*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b*c*tan(1/2*a)^2 + 12*d*tan(1/2*b*x)*tan
(1/2*a)^2 + 2*d*tan(1/2*a)^3 + b*d*x + b*c - 2*d*tan(1/2*b*x) - 2*d*tan(1/2
*a))/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^4 - 2*b^2*tan(1/2*b*x)^4*tan(1/2*a)^2 -
```

```
 8*b^2*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*b^2*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^
2*tan(1/2*b*x)^4 + 8*b^2*tan(1/2*b*x)^3*tan(1/2*a) + 20*b^2*tan(1/2*b*x)^2*
tan(1/2*a)^2 + 8*b^2*tan(1/2*b*x)*tan(1/2*a)^3 + b^2*tan(1/2*a)^4 - 2*b^2*t
an(1/2*b*x)^2 - 8*b^2*tan(1/2*b*x)*tan(1/2*a) - 2*b^2*tan(1/2*a)^2 + b^2)
```

Mupad [B]
time = 2.19, size = 53, normalized size = 1.51

$$-\frac{d\,1\mathrm{i}+\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,\left(-b\,\left(2\,c+2\,d\,x\right)+d\,1\mathrm{i}\right)}{b^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x))/cos(a + b*x)^2,x)
```

```
[Out] -(d*1i + exp(a*2i + b*x*2i)*(d*1i - b*(2*c + 2*d*x)))/(b^2*(exp(a*2i + b*x*
2i) + 1)^2)
```

3.295 $\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sec^2(a+bx)\tan(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] Defer[Int][(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 6.79, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(\sec^2(bx+a))\tan(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] (4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
(2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) +
2*(b*d*x + b*c)*cos(2*b*x + 2*a) + 2*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2
*d^2 + (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(4*b*x + 4*a)^2 + 4*(
b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^
2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^4*x^2 + 2*b^
2*c*d^3*x + b^2*c^2*d^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^4*x^2
 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4*x^2 + 2*b^2
*c*d^3*x + b^2*c^2*d^2 + 2*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(
2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d
^2)*cos(2*b*x + 2*a))*integrate(sin(2*b*x + 2*a)/(b^2*d^3*x^3 + 3*b^2*c*d^2
*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2
*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2
*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) + (d*cos(2*b*x + 2*a) + 2*(
b*d*x + b*c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4*a) + d*sin(2*b*x + 2*a))/(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2
*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^
2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a
))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2
*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^2*tan(b*x + a)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*tan(b*x+a)/(d*x+c),x)
[Out] Integral(tan(a + b*x)*sec(a + b*x)**2/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(sec(b*x + a)^2*tan(b*x + a)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(a+b\,x)}{\cos\left(a+b\,x\right)^2\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)),x)
[Out] int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)), x)
```

3.296 $\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
[Out] Defer[Int][(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 10.23, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
[Out] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(\sec^2\left(bx+a\right)\right)\tan\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 2*(2*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 2*(b*d*x + b*c)*sin(2*b*x + 2*a)^2
+ ((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) +
(b*d*x + b*c)*cos(2*b*x + 2*a) + 3*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c
^2*d^3*x + b^2*c^3*d^2 + (b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x +
 b^2*c^3*d^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2
*c^2*d^3*x + b^2*c^3*d^2)*cos(2*b*x + 2*a)^2 + (b^2*d^5*x^3 + 3*b^2*c*d^4*x
^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^5*x^3 + 3
*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*sin(4*b*x + 4*a)*sin(2*b*x
+ 2*a) + 4*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*
sin(2*b*x + 2*a)^2 + 2*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b
^2*c^3*d^2 + 2*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d
^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 +
 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*cos(2*b*x + 2*a))*integrate(sin(2*b*x + 2*a
)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*
c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x +
b^2*c^4)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^
2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4*x^4 + 4*b^
2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)
), x) + (d*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(2*b*x + 2*a) + d)*sin(4*b*x
 + 4*a) + d*sin(2*b*x + 2*a))/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*
x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos
(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^
3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^
2*c^3)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*
x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d
^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b
^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^2*tan(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*tan(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(tan(a + b*x)*sec(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2*tan(b*x + a)/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(a+b\,x)}{\cos\left(a+b\,x\right)^2\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)^2),x)
```

```
[Out] int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)^2), x)
```

3.297 $\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=39

$$-\text{Int}((c+dx)^m\sec(a+bx),x)+\text{Int}\left((c+dx)^m\sec^3(a+bx),x\right)$$

```
[Out] -Unintegrable((d*x+c)^m*sec(b*x+a),x)+Unintegrable((d*x+c)^m*sec(b*x+a)^3,x
)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m\sec(a+bx)\tan^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] -Defer[Int][(c + d*x)^m*Sec[a + b*x], x] + Defer[Int][(c + d*x)^m*Sec[a + b
*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m\sec(a+bx)\tan^2(a+bx)\,dx=-\int (c+dx)^m\sec(a+bx)\,dx+\int (c+dx)^m\sec^3(a+bx)\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$$\$\text{Aborted}$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] $Aborted
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int (dx+c)^m\sec(bx+a)\left(\tan^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \tan^2(a+bx) \sec(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*tan(a + b*x)**2*sec(a + b*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tan(a+b\,x)^2\,(c+d\,x)^m}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)^2*(c + d*x)^m)/cos(a + b*x),x)
[Out] int((tan(a + b*x)^2*(c + d*x)^m)/cos(a + b*x), x)
```

3.298 $\int (c+dx)^3 \sec(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=337

$$-\frac{6id^2(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^3}+\frac{i(c+dx)^3\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{3id^3\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^4}-\frac{3id(c+dx)^2\text{Poly}}{2}$$

```
[Out] -6*I*d^2*(d*x+c)*arctan(exp(I*(b*x+a)))/b^3+I*(d*x+c)^3*arctan(exp(I*(b*x+a
)))/b+3*I*d^3*polylog(2,-I*exp(I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polylog(2,
-I*exp(I*(b*x+a)))/b^2-3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4+3/2*I*d*(d*x
+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^2+3*d^2*(d*x+c)*polylog(3,-I*exp(I*(b*x
+a)))/b^3-3*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^3+3*I*d^3*polylog(4,-
I*exp(I*(b*x+a)))/b^4-3*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-3/2*d*(d*x+c)
^2*sec(b*x+a)/b^2+1/2*(d*x+c)^3*sec(b*x+a)*tan(b*x+a)/b
```

Rubi [A]
time = 0.28, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {4498, 4266, 2611, 6744, 2320, 6724, 4271, 2317, 2438}

$$-\frac{6id^2(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^3}+\frac{i(c+dx)^3\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{3id^3\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^4}+\frac{3id^3\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{Li}_4\left(ie^{i(a+bx)}\right)}{b^4}+\frac{3d^2(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}-\frac{3d^2(c+dx)\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}-\frac{3id(c+dx)^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2}+\frac{3id(c+dx)^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{2b^2}-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] ((-6*I)*d^2*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^3 + (I*(c + d*x)^3*ArcTan[
E^(I*(a + b*x))])/b + ((3*I)*d^3*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^4 - ((
(3*I)/2)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d^3*P
olyLog[2, I*E^(I*(a + b*x))])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E
^(I*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b
^3 - (3*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + ((3*I)*d^3*PolyL
og[4, (-I)*E^(I*(a + b*x))])/b^4 - ((3*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))]
)/b^4 - (3*d*(c + d*x)^2*Sec[a + b*x])/(2*b^2) + ((c + d*x)^3*Sec[a + b*x]*
Tan[a + b*x])/(2*b)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4498

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\sec(a+bx)\tan^2(a+bx)\,dx &= -\int (c+dx)^3\sec(a+bx)\,dx+\int (c+dx)^3\sec^3(a+bx)\,dx\\ &= \frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3}{}\\ &= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3}+\frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{3id}{}\\ &= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3}+\frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{3id}{}\\ &= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3}+\frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id}{}\\ &= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3}+\frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id}{}\\ &= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3}+\frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id}{}\end{aligned}$$

Mathematica [A]
time = 3.09, size = 530, normalized size = 1.57

$$\frac{2ib^3c^3\text{ArcTan}\left(e^{i(a+bx)}\right)-12ibcd^2\text{ArcTan}\left(e^{i(a+bx)}\right)-3b^3c^2dx\log\left(1-ie^{i(a+bx)}\right)+6bd^3x\log\left(1-ie^{i(a+bx)}\right)-3b^3cd^2x^2\log\left(1-ie^{i(a+bx)}\right)-b^3d^3x^3\log\left(1-ie^{i(a+bx)}\right)+3b^3c^2dx\log\left(1+ie^{i(a+bx)}\right)-6bd^3x\log\left(1+ie^{i(a+bx)}\right)+3b^3cd^2x^2\log\left(1+ie^{i(a+bx)}\right)+b^3d^3x^3\log\left(1+ie^{i(a+bx)}\right)-3id(-2d^2+b^2(c+dx)^2)\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)+3id(-2d^2+b^2(c+dx)^2)\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)+6bcd^2\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)+6bd^3x\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)-6bcd^2\text{PolyLog}\left(3,ie^{i(a+bx)}\right)-6bd^3x\text{PolyLog}\left(3,ie^{i(a+bx)}\right)+6id^3\text{PolyLog}\left(4,-ie^{i(a+bx)}\right)-6id^3\text{PolyLog}\left(4,ie^{i(a+bx)}\right)+b^2(c+dx)^2\sec(a+bx)(-3d+b(c+dx)\tan(a+bx))}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] ((2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] - (12*I)*b*c*d^2*ArcTan[E^(I*(a + b*
x))] - 3*b^3*c^2*d*x*Log[1 - I*E^(I*(a + b*x))] + 6*b*d^3*x*Log[1 - I*E^(I*
(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[
1 - I*E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 6*b*d^3
*x*Log[1 + I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))]
+ b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x))] - (3*I)*d*(-2*d^2 + b^2*(c + d*x)^
2)*PolyLog[2, (-I)*E^(I*(a + b*x))] + (3*I)*d*(-2*d^2 + b^2*(c + d*x)^2)*Po
```

```
lyLog[2, I*E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] +
6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(a
 + b*x))] - 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4,
(-I)*E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))] + b^2*(c +
d*x)^2*Sec[a + b*x]*(-3*d + b*(c + d*x)*Tan[a + b*x]))/(2*b^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1126 vs. 2(293) = 586.
time = 0.24, size = 1127, normalized size = 3.34

method	result	size
risch	Expression too large to display	1127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3*I/b^2*c*d^2*polylog(2,I*exp(I*(b*x+a)))*x-3*I/b^2*c^2*d*a*arctan(exp(I*(b
*x+a)))-3*I/b^2*c*d^2*polylog(2,-I*exp(I*(b*x+a)))*x+3*I/b^3*c*d^2*a^2*arct
an(exp(I*(b*x+a)))-6*I/b^3*c*d^2*arctan(exp(I*(b*x+a)))-3/2*I/b^2*d^3*polyl
og(2,-I*exp(I*(b*x+a)))*x^2+3/2*I/b^2*d^3*polylog(2,I*exp(I*(b*x+a)))*x^2+3
/2*I/b^2*c^2*d*polylog(2,I*exp(I*(b*x+a)))-3/2*I/b^2*c^2*d*polylog(2,-I*exp
(I*(b*x+a)))-I/b^4*d^3*a^3*arctan(exp(I*(b*x+a)))+6*I/b^4*d^3*a*arctan(exp(
I*(b*x+a)))+3*I*d^3*polylog(2,-I*exp(I*(b*x+a)))/b^4+3*I*d^3*polylog(4,-I*e
xp(I*(b*x+a)))/b^4-3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4-3*I*d^3*polylog(
4,I*exp(I*(b*x+a)))/b^4-I/b^2/(1+exp(2*I*(b*x+a)))^2*(d^3*x^3*b*exp(3*I*(b*
x+a))+3*c*d^2*x^2*b*exp(3*I*(b*x+a))+3*c^2*d*x*b*exp(3*I*(b*x+a))-d^3*x^3*b
*exp(I*(b*x+a))+c^3*b*exp(3*I*(b*x+a))-3*c*d^2*x^2*b*exp(I*(b*x+a))-3*I*d^3
*x^2*exp(3*I*(b*x+a))-3*c^2*d*x*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(3*I*(b*x+a
))-c^3*b*exp(I*(b*x+a))-3*I*c^2*d*exp(3*I*(b*x+a))-3*I*d^3*x^2*exp(I*(b*x+a
))-6*I*c*d^2*x*exp(I*(b*x+a))-3*I*c^2*d*exp(I*(b*x+a)))+3/b^3*d^3*polylog(3
,-I*exp(I*(b*x+a)))*x-3/b^3*d^3*polylog(3,I*exp(I*(b*x+a)))*x-3/2/b^3*a^2*d
^2*c*ln(1+I*exp(I*(b*x+a)))-3/2/b*c^2*d*ln(1-I*exp(I*(b*x+a)))*x-3/2/b^2*c^
2*d*ln(1-I*exp(I*(b*x+a)))*a+3/2/b^3*a^2*d^2*c*ln(1-I*exp(I*(b*x+a)))+3/2/b
*d^2*c*ln(1+I*exp(I*(b*x+a)))*x^2-3/2/b*d^2*c*ln(1-I*exp(I*(b*x+a)))*x^2+3/
2/b*c^2*d*ln(1+I*exp(I*(b*x+a)))*x+3/2/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a+3
/b^3*d^3*ln(1-I*exp(I*(b*x+a)))*x+3/b^4*d^3*ln(1-I*exp(I*(b*x+a)))*a-3/b^3*
d^3*ln(1+I*exp(I*(b*x+a)))*x-3/b^4*d^3*ln(1+I*exp(I*(b*x+a)))*a+I/b*c^3*arc
tan(exp(I*(b*x+a)))-1/2/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3+1/2/b*d^3*ln(1+I*e
xp(I*(b*x+a)))*x^3-1/2/b^4*a^3*d^3*ln(1-I*exp(I*(b*x+a)))+1/2/b^4*a^3*d^3*l
n(1+I*exp(I*(b*x+a)))+3/b^3*d^2*c*polylog(3,-I*exp(I*(b*x+a)))-3/b^3*d^2*c*
polylog(3,I*exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3831 vs. 2(273) = 546.
time = 1.60, size = 3831, normalized size = 11.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")

[Out] -1/4*(c^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - lo
g(sin(b*x + a) - 1)) - 3*a*c^2*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log
(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*sin(b*x + a)
/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b^2
- a^3*d^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - lo
g(sin(b*x + a) - 1))/b^3 - 4*(2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*
(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)
*(b*x + a) + ((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(
b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(4*b
*x + 4*a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*
(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(2*
b*x + 2*a) + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + 3*(I*b*c*d^2 -
I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 - 2*I)*d^3)*
(b*x + a))*sin(4*b*x + 4*a) + 2*(I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^
3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 +
(I*a^2 - 2*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b
x + a) + 1) + 2((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^
3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a) + ((
b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*
(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 2*(
(b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3
*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (I
*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x +
 a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 - 2*I)*d^3)*(b*x + a))*sin(
4*b*x + 4*a) + 2*(I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + 3*(I*b*c*d^
2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 - 2*I)*d
^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) -
 4*((b*x + a)^3*d^3 - 3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*a^2*d^3 + 3*(b*c*
d^2 - (a + I)*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*(a + I)*b*c*d^2 + (a^2 +
2*I*a)*d^3)*(b*x + a))*cos(3*b*x + 3*a) + 4*((b*x + a)^3*d^3 + 3*I*b^2*c^2*
d - 6*I*a*b*c*d^2 + 3*I*a^2*d^3 + 3*(b*c*d^2 - (a - I)*d^3)*(b*x + a)^2 + 3
*(b^2*c^2*d - 2*(a - I)*b*c*d^2 + (a^2 - 2*I*a)*d^3)*(b*x + a))*cos(b*x + a
) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d
^2 - a*d^3)*(b*x + a) + (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 -
 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 2*(b^2*c^2*d -
2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x +
a))*cos(2*b*x + 2*a) + (I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + (
I*a^2 - 2*I)*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*
(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + (I*a^2 - 2*I)*d^3 + 2*(I
*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) -
 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2

```
- a*d^3)*(b*x + a) + (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)
*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 2*(b^2*c^2*d - 2*a
*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))
*cos(2*b*x + 2*a) - (-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 + (-I
*a^2 + 2*I)*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*
(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 + (-I*a^2 + 2*I)*d^3 + 2*
(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a
)) - (-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 - 3*(I*b*c*d^2 - I*a*d^3
)*(b*x + a)^2 - 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 - 2*I)*d^3)*(b*x +
a) + (-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 - 3*(I*b*c*d^2 - I*a*d^3
)*(b*x + a)^2 - 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 - 2*I)*d^3)*(b*x +
a))*cos(4*b*x + 4*a) - 2*(I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + 3*(
I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2
- 2*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*
a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2
 - 2)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6
*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^
2 - 2)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^
2 + 2*sin(b*x + a) + 1) - (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 - 3*
(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 - 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*
a^2 + 2*I)*d^3)*(b*x + a) + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 -
3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 - 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-
I*a^2 + 2*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 2*(-I*(b*x + a)^3*d^3 + 6*I
*b*c*d^2 - 6*I*a*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2
*d + 2*I*a*b*c*d^2 + (-I*a^2 + 2*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - ((b*
x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1315 vs. 2(273) = 546.
time = 3.72, size = 1315, normalized size = 3.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(-6*I*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 6*
I*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 6*I*d^3*co
s(b*x + a)^2*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*cos(b*x +
 a)^2*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*
b^2*c*d^2*x - I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) +
sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d + 2*I*d^3
)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*d^3*x^2 +
2*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x +
a) + sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I
```

```
*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^3*c^3 - 3*a
*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2*log(cos(
b*x + a) + I*sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c
*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) +
I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3
- 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2*log(I*cos(b*x + a) +
 sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2*log(
I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos
(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*
c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d
- 2*b*d^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3
*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^
2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a
^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin
(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, I*cos(b*x
+ a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, I*co
s(b*x + a) - sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3
, -I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*po
lylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x +
 b^2*c^2*d)*cos(b*x + a) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x
 + b^3*c^3)*sin(b*x + a))/(b^4*cos(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \tan^2 (a + bx) \sec (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*tan(b*x+a)**2,x)
[Out] Integral((c + d*x)**3*tan(a + b*x)**2*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^3*sec(b*x + a)*tan(b*x + a)^2, x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)^2*(c + d*x)^3)/cos(a + b*x),x)
[Out] \text{Hanged}
```

3.299 $\int (c+dx)^2 \sec(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=193

$$\frac{i(c+dx)^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3}-\frac{id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}+\frac{id(c+dx)\text{PolyLog}}{b^2}$$

```
[Out] I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b+d^2*arctanh(sin(b*x+a))/b^3-I*d*(d*x+c
)*polylog(2,-I*exp(I*(b*x+a)))/b^2+I*d*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/
b^2+d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3-d^2*polylog(3,I*exp(I*(b*x+a)))/b^
3-d*(d*x+c)*sec(b*x+a)/b^2+1/2*(d*x+c)^2*sec(b*x+a)*tan(b*x+a)/b
```

Rubi [A]
time = 0.18, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4498, 4266, 2611, 2320, 6724, 4271, 3855}

$$\frac{i(c+dx)^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}-\frac{d^2\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3}-\frac{id(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}+\frac{id(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\sec(a+bx)}{b^2}+\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (I*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (d^2*ArcTanh[Sin[a + b*x]])/b^3
 - (I*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 + (I*d*(c + d*x)*Po
lyLog[2, I*E^(I*(a + b*x))])/b^2 + (d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b
^3 - (d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - (d*(c + d*x)*Sec[a + b*x])/b
^2 + ((c + d*x)^2*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4498

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2 \sec(a+bx)\tan^2(a+bx)\,dx &= -\int (c+dx)^2\sec(a+bx)\,dx + \int (c+dx)^2\sec^3(a+bx)\,dx \\ &= \frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d(c+dx)\sec(a+bx)}{b^2} + \frac{(c+dx)^2 \mathrm{s}}{} \\ &= \frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3} - \frac{2id(c+dx}{} \\ &= \frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3} - \frac{id(c+dx)}{} \\ &= \frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3} - \frac{id(c+dx)}{} \\ &= \frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3} - \frac{id(c+dx)}{}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 526 vs. $2(193) = 386$.
time = 6.92, size = 526, normalized size = 2.73

$$\frac{ibc^2\mathrm{ArcTan}\left(e^{i(a+bx)}\right) - \frac{2id^2\mathrm{ArcTan}\left(e^{i(a+bx)}\right)}{b} - bcdx\log\left(1-ie^{i(a+bx)}\right) - \frac{1}{2}bd^2x^2\log\left(1-ie^{i(a+bx)}\right) + bcdx\log\left(1+ie^{i(a+bx)}\right) + \frac{1}{2}bd^2x^2\log\left(1+ie^{i(a+bx)}\right) - id(c+dx)\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right) + id(c+dx)\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right) + \frac{d^2\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b} - \frac{d^2\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b}}{b^2} - \frac{d(c+dx)\sec(a)}{b^2} + \frac{c^2+2cdx+d^2x^2}{4b\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)^2} + \frac{-cd\sin\left(\frac{bx}{2}\right)-d^2x\sin\left(\frac{bx}{2}\right)}{b^2\left(\cos\left(\frac{a}{2}\right)-\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)} + \frac{-c^2-2cdx-d^2x^2}{4b\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)^2} + \frac{cd\sin\left(\frac{bx}{2}\right)+d^2x\sin\left(\frac{bx}{2}\right)}{b^2\left(\cos\left(\frac{a}{2}\right)+\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (I*b*c^2*ArcTan[E^(I*(a + b*x))] - ((2*I)*d^2*ArcTan[E^(I*(a + b*x))])/b -
b*c*d*x*Log[1 - I*E^(I*(a + b*x))] - (b*d^2*x^2*Log[1 - I*E^(I*(a + b*x))])
/2 + b*c*d*x*Log[1 + I*E^(I*(a + b*x))] + (b*d^2*x^2*Log[1 + I*E^(I*(a + b*
x))])/2 - I*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] + I*d*(c + d*x)*Po
lyLog[2, I*E^(I*(a + b*x))] + (d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b - (d
^2*PolyLog[3, I*E^(I*(a + b*x))])/b)/b^2 - (d*(c + d*x)*Sec[a])/b^2 + (c^2
+ 2*c*d*x + d^2*x^2)/(4*b*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])^2) + (-
(c*d*Sin[(b*x)/2]) - d^2*x*Sin[(b*x)/2])/(b^2*(Cos[a/2] - Sin[a/2])*(Cos[a/
2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) + (-c^2 - 2*c*d*x - d^2*x^2)/(4*b*(Cos[
a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2])^2) + (c*d*Sin[(b*x)/2] + d^2*x*Sin[(b*
x)/2])/(b^2*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2])
)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(174) = 348$.
time = 0.19, size = 584, normalized size = 3.03

method	result

risch	$\frac{ic^2 \arctan(e^{i(bx+a)})}{b} + \frac{id^2a^2 \arctan(e^{i(bx+a)})}{b^3} - \frac{id^2 \operatorname{polylog}(2,-ie^{i(bx+a)})x}{b^2} + \frac{id^2 \operatorname{polylog}(2,ie^{i(bx+a)})x}{b^2} - \frac{d^2 \ln(1-ie^{i(bx+a}}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] I/b*c^2*arctan(exp(I*(b*x+a)))+I/b^3*a^2*d^2*arctan(exp(I*(b*x+a)))-I/b^2*d
^2*polylog(2,-I*exp(I*(b*x+a)))*x+I/b^2*d^2*polylog(2,I*exp(I*(b*x+a)))*x-1
/2/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2+d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+1/
2/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^2+1/b^2*c*d*ln(1+I*exp(I*(b*x+a)))*a+1/b*c
*d*ln(1+I*exp(I*(b*x+a)))*x+1/2/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))-1/b*c*d*
ln(1-I*exp(I*(b*x+a)))*x-2*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))+I/b^2*c*d*pol
ylog(2,I*exp(I*(b*x+a)))-I/b^2*c*d*polylog(2,-I*exp(I*(b*x+a)))-I/b^2/(1+ex
p(2*I*(b*x+a)))^2*(d^2*x^2*b*exp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I*(b*x+a))+c^
2*b*exp(3*I*(b*x+a))-d^2*x^2*b*exp(I*(b*x+a))-2*c*d*x*b*exp(I*(b*x+a))-2*I*
d^2*x*exp(3*I*(b*x+a))-c^2*b*exp(I*(b*x+a))-2*I*d*c*exp(3*I*(b*x+a))-2*I*d^
2*x*exp(I*(b*x+a))-2*I*c*d*exp(I*(b*x+a)))-2*I/b^3*d^2*arctan(exp(I*(b*x+a)
))-1/2/b^3*a^2*d^2*ln(1+I*exp(I*(b*x+a)))-d^2*polylog(3,I*exp(I*(b*x+a)))/b
^3-1/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1891 vs. $2(165) = 330$.
time = 0.75, size = 1891, normalized size = 9.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(c^2*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - lo
g(sin(b*x + a) - 1)) - 2*a*c*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(s
in(b*x + a) + 1) - log(sin(b*x + a) - 1))/b + a^2*d^2*(2*sin(b*x + a)/(sin(
b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b^2 - 4*(2
*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2 + ((b*x + a)^2*d^2
+ 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(4*b*x + 4*a) + 2*((b*x + a)^2*d^
2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(2*b*x + 2*a) + (I*(b*x + a)^2*
d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(4*b*x + 4*a) + 2*(I*(b
*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(2*b*x + 2*a)
)*arctan2(cos(b*x + a), sin(b*x + a) + 1) + 2*((b*x + a)^2*d^2 + 2*(b*c*d -
 a*d^2)*(b*x + a) - 2*d^2 + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a)
- 2*d^2)*cos(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a
) - 2*d^2)*cos(2*b*x + 2*a) + (I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b
*x + a) - 2*I*d^2)*sin(4*b*x + 4*a) + 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I
*a*d^2)*(b*x + a) - 2*I*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b
```

```
*x + a) + 1) - 4*((b*x + a)^2*d^2 - 2*I*b*c*d + 2*I*a*d^2 + 2*(b*c*d - (a +
 I)*d^2)*(b*x + a))*cos(3*b*x + 3*a) + 4*((b*x + a)^2*d^2 + 2*I*b*c*d - 2*I
*a*d^2 + 2*(b*c*d - (a - I)*d^2)*(b*x + a))*cos(b*x + a) + 4*(b*c*d + (b*x
+ a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + 2*(b*
c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (I*b*c*d + I*(b*x + a)*d^2
- I*a*d^2)*sin(4*b*x + 4*a) + 2*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(2
*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2 +
(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + 2*(b*c*d + (b*x + a)*d^2
 - a*d^2)*cos(2*b*x + 2*a) - (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(4*b
*x + 4*a) - 2*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(2*b*x + 2*a))*dilo
g(-I*e^(I*b*x + I*a)) - (-I*(b*x + a)^2*d^2 - 2*(I*b*c*d - I*a*d^2)*(b*x +
a) + 2*I*d^2 + (-I*(b*x + a)^2*d^2 - 2*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*
d^2)*cos(4*b*x + 4*a) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x +
 a) - 2*I*d^2)*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
 + a) - 2*d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b
*x + a) - 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*
sin(b*x + a) + 1) - (I*(b*x + a)^2*d^2 - 2*(-I*b*c*d + I*a*d^2)*(b*x + a) -
 2*I*d^2 + (I*(b*x + a)^2*d^2 - 2*(-I*b*c*d + I*a*d^2)*(b*x + a) - 2*I*d^2)
*cos(4*b*x + 4*a) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a
) + 2*I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x +
 a) - 2*d^2)*sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
 + a) - 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*si
n(b*x + a) + 1) + 4*(I*d^2*cos(4*b*x + 4*a) + 2*I*d^2*cos(2*b*x + 2*a) - d^
2*sin(4*b*x + 4*a) - 2*d^2*sin(2*b*x + 2*a) + I*d^2)*polylog(3, I*e^(I*b*x
+ I*a)) + 4*(-I*d^2*cos(4*b*x + 4*a) - 2*I*d^2*cos(2*b*x + 2*a) + d^2*sin(4
*b*x + 4*a) + 2*d^2*sin(2*b*x + 2*a) - I*d^2)*polylog(3, -I*e^(I*b*x + I*a)
) + 4*(-I*(b*x + a)^2*d^2 - 2*b*c*d + 2*a*d^2 + 2*(-I*b*c*d + (I*a - 1)*d^2
)*(b*x + a))*sin(3*b*x + 3*a) + 4*(I*(b*x + a)^2*d^2 - 2*b*c*d + 2*a*d^2 +
2*(I*b*c*d + (-I*a - 1)*d^2)*(b*x + a))*sin(b*x + a))/(-4*I*b^2*cos(4*b*x +
 4*a) - 8*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(4*b*x + 4*a) + 8*b^2*sin(2*b*x
 + 2*a) - 4*I*b^2))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(165) = 330$.
time = 3.32, size = 795, normalized size = 4.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^2
*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b*x +
 a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*cos(b*x + a)^2*pol
ylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*cos(b*x
```

```
+ a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*cos(
b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*c
os(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*
d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^2*c^2 - 2*a*b*
c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I)
+ (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) - I
*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(
b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*
d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a)
 + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*lo
g(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*
c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b^
2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin
(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(-
cos(b*x + a) - I*sin(b*x + a) + I) - 4*(b*d^2*x + b*c*d)*cos(b*x + a) + 2*(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(b*x + a))/(b^3*cos(b*x + a)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \tan^2(a+bx) \sec(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*tan(b*x+a)**2,x)
[Out] Integral((c + d*x)**2*tan(a + b*x)**2*sec(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^2*sec(b*x + a)*tan(b*x + a)^2, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)^2*(c + d*x)^2)/cos(a + b*x),x)
[Out] \text{Hanged}
```

3.300 $\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=117

$$\frac{i(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}-\frac{id\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}+\frac{id\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\sec(\cdots}{\cdots}$$

```
[Out] I*(d*x+c)*arctan(exp(I*(b*x+a)))/b-1/2*I*d*polylog(2,-I*exp(I*(b*x+a)))/b^2
+1/2*I*d*polylog(2,I*exp(I*(b*x+a)))/b^2-1/2*d*sec(b*x+a)/b^2+1/2*(d*x+c)*s
ec(b*x+a)*tan(b*x+a)/b
```

Rubi [A]
time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4498, 4266, 2317, 2438, 4270}

$$\frac{i(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}-\frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2}+\frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (I*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b - ((I/2)*d*PolyLog[2, (-I)*E^(I*(a
+ b*x))])/b^2 + ((I/2)*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (d*Sec[a + b*
x])/(2*b^2) + ((c + d*x)*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4498

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rubi steps

$$\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)\sec(a+bx)\,dx + \int (c+dx)\sec^3(a+bx)\,dx$$

$$= \frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\sec(a+bx)}{2b^2} + \frac{(c+dx)\sec(a+bx)}{2b}$$

$$= \frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\sec(a+bx)}{2b^2} + \frac{(c+dx)\sec(a+bx)}{2b}$$

$$= \frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2} + \frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2} +$$

$$= \frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2} + \frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{2b^2} +$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 538 vs. $2(117) = 234$.
time = 6.63, size = 538, normalized size = 4.60

$$-\frac{c\tanh^{-1}(\sin(a+bx))}{2b} - \frac{dx\left(a\left(-\log\left(1-\tan\left(\frac{1}{2}(a+bx)\right)\right)+\log\left(1+\tan\left(\frac{1}{2}(a+bx)\right)\right)\right) - i\left(\log\left(1+i\tan\left(\frac{1}{2}(a+bx)\right)\right)\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)\left(-1+\tan\left(\frac{1}{2}(a+bx)\right)\right)\right) - \log\left(1-i\tan\left(\frac{1}{2}(a+bx)\right)\right)\log\left(\left(-\frac{1}{2}+\frac{i}{2}\right)\left(-1+\tan\left(\frac{1}{2}(a+bx)\right)\right)\right) - \log\left(1+i\tan\left(\frac{1}{2}(a+bx)\right)\right)\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)\left(1+\tan\left(\frac{1}{2}(a+bx)\right)\right)\right) + \log\left(1-i\tan\left(\frac{1}{2}(a+bx)\right)\right)\log\left(\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+\tan\left(\frac{1}{2}(a+bx)\right)\right)\right) - \text{PolyLog}\left(2,\frac{1}{2}\left((1+i)-(1-i)\tan\left(\frac{1}{2}(a+bx)\right)\right)\right) + \text{PolyLog}\left(2,\left(-\frac{1}{2}-\frac{i}{2}\right)\left(i+\tan\left(\frac{1}{2}(a+bx)\right)\right)\right) - \text{PolyLog}\left(2,\frac{1}{2}\left((1+i)+(1-i)\tan\left(\frac{1}{2}(a+bx)\right)\right)\right) + \text{PolyLog}\left(2,\frac{1}{2}\left((1-i)+(1+i)\tan\left(\frac{1}{2}(a+bx)\right)\right)\right)\right)\right)}{2b\left(a-i\log\left(1-i\tan\left(\frac{1}{2}(a+bx)\right)\right)+i\log\left(1+i\tan\left(\frac{1}{2}(a+bx)\right)\right)\right)} + \frac{dx}{4b\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)^2} - \frac{d\sin\left(\frac{1}{2}(a+bx)\right)}{2b^2\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)} - \frac{dx}{4b\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)^2} + \frac{d\sin\left(\frac{1}{2}(a+bx)\right)}{2b^2\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)} + \frac{c\sec(a+bx)\tan(a+bx)}{2b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] -1/2*(c*ArcTanh[Sin[a + b*x]])/b - (d*x*(a*(-Log[1 - Tan[(a + b*x)/2]] + Lo
g[1 + Tan[(a + b*x)/2]]) - I*(Log[1 + I*Tan[(a + b*x)/2]]*Log[(-1/2 - I/2)*
(-1 + Tan[(a + b*x)/2])] - Log[1 - I*Tan[(a + b*x)/2]]*Log[(-1/2 + I/2)*(-1
 + Tan[(a + b*x)/2])] - Log[1 + I*Tan[(a + b*x)/2]]*Log[(1/2 - I/2)*(1 + Ta
n[(a + b*x)/2])] + Log[1 - I*Tan[(a + b*x)/2]]*Log[(1/2 + I/2)*(1 + Tan[(a
```

```
+ b*x)/2])] - PolyLog[2, ((1 + I) - (1 - I)*Tan[(a + b*x)/2])/2] + PolyLog[
2, (-1/2 - I/2)*(I + Tan[(a + b*x)/2])] - PolyLog[2, ((1 + I) + (1 - I)*Tan
[(a + b*x)/2])/2] + PolyLog[2, ((1 - I) + (1 + I)*Tan[(a + b*x)/2])/2])))/(
2*b*(a - I*Log[1 - I*Tan[(a + b*x)/2]] + I*Log[1 + I*Tan[(a + b*x)/2]])) +
(d*x)/(4*b*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])^2) - (d*Sin[(a + b*x)/2])/
(2*b^2*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - (d*x)/(4*b*(Cos[(a + b*x)/2
] + Sin[(a + b*x)/2])^2) + (d*Sin[(a + b*x)/2])/(2*b^2*(Cos[(a + b*x)/2] +
Sin[(a + b*x)/2])) + (c*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(98) = 196$.
time = 0.12, size = 267, normalized size = 2.28

method	result
risch	$-\frac{i\left(dxb\,\mathrm{e}^{3i(bx+a)}-id\,\mathrm{e}^{3i(bx+a)}+cb\,\mathrm{e}^{3i(bx+a)}-dxb\,\mathrm{e}^{i(bx+a)}-id\,\mathrm{e}^{i(bx+a)}-cb\,\mathrm{e}^{i(bx+a)}\right)}{b^2\left(1+\mathrm{e}^{2i(bx+a)}\right)^2}+\frac{ic\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b}+\frac{d\ln\left(1+i\mathrm{e}^{i(bx+a)}\right)x}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -I/b^2/(1+exp(2*I*(b*x+a)))^2*(d*x*b*exp(3*I*(b*x+a))-I*d*exp(3*I*(b*x+a))+
c*b*exp(3*I*(b*x+a))-d*x*b*exp(I*(b*x+a))-I*d*exp(I*(b*x+a))-c*b*exp(I*(b*x
+a)))+I/b*c*arctan(exp(I*(b*x+a)))+1/2/b*d*ln(1+I*exp(I*(b*x+a)))*x+1/2/b^2
*d*ln(1+I*exp(I*(b*x+a)))*a-1/2/b*d*ln(1-I*exp(I*(b*x+a)))*x-1/2/b^2*d*ln(1
-I*exp(I*(b*x+a)))*a-1/2*I/b^2*d*dilog(1+I*exp(I*(b*x+a)))+1/2*I/b^2*d*dilo
g(1-I*exp(I*(b*x+a)))-I/b^2*d*a*arctan(exp(I*(b*x+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(d*cos(3*b*x + 3*a) + d*cos(b*x + a) - (b*d*x + b*c)*sin(3*b*x + 3*
a) + (b*d*x + b*c)*sin(b*x + a))*cos(4*b*x + 4*a) + 4*(2*d*cos(2*b*x + 2*a)
 + 2*(b*d*x + b*c)*sin(2*b*x + 2*a) + d)*cos(3*b*x + 3*a) + 8*(d*cos(b*x +
a) + (b*d*x + b*c)*sin(b*x + a))*cos(2*b*x + 2*a) + 4*d*cos(b*x + a) + 4*(b
^2*d*cos(4*b*x + 4*a)^2 + 4*b^2*d*cos(2*b*x + 2*a)^2 + b^2*d*sin(4*b*x + 4*
a)^2 + 4*b^2*d*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*d*sin(2*b*x + 2*a)
^2 + 4*b^2*d*cos(2*b*x + 2*a) + b^2*d + 2*(2*b^2*d*cos(2*b*x + 2*a) + b^2*d
)*cos(4*b*x + 4*a))*integrate((x*cos(2*b*x + 2*a)*cos(b*x + a) + x*sin(2*b*
x + 2*a)*sin(b*x + a) + x*cos(b*x + a))/(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2
*a)^2 + 2*cos(2*b*x + 2*a) + 1), x) + (b*c*cos(4*b*x + 4*a)^2 + 4*b*c*cos(2
```

```
*b*x + 2*a)^2 + b*c*sin(4*b*x + 4*a)^2 + 4*b*c*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) + 4*b*c*sin(2*b*x + 2*a)^2 + 4*b*c*cos(2*b*x + 2*a) + b*c + 2*(2*b*c*
cos(2*b*x + 2*a) + b*c)*cos(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)
^2 + 2*sin(b*x + a) + 1) - (b*c*cos(4*b*x + 4*a)^2 + 4*b*c*cos(2*b*x + 2*a)
^2 + b*c*sin(4*b*x + 4*a)^2 + 4*b*c*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b
*c*sin(2*b*x + 2*a)^2 + 4*b*c*cos(2*b*x + 2*a) + b*c + 2*(2*b*c*cos(2*b*x +
 2*a) + b*c)*cos(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(
b*x + a) + 1) + 4*((b*d*x + b*c)*cos(3*b*x + 3*a) - (b*d*x + b*c)*cos(b*x +
 a) + d*sin(3*b*x + 3*a) + d*sin(b*x + a))*sin(4*b*x + 4*a) - 4*(b*d*x + b*
c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - 2*d*sin(2*b*x + 2*a))*sin(3*b*x + 3*
a) - 8*((b*d*x + b*c)*cos(b*x + a) - d*sin(b*x + a))*sin(2*b*x + 2*a) + 4*(
b*d*x + b*c)*sin(b*x + a))/(b^2*cos(4*b*x + 4*a)^2 + 4*b^2*cos(2*b*x + 2*a)
^2 + b^2*sin(4*b*x + 4*a)^2 + 4*b^2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b
^2*sin(2*b*x + 2*a)^2 + 4*b^2*cos(2*b*x + 2*a) + b^2 + 2*(2*b^2*cos(2*b*x +
 2*a) + b^2)*cos(4*b*x + 4*a))
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(93) = 186$.
time = 2.72, size = 435, normalized size = 3.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*cos(b*x
+ a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(-I*c
os(b*x + a) + sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - si
n(b*x + a)) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a)
+ I) + (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) -
(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*
x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b*d*x + a
*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a*d)*
cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b*c - a*d)*cos(b*
x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*cos(b*x + a)
^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 2*d*cos(b*x + a) + 2*(b*d*x +
b*c)*sin(b*x + a))/(b^2*cos(b*x + a)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \tan^2 (a + bx) \sec (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*tan(a + b*x)**2*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)*sec(b*x + a)*tan(b*x + a)^2, x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)^2*(c + d*x))/cos(a + b*x),x)
[Out] \text{Hanged}
```

3.301 $\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=39

$$-\text{Int}\left(\frac{\sec(a+bx)}{c+dx},x\right)+\text{Int}\left(\frac{\sec^3(a+bx)}{c+dx},x\right)$$

```
[Out] -Unintegrable(sec(b*x+a)/(d*x+c),x)+Unintegrable(sec(b*x+a)^3/(d*x+c),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

```
[Out] -Defer[Int][Sec[a + b*x]/(c + d*x), x] + Defer[Int][Sec[a + b*x]^3/(c + d*x
), x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx=-\int \frac{\sec(a+bx)}{c+dx}\,dx+\int \frac{\sec^3(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 26.39, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

```
[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x), x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\left(\tan^2(bx+a)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] ((d*cos(3*b*x + 3*a) + d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a) - (b
*d*x + b*c)*sin(b*x + a))*cos(4*b*x + 4*a) + (2*d*cos(2*b*x + 2*a) - 2*(b*d
*x + b*c)*sin(2*b*x + 2*a) + d)*cos(3*b*x + 3*a) + 2*(d*cos(b*x + a) - (b*d
*x + b*c)*sin(b*x + a))*cos(2*b*x + 2*a) + d*cos(b*x + a) - (b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4
*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^
2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*
c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x +
 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate
(((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(2*b*x + 2*a)*cos(b*x +
a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(2*b*x + 2*a)*sin(b*x
 + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a))/(b^2*d^
3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*
d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^
2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) - ((b*d*
x + b*c)*cos(3*b*x + 3*a) - (b*d*x + b*c)*cos(b*x + a) - d*sin(3*b*x + 3*a)
 - d*sin(b*x + a))*sin(4*b*x + 4*a) + (b*d*x + b*c + 2*(b*d*x + b*c)*cos(2*
b*x + 2*a) + 2*d*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) + 2*((b*d*x + b*c)*cos(
b*x + a) + d*sin(b*x + a))*sin(2*b*x + 2*a) - (b*d*x + b*c)*sin(b*x + a))/(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2
*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^
2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a
))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2
*a))
```

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")
[Out] integral(sec(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2{(a+bx)}\sec{(a+bx)}}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)**2/(d*x+c),x)
[Out] Integral(tan(a + b*x)**2*sec(a + b*x)/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(sec(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\tan(a+b\,x)}^2}{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)),x)
[Out] int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)), x)
```

3.302 $\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=39

$$-\mathrm{Int}\left(\frac{\sec(a+bx)}{(c+dx)^2},x\right)+\mathrm{Int}\left(\frac{\sec^3(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] -Unintegrable(sec(b*x+a)/(d*x+c)^2,x)+Unintegrable(sec(b*x+a)^3/(d*x+c)^2,x
)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -Defer[Int][Sec[a + b*x]/(c + d*x)^2, x] + Defer[Int][Sec[a + b*x]^3/(c + d
*x)^2, x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx=-\int \frac{\sec(a+bx)}{(c+dx)^2}\,dx+\int \frac{\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 28.93, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2, x]
```

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\left(\tan^2(bx+a)\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] ((2*d*cos(3*b*x + 3*a) + 2*d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a)
- (b*d*x + b*c)*sin(b*x + a))*cos(4*b*x + 4*a) + 2*(2*d*cos(2*b*x + 2*a) -
(b*d*x + b*c)*sin(2*b*x + 2*a) + d)*cos(3*b*x + 3*a) + 2*(2*d*cos(b*x + a)
- (b*d*x + b*c)*sin(b*x + a))*cos(2*b*x + 2*a) + 2*d*cos(b*x + a) - (b^2*d^
3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*
d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*
b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^
3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b
*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin
(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^
3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x +
 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*cos(2*b*x + 2*a))*integrate(((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2 - 6*d^2)*cos(2*b*x + 2*a)*cos(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2 - 6*d^2)*sin(2*b*x + 2*a)*sin(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2 - 6*d^2)*cos(b*x + a))/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d
^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c
^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4
*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(2*b*x + 2
*a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*
x + b^2*c^4)*cos(2*b*x + 2*a)), x) - ((b*d*x + b*c)*cos(3*b*x + 3*a) - (b*d
*x + b*c)*cos(b*x + a) - 2*d*sin(3*b*x + 3*a) - 2*d*sin(b*x + a))*sin(4*b*x
 + 4*a) + (b*d*x + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + 4*d*sin(2*b*x +
 2*a))*sin(3*b*x + 3*a) + 2*((b*d*x + b*c)*cos(b*x + a) + 2*d*sin(b*x + a))
*sin(2*b*x + 2*a) - (b*d*x + b*c)*sin(b*x + a))/(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*
d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d
^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*
```

```
(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 +
 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4
*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x
 + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(sec(b*x + a)*tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)**2/(d*x+c)**2,x)
[Out] Integral(tan(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sec(b*x + a)*tan(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tan(a+b\,x)^2}{\cos(a+b\,x)\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)^2),x)
[Out] int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)^2), x)
```

3.303 $\int (c+dx)^m \tan^3(a+bx)\,dx$

Optimal. Leaf size=19
$$\text{Int}\left((c+dx)^m \tan^3(a+bx),x\right)$$

```
[Out] Unintegrable((d*x+c)^m*tan(b*x+a)^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \tan^3(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Tan[a + b*x]^3,x]
[Out] Defer[Int][(c + d*x)^m*Tan[a + b*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \tan^3(a+bx)\,dx = \int (c+dx)^m \tan^3(a+bx)\,dx$$

Mathematica [A]
time = 9.12, size = 0, normalized size = 0.00

$$\int (c+dx)^m \tan^3(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Tan[a + b*x]^3,x]
[Out] Integrate[(c + d*x)^m*Tan[a + b*x]^3, x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\tan^3(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*tan(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*tan(b*x+a)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*tan(b*x + a)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="fricas")
[Out] integral((d*x + c)^m*tan(b*x + a)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \tan^3 (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*tan(b*x+a)**3,x)
[Out] Integral((c + d*x)**m*tan(a + b*x)**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*tan(b*x + a)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \tan(a + b\,x)^3\,(c + d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3*(c + d*x)^m,x)
[Out] int(tan(a + b*x)^3*(c + d*x)^m, x)
```

3.304 $\int (c+dx)^3 \tan^3(a+bx)\,dx$

Optimal. Leaf size=259

$$\frac{3id(c+dx)^2}{2b^2}+\frac{(c+dx)^3}{2b}-\frac{i(c+dx)^4}{4d}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}+\frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{3id^3\text{Po}}{}$$

```
[Out] 3/2*I*d*(d*x+c)^2/b^2+1/2*(d*x+c)^3/b-1/4*I*(d*x+c)^4/d-3*d^2*(d*x+c)*ln(1+
exp(2*I*(b*x+a)))/b^3+(d*x+c)^3*ln(1+exp(2*I*(b*x+a)))/b+3/2*I*d^3*polylog(
2,-exp(2*I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/b^2
+3/2*d^2*(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))/b^3+3/4*I*d^3*polylog(4,-exp(
2*I*(b*x+a)))/b^4-3/2*d*(d*x+c)^2*tan(b*x+a)/b^2+1/2*(d*x+c)^3*tan(b*x+a)^2
/b
```

Rubi [A]

time = 0.23, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3801, 3800, 2221, 2317, 2438, 32, 2611, 6744, 2320, 6724}

$$\frac{3id^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^4}+\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}-\frac{3id(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{3d(c+dx)^2\tan(a+bx)}{2b^2}+\frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}+\frac{3id(c+dx)^2}{2b^2}+\frac{(c+dx)^3}{2b}-\frac{i(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Tan[a + b*x]^3,x]
```

```
[Out] (((3*I)/2)*d*(c + d*x)^2)/b^2 + (c + d*x)^3/(2*b) - ((I/4)*(c + d*x)^4)/d -
 (3*d^2*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^3 + ((c + d*x)^3*Log[1 +
E^((2*I)*(a + b*x))])/b + (((3*I)/2)*d^3*PolyLog[2, -E^((2*I)*(a + b*x))])/
b^4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + (3*d
^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (((3*I)/4)*d^3*Pol
yLog[4, -E^((2*I)*(a + b*x))])/b^4 - (3*d*(c + d*x)^2*Tan[a + b*x])/(2*b^2)
 + ((c + d*x)^3*Tan[a + b*x]^2)/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\tan^3(a+bx)\,dx = \frac{(c+dx)^3\tan^2(a+bx)}{2b} - \frac{(3d)\int (c+dx)^2\tan^2(a+bx)\,dx}{2b} - \int (c+dx)^3\tan$$

$$= -\frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\tan(a+bx)}{2b^2} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} + 2i\int \frac{e^{2i}}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3d(}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} +$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} +$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} +$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} +$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 817 vs. 2(259) = 518.
time = 6.81, size = 817, normalized size = 3.15

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Tan[a + b*x]^3,x]
```

```
[Out] -1/4*(c*d^2*((2*I)*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(1 + E^((2*I)*a))*Log
[1 + E^((2*I)*(a + b*x))]) + (6*I)*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((2*
I)*(a + b*x))] - 3*(1 + E^((2*I)*a))*PolyLog[3, -E^((2*I)*(a + b*x))])*Sec[
a])/(b^3*E^(I*a)) + (I/4)*d^3*E^(I*a)*(-x^4 + (1 + E^((-2*I)*a))*x^4 - ((1
+ E^((2*I)*a))*(2*b^4*x^4 + (4*I)*b^3*x^3*Log[1 + E^((2*I)*(a + b*x))] + 6*
b^2*x^2*PolyLog[2, -E^((2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[3, -E^((2*I)*(
```

```
a + b*x))] - 3*PolyLog[4, -E^((2*I)*(a + b*x))]))/(2*b^4*E^((2*I)*a)))*Sec[
a] + ((c + d*x)^3*Sec[a + b*x]^2)/(2*b) + (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Co
s[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (3*c*d
^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^
3*(Cos[a]^2 + Sin[a]^2)) + (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]])
- (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*
(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[C
os[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E
^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[
Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*d^3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot
[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)
] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi
*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLo
g[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^4
*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*Sec[a]*Sec[a + b*x]*(c^2*d*Sin[
b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[b*x]))/(2*b^2) - (x*(4*c^3 + 6*c^2*
d*x + 4*c*d^2*x^2 + d^3*x^3)*Tan[a])/4
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(224) = 448$.
time = 0.14, size = 729, normalized size = 2.81

method	result
risch	$\frac{6id^3ax}{b^3} - \frac{id^3x^4}{4} - \frac{2c^3\ln\left(e^{i(bx+a)}\right)}{b} + ic^3x + \frac{ic^4}{4d} + \frac{6ic\,d^2a^2x}{b^2} - \frac{6ic^2dax}{b} - \frac{3id^3a^4}{2b^4} - id^2c\,x^3 - \frac{3id\,c^2x^2}{2} + \frac{2d^3a^3\ln(}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 6*I/b^3*d^3*a*x-3/2*I/b^4*d^3*a^4-I*d^2*c*x^3-3/2*I*d*c^2*x^2+2/b^4*d^3*a^3
*ln(exp(I*(b*x+a)))-1/4*I*d^3*x^4-2/b*c^3*ln(exp(I*(b*x+a)))+I*c^3*x+1/4*I/
d*c^4+6*d^2/b^3*c*ln(exp(I*(b*x+a)))-6*d^3/b^4*a*ln(exp(I*(b*x+a)))+6/b^2*c
^2*d*a*ln(exp(I*(b*x+a)))+1/b*c^3*ln(1+exp(2*I*(b*x+a)))+(2*b*d^3*x^3*exp(2
*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-6*I
*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*I*c^2*d*exp(2*I*(b
*x+a))-3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))-6*I*c*d^2*x-3*I*c^2*d)/b^2/(1+e
xp(2*I*(b*x+a)))^2+3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))+3/2/b^3*d^3*p
olylog(3,-exp(2*I*(b*x+a)))*x-3/b^3*d^2*c*ln(1+exp(2*I*(b*x+a)))-3/b^3*d^3*
ln(1+exp(2*I*(b*x+a)))*x+3*I/b^2*d^3*x^2+3*I/b^4*d^3*a^2-6/b^3*c*d^2*a^2*ln
(exp(I*(b*x+a)))+4*I/b^3*c*d^2*a^3-3*I/b^2*c^2*d*a^2-2*I/b^3*d^3*a^3*x-3*I/
b^2*polylog(2,-exp(2*I*(b*x+a)))*c*d^2*x-3/2*I/b^2*c^2*d*polylog(2,-exp(2*I
*(b*x+a)))-3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2+3/2*I*d^3*polylog
(2,-exp(2*I*(b*x+a)))/b^4+3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/b*c^
2*d*ln(1+exp(2*I*(b*x+a)))*x+3/b*c*d^2*ln(1+exp(2*I*(b*x+a)))*x^2+1/b*d^3*l
n(1+exp(2*I*(b*x+a)))*x^3+6*I/b^2*c*d^2*a^2*x-6*I/b*c^2*d*a*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2413 vs. 2(217) = 434.
time = 1.13, size = 2413, normalized size = 9.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1)) - 3*a*c^2*d*(1
/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b + 3*a^2*c*d^2*(1/(sin(b*
x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b^2 - a^3*d^3*(1/(sin(b*x + a)^2 -
 1) - log(sin(b*x + a)^2 - 1))/b^3 + 2*(3*(b*x + a)^4*d^3 + 36*b^2*c^2*d -
72*a*b*c*d^2 + 36*a^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a)^3 + 18*(b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a)^2 - 4*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 +
 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (
a^2 - 1)*d^3)*(b*x + a) + (4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3 + 9*(b*c
*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*
x + a))*cos(4*b*x + 4*a) + 2*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3 + 9*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*
(b*x + a))*cos(2*b*x + 2*a) - (-4*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d
^3 + 9*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 9*(-I*b^2*c^2*d + 2*I*a*b*c*d^2
 + (-I*a^2 + I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(-4*I*(b*x + a)^3*d^3
+ 9*I*b*c*d^2 - 9*I*a*d^3 + 9*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 9*(-I*b^
2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 + I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*ar
ctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + 3*((b*x + a)^4*d^3 + 4*(b*c
*d^2 - a*d^3)*(b*x + a)^3 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*
x + a)^2 - 24*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 6*((b*x + a)^
4*d^3 + 6*b^2*c^2*d - 12*a*b*c*d^2 + 6*a^2*d^3 + 4*(b*c*d^2 - (a - I)*d^3)*
(b*x + a)^3 + 6*(b^2*c^2*d - 2*(a - I)*b*c*d^2 + (a^2 - 2*I*a - 1)*d^3)*(b*
x + a)^2 + 12*(I*b^2*c^2*d + (-2*I*a - 1)*b*c*d^2 + (I*a^2 + a)*d^3)*(b*x +
 a))*cos(2*b*x + 2*a) + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 +
3*(a^2 - 1)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a) + (3*b^2*c^2*d - 6*a*b*c*d^
2 + 4*(b*x + a)^2*d^3 + 3*(a^2 - 1)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*co
s(4*b*x + 4*a) + 2*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*(a^2
- 1)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (3*I*b^2*c^2*d
 - 6*I*a*b*c*d^2 + 4*I*(b*x + a)^2*d^3 + 3*(I*a^2 - I)*d^3 + 6*(I*b*c*d^2 -
 I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*(3*I*b^2*c^2*d - 6*I*a*b*c*d^2 +
4*I*(b*x + a)^2*d^3 + 3*(I*a^2 - I)*d^3 + 6*(I*b*c*d^2 - I*a*d^3)*(b*x + a)
)*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) + 2*(4*I*(b*x + a)^3*d^3 -
9*I*b*c*d^2 + 9*I*a*d^3 + 9*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 9*(I*b^2*c^
2*d - 2*I*a*b*c*d^2 + (I*a^2 - I)*d^3)*(b*x + a) + (4*I*(b*x + a)^3*d^3 - 9
*I*b*c*d^2 + 9*I*a*d^3 + 9*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 9*(I*b^2*c^2
*d - 2*I*a*b*c*d^2 + (I*a^2 - I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 2*(4*I*
(b*x + a)^3*d^3 - 9*I*b*c*d^2 + 9*I*a*d^3 + 9*(I*b*c*d^2 - I*a*d^3)*(b*x +
```

```
a)^2 + 9*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 - I)*d^3)*(b*x + a))*cos(2*b
*x + 2*a) - (4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*
(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(4*
b*x + 4*a) - 2*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3 + 9*(b*c*d^2 - a*d^
3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin
(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
2*a) + 1) - 12*(d^3*cos(4*b*x + 4*a) + 2*d^3*cos(2*b*x + 2*a) + I*d^3*sin(4
*b*x + 4*a) + 2*I*d^3*sin(2*b*x + 2*a) + d^3)*polylog(4, -e^(2*I*b*x + 2*I*
a)) + 6*(3*I*b*c*d^2 + 4*I*(b*x + a)*d^3 - 3*I*a*d^3 + (3*I*b*c*d^2 + 4*I*(
b*x + a)*d^3 - 3*I*a*d^3)*cos(4*b*x + 4*a) + 2*(3*I*b*c*d^2 + 4*I*(b*x + a)
*d^3 - 3*I*a*d^3)*cos(2*b*x + 2*a) - (3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3
)*sin(4*b*x + 4*a) - 2*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*sin(2*b*x +
2*a))*polylog(3, -e^(2*I*b*x + 2*I*a)) + 3*(I*(b*x + a)^4*d^3 + 4*(I*b*c*d^
2 - I*a*d^3)*(b*x + a)^3 + 6*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 - 2*I)*d
^3)*(b*x + a)^2 + 24*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 6
*(I*(b*x + a)^4*d^3 + 6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*a^2*d^3 + 4*(I*b
*c*d^2 + (-I*a - 1)*d^3)*(b*x + a)^3 + 6*(I*b^2*c^2*d + 2*(-I*a - 1)*b*c*d^
2 + (I*a^2 + 2*a - I)*d^3)*(b*x + a)^2 - 12*(b^2*c^2*d - (2*a - I)*b*c*d^2
+ (a^2 - I*a)*d^3)*(b*x + a))*sin(2*b*x + 2*a))/(-12*I*b^3*cos(4*b*x + 4*a)
 - 24*I*b^3*cos(2*b*x + 2*a) + 12*b^3*sin(4*b*x + 4*a) + 24*b^3*sin(2*b*x +
 2*a) - 12*I*b^3))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(217) = 434$.
time = 3.26, size = 592, normalized size = 2.29

$$\frac{4\,b^3d^3x^3+12\,b^3cd^2x^2+12\,b^3c^2dx-3i\,d^3\mathrm{polylog}\left(4,\frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)+3i\,d^3\mathrm{polylog}\left(4,\frac{\tan(bx+a)^2-2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)+4\,(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\tan(bx+a)^2-6\,(-i\,b^2d^3x^2-2i\,b^2cd^2x-i\,b^2c^2d+i\,d^3)\mathrm{Li}_2\left(\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)-6\,(i\,b^2d^3x^2+2i\,b^2cd^2x+i\,b^2c^2d-i\,d^3)\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)+4\,(b^3d^3x^3+3\,b^3cd^2x^2+b^3c^3-3\,bcd^2+3\,(b^3c^2d-bd^3)x)\log\left(-\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right)+4\,(b^3d^3x^3+3\,b^3cd^2x^2+b^3c^3-3\,bcd^2+3\,(b^3c^2d-bd^3)x)\log\left(-\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right)+6\,(bd^3x+bcd^2)\mathrm{polylog}\left(3,\frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)+6\,(bd^3x+bcd^2)\mathrm{polylog}\left(3,\frac{\tan(bx+a)^2-2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)-12\,(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d)\tan(bx+a)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x - 3*I*d^3*polylog(4,
 (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 3*I*d^3*po
lylog(4, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 4*
(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*tan(b*x + a)^2 -
6*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d + I*d^3)*dilog(2*(I*tan(b
*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) - 6*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x
 + I*b^2*c^2*d - I*d^3)*dilog(2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)
+ 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 3*b*c*d^2 + 3*(b^3*c^2*
d - b*d^3)*x)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 4*(b^3*d^
3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 3*b*c*d^2 + 3*(b^3*c^2*d - b*d^3)*x)*lo
g(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 6*(b*d^3*x + b*c*d^2)*po
lylog(3, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 6*
(b*d^3*x + b*c*d^2)*polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan
(b*x + a)^2 + 1)) - 12*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*tan(b*x +
a))/b^4
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \tan^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*tan(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**3*tan(a + b*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*tan(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(a+b\,x)^3\,(c+d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] int(tan(a + b*x)^3*(c + d*x)^3, x)
```

3.305 $\int (c+dx)^2 \tan^3(a+bx)\,dx$

Optimal. Leaf size=169

$$\frac{cdx}{b}+\frac{d^2x^2}{2b}-\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{d^2\log(\cos(a+bx))}{b^3}-\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}$$

```
[Out] c*d*x/b+1/2*d^2*x^2/b-1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b-
d^2*ln(cos(b*x+a))/b^3-I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/2*d^2
*polylog(3,-exp(2*I*(b*x+a)))/b^3-d*(d*x+c)*tan(b*x+a)/b^2+1/2*(d*x+c)^2*ta
n(b*x+a)^2/b
```

Rubi [A]
time = 0.15, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3801, 3556, 3800, 2221, 2611, 2320, 6724}

$$\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}-\frac{d^2\log(\cos(a+bx))}{b^3}-\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\tan(a+bx)}{b^2}+\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}+\frac{cdx}{b}+\frac{d^2x^2}{2b}-\frac{i(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Tan[a + b*x]^3,x]
```

```
[Out] (c*d*x)/b + (d^2*x^2)/(2*b) - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[1 +
E^((2*I)*(a + b*x))])/b - (d^2*Log[Cos[a + b*x]])/b^3 - (I*d*(c + d*x)*Poly
Log[2, -E^((2*I)*(a + b*x))])/b^2 + (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/
(2*b^3) - (d*(c + d*x)*Tan[a + b*x])/b^2 + ((c + d*x)^2*Tan[a + b*x]^2)/(2*
b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \tan^3(a+bx)\,dx = \frac{(c+dx)^2\tan^2(a+bx)}{2b} - \frac{d\int (c+dx)\tan^2(a+bx)\,dx}{b} - \int (c+dx)^2\tan(a+$$

$$= -\frac{i(c+dx)^3}{3d} - \frac{d(c+dx)\tan(a+bx)}{b^2} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+b}}{1+}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 461 vs. $2(169) = 338$.
time = 6.63, size = 461, normalized size = 2.73

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Tan[a + b*x]^3,x]
```

```
[Out] -1/12*(d^2*((2*I)*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(1 + E^((2*I)*a))*Log[
1 + E^((2*I)*(a + b*x))]) + (6*I)*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((2*I
)*(a + b*x))] - 3*(1 + E^((2*I)*a))*PolyLog[3, -E^((2*I)*(a + b*x))])*Sec[a
])/(b^3*E^(I*a)) + ((c + d*x)^2*Sec[a + b*x]^2)/(2*b) + (c^2*Sec[a]*(Cos[a]
*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a
]^2)) - (d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Si
n[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot
[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)
] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi
*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLo
g[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*S
qrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Sec[a]*Sec[a + b*x]*(-(c*d*Sin[b*x]
) - d^2*x*Sin[b*x]))/b^2 - (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tan[a])/3
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(153) = 306$.
time = 0.11, size = 409, normalized size = 2.42

method	result
risch	$-\frac{id^2x^3}{3}+\frac{ic^3}{3d}+\frac{2id^2a^2x}{b^2}+\frac{4id^2a^3}{3b^3}+\frac{2b\,d^2x^2\mathrm{e}^{2i(bx+a)}+4bcdx\,\mathrm{e}^{2i(bx+a)}+2b\,c^2\mathrm{e}^{2i(bx+a)}-2id^2x\,\mathrm{e}^{2i(bx+a)}-2icd\,\mathrm{e}^{2i(bx+a)}-2ic}{b^2\left(1+\mathrm{e}^{2i(bx+a)}\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I*d^2*x^3+1/3*I/d*c^3+2*I/b^2*d^2*a^2*x+4/3*I/b^3*d^2*a^3+2*(b*d^2*x^2
*exp(2*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x
*exp(2*I*(b*x+a))-I*c*d*exp(2*I*(b*x+a))-I*d^2*x-I*d*c)/b^2/(1+exp(2*I*(b*x
+a)))^2-I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+2/b*c*d*ln(1+exp(2*I*(b*x+
a)))*x+4/b^2*c*d*a*ln(exp(I*(b*x+a)))-2*I/b^2*c*d*a^2+1/2*d^2*polylog(3,-ex
p(2*I*(b*x+a)))/b^3-d^2/b^3*ln(1+exp(2*I*(b*x+a)))+2/b^3*d^2*ln(exp(I*(b*x+
a)))-4*I/b*c*d*a*x-I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))+1/b*d^2*ln(1+exp(
2*I*(b*x+a)))*x^2-I*d*c*x^2+I*c^2*x+1/b*c^2*ln(1+exp(2*I*(b*x+a)))-2/b*c^2*
ln(exp(I*(b*x+a)))-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1228 vs. $2(150) = 300$.
time = 0.68, size = 1228, normalized size = 7.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1)) - 2*a*c*d*(1/(
sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b + a^2*d^2*(1/(sin(b*x + a)
^2 - 1) - log(sin(b*x + a)^2 - 1))/b^2 + 2*(2*(b*x + a)^3*d^2 + 6*(b*c*d -
a*d^2)*(b*x + a)^2 + 12*b*c*d - 12*a*d^2 - 6*((b*x + a)^2*d^2 + 2*(b*c*d -
a*d^2)*(b*x + a) - d^2 + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d
^2)*cos(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d
^2)*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x +
a) + I*d^2)*sin(4*b*x + 4*a) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^
2)*(b*x + a) + I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x
 + 2*a) + 1) + 2*((b*x + a)^3*d^2 + 3*(b*c*d - a*d^2)*(b*x + a)^2 - 6*(b*x
+ a)*d^2)*cos(4*b*x + 4*a) + 4*((b*x + a)^3*d^2 + 3*(b*c*d - (a - I)*d^2)*(
b*x + a)^2 + 3*b*c*d - 3*a*d^2 + 3*(2*I*b*c*d + (-2*I*a - 1)*d^2)*(b*x + a)
)*cos(2*b*x + 2*a) + 6*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*
d^2 - a*d^2)*cos(4*b*x + 4*a) + 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x
 + 2*a) + (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(4*b*x + 4*a) + 2*(I*b*c
*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a
)) + 3*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - I*d^2 + (I*(b
*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - I*d^2)*cos(4*b*x + 4*a) +
```

```
 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - I*d^2)*cos(2*b*x
+ 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(4*b*x +
4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(2*b*x +
2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)
 + 3*(I*d^2*cos(4*b*x + 4*a) + 2*I*d^2*cos(2*b*x + 2*a) - d^2*sin(4*b*x + 4
*a) - 2*d^2*sin(2*b*x + 2*a) + I*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) + 2*
(I*(b*x + a)^3*d^2 + 3*(I*b*c*d - I*a*d^2)*(b*x + a)^2 - 6*I*(b*x + a)*d^2)
*sin(4*b*x + 4*a) + 4*(I*(b*x + a)^3*d^2 + 3*(I*b*c*d + (-I*a - 1)*d^2)*(b*
x + a)^2 + 3*I*b*c*d - 3*I*a*d^2 - 3*(2*b*c*d - (2*a - I)*d^2)*(b*x + a))*s
in(2*b*x + 2*a))/(-6*I*b^2*cos(4*b*x + 4*a) - 12*I*b^2*cos(2*b*x + 2*a) + 6
*b^2*sin(4*b*x + 4*a) + 12*b^2*sin(2*b*x + 2*a) - 6*I*b^2))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(150) = 300$.
time = 3.49, size = 354, normalized size = 2.09

$$\frac{2\,b^2d^2x^2 + 4\,b^2cdx + d^2\mathrm{polylog}\left(3, \frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) + d^2\mathrm{polylog}\left(3, \frac{\tan(bx+a)^2-2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) + 2\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\tan\left(bx+a\right)^2 - 2\left(-i\,bd^2x - i\,bcd\right)\mathrm{Li}_2\left(\frac{2\left(i\,\tan(bx+a)-1\right)}{\tan(bx+a)^2+1} + 1\right) - 2\left(i\,bd^2x + i\,bcd\right)\mathrm{Li}_2\left(\frac{2\left(-i\,\tan(bx+a)-1\right)}{\tan(bx+a)^2+1} + 1\right) + 2\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 - d^2\right)\log\left(-\frac{2\left(i\,\tan(bx+a)-1\right)}{\tan(bx+a)^2+1}\right) + 2\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 - d^2\right)\log\left(-\frac{2\left(-i\,\tan(bx+a)-1\right)}{\tan(bx+a)^2+1}\right) - 4\left(bd^2x + bcd\right)\tan\left(bx+a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + d^2*polylog(3, (tan(b*x + a)^2 + 2*I*tan
(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + d^2*polylog(3, (tan(b*x + a)^2 - 2*I
*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*tan(b*x + a)^2 - 2*(-I*b*d^2*x - I*b*c*d)*dilog(2*(I*tan(b*x + a) -
 1)/(tan(b*x + a)^2 + 1) + 1) - 2*(I*b*d^2*x + I*b*c*d)*dilog(2*(-I*tan(b*x
 + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2 - d^2)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2 - d^2)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)
^2 + 1)) - 4*(b*d^2*x + b*c*d)*tan(b*x + a))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + dx\right)^2 \tan^3\left(a + bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*tan(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*tan(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*tan(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(a+b\,x)^3\,(c+d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] int(tan(a + b*x)^3*(c + d*x)^2, x)
```

3.306 $\int (c+dx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=108

$$\frac{dx}{2b}-\frac{i(c+dx)^2}{2d}+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\tan(a+bx)}{2b^2}+\frac{(c+dx)\tan^2(a+bx)}{2b}$$

```
[Out] 1/2*d*x/b-1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b-1/2*I*d*polylo
g(2,-exp(2*I*(b*x+a)))/b^2-1/2*d*tan(b*x+a)/b^2+1/2*(d*x+c)*tan(b*x+a)^2/b
```

Rubi [A]
time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3801, 3554, 8, 3800, 2221, 2317, 2438}

$$-\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\tan(a+bx)}{2b^2}+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{(c+dx)\tan^2(a+bx)}{2b}+\frac{dx}{2b}-\frac{i(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Tan[a + b*x]^3,x]
```

```
[Out] (d*x)/(2*b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 + E^((2*I)*(a + b*x)
)])/b - ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (d*Tan[a + b*x])/(
2*b^2) + ((c + d*x)*Tan[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)\tan^3(a+bx)\,dx &= \frac{(c+dx)\tan^2(a+bx)}{2b} - \frac{d\int \tan^2(a+bx)\,dx}{2b} - \int (c+dx)\tan(a+bx)\,dx \\
&= -\frac{i(c+dx)^2}{2d} - \frac{d\tan(a+bx)}{2b^2} + \frac{(c+dx)\tan^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}(c+dx}{1+e^{2i(a+bx)}} \\
&= \frac{dx}{2b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d\tan(a+bx)}{2b^2} + \frac{(c+dx)\tan}{2b} \\
&= \frac{dx}{2b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d\tan(a+bx)}{2b^2} + \frac{(c+dx)\tan}{2b} \\
&= \frac{dx}{2b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{d\tan(a}{2b}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 240 vs. 2(108) = 216.
time = 6.18, size = 240, normalized size = 2.22

$$\frac{dx\sec^2(a+bx)}{2b} + \frac{d\csc(a)\left(b^2e^{-i\text{ArcTan}(\cot(a))}x^2 - \frac{\cot(a)\left(ibx(-\pi-2\text{ArcTan}(\cot(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx-\text{ArcTan}(\cot(a)))\log\left(1-e^{2i(bx-\text{ArcTan}(\cot(a)))}\right)+\pi\log(\cos(bx))-2\text{ArcTan}(\cot(a))\log(\sin(bx-\text{ArcTan}(\cot(a))))+i\text{PolyLog}\left(2,e^{2i(bx-\text{ArcTan}(\cot(a)))}\right)\right)}{\sqrt{1+\cot^2(a)}}\right)\sec(a)}{2b^2\sqrt{\csc^2(a)\left(\cos^2(a)+\sin^2(a)\right)}} - \frac{d\sec(a)\sec(a+bx)\sin(bx)}{2b^2} - \frac{1}{2}dx^2\tan(a) + \frac{c\left(2\log(\cos(a+bx))+\tan^2(a+bx)\right)}{2b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)*Tan[a + b*x]^3,x]

[Out] (d*x*Sec[a + b*x]^2)/(2*b) + (d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (d*Sec[a]*Sec[a + b*x]*Sin[b*x])/(2*b^2) - (d*x^2*Tan[a])/2 + (c*(2*Log[Cos[a + b*x]] + Tan[a + b*x]^2))/(2*b)

Maple [A]
time = 0.08, size = 183, normalized size = 1.69

method	result
risch	$-\frac{id\,x^{2}}{2}+icx+\frac{2bdx\,{\rm e}^{2i(bx+a)}-id\,{\rm e}^{2i(bx+a)}+2bc\,{\rm e}^{2i(bx+a)}-id}{b^{2}\left(1+{\rm e}^{2i(bx+a)}\right)^{2}}+\frac{c\ln\left(1+{\rm e}^{2i(bx+a)}\right)}{b}-\frac{2c\ln\left({\rm e}^{i(bx+a)}\right)}{b}-\frac{2idax}{b}-\frac{id\,a^{2}}{b^{2}}+$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*tan(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] -1/2*I*d*x^2+I*c*x+(2*b*d*x*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))-I*d)/b^2/(1+exp(2*I*(b*x+a)))^2+1/b*c*ln(1+exp(2*I*(b*x+a)))-2/b*c*ln(exp(I*(b*x+a)))-2*I/b*d*a*x-I/b^2*d*a^2+1/b*d*ln(1+exp(2*I*(b*x+a)))*x-1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b^2*d*a*ln(exp(I*(b*x+a)))

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 517 vs. 2(89) = 178.
time = 0.58, size = 517, normalized size = 4.79

$$-\frac{b^{2}dx^{2}+2b^{2}cx-2\left(bdx+bc+(bdx+bc)\cos(4bx+4a)+2(bdx+bc)\cos(2bx+2a)-(-ibdx-ibc)\sin(4bx+4a)-2(-ibdx-ibc)\sin(2bx+2a)\right)\arctan\left(\sin(2bx+2a),\cos(2bx+2a)+1\right)+(b^{2}dx^{2}+2b^{2}cx)\cos(4bx+4a)+2(b^{2}dx^{2}+2ibc+2(b^{2}c+ibd)x+d)\cos(2bx+2a)+(d\cos(4bx+4a)+2d\cos(2bx+2a)+id\sin(4bx+4a)+2id\sin(2bx+2a)+d){\rm Li}_2(-e^{(2ibx+2ia)})-(-ibdx-ibc+(-ibdx-ibc)\cos(4bx+4a)-2(ibdx+ibc)\cos(2bx+2a)+(bdx+bc)\sin(4bx+4a)+2(bdx+bc)\sin(2bx+2a))\log(\cos(2bx+2a)^{2}+\sin(2bx+2a)^{2}+2\cos(2bx+2a)+1)-(-ib^{2}dx^{2}-2ib^{2}cx)\sin(4bx+4a)+2(ib^{2}dx^{2}-2bc+2(ib^{2}c-bd)x+id)\sin(2bx+2a)+2d}{-2ib^{2}\cos(4bx+4a)-4ib^{2}\cos(2bx+2a)+2b^{2}\sin(4bx+4a)+4b^{2}\sin(2bx+2a)-2ib^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="maxima")

[Out] -(b^2*d*x^2 + 2*b^2*c*x - 2*(b*d*x + b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (-I*b*d*x - I*b*c)*sin(4*b*x + 4*a) - 2*(-I*b*d*x - I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + (b^2*d*x^2 + 2*b^2*c*x)*cos(4*b*x + 4*a) + 2*(b^2*d*x^2 + 2*I*b*c + 2*(b^2*c + I*b*d)*x + d)*cos(2*b*x + 2*a) + (d*cos(4*b*x + 4*a) + 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) + 2*I*d*sin(2*b*x + 2*a) + d)*dilog(-e^(2*I*b*x + 2*I*a)) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(4*b*x + 4*a) - 2*(I*b*d*x + I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (-I*b^2*d*x^2 - 2*I*b^2*c*x)*sin(4*b*x + 4*a) + 2*(I*b^2*d*x^2 - 2*b*c + 2*(I*b^2*c - b*d)*x + I*d)*sin(2*b*x + 2*a) + 2*d)/(-2*I*b^2*cos(4*b*x + 4*a) - 4*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4*b*x + 4*a) + 4*b^2*sin(2*b*x + 2*a) - 2*I*b^2)

Fricas [A]
time = 3.05, size = 168, normalized size = 1.56

$$\frac{2\,bdx+2\,(bdx+bc)\tan\left(bx+a\right)^{2}+i\,d\mathrm{Li}_{2}\left(\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^{2}+1}+1\right)-i\,d\mathrm{Li}_{2}\left(\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^{2}+1}+1\right)+2\,(bdx+bc)\log\left(-\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^{2}+1}\right)+2\,(bdx+bc)\log\left(-\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^{2}+1}\right)-2\,d\tan\left(bx+a\right)}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*d*x + 2*(b*d*x + b*c)*tan(b*x + a)^2 + I*d*dilog(2*(I*tan(b*x + a)
 - 1)/(tan(b*x + a)^2 + 1) + 1) - I*d*dilog(2*(-I*tan(b*x + a) - 1)/(tan(b*
x + a)^2 + 1) + 1) + 2*(b*d*x + b*c)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x +
 a)^2 + 1)) + 2*(b*d*x + b*c)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2
+ 1)) - 2*d*tan(b*x + a))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\tan^{3}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*tan(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*tan(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(a+b\,x)^{3}\,(c+d\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3*(c + d*x),x)
```

```
[Out] int(tan(a + b*x)^3*(c + d*x), x)
```

3.307 $\int \frac{\tan^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\tan^3(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(tan(b*x+a)^3/(d*x+c),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Tan[a + b*x]^3/(c + d*x),x]
```

```
[Out] Defer[Int][Tan[a + b*x]^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx = \int \frac{\tan^3(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 6.68, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[a + b*x]^3/(c + d*x),x]
```

```
[Out] Integrate[Tan[a + b*x]^3/(c + d*x), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^3/(d*x+c),x)
[Out] int(tan(b*x+a)^3/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="maxima")
[Out] (4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
(2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) +
2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b
*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*
b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate(2*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2 - d^2)*sin(2*b*x + 2*a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2
*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) + (d*cos(2*b*x + 2*a) + 2*(b*d*x + b*
c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4*a) + d*sin(2*b*x + 2*a))/(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x
 + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b
*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="fricas")
[Out] integral(tan(b*x + a)^3/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(tan(a + b*x)**3/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(tan(b*x + a)^3/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\tan(a+b\,x)^3}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3/(c + d*x),x)
```

```
[Out] int(tan(a + b*x)^3/(c + d*x), x)
```

3.308 $\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left(\frac{\tan^3(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(tan(b*x+a)^3/(d*x+c)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Tan[a + b*x]^3/(c + d*x)^2,x]
[Out] Defer[Int][Tan[a + b*x]^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 7.37, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[a + b*x]^3/(c + d*x)^2,x]
[Out] Integrate[Tan[a + b*x]^3/(c + d*x)^2, x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\tan^3\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(tan(b*x+a)^3/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) +
2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
 + b^2*c^3)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^
2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*inte
grate(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 3*d^2)*sin(2*b*x + 2*a)/(b^2
*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 +
(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^
4)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2
+ 4*b^2*c^3*d*x + b^2*c^4)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^
3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)), x)
+ 2*(d*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4
*a) + 2*d*sin(2*b*x + 2*a))/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4
*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*
c^3)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2
*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2
*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(tan(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(tan(a + b*x)**3/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(tan(b*x + a)^3/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{{\tan(a+b\,x)}^3}{{(c+d\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3/(c + d*x)^2,x)
```

```
[Out] int(tan(a + b*x)^3/(c + d*x)^2, x)
```

3.309 $\int (c+dx)^m \csc(a+bx) \sec^3(a+bx) \, dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left((c+dx)^m \csc(a+bx) \sec^3(a+bx), x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x)
```

Rubi [A]
time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \csc(a+bx) \sec^3(a+bx) \, dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \csc(a+bx) \sec^3(a+bx) \, dx = \int (c+dx)^m \csc(a+bx) \sec^3(a+bx) \, dx$$

Mathematica [A]
time = 12.23, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc(a+bx) \sec^3(a+bx) \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3, x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a) \left(\sec^3(bx+a)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x)
[Out] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
[Out] integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a)**3,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)),x)
[Out] int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)), x)
```

3.310 $\int (c+dx)^4 \csc(a+bx) \sec^3(a+bx) \, dx$

Optimal. Leaf size=399

$$\frac{2id(c+dx)^3}{b^2}+\frac{(c+dx)^4}{2b}-\frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{6d^2(c+dx)^2 \log\left(1+e^{2i(a+bx)}\right)}{b^3}+\frac{6id^3(c+dx)\text{PolyL}}{b^4}$$

```
[Out] -3*I*d^3*(d*x+c)*polylog(4,-exp(2*I*(b*x+a)))/b^4+1/2*(d*x+c)^4/b-2*(d*x+c)
^4*arctanh(exp(2*I*(b*x+a)))/b-6*d^2*(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b^3+2
*I*d*(d*x+c)^3*polylog(2,-exp(2*I*(b*x+a)))/b^2+2*I*d*(d*x+c)^3/b^2+3*I*d^3
*(d*x+c)*polylog(4,exp(2*I*(b*x+a)))/b^4-3*d^4*polylog(3,-exp(2*I*(b*x+a)))
/b^5-3*d^2*(d*x+c)^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+3*d^2*(d*x+c)^2*polyl
og(3,exp(2*I*(b*x+a)))/b^3-2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^2+
6*I*d^3*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^4+3/2*d^4*polylog(5,-exp(2*I
*(b*x+a)))/b^5-3/2*d^4*polylog(5,exp(2*I*(b*x+a)))/b^5-2*d*(d*x+c)^3*tan(b*
x+a)/b^2+1/2*(d*x+c)^4*tan(b*x+a)^2/b
```

Rubi [A]
time = 0.65, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {2700, 14, 4505, 6873, 12, 6874, 2631, 4268, 2611, 6744, 2320, 6724, 3801, 3800, 2221, 32}

$$-\frac{3d^4\text{Li}_3\left(-e^{2i(a+bx)}\right)}{b^5}+\frac{3d^4\text{Li}_5\left(-e^{2i(a+bx)}\right)}{2b^5}-\frac{3d^4\text{Li}_5\left(e^{2i(a+bx)}\right)}{2b^5}+\frac{6id^3(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^4}-\frac{3id^3(c+dx)\text{Li}_4\left(-e^{2i(a+bx)}\right)}{b^4}+\frac{3id^3(c+dx)\text{Li}_4\left(e^{2i(a+bx)}\right)}{b^4}-\frac{3d^2(c+dx)^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{3d^2(c+dx)^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{b^3}-\frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3}+\frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{2d(c+dx)^3\tan(a+bx)}{b^2}+\frac{(c+dx)^4\tan^2(a+bx)}{2b}-\frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}+\frac{2id(c+dx)^3}{b^2}+\frac{(c+dx)^4}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] ((2*I)*d*(c + d*x)^3)/b^2 + (c + d*x)^4/(2*b) - (2*(c + d*x)^4*ArcTanh[E^((
2*I)*(a + b*x))])/b - (6*d^2*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b^3
+ ((6*I)*d^3*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + ((2*I)*d*(c
+ d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)^3*PolyL
og[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^4*PolyLog[3, -E^((2*I)*(a + b*x))])/
b^5 - (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 + (3*d^2*(c
+ d*x)^2*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*PolyLo
g[4, -E^((2*I)*(a + b*x))])/b^4 + ((3*I)*d^3*(c + d*x)*PolyLog[4, E^((2*I)*
(a + b*x))])/b^4 + (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5) - (3*d^
4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5) - (2*d*(c + d*x)^3*Tan[a + b*x])
/b^2 + ((c + d*x)^4*Tan[a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^4 \csc(a+bx) \sec^3(a+bx)\,dx = \frac{(c+dx)^4 \log(\tan(a+bx))}{b} + \frac{(c+dx)^4 \tan^2(a+bx)}{2b} - (4d)\int (c$$

$$= \frac{(c+dx)^4 \log(\tan(a+bx))}{b} + \frac{(c+dx)^4 \tan^2(a+bx)}{2b} - (4d)\int \frac{(c}{}$$

$$= \frac{(c+dx)^4 \log(\tan(a+bx))}{b} + \frac{(c+dx)^4 \tan^2(a+bx)}{2b} - \frac{(2d)\int (c-}{}$$

$$= \frac{(c+dx)^4 \log(\tan(a+bx))}{b} + \frac{(c+dx)^4 \tan^2(a+bx)}{2b} - \frac{(2d)\int (2(}{}$$

$$= \frac{(c+dx)^4 \log(\tan(a+bx))}{b} + \frac{(c+dx)^4 \tan^2(a+bx)}{2b} - \frac{(2d)\int (c-}{}$$

$$= -\frac{2d(c+dx)^3 \tan(a+bx)}{b^2} + \frac{(c+dx)^4 \tan^2(a+bx)}{2b} + \frac{\int 2b(c+d}{}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2d(c+dx)^3 \tan(a+bx)}{b^2} + \frac{(c+dx)^4}{}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(}{}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(}{}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(}{}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(}{}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(}{}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1790 vs. $2(399) = 798$.

time = 7.36, size = 1790, normalized size = 4.49

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] -1/2*(c^2*d^2*Csc[a]*(2*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(-1 + E^((2*I)*a
))*Log[1 - E^((2*I)*(a + b*x))]) + 6*b*(-1 + E^((2*I)*a))*x*PolyLog[2, E^((
2*I)*(a + b*x))] + (3*I)*(-1 + E^((2*I)*a))*PolyLog[3, E^((2*I)*(a + b*x))]
))/(b^3*E^(I*a)) - c*d^3*E^(I*a)*Csc[a]*(x^4 + (-1 + E^((-2*I)*a))*x^4 + ((
-1 + E^((2*I)*a))*(2*b^4*x^4 + (4*I)*b^3*x^3*Log[1 - E^((2*I)*(a + b*x))] +
 6*b^2*x^2*PolyLog[2, E^((2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[3, E^((2*I)*
(a + b*x))] - 3*PolyLog[4, E^((2*I)*(a + b*x))]))/(2*b^4*E^((2*I)*a))) - (d
^4*E^(I*a)*Csc[a]*(x^5 + (-1 + E^((-2*I)*a))*x^5 + ((-1 + E^((2*I)*a))*(4*b
^5*x^5 + (10*I)*b^4*x^4*Log[1 - E^((2*I)*(a + b*x))] + 20*b^3*x^3*PolyLog[2
, E^((2*I)*(a + b*x))] + (30*I)*b^2*x^2*PolyLog[3, E^((2*I)*(a + b*x))] - 3
0*b*x*PolyLog[4, E^((2*I)*(a + b*x))] - (15*I)*PolyLog[5, E^((2*I)*(a + b*x
))]))/(4*b^5*E^((2*I)*a))))/5 + (x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5
*c*d^3*x^3 + d^4*x^4)*Csc[a]*Sec[a])/5 + (c^2*d^2*((2*I)*b^2*x^2*(2*b*E^((2
*I)*a)*x + (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((2*I)*(a + b*x))]) + (6*I)*b*
(1 + E^((2*I)*a))*x*PolyLog[2, -E^((2*I)*(a + b*x))] - 3*(1 + E^((2*I)*a))*
PolyLog[3, -E^((2*I)*(a + b*x))])*Sec[a])/(2*b^3*E^(I*a)) + (d^4*((2*I)*b^2
*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((2*I)*(a + b*x
))]) + (6*I)*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((2*I)*(a + b*x))] - 3*(1
+ E^((2*I)*a))*PolyLog[3, -E^((2*I)*(a + b*x))])*Sec[a])/(2*b^5*E^(I*a)) -
I*c*d^3*E^(I*a)*(-x^4 + (1 + E^((-2*I)*a))*x^4 - ((1 + E^((2*I)*a))*(2*b^4*
x^4 + (4*I)*b^3*x^3*Log[1 + E^((2*I)*(a + b*x))] + 6*b^2*x^2*PolyLog[2, -E^
((2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[3, -E^((2*I)*(a + b*x))] - 3*PolyLog
[4, -E^((2*I)*(a + b*x))]))/(2*b^4*E^((2*I)*a)))*Sec[a] - (I/5)*d^4*E^(I*a)
*(-x^5 + (1 + E^((-2*I)*a))*x^5 - ((1 + E^((2*I)*a))*(4*b^5*x^5 + (10*I)*b^
4*x^4*Log[1 + E^((2*I)*(a + b*x))] + 20*b^3*x^3*PolyLog[2, -E^((2*I)*(a + b
*x))] + (30*I)*b^2*x^2*PolyLog[3, -E^((2*I)*(a + b*x))] - 30*b*x*PolyLog[4,
 -E^((2*I)*(a + b*x))] - (15*I)*PolyLog[5, -E^((2*I)*(a + b*x))]))/(4*b^5*E
^((2*I)*a)))*Sec[a] + ((c + d*x)^4*Sec[a + b*x]^2)/(2*b) - (c^4*Sec[a]*(Cos
[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Si
n[a]^2)) - (6*c^2*d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]]
 + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (c^4*Csc[a]*(-(b*x*Cos[a]) +
Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) -
 (2*c^3*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*
ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log
[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]
]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[
a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^
2)]) - (6*c*d^3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-P
```

```
i - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]
])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[
Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTa
n[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + S
in[a]^2)]) - (2*Sec[a]*Sec[a + b*x]*(c^3*d*Sin[b*x] + 3*c^2*d^2*x*Sin[b*x]
+ 3*c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin[b*x]))/b^2 - (2*c^3*d*Csc[a]*Sec[a]*(b
^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 +
 E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[
Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a
]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan
[a]^2]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1728 vs. $2(361) = 722$.
time = 0.26, size = 1729, normalized size = 4.33

method	result	size
risch	Expression too large to display	1729

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3+4/b
*c*d^3*ln(exp(I*(b*x+a))+1)*x^3-12*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*
x-12*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-12*I/b^2*c*d^3*polylog(2,-e
xp(I*(b*x+a)))*x^2-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2+2*I/b^2*d^4
*polylog(2,-exp(2*I*(b*x+a)))*x^3-3*I/b^4*d^4*polylog(4,-exp(2*I*(b*x+a)))*
x+2*I/b^2*c^3*d*polylog(2,-exp(2*I*(b*x+a)))-3*I/b^4*c*d^3*polylog(4,-exp(2
*I*(b*x+a)))-4/b*c*d^3*ln(1+exp(2*I*(b*x+a)))*x^3+6*I/b^2*c^2*d^2*polylog(2
,-exp(2*I*(b*x+a)))*x+3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5-3*d^4*polylo
g(3,-exp(2*I*(b*x+a)))/b^5-4/b^2*c^3*d*a*ln(exp(I*(b*x+a))-1)-4/b^4*c*d^3*a
^3*ln(exp(I*(b*x+a))-1)-4*I/b^2*d^4*polylog(2,exp(I*(b*x+a)))*x^3-4*I/b^2*d
^4*polylog(2,-exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x
+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x+24*I/b^4*c*d^3*polylog(4,-exp(I*(
b*x+a)))-4*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))+24*I/b^4*c*d^3*polylog(4,e
xp(I*(b*x+a)))-4*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))-24*d^4*polylog(5,-e
xp(I*(b*x+a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5+1/b*c^4*ln(exp(I*(b
*x+a))-1)+1/b*c^4*ln(exp(I*(b*x+a))+1)+2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*
d^3*x^3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x^2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2
*I*(b*x+a))-2*I*d^4*x^3*exp(2*I*(b*x+a))+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x
^2*exp(2*I*(b*x+a))-6*I*c^2*d^2*x*exp(2*I*(b*x+a))-2*I*d^4*x^3-2*I*c^3*d*ex
p(2*I*(b*x+a))-6*I*c*d^3*x^2-6*I*c^2*d^2*x-2*I*c^3*d)/b^2/(1+exp(2*I*(b*x+a
)))^2+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))+12/b^3*c^2*d^2*polylog(3,-ex
p(I*(b*x+a)))+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)-6/b^3*d^2*c^2*ln(1+exp(2*I
*(b*x+a)))-6/b^3*d^4*ln(1+exp(2*I*(b*x+a)))*x^2-8*I/b^5*d^4*a^3+4*I/b^2*d^4
```

```
*x^3-1/b*c^4*ln(1+exp(2*I*(b*x+a)))-1/b^5*d^4*a^4*ln(1-exp(I*(b*x+a)))+12/b
^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2+12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*
x^2-3/b^3*c^2*d^2*polylog(3,-exp(2*I*(b*x+a)))-3/b^3*d^4*polylog(3,-exp(2*I
*(b*x+a)))*x^2+12*d^2/b^3*c^2*ln(exp(I*(b*x+a)))+12*d^4/b^5*a^2*ln(exp(I*(b
*x+a)))+12*I/b^4*d^3*c*a^2-12*I/b^4*d^4*a^2*x+6*I/b^4*d^4*polylog(2,-exp(2*
I*(b*x+a)))*x+6*I/b^4*d^3*c*polylog(2,-exp(2*I*(b*x+a)))+12*I/b^2*d^3*c*x^2
-4/b*c^3*d*ln(1+exp(2*I*(b*x+a)))*x-6/b*c^2*d^2*ln(1+exp(2*I*(b*x+a)))*x^2-
1/b*d^4*ln(1+exp(2*I*(b*x+a)))*x^4-6/b^3*c*d^3*polylog(3,-exp(2*I*(b*x+a)))
*x-24*d^3/b^4*c*a*ln(exp(I*(b*x+a)))+6*I/b^2*c*d^3*polylog(2,-exp(2*I*(b*x+
a)))*x^2-12/b^3*d^3*c*ln(1+exp(2*I*(b*x+a)))*x+24*I/b^3*d^3*c*a*x+4/b*c^3*d
*ln(1-exp(I*(b*x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*(b*x+a)))*a+4/b*c^3*d*ln(exp
(I*(b*x+a))+1)*x+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+1/b*d^4*ln(exp(I*(b*x+a))
+1)*x^4+6/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a))-1)+6/b*c^2*d^2*ln(exp(I*(b*x+a)
)+1)*x^2+6/b*c^2*d^2*ln(1-exp(I*(b*x+a)))*x^2-6/b^3*c^2*d^2*ln(1-exp(I*(b*x
+a)))*a^2+24/b^3*c*d^3*polylog(3,-exp(I*(b*x+a)))*x+24/b^3*c*d^3*polylog(3,
exp(I*(b*x+a)))*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8853 vs. 2(352) = 704.
time = 5.37, size = 8853, normalized size = 22.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^4*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x +
 a)^2)) - 4*a*c^3*d*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log
(sin(b*x + a)^2))/b + 6*a^2*c^2*d^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x +
 a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 4*a^3*c*d^3*(1/(sin(b*x + a)^2 - 1)
 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^3 + a^4*d^4*(1/(sin(b*x
 + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^4 + 2*(24*b
^3*c^3*d - 72*a*b^2*c^2*d^2 + 72*a^2*b*c*d^3 - 24*a^3*d^4 + 4*(3*(b*x + a)^
4*d^4 + 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x
 + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a)^2 + 6*(b^
3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4)*(b*x + a
) + (3*(b*x + a)^4*d^4 + 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^4 + 8*(b*c*
d^3 - a*d^4)*(b*x + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b
*x + a)^2 + 6*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3
*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 2*(3*(b*x + a)^4*d^4 + 9*b^2*c^2*d^2
 - 18*a*b*c*d^3 + 9*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^2*c^2*
d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b^2*c^2
*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) -
 (-3*I*(b*x + a)^4*d^4 - 9*I*b^2*c^2*d^2 + 18*I*a*b*c*d^3 - 9*I*a^2*d^4 + 8
*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 9*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 + (
```

-I*a^2 - I)*d^4)*(b*x + a)^2 + 6*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 + 3*(-I*
a^2 - I)*b*c*d^3 + (I*a^3 + 3*I*a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 2*(-3
I(b*x + a)^4*d^4 - 9*I*b^2*c^2*d^2 + 18*I*a*b*c*d^3 - 9*I*a^2*d^4 + 8*(-I
*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 9*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 + (-I*a
^2 - I)*d^4)*(b*x + a)^2 + 6*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 + 3*(-I*a^2
 - I)*b*c*d^3 + (I*a^3 + 3*I*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(si
n(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a
d^4)(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4
*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a) + ((b*x
 + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3
 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 -
a^3*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 2*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*
d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*
(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*cos(2*b*
x + 2*a) + (I*(b*x + a)^4*d^4 + 4*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^3 + 6*(I*
b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a)^2 + 4*(I*b^3*c^3*d - 3*I
*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 - I*a^3*d^4)*(b*x + a))*sin(4*b*x + 4*a) +
 2*(I*(b*x + a)^4*d^4 + 4*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^3 + 6*(I*b^2*c^2*
d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a)^2 + 4*(I*b^3*c^3*d - 3*I*a*b^2*c
^2*d^2 + 3*I*a^2*b*c*d^3 - I*a^3*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(
sin(b*x + a), cos(b*x + a) + 1) + 6*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*
(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*
c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a) + ((b*x + a)^4
d^4 + 4(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2
d^4)(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^
4)*(b*x + a))*cos(4*b*x + 4*a) + 2*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(
b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c
^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*cos(2*b*x + 2*
a) - (-I*(b*x + a)^4*d^4 + 4*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2
*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a)^2 + 4*(-I*b^3*c^3*d + 3*I*a
*b^2*c^2*d^2 - 3*I*a^2*b*c*d^3 + I*a^3*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 2
(-I(b*x + a)^4*d^4 + 4*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c^2
*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a)^2 + 4*(-I*b^3*c^3*d + 3*I*a*b^2
*c^2*d^2 - 3*I*a^2*b*c*d^3 + I*a^3*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan
2(sin(b*x + a), -cos(b*x + a) + 1) - 24*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d
^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(4*
b*x + 4*a) - 12*(-I*(b*x + a)^4*d^4 - 2*b^3*c^3*d + 6*a*b^2*c^2*d^2 - 6*a^2
*b*c*d^3 + 2*a^3*d^4 + 2*(-2*I*b*c*d^3 + (2*I*a + 1)*d^4)*(b*x + a)^3 + 6*(
-I*b^2*c^2*d^2 + (2*I*a + 1)*b*c*d^3 + (-I*a^2 - a)*d^4)*(b*x + a)^2 + 2*(-
2*I*b^3*c^3*d + 3*(2*I*a + 1)*b^2*c^2*d^2 + 6*(-I*a^2 - a)*b*c*d^3 + (2*I*a
^3 + 3*a^2)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - 12*(b^3*c^3*d - 3*a*b^2*c^2*
d^2 + 2*(b*x + a)^3*d^4 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4 + 4*(b*c*d^
3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x
 + a) + (b^3*c^3*d - 3*a*b^2*c^2*d^2 + 2*(b*x + a)^3*d^4 + 3*(a^2 + 1)*b*c*
d^3 - (a^3 + 3*a)*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 -

```
2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 2*(b^3*c^3*d - 3
*a*b^2*c^2*d^2 + 2*(b*x + a)^3*d^4 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4
+ 4*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3324 vs. $2(352) = 704$.
time = 4.31, size = 3324, normalized size = 8.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^
4*c^4 - 24*d^4*cos(b*x + a)^2*polylog(5, cos(b*x + a) + I*sin(b*x + a)) - 2
4*d^4*cos(b*x + a)^2*polylog(5, cos(b*x + a) - I*sin(b*x + a)) + 24*d^4*cos
(b*x + a)^2*polylog(5, I*cos(b*x + a) + sin(b*x + a)) + 24*d^4*cos(b*x + a)
^2*polylog(5, I*cos(b*x + a) - sin(b*x + a)) + 24*d^4*cos(b*x + a)^2*polylo
g(5, -I*cos(b*x + a) + sin(b*x + a)) + 24*d^4*cos(b*x + a)^2*polylog(5, -I*
cos(b*x + a) - sin(b*x + a)) - 24*d^4*cos(b*x + a)^2*polylog(5, -cos(b*x +
a) + I*sin(b*x + a)) - 24*d^4*cos(b*x + a)^2*polylog(5, -cos(b*x + a) - I*s
in(b*x + a)) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I
*b^3*c^3*d)*cos(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x + a)) - 4*(-I*b^3
*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*cos(b*x + a
)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3
*x^2 + I*b^3*c^3*d + 3*I*b*c*d^3 + 3*I*(b^3*c^2*d^2 + b*d^4)*x)*cos(b*x + a
)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) - 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^
3*x^2 - I*b^3*c^3*d - 3*I*b*c*d^3 - 3*I*(b^3*c^2*d^2 + b*d^4)*x)*cos(b*x +
a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d
^3*x^2 - I*b^3*c^3*d - 3*I*b*c*d^3 - 3*I*(b^3*c^2*d^2 + b*d^4)*x)*cos(b*x +
 a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*
d^3*x^2 + I*b^3*c^3*d + 3*I*b*c*d^3 + 3*I*(b^3*c^2*d^2 + b*d^4)*x)*cos(b*x
+ a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 4*(-I*b^3*d^4*x^3 - 3*I*b^3*
c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*cos(b*x + a)^2*dilog(-cos(b*x
+ a) + I*sin(b*x + a)) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2
*d^2*x + I*b^3*c^3*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*sin(b*x + a))
+ (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*
c^4)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*c^4 - 4*a
*b^3*c^3*d + 6*(a^2 + 1)*b^2*c^2*d^2 - 4*(a^3 + 3*a)*b*c*d^3 + (a^4 + 6*a^2
)*d^4)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^4*d^4*x^4
 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*cos(b*x +
 a)^2*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6
*(a^2 + 1)*b^2*c^2*d^2 - 4*(a^3 + 3*a)*b*c*d^3 + (a^4 + 6*a^2)*d^4)*cos(b*x
 + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^4*d^4*x^4 + 4*b^4*c*d^3
*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 + 3*a)*b*c*d^3 - (a^4 + 6
```

```
*a^2)*d^4 + 6*(b^4*c^2*d^2 + b^2*d^4)*x^2 + 4*(b^4*c^3*d + 3*b^2*c*d^3)*x)*
cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^
4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 + 3*a)*b*c*d^3 - (
a^4 + 6*a^2)*d^4 + 6*(b^4*c^2*d^2 + b^2*d^4)*x^2 + 4*(b^4*c^3*d + 3*b^2*c*d
^3)*x)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^4*d^4*x^4
 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 + 3*a)*b*c*
d^3 - (a^4 + 6*a^2)*d^4 + 6*(b^4*c^2*d^2 + b^2*d^4)*x^2 + 4*(b^4*c^3*d + 3*
b^2*c*d^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^4
*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 + 3
*a)*b*c*d^3 - (a^4 + 6*a^2)*d^4 + 6*(b^4*c^2*d^2 + b^2*d^4)*x^2 + 4*(b^4*c^
3*d + 3*b^2*c*d^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1
) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*cos(b*x + a)^2*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^4*c^
4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*cos(b*x +
a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^4*d^4*x^4 + 4*b
^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^
2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*cos(b*x + a)^2*log(-cos(b*x + a) + I*s
in(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 + 1)*b^2*c^2*d^2 - 4*(
a^3 + 3*a)*b*c*d^3 + (a^4 + 6*a^2)*d^4)*cos(b*x + a)^2*log(-cos(b*x + a) +
I*sin(b*x + a) + I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 +
4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4
)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b
^3*c^3*d + 6*(a^2 + 1)*b^2*c^2*d^2 - 4*(a^3 + 3*a)*b*c*d^3 + (a^4 + 6*a^2)*
d^4)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 24*(-I*b*d^4*
x - I*b*c*d^3)*cos(b*x + a)^2*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 2
4*(I*b*d^4*x + I*b*c*d^3)*cos(b*x + a)^2*polylog(4, cos(b*x + a) - I*sin(b*
x + a)) - 24*(-I*b*d^4*x - I*b*c*d^3)*cos(b*x + a)^2*polylog(4, I*cos(b*x +
 a) + sin(b*x + a)) - 24*(I*b*d^4*x + I*b*c*d^3)*cos(b*x + a)^2*polylog(4,
I*cos(b*x + a) - sin(b*x + a)) - 24*(I*b*d^4*x + I*b*c*d^3)*cos(b*x + a)^2*
polylog(4, -I*cos(b*x + a) + sin(b*x + a)) - 24*(-I*b*d^4*x - I*b*c*d^3)*co
s(b*x + a)^2*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 24*(I*b*d^4*x + I
*b*c*d^3)*cos(b*x + a)^2*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) - 24*(-
I*b*d^4*x - I*b*c*d^3)*cos(b*x + a)^2*polylog(4, -cos(b*x + a) - I*sin(b*x
+ a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2*polyl
og(3, cos(b*x + a) + I*sin(b*x + a)) + 12*(b^2*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^4*csc(b*x + a)*sec(b*x + a)^3, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^4/(cos(a + b*x)^3*sin(a + b*x)),x)
[Out] \text{Hanged}
```

3.311 $\int (c+dx)^3 \csc(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=325

$$\frac{3id(c+dx)^2}{2b^2}+\frac{(c+dx)^3}{2b}-\frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}+\frac{3id^3\text{PolyLog}(2,-}{2b^4}$$

```
[Out] 3/2*I*d*(d*x+c)^2/b^2+1/2*(d*x+c)^3/b-2*(d*x+c)^3*arctanh(exp(2*I*(b*x+a)))
/b-3*d^2*(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b^3+3/2*I*d^3*polylog(2,-exp(2*I*(b
*x+a)))/b^4+3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/b^2-3/2*I*d*(d*x
+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)*polylog(3,-exp(2*I*(b
*x+a)))/b^3+3/2*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*polyl
og(4,-exp(2*I*(b*x+a)))/b^4+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4-3/2*d
*(d*x+c)^2*tan(b*x+a)/b^2+1/2*(d*x+c)^3*tan(b*x+a)^2/b
```

Rubi [A]
time = 0.45, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {2700, 14, 4505, 6873, 12, 6874, 2631, 4268, 2611, 6744, 2320, 6724, 3801, 3800, 2221, 2317, 2438, 32}

$$\frac{3id^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}+\frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{3id(c+dx)^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{3d(c+dx)^2\tan(a+bx)}{2b^2}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2}{2b^2}+\frac{(c+dx)^3}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] (((3*I)/2)*d*(c + d*x)^2)/b^2 + (c + d*x)^3/(2*b) - (2*(c + d*x)^3*ArcTanh[
E^((2*I)*(a + b*x))])/b - (3*d^2*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^
3 + (((3*I)/2)*d^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/2)*d*(c
+ d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (((3*I)/2)*d*(c + d*x)^2*P
olyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)
*(a + b*x))])/(2*b^3) + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/(
2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/4)*
d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4 - (3*d*(c + d*x)^2*Tan[a + b*x])/(
2*b^2) + ((c + d*x)^3*Tan[a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
```

```
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^3 \csc(a+bx)\sec^3(a+bx)\,dx = \frac{(c+dx)^3\log(\tan(a+bx))}{b} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} - (3d)\int (c+$$

$$= \frac{(c+dx)^3\log(\tan(a+bx))}{b} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} - (3d)\int \frac{(c+}{}$$

$$= \frac{(c+dx)^3\log(\tan(a+bx))}{b} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} - \frac{(3d)\int(c+}{}$$

$$= \frac{(c+dx)^3\log(\tan(a+bx))}{b} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} - \frac{(3d)\int (2(c}{}$$

$$= \frac{(c+dx)^3\log(\tan(a+bx))}{b} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} - \frac{(3d)\int(c+}{}$$

$$= -\frac{3d(c+dx)^2\tan(a+bx)}{2b^2} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} + \frac{\int 2b(c+dx}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{3d(c+dx)^2\tan(a+bx)}{2b^2} + \frac{(c+dx)^3\,\mathrm{t}}{2}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d^2(c}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d^2(c}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d^2(c}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d^2(c}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d^2(c}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1294 vs. $2(325) = 650$.
time = 6.81, size = 1294, normalized size = 3.98

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] -1/4*(c*d^2*Csc[a]*(2*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(-1 + E^((2*I)*a))
*Log[1 - E^((2*I)*(a + b*x))]) + 6*b*(-1 + E^((2*I)*a))*x*PolyLog[2, E^((2*
I)*(a + b*x))] + (3*I)*(-1 + E^((2*I)*a))*PolyLog[3, E^((2*I)*(a + b*x))]))
/(b^3*E^(I*a)) - (d^3*E^(I*a)*Csc[a]*(x^4 + (-1 + E^((-2*I)*a))*x^4 + ((-1
+ E^((2*I)*a))*(2*b^4*x^4 + (4*I)*b^3*x^3*Log[1 - E^((2*I)*(a + b*x))] + 6*
b^2*x^2*PolyLog[2, E^((2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[3, E^((2*I)*(a
+ b*x))] - 3*PolyLog[4, E^((2*I)*(a + b*x))]))/(2*b^4*E^((2*I)*a))))/4 + (x
*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Csc[a]*Sec[a])/4 + (c*d^2*((2*
I)*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((2*I)*(a
 + b*x))]) + (6*I)*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((2*I)*(a + b*x))] -
 3*(1 + E^((2*I)*a))*PolyLog[3, -E^((2*I)*(a + b*x))])*Sec[a])/(4*b^3*E^(I*
a)) - (I/4)*d^3*E^(I*a)*(-x^4 + (1 + E^((-2*I)*a))*x^4 - ((1 + E^((2*I)*a))
*(2*b^4*x^4 + (4*I)*b^3*x^3*Log[1 + E^((2*I)*(a + b*x))] + 6*b^2*x^2*PolyLo
g[2, -E^((2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[3, -E^((2*I)*(a + b*x))] - 3
*PolyLog[4, -E^((2*I)*(a + b*x))]))/(2*b^4*E^((2*I)*a)))*Sec[a] + ((c + d*x
)^3*Sec[a + b*x]^2)/(2*b) - (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a
]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (3*c*d^2*Sec[a]*(Cos
[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 +
Sin[a]^2)) + (c^3*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[
b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*
ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((
-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a
]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]]
+ I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[
a])/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*d^3*Csc[a]*((b^2*x^2)
/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1
+ E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan
[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[
a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2]
)*Sec[a])/(2*b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*Sec[a]*Sec[a +
b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[b*x]))/(2*b^2) - (3
*c^2*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTa
n[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 -
E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log
[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))
```

```
])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)]
)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1114 vs. 2(279) = 558.
time = 0.17, size = 1115, normalized size = 3.43

method	result	size
risch	Expression too large to display	1115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 6*I/b^3*d^3*a*x+6*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)))-1/b^4*d^3*a^3*ln(exp
(I*(b*x+a))-1)+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,
-exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3
,exp(I*(b*x+a)))*x+1/b*c^3*ln(exp(I*(b*x+a))+1)+1/b*c^3*ln(exp(I*(b*x+a))-1
)-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x+6*I*d^3*polylog(4,exp(I*(b*x+a)
))/b^4+6*d^2/b^3*c*ln(exp(I*(b*x+a)))-6*d^3/b^4*a*ln(exp(I*(b*x+a)))+1/b*d^
3*ln(exp(I*(b*x+a))+1)*x^3+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1-
exp(I*(b*x+a)))*a^3+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)-3/4*I*d^3*polylog(
4,-exp(2*I*(b*x+a)))/b^4-1/b*c^3*ln(1+exp(2*I*(b*x+a)))+(2*b*d^3*x^3*exp(2*
I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-6*I*
c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*I*c^2*d*exp(2*I*(b*
x+a))-3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))-6*I*c*d^2*x-3*I*c^2*d)/b^2/(1+ex
p(2*I*(b*x+a)))^2-3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))-3/2/b^3*d^3*po
lylog(3,-exp(2*I*(b*x+a)))*x-3/b^3*d^2*c*ln(1+exp(2*I*(b*x+a)))-3/b^3*d^3*l
n(1+exp(2*I*(b*x+a)))*x+3*I/b^2*d^3*x^2+3*I/b^4*d^3*a^2+3/b*c^2*d*ln(exp(I*
(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-exp(I*(b*x+
a)))*a+3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^
2-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)-3
*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+
a)))-3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2,exp(I
*(b*x+a)))*x^2+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2+3/2*I/b^2*c^2
*d*polylog(2,-exp(2*I*(b*x+a)))+3/2*I*d^3*polylog(2,-exp(2*I*(b*x+a)))/b^4-
3/b*c^2*d*ln(1+exp(2*I*(b*x+a)))*x-3/b*c*d^2*ln(1+exp(2*I*(b*x+a)))*x^2-1/b
*d^3*ln(1+exp(2*I*(b*x+a)))*x^3-6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x+
3*I/b^2*c*d^2*polylog(2,-exp(2*I*(b*x+a)))*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4954 vs. 2(270) = 540.
time = 1.77, size = 4954, normalized size = 15.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")

[Out] -1/2*(c^3*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x +
 a)^2)) - 3*a*c^2*d*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log
(sin(b*x + a)^2))/b + 3*a^2*c*d^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a
)^2 - 1) - log(sin(b*x + a)^2))/b^2 - a^3*d^3*(1/(sin(b*x + a)^2 - 1) + log
(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^3 + 2*(18*b^2*c^2*d - 36*a*b*
c*d^2 + 18*a^2*d^3 + 2*(4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 9*(b*c*d^
2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x +
 a) + (4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x +
 a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x +
4*a) + 2*(4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*
x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(2*b*x
 + 2*a) - (-4*I*(b*x + a)^3*d^3 - 9*I*b*c*d^2 + 9*I*a*d^3 + 9*(-I*b*c*d^2 +
 I*a*d^3)*(b*x + a)^2 + 9*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 - I)*d^3)
*(b*x + a))*sin(4*b*x + 4*a) - 2*(-4*I*(b*x + a)^3*d^3 - 9*I*b*c*d^2 + 9*I*
a*d^3 + 9*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 9*(-I*b^2*c^2*d + 2*I*a*b*c*
d^2 + (-I*a^2 - I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*
a), cos(2*b*x + 2*a) + 1) - 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x +
 a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) + ((b*x + a)^3*d^3
+ 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(
b*x + a))*cos(4*b*x + 4*a) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x
+ a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a)
+ (I*(b*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d
 - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*(I*(b*x + a)^
3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^
2 + I*a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x +
 a) + 1) + 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*
d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(4*b
*x + 4*a) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c
^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^3
*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d
^2 - I*a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(-I*(b*x + a)^3*d^3 + 3*(-I
*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d
^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) -
 18*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 6*
(-2*I*(b*x + a)^3*d^3 - 3*b^2*c^2*d + 6*a*b*c*d^2 - 3*a^2*d^3 + 3*(-2*I*b*c
*d^2 + (2*I*a + 1)*d^3)*(b*x + a)^2 + 6*(-I*b^2*c^2*d + (2*I*a + 1)*b*c*d^2
 + (-I*a^2 - a)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*(3*b^2*c^2*d - 6*a*b*c
*d^2 + 4*(b*x + a)^2*d^3 + 3*(a^2 + 1)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)
+ (3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*(a^2 + 1)*d^3 + 6*(b*c
*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 2*(3*b^2*c^2*d - 6*a*b*c*d^2 +
4*(b*x + a)^2*d^3 + 3*(a^2 + 1)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*
b*x + 2*a) + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 4*I*(b*x + a)^2*d^3 + 3*(I*a^
```

```
2 + I)*d^3 + 6*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*(3*I*b
^2*c^2*d - 6*I*a*b*c*d^2 + 4*I*(b*x + a)^2*d^3 + 3*(I*a^2 + I)*d^3 + 6*(I*b
*c*d^2 - I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a))
+ 18*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*
d^3)*(b*x + a) + (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(
b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 2*(b^2*c^2*d - 2*a*b*c*d^2 +
 (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a
) - (-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b
*c*d^2 + I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(-I*b^2*c^2*d + 2*I*a*b*c
*d^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*
sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + 18*(b^2*c^2*d - 2*a*b*c*d^2 + (
b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) + (b^2*c^2*d - 2*a
*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4
*b*x + 4*a) + 2*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b
*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-I*b^2*c^2*d + 2*I*a*b*c*d^2
 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(
4*b*x + 4*a) - 2*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 - I*a^2*
d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x
+ I*a)) + (-4*I*(b*x + a)^3*d^3 - 9*I*b*c*d^2 + 9*I*a*d^3 - 9*(I*b*c*d^2 -
I*a*d^3)*(b*x + a)^2 - 9*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 + I)*d^3)*(b
*x + a) + (-4*I*(b*x + a)^3*d^3 - 9*I*b*c*d^2 + 9*I*a*d^3 - 9*(I*b*c*d^2 -
I*a*d^3)*(b*x + a)^2 - 9*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 + I)*d^3)*(b
*x + a))*cos(4*b*x + 4*a) - 2*(4*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^
3 + 9*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 9*(I*...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2268 vs. $2(270) = 540$.
time = 5.33, size = 2268, normalized size = 6.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 + 6*I*d^3*cos(
b*x + a)^2*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*cos(b*x + a)
^2*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*cos(b*x + a)^2*polyl
og(4, I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*cos(b*x + a)^2*polylog(4, I*
cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*cos(b*x + a)^2*polylog(4, -I*cos(b*x
 + a) + sin(b*x + a)) + 6*I*d^3*cos(b*x + a)^2*polylog(4, -I*cos(b*x + a) -
 sin(b*x + a)) - 6*I*d^3*cos(b*x + a)^2*polylog(4, -cos(b*x + a) + I*sin(b*
x + a)) + 6*I*d^3*cos(b*x + a)^2*polylog(4, -cos(b*x + a) - I*sin(b*x + a))
 - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*cos(b*x + a)^2*dilog(c
os(b*x + a) + I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2
*c^2*d)*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*(I*b^2*d^3*
```

```
x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d + I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x
 + a) + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d -
 I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - 3*(-I*b^2*d^3
*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d - I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b
*x + a) + sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d
+ I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(-I*b^2*d
^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*cos(b*x + a)^2*dilog(-cos(b*x + a)
+ I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*cos(b
*x + a)^2*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b^3*d^3*x^3 + 3*b^3*c*d^
2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*
x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*
d^3)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*d^3*x^3 +
 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)^2*log(cos(b*x + a)
 - I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (
a^3 + 3*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^
3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d
^3 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x +
 a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
 (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2*log(I*cos(b*x +
a) - sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2*lo
g(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*co
s(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) + 1/2*
I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^3*d^
3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a
^3*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^3*c^3 -
 3*a*b^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x + a)^2*log(
-cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)^2*log(-co
s(b*x + a) - I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 1)*b
*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a)
 + I) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, cos(b*x + a) + I*si
n(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, cos(b*x + a)
- I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, I*cos(b
*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, I
*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylo
g(3, -I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2
*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x
 + a)^2*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*
cos(b*x + a)^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) - 3*(b^2*d^3*x^2
+ 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a))/(b^4*cos(b*x + a)^2
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a)^3, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)^3*sin(a + b*x)),x)
```

```
[Out] \text{Hanged}
```

3.312 $\int (c+dx)^2 \csc(a+bx) \sec^3(a+bx) \, dx$

Optimal. Leaf size=201

$$\frac{cdx}{b}+\frac{d^2x^2}{2b}-\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{d^2\log(\cos(a+bx))}{b^3}+\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+$$

```
[Out] c*d*x/b+1/2*d^2*x^2/b-2*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b-d^2*ln(cos(b*
x+a))/b^3+I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2-I*d*(d*x+c)*polylog(
2,exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+1/2*d^2*po
lylog(3,exp(2*I*(b*x+a)))/b^3-d*(d*x+c)*tan(b*x+a)/b^2+1/2*(d*x+c)^2*tan(b*
x+a)^2/b
```

Rubi [A]

time = 0.27, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {2700, 14, 4505, 6873, 12, 6874, 2631, 4268, 2611, 2320, 6724, 3801, 3556}

$$-\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}-\frac{d^2\log(\cos(a+bx))}{b^3}+\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\tan(a+bx)}{b^2}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}-\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}+\frac{cdx}{b}+\frac{d^2x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] (c*d*x)/b + (d^2*x^2)/(2*b) - (2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/
b - (d^2*Log[Cos[a + b*x]])/b^3 + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a +
b*x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d^2*Po
lyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E^((2*I)*(a + b*x
))])/(2*b^3) - (d*(c + d*x)*Tan[a + b*x])/b^2 + ((c + d*x)^2*Tan[a + b*x]^2
)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^2 \csc(a+bx)\sec^3(a+bx)\,dx &= \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - (2d)\int (c+ \\
&= \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - (2d)\int (c+ \\
&= \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - \frac{d\int (c+dx}{} \\
&= \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - \frac{d\int (2(c+}{} \\
&= \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - \frac{d\int (c+dx}{} \\
&= -\frac{d(c+dx)\tan(a+bx)}{b^2} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} + \frac{\int 2b(c+dx)^2}{} \\
&= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{d^2\log(\cos(a+bx))}{b^3} - \frac{d(c+dx)\tan(a+bx)}{b^2} + \frac{(c+}{} \\
&= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} \\
&= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} \\
&= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} \\
&= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 788 vs. $2(201) = 402$.
time = 6.69, size = 788, normalized size = 3.92

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] -1/12*(d^2*Csc[a]*(2*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(-1 + E^((2*I)*a))*
Log[1 - E^((2*I)*(a + b*x))]) + 6*b*(-1 + E^((2*I)*a))*x*PolyLog[2, E^((2*I
)*(a + b*x))] + (3*I)*(-1 + E^((2*I)*a))*PolyLog[3, E^((2*I)*(a + b*x))]))/
(b^3*E^(I*a)) + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Csc[a]*Sec[a])/3 + (d^2*((2*
I)*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((2*I)*(a
 + b*x))]) + (6*I)*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((2*I)*(a + b*x))] -
 3*(1 + E^((2*I)*a))*PolyLog[3, -E^((2*I)*(a + b*x))])*Sec[a])/(12*b^3*E^(I
*a)) + ((c + d*x)^2*Sec[a + b*x]^2)/(2*b) - (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*
```

```
Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (d^2
*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*
(Cos[a]^2 + Sin[a]^2)) + (c^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] +
 Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (c*d*Csc[a]*((b^2*x^
2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[
1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcT
an[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Co
t[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^
2])*Sec[a])/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Sec[a]*Sec[a + b*
x]*(-(c*d*Sin[b*x]) - d^2*x*Sin[b*x]))/b^2 - (c*d*Csc[a]*Sec[a]*(b^2*E^(I*A
rcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I
)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))
] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*
PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/
(b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(181) = 362$.
time = 0.13, size = 614, normalized size = 3.05

method	result
risch	$-\frac{d^2 \ln\left(1+e^{2i(bx+a)}\right)x^2}{b} - \frac{2cda \ln\left(e^{i(bx+a)}-1\right)}{b^2} - \frac{c^2 \ln\left(1+e^{2i(bx+a)}\right)}{b} - \frac{d^2 \operatorname{polylog}\left(3,-e^{2i(bx+a)}\right)}{2b^3} - \frac{2id^2 \operatorname{polylog}\left(2,e^{i(bx+a)}\right)}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d
^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2-1/b*d^2*ln(1+e
xp(2*I*(b*x+a)))*x^2-1/b*c^2*ln(1+exp(2*I*(b*x+a)))+2/b^2*c*d*ln(1-exp(I*(b
*x+a)))*a+2/b*c*d*ln(exp(I*(b*x+a))+1)*x+2/b*c*d*ln(1-exp(I*(b*x+a)))*x-2/b
^2*c*d*a*ln(exp(I*(b*x+a))-1)-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b
^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-
2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))-2/b*c*d*ln(1+exp(2*I*(b*x+a)))*x+2*(b
*d^2*x^2*exp(2*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))
-I*d^2*x*exp(2*I*(b*x+a))-I*c*d*exp(2*I*(b*x+a))-I*d^2*x-I*d*c)/b^2/(1+exp(
2*I*(b*x+a)))^2-d^2/b^3*ln(1+exp(2*I*(b*x+a)))+1/b*c^2*ln(exp(I*(b*x+a))-1)
+1/b*c^2*ln(exp(I*(b*x+a))+1)-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+2*d^
2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3+2/b^3*
d^2*ln(exp(I*(b*x+a)))+I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+I/b^2*c*d*p
olylog(2,-exp(2*I*(b*x+a)))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2447 vs. $2(177) = 354$.
time = 0.76, size = 2447, normalized size = 12.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")

[Out] -1/2*(c^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 2*a*c*d*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + a^2*d^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 2*(4*(b*x + a)*d^2*cos(4*b*x + 4*a) + 4*I*(b*x + a)*d^2*sin(4*b*x + 4*a) - 4*b*c*d + 4*a*d^2 - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2 + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*cos(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a) - I*d^2)*sin(4*b*x + 4*a) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a) - I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a) + 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 4*(-I*(b*x + a)^2*d^2 - b*c*d + a*d^2 + (-2*I*b*c*d + (2*I*a + 1)*d^2)*(b*x + a))*cos(2*b*x + 2*a) + 2*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(4*b*x + 4*a) + 2*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(4*b*x + 4*a) - 2*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(4*b*x + 4*a) - 2*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-I*(b*x + a)^2*d^2 - 2*(I*b*c*d - I*a*d^2)*(b*x + a) - I*d^2 + (-I*(b*x + a)^2*d^2 - 2*(I*b*c*d - I*a*d^2)*(b*x + a) - I*d^2)*cos(4*b*x + 4*a) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) + I*d^2)*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (I*(b*x + a)^2*d^2 - 2*(-I*b*c*d + I*a*d^2)*(b*x + a) + (I*(b*x + a)^2*d^2 - 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*cos(4*b*x + 4*a) - 2*(-I*(b*x + a)^2*

```
d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(
b*c*d - a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
a)^2 + 2*cos(b*x + a) + 1) - (I*(b*x + a)^2*d^2 - 2*(-I*b*c*d + I*a*d^2)*(b
*x + a) + (I*(b*x + a)^2*d^2 - 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*cos(4*b*x
+ 4*a) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*cos(2*b*
x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(4*b*x + 4*a)
 - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*log(
cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-I*d^2*cos(4*b*x +
 4*a) - 2*I*d^2*cos(2*b*x + 2*a) + d^2*sin(4*b*x + 4*a) + 2*d^2*sin(2*b*x +
 2*a) - I*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) + 4*(-I*d^2*cos(4*b*x + 4*a
) - 2*I*d^2*cos(2*b*x + 2*a) + d^2*sin(4*b*x + 4*a) + 2*d^2*sin(2*b*x + 2*a
) - I*d^2)*polylog(3, -e^(I*b*x + I*a)) + 4*(-I*d^2*cos(4*b*x + 4*a) - 2*I*
d^2*cos(2*b*x + 2*a) + d^2*sin(4*b*x + 4*a) + 2*d^2*sin(2*b*x + 2*a) - I*d^
2)*polylog(3, e^(I*b*x + I*a)) + 4*((b*x + a)^2*d^2 - I*b*c*d + I*a*d^2 + (
2*b*c*d - (2*a - I)*d^2)*(b*x + a))*sin(2*b*x + 2*a))/(-2*I*b^2*cos(4*b*x +
 4*a) - 4*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4*b*x + 4*a) + 4*b^2*sin(2*b*x
 + 2*a) - 2*I*b^2))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1404 vs. $2(177) = 354$.
time = 3.58, size = 1404, normalized size = 6.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*d^2*cos(b*x + a)^2*polylog(3, cos(b*x +
a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)^2*polylog(3, cos(b*x + a) - I*sin
(b*x + a)) - 2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a))
 - 2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 2*d^2*c
os(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*cos(b*x +
a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b*x + a)^2*poly
log(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)^2*polylog(3, -c
os(b*x + a) - I*sin(b*x + a)) + b^2*c^2 - 2*(I*b*d^2*x + I*b*c*d)*cos(b*x +
 a)^2*dilog(cos(b*x + a) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*cos(b
*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*co
s(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d
)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - 2*(-I*b*d^2*x - I*b
*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 2*(I*b*d^2*x +
 I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 2*(-I*b*d^
2*x - I*b*c*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 2*(I*
b*d^2*x + I*b*c*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a)^2*log(cos(b*x + a) + I*si
```

```
n(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(b*x + a)^2*log(
cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*
cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*
d + (a^2 + 1)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) -
(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(
b*x + a) + sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2
*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*d^2*x^2
+ 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x + a) + s
in(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*
x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) +
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*
I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*c
os(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*
d + (a^2 + 1)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) +
 (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(-cos(
b*x + a) - I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(
b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 2*(b*d^2*x + b*c*d)*co
s(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \csc (a+bx) \sec ^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a)**3,x)
[Out] Integral((c + d*x)**2*csc(a + b*x)*sec(a + b*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^2*csc(b*x + a)*sec(b*x + a)^3, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)^3*sin(a + b*x)),x)
[Out] \text{Hanged}
```

3.313 $\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=139

$$\frac{dx}{2b}-\frac{2dx\tanh^{-1}\left(e^{2ia+2ibx}\right)}{b}+\frac{c\log(\tan(a+bx))}{b}+\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{d\tan(}{2}$$

```
[Out] 1/2*d*x/b-2*d*x*arctanh(exp(2*I*a+2*I*b*x))/b+c*ln(tan(b*x+a))/b+1/2*I*d*po
lylog(2,-exp(2*I*(b*x+a)))/b^2-1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^2-1/2*
d*tan(b*x+a)/b^2+1/2*c*tan(b*x+a)^2/b+1/2*d*x*tan(b*x+a)^2/b
```

Rubi [A]
time = 0.10, antiderivative size = 141, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$,
Rules used = {2700, 14, 4505, 2628, 12, 4268, 2317, 2438, 3554, 8}

$$\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{d\tan(a+bx)}{2b^2}+\frac{(c+dx)\tan^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+bx))}{b}-\frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{dx\log(\tan(a+bx))}{b}+\frac{dx}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] (d*x)/(2*b) - (2*d*x*ArcTanh[E^((2*I)*(a + b*x))])/b - (d*x*Log[Tan[a + b*x
]])/b + ((c + d*x)*Log[Tan[a + b*x]])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a
+ b*x))])/b^2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d*Tan[a +
b*x])/(2*b^2) + ((c + d*x)*Tan[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rubi steps

$$\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx = \frac{(c+dx)\log(\tan(a+bx))}{b}+\frac{(c+dx)\tan^2(a+bx)}{2b}-d\int\left(\frac{\log(\text{ta}\right.$$
$$=\frac{(c+dx)\log(\tan(a+bx))}{b}+\frac{(c+dx)\tan^2(a+bx)}{2b}-\frac{d\int\tan^2(a+}{2b}$$
$$=-\frac{dx\log(\tan(a+bx))}{b}+\frac{(c+dx)\log(\tan(a+bx))}{b}-\frac{d\tan(a+bx}{2b^2}$$
$$=\frac{dx}{2b}-\frac{dx\log(\tan(a+bx))}{b}+\frac{(c+dx)\log(\tan(a+bx))}{b}-\frac{d\tan(a\cdot}{2b^2}$$
$$=\frac{dx}{2b}-\frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{dx\log(\tan(a+bx))}{b}+\frac{(c+dx)\log}{}$$
$$=\frac{dx}{2b}-\frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{dx\log(\tan(a+bx))}{b}+\frac{(c+dx)\log}{}$$
$$=\frac{dx}{2b}-\frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{dx\log(\tan(a+bx))}{b}+\frac{(c+dx)\log}{}$$

Mathematica [A]
time = 0.59, size = 212, normalized size = 1.53

$$\frac{ad\log(\cos(a+bx))}{b^2}-\frac{ad\log(\sin(a+bx))}{b^2}+\frac{d\left(\frac{1}{2}i(a+bx)^2-(a+bx)\log\left(1+e^{2i(a+bx)}\right)+\frac{1}{2}i\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)\right)}{b^2}+\frac{d\left((a+bx)\log\left(1-e^{2i(a+bx)}\right)-\frac{1}{2}i\left((a+bx)^2+\text{PolyLog}\left(2,e^{2i(a+bx)}\right)\right)\right)}{b^2}+\frac{dx\sec^2(a+bx)}{2b}-\frac{c\left(2\log(\cos(a+bx))-2\log(\sin(a+bx))-\sec^2(a+bx)\right)}{2b}-\frac{d\tan(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] (a*d*Log[Cos[a + b*x]])/b^2 - (a*d*Log[Sin[a + b*x]])/b^2 + (d*((I/2)*(a +
b*x)^2 - (a + b*x)*Log[1 + E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^((2*I
)*(a + b*x))]))/b^2 + (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x))] - (I/2)*((
a + b*x)^2 + PolyLog[2, E^((2*I)*(a + b*x))])))/b^2 + (d*x*Sec[a + b*x]^2)/
(2*b) - (c*(2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]] - Sec[a + b*x]^2))/(2
*b) - (d*Tan[a + b*x])/(2*b^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(118) = 236.
time = 0.11, size = 270, normalized size = 1.94

method	result
risch	$\frac{2bdx\,e^{2i(bx+a)}-id\,e^{2i(bx+a)}+2bc\,e^{2i(bx+a)}-id}{b^2\left(1+e^{2i(bx+a)}\right)^2}+\frac{c\ln\left(e^{i(bx+a)}-1\right)}{b}+\frac{c\ln\left(e^{i(bx+a)}+1\right)}{b}-\frac{c\ln\left(1+e^{2i(bx+a)}\right)}{b}+\frac{d\ln\left(1-e^{i(bx+a}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2*b*d*x*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))-I*d)/
b^2/(1+exp(2*I*(b*x+a)))^2+1/b*c*ln(exp(I*(b*x+a))-1)+1/b*c*ln(exp(I*(b*x+a
))+1)-1/b*c*ln(1+exp(2*I*(b*x+a)))+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(
1-exp(I*(b*x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/b*d*ln(1+exp(2*I*(b
*x+a)))*x+1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+
1)*x-I*d*polylog(2,-exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(112) = 224$.
time = 0.63, size = 1028, normalized size = 7.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -(2*(b*d*x + b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) + 2*(b*d*x + b*c)*cos(2*b
*x + 2*a) - (-I*b*d*x - I*b*c)*sin(4*b*x + 4*a) - 2*(-I*b*d*x - I*b*c)*sin(
2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 2*(b*d*x +
b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (
I*b*d*x + I*b*c)*sin(4*b*x + 4*a) + 2*(I*b*d*x + I*b*c)*sin(2*b*x + 2*a))*a
rctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(b*c*cos(4*b*x + 4*a) + 2*b*c*co
s(2*b*x + 2*a) + I*b*c*sin(4*b*x + 4*a) + 2*I*b*c*sin(2*b*x + 2*a) + b*c)*a
rctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*(b*d*x*cos(4*b*x + 4*a) + 2*b*d*
x*cos(2*b*x + 2*a) + I*b*d*x*sin(4*b*x + 4*a) + 2*I*b*d*x*sin(2*b*x + 2*a)
+ b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*(-2*I*b*d*x - 2*I*b*c
 - d)*cos(2*b*x + 2*a) - (d*cos(4*b*x + 4*a) + 2*d*cos(2*b*x + 2*a) + I*d*s
in(4*b*x + 4*a) + 2*I*d*sin(2*b*x + 2*a) + d)*dilog(-e^(2*I*b*x + 2*I*a)) +
 2*(d*cos(4*b*x + 4*a) + 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) + 2*I*
d*sin(2*b*x + 2*a) + d)*dilog(-e^(I*b*x + I*a)) + 2*(d*cos(4*b*x + 4*a) + 2
*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) + 2*I*d*sin(2*b*x + 2*a) + d)*di
log(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(4*b*x + 4
*a) - 2*(I*b*d*x + I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*b*x + 4*a)
 + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2
*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(
4*b*x + 4*a) - 2*(-I*b*d*x - I*b*c)*cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*
b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x
 + a)^2 + 2*cos(b*x + a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(4*
b*x + 4*a) - 2*(-I*b*d*x - I*b*c)*cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*b*
x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
 a)^2 - 2*cos(b*x + a) + 1) - 2*(2*b*d*x + 2*b*c - I*d)*sin(2*b*x + 2*a) +
2*d)/(-2*I*b^2*cos(4*b*x + 4*a) - 4*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4*b*
x + 4*a) + 4*b^2*sin(2*b*x + 2*a) - 2*I*b^2)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(112) = 224$.

time = 3.09, size = 760, normalized size = 5.47

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")

[Out] 1/2*(-I*d*cos(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) + I*d*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b*d*x + b*c)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b*d*x + b*c)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b*d*x + a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b*c - a*d)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b*d*x + a*d)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b*c - a*d)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*d*x + a*d)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b*c - a*d)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) + b*d*x - d*cos(b*x + a)*sin(b*x + a) + b*c)/(b^2*cos(b*x + a)^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)**3,x)

[Out] Integral((c + d*x)*csc(a + b*x)*sec(a + b*x)**3, x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)*sec(b*x + a)^3, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)^3*sin(a + b*x)),x)
```

```
[Out] \text{Hanged}
```

3.314 $\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{\csc(a+bx)\sec^3(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x),x]
[Out] Defer[Int][(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 8.79, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x),x]
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)\left(\sec^3\left(bx+a\right)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] (4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
(2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) +
2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b
*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*
b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate(2*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2 + d^2)*sin(2*b*x + 2*a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2
*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*si
n(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*s
in(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/((d*x
+ c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*cos(b*x
+ a) + c), x) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x +
4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x
 + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d
*x + c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) + (d*cos(2
*b*x + 2*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4*a) + d*si
n(2*b*x + 2*a))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2
```

```
*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x +
4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x
 + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos(2*b*x + 2*a))
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")
[Out] integral(csc(b*x + a)*sec(b*x + a)^3/(d*x + c), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(a+bx\right)\sec^{3}\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)**3/(d*x+c),x)
[Out] Integral(csc(a + b*x)*sec(a + b*x)**3/(c + d*x), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)*sec(b*x + a)^3/(d*x + c), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\cos\left(a+b\,x\right)}^3\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)),x)
[Out] int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)), x)
```

3.315 $\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2,x]
[Out] Defer[Int][(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 6.44, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2,x]
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)\left(\sec^3\left(bx+a\right)\right)}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) +
2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
 + b^2*c^3)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^
2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*inte
grate(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 3*d^2)*sin(2*b*x + 2*a)/(b^2
*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 +
(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^
4)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2
+ 4*b^2*c^3*d*x + b^2*c^4)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^
3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)), x)
- (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 +
 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3
*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2
*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 +
4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a
)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
 + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*co
s(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2
*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(d^2*x^2 + 2*c
*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x + c^2)
*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x) + (b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^
2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*
x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 + 4*(b^
```

```
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^
2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b
*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*
d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(d^2*x^2 + 2*c*d*x
+ (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(
b*x + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x) + 2*(d*cos
(2*b*x + 2*a) + (b*d*x + b*c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4*a) + 2*d*
sin(2*b*x + 2*a))/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
+ (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a
)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x
 + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4
*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^3*x^3 + 3*b^2*c
*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc (a+bx)\sec ^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)**3/(c + d*x)**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^2), x)
```

3.316 $\int (c+dx)^m \csc^2(a+bx) \sec^3(a+bx) \, dx$

Optimal. Leaf size=27

$$\text{Int}\left((c+dx)^m \csc^2(a+bx) \sec^3(a+bx),x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

Rubi [A]
time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \csc^2(a+bx) \sec^3(a+bx) \, dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \csc^2(a+bx) \sec^3(a+bx) \, dx = \int (c+dx)^m \csc^2(a+bx) \sec^3(a+bx) \, dx$$

Mathematica [A]
time = 27.77, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^2(a+bx) \sec^3(a+bx) \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3, x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\csc^2(bx+a)\right) \left(\sec^3(bx+a)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x)
[Out] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
[Out] integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a)**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^2),x)
[Out] int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^2), x)
```

3.317 $\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx) \, dx$

Optimal. Leaf size=486

$$-\frac{6id^2(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^3}-\frac{3i(c+dx)^3\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}-\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3(c+dx)^3\csc}{2b}$$

```
[Out] -3*I*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b-9/2*I*d*(d*x+c)^2*polylog(2,I*exp(I
*(b*x+a)))/b^2-6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^2-3/2*(d*x+c)^3*csc(
b*x+a)/b+9/2*I*d*(d*x+c)^2*polylog(2,-I*exp(I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)
*polylog(2,exp(I*(b*x+a)))/b^3+9*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4+6*I*
d^2*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^3-3*I*d^3*polylog(2,I*exp(I*(b*x+a
)))/b^4-9*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,-exp(I*(b*
x+a)))/b^4-9*d^2*(d*x+c)*polylog(3,-I*exp(I*(b*x+a)))/b^3+9*d^2*(d*x+c)*pol
ylog(3,I*exp(I*(b*x+a)))/b^3+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4-6*I*d^2*(d
*x+c)*arctan(exp(I*(b*x+a)))/b^3+3*I*d^3*polylog(2,-I*exp(I*(b*x+a)))/b^4-3
/2*d*(d*x+c)^2*sec(b*x+a)/b^2+1/2*(d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2/b
```

Rubi [A]

time = 0.86, antiderivative size = 486, normalized size of antiderivative = 1.00, number of steps used = 44, number of rules used = 19, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.792$, Rules used = {2701, 294, 327, 213, 4505, 6820, 12, 6874, 6408, 4266, 2611, 6744, 2320, 6724, 4268, 2702, 6873, 2317, 2438}

$$-\frac{6id^2(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^3}-\frac{3i(c+dx)^3\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{3id^3\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{Li}_3\left(e^{i(a+bx)}\right)}{b^4}-\frac{9id^3\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^4}+\frac{9id^3\text{Li}_4\left(ie^{i(a+bx)}\right)}{b^4}+\frac{6id^2(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}-\frac{9d^2(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{9d^2(c+dx)\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{9id(c+dx)^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2}-\frac{9id(c+dx)^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{2b^2}-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}-\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3(c+dx)^3\csc(a+bx)}{2b}+\frac{(c+dx)^3\csc(a+bx)\sec^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] ((-6*I)*d^2*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^3 - ((3*I)*(c + d*x)^3*Arc
Tan[E^(I*(a + b*x))])/b - (6*d*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^2 -
(3*(c + d*x)^3*Csc[a + b*x])/(2*b) + ((6*I)*d^2*(c + d*x)*PolyLog[2, -E^(I*
(a + b*x))])/b^3 + ((3*I)*d^3*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^4 + (((9*
I)/2)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d^3*Poly
Log[2, I*E^(I*(a + b*x))])/b^4 - (((9*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E^(I
*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^3 -
 (6*d^3*PolyLog[3, -E^(I*(a + b*x))])/b^4 - (9*d^2*(c + d*x)*PolyLog[3, (-I
)*E^(I*(a + b*x))])/b^3 + (9*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b
^3 + (6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4 - ((9*I)*d^3*PolyLog[4, (-I)*E
^(I*(a + b*x))])/b^4 + ((9*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4 - (3*d
*(c + d*x)^2*Sec[a + b*x])/(2*b^2) + ((c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]
^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
```

```
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx) \, dx = \frac{3(c+dx)^3 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3 \csc(a+bx)}{2b} + \frac{(c+}{}$$

$$= \frac{3(c+dx)^3 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3 \csc(a+bx)}{2b} + \frac{(c+}{}$$

$$= \frac{3(c+dx)^3 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3 \csc(a+bx)}{2b} + \frac{(c+}{}$$

$$= \frac{3(c+dx)^3 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3 \csc(a+bx)}{2b} + \frac{(c+}{}$$

$$= \frac{3(c+dx)^3 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3 \csc(a+bx)}{2b} + \frac{(c+}{}$$

$$= \frac{3d(c+dx)^2 \tanh^{-1}(\cos(a+bx))}{2b^2} + \frac{3(c+dx)^3 \tanh^{-1}(\sin(a+bx}{2b}$$

$$= \frac{3d(c+dx)^2 \tanh^{-1}(\cos(a+bx))}{2b^2} + \frac{3(c+dx)^3 \tanh^{-1}(\sin(a+bx}{2b}$$

$$= -\frac{9d(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{3d(c+dx)^2 \tanh^{-1}(\cos(a+bx)}{2b^2}$$

$$= -\frac{9d(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{3d(c+dx)^2 \tanh^{-1}(\cos(a+bx)}{2b^2}$$

$$= -\frac{3i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{9d(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + 3$$

$$= -\frac{6id^2(c+dx) \tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - 9$$

$$= -\frac{6id^2(c+dx) \tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - 9$$

$$= -\frac{6id^2(c+dx) \tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - 6$$

$$= -\frac{6id^2(c+dx) \tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - 6$$

$$= -\frac{6id^2(c+dx) \tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - 6$$

$$= -\frac{6id^2(c+dx) \tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - 6$$

Mathematica [A]
time = 7.36, size = 881, normalized size = 1.81

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] (3*d*(-2*b^2*c^2*ArcTanh[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - E^(I*(a + b
*x))] + b^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1 + E^(I*(a
+ b*x))] - b^2*d^2*x^2*Log[1 + E^(I*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyL
og[2, -E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] -
 2*d^2*PolyLog[3, -E^(I*(a + b*x))] + 2*d^2*PolyLog[3, E^(I*(a + b*x))]))/b
^4 - (3*((2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + (4*I)*b*c*d^2*ArcTan[E^(I*
(a + b*x))] - 3*b^3*c^2*d*x*Log[1 - I*E^(I*(a + b*x))] - 2*b*d^3*x*Log[1 -
I*E^(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - b^3*d^3*x
^3*Log[1 - I*E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] +
2*b*d^3*x*Log[1 + I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a +
b*x))] + b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x))] - I*d*(2*d^2 + 3*b^2*(c + d
*x)^2)*PolyLog[2, (-I)*E^(I*(a + b*x))] + I*d*(2*d^2 + 3*b^2*(c + d*x)^2)*P
olyLog[2, I*E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] +
 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(
a + b*x))] - 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4,
 (-I)*E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))]))/(2*b^4)
- (Csc[a + b*x]*Sec[a + b*x]^2*(b*c^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2 + b*d^3
*x^3 + 3*b*c^3*Cos[2*a + 2*b*x] + 9*b*c^2*d*x*Cos[2*a + 2*b*x] + 9*b*c*d^2*
x^2*Cos[2*a + 2*b*x] + 3*b*d^3*x^3*Cos[2*a + 2*b*x] + 3*c^2*d*Sin[2*a + 2*b
*x] + 6*c*d^2*x*Sin[2*a + 2*b*x] + 3*d^3*x^2*Sin[2*a + 2*b*x]))/(4*b^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1628 vs. $2(428) = 856$.
time = 0.61, size = 1629, normalized size = 3.35

method	result	size
risch	Expression too large to display	1629

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -9/2*I/b^2*c^2*d*polylog(2,I*exp(I*(b*x+a)))+3*I/b^4*a^3*d^3*arctan(exp(I*(
b*x+a)))-9/2*I/b^2*d^3*polylog(2,I*exp(I*(b*x+a)))*x^2+9/2*I/b^2*d^3*polylo
g(2,-I*exp(I*(b*x+a)))*x^2-6*I/b^3*c*d^2*arctan(exp(I*(b*x+a)))+6*I/b^4*d^3
*a*arctan(exp(I*(b*x+a)))+6/b^3*d^3*ln(1-exp(I*(b*x+a)))*a*x+3*I*d^3*polylo
g(2,-I*exp(I*(b*x+a)))/b^4-6*d^2/b^2*c*ln(exp(I*(b*x+a))+1)*x+6*I*d^3/b^3*p
olylog(2,-exp(I*(b*x+a)))*x-6*I*d^3/b^3*polylog(2,exp(I*(b*x+a)))*x-6*I/b^4
*d^3*a*dilog(exp(I*(b*x+a)))+6*I/b^3*c*d^2*dilog(exp(I*(b*x+a)))+6*I/b^4*d^
3*polylog(2,-exp(I*(b*x+a)))*a-6*I/b^4*d^3*polylog(2,exp(I*(b*x+a)))*a+6*I/
b^3*c*d^2*dilog(exp(I*(b*x+a))+1)-6*I/b^4*a*d^3*dilog(exp(I*(b*x+a))+1)+6*d
^3*polylog(3,exp(I*(b*x+a)))/b^4-3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4-6*
d^2/b^3*c*a*ln(exp(I*(b*x+a))-1)+9*I/b^2*c*d^2*polylog(2,-I*exp(I*(b*x+a)))
```

```
*x-9*I/b^2*c*d^2*polylog(2,I*exp(I*(b*x+a)))*x+9*I/b^2*a*c^2*d*arctan(exp(I
*(b*x+a)))-9*I/b^3*a^2*c*d^2*arctan(exp(I*(b*x+a)))+9/2*I/b^2*c^2*d*polylog
(2,-I*exp(I*(b*x+a)))-9/b^3*d^3*polylog(3,-I*exp(I*(b*x+a)))*x+9/b^3*d^3*po
lylog(3,I*exp(I*(b*x+a)))*x+9/2/b^3*a^2*d^2*c*ln(1+I*exp(I*(b*x+a)))+9/2/b*
c^2*d*ln(1-I*exp(I*(b*x+a)))*x+9/2/b^2*c^2*d*ln(1-I*exp(I*(b*x+a)))*a-9/2/b
^3*a^2*d^2*c*ln(1-I*exp(I*(b*x+a)))-9/2/b*d^2*c*ln(1+I*exp(I*(b*x+a)))*x^2+
9/2/b*d^2*c*ln(1-I*exp(I*(b*x+a)))*x^2-9/2/b*c^2*d*ln(1+I*exp(I*(b*x+a)))*x
-9/2/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a+3*d/b^2*c^2*ln(exp(I*(b*x+a))-1)-3*
d/b^2*c^2*ln(exp(I*(b*x+a))+1)+3*d^3/b^4*a^2*ln(exp(I*(b*x+a))-1)+3*d^3/b^2
*ln(1-exp(I*(b*x+a)))*x^2+3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2-3*d^3/b^2*ln(e
xp(I*(b*x+a))+1)*x^2-3*I/b*c^3*arctan(exp(I*(b*x+a)))+3/b^3*d^3*ln(1-I*exp(
I*(b*x+a)))*x+3/b^4*d^3*ln(1-I*exp(I*(b*x+a)))*a-3/b^3*d^3*ln(1+I*exp(I*(b*
x+a)))*x-3/b^4*d^3*ln(1+I*exp(I*(b*x+a)))*a-9*I*d^3*polylog(4,-I*exp(I*(b*x
+a)))/b^4-I/b^2/(1+exp(2*I*(b*x+a)))^2/(exp(2*I*(b*x+a))-1)*(3*d^3*x^3*b*ex
p(5*I*(b*x+a))+9*c*d^2*x^2*b*exp(5*I*(b*x+a))+9*c^2*d*x*b*exp(5*I*(b*x+a))+
2*d^3*x^3*b*exp(3*I*(b*x+a))+3*c^3*b*exp(5*I*(b*x+a))+6*c*d^2*x^2*b*exp(3*I
*(b*x+a))+3*I*d^3*x^2*exp(I*(b*x+a))+6*c^2*d*x*b*exp(3*I*(b*x+a))+3*d^3*x^3
*b*exp(I*(b*x+a))-3*I*c^2*d*exp(5*I*(b*x+a))+2*c^3*b*exp(3*I*(b*x+a))+9*c*d
^2*x^2*b*exp(I*(b*x+a))-3*I*d^3*x^2*exp(5*I*(b*x+a))+9*c^2*d*x*b*exp(I*(b*x
+a))+3*c^3*b*exp(I*(b*x+a))+3*I*c^2*d*exp(I*(b*x+a))+6*I*c*d^2*x*exp(I*(b*x
+a))-6*I*c*d^2*x*exp(5*I*(b*x+a)))+3/2/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3-3/2
/b*d^3*ln(1+I*exp(I*(b*x+a)))*x^3+3/2/b^4*a^3*d^3*ln(1-I*exp(I*(b*x+a)))-3/
2/b^4*a^3*d^3*ln(1+I*exp(I*(b*x+a)))-9/b^3*d^2*c*polylog(3,-I*exp(I*(b*x+a)
))+9/b^3*d^2*c*polylog(3,I*exp(I*(b*x+a)))-6*d^3*polylog(3,-exp(I*(b*x+a)))
/b^4+9*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8046 vs. $2(404) = 808$.
time = 4.71, size = 8046, normalized size = 16.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(c^3*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log
(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1)) - 3*a*c^2*d*(2*(3*sin(b*x + a
)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(
sin(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^
3 - sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b^2
- a^3*d^3*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log
(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b^3 - 4*(6*((b*x + a)^3*d^3 +
 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a
*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a) - ((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*
d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2
```

+ 2)*d^3)*(b*x + a))*cos(6*b*x + 6*a) - ((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a
d^3 + 3(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^
2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + ((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*
a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a
^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (I*(b*x + a)^3*d^3 + 2*I*b*c*d^2
 - 2*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a
*b*c*d^2 + (3*I*a^2 + 2*I)*d^3)*(b*x + a))*sin(6*b*x + 6*a) - (I*(b*x + a)^
3*d^3 + 2*I*b*c*d^2 - 2*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + (3*
I*b^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 + 2*I)*d^3)*(b*x + a))*sin(4*b*x + 4
a) - (-I(b*x + a)^3*d^3 - 2*I*b*c*d^2 + 2*I*a*d^3 + 3*(-I*b*c*d^2 + I*a*d
^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 + (-3*I*a^2 - 2*I)*d^3)*(
b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + 6*((b
*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*
b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a) - ((b*x + a)^3*d^3 + 2
*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b
*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(6*b*x + 6*a) - ((b*x + a)^3*d^3 +
2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*
b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + ((b*x + a)^3*d^3 +
 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a
*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (I*(b*x + a)^3*d^
3 + 2*I*b*c*d^2 - 2*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + (3*I*b^
2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 + 2*I)*d^3)*(b*x + a))*sin(6*b*x + 6*a)
- (I*(b*x + a)^3*d^3 + 2*I*b*c*d^2 - 2*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b
*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 + 2*I)*d^3)*(b*x + a)
)*sin(4*b*x + 4*a) - (-I*(b*x + a)^3*d^3 - 2*I*b*c*d^2 + 2*I*a*d^3 + 3*(-I*
b*c*d^2 + I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 + (-3*I*a^
2 - 2*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x +
 a) + 1) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c
*d^2 - a*d^3)*(b*x + a) - (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*
d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6*a) - (b^2*c^2*d - 2*a*b*
c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*
x + 4*a) + (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^
2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(
b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(6*b*x +
 6*a) - (I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I
*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (-I*b^2*c^2*d + 2*I*a*b*c
d^2 - I(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*
sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 12*(b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3 - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(6*b*x + 6*
a) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(4*b*x + 4*a) + (b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*cos(2*b*x + 2*a) + (-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a
^2*d^3)*sin(6*b*x + 6*a) + (-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*sin(4
*b*x + 4*a) + (I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*sin(2*b*x + 2*a))*a
rctan2(sin(b*x + a), cos(b*x + a) - 1) + 12*((b*x + a)^2*d^3 + 2*(b*c*d^2 -
 a*d^3)*(b*x + a) - ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6

```
*b*x + 6*a) - ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x +
 4*a) + ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a)
- (I*(b*x + a)^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a)
- (I*(b*x + a)^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a)
- (-I*(b*x + a)^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a
))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 12*((b*x + a)^3*d^3 - I*b^2*c
^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3 + (3*b*c*d^2 - (3*a + I)*d^3)*(b*x + a)^2
+ (3*b^2*c^2*d - 2*(3*a + I)*b*c*d^2 + (3*a^2 + 2*I*a)*d^3)*(b*x + a))*cos(
5*b*x + 5*a) - 8*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^
2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(3*b*x + 3*a) - 12*((b*x + a
)^3*d^3 + I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3 + (3*b*c*d^2 - (3*a - I)*
d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 2*(3*a - I)*b...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2226 vs. $2(404) = 808$.
time = 3.46, size = 2226, normalized size = 4.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 18*I*d^3*cos(b*x + a
)^2*polylog(4, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + 18*I*d^3*cos(b
*x + a)^2*polylog(4, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 18*I*d^3
*cos(b*x + a)^2*polylog(4, -I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 1
8*I*d^3*cos(b*x + a)^2*polylog(4, -I*cos(b*x + a) - sin(b*x + a))*sin(b*x +
 a) + 12*d^3*cos(b*x + a)^2*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b
*x + a) + 12*d^3*cos(b*x + a)^2*polylog(3, cos(b*x + a) - I*sin(b*x + a))*s
in(b*x + a) - 12*d^3*cos(b*x + a)^2*polylog(3, -cos(b*x + a) + I*sin(b*x +
a))*sin(b*x + a) - 12*d^3*cos(b*x + a)^2*polylog(3, -cos(b*x + a) - I*sin(b
*x + a))*sin(b*x + a) + 2*b^3*c^3 - 12*(I*b*d^3*x + I*b*c*d^2)*cos(b*x + a)
^2*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 12*(-I*b*d^3*x - I*b
*c*d^2)*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) -
3*(3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a
)^2*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 3*(3*I*b^2*d^3*x^2
+ 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x
 + a) - sin(b*x + a))*sin(b*x + a) - 3*(-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x
- 3*I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x +
 a))*sin(b*x + a) - 3*(-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d -
 2*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a)
 - 12*(I*b*d^3*x + I*b*c*d^2)*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*sin(b*
x + a))*sin(b*x + a) - 12*(-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a)^2*dilog(-co
s(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
+ b^2*c^2*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x
```

```
+ a) + 3*(b^3*c^3 - 3*a*b^2*c^2*d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*
cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) - 6*(b^2
*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*s
in(b*x + a) + 1)*sin(b*x + a) - 3*(b^3*c^3 - 3*a*b^2*c^2*d + (3*a^2 + 2)*b*
c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) +
 I)*sin(b*x + a) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2*log(
I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^3*d^3*x^3 + 3*b^3*c*
d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d +
2*b*d^3)*x)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x +
 a) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a
^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x +
a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*
cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) + 6*(b^
2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) + 1/2
*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)
*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x +
 a) + 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)^
2*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^3*c^3 - 3*a*b
^2*c^2*d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a)^2*log(-cos(b
*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
 + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a)
 + 1)*sin(b*x + a) - 3*(b^3*c^3 - 3*a*b^2*c^2*d + (3*a^2 + 2)*b*c*d^2 - (a^
3 + 2*a)*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*
x + a) - 18*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) +
sin(b*x + a))*sin(b*x + a) + 18*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(
3, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 18*(b*d^3*x + b*c*d^2)*cos
(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + 18*(b
*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a))
*sin(b*x + a) - 6*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)
*cos(b*x + a)^2 - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*
sin(b*x + a))/(b^4*cos(b*x + a)^2*sin(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)^3*sin(a + b*x)^2),x)
```

```
[Out] \text{Hanged}
```

3.318 $\int (c+dx)^2 \csc^2(a+bx) \sec^3(a+bx) \, dx$

Optimal. Leaf size=341

$$-\frac{3i(c+dx)^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{2d^2x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{6d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{d^2x\tanh^{-1}(\cos(a+b$$

```
[Out] -3*I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b+2*d^2*x*arctanh(exp(I*(b*x+a)))/b^2
-6*d*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^2-d^2*x*arctanh(cos(b*x+a))/b^2+d*(d
*x+c)*arctanh(cos(b*x+a))/b^2+d^2*arctanh(sin(b*x+a))/b^3-3/2*(d*x+c)^2*csc
(b*x+a)/b+2*I*d^2*polylog(2,-exp(I*(b*x+a)))/b^3+3*I*d*(d*x+c)*polylog(2,-I
*exp(I*(b*x+a)))/b^2-3*I*d*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^2-2*I*d^2*
polylog(2,exp(I*(b*x+a)))/b^3-3*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+3*d^2*
polylog(3,I*exp(I*(b*x+a)))/b^3-d*(d*x+c)*sec(b*x+a)/b^2+1/2*(d*x+c)^2*csc(
b*x+a)*sec(b*x+a)^2/b
```

Rubi [A]

time = 0.44, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 19, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.792$, Rules used = {2701, 294, 327, 213, 4505, 6820, 12, 6874, 6408, 4266, 2611, 2320, 6724, 4268, 2317, 2438, 2702, 6406, 3855}

$$-\frac{3i(c+dx)^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{2id^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}-\frac{3d^2\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{3d^2\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3}+\frac{3id(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\sec(a+bx)}{b^2}-\frac{6d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{d(c+dx)\tanh^{-1}(\cos(a+bx))}{b^2}+\frac{2d^2x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{d^2x\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{3(c+dx)^2\csc(a+bx)}{2b}+\frac{(c+dx)^2\csc(a+bx)\sec^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] ((-3*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (2*d^2*x*ArcTanh[E^(I*(a +
 b*x))])/b^2 - (6*d*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^2 - (d^2*x*ArcTan
h[Cos[a + b*x]])/b^2 + (d*(c + d*x)*ArcTanh[Cos[a + b*x]])/b^2 + (d^2*ArcTa
nh[Sin[a + b*x]])/b^3 - (3*(c + d*x)^2*Csc[a + b*x])/(2*b) + ((2*I)*d^2*Pol
yLog[2, -E^(I*(a + b*x))])/b^3 + ((3*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a
 + b*x))])/b^2 - ((3*I)*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((
2*I)*d^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (3*d^2*PolyLog[3, (-I)*E^(I*(a
+ b*x))])/b^3 + (3*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - (d*(c + d*x)*Se
c[a + b*x])/b^2 + ((c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6406

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^2 \csc^2(a+bx) \sec^3(a+bx)\, dx = \frac{3(c+dx)^2 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2 \csc(a+bx)}{2b} + \frac{(c+\cdots}{\cdots}$$

$$= \frac{3(c+dx)^2 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2 \csc(a+bx)}{2b} + \frac{(c+\cdots}{\cdots}$$

$$= \frac{3(c+dx)^2 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2 \csc(a+bx)}{2b} + \frac{(c+\cdots}{\cdots}$$

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$$= \frac{3(c+dx)^2 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2 \csc(a+bx)}{2b} + \frac{(c+\cdots}{\cdots}$$

$$= \frac{d(c+dx) \tanh^{-1}(\cos(a+bx))}{b^2} + \frac{3(c+dx)^2 \tanh^{-1}(\sin(a+bx)\cdots}{2b}$$

$$= \frac{d(c+dx) \tanh^{-1}(\cos(a+bx))}{b^2} + \frac{3(c+dx)^2 \tanh^{-1}(\sin(a+bx)\cdots}{2b}$$

$$= -\frac{6d(c+dx) \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{d^2 x \tanh^{-1}(\cos(a+bx))}{b^2} + \frac{d(c\cdots}{\cdots}$$

$$= -\frac{6d(c+dx) \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{d^2 x \tanh^{-1}(\cos(a+bx))}{b^2} + \frac{d(c\cdots}{\cdots}$$

$$= -\frac{3i(c+dx)^2 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2d^2 x \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d(c+\cdots}{\cdots}$$

$$= -\frac{3i(c+dx)^2 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2d^2 x \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d(c+\cdots}{\cdots}$$

$$= -\frac{3i(c+dx)^2 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2d^2 x \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d(c+\cdots}{\cdots}$$

$$= -\frac{3i(c+dx)^2 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2d^2 x \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d(c+\cdots}{\cdots}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 889 vs. $2(341) = 682$.
time = 7.19, size = 889, normalized size = 2.61

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] -1/2*((6*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + (4*I)*d^2*ArcTan[E^(I*(a + b*
x))] - 6*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] - 3*b^2*d^2*x^2*Log[1 - I*E^(
```

```
I*(a + b*x))] + 6*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] + 3*b^2*d^2*x^2*Log[
1 + I*E^(I*(a + b*x))] - (6*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x)
)] + (6*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] + 6*d^2*PolyLog[3, (
-I)*E^(I*(a + b*x))] - 6*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((c + d*x
)*Csc[a]*Sec[a]*(b*c*Cos[a] + b*d*x*Cos[a] + d*Sin[a]))/b^2 + ((4*I)*c*d*Ar
cTan[(I*Cos[a] - I*Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/(b^2*Sq
rt[Cos[a]^2 + Sin[a]^2]) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c^2*Sin[(b*x)/2]
) - 2*c*d*x*Sin[(b*x)/2] - d^2*x^2*Sin[(b*x)/2]))/(2*b) + (Csc[a/2]*Csc[a/2
 + (b*x)/2]*(c^2*Sin[(b*x)/2] + 2*c*d*x*Sin[(b*x)/2] + d^2*x^2*Sin[(b*x)/2]
))/(2*b) + (c^2 + 2*c*d*x + d^2*x^2)/(4*b*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (
b*x)/2])^2) + (-(c*d*Sin[(b*x)/2]) - d^2*x*Sin[(b*x)/2])/(b^2*(Cos[a/2] - S
in[a/2])*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) + (-c^2 - 2*c*d*x - d^2
*x^2)/(4*b*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2])^2) + (c*d*Sin[(b*x)/2]
 + d^2*x*Sin[(b*x)/2])/(b^2*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin
[a/2 + (b*x)/2])) + (2*d^2*((-2*ArcTan[Tan[a]]*ArcTanh[(-Cos[a] + Sin[a]*Ta
n[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/Sqrt[Cos[a]^2 + Sin[a]^2] + (((b*x
+ ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(I*(b*
x + ArcTan[Tan[a]]))]) + I*(PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))] - Pol
yLog[2, E^(I*(b*x + ArcTan[Tan[a]]))]))*Sec[a])/Sqrt[1 + Tan[a]^2]))/b^3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 769 vs. $2(310) = 620$.
time = 0.34, size = 770, normalized size = 2.26

method	result
risch	$-\frac{i\left(3d^2x^2b\,e^{5i(bx+a)}+6cdxb\,e^{5i(bx+a)}+3c^2b\,e^{5i(bx+a)}+2d^2x^2b\,e^{3i(bx+a)}+4cdxb\,e^{3i(bx+a)}-2id^2x\,e^{5i(bx+a)}+2c^2b\,e^{3i(bx+a)}+3d^2x^2b\,e^{\cdots}\right.}{b^2\left(1+e^{2i(bx+a)}\right)^2\left(e^{2i(bx+a)}-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3*I/b^2*d^2*polylog(2,I*exp(I*(b*x+a)))*x+3*I/b^2*d^2*polylog(2,-I*exp(I*(
b*x+a)))*x+3*I/b^2*c*d*polylog(2,-I*exp(I*(b*x+a)))-3*I/b^2*c*d*polylog(2,I
*exp(I*(b*x+a)))-3*I/b^3*a^2*d^2*arctan(exp(I*(b*x+a)))+6*I/b^2*c*d*a*arcta
n(exp(I*(b*x+a)))-I/b^2/(1+exp(2*I*(b*x+a)))^2/(exp(2*I*(b*x+a))-1)*(3*d^2*
x^2*b*exp(5*I*(b*x+a))+6*c*d*x*b*exp(5*I*(b*x+a))+3*c^2*b*exp(5*I*(b*x+a))+
2*d^2*x^2*b*exp(3*I*(b*x+a))+4*c*d*x*b*exp(3*I*(b*x+a))-2*I*d^2*x*exp(5*I*(
b*x+a))+2*c^2*b*exp(3*I*(b*x+a))+3*d^2*x^2*b*exp(I*(b*x+a))-2*I*c*d*exp(5*I
*(b*x+a))+6*c*d*x*b*exp(I*(b*x+a))+3*c^2*b*exp(I*(b*x+a))+2*I*d^2*x*exp(I*(
b*x+a))+2*I*d*c*exp(I*(b*x+a)))-2*d^2/b^2*ln(exp(I*(b*x+a))+1)*x-3*d^2*poly
log(3,-I*exp(I*(b*x+a)))/b^3+3*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3+2*I/b^3*
d^2*dilog(exp(I*(b*x+a))+1)+2*I/b^3*d^2*dilog(exp(I*(b*x+a)))+2*d/b^2*c*ln(
exp(I*(b*x+a))-1)-2*d/b^2*c*ln(exp(I*(b*x+a))+1)-2*d^2/b^3*a*ln(exp(I*(b*x+
a))-1)-3/b^2*c*d*ln(1+I*exp(I*(b*x+a)))*a+3/b*c*d*ln(1-I*exp(I*(b*x+a)))*x-
3/b*c*d*ln(1+I*exp(I*(b*x+a)))*x+3/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a-3/2/b^3
```

```
*a^2*d^2*ln(1-I*exp(I*(b*x+a)))-3/2/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^2+3/2/b^
3*a^2*d^2*ln(1+I*exp(I*(b*x+a)))+3/2/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2-2*I/b
^3*d^2*arctan(exp(I*(b*x+a)))-3*I/b*c^2*arctan(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3828 vs. $2(297) = 594$.
time = 1.27, size = 3828, normalized size = 11.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(c^2*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log
(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1)) - 2*a*c*d*(2*(3*sin(b*x + a)^
2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(si
n(b*x + a) - 1))/b + a^2*d^2*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - si
n(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b^2 - 4*(2
*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 - (3*(b*x + a)^2*
d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(6*b*x + 6*a) - (3*(b*x + a)^
2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) + (3*(b*x + a
)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) - (3*I*(b*x
 + a)^2*d^2 + 6*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(6*b*x + 6*a) -
 (3*I*(b*x + a)^2*d^2 + 6*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(4*b*
x + 4*a) - (-3*I*(b*x + a)^2*d^2 + 6*(-I*b*c*d + I*a*d^2)*(b*x + a) - 2*I*d
^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + 2*(3*(b*x +
 a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 - (3*(b*x + a)^2*d^2 + 6*(b
*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(6*b*x + 6*a) - (3*(b*x + a)^2*d^2 + 6*
(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) + (3*(b*x + a)^2*d^2 +
6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) - (3*I*(b*x + a)^2*d^
2 + 6*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(6*b*x + 6*a) - (3*I*(b*x
 + a)^2*d^2 + 6*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(4*b*x + 4*a) -
 (-3*I*(b*x + a)^2*d^2 + 6*(-I*b*c*d + I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(2*
b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) + 8*(b*c*d + (b*x + a)
*d^2 - a*d^2 - (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - (b*c*d +
(b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + (b*c*d + (b*x + a)*d^2 - a*d^2)*c
os(2*b*x + 2*a) - (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(6*b*x + 6*a) -
(I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(4*b*x + 4*a) - (-I*b*c*d - I*(b*x
 + a)*d^2 + I*a*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) +
 1) - 8*(b*c*d - a*d^2 - (b*c*d - a*d^2)*cos(6*b*x + 6*a) - (b*c*d - a*d^2)
*cos(4*b*x + 4*a) + (b*c*d - a*d^2)*cos(2*b*x + 2*a) + (-I*b*c*d + I*a*d^2)
*sin(6*b*x + 6*a) + (-I*b*c*d + I*a*d^2)*sin(4*b*x + 4*a) + (I*b*c*d - I*a*
d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) - 8*((b*x +
a)*d^2*cos(6*b*x + 6*a) + (b*x + a)*d^2*cos(4*b*x + 4*a) - (b*x + a)*d^2*co
s(2*b*x + 2*a) + I*(b*x + a)*d^2*sin(6*b*x + 6*a) + I*(b*x + a)*d^2*sin(4*b
```

```
*x + 4*a) - I*(b*x + a)*d^2*sin(2*b*x + 2*a) - (b*x + a)*d^2)*arctan2(sin(b
*x + a), -cos(b*x + a) + 1) - 4*(3*(b*x + a)^2*d^2 - 2*I*b*c*d + 2*I*a*d^2
+ 2*(3*b*c*d - (3*a + I)*d^2)*(b*x + a))*cos(5*b*x + 5*a) - 8*((b*x + a)^2*
d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(3*b*x + 3*a) - 4*(3*(b*x + a)^2*d^2
+ 2*I*b*c*d - 2*I*a*d^2 + 2*(3*b*c*d - (3*a - I)*d^2)*(b*x + a))*cos(b*x +
a) + 12*(b*c*d + (b*x + a)*d^2 - a*d^2 - (b*c*d + (b*x + a)*d^2 - a*d^2)*co
s(6*b*x + 6*a) - (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + (b*c*d
+ (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (I*b*c*d + I*(b*x + a)*d^2 - I*
a*d^2)*sin(6*b*x + 6*a) - (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(4*b*x +
 4*a) - (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(2*b*x + 2*a))*dilog(I*e^
(I*b*x + I*a)) - 12*(b*c*d + (b*x + a)*d^2 - a*d^2 - (b*c*d + (b*x + a)*d^2
 - a*d^2)*cos(6*b*x + 6*a) - (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*
a) + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (-I*b*c*d - I*(b*x
+ a)*d^2 + I*a*d^2)*sin(6*b*x + 6*a) + (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^
2)*sin(4*b*x + 4*a) + (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(2*b*x + 2*a
))*dilog(-I*e^(I*b*x + I*a)) + 8*(d^2*cos(6*b*x + 6*a) + d^2*cos(4*b*x + 4*
a) - d^2*cos(2*b*x + 2*a) + I*d^2*sin(6*b*x + 6*a) + I*d^2*sin(4*b*x + 4*a)
 - I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(-e^(I*b*x + I*a)) - 8*(d^2*cos(6*b*x
 + 6*a) + d^2*cos(4*b*x + 4*a) - d^2*cos(2*b*x + 2*a) + I*d^2*sin(6*b*x + 6
*a) + I*d^2*sin(4*b*x + 4*a) - I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(e^(I*b*x
 + I*a)) - 4*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2 + (-I*b*c*d - I*(b*x + a)
*d^2 + I*a*d^2)*cos(6*b*x + 6*a) + (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*c
os(4*b*x + 4*a) + (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*cos(2*b*x + 2*a) +
(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(6*b*x + 6*a) + (b*c*d + (b*x + a)*d^2 -
 a*d^2)*sin(4*b*x + 4*a) - (b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a)
)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - 4*(-I*b*c*d -
 I*(b*x + a)*d^2 + I*a*d^2 + (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*cos(6*b*
x + 6*a) + (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*cos(4*b*x + 4*a) + (-I*b*c
*d - I*(b*x + a)*d^2 + I*a*d^2)*cos(2*b*x + 2*a) - (b*c*d + (b*x + a)*d^2 -
 a*d^2)*sin(6*b*x + 6*a) - (b*c*d + (b*x + a)*d^2 - a*d^2)*sin(4*b*x + 4*a)
 + (b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + s
in(b*x + a)^2 - 2*cos(b*x + a) + 1) + (3*I*(b*x + a)^2*d^2 - 6*(-I*b*c*d +
I*a*d^2)*(b*x + a) + 2*I*d^2 + (-3*I*(b*x + a)^2*d^2 - 6*(I*b*c*d - I*a*d^2
)*(b*x + a) - 2*I*d^2)*cos(6*b*x + 6*a) + (-3*I*(b*x + a)^2*d^2 - 6*(I*b*c*
d - I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(4*b*x + 4...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1366 vs. $2(297) = 594$.
time = 2.82, size = 1366, normalized size = 4.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*d^2*x^2 - 4*I*d^2*cos(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x
+ a))*sin(b*x + a) + 4*I*d^2*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x
+ a))*sin(b*x + a) - 4*I*d^2*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*sin(b*x
 + a))*sin(b*x + a) + 4*I*d^2*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*sin(b*
x + a))*sin(b*x + a) - 6*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin
(b*x + a))*sin(b*x + a) + 6*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) -
sin(b*x + a))*sin(b*x + a) - 6*d^2*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a
) + sin(b*x + a))*sin(b*x + a) + 6*d^2*cos(b*x + a)^2*polylog(3, -I*cos(b*x
 + a) - sin(b*x + a))*sin(b*x + a) + 4*b^2*c*d*x - 6*(I*b*d^2*x + I*b*c*d)*
cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 6*(I*b*d
^2*x + I*b*c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x
 + a) - 6*(-I*b*d^2*x - I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin
(b*x + a))*sin(b*x + a) - 6*(-I*b*d^2*x - I*b*c*d)*cos(b*x + a)^2*dilog(-I*
cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 4*(b*d^2*x + b*c*d)*cos(b*x + a
)^2*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + (3*b^2*c^2 - 6*a*
b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) +
 I)*sin(b*x + a) - 4*(b*d^2*x + b*c*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*
sin(b*x + a) + 1)*sin(b*x + a) - (3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*
cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 3*(b^2
*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x
+ a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*
b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1)*sin(
b*x + a) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)
^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^2*x^2 +
2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin
(b*x + a) + 1)*sin(b*x + a) + 4*(b*c*d - a*d^2)*cos(b*x + a)^2*log(-1/2*cos
(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 4*(b*c*d - a*d^2)*cos(
b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) +
 4*(b*d^2*x + a*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + 1)
*sin(b*x + a) + (3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a)^2*lo
g(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 4*(b*d^2*x + a*d^2)*co
s(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - (3*b^2*
c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin
(b*x + a) + I)*sin(b*x + a) + 2*b^2*c^2 - 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2)*cos(b*x + a)^2 - 4*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3
*cos(b*x + a)^2*sin(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^2*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)^3*sin(a + b*x)^2),x)
[Out] \text{Hanged}
```

3.319 $\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=162

$$-\frac{3idx\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}+\frac{3c\tanh^{-1}(\sin(a+bx))}{2b}-\frac{3(c+dx)\csc(a+bx)}{2b}+\frac{3id\text{PolyI}}{}$$

```
[Out] -3*I*d*x*arctan(exp(I*(b*x+a)))/b-d*arctanh(cos(b*x+a))/b^2+3/2*c*arctanh(s
in(b*x+a))/b-3/2*(d*x+c)*csc(b*x+a)/b+3/2*I*d*polylog(2,-I*exp(I*(b*x+a)))/
b^2-3/2*I*d*polylog(2,I*exp(I*(b*x+a)))/b^2-1/2*d*sec(b*x+a)/b^2+1/2*(d*x+c
)*csc(b*x+a)*sec(b*x+a)^2/b
```

Rubi [A]
time = 0.12, antiderivative size = 182, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2701, 294, 327, 213, 4505, 6406, 12, 4266, 2317, 2438, 3855, 2702}

$$-\frac{3idx\text{ArcTan}\left(e^{i(a+bx)}\right)}{b}+\frac{3id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2}-\frac{3id\text{Li}_2\left(ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{3(c+dx)\csc(a+bx)}{2b}+\frac{3(c+dx)\tanh^{-1}(\sin(a+bx))}{2b}+\frac{(c+dx)\csc(a+bx)\sec^2(a+bx)}{2b}-\frac{3dx\tanh^{-1}(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] ((-3*I)*d*x*ArcTan[E^(I*(a + b*x))])/b - (d*ArcTanh[Cos[a + b*x]])/b^2 - (3
*d*x*ArcTanh[Sin[a + b*x]])/(2*b) + (3*(c + d*x)*ArcTanh[Sin[a + b*x]])/(2*
b) - (3*(c + d*x)*Csc[a + b*x])/(2*b) + (((3*I)/2)*d*PolyLog[2, (-I)*E^(I*(
a + b*x))])/b^2 - (((3*I)/2)*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (d*Sec[
a + b*x])/(2*b^2) + ((c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6406

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx = \frac{3(c+dx)\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)\csc(a+bx)}{2b} + \frac{(c+dx \ldots}{}$$

$$= \frac{3(c+dx)\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)\csc(a+bx)}{2b} + \frac{(c+dx \ldots}{}$$

$$= -\frac{3d\tanh^{-1}(\cos(a+bx))}{2b^2} - \frac{3dx\tanh^{-1}(\sin(a+bx))}{2b} + \frac{3(c+dx \ldots}{}$$

$$= -\frac{3d\tanh^{-1}(\cos(a+bx))}{2b^2} - \frac{3dx\tanh^{-1}(\sin(a+bx))}{2b} + \frac{3(c+dx \ldots}{}$$

$$= -\frac{3idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{3dx\tanh^{-1}(\mathrm{si} \ldots}{2b}$$

$$= -\frac{3idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{3dx\tanh^{-1}(\mathrm{si} \ldots}{2b}$$

$$= -\frac{3idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{3dx\tanh^{-1}(\mathrm{si} \ldots}{2b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.68, size = 660, normalized size = 4.07

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] (d*(a*Cos[(a + b*x)/2] - (a + b*x)*Cos[(a + b*x)/2])*Csc[(a + b*x)/2])/(2*b
^2) - (c*Csc[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, Sin[a + b*x]^2])/b -
```

```
(d*Log[Cos[(a + b*x)/2]])/b^2 + (d*Log[Sin[(a + b*x)/2]])/b^2 - (3*d*x*(a*(
Log[1 - Tan[(a + b*x)/2]] - Log[1 + Tan[(a + b*x)/2]]) + I*(Log[1 + I*Tan[(
a + b*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(a + b*x)/2])] - Log[1 - I*Tan[(a +
 b*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(a + b*x)/2])] - Log[1 + I*Tan[(a + b*
x)/2]]*Log[(1/2 - I/2)*(1 + Tan[(a + b*x)/2])] + Log[1 - I*Tan[(a + b*x)/2]
]*Log[(1/2 + I/2)*(1 + Tan[(a + b*x)/2])] - PolyLog[2, ((1 + I) - (1 - I)*T
an[(a + b*x)/2])/2] + PolyLog[2, (-1/2 - I/2)*(I + Tan[(a + b*x)/2])] - Pol
yLog[2, ((1 + I) + (1 - I)*Tan[(a + b*x)/2])/2] + PolyLog[2, ((1 - I) + (1
+ I)*Tan[(a + b*x)/2])/2])))/(2*b*(a - I*Log[1 - I*Tan[(a + b*x)/2]] + I*Lo
g[1 + I*Tan[(a + b*x)/2]])) + (d*x)/(4*b*(Cos[(a + b*x)/2] - Sin[(a + b*x)/
2])^2) - (d*Sin[(a + b*x)/2])/(2*b^2*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2]))
 - (d*x)/(4*b*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])^2) + (d*Sin[(a + b*x)/2
])/(2*b^2*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])) + (d*Sec[(a + b*x)/2]*(a*S
in[(a + b*x)/2] - (a + b*x)*Sin[(a + b*x)/2]))/(2*b^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(139) = 278$.
time = 0.17, size = 344, normalized size = 2.12

method	result
risch	$-\frac{i\left(3dxb\,e^{5i(bx+a)}+3cb\,e^{5i(bx+a)}+2dxb\,e^{3i(bx+a)}+2cb\,e^{3i(bx+a)}-id\,e^{5i(bx+a)}+3dxb\,e^{i(bx+a)}+3cb\,e^{i(bx+a)}+id\,e^{i(bx+a)}\right)}{b^{2}\left(1+e^{2i(bx+a)}\right)^{2}\left(e^{2i(bx+a)}-1\right)}-\frac{3ic\,\mathrm{arcta}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -I/b^2/(1+exp(2*I*(b*x+a)))^2/(exp(2*I*(b*x+a))-1)*(3*d*x*b*exp(5*I*(b*x+a)
)+3*c*b*exp(5*I*(b*x+a))+2*d*x*b*exp(3*I*(b*x+a))+2*c*b*exp(3*I*(b*x+a))-I*
d*exp(5*I*(b*x+a))+3*d*x*b*exp(I*(b*x+a))+3*c*b*exp(I*(b*x+a))+I*d*exp(I*(b
*x+a)))-3*I/b*c*arctan(exp(I*(b*x+a)))+3*I/b^2*d*a*arctan(exp(I*(b*x+a)))+d
/b^2*ln(exp(I*(b*x+a))-1)-d/b^2*ln(exp(I*(b*x+a))+1)+3/2/b*d*ln(1-I*exp(I*(
b*x+a)))*x+3/2/b^2*d*ln(1-I*exp(I*(b*x+a)))*a-3/2/b*d*ln(1+I*exp(I*(b*x+a))
)*x-3/2/b^2*d*ln(1+I*exp(I*(b*x+a)))*a-3/2*I/b^2*d*dilog(1-I*exp(I*(b*x+a))
)+3/2*I/b^2*d*dilog(1+I*exp(I*(b*x+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(8*(b*d*x + b*c)*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) - 4*(d*cos(5*b*x + 5
*a) - d*cos(b*x + a) - 3*(b*d*x + b*c)*sin(5*b*x + 5*a) - 2*(b*d*x + b*c)*s
```

```
in(3*b*x + 3*a) - 3*(b*d*x + b*c)*sin(b*x + a))*cos(6*b*x + 6*a) - 4*(d*cos
(4*b*x + 4*a) - d*cos(2*b*x + 2*a) + 3*(b*d*x + b*c)*sin(4*b*x + 4*a) - 3*(
b*d*x + b*c)*sin(2*b*x + 2*a) - d)*cos(5*b*x + 5*a) + 4*(d*cos(b*x + a) + 2
*(b*d*x + b*c)*sin(3*b*x + 3*a) + 3*(b*d*x + b*c)*sin(b*x + a))*cos(4*b*x +
 4*a) - 4*(d*cos(b*x + a) + 3*(b*d*x + b*c)*sin(b*x + a))*cos(2*b*x + 2*a)
 - 4*d*cos(b*x + a) + 12*(b^2*d*cos(6*b*x + 6*a)^2 + b^2*d*cos(4*b*x + 4*a)^
2 + b^2*d*cos(2*b*x + 2*a)^2 + b^2*d*sin(6*b*x + 6*a)^2 + b^2*d*sin(4*b*x +
 4*a)^2 - 2*b^2*d*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b^2*d*sin(2*b*x + 2*a
)^2 + 2*b^2*d*cos(2*b*x + 2*a) + b^2*d + 2*(b^2*d*cos(4*b*x + 4*a) - b^2*d*
cos(2*b*x + 2*a) - b^2*d)*cos(6*b*x + 6*a) - 2*(b^2*d*cos(2*b*x + 2*a) + b^
2*d)*cos(4*b*x + 4*a) + 2*(b^2*d*sin(4*b*x + 4*a) - b^2*d*sin(2*b*x + 2*a))
*sin(6*b*x + 6*a))*integrate((x*cos(2*b*x + 2*a)*cos(b*x + a) + x*sin(2*b*x
 + 2*a)*sin(b*x + a) + x*cos(b*x + a))/(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*
a)^2 + 2*cos(2*b*x + 2*a) + 1), x) + 3*(b*c*cos(6*b*x + 6*a)^2 + b*c*cos(4*
b*x + 4*a)^2 + b*c*cos(2*b*x + 2*a)^2 + b*c*sin(6*b*x + 6*a)^2 + b*c*sin(4*
b*x + 4*a)^2 - 2*b*c*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*c*sin(2*b*x + 2*
a)^2 + 2*b*c*cos(2*b*x + 2*a) + b*c + 2*(b*c*cos(4*b*x + 4*a) - b*c*cos(2*b
*x + 2*a) - b*c)*cos(6*b*x + 6*a) - 2*(b*c*cos(2*b*x + 2*a) + b*c)*cos(4*b*
x + 4*a) + 2*(b*c*sin(4*b*x + 4*a) - b*c*sin(2*b*x + 2*a))*sin(6*b*x + 6*a)
)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - 3*(b*c*cos(6*
b*x + 6*a)^2 + b*c*cos(4*b*x + 4*a)^2 + b*c*cos(2*b*x + 2*a)^2 + b*c*sin(6*
b*x + 6*a)^2 + b*c*sin(4*b*x + 4*a)^2 - 2*b*c*sin(4*b*x + 4*a)*sin(2*b*x +
2*a) + b*c*sin(2*b*x + 2*a)^2 + 2*b*c*cos(2*b*x + 2*a) + b*c + 2*(b*c*cos(4
*b*x + 4*a) - b*c*cos(2*b*x + 2*a) - b*c)*cos(6*b*x + 6*a) - 2*(b*c*cos(2*b
*x + 2*a) + b*c)*cos(4*b*x + 4*a) + 2*(b*c*sin(4*b*x + 4*a) - b*c*sin(2*b*x
 + 2*a))*sin(6*b*x + 6*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x
+ a) + 1) - 2*(d*cos(6*b*x + 6*a)^2 + d*cos(4*b*x + 4*a)^2 + d*cos(2*b*x +
2*a)^2 + d*sin(6*b*x + 6*a)^2 + d*sin(4*b*x + 4*a)^2 - 2*d*sin(4*b*x + 4*a)
*sin(2*b*x + 2*a) + d*sin(2*b*x + 2*a)^2 + 2*(d*cos(4*b*x + 4*a) - d*cos(2*
b*x + 2*a) - d)*cos(6*b*x + 6*a) - 2*(d*cos(2*b*x + 2*a) + d)*cos(4*b*x + 4
*a) + 2*d*cos(2*b*x + 2*a) + 2*(d*sin(4*b*x + 4*a) - d*sin(2*b*x + 2*a))*si
n(6*b*x + 6*a) + d)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x
)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 2*(d*cos(6*b*x + 6*a)^2 + d*cos(4*b*x
 + 4*a)^2 + d*cos(2*b*x + 2*a)^2 + d*sin(6*b*x + 6*a)^2 + d*sin(4*b*x + 4*a
)^2 - 2*d*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + d*sin(2*b*x + 2*a)^2 + 2*(d*c
os(4*b*x + 4*a) - d*cos(2*b*x + 2*a) - d)*cos(6*b*x + 6*a) - 2*(d*cos(2*b*x
 + 2*a) + d)*cos(4*b*x + 4*a) + 2*d*cos(2*b*x + 2*a) + 2*(d*sin(4*b*x + 4*a
) - d*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + d)*log(cos(b*x)^2 - 2*cos(b*x)*c
os(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) - 4*(3*(b*d*x
 + b*c)*cos(5*b*x + 5*a) + 2*(b*d*x + b*c)*cos(3*b*x + 3*a) + 3*(b*d*x + b*
c)*cos(b*x + a) + d*sin(5*b*x + 5*a) - d*sin(b*x + a))*sin(6*b*x + 6*a) - 4
*(3*b*d*x + 3*b*c - 3*(b*d*x + b*c)*cos(4*b*x + 4*a) + 3*(b*d*x + b*c)*cos(
2*b*x + 2*a) + d*sin(4*b*x + 4*a) - d*sin(2*b*x + 2*a))*sin(5*b*x + 5*a) -
4*(2*(b*d*x + b*c)*cos(3*b*x + 3*a) + 3*(b*d*x + b*c)*cos(b*x + a) - d*sin(
b*x + a))*sin(4*b*x + 4*a) - 8*(b*d*x + b*c + (b*d*x + b*c)*cos(2*b*x + 2*a
```

```
))*sin(3*b*x + 3*a) + 4*(3*(b*d*x + b*c)*cos(b*x + a) - d*sin(b*x + a))*sin
(2*b*x + 2*a) - 12*(b*d*x + b*c)*sin(b*x + a))/(b^2*cos(6*b*x + 6*a)^2 + b^
2*cos(4*b*x + 4*a)^2 + b^2*cos(2*b*x + 2*a)^2 + b^2*sin(6*b*x + 6*a)^2 + b^
2*sin(4*b*x + 4*a)^2 - 2*b^2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b^2*sin(2*
b*x + 2*a)^2 + 2*b^2*cos(2*b*x + 2*a) + b^2 + 2*(b^2*cos(4*b*x + 4*a) - b^2
*cos(2*b*x + 2*a) - b^2)*cos(6*b*x + 6*a) - 2*(b^2*cos(2*b*x + 2*a) + b^2)*
cos(4*b*x + 4*a) + 2*(b^2*sin(4*b*x + 4*a) - b^2*sin(2*b*x + 2*a))*sin(6*b*
x + 6*a))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(132) = 264$.
time = 2.56, size = 592, normalized size = 3.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(-3*I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a
) - 3*I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a)
+ 3*I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) +
 3*I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) +
3*(b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x
 + a) - 3*(b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I)
*sin(b*x + a) - 2*d*cos(b*x + a)^2*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a)
 + 3*(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1)*si
n(b*x + a) - 3*(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x +
a) + 1)*sin(b*x + a) + 3*(b*d*x + a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) +
 sin(b*x + a) + 1)*sin(b*x + a) - 3*(b*d*x + a*d)*cos(b*x + a)^2*log(-I*cos
(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) + 2*d*cos(b*x + a)^2*log(-1/2*co
s(b*x + a) + 1/2)*sin(b*x + a) + 3*(b*c - a*d)*cos(b*x + a)^2*log(-cos(b*x
+ a) + I*sin(b*x + a) + I)*sin(b*x + a) - 3*(b*c - a*d)*cos(b*x + a)^2*log(
-cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 2*b*d*x - 6*(b*d*x + b*c
)*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x + a) + 2*b*c)/(b^2*cos(b*x + a)
^2*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)**2*sec(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)^3*sin(a + b*x)^2),x)
[Out] \text{Hanged}
```

3.320 $\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x),x]
[Out] Defer[Int][(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 20.26, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x),x]
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x), x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^2\left(bx+a\right)\right)\left(\sec^3\left(bx+a\right)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] (2*(b*d*x + b*c)*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) + (d*cos(5*b*x + 5*a) -
d*cos(b*x + a) + 3*(b*d*x + b*c)*sin(5*b*x + 5*a) + 2*(b*d*x + b*c)*sin(3*b
*x + 3*a) + 3*(b*d*x + b*c)*sin(b*x + a))*cos(6*b*x + 6*a) + (d*cos(4*b*x +
 4*a) - d*cos(2*b*x + 2*a) - 3*(b*d*x + b*c)*sin(4*b*x + 4*a) + 3*(b*d*x +
b*c)*sin(2*b*x + 2*a) - d)*cos(5*b*x + 5*a) - (d*cos(b*x + a) - 2*(b*d*x +
b*c)*sin(3*b*x + 3*a) - 3*(b*d*x + b*c)*sin(b*x + a))*cos(4*b*x + 4*a) + (d
*cos(b*x + a) - 3*(b*d*x + b*c)*sin(b*x + a))*cos(2*b*x + 2*a) + d*cos(b*x
+ a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos(6*b*x + 6*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b
*x + 4*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(6*b*x + 6*a)^2 + (b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*
c^2)*sin(2*b*x + 2*a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a) + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*cos(2*b*x + 2*a))*cos(6*b*x + 6*a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(
4*b*x + 4*a) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a) + 2
*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a) - (b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a))*sin(6*b*x + 6*a))*integrate(((3*b^2
*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(2*b*x + 2*a)*cos(b*x + a) +
 (3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*sin(2*b*x + 2*a)*sin(b*x
 + a) + (3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(b*x + a))/(b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^
2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*
x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) - (b
^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2
*c^2*d)*cos(6*b*x + 6*a)^2 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(
4*b*x + 4*a)^2 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(2*b*x + 2*a)
^2 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(6*b*x + 6*a)^2 + (b^2*d^
3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(4*b*x + 4*a)^2 - 2*(b^2*d^3*x^2 + 2*
b^2*c*d^2*x + b^2*c^2*d)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b^2*d^3*x^2 +
 2*b^2*c*d^2*x + b^2*c^2*d)*sin(2*b*x + 2*a)^2 - 2*(b^2*d^3*x^2 + 2*b^2*c*d
```

```
^2*x + b^2*c^2*d - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(4*b*x + 4*
a) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(2*b*x + 2*a))*cos(6*b*x
+ 6*a) - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d + (b^2*d^3*x^2 + 2*b^2*
c*d^2*x + b^2*c^2*d)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b^2*d^3*x^2 +
2*b^2*c*d^2*x + b^2*c^2*d)*cos(2*b*x + 2*a) + 2*((b^2*d^3*x^2 + 2*b^2*c*d^2
*x + b^2*c^2*d)*sin(4*b*x + 4*a) - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d
)*sin(2*b*x + 2*a))*sin(6*b*x + 6*a))*integrate(sin(b*x + a)/(b*d^2*x^2 + 2
*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 + (b*d^2*
x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)
*cos(b*x + a)), x) - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d + (b^2*d^3*x^
2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(6*b*x + 6*a)^2 + (b^2*d^3*x^2 + 2*b^2*c*
d^2*x + b^2*c^2*d)*cos(4*b*x + 4*a)^2 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*
c^2*d)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(6
*b*x + 6*a)^2 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(4*b*x + 4*a)^
2 - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(4*b*x + 4*a)*sin(2*b*x
+ 2*a) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(2*b*x + 2*a)^2 - 2*(
b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^
2*c^2*d)*cos(4*b*x + 4*a) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(2
*b*x + 2*a))*cos(6*b*x + 6*a) - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d
+ (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(2*b*x + 2*a))*cos(4*b*x + 4
*a) + 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(2*b*x + 2*a) + 2*((b^
2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(4*b*x + 4*a) - (b^2*d^3*x^2 + 2*
b^2*c*d^2*x + b^2*c^2*d)*sin(2*b*x + 2*a))*sin(6*b*x + 6*a))*integrate(sin(
b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*c
os(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x
^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) - (3*(b*d*x + b*c)*cos(5*b*x + 5*
a) + 2*(b*d*x + b*c)*cos(3*b*x + 3*a) + 3*(b*d*x + b*c)*cos(b*x + a) - d*si
n(5*b*x + 5*a) + d*sin(b*x + a))*sin(6*b*x + 6*a) - (3*b*d*x + 3*b*c - 3*(b
*d*x + b*c)*cos(4*b*x + 4*a) + 3*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(4*b
*x + 4*a) + d*sin(2*b*x + 2*a))*sin(5*b*x + 5*a) - (2*(b*d*x + b*c)*cos(3*b
*x + 3*a) + 3*(b*d*x + b*c)*cos(b*x + a) + d*sin(b*x + a))*sin(4*b*x + 4*a)
 - 2*(b*d*x + b*c + (b*d*x + b*c)*cos(2*b*x + 2*a))*sin(3*b*x + 3*a) + (3*(
b*d*x + b*c)*cos(b*x + a) + d*sin(b*x + a))*sin...
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sec(b*x + a)^3/(d*x + c), x)
```

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)**3/(d*x+c),x)
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)**3/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^2*sec(b*x + a)^3/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)),x)
[Out] int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)), x)
```

3.321 $\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)
```

Rubi [A]
time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2,x]
[Out] Defer[Int][(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 25.57, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2,x]
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(\csc^2(bx+a))(\sec^3(bx+a))}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)
[Out] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
[Out] (2*(b*d*x + b*c)*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) + (2*d*cos(5*b*x + 5*a)
- 2*d*cos(b*x + a) + 3*(b*d*x + b*c)*sin(5*b*x + 5*a) + 2*(b*d*x + b*c)*sin
(3*b*x + 3*a) + 3*(b*d*x + b*c)*sin(b*x + a))*cos(6*b*x + 6*a) + (2*d*cos(4
*b*x + 4*a) - 2*d*cos(2*b*x + 2*a) - 3*(b*d*x + b*c)*sin(4*b*x + 4*a) + 3*(
b*d*x + b*c)*sin(2*b*x + 2*a) - 2*d)*cos(5*b*x + 5*a) - (2*d*cos(b*x + a) -
 2*(b*d*x + b*c)*sin(3*b*x + 3*a) - 3*(b*d*x + b*c)*sin(b*x + a))*cos(4*b*x
 + 4*a) + (2*d*cos(b*x + a) - 3*(b*d*x + b*c)*sin(b*x + a))*cos(2*b*x + 2*a
) + 2*d*cos(b*x + a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(6*b*x
+ 6*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*
b*x + 4*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*co
s(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
)*sin(6*b*x + 6*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*sin(4*b*x + 4*a)^2 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
 + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 - 2*(b^2*d^3*x^3 + 3*b^2*
c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(6*b*x + 6*a) - 2*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^
2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b^2*d^
3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a) + 2*((b
^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a) -
(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a))
*sin(6*b*x + 6*a))*integrate(3*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*d^
2)*cos(2*b*x + 2*a)*cos(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2
*d^2)*sin(2*b*x + 2*a)*sin(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
+ 2*d^2)*cos(b*x + a))/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 +
 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x
^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d
^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(2*b*x + 2*a)^2 +
2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*
c^4)*cos(2*b*x + 2*a)), x) - 2*(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d
^2*x + b^2*c^3*d + (b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c
```

```
^3*d)*cos(6*b*x + 6*a)^2 + (b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x
 + b^2*c^3*d)*cos(4*b*x + 4*a)^2 + (b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c
^2*d^2*x + b^2*c^3*d)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 +
 3*b^2*c^2*d^2*x + b^2*c^3*d)*sin(6*b*x + 6*a)^2 + (b^2*d^4*x^3 + 3*b^2*c*d
^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*sin(4*b*x + 4*a)^2 - 2*(b^2*d^4*x^3 +
 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*sin(4*b*x + 4*a)*sin(2*b*x
+ 2*a) + (b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*sin(
2*b*x + 2*a)^2 - 2*(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c
^3*d - (b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*cos(4*
b*x + 4*a) + (b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*
cos(2*b*x + 2*a))*cos(6*b*x + 6*a) - 2*(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b
^2*c^2*d^2*x + b^2*c^3*d + (b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x
 + b^2*c^3*d)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b^2*d^4*x^3 + 3*b^2*c
*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*cos(2*b*x + 2*a) + 2*((b^2*d^4*x^3
+ 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*sin(4*b*x + 4*a) - (b^2*d^
4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*sin(2*b*x + 2*a))*si
n(6*b*x + 6*a))*integrate(sin(b*x + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2
*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x +
a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(b*x + a)^2 + 2
*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)), x) - 2*(b
^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d + (b^2*d^4*x^3 +
 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*cos(6*b*x + 6*a)^2 + (b^2*d
^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*cos(4*b*x + 4*a)^2
+ (b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*cos(2*b*x +
 2*a)^2 + (b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*sin
(6*b*x + 6*a)^2 + (b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^
3*d)*sin(4*b*x + 4*a)^2 - 2*(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*
x + b^2*c^3*d)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b^2*d^4*x^3 + 3*b^2*c*d
^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*sin(2*b*x + 2*a)^2 - 2*(b^2*d^4*x^3 +
 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d - (b^2*d^4*x^3 + 3*b^2*c*d^3
*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*cos(4*b*x + 4*a) + (b^2*d^4*x^3 + 3*b^2
*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*cos(2*b*x + 2*a))*cos(6*b*x + 6*a
) - 2*(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^...
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)**3/(c + d*x)**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+b\,x)^3\,\sin(a+b\,x)^2\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^2),x)
```

```
[Out] int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^2), x)
```

3.322 $\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\text{Int}\left((c+dx)^m \csc^3(a+bx)\sec^3(a+bx),x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

Rubi [A]
time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx = \int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx$$

Mathematica [A]
time = 29.49, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3, x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\csc^3(bx+a)\right)\left(\sec^3(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x)
[Out] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
[Out] integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a)**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^3),x)
[Out] int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^3), x)
```

3.323 $\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx) \, dx$

Optimal. Leaf size=318

$$-\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3}-\frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\csc(2a+2bx)}{b^2}-\frac{2(c+dx)^3\cot(2$$

```
[Out] -6*d^2*(d*x+c)*arctanh(exp(2*I*(b*x+a)))/b^3-4*(d*x+c)^3*arctanh(exp(2*I*(b
*x+a)))/b-3*d*(d*x+c)^2*csc(2*b*x+2*a)/b^2-2*(d*x+c)^3*cot(2*b*x+2*a)*csc(2
*b*x+2*a)/b+3/2*I*d^3*polylog(2,-exp(2*I*(b*x+a)))/b^4+3*I*d*(d*x+c)^2*poly
log(2,-exp(2*I*(b*x+a)))/b^2-3/2*I*d^3*polylog(2,exp(2*I*(b*x+a)))/b^4-3*I*
d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3*d^2*(d*x+c)*polylog(3,-exp(2*
I*(b*x+a)))/b^3+3*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^3-3/2*I*d^3*pol
ylog(4,-exp(2*I*(b*x+a)))/b^4+3/2*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4
```

Rubi [A]

time = 0.21, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4504, 4271, 4268, 2317, 2438, 2611, 6744, 2320, 6724}

$$\frac{3id^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{2b^4}+\frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{2b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{3d^2(c+dx)\text{Li}_3\left(e^{2i(a+bx)}\right)}{b^3}-\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{3d(c+dx)^2\csc(2a+2bx)}{b^2}-\frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{2(c+dx)^3\cot(2a+2bx)\csc(2a+2bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] (-6*d^2*(c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b^3 - (4*(c + d*x)^3*ArcTan
h[E^((2*I)*(a + b*x))])/b - (3*d*(c + d*x)^2*Csc[2*a + 2*b*x])/b^2 - (2*(c
+ d*x)^3*Cot[2*a + 2*b*x]*Csc[2*a + 2*b*x])/b + (((3*I)/2)*d^3*PolyLog[2, -
E^((2*I)*(a + b*x))])/b^4 + ((3*I)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a +
b*x))])/b^2 - (((3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^4 - ((3*I)*
d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyL
og[3, -E^((2*I)*(a + b*x))])/b^3 + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a
+ b*x))])/b^3 - (((3*I)/2)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3
*I)/2)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \csc^3(a+bx)\sec^3(a+bx)\,dx = 8\int (c+dx)^3 \csc^3(2a+2bx)\,dx$$

$$= -\frac{3d(c+dx)^2\csc(2a+2bx)}{b^2} - \frac{2(c+dx)^3\cot(2a+2bx)\csc(2a+$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} -$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} -$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} -$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} -$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} -$$

Mathematica [A]
time = 7.24, size = 582, normalized size = 1.83

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] 8*(-1/16*(8*b^3*c^3*ArcTanh[E^((2*I)*(a + b*x))] + 12*b*c*d^2*ArcTanh[E^((2
*I)*(a + b*x))] - 12*b^3*c^2*d*x*Log[1 - E^((2*I)*(a + b*x))] - 6*b*d^3*x*L
og[1 - E^((2*I)*(a + b*x))] - 12*b^3*c*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))]
 - 4*b^3*d^3*x^3*Log[1 - E^((2*I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 + E^((
2*I)*(a + b*x))] + 6*b*d^3*x*Log[1 + E^((2*I)*(a + b*x))] + 12*b^3*c*d^2*x^
2*Log[1 + E^((2*I)*(a + b*x))] + 4*b^3*d^3*x^3*Log[1 + E^((2*I)*(a + b*x))]
 - (3*I)*d*(d^2 + 2*b^2*(c + d*x)^2)*PolyLog[2, -E^((2*I)*(a + b*x))] + (3*
I)*d*(d^2 + 2*b^2*(c + d*x)^2)*PolyLog[2, E^((2*I)*(a + b*x))] + 6*b*c*d^2*
```

```
PolyLog[3, -E^((2*I)*(a + b*x))] + 6*b*d^3*x*PolyLog[3, -E^((2*I)*(a + b*x)
)] - 6*b*c*d^2*PolyLog[3, E^((2*I)*(a + b*x))] - 6*b*d^3*x*PolyLog[3, E^((2
*I)*(a + b*x))] + (3*I)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))] - (3*I)*d^3*Po
lyLog[4, E^((2*I)*(a + b*x))])/b^4 - (Csc[2*a + 2*b*x]^2*(2*b*c^3*Cos[2*a +
 2*b*x] + 6*b*c^2*d*x*Cos[2*a + 2*b*x] + 6*b*c*d^2*x^2*Cos[2*a + 2*b*x] + 2
*b*d^3*x^3*Cos[2*a + 2*b*x] + 3*c^2*d*Sin[2*a + 2*b*x] + 6*c*d^2*x*Sin[2*a
+ 2*b*x] + 3*d^3*x^2*Sin[2*a + 2*b*x]))/(8*b^2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1328 vs. $2(284) = 568$.
time = 0.21, size = 1329, normalized size = 4.18

method	result	size
risch	Expression too large to display	1329

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -12*I/b^2*polylog(2,exp(I*(b*x+a)))*c*d^2*x-12*I/b^2*polylog(2,-exp(I*(b*x+
a)))*c*d^2*x+6*I/b^2*polylog(2,-exp(2*I*(b*x+a)))*c*d^2*x-2/b^4*d^3*a^3*ln(
exp(I*(b*x+a))-1)+12/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+12/b^3*c*d^2*polyl
og(3,-exp(I*(b*x+a)))+12/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+12/b^3*d^3*po
lylog(3,exp(I*(b*x+a)))*x+2/b*c^3*ln(exp(I*(b*x+a))+1)+2/b*c^3*ln(exp(I*(b*
x+a))-1)-6*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-6*I/b^2*d^3*polylog(2,-exp
(I*(b*x+a)))*x^2+3*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2-6*I/b^2*c^2*d
*polylog(2,-exp(I*(b*x+a)))+3*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+a)))-6*I/
b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2+2/b^2/(1+exp(2*I*(b*x+a)))^2/(exp(2*I
*(b*x+a))-1)^2*(2*d^3*x^3*b*exp(6*I*(b*x+a))+6*c*d^2*x^2*b*exp(6*I*(b*x+a))
+6*c^2*d*x*b*exp(6*I*(b*x+a))+2*c^3*b*exp(6*I*(b*x+a))-3*I*d^3*x^2*exp(6*I*
(b*x+a))+2*b*d^3*x^3*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(6*I*(b*x+a))+6*b*c*d^
2*x^2*exp(2*I*(b*x+a))-3*I*c^2*d*exp(6*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+
a))+2*b*c^3*exp(2*I*(b*x+a))+3*I*d^3*x^2*exp(2*I*(b*x+a))+6*I*c*d^2*x*exp(2
*I*(b*x+a))+3*I*c^2*d*exp(2*I*(b*x+a)))+3*d^2/b^3*c*ln(exp(I*(b*x+a))-1)+3*
d^2/b^3*c*ln(exp(I*(b*x+a))+1)+3*d^3/b^3*ln(1-exp(I*(b*x+a)))*x+3*d^3/b^4*l
n(1-exp(I*(b*x+a)))*a+3*d^3/b^3*ln(exp(I*(b*x+a))+1)*x-3*d^3/b^4*a*ln(exp(I
*(b*x+a))-1)-3*I*d^3/b^4*polylog(2,-exp(I*(b*x+a)))+2/b*d^3*ln(exp(I*(b*x+a
))+1)*x^3+2/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+2/b^4*d^3*ln(1-exp(I*(b*x+a)))*a
^3-3/2*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+6/b^3*c*d^2*a^2*ln(exp(I*(b*x
+a))-1)-2/b*c^3*ln(1+exp(2*I*(b*x+a)))-3/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+
a)))-3/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x-3/b^3*d^2*c*ln(1+exp(2*I*(b*x
+a)))-3/b^3*d^3*ln(1+exp(2*I*(b*x+a)))*x+6/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+6
/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+6/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a+6/b*c*d^
2*ln(exp(I*(b*x+a))+1)*x^2+6/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-6/b^3*c*d^2*l
n(1-exp(I*(b*x+a)))*a^2-6/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+3/2*I*d^3*polylo
g(2,-exp(2*I*(b*x+a)))/b^4+12*I/b^4*d^3*polylog(4,exp(I*(b*x+a)))+12*I/b^4*
```

```
d^3*polylog(4,-exp(I*(b*x+a)))-6/b*c^2*d*ln(1+exp(2*I*(b*x+a)))*x-6/b*c*d^2
*ln(1+exp(2*I*(b*x+a)))*x^2-2/b*d^3*ln(1+exp(2*I*(b*x+a)))*x^3-3*I*d^3*poly
log(2,exp(I*(b*x+a)))/b^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5646 vs. 2(274) = 548.
time = 2.68, size = 5646, normalized size = 17.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log
(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2)) - 3*a*c^2*d*((2*sin(b*x + a)^
2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*lo
g(sin(b*x + a)^2))/b + 3*a^2*c*d^2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4
- sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b^2
- a^3*d^3*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log
(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b^3 + 2*(2*(8*(b*x + a)^3*d^3
 + 9*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(2*b^2*c^2*d
- 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a) + (8*(b*x + a)^3*d^3 + 9*b*c*d^2
 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(2*b^2*c^2*d - 4*a*b*c*d^
2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 2*(8*(b*x + a)^3*d^3 + 9
*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(2*b^2*c^2*d - 4*
a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - (-8*I*(b*x + a)^
3*d^3 - 9*I*b*c*d^2 + 9*I*a*d^3 + 18*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 9
*(-2*I*b^2*c^2*d + 4*I*a*b*c*d^2 + (-2*I*a^2 - I)*d^3)*(b*x + a))*sin(8*b*x
 + 8*a) - 2*(8*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + 18*(I*b*c*d^2
- I*a*d^3)*(b*x + a)^2 + 9*(2*I*b^2*c^2*d - 4*I*a*b*c*d^2 + (2*I*a^2 + I)*d
^3)*(b*x + a))*sin(4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a)
 + 1) - 6*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b
*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a) + (2*
(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3
*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(8*b*x + 8*a)
- 2*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a
)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x
+ 4*a) + (2*I*(b*x + a)^3*d^3 + 3*I*b*c*d^2 - 3*I*a*d^3 + 6*(I*b*c*d^2 - I*
a*d^3)*(b*x + a)^2 + 3*(2*I*b^2*c^2*d - 4*I*a*b*c*d^2 + (2*I*a^2 + I)*d^3)*
(b*x + a))*sin(8*b*x + 8*a) + 2*(-2*I*(b*x + a)^3*d^3 - 3*I*b*c*d^2 + 3*I*a
*d^3 + 6*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-2*I*b^2*c^2*d + 4*I*a*b*c
*d^2 + (-2*I*a^2 - I)*d^3)*(b*x + a))*sin(4*b*x + 4*a))*arctan2(sin(b*x + a
), cos(b*x + a) + 1) - 18*(b*c*d^2 - a*d^3 + (b*c*d^2 - a*d^3)*cos(8*b*x +
8*a) - 2*(b*c*d^2 - a*d^3)*cos(4*b*x + 4*a) + (I*b*c*d^2 - I*a*d^3)*sin(8*b
*x + 8*a) + 2*(-I*b*c*d^2 + I*a*d^3)*sin(4*b*x + 4*a))*arctan2(sin(b*x + a)
```

```
, cos(b*x + a) - 1) + 6*(2*(b*x + a)^3*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^
2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a) + (2*(b*x + a
)^3*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 +
(2*a^2 + 1)*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 2*(2*(b*x + a)^3*d^3 + 6*(b*
c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3
)*(b*x + a))*cos(4*b*x + 4*a) - (-2*I*(b*x + a)^3*d^3 + 6*(-I*b*c*d^2 + I*a
*d^3)*(b*x + a)^2 + 3*(-2*I*b^2*c^2*d + 4*I*a*b*c*d^2 + (-2*I*a^2 - I)*d^3)
*(b*x + a))*sin(8*b*x + 8*a) - 2*(2*I*(b*x + a)^3*d^3 + 6*(I*b*c*d^2 - I*a*
d^3)*(b*x + a)^2 + 3*(2*I*b^2*c^2*d - 4*I*a*b*c*d^2 + (2*I*a^2 + I)*d^3)*(b
*x + a))*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 12*(-
2*I*(b*x + a)^3*d^3 - 3*b^2*c^2*d + 6*a*b*c*d^2 - 3*a^2*d^3 + 3*(-2*I*b*c*d
^2 + (2*I*a - 1)*d^3)*(b*x + a)^2 + 6*(-I*b^2*c^2*d + (2*I*a - 1)*b*c*d^2 +
 (-I*a^2 + a)*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 12*(-2*I*(b*x + a)^3*d^3 +
 3*b^2*c^2*d - 6*a*b*c*d^2 + 3*a^2*d^3 + 3*(-2*I*b*c*d^2 + (2*I*a + 1)*d^3)
*(b*x + a)^2 + 6*(-I*b^2*c^2*d + (2*I*a + 1)*b*c*d^2 + (-I*a^2 - a)*d^3)*(b
*x + a))*cos(2*b*x + 2*a) - 3*(6*b^2*c^2*d - 12*a*b*c*d^2 + 8*(b*x + a)^2*d
^3 + 3*(2*a^2 + 1)*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + (6*b^2*c^2*d - 12
*a*b*c*d^2 + 8*(b*x + a)^2*d^3 + 3*(2*a^2 + 1)*d^3 + 12*(b*c*d^2 - a*d^3)*(
b*x + a))*cos(8*b*x + 8*a) - 2*(6*b^2*c^2*d - 12*a*b*c*d^2 + 8*(b*x + a)^2*
d^3 + 3*(2*a^2 + 1)*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a)
+ (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 8*I*(b*x + a)^2*d^3 + 3*(2*I*a^2 + I)*d
^3 + 12*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(8*b*x + 8*a) + 2*(-6*I*b^2*c^2
*d + 12*I*a*b*c*d^2 - 8*I*(b*x + a)^2*d^3 + 3*(-2*I*a^2 - I)*d^3 + 12*(-I*b
*c*d^2 + I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a))*dilog(-e^(2*I*b*x + 2*I*a))
+ 18*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a^2 + 1)*d^3 + 4*(
b*c*d^2 - a*d^3)*(b*x + a) + (2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3
 + (2*a^2 + 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 2*(2
*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a^2 + 1)*d^3 + 4*(b*c*d^2
 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - (-2*I*b^2*c^2*d + 4*I*a*b*c*d^2 - 2
*I*(b*x + a)^2*d^3 + (-2*I*a^2 - I)*d^3 + 4*(-I*b*c*d^2 + I*a*d^3)*(b*x + a
))*sin(8*b*x + 8*a) - 2*(2*I*b^2*c^2*d - 4*I*a*b*c*d^2 + 2*I*(b*x + a)^2*d^
3 + (2*I*a^2 + I)*d^3 + 4*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a)
)*dilog(-e^(I*b*x + I*a)) + 18*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*d
^3 + (2*a^2 + 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a) + (2*b^2*c^2*d - 4*a*b
*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a^2 + 1)*d^3 + ...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4193 vs. $2(274) = 548$.
time = 4.51, size = 4193, normalized size = 13.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

[Out] -1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)^2 - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) + 3*((2*I*b^2*d^3*x^2 + 4*I*b^2*c*d^2*x + 2*I*b^2*c^2*d + I*d^3)*cos(b*x + a)^4 + (-2*I*b^2*d^3*x^2 - 4*I*b^2*c*d^2*x - 2*I*b^2*c^2*d - I*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) + 3*((-2*I*b^2*d^3*x^2 - 4*I*b^2*c*d^2*x - 2*I*b^2*c^2*d - I*d^3)*cos(b*x + a)^4 + (2*I*b^2*d^3*x^2 + 4*I*b^2*c*d^2*x + 2*I*b^2*c^2*d + I*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) + 3*((2*I*b^2*d^3*x^2 + 4*I*b^2*c*d^2*x + 2*I*b^2*c^2*d + I*d^3)*cos(b*x + a)^4 + (-2*I*b^2*d^3*x^2 - 4*I*b^2*c*d^2*x - 2*I*b^2*c^2*d - I*d^3)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) + sin(b*x + a)) + 3*((-2*I*b^2*d^3*x^2 - 4*I*b^2*c*d^2*x - 2*I*b^2*c^2*d - I*d^3)*cos(b*x + a)^4 + (2*I*b^2*d^3*x^2 + 4*I*b^2*c*d^2*x + 2*I*b^2*c^2*d + I*d^3)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a)) + 3*((-2*I*b^2*d^3*x^2 - 4*I*b^2*c*d^2*x - 2*I*b^2*c^2*d - I*d^3)*cos(b*x + a)^4 + (2*I*b^2*d^3*x^2 + 4*I*b^2*c*d^2*x + 2*I*b^2*c^2*d + I*d^3)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 3*((2*I*b^2*d^3*x^2 + 4*I*b^2*c*d^2*x + 2*I*b^2*c^2*d + I*d^3)*cos(b*x + a)^4 + (-2*I*b^2*d^3*x^2 - 4*I*b^2*c*d^2*x - 2*I*b^2*c^2*d - I*d^3)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 3*((-2*I*b^2*d^3*x^2 - 4*I*b^2*c*d^2*x - 2*I*b^2*c^2*d - I*d^3)*cos(b*x + a)^4 + (2*I*b^2*d^3*x^2 + 4*I*b^2*c*d^2*x + 2*I*b^2*c^2*d + I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + 3*((2*I*b^2*d^3*x^2 + 4*I*b^2*c*d^2*x + 2*I*b^2*c^2*d + I*d^3)*cos(b*x + a)^4 + (-2*I*b^2*d^3*x^2 - 4*I*b^2*c*d^2*x - 2*I*b^2*c^2*d - I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - ((2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + ((2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^4 - (2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + I) - ((2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + ((2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^4 - (2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) + ((2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + ((2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + ((2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*d -

```
6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)
^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*
a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2)*log(-I*cos(b*x
+ a) + sin(b*x + a) + 1) + ((2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*
d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x
+ a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 +
 (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2)*log(-I*cos(
b*x + a) - sin(b*x + a) + 1) - ((2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*
b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^4 - (2*b^3*c^3 - 6*a*b^2*c^2*d +
3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x
 + a) + 1/2*I*sin(b*x + a) + 1/2) - ((2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2
+ 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^4 - (2*b^3*c^3 - 6*a*b^2*c^2
*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^2)*log(-1/2*co
s(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - ((2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2
+ 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^
3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6
*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^
2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + ((2*b^3*c^3 - 6*a*b^2*c^2*d +
3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^4 - (2*b^3*c^3 - 6*
a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^2)*lo
g(-cos(b*x + a) + I*sin(b*x + a) + I) - ((2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 +
 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*...
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a)**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^3*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

3.324 $\int (c+dx)^2 \csc^3(a+bx) \sec^3(a+bx)\,dx$

Optimal. Leaf size=190

$$-\frac{4(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{d^2\tanh^{-1}(\cos(2a+2bx))}{b^3}-\frac{2d(c+dx)\csc(2a+2bx)}{b^2}-\frac{2(c+dx)^2\cot(2a+}{b}$$

```
[Out] -4*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b-d^2*arctanh(cos(2*b*x+2*a))/b^3-2*
d*(d*x+c)*csc(2*b*x+2*a)/b^2-2*(d*x+c)^2*cot(2*b*x+2*a)*csc(2*b*x+2*a)/b+2*
I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylog(2,exp(2*
I*(b*x+a)))/b^2-d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+d^2*polylog(3,exp(2*I*
(b*x+a)))/b^3
```

Rubi [A]
time = 0.14, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4504, 4271, 3855, 4268, 2611, 2320, 6724}

$$-\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{b^3}-\frac{d^2\tanh^{-1}(\cos(2a+2bx))}{b^3}+\frac{2id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{2d(c+dx)\csc(2a+2bx)}{b^2}-\frac{4(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{2(c+dx)^2\cot(2a+2bx)\csc(2a+2bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] (-4*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/b - (d^2*ArcTanh[Cos[2*a + 2*
b*x]])/b^3 - (2*d*(c + d*x)*Csc[2*a + 2*b*x])/b^2 - (2*(c + d*x)^2*Cot[2*a
+ 2*b*x]*Csc[2*a + 2*b*x])/b + ((2*I)*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a +
 b*x))])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d
^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 + (d^2*PolyLog[3, E^((2*I)*(a + b*
x))])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \csc^3(a+bx) \sec^3(a+bx) \, dx = 8 \int (c+dx)^2 \csc^3(2a+2bx) \, dx$$

$$= -\frac{2d(c+dx)\csc(2a+2bx)}{b^2} - \frac{2(c+dx)^2 \cot(2a+2bx)\csc(2a+$$

$$= -\frac{4(c+dx)^2 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2 \tanh^{-1}(\cos(2a+2bx))}{b^3} - 2c$$

$$= -\frac{4(c+dx)^2 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2 \tanh^{-1}(\cos(2a+2bx))}{b^3} - 2c$$

$$= -\frac{4(c+dx)^2 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2 \tanh^{-1}(\cos(2a+2bx))}{b^3} - 2c$$

$$= -\frac{4(c+dx)^2 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2 \tanh^{-1}(\cos(2a+2bx))}{b^3} - 2c$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 381 vs. 2(190) = 380.
time = 6.94, size = 381, normalized size = 2.01

$$8\left(-\frac{d(c+dx)\csc(2a)}{4b^2} + \frac{(-c^2-2cdx-d^2x^2)\csc^2(a+bx)}{16b} - \frac{4b^2c^2\tanh^{-1}\left(e^{2i(a+bx)}\right) + 2d^2\tanh^{-1}\left(e^{2i(a+bx)}\right) - 4b^2cdx\log\left(1-e^{2i(a+bx)}\right) - 2b^2d^2x^2\log\left(1-e^{2i(a+bx)}\right) + 4b^2cdx\log\left(1+e^{2i(a+bx)}\right) + 2b^2d^2x^2\log\left(1+e^{2i(a+bx)}\right) - 2ibd(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right) + 2ibd(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right) + d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right) - d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{8b^3} + \frac{(c^2+2cdx+d^2x^2)\sec^2(a+bx)}{16b} + \frac{\sec(a)\sec(a+bx)\left(-cd\sin(bx)-d^2x\sin(bx)\right)}{8b^2} + \frac{\csc(a)\csc(a+bx)\left(cd\sin(bx)+d^2x\sin(bx)\right)}{8b^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] 8*(-1/4*(d*(c + d*x)*Csc[2*a])/b^2 + ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a + b*
x]^2)/(16*b) - (4*b^2*c^2*ArcTanh[E^((2*I)*(a + b*x))] + 2*d^2*ArcTanh[E^((
2*I)*(a + b*x))] - 4*b^2*c*d*x*Log[1 - E^((2*I)*(a + b*x))] - 2*b^2*d^2*x^2
*Log[1 - E^((2*I)*(a + b*x))] + 4*b^2*c*d*x*Log[1 + E^((2*I)*(a + b*x))] +
2*b^2*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2,
 -E^((2*I)*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
] + d^2*PolyLog[3, -E^((2*I)*(a + b*x))] - d^2*PolyLog[3, E^((2*I)*(a + b*x
))])/(8*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*Sec[a + b*x]^2)/(16*b) + (Sec[a]*
Sec[a + b*x]*(-(c*d*Sin[b*x]) - d^2*x*Sin[b*x]))/(8*b^2) + (Csc[a]*Csc[a +
b*x]*(c*d*Sin[b*x] + d^2*x*Sin[b*x]))/(8*b^2))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 715 vs. 2(178) = 356.
time = 0.16, size = 716, normalized size = 3.77

method	result
risch	$-\frac{2d^2 \ln(1+e^{2i(bx+a)})x^2}{b} - \frac{4cda \ln(e^{i(bx+a)}-1)}{b^2} - \frac{2c^2 \ln(1+e^{2i(bx+a)})}{b} - \frac{d^2 \operatorname{polylog}(3,-e^{2i(bx+a)})}{b^3} - \frac{4id^2 \operatorname{polylog}(2,e^{i(bx+}}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+2/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-2/b^3*d
^2*ln(1-exp(I*(b*x+a)))*a^2+2/b*d^2*ln(exp(I*(b*x+a))+1)*x^2-2/b*d^2*ln(1+e
xp(2*I*(b*x+a)))*x^2-2/b*c^2*ln(1+exp(2*I*(b*x+a)))+4/b^2*c*d*ln(1-exp(I*(b
*x+a)))*a+4/b*c*d*ln(exp(I*(b*x+a))+1)*x+4/b*c*d*ln(1-exp(I*(b*x+a)))*x-4/b
^2*c*d*a*ln(exp(I*(b*x+a))-1)-4/b*c*d*ln(1+exp(2*I*(b*x+a)))*x-d^2/b^3*ln(1
+exp(2*I*(b*x+a)))+2/b*c^2*ln(exp(I*(b*x+a))-1)+2/b*c^2*ln(exp(I*(b*x+a))+1
)-d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+4*d^2*polylog(3,-exp(I*(b*x+a)))/b^3
+4*d^2*polylog(3,exp(I*(b*x+a)))/b^3+4/b^2/(1+exp(2*I*(b*x+a)))^2/(exp(2*I*
(b*x+a))-1)^2*(d^2*x^2*b*exp(6*I*(b*x+a))+2*c*d*x*b*exp(6*I*(b*x+a))+c^2*b*
exp(6*I*(b*x+a))-I*d^2*x*exp(6*I*(b*x+a))+b*d^2*x^2*exp(2*I*(b*x+a))-I*c*d*
exp(6*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))+I*d^2*x*
exp(2*I*(b*x+a))+I*c*d*exp(2*I*(b*x+a)))+1/b^3*d^2*ln(exp(I*(b*x+a))-1)+1/b
^3*d^2*ln(exp(I*(b*x+a))+1)+2*I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+2*I/
b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))-4*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))-
4*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-4*I/b^2*polylog(2,exp(I*(b*x+a)))*d^
2*x-4*I/b^2*polylog(2,-exp(I*(b*x+a)))*d^2*x
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2728 vs. $2(174) = 348$.
time = 0.93, size = 2728, normalized size = 14.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log
(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2)) - 2*a*c*d*((2*sin(b*x + a)^2
- 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*log(
sin(b*x + a)^2))/b + a^2*d^2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(
b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b^2 + 2*(2
*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2 + (2*(b*x + a)^2*d^
2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*cos(8*b*x + 8*a) - 2*(2*(b*x + a)^2*
d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*cos(4*b*x + 4*a) - (-2*I*(b*x + a)
^2*d^2 + 4*(-I*b*c*d + I*a*d^2)*(b*x + a) - I*d^2)*sin(8*b*x + 8*a) - 2*(2*
I*(b*x + a)^2*d^2 + 4*(I*b*c*d - I*a*d^2)*(b*x + a) + I*d^2)*sin(4*b*x + 4*
a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 2*(2*(b*x + a)^2*d^2
+ 4*(b*c*d - a*d^2)*(b*x + a) + d^2 + (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2
)*(b*x + a) + d^2)*cos(8*b*x + 8*a) - 2*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d
^2)*(b*x + a) + d^2)*cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2 + 4*(I*b*c*d -
```

```
 I*a*d^2)*(b*x + a) + I*d^2)*sin(8*b*x + 8*a) + 2*(-2*I*(b*x + a)^2*d^2 + 4
*(-I*b*c*d + I*a*d^2)*(b*x + a) - I*d^2)*sin(4*b*x + 4*a))*arctan2(sin(b*x
+ a), cos(b*x + a) + 1) - 2*(d^2*cos(8*b*x + 8*a) - 2*d^2*cos(4*b*x + 4*a)
+ I*d^2*sin(8*b*x + 8*a) - 2*I*d^2*sin(4*b*x + 4*a) + d^2)*arctan2(sin(b*x
+ a), cos(b*x + a) - 1) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a)
+ ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(8*b*x + 8*a) - 2*((b*
x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a) - (-I*(b*x + a
)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*sin(8*b*x + 8*a) - 2*(I*(b*x +
a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a))*arctan2(sin(b
*x + a), -cos(b*x + a) + 1) - 8*(-I*(b*x + a)^2*d^2 - b*c*d + a*d^2 + (-2*I
*b*c*d + (2*I*a - 1)*d^2)*(b*x + a))*cos(6*b*x + 6*a) - 8*(-I*(b*x + a)^2*d
^2 + b*c*d - a*d^2 + (-2*I*b*c*d + (2*I*a + 1)*d^2)*(b*x + a))*cos(2*b*x +
2*a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*c
os(8*b*x + 8*a) - 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + (I*b
*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(8*b*x + 8*a) + 2*(-I*b*c*d - I*(b*x +
 a)*d^2 + I*a*d^2)*sin(4*b*x + 4*a))*dilog(-e^(2*I*b*x + 2*I*a)) + 8*(b*c*d
 + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(8*b*x + 8*a)
 - 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - (-I*b*c*d - I*(b*x
+ a)*d^2 + I*a*d^2)*sin(8*b*x + 8*a) - 2*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d
^2)*sin(4*b*x + 4*a))*dilog(-e^(I*b*x + I*a)) + 8*(b*c*d + (b*x + a)*d^2 -
a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(8*b*x + 8*a) - 2*(b*c*d + (b*x
+ a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)
*sin(8*b*x + 8*a) - 2*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(4*b*x + 4*a
))*dilog(e^(I*b*x + I*a)) + (-2*I*(b*x + a)^2*d^2 - 4*(I*b*c*d - I*a*d^2)*(
b*x + a) - I*d^2 + (-2*I*(b*x + a)^2*d^2 - 4*(I*b*c*d - I*a*d^2)*(b*x + a)
- I*d^2)*cos(8*b*x + 8*a) - 2*(-2*I*(b*x + a)^2*d^2 + 4*(-I*b*c*d + I*a*d^2
)*(b*x + a) - I*d^2)*cos(4*b*x + 4*a) + (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d
^2)*(b*x + a) + d^2)*sin(8*b*x + 8*a) - 2*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a
*d^2)*(b*x + a) + d^2)*sin(4*b*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (2*I*(b*x + a)^2*d^2 - 4*(-I*b*c*d +
I*a*d^2)*(b*x + a) + I*d^2 + (2*I*(b*x + a)^2*d^2 - 4*(-I*b*c*d + I*a*d^2)*
(b*x + a) + I*d^2)*cos(8*b*x + 8*a) - 2*(2*I*(b*x + a)^2*d^2 + 4*(I*b*c*d -
 I*a*d^2)*(b*x + a) + I*d^2)*cos(4*b*x + 4*a) - (2*(b*x + a)^2*d^2 + 4*(b*c
*d - a*d^2)*(b*x + a) + d^2)*sin(8*b*x + 8*a) + 2*(2*(b*x + a)^2*d^2 + 4*(b
*c*d - a*d^2)*(b*x + a) + d^2)*sin(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b
*x + a)^2 + 2*cos(b*x + a) + 1) + (2*I*(b*x + a)^2*d^2 - 4*(-I*b*c*d + I*a*
d^2)*(b*x + a) + I*d^2 + (2*I*(b*x + a)^2*d^2 - 4*(-I*b*c*d + I*a*d^2)*(b*x
 + a) + I*d^2)*cos(8*b*x + 8*a) - 2*(2*I*(b*x + a)^2*d^2 + 4*(I*b*c*d - I*a
*d^2)*(b*x + a) + I*d^2)*cos(4*b*x + 4*a) - (2*(b*x + a)^2*d^2 + 4*(b*c*d -
 a*d^2)*(b*x + a) + d^2)*sin(8*b*x + 8*a) + 2*(2*(b*x + a)^2*d^2 + 4*(b*c*d
 - a*d^2)*(b*x + a) + d^2)*sin(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x +
 a)^2 - 2*cos(b*x + a) + 1) - 2*(I*d^2*cos(8*b*x + 8*a) - 2*I*d^2*cos(4*b*x
 + 4*a) - d^2*sin(8*b*x + 8*a) + 2*d^2*sin(4*b*x + 4*a) + I*d^2)*polylog(3,
 -e^(2*I*b*x + 2*I*a)) - 8*(-I*d^2*cos(8*b*x + 8*a) + 2*I*d^2*cos(4*b*x + 4
*a) + d^2*sin(8*b*x + 8*a) - 2*d^2*sin(4*b*x + 4*a) - I*d^2)*polylog(3, -e^
```

```
(I*b*x + I*a)) - 8*(-I*d^2*cos(8*b*x + 8*a) + 2*I*d^2*cos(4*b*x + 4*a) + d^
2*sin(8*b*x + 8*a) - 2*d^2*sin(4*b*x + 4*a) - I*d^2)*polylog(3, e^(I*b*x +
I*a)) - 8*((b*x + a)^2*d^2 - I*b*c*d + I*a*d^2 + (2*b*c*d - (2*a + I)*d^2)*
(b*x + a))*sin(6*b*x + 6*a) - 8*((b*x + a)^2*d^2 + I*b*c*d - I*a*d^2 + (2*b
*c*d - (2*a - I)*d^2)*(b*x + a))*sin(2*b*x + 2*a))/(-2*I*b^2*cos(8*b*x + 8*
a) + 4*I*b^2*cos(4*b*x + 4*a) + 2*b^2*sin(8*b*x + 8*a) - 4*b^2*sin(4*b*x +
4*a) - 2*I*b^2))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2387 vs. $2(174) = 348$.
time = 3.42, size = 2387, normalized size = 12.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos(b*x + a)^2 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) + 4
*((I*b*d^2*x + I*b*c*d)*cos(b*x + a)^4 + (-I*b*d^2*x - I*b*c*d)*cos(b*x + a
)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) + 4*((-I*b*d^2*x - I*b*c*d)*cos(b
*x + a)^4 + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*si
n(b*x + a)) + 4*((I*b*d^2*x + I*b*c*d)*cos(b*x + a)^4 + (-I*b*d^2*x - I*b*c
*d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) + sin(b*x + a)) + 4*((-I*b*d^2*x -
 I*b*c*d)*cos(b*x + a)^4 + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(I*co
s(b*x + a) - sin(b*x + a)) + 4*((-I*b*d^2*x - I*b*c*d)*cos(b*x + a)^4 + (I*
b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + sin(b*x + a)) +
4*((I*b*d^2*x + I*b*c*d)*cos(b*x + a)^4 + (-I*b*d^2*x - I*b*c*d)*cos(b*x +
a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 4*((-I*b*d^2*x - I*b*c*d)*cos
(b*x + a)^4 + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I
*sin(b*x + a)) + 4*((I*b*d^2*x + I*b*c*d)*cos(b*x + a)^4 + (-I*b*d^2*x - I*
b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - ((2*b^2*d^2*
x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*cos(b*x + a)^4 - (2*b^2*d^2*x^2 + 4*b^
2*c*d*x + 2*b^2*c^2 + d^2)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a
) + 1) + ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*b^2
*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) + I*si
n(b*x + a) + I) - ((2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*cos(b*x
+ a)^4 - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*cos(b*x + a)^2)*lo
g(cos(b*x + a) - I*sin(b*x + a) + 1) + ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1
)*d^2)*cos(b*x + a)^4 - (2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x +
 a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) + 2*((b^2*d^2*x^2 + 2*b^2*c*d
*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a
*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) +
2*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^4 - (b^2*
d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(I*cos(b*x
```

```
+ a) - sin(b*x + a) + 1) + 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*
d^2)*cos(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos
(b*x + a)^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 2*((b^2*d^2*x^2 + 2*
b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*
x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-I*cos(b*x + a) - sin(b*x + a)
 + 1) - ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*b^2*
c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) +
1/2*I*sin(b*x + a) + 1/2) - ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(
b*x + a)^4 - (2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^2)*log(
-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 2*((b^2*d^2*x^2 + 2*b^2*c*d
*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a
*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) +
 ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*b^2*c^2 - 4
*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x +
 a) + I) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a
)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log
(-cos(b*x + a) - I*sin(b*x + a) + 1) + ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1
)*d^2)*cos(b*x + a)^4 - (2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x +
 a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 4*(d^2*cos(b*x + a)^4 - d^
2*cos(b*x + a)^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 4*(d^2*cos(b*
x + a)^4 - d^2*cos(b*x + a)^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) +
4*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*polylog(3, I*cos(b*x + a) + sin
(b*x + a)) + 4*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*polylog(3, I*cos(b
*x + a) - sin(b*x + a)) + 4*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*polyl
og(3, -I*cos(b*x + a) + sin(b*x + a)) + 4*(d^2*cos(b*x + a)^4 - d^2*cos(b*x
 + a)^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 4*(d^2*cos(b*x + a)^4
 - d^2*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 4*(d^2*
cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) - I*sin(b*x +
 a)))/(b^3*cos(b*x + a)^4 - b^3*cos(b*x + a)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

3.325 $\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=110

$$-\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{d\csc(2a+2bx)}{b^2}-\frac{2(c+dx)\cot(2a+2bx)\csc(2a+2bx)}{b}+\frac{id\text{PolyLog}(2,-e^2}{b^2}$$

[Out] -4*(d*x+c)*arctanh(exp(2*I*(b*x+a)))/b-d*csc(2*b*x+2*a)/b^2-2*(d*x+c)*cot(2*b*x+2*a)*csc(2*b*x+2*a)/b+I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2-I*d*polylog(2,exp(2*I*(b*x+a)))/b^2

Rubi [A]
time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4504, 4270, 4268, 2317, 2438}

$$\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{d\csc(2a+2bx)}{b^2}-\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{2(c+dx)\cot(2a+2bx)\csc(2a+2bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^3,x]

[Out] (-4*(c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b - (d*Csc[2*a + 2*b*x])/b^2 - (2*(c + d*x)*Cot[2*a + 2*b*x]*Csc[2*a + 2*b*x])/b + (I*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx = 8\int (c+dx)\csc^3(2a+2bx)\,dx$$
$$= -\frac{d\csc(2a+2bx)}{b^2} - \frac{2(c+dx)\cot(2a+2bx)\csc(2a+2bx)}{b} + 4\int ($$
$$= -\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\csc(2a+2bx)}{b^2} - \frac{2(c+dx)\cot(2$$
$$= -\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\csc(2a+2bx)}{b^2} - \frac{2(c+dx)\cot(2$$
$$= -\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\csc(2a+2bx)}{b^2} - \frac{2(c+dx)\cot(2$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 236 vs. 2(110) = 220.
time = 2.23, size = 236, normalized size = 2.15

$$-\frac{d\cot(a+bx)}{2b^2} - \frac{c\csc^2(a+bx)}{2b} + \frac{d(2a-2(a+bx))\csc^2(a+bx)}{4b^2} - \frac{2c\log(\cos(a+bx))}{b} + \frac{2c\log(\sin(a+bx))}{b} - \frac{2ad\log(\tan(a+bx))}{b^2} + \frac{d\left(2(a+bx)\left(\log\left(1-e^{2i(a+bx)}\right)-\log\left(1+e^{2i(a+bx)}\right)\right)+i\left(\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)-\text{PolyLog}\left(2,e^{2i(a+bx)}\right)\right)\right)}{b^2} + \frac{c\sec^2(a+bx)}{2b} + \frac{d(-2a+2(a+bx))\sec^2(a+bx)}{4b^2} - \frac{d\tan(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] -1/2*(d*Cot[a + b*x])/b^2 - (c*Csc[a + b*x]^2)/(2*b) + (d*(2*a - 2*(a + b*x
))*Csc[a + b*x]^2)/(4*b^2) - (2*c*Log[Cos[a + b*x]])/b + (2*c*Log[Sin[a + b
*x]])/b - (2*a*d*Log[Tan[a + b*x]])/b^2 + (d*(2*(a + b*x)*(Log[1 - E^((2*I)
*(a + b*x))] - Log[1 + E^((2*I)*(a + b*x))]) + I*(PolyLog[2, -E^((2*I)*(a +
 b*x))] - PolyLog[2, E^((2*I)*(a + b*x))])))/b^2 + (c*Sec[a + b*x]^2)/(2*b)
 + (d*(-2*a + 2*(a + b*x))*Sec[a + b*x]^2)/(4*b^2) - (d*Tan[a + b*x])/(2*b^
2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(102) = 204$.
time = 0.17, size = 325, normalized size = 2.95

method	result
risch	$\frac{4dxb\,e^{6i(bx+a)}+4cb\,e^{6i(bx+a)}-2id\,e^{6i(bx+a)}+4bdx\,e^{2i(bx+a)}+4bc\,e^{2i(bx+a)}+2id\,e^{2i(bx+a)}}{b^2\left(e^{2i(bx+a)}-1\right)^2\left(1+e^{2i(bx+a)}\right)^2}+\frac{2c\ln\left(e^{i(bx+a)}-1\right)}{b}+\frac{2c\ln\left(e^{i(bx+a)}+\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2/(exp(2*I*(b*x+a))-1)^2/(1+exp(2*I*(b*x+a)))^2*(2*d*x*b*exp(6*I*(b*x+a
))+2*c*b*exp(6*I*(b*x+a))-I*d*exp(6*I*(b*x+a))+2*b*d*x*exp(2*I*(b*x+a))+2*b
*c*exp(2*I*(b*x+a))+I*d*exp(2*I*(b*x+a)))+2/b*c*ln(exp(I*(b*x+a))-1)+2/b*c*
ln(exp(I*(b*x+a))+1)-2/b*c*ln(1+exp(2*I*(b*x+a)))+2/b*d*ln(1-exp(I*(b*x+a))
)*x+2/b^2*d*ln(1-exp(I*(b*x+a)))*a-2*I/b^2*d*polylog(2,exp(I*(b*x+a)))-2/b*
d*ln(1+exp(2*I*(b*x+a)))*x+I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b*d*ln(ex
p(I*(b*x+a))+1)*x-2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2-2/b^2*d*a*ln(exp(I*(
b*x+a))-1)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1070 vs. $2(98) = 196$.
time = 0.66, size = 1070, normalized size = 9.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -(2*(b*d*x + b*c + (b*d*x + b*c)*cos(8*b*x + 8*a) - 2*(b*d*x + b*c)*cos(4*b
*x + 4*a) - (-I*b*d*x - I*b*c)*sin(8*b*x + 8*a) - 2*(I*b*d*x + I*b*c)*sin(4
*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 2*(b*d*x + b
*c + (b*d*x + b*c)*cos(8*b*x + 8*a) - 2*(b*d*x + b*c)*cos(4*b*x + 4*a) + (I
*b*d*x + I*b*c)*sin(8*b*x + 8*a) + 2*(-I*b*d*x - I*b*c)*sin(4*b*x + 4*a))*a
rctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(b*c*cos(8*b*x + 8*a) - 2*b*c*co
s(4*b*x + 4*a) + I*b*c*sin(8*b*x + 8*a) - 2*I*b*c*sin(4*b*x + 4*a) + b*c)*a
rctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*(b*d*x*cos(8*b*x + 8*a) - 2*b*d*
x*cos(4*b*x + 4*a) + I*b*d*x*sin(8*b*x + 8*a) - 2*I*b*d*x*sin(4*b*x + 4*a)
+ b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*(-2*I*b*d*x - 2*I*b*c
 - d)*cos(6*b*x + 6*a) - 2*(-2*I*b*d*x - 2*I*b*c + d)*cos(2*b*x + 2*a) - (d
*cos(8*b*x + 8*a) - 2*d*cos(4*b*x + 4*a) + I*d*sin(8*b*x + 8*a) - 2*I*d*sin
(4*b*x + 4*a) + d)*dilog(-e^(2*I*b*x + 2*I*a)) + 2*(d*cos(8*b*x + 8*a) - 2*
d*cos(4*b*x + 4*a) + I*d*sin(8*b*x + 8*a) - 2*I*d*sin(4*b*x + 4*a) + d)*dil
og(-e^(I*b*x + I*a)) + 2*(d*cos(8*b*x + 8*a) - 2*d*cos(4*b*x + 4*a) + I*d*s
in(8*b*x + 8*a) - 2*I*d*sin(4*b*x + 4*a) + d)*dilog(e^(I*b*x + I*a)) + (-I*
```

```
b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(8*b*x + 8*a) - 2*(-I*b*d*x - I*b*c)*
cos(4*b*x + 4*a) + (b*d*x + b*c)*sin(8*b*x + 8*a) - 2*(b*d*x + b*c)*sin(4*b
*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)
 + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(8*b*x + 8*a) - 2*(I*b*d*x
+ I*b*c)*cos(4*b*x + 4*a) - (b*d*x + b*c)*sin(8*b*x + 8*a) + 2*(b*d*x + b*c
)*sin(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) +
1) + (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(8*b*x + 8*a) - 2*(I*b*d*x + I
*b*c)*cos(4*b*x + 4*a) - (b*d*x + b*c)*sin(8*b*x + 8*a) + 2*(b*d*x + b*c)*s
in(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1)
- 2*(2*b*d*x + 2*b*c - I*d)*sin(6*b*x + 6*a) - 2*(2*b*d*x + 2*b*c + I*d)*si
n(2*b*x + 2*a))/(-I*b^2*cos(8*b*x + 8*a) + 2*I*b^2*cos(4*b*x + 4*a) + b^2*s
in(8*b*x + 8*a) - 2*b^2*sin(4*b*x + 4*a) - I*b^2)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1193 vs. 2(98) = 196.
time = 2.59, size = 1193, normalized size = 10.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 - d*cos(b*x + a)*sin(b*x + a)
+ b*c + 2*(I*d*cos(b*x + a)^4 - I*d*cos(b*x + a)^2)*dilog(cos(b*x + a) + I*
sin(b*x + a)) + 2*(-I*d*cos(b*x + a)^4 + I*d*cos(b*x + a)^2)*dilog(cos(b*x
+ a) - I*sin(b*x + a)) + 2*(I*d*cos(b*x + a)^4 - I*d*cos(b*x + a)^2)*dilog(
I*cos(b*x + a) + sin(b*x + a)) + 2*(-I*d*cos(b*x + a)^4 + I*d*cos(b*x + a)^
2)*dilog(I*cos(b*x + a) - sin(b*x + a)) + 2*(-I*d*cos(b*x + a)^4 + I*d*cos(
b*x + a)^2)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 2*(I*d*cos(b*x + a)^4 -
 I*d*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 2*(-I*d*cos(b*
x + a)^4 + I*d*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + 2*(I
*d*cos(b*x + a)^4 - I*d*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a
)) - 2*((b*d*x + b*c)*cos(b*x + a)^4 - (b*d*x + b*c)*cos(b*x + a)^2)*log(co
s(b*x + a) + I*sin(b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x + a)^4 - (b*c - a
*d)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + I) - 2*((b*d*x + b*
c)*cos(b*x + a)^4 - (b*d*x + b*c)*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin(
b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x + a)^4 - (b*c - a*d)*cos(b*x + a)^2)
*log(cos(b*x + a) - I*sin(b*x + a) + I) + 2*((b*d*x + a*d)*cos(b*x + a)^4 -
 (b*d*x + a*d)*cos(b*x + a)^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(
(b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2)*log(I*cos(b*x
+ a) - sin(b*x + a) + 1) + 2*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d)*
cos(b*x + a)^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 2*((b*d*x + a*d)*
cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2)*log(-I*cos(b*x + a) - sin(b*
x + a) + 1) - 2*((b*c - a*d)*cos(b*x + a)^4 - (b*c - a*d)*cos(b*x + a)^2)*l
og(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*((b*c - a*d)*cos(b*x +
```

```
 a)^4 - (b*c - a*d)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x +
 a) + 1/2) - 2*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2
)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x + a)^4 -
 (b*c - a*d)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - 2*((
b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2)*log(-cos(b*x +
a) - I*sin(b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x + a)^4 - (b*c - a*d)*cos(
b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/(b^2*cos(b*x + a)^4 -
b^2*cos(b*x + a)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

3.326 $\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=24

$$8\text{Int}\left(\frac{\csc^3(2a+2bx)}{c+dx},x\right)$$

```
[Out] 8*Unintegrable(csc(2*b*x+2*a)^3/(d*x+c),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x),x]
[Out] 8*Defer[Int][Csc[2*a + 2*b*x]^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx = 8\int \frac{\csc^3(2a+2bx)}{c+dx}\,dx$$

Mathematica [A]
time = 25.27, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x),x]
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x), x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^3\left(bx+a\right)\right)\left(\sec^3\left(bx+a\right)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] (2*(2*(b*d*x + b*c)*cos(6*b*x + 6*a) + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d
*sin(6*b*x + 6*a) + d*sin(2*b*x + 2*a))*cos(8*b*x + 8*a) + 4*(b*d*x + b*c -
 2*(b*d*x + b*c)*cos(4*b*x + 4*a) - d*sin(4*b*x + 4*a))*cos(6*b*x + 6*a) -
4*(2*(b*d*x + b*c)*cos(2*b*x + 2*a) + d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
+ 4*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 +
 (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(8*b*x + 8*a)^2 + 4*(b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*sin(8*b*x + 8*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(8
*b*x + 8*a)*sin(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(
4*b*x + 4*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a))*cos(8*b*x + 8*a) - 4*(b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a))*integrate(2*(2*b^2*d^2*x^2 + 4*b
^2*c*d*x + 2*b^2*c^2 + d^2)*sin(2*b*x + 2*a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
 + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2
*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) - (b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(8*b*x + 8*a)^2 + 4*(b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + (b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*sin(8*b*x + 8*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*
c^2)*sin(4*b*x + 4*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a))*cos(8*b*x + 8*a) - 4*(b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a))*integrate((2*b^2*d^2*x^
2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*
d*x + b^2*c^3)*cos(b*x + a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*
d*x + b^2*c^3)*sin(b*x + a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^
2*d*x + b^2*c^3)*cos(b*x + a)), x) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 +
 (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(8*b*x + 8*a)^2 + 4*(b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*sin(8*b*x + 8*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(8
*b*x + 8*a)*sin(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(
4*b*x + 4*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 +
```

```
2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a))*cos(8*b*x + 8*a) - 4*(b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a))*integrate((2*b^2*d^2*x^2 + 4*b^2
*c*d*x + 2*b^2*c^2 + d^2)*sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*cos(b*x + a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*sin(b*x + a)^2 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*cos(b*x + a)), x) + 2*(d*cos(6*b*x + 6*a) - d*cos(2*b*x + 2*a) + 2*
(b*d*x + b*c)*sin(6*b*x + 6*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*sin(8*b*
x + 8*a) + 2*(2*d*cos(4*b*x + 4*a) - 4*(b*d*x + b*c)*sin(4*b*x + 4*a) - d)*
sin(6*b*x + 6*a) + 4*(d*cos(2*b*x + 2*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a)
)*sin(4*b*x + 4*a) + 2*d*sin(2*b*x + 2*a))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(8*b*x + 8*a)^2 + 4*(b^2*d^
2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2)*sin(8*b*x + 8*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*sin(4*b*x + 4*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a))*cos(8*b*x + 8*a) - 4*(b^2*d^
2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^3/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**3/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^3/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^3\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)),x)
```

```
[Out] int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)), x)
```

3.327 $\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=24

$$8\text{Int}\left(\frac{\csc^3(2a+2bx)}{(c+dx)^2},x\right)$$

[Out] 8*Unintegrable(csc(2*b*x+2*a)^3/(d*x+c)^2,x)

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2,x]

[Out] 8*Defer[Int][Csc[2*a + 2*b*x]^3/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = 8\int \frac{\csc^3(2a+2bx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 29.66, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2,x]

[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2, x]

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^3(bx+a)\right)\left(\sec^3(bx+a)\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x)

[Out] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")

[Out] (4*((b*d*x + b*c)*cos(6*b*x + 6*a) + (b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(6*b*x + 6*a) + d*sin(2*b*x + 2*a))*cos(8*b*x + 8*a) + 4*(b*d*x + b*c - 2*(b*d*x + b*c)*cos(4*b*x + 4*a) - 2*d*sin(4*b*x + 4*a))*cos(6*b*x + 6*a) - 8*((b*d*x + b*c)*cos(2*b*x + 2*a) + d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(8*b*x + 8*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(8*b*x + 8*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a))*cos(8*b*x + 8*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a))*integrate(2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 3*d^2)*sin(2*b*x + 2*a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)), x) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(8*b*x + 8*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(8*b*x + 8*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a))*cos(8*b*x + 8*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a))*integrate((2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 3*d^2)*sin(b*x + a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x +

```
a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x
 + b^2*c^4)*cos(b*x + a)), x) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*
d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*c
os(8*b*x + 8*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*
c^3)*cos(4*b*x + 4*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
b^2*c^3)*sin(8*b*x + 8*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*
d*x + b^2*c^3)*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + 4*(b^2*d^3*x^3 + 3*b^2*c
*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 + 2*(b^2*d^3*x^3 + 3
*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a))*cos(8*b*x + 8*a) - 4*(b^2*d^3
*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a))*integra
te((2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 3*d^2)*sin(b*x + a)/(b^2*d^4*
x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*
d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*co
s(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c
^3*d*x + b^2*c^4)*sin(b*x + a)^2 - 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2
*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) + 4*(d*cos(6*b*x
+ 6*a) - d*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(6*b*x + 6*a) + (b*d*x + b*c
)*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + 4*(2*d*cos(4*b*x + 4*a) - 2*(b*d*x +
 b*c)*sin(4*b*x + 4*a) - d)*sin(6*b*x + 6*a) + 8*(d*cos(2*b*x + 2*a) - (b*d
*x + b*c)*sin(2*b*x + 2*a))*sin(4*b*x + 4*a) + 4*d*sin(2*b*x + 2*a))/(b^2*d
^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c
*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(8*b*x + 8*a)^2 + 4*(b^2*d^3*x^3 + 3
*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(8*b*x + 8*a)^2 - 4*(b^2*d
^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(8*b*x + 8*a)*sin(4*
b*x + 4*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*si
n(4*b*x + 4*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c
^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x
+ 4*a))*cos(8*b*x + 8*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
 + b^2*c^3)*cos(4*b*x + 4*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)**3*sec(b*x+a)**3/(d*x+c)**2,x)

[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**3/(c + d*x)**2, x)

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")

[Out] Timed out

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^2),x)

[Out] int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^2), x)

3.328 $\int x\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=83

$$-\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}+\frac{20F\left(\frac{1}{2}(a+bx)\middle|2\right)}{147b^2}+\frac{20\sqrt{\cos(a+bx)}\,\sin(a+bx)}{147b^2}+\frac{4\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)}{49b^2}$$

```
[Out] -2/7*x*cos(b*x+a)^(7/2)/b+20/147*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2
*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))/b^2+4/49*cos(b*x+a)^(5/2)*sin(b
*x+a)/b^2+20/147*sin(b*x+a)*cos(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3525, 2715, 2720}

$$\frac{20F\left(\frac{1}{2}(a+bx)\middle|2\right)}{147b^2}+\frac{4\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{49b^2}+\frac{20\sin(a+bx)\sqrt{\cos(a+bx)}}{147b^2}-\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]^(5/2)*Sin[a + b*x],x]
```

```
[Out] (-2*x*Cos[a + b*x]^(7/2))/(7*b) + (20*EllipticF[(a + b*x)/2, 2])/(147*b^2)
+ (20*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(147*b^2) + (4*Cos[a + b*x]^(5/2)*Si
n[a + b*x])/(49*b^2)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3525

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p
 + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx = -\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b} + \frac{2\int\cos^{\frac{7}{2}}(a+bx)\,dx}{7b}$$
$$= -\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b} + \frac{4\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)}{49b^2} + \frac{10\int\cos^{\frac{3}{2}}(a+bx)\,dx}{49b}$$
$$= -\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b} + \frac{20\sqrt{\cos(a+bx)}\,\sin(a+bx)}{147b^2} + \frac{4\cos^{\frac{5}{2}}(a+bx)\sin(}{49b^2}$$
$$= -\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b} + \frac{20F\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{147b^2} + \frac{20\sqrt{\cos(a+bx)}\,\sin(a+bx)}{147b^2}$$

Mathematica [A]
time = 0.36, size = 73, normalized size = 0.88

$$\frac{40F\left(\frac{1}{2}(a+bx)\middle|\,2\right)+\sqrt{\cos(a+bx)}\,(-63bx\cos(a+bx)-21bx\cos(3(a+bx))+46\sin(a+bx)+6\sin(3(a+bx)))}{294b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]^(5/2)*Sin[a + b*x],x]
```

```
[Out] (40*EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*(-63*b*x*Cos[a + b*x] -
21*b*x*Cos[3*(a + b*x)] + 46*Sin[a + b*x] + 6*Sin[3*(a + b*x)]))/(294*b^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\left(\cos^{\frac{5}{2}}(bx+a)\right)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)^(5/2)*sin(b*x+a),x)
```

```
[Out] int(x*cos(b*x+a)^(5/2)*sin(b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)^(5/2)*sin(b*x + a), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)**(5/2)*sin(b*x+a),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4848 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)^(5/2)*sin(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\cos\left(a+b\,x\right)^{5/2}\,\sin\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)^(5/2)*sin(a + b*x),x)
```

```
[Out] int(x*cos(a + b*x)^(5/2)*sin(a + b*x), x)
```

3.329 $\int x \cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=60

$$-\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}+\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}+\frac{4\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)}{25b^2}$$

```
[Out] -2/5*x*cos(b*x+a)^(5/2)/b+12/25*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*
b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b^2+4/25*cos(b*x+a)^(3/2)*sin(b*
x+a)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3525, 2715, 2719}

$$\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}+\frac{4\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{25b^2}-\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]^(3/2)*Sin[a + b*x],x]
```

```
[Out] (-2*x*Cos[a + b*x]^(5/2))/(5*b) + (12*EllipticE[(a + b*x)/2, 2])/(25*b^2) +
 (4*Cos[a + b*x]^(3/2)*Sin[a + b*x])/(25*b^2)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3525

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p
 + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = -\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\int\cos^{\frac{5}{2}}(a+bx)\,dx}{5b}$$
$$= -\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)}{25b^2}+\frac{6\int\sqrt{\cos(a+bx)}\,dx}{25b}$$
$$= -\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}+\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}+\frac{4\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)}{25b^2}$$

Mathematica [A]
time = 0.41, size = 51, normalized size = 0.85

$$-\frac{2\left(-6E\left(\frac{1}{2}(a+bx)\middle|2\right)+\cos^{\frac{3}{2}}(a+bx)(5bx\cos(a+bx)-2\sin(a+bx))\right)}{25b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]^(3/2)*Sin[a + b*x],x]
[Out] (-2*(-6*EllipticE[(a + b*x)/2, 2] + Cos[a + b*x]^(3/2)*(5*b*x*Cos[a + b*x]
- 2*Sin[a + b*x])))/(25*b^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\left(\cos^{\frac{3}{2}}(bx+a)\right)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)^(3/2)*sin(b*x+a),x)
[Out] int(x*cos(b*x+a)^(3/2)*sin(b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)^(3/2)*sin(b*x + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)**(3/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)^(3/2)*sin(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x \cos\left(a+b\,x\right)^{3/2}\,\sin\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)^(3/2)*sin(a + b*x),x)
```

```
[Out] int(x*cos(a + b*x)^(3/2)*sin(a + b*x), x)
```

3.330 $\int x\sqrt{\cos(a+bx)}\,\sin(a+bx)\,dx$

Optimal. Leaf size=60

$$-\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}+\frac{4F\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{9b^2}+\frac{4\sqrt{\cos(a+bx)}\,\sin(a+bx)}{9b^2}$$

```
[Out] -2/3*x*cos(b*x+a)^(3/2)/b+4/9*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*
x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))/b^2+4/9*sin(b*x+a)*cos(b*x+a)^(1/2
)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3525, 2715, 2720}

$$\frac{4F\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{9b^2}+\frac{4\sin(a+bx)\sqrt{\cos(a+bx)}}{9b^2}-\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[Cos[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (-2*x*Cos[a + b*x]^(3/2))/(3*b) + (4*EllipticF[(a + b*x)/2, 2])/(9*b^2) + (
4*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(9*b^2)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3525

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p
 + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx = -\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b} + \frac{2\int\cos^{\frac{3}{2}}(a+bx)\,dx}{3b}$$
$$= -\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b} + \frac{4\sqrt{\cos(a+bx)}\sin(a+bx)}{9b^2} + \frac{2\int\frac{1}{\sqrt{\cos(a+bx)}}}{9b}$$
$$= -\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b} + \frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{9b^2} + \frac{4\sqrt{\cos(a+bx)}\sin(a+bx)}{9b^2}$$

Mathematica [A]
time = 0.15, size = 52, normalized size = 0.87

$$\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)+2\sqrt{\cos(a+bx)}\,(-3bx\cos(a+bx)+2\sin(a+bx))}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[Cos[a + b*x]]*Sin[a + b*x],x]
[Out] (4*EllipticF[(a + b*x)/2, 2] + 2*Sqrt[Cos[a + b*x]]*(-3*b*x*Cos[a + b*x] +
2*Sin[a + b*x]))/(9*b^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\sin(bx+a)\left(\sqrt{\cos}\,(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)*cos(b*x+a)^(1/2),x)
[Out] int(x*sin(b*x+a)*cos(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*cos(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x*sqrt(cos(b*x + a))*sin(b*x + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin{(a+bx)}\sqrt{\cos{(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*cos(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sin(a + b*x)*sqrt(cos(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(cos(b*x + a))*sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sqrt{\cos\left(a+b\,x\right)}\,\sin\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)^(1/2)*sin(a + b*x),x)
```

```
[Out] int(x*cos(a + b*x)^(1/2)*sin(a + b*x), x)
```

3.331 $\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx$

Optimal. Leaf size=33

$$-\frac{2x\sqrt{\cos(a+bx)}}{b}+\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

```
[Out] 4*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b
*x),2^(1/2))/b^2-2*x*cos(b*x+a)^(1/2)/b
```

Rubi [A]

time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3525, 2719}

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}-\frac{2x\sqrt{\cos(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Sqrt[Cos[a + b*x]],x]
```

```
[Out] (-2*x*Sqrt[Cos[a + b*x]])/b + (4*EllipticE[(a + b*x)/2, 2])/b^2
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3525

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p
 + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx &= -\frac{2x\sqrt{\cos(a+bx)}}{b}+\frac{2\int \sqrt{\cos(a+bx)}\,dx}{b}\\ &= -\frac{2x\sqrt{\cos(a+bx)}}{b}+\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(33) = 66.
time = 1.92, size = 181, normalized size = 5.48

$$\frac{4\cos^2\left(\frac{1}{2}(a+bx)\right)^{3/2}\sqrt{\frac{\cos(a+bx)}{(1+\cos(a+bx))^2}}\sqrt{\frac{1}{1+\cos(a+bx)}}\left(2E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)|-1\right)\sqrt{\sec^2\left(\frac{1}{2}(a+bx)\right)}-2F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)|-1\right)\sqrt{\sec^2\left(\frac{1}{2}(a+bx)\right)}+\sqrt{\cos(a+bx)\sec^2\left(\frac{1}{2}(a+bx)\right)}\left(-bx+2\tan\left(\frac{1}{2}(a+bx)\right)\right)\right)}{b^2\sqrt{\frac{\cos(a+bx)}{1+\cos(a+bx)}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sin[a + b*x])/Sqrt[Cos[a + b*x]],x]
```

```
[Out] (4*(Cos[(a + b*x)/2]^2)^(3/2)*Sqrt[Cos[a + b*x]/(1 + Cos[a + b*x])^2]*Sqrt[
(1 + Cos[a + b*x])^(-1)]*(2*EllipticE[ArcSin[Tan[(a + b*x)/2]], -1]*Sqrt[Se
c[(a + b*x)/2]^2] - 2*EllipticF[ArcSin[Tan[(a + b*x)/2]], -1]*Sqrt[Sec[(a +
 b*x)/2]^2] + Sqrt[Cos[a + b*x]*Sec[(a + b*x)/2]^2]*(-(b*x) + 2*Tan[(a + b*
x)/2])))/(b^2*Sqrt[Cos[a + b*x]/(1 + Cos[a + b*x])])
```

Maple [C] Result contains complex when optimal does not.
time = 0.15, size = 310, normalized size = 9.39

method	result
risch	$-\frac{(bx+2i)\left(1+e^{2i(bx+a)}\right)\sqrt{2}\,e^{-i(bx+a)}}{b^2\sqrt{\left(1+e^{2i(bx+a)}\right)e^{-i(bx+a)}}}-\frac{2i\left(-\frac{2\left(1+e^{2i(bx+a)}\right)}{\sqrt{\left(1+e^{2i(bx+a)}\right)e^{i(bx+a)}}}+\frac{i\sqrt{-i\left(e^{i(bx+a)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(bx+}\right.}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+2*I)*(exp(I*(b*x+a))^2+1)/b^2*2^(1/2)/((exp(I*(b*x+a))^2+1)/exp(I*(b*
x+a)))^(1/2)/exp(I*(b*x+a))-2*I/b^2*(-2*(exp(I*(b*x+a))^2+1)/((exp(I*(b*x+a
))^2+1)*exp(I*(b*x+a)))^(1/2)+I*(-I*(exp(I*(b*x+a))+I))^(1/2)*2^(1/2)*(I*(e
xp(I*(b*x+a))-I))^(1/2)*(I*exp(I*(b*x+a)))^(1/2)/(exp(I*(b*x+a))^3+exp(I*(b
*x+a)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1/2))+I*
EllipticF((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1/2))))*2^(1/2)/((exp(I*(b*x
+a))^2+1)/exp(I*(b*x+a)))^(1/2)*((exp(I*(b*x+a))^2+1)*exp(I*(b*x+a)))^(1/2)
/exp(I*(b*x+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/sqrt(cos(b*x + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin{(a+bx)}}{\sqrt{\cos{(a+bx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sin(a + b*x)/sqrt(cos(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/sqrt(cos(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\,\sin\left(a+b\,x\right)}{\sqrt{\cos\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/cos(a + b*x)^(1/2),x)
```

```
[Out] int((x*sin(a + b*x))/cos(a + b*x)^(1/2), x)
```

3.332 $\int \frac{x \sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)} \, dx$

Optimal. Leaf size=33

$$\frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

```
[Out] -4*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*
b*x),2^(1/2))/b^2+2*x/b/cos(b*x+a)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3525, 2720}

$$\frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(3/2),x]
```

```
[Out] (2*x)/(b*Sqrt[Cos[a + b*x]]) - (4*EllipticF[(a + b*x)/2, 2])/b^2
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3525

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p
 + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x \sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)} \, dx &= \frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{2\int \frac{1}{\sqrt{\cos(a+bx)}} \, dx}{b} \\
&= \frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 33, normalized size = 1.00

$$\frac{2x}{b\sqrt{\cos(a+bx)}}-\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(3/2),x]
[Out] (2*x)/(b*Sqrt[Cos[a + b*x]]) - (4*EllipticF[(a + b*x)/2, 2])/b^2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x\sin\left(bx+a\right)}{\cos\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(3/2),x)
[Out] int(x*sin(b*x+a)/cos(b*x+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin\left(a+bx\right)}{\cos^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(3/2),x)
```

```
[Out] Integral(x*sin(a + b*x)/cos(a + b*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\,\sin\left(a+b\,x\right)}{{\cos\left(a+b\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/cos(a + b*x)^(3/2),x)
```

```
[Out] int((x*sin(a + b*x))/cos(a + b*x)^(3/2), x)
```

3.333 $\int \frac{x \sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)} dx$

Optimal. Leaf size=60

$$\frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}+\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2}-\frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}}$$

```
[Out] 2/3*x/b/cos(b*x+a)^(3/2)+4/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x
)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b^2-4/3*sin(b*x+a)/b^2/cos(b*x+a)^(
1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3525, 2716, 2719}

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2}-\frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}}+\frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(5/2),x]
```

```
[Out] (2*x)/(3*b*Cos[a + b*x]^(3/2)) + (4*EllipticE[(a + b*x)/2, 2])/(3*b^2) - (4
*Sin[a + b*x])/(3*b^2*Sqrt[Cos[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3525

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p
 + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)} - \frac{2\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx}{3b}$$
$$= \frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)} - \frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}} + \frac{2\int \sqrt{\cos(a+bx)}\,dx}{3b}$$
$$= \frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)} + \frac{4E\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{3b^2} - \frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}}$$

Mathematica [**A**]
time = 0.20, size = 54, normalized size = 0.90

$$\frac{2\left(bx+2\cos^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|\,2\right)-\sin(2(a+bx))\right)}{3b^2\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(5/2),x]
[Out] (2*(b*x + 2*Cos[a + b*x]^(3/2)*EllipticE[(a + b*x)/2, 2] - Sin[2*(a + b*x)]
))/(3*b^2*Cos[a + b*x]^(3/2))
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(5/2),x)
[Out] int(x*sin(b*x+a)/cos(b*x+a)^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(5/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\sin\left(a+b\,x\right)}{{\cos\left(a+b\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/cos(a + b*x)^(5/2),x)
[Out] int((x*sin(a + b*x))/cos(a + b*x)^(5/2), x)
```

3.334 $\int \frac{x\sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=60

$$\frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b^2}-\frac{4\sin(a+bx)}{15b^2\cos^{\frac{3}{2}}(a+bx)}$$

```
[Out] 2/5*x/b/cos(b*x+a)^(5/2)-4/15*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*
x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))/b^2-4/15*sin(b*x+a)/b^2/cos(b*x+a)
^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3525, 2716, 2720}

$$-\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b^2}-\frac{4\sin(a+bx)}{15b^2\cos^{\frac{3}{2}}(a+bx)}+\frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(7/2),x]
```

```
[Out] (2*x)/(5*b*Cos[a + b*x]^(5/2)) - (4*EllipticF[(a + b*x)/2, 2])/(15*b^2) - (
4*Sin[a + b*x])/(15*b^2*Cos[a + b*x]^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3525

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p
 + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{2\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}\,dx}{5b}$$
$$= \frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{4\sin(a+bx)}{15b^2\cos^{\frac{3}{2}}(a+bx)} - \frac{2\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{15b}$$
$$= \frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{4F\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{15b^2} - \frac{4\sin(a+bx)}{15b^2\cos^{\frac{3}{2}}(a+bx)}$$

Mathematica [A]
time = 0.25, size = 53, normalized size = 0.88

$$-\frac{2\left(-3bx+2\cos^{\frac{5}{2}}(a+bx)F\left(\frac{1}{2}(a+bx)\middle|\,2\right)+\sin(2(a+bx))\right)}{15b^2\cos^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(7/2),x]
[Out] (-2*(-3*b*x + 2*Cos[a + b*x]^(5/2)*EllipticF[(a + b*x)/2, 2] + Sin[2*(a + b
*x)]))/(15*b^2*Cos[a + b*x]^(5/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(7/2),x)
[Out] int(x*sin(b*x+a)/cos(b*x+a)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="maxima")
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(7/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\sin\left(a+b\,x\right)}{{\cos\left(a+b\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/cos(a + b*x)^(7/2),x)
```

```
[Out] int((x*sin(a + b*x))/cos(a + b*x)^(7/2), x)
```

3.335 $\int \frac{x \sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)} dx$

Optimal. Leaf size=83

$$\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} + \frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2} - \frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)} - \frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}}$$

```
[Out] 2/7*x/b/cos(b*x+a)^(7/2)+12/35*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b
*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b^2-4/35*sin(b*x+a)/b^2/cos(b*x+a
)^(5/2)-12/35*sin(b*x+a)/b^2/cos(b*x+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3525, 2716, 2719}

$$\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2} - \frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)} - \frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}} + \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(9/2),x]
```

```
[Out] (2*x)/(7*b*Cos[a + b*x]^(7/2)) + (12*EllipticE[(a + b*x)/2, 2])/(35*b^2) -
(4*Sin[a + b*x])/(35*b^2*Cos[a + b*x]^(5/2)) - (12*Sin[a + b*x])/(35*b^2*Sq
rt[Cos[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3525

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p
 + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx = \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} - \frac{2\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)}\,dx}{7b}$$

$$= \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} - \frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)} - \frac{6\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx}{35b}$$

$$= \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} - \frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)} - \frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}} + \frac{6\int \sqrt{\cos(a+bx)}\,dx}{35b}$$

$$= \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} + \frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2} - \frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)} - \frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}}$$

Mathematica [**A**]
time = 0.28, size = 65, normalized size = 0.78

$$\frac{20bx + 24\cos^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right) - 10\sin(2(a+bx)) - 3\sin(4(a+bx))}{70b^2\cos^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(9/2),x]
```

```
[Out] (20*b*x + 24*Cos[a + b*x]^(7/2)*EllipticE[(a + b*x)/2, 2] - 10*Sin[2*(a + b
*x)] - 3*Sin[4*(a + b*x)])/(70*b^2*Cos[a + b*x]^(7/2))
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(9/2),x)
```

```
[Out] int(x*sin(b*x+a)/cos(b*x+a)^(9/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(9/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin\left(a+b\,x\right)}{{\cos\left(a+b\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/cos(a + b*x)^(9/2),x)
```

```
[Out] int((x*sin(a + b*x))/cos(a + b*x)^(9/2), x)
```

3.336 $\int x \sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=103

$$\frac{12\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{35b^2}+\frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b}-\frac{12\sqrt{\sec(a+bx)}\,\sin(a+bx)}{35b^2}-\frac{4\sec^{\frac{5}{2}}(a$$

```
[Out] 2/7*x*sec(b*x+a)^(7/2)/b-4/35*sec(b*x+a)^(5/2)*sin(b*x+a)/b^2-12/35*sin(b*x
+a)*sec(b*x+a)^(1/2)/b^2+12/35*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b
*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)
/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4297, 3853, 3856, 2719}

$$-\frac{4\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{12\sin(a+bx)\sqrt{\sec(a+bx)}}{35b^2}+\frac{12\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2}+\frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*x]^(9/2)*Sin[a + b*x],x]
```

```
[Out] (12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(35*b^
2) + (2*x*Sec[a + b*x]^(7/2))/(7*b) - (12*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/
(35*b^2) - (4*Sec[a + b*x]^(5/2)*Sin[a + b*x])/(35*b^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4297

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\begin{aligned}\int x\sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx &= \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\int\sec^{\frac{7}{2}}(a+bx)\,dx}{7b}\\ &= \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b}-\frac{4\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)}{35b^2}-\frac{6\int\sec^{\frac{3}{2}}(a+bx)\,dx}{35b}\\ &= \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b}-\frac{12\sqrt{\sec(a+bx)}\,\sin(a+bx)}{35b^2}-\frac{4\sec^{\frac{5}{2}}(a+bx)\sin(a-}{35b^2}\\ &= \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b}-\frac{12\sqrt{\sec(a+bx)}\,\sin(a+bx)}{35b^2}-\frac{4\sec^{\frac{5}{2}}(a+bx)\sin(a-}{35b^2}\\ &= \frac{12\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{35b^2}+\frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b}-1\end{aligned}$$

Mathematica [A]
time = 0.30, size = 65, normalized size = 0.63

$$\frac{\sec^{\frac{7}{2}}(a+bx)\left(20bx+24\cos^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)-10\sin(2(a+bx))-3\sin(4(a+bx))\right)}{70b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*x]^(9/2)*Sin[a + b*x],x]
[Out] (Sec[a + b*x]^(7/2)*(20*b*x + 24*Cos[a + b*x]^(7/2)*EllipticE[(a + b*x)/2,
2] - 10*Sin[2*(a + b*x)] - 3*Sin[4*(a + b*x)]))/(70*b^2)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x\left(\sec^{\frac{9}{2}}(bx+a)\right)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(b*x+a)^(9/2)*sin(b*x+a),x)
[Out] int(x*sec(b*x+a)^(9/2)*sin(b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="maxima")
[Out] integrate(x*sec(b*x + a)^(9/2)*sin(b*x + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(9/2)*sin(b*x+a),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="giac")
[Out] integrate(x*sec(b*x + a)^(9/2)*sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sin\left(a+b\,x\right)\,{\left(\frac{1}{\cos\left(a+b\,x\right)}\right)}^{9/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*x)*(1/cos(a + b*x))^(9/2),x)
[Out] int(x*sin(a + b*x)*(1/cos(a + b*x))^(9/2), x)
```

3.337 $\int x\sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=80

$$-\frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|\,2\right)\sqrt{\sec(a+bx)}}{15b^2}+\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{4\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)}{15b^2}$$

```
[Out] 2/5*x*sec(b*x+a)^(5/2)/b-4/15*sec(b*x+a)^(3/2)*sin(b*x+a)/b^2-4/15*(cos(1/2
*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2
))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4297, 3853, 3856, 2720}

$$-\frac{4\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{15b^2}+\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*x]^(7/2)*Sin[a + b*x],x]
```

```
[Out] (-4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(15*b^
2) + (2*x*Sec[a + b*x]^(5/2))/(5*b) - (4*Sec[a + b*x]^(3/2)*Sin[a + b*x])/(
15*b^2)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4297

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\begin{aligned}\int x\sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx &= \frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\int\sec^{\frac{5}{2}}(a+bx)\,dx}{5b}\\ &= \frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{4\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)}{15b^2}-\frac{2\int\sqrt{\sec(a+bx)}\,dx}{15b}\\ &= \frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{4\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)}{15b^2}-\frac{\left(2\sqrt{\cos(a+bx)}\sqrt{\sec(a\right.}{}\\ &= -\frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{15b^2}+\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{4}{}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 61, normalized size = 0.76

$$\frac{2\sqrt{\sec(a+bx)}\left(-2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)+3bx\sec^2(a+bx)-2\tan(a+bx)\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*x]^(7/2)*Sin[a + b*x],x]
```

```
[Out] (2*Sqrt[Sec[a + b*x]]*(-2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 3*
b*x*Sec[a + b*x]^2 - 2*Tan[a + b*x]))/(15*b^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x\left(\sec^{\frac{7}{2}}(bx+a)\right)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(b*x+a)^(7/2)*sin(b*x+a),x)
```

```
[Out] int(x*sec(b*x+a)^(7/2)*sin(b*x+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="maxima")
[Out] integrate(x*sec(b*x + a)^(7/2)*sin(b*x + a), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(7/2)*sin(b*x+a),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="giac")
[Out] integrate(x*sec(b*x + a)^(7/2)*sin(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sin\left(a+b\,x\right)\,{\left(\frac{1}{\cos\left(a+b\,x\right)}\right)}^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*x)*(1/cos(a + b*x))^(7/2),x)
[Out] int(x*sin(a + b*x)*(1/cos(a + b*x))^(7/2), x)
```

3.338 $\int x \sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=80

$$\frac{4\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{3b^2}+\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}-\frac{4\sqrt{\sec(a+bx)}\,\sin(a+bx)}{3b^2}$$

```
[Out] 2/3*x*sec(b*x+a)^(3/2)/b-4/3*sin(b*x+a)*sec(b*x+a)^(1/2)/b^2+4/3*(cos(1/2*a
+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))
*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4297, 3853, 3856, 2719}

$$-\frac{4\sin(a+bx)\sqrt{\sec(a+bx)}}{3b^2}+\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2}+\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*x]^(5/2)*Sin[a + b*x],x]
```

```
[Out] (4*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b^2)
 + (2*x*Sec[a + b*x]^(3/2))/(3*b) - (4*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/(3*
b^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4297

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
```

```
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\int x\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx = \frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\int\sec^{\frac{3}{2}}(a+bx)\,dx}{3b}$$

$$= \frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{4\sqrt{\sec(a+bx)}\,\sin(a+bx)}{3b^2} + \frac{2\int\frac{1}{\sqrt{\sec(a+bx)}}\,dx}{3b}$$

$$= \frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{4\sqrt{\sec(a+bx)}\,\sin(a+bx)}{3b^2} + \frac{\left(2\sqrt{\cos(a+bx)}\,\sqrt{\text{se}\ldots}\right.}{\ldots}$$

$$= \frac{4\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{3b^2} + \frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{4\ldots}{\ldots}$$

Mathematica [A]
time = 0.20, size = 54, normalized size = 0.68

$$\frac{2\sec^{\frac{3}{2}}(a+bx)\left(bx+2\cos^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)-\sin(2(a+bx))\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*x]^(5/2)*Sin[a + b*x],x]
[Out] (2*Sec[a + b*x]^(3/2)*(b*x + 2*Cos[a + b*x]^(3/2)*EllipticE[(a + b*x)/2, 2]
 - Sin[2*(a + b*x)]))/(3*b^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x\left(\sec^{\frac{5}{2}}(bx+a)\right)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(b*x+a)^(5/2)*sin(b*x+a),x)
[Out] int(x*sec(b*x+a)^(5/2)*sin(b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*sec(b*x + a)^(5/2)*sin(b*x + a), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(5/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*x + a)^(5/2)*sin(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \sin(a+b\,x)\,{\left(\frac{1}{\cos(a+b\,x)}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*x)*(1/cos(a + b*x))^(5/2),x)
```

```
[Out] int(x*sin(a + b*x)*(1/cos(a + b*x))^(5/2), x)
```

3.339 $\int x\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=53

$$\frac{2x\sqrt{\sec(a+bx)}}{b}-\frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b^2}$$

```
[Out] 2*x*sec(b*x+a)^(1/2)/b-4*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*El
lipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4297, 3856, 2720}

$$\frac{2x\sqrt{\sec(a+bx)}}{b}-\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*x]^(3/2)*Sin[a + b*x],x]
```

```
[Out] (2*x*Sqrt[Sec[a + b*x]])/b - (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2
]*Sqrt[Sec[a + b*x]])/b^2
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4297

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\int x\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx=\frac{2x\sqrt{\sec(a+bx)}}{b}-\frac{2\int\sqrt{\sec(a+bx)}\,dx}{b}$$
$$=\frac{2x\sqrt{\sec(a+bx)}}{b}-\frac{\left(2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}}\,d}{b}$$
$$=\frac{2x\sqrt{\sec(a+bx)}}{b}-\frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b^2}$$

Mathematica [A]
time = 0.14, size = 42, normalized size = 0.79

$$\frac{2\left(bx-2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\right)\sqrt{\sec(a+bx)}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*x]^(3/2)*Sin[a + b*x],x]
[Out] (2*(b*x - 2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])*Sqrt[Sec[a + b*x]
])/b^2
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x\left(\sec^{\frac{3}{2}}(bx+a)\right)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(b*x+a)^(3/2)*sin(b*x+a),x)
[Out] int(x*sec(b*x+a)^(3/2)*sin(b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="maxima")
[Out] integrate(x*sec(b*x + a)^(3/2)*sin(b*x + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(3/2)*sin(b*x+a),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*x + a)^(3/2)*sin(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sin\left(a+b\,x\right)\,{\left(\frac{1}{\cos\left(a+b\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*x)*(1/cos(a + b*x))^(3/2),x)
```

```
[Out] int(x*sin(a + b*x)*(1/cos(a + b*x))^(3/2), x)
```

3.340 $\int x\sqrt{\sec(a+bx)}\,\sin(a+bx)\,dx$

Optimal. Leaf size=53

$$-\frac{2x}{b\sqrt{\sec(a+bx)}}+\frac{4\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b^2}$$

```
[Out] -2*x/b/sec(b*x+a)^(1/2)+4*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*E
llipticE(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {4297, 3856, 2719}

$$\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}-\frac{2x}{b\sqrt{\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[Sec[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (-2*x)/(b*Sqrt[Sec[a + b*x]]) + (4*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2
, 2]*Sqrt[Sec[a + b*x]])/b^2
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4297

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\int x\sqrt{\sec(a+bx)}\,\sin(a+bx)\,dx = -\frac{2x}{b\sqrt{\sec(a+bx)}} + \frac{2\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx}{b}$$
$$= -\frac{2x}{b\sqrt{\sec(a+bx)}} + \frac{\left(2\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\right)\int \sqrt{\cos(a+bx}}{b}$$
$$= -\frac{2x}{b\sqrt{\sec(a+bx)}} + \frac{4\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|\,2\right)\sqrt{\sec(a+bx)}}{b^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 132 vs. 2(53) = 106.
time = 2.42, size = 132, normalized size = 2.49

$$\frac{2\left(-bx+\frac{2E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)\middle|-1\right)\sec^2\left(\frac{1}{2}(a+bx)\right)}{\sqrt{\cos(a+bx)\sec^4\left(\frac{1}{2}(a+bx)\right)}}-\frac{2F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)\middle|-1\right)\sec^2\left(\frac{1}{2}(a+bx)\right)}{\sqrt{\cos(a+bx)\sec^4\left(\frac{1}{2}(a+bx)\right)}}+2\tan\left(\frac{1}{2}(a+bx)\right)\right)}{b^2\sqrt{\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[Sec[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (2*(-(b*x) + (2*EllipticE[ArcSin[Tan[(a + b*x)/2]], -1]*Sec[(a + b*x)/2]^2)
/Sqrt[Cos[a + b*x]*Sec[(a + b*x)/2]^4] - (2*EllipticF[ArcSin[Tan[(a + b*x)/
2]], -1]*Sec[(a + b*x)/2]^2)/Sqrt[Cos[a + b*x]*Sec[(a + b*x)/2]^4] + 2*Tan[
(a + b*x)/2]))/(b^2*Sqrt[Sec[a + b*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.08, size = 310, normalized size = 5.85

method	result
risch	$-\frac{(bx+2i)\left(1+e^{2i(bx+a)}\right)\sqrt{2}\,\sqrt{\frac{e^{i(bx+a)}}{1+e^{2i(bx+a)}}}\,e^{-i(bx+a)}}{b^2} - \frac{2i\left(-\frac{2\left(1+e^{2i(bx+a)}\right)}{\sqrt{\left(1+e^{2i(bx+a)}\right)e^{i(bx+a)}}}+\frac{i\sqrt{-i\left(e^{i(bx+a)}+i\right)}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)*sec(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+2*I)*(exp(I*(b*x+a))^2+1)/b^2*2^(1/2)*(exp(I*(b*x+a))/(exp(I*(b*x+a))
^2+1))^(1/2)/exp(I*(b*x+a))-2*I/b^2*(-2*(exp(I*(b*x+a))^2+1)/((exp(I*(b*x+a
```

```
))^2+1)*exp(I*(b*x+a)))^(1/2)+I*(-I*(exp(I*(b*x+a))+I))^(1/2)*2^(1/2)*(I*(e
xp(I*(b*x+a))-I))^(1/2)*(I*exp(I*(b*x+a)))^(1/2)/(exp(I*(b*x+a))^3+exp(I*(b
*x+a)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1/2))+I*
EllipticF((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1/2))))*2^(1/2)*(exp(I*(b*x+
a))/(exp(I*(b*x+a))^2+1))^(1/2)*((exp(I*(b*x+a))^2+1)*exp(I*(b*x+a)))^(1/2)
/exp(I*(b*x+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*sec(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x*sqrt(sec(b*x + a))*sin(b*x + a), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*sec(b*x+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin(a+bx)\sqrt{\sec(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*sec(b*x+a)**(1/2),x)
[Out] Integral(x*sin(a + b*x)*sqrt(sec(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*sec(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(sec(b*x + a))*sin(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sin\left(a+b\,x\right)\,\sqrt{\frac{1}{\cos\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*x)*(1/cos(a + b*x))^(1/2),x)
[Out] int(x*sin(a + b*x)*(1/cos(a + b*x))^(1/2), x)
```

3.341 $\int \frac{x \sin(a+bx)}{\sqrt{\sec(a+bx)}} \, dx$

Optimal. Leaf size=80

$$-\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}+\frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{9b^2}+\frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}}$$

```
[Out] -2/3*x/b/sec(b*x+a)^(3/2)+4/9*sin(b*x+a)/b^2/sec(b*x+a)^(1/2)+4/9*(cos(1/2*
a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2)
)*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4297, 3854, 3856, 2720}

$$\frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}}+\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{9b^2}-\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Sqrt[Sec[a + b*x]],x]
```

```
[Out] (-2*x)/(3*b*Sec[a + b*x]^(3/2)) + (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)
/2, 2]*Sqrt[Sec[a + b*x]])/(9*b^2) + (4*Sin[a + b*x])/(9*b^2*Sqrt[Sec[a + b
*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4297

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx &= -\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}+\frac{2\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx}{3b}\\ &= -\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}+\frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}}+\frac{2\int \sqrt{\sec(a+bx)}\,dx}{9b}\\ &= -\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}+\frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}}+\frac{\left(2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int \frac{}{\sqrt{\text{co}}}}{9b}\\ &= -\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}+\frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{9b^2}+\frac{4\sin(a+}{9b^2\sqrt{\sec(a}}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 63, normalized size = 0.79

$$\frac{\sqrt{\sec(a+bx)}\left(-6bx\cos^2(a+bx)+4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)+2\sin(2(a+bx))\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Sqrt[Sec[a + b*x]],x]
[Out] (Sqrt[Sec[a + b*x]]*(-6*b*x*Cos[a + b*x]^2 + 4*Sqrt[Cos[a + b*x]]*EllipticF
[(a + b*x)/2, 2] + 2*Sin[2*(a + b*x)]))/(9*b^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x\sin\left(bx+a\right)}{\sqrt{\sec\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/sec(b*x+a)^(1/2),x)
[Out] int(x*sin(b*x+a)/sec(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x*sin(b*x + a)/sqrt(sec(b*x + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin{\left(a + bx \right)}}{\sqrt{\sec{\left(a + bx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)**(1/2),x)
[Out] Integral(x*sin(a + b*x)/sqrt(sec(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(x*sin(b*x + a)/sqrt(sec(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin\left(a+b\,x\right)}{\sqrt{\frac{1}{\cos\left(a+b\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/(1/cos(a + b*x))^(1/2),x)
[Out] int((x*sin(a + b*x))/(1/cos(a + b*x))^(1/2), x)
```

3.342 $\int \frac{x \sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)} dx$

Optimal. Leaf size=80

$$-\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}+\frac{12\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{25b^2}+\frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)}$$

```
[Out] -2/5*x/b/sec(b*x+a)^(5/2)+4/25*sin(b*x+a)/b^2/sec(b*x+a)^(3/2)+12/25*(cos(1
/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1
/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4297, 3854, 3856, 2719}

$$\frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)}+\frac{12\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}-\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Sec[a + b*x]^(3/2),x]
```

```
[Out] (-2*x)/(5*b*Sec[a + b*x]^(5/2)) + (12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x
)/2, 2]*Sqrt[Sec[a + b*x]])/(25*b^2) + (4*Sin[a + b*x])/(25*b^2*Sec[a + b*x
]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4297

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)} + \frac{2\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx}{5b}$$

$$= -\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)} + \frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)} + \frac{6\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx}{25b}$$

$$= -\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)} + \frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)} + \frac{\left(6\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\right)\int \sqrt{\cos(a}}{25b}$$

$$= -\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)} + \frac{12\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{25b^2} + \frac{4\sin(a+}{25b^2\sec^{\frac{3}{2}}(a+}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 212 vs. 2(80) = 160.
time = 8.25, size = 212, normalized size = 2.65

$$\frac{\sqrt{\sec(a+bx)}\left(-\frac{1}{10}x\cos(a+bx)-\frac{1}{10}x\cos(3(a+bx))+\frac{\sin(a+bx)}{25b}+\frac{\sin(3(a+bx))}{25b}\right)}{b}+\frac{\cos^2\left(\frac{1}{2}(a+bx)\right)\sqrt{\sec(a+bx)}\left(12E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)\middle|-1\right)\sqrt{\cos(a+bx)\sec^4\left(\frac{1}{2}(a+bx)\right)}-12F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)\middle|-1\right)\sqrt{\cos(a+bx)\sec^4\left(\frac{1}{2}(a+bx)\right)}+\left(-5a+5(a+bx)-12\tan\left(\frac{1}{2}(a+bx)\right)\right)\left(-1+\tan^2\left(\frac{1}{2}(a+bx)\right)\right)\right)}{25b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Sec[a + b*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(-1/10*(x*Cos[a + b*x]) - (x*Cos[3*(a + b*x)])/10 + Sin
[a + b*x]/(25*b) + Sin[3*(a + b*x)]/(25*b)))/b + (Cos[(a + b*x)/2]^2*Sqrt[S
ec[a + b*x]]*(12*EllipticE[ArcSin[Tan[(a + b*x)/2]], -1]*Sqrt[Cos[a + b*x]*
Sec[(a + b*x)/2]^4] - 12*EllipticF[ArcSin[Tan[(a + b*x)/2]], -1]*Sqrt[Cos[a
 + b*x]*Sec[(a + b*x)/2]^4] + (-5*a + 5*(a + b*x) - 12*Tan[(a + b*x)/2])*(-
1 + Tan[(a + b*x)/2]^2)))/(25*b^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\sec(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/sec(b*x+a)^(3/2),x)
[Out] int(x*sin(b*x+a)/sec(b*x+a)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin\left(a+bx\right)}{\sec^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)**(3/2),x)
[Out] Integral(x*sin(a + b*x)/sec(a + b*x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin\left(a+b\,x\right)}{{\left(\frac{1}{\cos\left(a+b\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/(1/cos(a + b*x))^(3/2),x)
[Out] int((x*sin(a + b*x))/(1/cos(a + b*x))^(3/2), x)
```

3.343 $\int \frac{x\sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=103

$$-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}+\frac{20\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{147b^2}+\frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)}+\frac{20\sin(a+bx)}{147b^2\sqrt{\sec(a+bx)}}$$

```
[Out] -2/7*x/b/sec(b*x+a)^(7/2)+4/49*sin(b*x+a)/b^2/sec(b*x+a)^(5/2)+20/147*sin(b
*x+a)/b^2/sec(b*x+a)^(1/2)+20/147*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/
2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1
/2)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4297, 3854, 3856, 2720}

$$\frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)}+\frac{20\sin(a+bx)}{147b^2\sqrt{\sec(a+bx)}}+\frac{20\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{147b^2}-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Sec[a + b*x]^(5/2),x]
```

```
[Out] (-2*x)/(7*b*Sec[a + b*x]^(7/2)) + (20*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x
)/2, 2]*Sqrt[Sec[a + b*x]])/(147*b^2) + (4*Sin[a + b*x])/(49*b^2*Sec[a + b*
x]^(5/2)) + (20*Sin[a + b*x])/(147*b^2*Sqrt[Sec[a + b*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4297

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx &= -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}+\frac{2\int\frac{1}{\sec^{\frac{7}{2}}(a+bx)}\,dx}{7b} \\
&= -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}+\frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)}+\frac{10\int\frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx}{49b} \\
&= -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}+\frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)}+\frac{20\sin(a+bx)}{147b^2\sqrt{\sec(a+bx)}}+\frac{10\int\sqrt{\sec(a+bx)}}{147b} \\
&= -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}+\frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)}+\frac{20\sin(a+bx)}{147b^2\sqrt{\sec(a+bx)}}+\frac{\left(10\sqrt{\cos(a+bx)}\right.}{} \\
&= -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}+\frac{20\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|\,2\right)\sqrt{\sec(a+bx)}}{147b^2}+\frac{4\sin(a+}{49b^2\sec^{\frac{5}{2}}(a}
\end{aligned}$$

Mathematica [A]
time = 0.36, size = 89, normalized size = 0.86

$$\frac{\sqrt{\sec(a+bx)}\left(-63bx-84bx\cos(2(a+bx))-21bx\cos(4(a+bx))+80\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|\,2\right)+52\sin(2(a+bx))+6\sin(4(a+bx))\right)}{588b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Sec[a + b*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(-63*b*x - 84*b*x*Cos[2*(a + b*x)] - 21*b*x*Cos[4*(a +
b*x)] + 80*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 52*Sin[2*(a + b*x
)] + 6*Sin[4*(a + b*x)]))/(588*b^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\sec(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/sec(b*x+a)^(5/2),x)
```

```
[Out] int(x*sin(b*x+a)/sec(b*x+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin{(a+bx)}}{\sec^{\frac{5}{2}}{(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)**(5/2),x)
```

```
[Out] Integral(x*sin(a + b*x)/sec(a + b*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin\left(a+b\,x\right)}{{\left(\frac{1}{\cos\left(a+b\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/(1/cos(a + b*x))^(5/2),x)
[Out] int((x*sin(a + b*x))/(1/cos(a + b*x))^(5/2), x)
```

3.344 $\int x\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=88

$$-\frac{20F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{147b^2}+\frac{20\cos(a+bx)\sqrt{\sin(a+bx)}}{147b^2}+\frac{4\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{49b^2}+\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}$$

```
[Out] 20/147*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Ellipt
icF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2+4/49*cos(b*x+a)*sin(b*x+a)^(5/2)
/b^2+2/7*x*sin(b*x+a)^(7/2)/b+20/147*cos(b*x+a)*sin(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3524, 2715, 2720}

$$-\frac{20F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{147b^2}+\frac{4\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{49b^2}+\frac{20\sqrt{\sin(a+bx)}\cos(a+bx)}{147b^2}+\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Sin[a + b*x]^(5/2),x]
```

```
[Out] (-20*EllipticF[(a - Pi/2 + b*x)/2, 2])/(147*b^2) + (20*Cos[a + b*x]*Sqrt[Si
n[a + b*x]])/(147*b^2) + (4*Cos[a + b*x]*Sin[a + b*x]^(5/2))/(49*b^2) + (2*
x*Sin[a + b*x]^(7/2))/(7*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)\,dx = \frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\int\sin^{\frac{7}{2}}(a+bx)\,dx}{7b}$$
$$= \frac{4\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{49b^2} + \frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b} - \frac{10\int\sin^{\frac{3}{2}}(a+bx)\,dx}{49b}$$
$$= \frac{20\cos(a+bx)\sqrt{\sin(a+bx)}}{147b^2} + \frac{4\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{49b^2} + \frac{2x\sin^{\frac{7}{2}}(a-}{7b}$$
$$= -\frac{20F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{147b^2} + \frac{20\cos(a+bx)\sqrt{\sin(a+bx)}}{147b^2} + \frac{4\cos(a+b}{}$$

Mathematica [A]
time = 0.59, size = 67, normalized size = 0.76

$$\frac{40F\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)+\sqrt{\sin(a+bx)}\,(46\cos(a+bx)-6\cos(3(a+bx))+84bx\sin^3(a+bx))}{294b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Sin[a + b*x]^(5/2),x]
```

```
[Out] (40*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + Sqrt[Sin[a + b*x]]*(46*Cos[a + b*
x] - 6*Cos[3*(a + b*x)] + 84*b*x*Sin[a + b*x]^3))/(294*b^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\left(\sin^{\frac{5}{2}}(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*sin(b*x+a)^(5/2),x)
```

```
[Out] int(x*cos(b*x+a)*sin(b*x+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)**(5/2),x)
```

```
[Out] Integral(x*sin(a + b*x)**(5/2)*cos(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*sin(a + b*x)^(5/2),x)
```

```
[Out] int(x*cos(a + b*x)*sin(a + b*x)^(5/2), x)
```

3.345 $\int x \cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=65

$$-\frac{12E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{25b^2}+\frac{4\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)}{25b^2}+\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] 12/25*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Ellipti
cE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2+4/25*cos(b*x+a)*sin(b*x+a)^(3/2)/
b^2+2/5*x*sin(b*x+a)^(5/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3524, 2715, 2719}

$$-\frac{12E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2}+\frac{4\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{25b^2}+\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Sin[a + b*x]^(3/2),x]
```

```
[Out] (-12*EllipticE[(a - Pi/2 + b*x)/2, 2])/(25*b^2) + (4*Cos[a + b*x]*Sin[a + b
*x]^(3/2))/(25*b^2) + (2*x*Sin[a + b*x]^(5/2))/(5*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx = \frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\int \sin^{\frac{5}{2}}(a+bx)\,dx}{5b}$$
$$= \frac{4\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)}{25b^2} + \frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b} - \frac{6\int \sqrt{\sin(a+bx)}\,dx}{25b}$$
$$= -\frac{12E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{25b^2} + \frac{4\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)}{25b^2} + \frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.92, size = 108, normalized size = 1.66

$$\frac{\sqrt{\sin(a+bx)}\left(5bx-5bx\cos(2(a+bx))+2\sin(2(a+bx))-12\tan\left(\frac{1}{2}(a+bx)\right)+4\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)\sqrt{\sec^2\left(\frac{1}{2}(a+bx)\right)}\tan\left(\frac{1}{2}(a+bx)\right)\right)}{25b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Sin[a + b*x]^(3/2),x]
```

```
[Out] (Sqrt[Sin[a + b*x]]*(5*b*x - 5*b*x*Cos[2*(a + b*x)] + 2*Sin[2*(a + b*x)] -
12*Tan[(a + b*x)/2] + 4*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2]^
2]*Sqrt[Sec[(a + b*x)/2]^2]*Tan[(a + b*x)/2]))/(25*b^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\left(\sin^{\frac{3}{2}}(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*sin(b*x+a)^(3/2),x)
```

```
[Out] int(x*cos(b*x+a)*sin(b*x+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)**(3/2),x)
[Out] Integral(x*sin(a + b*x)**(3/2)*cos(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*sin(a + b*x)^(3/2),x)
[Out] int(x*cos(a + b*x)*sin(a + b*x)^(3/2), x)
```

3.346 $\int x \cos(a+bx) \sqrt{\sin(a+bx)}\, dx$

Optimal. Leaf size=65

$$-\frac{4F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{9b^2}+\frac{4\cos(a+bx)\sqrt{\sin(a+bx)}}{9b^2}+\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] 4/9*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF
(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2+2/3*x*sin(b*x+a)^(3/2)/b+4/9*cos(b*
x+a)*sin(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3524, 2715, 2720}

$$-\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{9b^2}+\frac{4\sqrt{\sin(a+bx)}\cos(a+bx)}{9b^2}+\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Sqrt[Sin[a + b*x]],x]
```

```
[Out] (-4*EllipticF[(a - Pi/2 + b*x)/2, 2])/(9*b^2) + (4*Cos[a + b*x]*Sqrt[Sin[a
+ b*x]])/(9*b^2) + (2*x*Sin[a + b*x]^(3/2))/(3*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx = \frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\int \sin^{\frac{3}{2}}(a+bx)\,dx}{3b}$$
$$= \frac{4\cos(a+bx)\sqrt{\sin(a+bx)}}{9b^2} + \frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\int \frac{1}{\sqrt{\sin(a+bx)}}\,d}{9b}$$
$$= -\frac{4F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|\,2\right)}{9b^2} + \frac{4\cos(a+bx)\sqrt{\sin(a+bx)}}{9b^2} + \frac{2x\sin^{\frac{3}{2}}(a+}{3b}$$

Mathematica [A]
time = 0.20, size = 56, normalized size = 0.86

$$\frac{4F\left(\frac{1}{4}(-2a+\pi-2bx)\middle|\,2\right)+2\sqrt{\sin(a+bx)}\,(2\cos(a+bx)+3bx\sin(a+bx))}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Sqrt[Sin[a + b*x]],x]
[Out] (4*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + 2*Sqrt[Sin[a + b*x]]*(2*Cos[a + b*
x] + 3*b*x*Sin[a + b*x]))/(9*b^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\cos\left(bx+a\right)\left(\sqrt{\sin}\left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*sin(b*x+a)^(1/2),x)
[Out] int(x*cos(b*x+a)*sin(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)*sqrt(sin(b*x + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\sin\left(a+bx\right)}\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(sin(a + b*x))*cos(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*sqrt(sin(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\cos\left(a+b\,x\right)\,\sqrt{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*sin(a + b*x)^(1/2),x)
```

```
[Out] int(x*cos(a + b*x)*sin(a + b*x)^(1/2), x)
```

3.347 $\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx$

Optimal. Leaf size=38

$$-\frac{4E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b^2}+\frac{2x\sqrt{\sin(a+bx)}}{b}$$

```
[Out] 4*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(c
os(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2+2*x*sin(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3524, 2719}

$$\frac{2x\sqrt{\sin(a+bx)}}{b}-\frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sqrt[Sin[a + b*x]],x]
```

```
[Out] (-4*EllipticE[(a - Pi/2 + b*x)/2, 2])/b^2 + (2*x*Sqrt[Sin[a + b*x]])/b
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx &= \frac{2x\sqrt{\sin(a+bx)}}{b}-\frac{2\int \sqrt{\sin(a+bx)}\,dx}{b}\\ &= -\frac{4E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b^2}+\frac{2x\sqrt{\sin(a+bx)}}{b}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.61, size = 86, normalized size = 2.26

$$\frac{2\sqrt{\sin(a+bx)}\left(3bx-6\tan\left(\frac{1}{2}(a+bx)\right)+2\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)\sqrt{\sec^2\left(\frac{1}{2}(a+bx)\right)}\tan\left(\frac{1}{2}(a+bx)\right)\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sqrt[Sin[a + b*x]],x]
```

```
[Out] (2*Sqrt[Sin[a + b*x]]*(3*b*x - 6*Tan[(a + b*x)/2] + 2*Hypergeometric2F1[1/2
, 3/4, 7/4, -Tan[(a + b*x)/2]^2]*Sqrt[Sec[(a + b*x)/2]^2]*Tan[(a + b*x)/2])
)/(3*b^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.11, size = 308, normalized size = 8.11

method	result
risch	$-\frac{i(bx+2i)\left(e^{2i(bx+a)}-1\right)\sqrt{2}\,e^{-i(bx+a)}}{b^2\sqrt{-i\left(e^{2i(bx+a)}-1\right)e^{-i(bx+a)}}}-\frac{2\left(\frac{2i\left(i-ie^{2i(bx+a)}\right)}{\sqrt{e^{i(bx+a)}\left(i-ie^{2i(bx+a)}\right)}}-\frac{\sqrt{e^{i(bx+a)}+1}\,\sqrt{-2\,e^{i(bx+a)}+2}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(b*x+2*I)*(exp(I*(b*x+a))^2-1)/b^2*2^(1/2)/(-I*(exp(I*(b*x+a))^2-1)/exp(
I*(b*x+a)))^(1/2)/exp(I*(b*x+a))-2/b^2*(2*I*(I-I*exp(I*(b*x+a))^2)/(exp(I*(
b*x+a))*(I-I*exp(I*(b*x+a))^2))^(1/2)-(exp(I*(b*x+a))+1)^(1/2)*(-2*exp(I*(b
*x+a))+2)^(1/2)*(-exp(I*(b*x+a)))^(1/2)/(-I*exp(I*(b*x+a))^3+I*exp(I*(b*x+a
)))^(1/2)*(-2*EllipticE((exp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))+EllipticF((ex
p(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))))*2^(1/2)/(-I*(exp(I*(b*x+a))^2-1)/exp(I
*(b*x+a)))^(1/2)*(-I*(exp(I*(b*x+a))^2-1)*exp(I*(b*x+a)))^(1/2)/exp(I*(b*x+
a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/sqrt(sin(b*x + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(1/2),x)
[Out] Integral(x*cos(a + b*x)/sqrt(sin(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(x*cos(b*x + a)/sqrt(sin(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\,\cos\left(a+b\,x\right)}{\sqrt{\sin\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/sin(a + b*x)^(1/2),x)
[Out] int((x*cos(a + b*x))/sin(a + b*x)^(1/2), x)
```

3.348 $\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=38

$$\frac{4F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b^2}-\frac{2x}{b\sqrt{\sin(a+bx)}}$$

```
[Out] -4*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(
cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2-2*x/b/sin(b*x+a)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3524, 2720}

$$\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}-\frac{2x}{b\sqrt{\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(3/2),x]
```

```
[Out] (4*EllipticF[(a - Pi/2 + b*x)/2, 2])/b^2 - (2*x)/(b*Sqrt[Sin[a + b*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx &= -\frac{2x}{b\sqrt{\sin(a+bx)}}+\frac{2\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx}{b}\\ &= \frac{4F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b^2}-\frac{2x}{b\sqrt{\sin(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 37, normalized size = 0.97

$$\frac{2\left(-2F\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)-\frac{bx}{\sqrt{\sin(a+bx)}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(3/2),x]
[Out] (2*(-2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] - (b*x)/Sqrt[Sin[a + b*x]]))/b^2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(3/2),x)
[Out] int(x*cos(b*x+a)/sin(b*x+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(3/2),x)
```

```
[Out] Integral(x*cos(a + b*x)/sin(a + b*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\,\cos\left(a+b\,x\right)}{{\sin\left(a+b\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/sin(a + b*x)^(3/2),x)
```

```
[Out] int((x*cos(a + b*x))/sin(a + b*x)^(3/2), x)
```

3.349 $\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=65

$$-\frac{4E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{3b^2}-\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}-\frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}}$$

```
[Out] 4/3*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE
(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2-2/3*x/b/sin(b*x+a)^(3/2)-4/3*cos(b*
x+a)/b^2/sin(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3524, 2716, 2719}

$$-\frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2}-\frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}}-\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(5/2),x]
```

```
[Out] (-4*EllipticE[(a - Pi/2 + b*x)/2, 2])/(3*b^2) - (2*x)/(3*b*Sin[a + b*x]^(3/
2)) - (4*Cos[a + b*x])/(3*b^2*Sqrt[Sin[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)} + \frac{2\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{3b}$$
$$= -\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)} - \frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}} - \frac{2\int \sqrt{\sin(a+bx)}\,dx}{3b}$$
$$= -\frac{4E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{3b^2} - \frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)} - \frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}}$$

Mathematica [A]
time = 0.20, size = 56, normalized size = 0.86

$$-\frac{2\left(bx-2E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sin^{\frac{3}{2}}(a+bx)+\sin(2(a+bx))\right)}{3b^2\sin^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(5/2),x]
[Out] (-2*(b*x - 2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(3/2) + Sin[2
*(a + b*x)]))/(3*b^2*Sin[a + b*x]^(3/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(5/2),x)
[Out] int(x*cos(b*x+a)/sin(b*x+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(5/2),x)
[Out] Integral(x*cos(a + b*x)/sin(a + b*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\cos(a+b\,x)}{{\sin(a+b\,x)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/sin(a + b*x)^(5/2),x)
[Out] int((x*cos(a + b*x))/sin(a + b*x)^(5/2), x)
```

3.350 $\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=65

$$\frac{4F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{15b^2}-\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}-\frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)}$$

```
[Out] -4/15*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Ellipti
cF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2-2/5*x/b/sin(b*x+a)^(5/2)-4/15*cos
(b*x+a)/b^2/sin(b*x+a)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3524, 2716, 2720}

$$\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2}-\frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)}-\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(7/2),x]
```

```
[Out] (4*EllipticF[(a - Pi/2 + b*x)/2, 2])/(15*b^2) - (2*x)/(5*b*Sin[a + b*x]^(5/
2)) - (4*Cos[a + b*x])/(15*b^2*Sin[a + b*x]^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx}{5b}$$
$$= -\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)} - \frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)} + \frac{2\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx}{15b}$$
$$= \frac{4F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|\,2\right)}{15b^2} - \frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)} - \frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)}$$

Mathematica [A]
time = 0.25, size = 57, normalized size = 0.88

$$-\frac{2\left(3bx+2F\left(\frac{1}{4}(-2a+\pi-2bx)\middle|\,2\right)\sin^{\frac{5}{2}}(a+bx)+\sin(2(a+bx))\right)}{15b^2\sin^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(7/2),x]
[Out] (-2*(3*b*x + 2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(5/2) + Sin
[2*(a + b*x)]))/(15*b^2*Sin[a + b*x]^(5/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(7/2),x)
[Out] int(x*cos(b*x+a)/sin(b*x+a)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos{(a+bx)}}{\sin^{\frac{7}{2}}{(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(7/2),x)
```

```
[Out] Integral(x*cos(a + b*x)/sin(a + b*x)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\cos\left(a+b\,x\right)}{{\sin\left(a+b\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/sin(a + b*x)^(7/2),x)
```

```
[Out] int((x*cos(a + b*x))/sin(a + b*x)^(7/2), x)
```

3.351 $\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx$

Optimal. Leaf size=88

$$-\frac{12E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{35b^2}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}-\frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)}-\frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}}$$

```
[Out] 12/35*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Ellipti
cE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2-2/7*x/b/sin(b*x+a)^(7/2)-4/35*cos
(b*x+a)/b^2/sin(b*x+a)^(5/2)-12/35*cos(b*x+a)/b^2/sin(b*x+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3524, 2716, 2719}

$$-\frac{12E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2}-\frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)}-\frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(9/2),x]
```

```
[Out] (-12*EllipticE[(a - Pi/2 + b*x)/2, 2])/(35*b^2) - (2*x)/(7*b*Sin[a + b*x]^(
7/2)) - (4*Cos[a + b*x])/(35*b^2*Sin[a + b*x]^(5/2)) - (12*Cos[a + b*x])/(3
5*b^2*Sqrt[Sin[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx = -\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)} + \frac{2\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx}{7b}$$

$$= -\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)} - \frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)} + \frac{6\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{35b}$$

$$= -\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)} - \frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)} - \frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}} - \frac{6\int \sqrt{\sin(a+bx)}\,dx}{35b}$$

$$= -\frac{12E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{35b^2} - \frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)} - \frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)} - \frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}}$$

Mathematica [A]
time = 0.31, size = 73, normalized size = 0.83

$$-\frac{2\left(5bx+6\cos(a+bx)\sin^3(a+bx)-6E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sin^{\frac{7}{2}}(a+bx)+\sin(2(a+bx))\right)}{35b^2\sin^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(9/2),x]
[Out] (-2*(5*b*x + 6*Cos[a + b*x]*Sin[a + b*x]^3 - 6*EllipticE[(-2*a + Pi - 2*b*x
)/4, 2]*Sin[a + b*x]^(7/2) + Sin[2*(a + b*x)]))/(35*b^2*Sin[a + b*x]^(7/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(9/2),x)
[Out] int(x*cos(b*x+a)/sin(b*x+a)^(9/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(9/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\cos\left(a+b\,x\right)}{{\sin\left(a+b\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/sin(a + b*x)^(9/2),x)
```

```
[Out] int((x*cos(a + b*x))/sin(a + b*x)^(9/2), x)
```

3.352 $\int x \cos(a+bx) \csc^{\frac{9}{2}}(a+bx)\, dx$

Optimal. Leaf size=108

$$-\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2}-\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}-\frac{12\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}\right.\right.}{35b^2}$$

```
[Out] -4/35*cos(b*x+a)*csc(b*x+a)^(5/2)/b^2-2/7*x*csc(b*x+a)^(7/2)/b-12/35*cos(b*
x+a)*csc(b*x+a)^(1/2)/b^2+12/35*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2
*a+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^
(1/2)*sin(b*x+a)^(1/2)/b^2
```

Rubi [A]

time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4298, 3853, 3856, 2719}

$$-\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2}-\frac{12\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2}-\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(9/2),x]
```

```
[Out] (-12*Cos[a + b*x]*Sqrt[Csc[a + b*x]])/(35*b^2) - (4*Cos[a + b*x]*Csc[a + b*
x]^(5/2))/(35*b^2) - (2*x*Csc[a + b*x]^(7/2))/(7*b) - (12*Sqrt[Csc[a + b*x]
]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(35*b^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4298

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p
- 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[
p, 1]
```

Rubi steps

$$\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx = -\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\int\csc^{\frac{7}{2}}(a+bx)\,dx}{7b}$$
$$= -\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}+\frac{6\int\csc^{\frac{3}{2}}(a+bx)\,dx}{35b}$$
$$= -\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2}-\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\csc^{\frac{7}{2}}(}{7}$$
$$= -\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2}-\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\csc^{\frac{7}{2}}(}{7}$$
$$= -\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2}-\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\csc^{\frac{7}{2}}(}{7}$$

Mathematica [A]
time = 0.28, size = 73, normalized size = 0.68

$$-\frac{2\csc^{\frac{7}{2}}(a+bx)\left(5bx+6\cos(a+bx)\sin^3(a+bx)-6E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sin^{\frac{7}{2}}(a+bx)+\sin(2(a+bx))\right)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(9/2),x]
[Out] (-2*Csc[a + b*x]^(7/2)*(5*b*x + 6*Cos[a + b*x]*Sin[a + b*x]^3 - 6*EllipticE
[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(7/2) + Sin[2*(a + b*x)]))/(35*b^2)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\left(\csc^{\frac{9}{2}}(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(9/2),x)
[Out] int(x*cos(b*x+a)*csc(b*x+a)^(9/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(9/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(9/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="giac")
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\cos(a+b\,x)\,\left(\frac{1}{\sin(a+b\,x)}\right)^{9/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*(1/sin(a + b*x))^(9/2),x)
[Out] int(x*cos(a + b*x)*(1/sin(a + b*x))^(9/2), x)
```

3.353 $\int x \cos(a+bx) \csc^{\frac{7}{2}}(a+bx) \, dx$

Optimal. Leaf size=85

$$-\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{15b^2}$$

```
[Out] -4/15*cos(b*x+a)*csc(b*x+a)^(3/2)/b^2-2/5*x*csc(b*x+a)^(5/2)/b-4/15*(sin(1/
2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+
1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4298, 3853, 3856, 2720}

$$-\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}+\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(7/2),x]
```

```
[Out] (-4*Cos[a + b*x]*Csc[a + b*x]^(3/2))/(15*b^2) - (2*x*Csc[a + b*x]^(5/2))/(5
*b) + (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b
*x]])/(15*b^2)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4298

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
```

```
1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p
- 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[
p, 1]
```

Rubi steps

$$\int x\cos(a+bx)\csc^{\frac{7}{2}}(a+bx)\,dx = -\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\int\csc^{\frac{5}{2}}(a+bx)\,dx}{5b}$$
$$= -\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\int\sqrt{\csc(a+bx)}\,dx}{15b}$$
$$= -\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}+\frac{\left(2\sqrt{\csc(a+bx)}\sqrt{\sin$$
$$= -\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\sqrt{\csc(a+bx)}F\left(\frac{1}{2}(a$$

Mathematica [A]
time = 0.32, size = 65, normalized size = 0.76

$$-\frac{2\sqrt{\csc(a+bx)}\left(2\cot(a+bx)+3bx\csc^2(a+bx)+2F\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sqrt{\sin(a+bx)}\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(7/2),x]
[Out] (-2*Sqrt[Csc[a + b*x]]*(2*Cot[a + b*x] + 3*b*x*Csc[a + b*x]^2 + 2*EllipticF
[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]]))/(15*b^2)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\left(\csc^{\frac{7}{2}}(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(7/2),x)
[Out] int(x*cos(b*x+a)*csc(b*x+a)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(7/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\cos\left(a+b\,x\right)\,{\left(\frac{1}{\sin\left(a+b\,x\right)}\right)}^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*(1/sin(a + b*x))^(7/2),x)
```

```
[Out] int(x*cos(a + b*x)*(1/sin(a + b*x))^(7/2), x)
```

3.354 $\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=85

$$-\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}-\frac{4\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{3b^2}$$

```
[Out] -2/3*x*csc(b*x+a)^(3/2)/b-4/3*cos(b*x+a)*csc(b*x+a)^(1/2)/b^2+4/3*(sin(1/2*
a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*a+1/
4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4298, 3853, 3856, 2719}

$$-\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2}-\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(5/2),x]
```

```
[Out] (-4*Cos[a + b*x]*Sqrt[Csc[a + b*x]])/(3*b^2) - (2*x*Csc[a + b*x]^(3/2))/(3*
b) - (4*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*
x]])/(3*b^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4298

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
```

```
1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p
- 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[
p, 1]
```

Rubi steps

$$\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx = -\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int \csc^{\frac{3}{2}}(a+bx)\,dx}{3b}$$
$$= -\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\int \frac{1}{\sqrt{\csc(a+bx)}}\,d}{3b}$$
$$= -\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}-\frac{\left(2\sqrt{\csc(a+bx)}\,\sqrt{\right.}{}$$
$$= -\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}-\frac{4\sqrt{\csc(a+bx)}\,E\left(\right.}{}$$

Mathematica [A]
time = 0.20, size = 56, normalized size = 0.66

$$-\frac{2\csc^{\frac{3}{2}}(a+bx)\left(bx-2E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sin^{\frac{3}{2}}(a+bx)+\sin(2(a+bx))\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(5/2),x]
```

```
[Out] (-2*Csc[a + b*x]^(3/2)*(b*x - 2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a +
 b*x]^(3/2) + Sin[2*(a + b*x)]))/(3*b^2)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x\cos\left(bx+a\right)\left(\csc^{\frac{5}{2}}\left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(5/2),x)
```

```
[Out] int(x*cos(b*x+a)*csc(b*x+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\cos\left(a+b\,x\right)\,{\left(\frac{1}{\sin\left(a+b\,x\right)}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*(1/sin(a + b*x))^(5/2),x)
```

```
[Out] int(x*cos(a + b*x)*(1/sin(a + b*x))^(5/2), x)
```

3.355 $\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=58

$$-\frac{2x\sqrt{\csc(a+bx)}}{b}+\frac{4\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b^2}$$

```
[Out] -2*x*csc(b*x+a)^(1/2)/b-4*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4
*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*
sin(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {4298, 3856, 2720}

$$\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}-\frac{2x\sqrt{\csc(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(3/2),x]
```

```
[Out] (-2*x*Sqrt[Csc[a + b*x]])/b + (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b
*x)/2, 2]*Sqrt[Sin[a + b*x]])/b^2
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4298

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p
- 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[
p, 1]
```

Rubi steps

$$\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx = -\frac{2x\sqrt{\csc(a+bx)}}{b}+\frac{2\int\sqrt{\csc(a+bx)}\,dx}{b}$$

$$= -\frac{2x\sqrt{\csc(a+bx)}}{b}+\frac{\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int\frac{1}{\sqrt{\sin(a+bx)}}}{b}$$

$$= -\frac{2x\sqrt{\csc(a+bx)}}{b}+\frac{4\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b^2}$$

Mathematica [A]
time = 0.15, size = 46, normalized size = 0.79

$$-\frac{2\sqrt{\csc(a+bx)}\left(bx+2F\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sqrt{\sin(a+bx)}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(3/2),x]
[Out] (-2*Sqrt[Csc[a + b*x]]*(b*x + 2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Si
n[a + b*x]]))/b^2
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\left(\csc^{\frac{3}{2}}(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(3/2),x)
[Out] int(x*cos(b*x+a)*csc(b*x+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\cos\left(a+b\,x\right)\,{\left(\frac{1}{\sin\left(a+b\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*(1/sin(a + b*x))^(3/2),x)
```

```
[Out] int(x*cos(a + b*x)*(1/sin(a + b*x))^(3/2), x)
```

3.356 $\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx$

Optimal. Leaf size=58

$$\frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{4\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b^2}$$

```
[Out] 2*x/b/csc(b*x+a)^(1/2)+4*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*
Pi+1/2*b*x)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*s
in(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {4298, 3856, 2719}

$$\frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{4\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Sqrt[Csc[a + b*x]],x]
```

```
[Out] (2*x)/(b*Sqrt[Csc[a + b*x]]) - (4*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 +
b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b^2
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4298

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p
- 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[
p, 1]
```

Rubi steps

$$\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx = \frac{2x}{b\sqrt{\csc(a+bx)}} - \frac{2\int \frac{1}{\sqrt{\csc(a+bx)}}\,dx}{b}$$
$$= \frac{2x}{b\sqrt{\csc(a+bx)}} - \frac{\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \sqrt{\sin(a+bx)}}{b}$$
$$= \frac{2x}{b\sqrt{\csc(a+bx)}} - \frac{4\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.79, size = 106, normalized size = 1.83

$$\frac{4\cos\left(\frac{1}{2}(a+bx)\right)\sqrt{\csc(a+bx)}\sin\left(\frac{1}{2}(a+bx)\right)\left(3bx-6\tan\left(\frac{1}{2}(a+bx)\right)+2\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)\sqrt{\sec^2\left(\frac{1}{2}(a+bx)\right)}\tan\left(\frac{1}{2}(a+bx)\right)\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Sqrt[Csc[a + b*x]],x]
```

```
[Out] (4*Cos[(a + b*x)/2]*Sqrt[Csc[a + b*x]]*Sin[(a + b*x)/2]*(3*b*x - 6*Tan[(a +
 b*x)/2] + 2*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2]^2]*Sqrt[Sec
[(a + b*x)/2]^2]*Tan[(a + b*x)/2]))/(3*b^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 308, normalized size = 5.31

method	result
risch	$-\frac{i(bx+2i)\left(e^{2i(bx+a)}-1\right)\sqrt{2}\sqrt{\frac{ie^{i(bx+a)}}{e^{2i(bx+a)}-1}}\,e^{-i(bx+a)}}{b^2} - \frac{2\left(-\frac{2i\left(-i+ie^{2i(bx+a)}\right)}{\sqrt{e^{i(bx+a)}\left(-i+ie^{2i(bx+a)}\right)}} - \frac{\sqrt{e^{i(bx+a)}+1}\sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(b*x+2*I)*(exp(I*(b*x+a))^2-1)/b^2*2^(1/2)*(I*exp(I*(b*x+a))/(exp(I*(b*x
+a))^2-1))^(1/2)/exp(I*(b*x+a))-2/b^2*(-2*I*(-I+I*exp(I*(b*x+a))^2)/(exp(I*
(b*x+a))*(-I+I*exp(I*(b*x+a))^2))^(1/2)-(exp(I*(b*x+a))+1)^(1/2)*(-2*exp(I*
(b*x+a))+2)^(1/2)*(-exp(I*(b*x+a)))^(1/2)/(I*exp(I*(b*x+a))^3-I*exp(I*(b*x+
a)))^(1/2)*(-2*EllipticE((exp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))+EllipticF((e
xp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))))*2^(1/2)*(I*exp(I*(b*x+a))/(exp(I*(b*x
```

```
+a))^2-1))^(1/2)*(I*(exp(I*(b*x+a))^2-1)*exp(I*(b*x+a)))^(1/2)/exp(I*(b*x+a
))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)*sqrt(csc(b*x + a)), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(a+bx\right)\sqrt{\csc\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(1/2),x)
[Out] Integral(x*cos(a + b*x)*sqrt(csc(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(x*cos(b*x + a)*sqrt(csc(b*x + a)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int x\cos(a+b\,x)\,\sqrt{\frac{1}{\sin(a+b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*(1/sin(a + b*x))^(1/2),x)
[Out] int(x*cos(a + b*x)*(1/sin(a + b*x))^(1/2), x)
```

3.357 $\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

Optimal. Leaf size=85

$$\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)}+\frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}}-\frac{4\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|\,2\right)\sqrt{\sin(a+bx)}}{9b^2}$$

```
[Out] 2/3*x/b/csc(b*x+a)^(3/2)+4/9*cos(b*x+a)/b^2/csc(b*x+a)^(1/2)+4/9*(sin(1/2*a
+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4
*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4298, 3854, 3856, 2720}

$$\frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}}-\frac{4\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|\,2\right)}{9b^2}+\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sqrt[Csc[a + b*x]],x]
```

```
[Out] (2*x)/(3*b*Csc[a + b*x]^(3/2)) + (4*Cos[a + b*x])/(9*b^2*Sqrt[Csc[a + b*x]]
) - (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x
]])/(9*b^2)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4298

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p
- 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[
p, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx &= \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{2\int \frac{1}{\csc^{\frac{3}{2}}(a+bx)}\,dx}{3b}\\ &= \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} + \frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}} - \frac{2\int \sqrt{\csc(a+bx)}\,dx}{9b}\\ &= \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} + \frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}} - \frac{\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \frac{1}{\sqrt{\sin(a}}}{9b}\\ &= \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} + \frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}} - \frac{4\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin}}{9b^2}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 65, normalized size = 0.76

$$\frac{2\sqrt{\csc(a+bx)}\left(2F\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sqrt{\sin(a+bx)}+3bx\sin^2(a+bx)+\sin(2(a+bx))\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sqrt[Csc[a + b*x]],x]
[Out] (2*Sqrt[Csc[a + b*x]]*(2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b
*x]] + 3*b*x*Sin[a + b*x]^2 + Sin[2*(a + b*x)]))/(9*b^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sqrt{\csc(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/csc(b*x+a)^(1/2),x)
[Out] int(x*cos(b*x+a)/csc(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/sqrt(csc(b*x + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos{(a+bx)}}{\sqrt{\csc{(a+bx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*cos(a + b*x)/sqrt(csc(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sqrt(csc(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\cos\left(a+b\,x\right)}{\sqrt{\frac{1}{\sin\left(a+b\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/(1/sin(a + b*x))^(1/2),x)
[Out] int((x*cos(a + b*x))/(1/sin(a + b*x))^(1/2), x)
```

3.358 $\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=85

$$\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)}+\frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)}-\frac{12\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{25b^2}$$

```
[Out] 2/5*x/b/csc(b*x+a)^(5/2)+4/25*cos(b*x+a)/b^2/csc(b*x+a)^(3/2)+12/25*(sin(1/
2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*a+
1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4298, 3854, 3856, 2719}

$$\frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)}-\frac{12\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2}+\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Csc[a + b*x]^(3/2),x]
```

```
[Out] (2*x)/(5*b*Csc[a + b*x]^(5/2)) + (4*Cos[a + b*x])/(25*b^2*Csc[a + b*x]^(3/2
)) - (12*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b
*x]])/(25*b^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4298

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p
- 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[
p, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx &= \frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)}-\frac{2\int \frac{1}{\csc^{\frac{5}{2}}(a+bx)}\,dx}{5b}\\ &= \frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)}+\frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)}-\frac{6\int \frac{1}{\sqrt{\csc(a+bx)}}\,dx}{25b}\\ &= \frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)}+\frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)}-\frac{\left(6\sqrt{\csc(a+bx)}\,\sqrt{\sin(a+bx)}\right)\int \sqrt{\sin(a+}}{25b}\\ &= \frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)}+\frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)}-\frac{12\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(}}{25b^2}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.03, size = 114, normalized size = 1.34

$$\frac{\left(-10+4\cos(a+bx)+2\cos(2(a+bx))+4\sqrt{2}\sqrt{\frac{1}{1+\cos(a+bx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+10bx\sin(a+bx)+5bx\sin(2(a+bx))\right)\tan\left(\frac{1}{2}(a+bx)\right)}{25b^2\sqrt{\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Csc[a + b*x]^(3/2),x]
```

```
[Out] ((-10 + 4*Cos[a + b*x] + 2*Cos[2*(a + b*x)] + 4*Sqrt[2]*Sqrt[(1 + Cos[a + b
*x])^(-1)]*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2]^2] + 10*b*x*S
in[a + b*x] + 5*b*x*Sin[2*(a + b*x)])*Tan[(a + b*x)/2])/(25*b^2*Sqrt[Csc[a
+ b*x]])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x\cos\left(bx+a\right)}{\csc\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/csc(b*x+a)^(3/2),x)
[Out] int(x*cos(b*x+a)/csc(b*x+a)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cos{\left(a + bx \right)}}{\csc^{\frac{3}{2}}{\left(a + bx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)**(3/2),x)
[Out] Integral(x*cos(a + b*x)/csc(a + b*x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\cos\left(a+b\,x\right)}{{\left(\frac{1}{\sin\left(a+b\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/(1/sin(a + b*x))^(3/2),x)
[Out] int((x*cos(a + b*x))/(1/sin(a + b*x))^(3/2), x)
```

3.359 $\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=108

$$\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)}+\frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)}+\frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}}-\frac{20\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+}}{147b^2}$$

```
[Out] 2/7*x/b/csc(b*x+a)^(7/2)+4/49*cos(b*x+a)/b^2/csc(b*x+a)^(5/2)+20/147*cos(b*
x+a)/b^2/csc(b*x+a)^(1/2)+20/147*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/
2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)
^(1/2)*sin(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4298, 3854, 3856, 2720}

$$\frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)}+\frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}}-\frac{20\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{147b^2}+\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Csc[a + b*x]^(5/2),x]
```

```
[Out] (2*x)/(7*b*Csc[a + b*x]^(7/2)) + (4*Cos[a + b*x])/(49*b^2*Csc[a + b*x]^(5/2
)) + (20*Cos[a + b*x])/(147*b^2*Sqrt[Csc[a + b*x]]) - (20*Sqrt[Csc[a + b*x]
]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(147*b^2)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4298

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p
- 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[
p, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx &= \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} - \frac{2\int \frac{1}{\csc^{\frac{7}{2}}(a+bx)}\,dx}{7b} \\
&= \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} + \frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)} - \frac{10\int \frac{1}{\csc^{\frac{3}{2}}(a+bx)}\,dx}{49b} \\
&= \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} + \frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)} + \frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}} - \frac{10\int \sqrt{\csc(a+bx)}\,d}{147b} \\
&= \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} + \frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)} + \frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}} - \frac{\left(10\sqrt{\csc(a+bx)}\,\sqrt{s}\right.}{} \\
&= \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} + \frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)} + \frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}} - \frac{20\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\right.}{}
\end{aligned}
$$

Mathematica [A]
time = 0.42, size = 93, normalized size = 0.86

$$\frac{\sqrt{\csc(a+bx)}\left(63bx-84bx\cos(2(a+bx))+21bx\cos(4(a+bx))+80F\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sqrt{\sin(a+bx)}+52\sin(2(a+bx))-6\sin(4(a+bx))\right)}{588b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Csc[a + b*x]^(5/2),x]
```

```
[Out] (Sqrt[Csc[a + b*x]]*(63*b*x - 84*b*x*Cos[2*(a + b*x)] + 21*b*x*Cos[4*(a + b
*x)] + 80*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]] + 52*Sin[2
*(a + b*x)] - 6*Sin[4*(a + b*x)]))/(588*b^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x\cos\left(bx+a\right)}{\csc\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/csc(b*x+a)^(5/2),x)
```

```
[Out] int(x*cos(b*x+a)/csc(b*x+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)**(5/2),x)
```

```
[Out] Integral(x*cos(a + b*x)/csc(a + b*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(5/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\cos\left(a+b\,x\right)}{{\left(\frac{1}{\sin\left(a+b\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/(1/sin(a + b*x))^(5/2),x)
[Out] int((x*cos(a + b*x))/(1/sin(a + b*x))^(5/2), x)
```

3.360 $\int x \csc(x) \sin(3x) \, dx$

Optimal. Leaf size=31

$$\frac{x^2}{2} + \frac{3\cos^2(x)}{4} + 2x\cos(x)\sin(x) - \frac{\sin^2(x)}{4}$$

```
[Out] 1/2*x^2+3/4*cos(x)^2+2*x*cos(x)*sin(x)-1/4*sin(x)^2
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4516, 3391, 30}

$$\frac{x^2}{2} - \frac{\sin^2(x)}{4} + \frac{3\cos^2(x)}{4} + 2x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[x]*Sin[3*x],x]
```

```
[Out] x^2/2 + (3*Cos[x]^2)/4 + 2*x*Cos[x]*Sin[x] - Sin[x]^2/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$
\begin{aligned}
\int x\csc(x)\sin(3x)\,dx &= \int \left(3x\cos^2(x)-x\sin^2(x)\right)\,dx\\
&= 3\int x\cos^2(x)\,dx-\int x\sin^2(x)\,dx\\
&= \frac{3\cos^2(x)}{4}+2x\cos(x)\sin(x)-\frac{\sin^2(x)}{4}-\frac{\int x\,dx}{2}+\frac{3\int x\,dx}{2}\\
&= \frac{x^2}{2}+\frac{3\cos^2(x)}{4}+2x\cos(x)\sin(x)-\frac{\sin^2(x)}{4}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.71

$$\frac{x^2}{2}+\frac{1}{2}\cos(2x)+x\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[x]*Sin[3*x],x]
[Out] x^2/2 + Cos[2*x]/2 + x*Sin[2*x]
```

Maple [A]
time = 0.08, size = 26, normalized size = 0.84

method	result	size
risch	$\frac{x^2}{2}+\frac{\cos(2x)}{2}+x\sin(2x)$	19
default	$4x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{3x^2}{2}-\left(\sin^2(x)\right)$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)*sin(3*x),x,method=_RETURNVERBOSE)
[Out] 4*x*(1/2*cos(x)*sin(x)+1/2*x)-3/2*x^2-sin(x)^2
```

Maxima [A]
time = 0.27, size = 18, normalized size = 0.58

$$\frac{1}{2}x^2+x\sin(2x)+\frac{1}{2}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sin(3*x),x, algorithm="maxima")
```

[Out] 1/2*x^2 + x*sin(2*x) + 1/2*cos(2*x)

Fricas [A]
time = 2.92, size = 17, normalized size = 0.55

$$2\,x\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,x^{2}+\cos\left(x\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)*sin(3*x),x, algorithm="fricas")

[Out] 2*x*cos(x)*sin(x) + 1/2*x^2 + cos(x)^2

Sympy [A]
time = 1.11, size = 37, normalized size = 1.19

$$-x^{2}\sin^{2}\left(x\right)-x^{2}\cos^{2}\left(x\right)+\frac{3x^{2}}{2}+2x\sin\left(x\right)\cos\left(x\right)+\cos^{2}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)*sin(3*x),x)

[Out] -x**2*sin(x)**2 - x**2*cos(x)**2 + 3*x**2/2 + 2*x*sin(x)*cos(x) + cos(x)**2

Giac [A]
time = 0.40, size = 18, normalized size = 0.58

$$\frac{1}{2}\,x^{2}+x\sin\left(2\,x\right)+\frac{1}{2}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)*sin(3*x),x, algorithm="giac")

[Out] 1/2*x^2 + x*sin(2*x) + 1/2*cos(2*x)

Mupad [B]
time = 0.09, size = 18, normalized size = 0.58

$$\frac{\cos\left(2\,x\right)}{2}+x\,\sin\left(2\,x\right)+\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*sin(3*x))/sin(x),x)

[Out] cos(2*x)/2 + x*sin(2*x) + x^2/2

3.361 $\int (c+dx)^4 \csc(x)\sin(3x)\,dx$

Optimal. Leaf size=131

$$\frac{3d^4x}{2}-d(c+dx)^3+\frac{(c+dx)^5}{5d}-\frac{9}{2}d^3(c+dx)\cos^2(x)+3d(c+dx)^3\cos^2(x)+3d^4\cos(x)\sin(x)-6d^2(c+dx)^2\cos(x)$$

```
[Out] 3/2*d^4*x-d*(d*x+c)^3+1/5*(d*x+c)^5/d-9/2*d^3*(d*x+c)*cos(x)^2+3*d*(d*x+c)^
3*cos(x)^2+3*d^4*cos(x)*sin(x)-6*d^2*(d*x+c)^2*cos(x)*sin(x)+2*(d*x+c)^4*co
s(x)*sin(x)+3/2*d^3*(d*x+c)*sin(x)^2-d*(d*x+c)^3*sin(x)^2
```

Rubi [A]
time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4516, 3392, 32, 2715, 8}

$$\frac{3}{2}d^3\sin^2(x)(c+dx)-\frac{9}{2}d^3\cos^2(x)(c+dx)-6d^2\sin(x)\cos(x)(c+dx)^2+\frac{(c+dx)^5}{5d}-d(c+dx)^3-d\sin^2(x)(c+dx)^3+3d\cos^2(x)(c+dx)^3+2\sin(x)\cos(x)(c+dx)^4+\frac{3d^4x}{2}+3d^4\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[x]*Sin[3*x],x]
```

```
[Out] (3*d^4*x)/2 - d*(c + d*x)^3 + (c + d*x)^5/(5*d) - (9*d^3*(c + d*x)*Cos[x]^2
)/2 + 3*d*(c + d*x)^3*Cos[x]^2 + 3*d^4*Cos[x]*Sin[x] - 6*d^2*(c + d*x)^2*Co
s[x]*Sin[x] + 2*(c + d*x)^4*Cos[x]*Sin[x] + (3*d^3*(c + d*x)*Sin[x]^2)/2 -
d*(c + d*x)^3*Sin[x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
```

```
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^4 \csc(x)\sin(3x)\,dx &= \int \left(3(c+dx)^4\cos^2(x)-(c+dx)^4\sin^2(x)\right)\,dx \\
&= 3\int (c+dx)^4\cos^2(x)\,dx-\int (c+dx)^4\sin^2(x)\,dx \\
&= 3d(c+dx)^3\cos^2(x)+2(c+dx)^4\cos(x)\sin(x)-d(c+dx)^3\sin^2(x)-\frac{1}{2}\int (c \\
&= \frac{(c+dx)^5}{5d}-\frac{9}{2}d^3(c+dx)\cos^2(x)+3d(c+dx)^3\cos^2(x)-6d^2(c+dx)^2\cos(x) \\
&= -d(c+dx)^3+\frac{(c+dx)^5}{5d}-\frac{9}{2}d^3(c+dx)\cos^2(x)+3d(c+dx)^3\cos^2(x)+3d^4 \\
&= \frac{3d^4x}{2}-d(c+dx)^3+\frac{(c+dx)^5}{5d}-\frac{9}{2}d^3(c+dx)\cos^2(x)+3d(c+dx)^3\cos^2(x)
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 154, normalized size = 1.18

$$c^4x+2c^3dx^2+2c^2d^2x^3+cd^3x^4+\frac{d^4x^5}{5}+d\left(2c^3+6c^2dx+d^3x\left(-3+2x^2\right)+3cd^2\left(-1+2x^2\right)\right)\cos(2x)+\frac{1}{2}\left(2c^4+8c^3dx+4cd^3x\left(-3+2x^2\right)+6c^2d^2\left(-1+2x^2\right)+d^4\left(3-6x^2+2x^4\right)\right)\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Csc[x]*Sin[3*x],x]
```

```
[Out] c^4*x + 2*c^3*d*x^2 + 2*c^2*d^2*x^3 + c*d^3*x^4 + (d^4*x^5)/5 + d*(2*c^3 +
6*c^2*d*x + d^3*x*(-3 + 2*x^2) + 3*c*d^2*(-1 + 2*x^2))*Cos[2*x] + ((2*c^4 +
 8*c^3*d*x + 4*c*d^3*x*(-3 + 2*x^2) + 6*c^2*d^2*(-1 + 2*x^2) + d^4*(3 - 6*x
^2 + 2*x^4))*Sin[2*x])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(123) = 246.
time = 0.12, size = 260, normalized size = 1.98

method	result
risch	$\frac{d^4x^5}{5}+d^3c\,x^4+2d^2c^2x^3+2d\,c^3x^2+c^4x+\frac{c^5}{5d}+d(2d^3x^3+6c\,d^2x^2+6c^2dx-3d^3x+2c^3-3c\,d^2)$
default	$4d^4\left(x^4\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+x^3(\cos^2(x))-3x^2\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{3x(\cos^2(x))}{2}+\frac{3\cos(x)\sin(x)}{4}+\frac{3x}{4}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 4*d^4*(x^4*(1/2*cos(x)*sin(x)+1/2*x)+x^3*cos(x)^2-3*x^2*(1/2*cos(x)*sin(x)+
1/2*x)-3/2*x*cos(x)^2+3/4*cos(x)*sin(x)+3/4*x+x^3-2/5*x^5)+16*d^3*c*(x^3*(1
/2*cos(x)*sin(x)+1/2*x)+3/4*x^2*cos(x)^2-3/2*x*(1/2*cos(x)*sin(x)+1/2*x)+3/
8*x^2+3/8*sin(x)^2-3/8*x^4)+24*c^2*d^2*(x^2*(1/2*cos(x)*sin(x)+1/2*x)+1/2*x
*cos(x)^2-1/4*cos(x)*sin(x)-1/4*x-1/3*x^3)-1/5*d^4*x^5+16*c^3*d*(x*(1/2*cos
(x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)-d^3*c*x^4+4*c^4*(1/2*cos(x)*sin(x)+
1/2*x)-2*d^2*c^2*x^3-2*d*c^3*x^2-c^4*x
```

Maxima [A]
time = 0.27, size = 146, normalized size = 1.11

$$2\left(x^2+2\,x\sin\left(2\,x\right)+\cos\left(2\,x\right)\right)c^3d+\left(2\,x^3+6\,x\cos\left(2\,x\right)+3\left(2\,x^2-1\right)\sin\left(2\,x\right)\right)c^2d^2+\left(x^4+3\left(2\,x^2-1\right)\cos\left(2\,x\right)+2\left(2\,x^3-3\,x\right)\sin\left(2\,x\right)\right)cd^3+\frac{1}{10}\left(2\,x^5+10\left(2\,x^3-3\,x\right)\cos\left(2\,x\right)+5\left(2\,x^4-6\,x^2+3\right)\sin\left(2\,x\right)\right)d^4+c^4(x+\sin\left(2\,x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 2*(x^2 + 2*x*sin(2*x) + cos(2*x))*c^3*d + (2*x^3 + 6*x*cos(2*x) + 3*(2*x^2
- 1)*sin(2*x))*c^2*d^2 + (x^4 + 3*(2*x^2 - 1)*cos(2*x) + 2*(2*x^3 - 3*x)*si
n(2*x))*c*d^3 + 1/10*(2*x^5 + 10*(2*x^3 - 3*x)*cos(2*x) + 5*(2*x^4 - 6*x^2
+ 3)*sin(2*x))*d^4 + c^4*(x + sin(2*x))
```

Fricas [A]
time = 2.52, size = 200, normalized size = 1.53

$$\frac{1}{5}\,d^4x^5+cd^3x^4+2\left(c^2d^2-d^4\right)x^3+2\left(c^3d-3\,cd^3\right)x^2+2\left(2\,d^4x^3+6\,cd^3x^2+2\,c^3d-3\,cd^3+3\left(2\,c^2d^2-d^4\right)x\right)\cos\left(x\right)^2+\left(2\,d^4x^4+8\,cd^3x^3+2\,c^4-6\,c^2d^2+3\,d^4+6\left(2\,c^2d^2-d^4\right)x^2+4\left(2\,c^3d-3\,cd^3\right)x\right)\cos\left(x\right)\sin\left(x\right)+\left(c^4-6\,c^2d^2+3\,d^4\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 1/5*d^4*x^5 + c*d^3*x^4 + 2*(c^2*d^2 - d^4)*x^3 + 2*(c^3*d - 3*c*d^3)*x^2 +
 2*(2*d^4*x^3 + 6*c*d^3*x^2 + 2*c^3*d - 3*c*d^3 + 3*(2*c^2*d^2 - d^4)*x)*co
s(x)^2 + (2*d^4*x^4 + 8*c*d^3*x^3 + 2*c^4 - 6*c^2*d^2 + 3*d^4 + 6*(2*c^2*d^
2 - d^4)*x^2 + 4*(2*c^3*d - 3*c*d^3)*x)*cos(x)*sin(x) + (c^4 - 6*c^2*d^2 +
3*d^4)*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(134) = 268$.
time = 15.49, size = 440, normalized size = 3.36

$$c^4x + c^4\sin(2x) - 4c^3dx^2\sin^2(x) - 4c^3dx^2\cos^2(x) + 6c^3dx^2 + 8c^3dx\sin(x)\cos(x) + 4c^3d\cos^2(x) - 4c^2d^2x^3\sin^2(x) - 4c^2d^2x^3\cos^2(x) + 6c^2d^2x^3 + 12c^2d^2x^2\sin(x)\cos(x) - 6c^2d^2x\sin^2(x) + 6c^2d^2x\cos^2(x) - 6c^2d^2\sin(x)\cos(x) - 2cd^3x^4\sin^2(x) - 2cd^3x^4\cos^2(x) + 3cd^3x^4 + 8cd^3x^3\sin(x)\cos(x) - 6cd^3x^2\sin^2(x) + 6cd^3x^2\cos^2(x) - 12cd^3x\sin(x)\cos(x) - 6cd^3\cos^2(x) - \frac{2d^4x^5\sin^2(x)}{5} - \frac{2d^4x^5\cos^2(x)}{5} + \frac{3d^4x^5}{5} + 2d^4x^4\sin(x)\cos(x) - 2d^4x^3\sin^2(x) + 2d^4x^3\cos^2(x) - 6d^4x^2\sin(x)\cos(x) + 3d^4x\sin^2(x) - 3d^4x\cos^2(x) + 3d^4\sin(x)\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(x)*sin(3*x),x)
```

```
[Out] c**4*x + c**4*sin(2*x) - 4*c**3*d*x**2*sin(x)**2 - 4*c**3*d*x**2*cos(x)**2
+ 6*c**3*d*x**2 + 8*c**3*d*x*sin(x)*cos(x) + 4*c**3*d*cos(x)**2 - 4*c**2*d*
*2*x**3*sin(x)**2 - 4*c**2*d**2*x**3*cos(x)**2 + 6*c**2*d**2*x**3 + 12*c**2
*d**2*x**2*sin(x)*cos(x) - 6*c**2*d**2*x*sin(x)**2 + 6*c**2*d**2*x*cos(x)**
2 - 6*c**2*d**2*sin(x)*cos(x) - 2*c*d**3*x**4*sin(x)**2 - 2*c*d**3*x**4*cos
(x)**2 + 3*c*d**3*x**4 + 8*c*d**3*x**3*sin(x)*cos(x) - 6*c*d**3*x**2*sin(x)
**2 + 6*c*d**3*x**2*cos(x)**2 - 12*c*d**3*x*sin(x)*cos(x) - 6*c*d**3*cos(x)
**2 - 2*d**4*x**5*sin(x)**2/5 - 2*d**4*x**5*cos(x)**2/5 + 3*d**4*x**5/5 + 2
*d**4*x**4*sin(x)*cos(x) - 2*d**4*x**3*sin(x)**2 + 2*d**4*x**3*cos(x)**2 -
6*d**4*x**2*sin(x)*cos(x) + 3*d**4*x*sin(x)**2 - 3*d**4*x*cos(x)**2 + 3*d**
4*sin(x)*cos(x)
```

Giac [A]
time = 0.39, size = 167, normalized size = 1.27

$$\frac{1}{5}d^4x^5 + cd^3x^4 + 2c^2d^2x^3 + 2c^3dx^2 + c^4x + \left(2d^4x^3 + 6cd^3x^2 + 6c^2d^2x - 3d^4x + 2c^3d - 3cd^3\right)\cos(2x) + \frac{1}{2}\left(2d^4x^4 + 8cd^3x^3 + 12c^2d^2x^2 - 6d^4x^2 + 8c^3dx - 12cd^3x + 2c^4 - 6c^2d^2 + 3d^4\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 1/5*d^4*x^5 + c*d^3*x^4 + 2*c^2*d^2*x^3 + 2*c^3*d*x^2 + c^4*x + (2*d^4*x^3
+ 6*c*d^3*x^2 + 6*c^2*d^2*x - 3*d^4*x + 2*c^3*d - 3*c*d^3)*cos(2*x) + 1/2*(
2*d^4*x^4 + 8*c*d^3*x^3 + 12*c^2*d^2*x^2 - 6*d^4*x^2 + 8*c^3*d*x - 12*c*d^3
*x + 2*c^4 - 6*c^2*d^2 + 3*d^4)*sin(2*x)
```

Mupad [B]
time = 2.26, size = 212, normalized size = 1.62

$$c^4\sin(2x) + \frac{3d^4\sin(2x)}{2} + c^4x + \frac{d^4x^5}{5} - 3c^2d^2\sin(2x) + 2d^4x^3\cos(2x) - 3d^4x^2\sin(2x) + d^4x^4\sin(2x) + 2c^3dx^2 + cd^3x^4 + 2c^2d^2x^3 - 3cd^3\cos(2x) + 2c^3d\cos(2x) - 3d^4x\cos(2x) + 6c^2d^2x^2\sin(2x) - 6cd^3x\sin(2x) + 4c^3dx\sin(2x) + 6c^2d^2x\cos(2x) + 6cd^3x^2\cos(2x) + 4cd^3x^3\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*x)*(c + d*x)^4)/sin(x),x)
```

```
[Out] c^4*sin(2*x) + (3*d^4*sin(2*x))/2 + c^4*x + (d^4*x^5)/5 - 3*c^2*d^2*sin(2*x
) + 2*d^4*x^3*cos(2*x) - 3*d^4*x^2*sin(2*x) + d^4*x^4*sin(2*x) + 2*c^3*d*x^
2 + c*d^3*x^4 + 2*c^2*d^2*x^3 - 3*c*d^3*cos(2*x) + 2*c^3*d*cos(2*x) - 3*d^4
*x*cos(2*x) + 6*c^2*d^2*x^2*sin(2*x) - 6*c*d^3*x*sin(2*x) + 4*c^3*d*x*sin(2
*x) + 6*c^2*d^2*x*cos(2*x) + 6*c*d^3*x^2*cos(2*x) + 4*c*d^3*x^3*sin(2*x)
```

3.362 $\int (c+dx)^3 \csc(x) \sin(3x) \, dx$

Optimal. Leaf size=115

$$-\frac{3}{2}cd^2x-\frac{3d^3x^2}{4}+\frac{(c+dx)^4}{4d}-\frac{9}{8}d^3\cos^2(x)+\frac{9}{4}d(c+dx)^2\cos^2(x)-3d^2(c+dx)\cos(x)\sin(x)+2(c+dx)^3\cos(x)\,\mathrm{si}$$

```
[Out] -3/2*c*d^2*x-3/4*d^3*x^2+1/4*(d*x+c)^4/d-9/8*d^3*cos(x)^2+9/4*d*(d*x+c)^2*c
os(x)^2-3*d^2*(d*x+c)*cos(x)*sin(x)+2*(d*x+c)^3*cos(x)*sin(x)+3/8*d^3*sin(x
)^2-3/4*d*(d*x+c)^2*sin(x)^2
```

Rubi [A]
time = 0.11, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4516, 3392, 32, 3391}

$$-\frac{3}{2}cd^2x-3d^2\sin(x)\cos(x)(c+dx)+\frac{(c+dx)^4}{4d}-\frac{3}{4}d\sin^2(x)(c+dx)^2+\frac{9}{4}d\cos^2(x)(c+dx)^2+2\sin(x)\cos(x)(c+dx)^3-\frac{3d^3x^2}{4}+\frac{3}{8}d^3\sin^2(x)-\frac{9}{8}d^3\cos^2(x)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[x]*Sin[3*x],x]
```

```
[Out] (-3*c*d^2*x)/2 - (3*d^3*x^2)/4 + (c + d*x)^4/(4*d) - (9*d^3*Cos[x]^2)/8 + (
9*d*(c + d*x)^2*Cos[x]^2)/4 - 3*d^2*(c + d*x)*Cos[x]*Sin[x] + 2*(c + d*x)^3
*Cos[x]*Sin[x] + (3*d^3*Sin[x]^2)/8 - (3*d*(c + d*x)^2*Sin[x]^2)/4
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int (c+dx)^3 \csc(x)\sin(3x)\,dx = \int \left(3(c+dx)^3\cos^2(x)-(c+dx)^3\sin^2(x)\right)\,dx$$
$$= 3\int (c+dx)^3\cos^2(x)\,dx-\int (c+dx)^3\sin^2(x)\,dx$$
$$= \frac{9}{4}d(c+dx)^2\cos^2(x)+2(c+dx)^3\cos(x)\sin(x)-\frac{3}{4}d(c+dx)^2\sin^2(x)-\frac{1}{2}\int ($$
$$= \frac{(c+dx)^4}{4d}-\frac{9}{8}d^3\cos^2(x)+\frac{9}{4}d(c+dx)^2\cos^2(x)-3d^2(c+dx)\cos(x)\sin(x)+$$
$$= -\frac{3}{2}cd^2x-\frac{3d^3x^2}{4}+\frac{(c+dx)^4}{4d}-\frac{9}{8}d^3\cos^2(x)+\frac{9}{4}d(c+dx)^2\cos^2(x)-3d^2(c+$$

Mathematica [A]
time = 0.15, size = 109, normalized size = 0.95

$$\frac{1}{4}\left(x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)+3d\left(2c^2+4cdx+d^2\left(-1+2x^2\right)\right)\cos(2x)+2\left(2c^3+6c^2dx+d^3x\left(-3+2x^2\right)+3cd^2\left(-1+2x^2\right)\right)\sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[x]*Sin[3*x],x]
```

```
[Out] (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 3*d*(2*c^2 + 4*c*d*x + d^2
*(-1 + 2*x^2))*Cos[2*x] + 2*(2*c^3 + 6*c^2*d*x + d^3*x*(-3 + 2*x^2) + 3*c*d
^2*(-1 + 2*x^2))*Sin[2*x])/4
```

Maple [A]
time = 0.09, size = 179, normalized size = 1.56

method	result
risch	$\frac{d^3x^4}{4}+d^2c\,x^3+\frac{3d\,c^2x^2}{2}+c^3x+\frac{c^4}{4d}+\frac{3d\left(2x^2d^2+4cdx+2c^2-d^2\right)\cos(2x)}{4}+\frac{\left(2d^3x^3+6c\,d^2x^2+6c^2dx-3d^3x+2c^3-3c\,d^2\right)\mathrm{si}}{2}$
default	$4d^3\left(x^3\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+\frac{3x^2\left(\cos^2(x)\right)}{4}-\frac{3x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)}{2}+\frac{3x^2}{8}+\frac{3\left(\sin^2(x)\right)}{8}-\frac{3x^4}{8}\right)+12c\,d^2\left(x^2\left(\frac{\cos}{}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 4*d^3*(x^3*(1/2*cos(x)*sin(x)+1/2*x)+3/4*x^2*cos(x)^2-3/2*x*(1/2*cos(x)*sin
(x)+1/2*x)+3/8*x^2+3/8*sin(x)^2-3/8*x^4)+12*c*d^2*(x^2*(1/2*cos(x)*sin(x)+1
/2*x)+1/2*x*cos(x)^2-1/4*cos(x)*sin(x)-1/4*x-1/3*x^3)+12*c^2*d*(x*(1/2*cos(
x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)-1/4*d^3*x^4+4*c^3*(1/2*cos(x)*sin(x)
+1/2*x)-d^2*c*x^3-3/2*d*c^2*x^2-c^3*x
```

Maxima [A]
time = 0.27, size = 101, normalized size = 0.88

$$\frac{3}{2}\left(x^{2}+2\,x\sin\left(2\,x\right)+\cos\left(2\,x\right)\right)c^{2}d+\frac{1}{2}\left(2\,x^{3}+6\,x\cos\left(2\,x\right)+3\left(2\,x^{2}-1\right)\sin\left(2\,x\right)\right)cd^{2}+\frac{1}{4}\left(x^{4}+3\left(2\,x^{2}-1\right)\cos\left(2\,x\right)+2\left(2\,x^{3}-3\,x\right)\sin\left(2\,x\right)\right)d^{3}+c^{3}(x+\sin\left(2\,x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 3/2*(x^2 + 2*x*sin(2*x) + cos(2*x))*c^2*d + 1/2*(2*x^3 + 6*x*cos(2*x) + 3*(
2*x^2 - 1)*sin(2*x))*c*d^2 + 1/4*(x^4 + 3*(2*x^2 - 1)*cos(2*x) + 2*(2*x^3 -
 3*x)*sin(2*x))*d^3 + c^3*(x + sin(2*x))
```

Fricas [A]
time = 3.13, size = 127, normalized size = 1.10

$$\frac{1}{4}\,d^{3}x^{4}+cd^{2}x^{3}+\frac{3}{2}\left(c^{2}d-d^{3}\right)x^{2}+\frac{3}{2}\left(2\,d^{3}x^{2}+4\,cd^{2}x+2\,c^{2}d-d^{3}\right)\cos\left(x\right)^{2}+\left(2\,d^{3}x^{3}+6\,cd^{2}x^{2}+2\,c^{3}-3\,cd^{2}+3\left(2\,c^{2}d-d^{3}\right)x\right)\cos\left(x\right)\sin\left(x\right)+\left(c^{3}-3\,cd^{2}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*(c^2*d - d^3)*x^2 + 3/2*(2*d^3*x^2 + 4*c*d^2*
x + 2*c^2*d - d^3)*cos(x)^2 + (2*d^3*x^3 + 6*c*d^2*x^2 + 2*c^3 - 3*c*d^2 +
3*(2*c^2*d - d^3)*x)*cos(x)*sin(x) + (c^3 - 3*c*d^2)*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(119) = 238$.
time = 7.62, size = 289, normalized size = 2.51

$$c^{3}x+c^{3}\sin\left(2x\right)-3c^{2}dx^{2}\sin^{2}\left(x\right)-3c^{2}dx^{2}\cos^{2}\left(x\right)+\frac{9c^{2}dx^{2}}{2}+6c^{2}dx\sin\left(x\right)\cos\left(x\right)+3c^{2}d\cos^{2}\left(x\right)-2cd^{2}x^{3}\sin^{2}\left(x\right)-2cd^{2}x^{3}\cos^{2}\left(x\right)+3cd^{2}x^{3}+6cd^{2}x^{2}\sin\left(x\right)\cos\left(x\right)-3cd^{2}x\sin^{2}\left(x\right)+3cd^{2}x\cos^{2}\left(x\right)-3cd^{2}\sin\left(x\right)\cos\left(x\right)-\frac{d^{3}x^{4}\sin^{2}\left(x\right)}{2}-\frac{d^{3}x^{4}\cos^{2}\left(x\right)}{2}+\frac{3d^{3}x^{4}}{4}+2d^{3}x^{3}\sin\left(x\right)\cos\left(x\right)-\frac{3d^{3}x^{2}\sin^{2}\left(x\right)}{2}+\frac{3d^{3}x^{2}\cos^{2}\left(x\right)}{2}-3d^{3}x\sin\left(x\right)\cos\left(x\right)-\frac{3d^{3}\cos^{2}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(x)*sin(3*x),x)
```

```
[Out] c**3*x + c**3*sin(2*x) - 3*c**2*d*x**2*sin(x)**2 - 3*c**2*d*x**2*cos(x)**2
+ 9*c**2*d*x**2/2 + 6*c**2*d*x*sin(x)*cos(x) + 3*c**2*d*cos(x)**2 - 2*c*d**
```

```
2*x**3*sin(x)**2 - 2*c*d**2*x**3*cos(x)**2 + 3*c*d**2*x**3 + 6*c*d**2*x**2*
sin(x)*cos(x) - 3*c*d**2*x*sin(x)**2 + 3*c*d**2*x*cos(x)**2 - 3*c*d**2*sin(
x)*cos(x) - d**3*x**4*sin(x)**2/2 - d**3*x**4*cos(x)**2/2 + 3*d**3*x**4/4 +
 2*d**3*x**3*sin(x)*cos(x) - 3*d**3*x**2*sin(x)**2/2 + 3*d**3*x**2*cos(x)**
2/2 - 3*d**3*x*sin(x)*cos(x) - 3*d**3*cos(x)**2/2
```

Giac [A]
time = 0.39, size = 112, normalized size = 0.97

$$\frac{1}{4}d^3x^4 + cd^2x^3 + \frac{3}{2}c^2dx^2 + c^3x + \frac{3}{4}\left(2\,d^3x^2 + 4\,cd^2x + 2\,c^2d - d^3\right)\cos\left(2\,x\right) + \frac{1}{2}\left(2\,d^3x^3 + 6\,cd^2x^2 + 6\,c^2dx - 3\,d^3x + 2\,c^3 - 3\,cd^2\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x + 3/4*(2*d^3*x^2 + 4*c*d^2*
x + 2*c^2*d - d^3)*cos(2*x) + 1/2*(2*d^3*x^3 + 6*c*d^2*x^2 + 6*c^2*d*x - 3*
d^3*x + 2*c^3 - 3*c*d^2)*sin(2*x)
```

Mupad [B]
time = 0.34, size = 136, normalized size = 1.18

$$c^3\,\sin\left(2\,x\right)-\frac{3\,d^3\,\cos\left(2\,x\right)}{4}+c^3\,x+\frac{d^3\,x^4}{4}+\frac{3\,d^3\,x^2\,\cos\left(2\,x\right)}{2}+d^3\,x^3\,\sin\left(2\,x\right)+\frac{3\,c^2\,d\,x^2}{2}+c\,d^2\,x^3+\frac{3\,c^2\,d\,\cos\left(2\,x\right)}{2}-\frac{3\,c\,d^2\,\sin\left(2\,x\right)}{2}-\frac{3\,d^3\,x\,\sin\left(2\,x\right)}{2}+3\,c\,d^2\,x\,\cos\left(2\,x\right)+3\,c^2\,d\,x\,\sin\left(2\,x\right)+3\,c\,d^2\,x^2\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*x)*(c + d*x)^3)/sin(x),x)
```

```
[Out] c^3*sin(2*x) - (3*d^3*cos(2*x))/4 + c^3*x + (d^3*x^4)/4 + (3*d^3*x^2*cos(2*
x))/2 + d^3*x^3*sin(2*x) + (3*c^2*d*x^2)/2 + c*d^2*x^3 + (3*c^2*d*cos(2*x))
/2 - (3*c*d^2*sin(2*x))/2 - (3*d^3*x*sin(2*x))/2 + 3*c*d^2*x*cos(2*x) + 3*c
^2*d*x*sin(2*x) + 3*c*d^2*x^2*sin(2*x)
```

3.363 $\int (c+dx)^2 \csc(x)\sin(3x)\,dx$

Optimal. Leaf size=73

$$-\frac{d^2x}{2}+\frac{(c+dx)^3}{3d}+\frac{3}{2}d(c+dx)\cos^2(x)-d^2\cos(x)\sin(x)+2(c+dx)^2\cos(x)\sin(x)-\frac{1}{2}d(c+dx)\sin^2(x)$$

```
[Out] -1/2*d^2*x+1/3*(d*x+c)^3/d+3/2*d*(d*x+c)*cos(x)^2-d^2*cos(x)*sin(x)+2*(d*x+
c)^2*cos(x)*sin(x)-1/2*d*(d*x+c)*sin(x)^2
```

Rubi [A]
time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4516, 3392, 32, 2715, 8}

$$\frac{(c+dx)^3}{3d}-\frac{1}{2}d\sin^2(x)(c+dx)+\frac{3}{2}d\cos^2(x)(c+dx)+2\sin(x)\cos(x)(c+dx)^2-\frac{d^2x}{2}-d^2\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[x]*Sin[3*x],x]
```

```
[Out] -1/2*(d^2*x) + (c + d*x)^3/(3*d) + (3*d*(c + d*x)*Cos[x]^2)/2 - d^2*Cos[x]*
Sin[x] + 2*(c + d*x)^2*Cos[x]*Sin[x] - (d*(c + d*x)*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2 \csc(x)\sin(3x)\,dx &= \int \left(3(c+dx)^2\cos^2(x)-(c+dx)^2\sin^2(x)\right)\,dx\\ &= 3\int (c+dx)^2\cos^2(x)\,dx-\int (c+dx)^2\sin^2(x)\,dx\\ &= \frac{3}{2}d(c+dx)\cos^2(x)+2(c+dx)^2\cos(x)\sin(x)-\frac{1}{2}d(c+dx)\sin^2(x)-\frac{1}{2}\int (c\\ &= \frac{(c+dx)^3}{3d}+\frac{3}{2}d(c+dx)\cos^2(x)-d^2\cos(x)\sin(x)+2(c+dx)^2\cos(x)\sin(x)\\ &= -\frac{d^2x}{2}+\frac{(c+dx)^3}{3d}+\frac{3}{2}d(c+dx)\cos^2(x)-d^2\cos(x)\sin(x)+2(c+dx)^2\cos(\end{aligned}$$

Mathematica [A]
time = 0.11, size = 60, normalized size = 0.82

$$c^2x+cdx^2+\frac{d^2x^3}{3}+d(c+dx)\cos(2x)+\left(2c^2+4cdx+d^2\left(-1+2x^2\right)\right)\cos(x)\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[x]*Sin[3*x],x]
```

```
[Out] c^2*x + c*d*x^2 + (d^2*x^3)/3 + d*(c + d*x)*Cos[2*x] + (2*c^2 + 4*c*d*x + d
^2*(-1 + 2*x^2))*Cos[x]*Sin[x]
```

Maple [A]
time = 0.09, size = 107, normalized size = 1.47

method	result
risch	$\frac{d^2x^3}{3}+cd\,x^2+c^2x+\frac{c^3}{3d}+d(dx+c)\cos(2x)+\frac{(2x^2d^2+4cdx+2c^2-d^2)\sin(2x)}{2}$

default	$4d^2\left(x^2\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+\frac{x(\cos^2(x))}{2}-\frac{\cos(x)\sin(x)}{4}-\frac{x}{4}-\frac{x^3}{3}\right)+8cd\left(x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{x^2}{4}-\frac{(\mathrm{si}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 4*d^2*(x^2*(1/2*cos(x)*sin(x)+1/2*x)+1/2*x*cos(x)^2-1/4*cos(x)*sin(x)-1/4*x
-1/3*x^3)+8*c*d*(x*(1/2*cos(x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)+4*c^2*(1
/2*cos(x)*sin(x)+1/2*x)-1/3*d^2*x^3-c*d*x^2-c^2*x
```

Maxima [A]
time = 0.27, size = 60, normalized size = 0.82

$$(x^2+2\,x\sin(2\,x)+\cos(2\,x))cd+\frac{1}{6}\left(2\,x^3+6\,x\cos(2\,x)+3\left(2\,x^2-1\right)\sin(2\,x)\right)d^2+c^2(x+\sin(2\,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] (x^2 + 2*x*sin(2*x) + cos(2*x))*c*d + 1/6*(2*x^3 + 6*x*cos(2*x) + 3*(2*x^2
- 1)*sin(2*x))*d^2 + c^2*(x + sin(2*x))
```

Fricas [A]
time = 2.54, size = 70, normalized size = 0.96

$$\frac{1}{3}\,d^2x^3+cdx^2+2\left(d^2x+cd\right)\cos(x)^2+\left(2\,d^2x^2+4\,cdx+2\,c^2-d^2\right)\cos(x)\sin(x)+\left(c^2-d^2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 1/3*d^2*x^3 + c*d*x^2 + 2*(d^2*x + c*d)*cos(x)^2 + (2*d^2*x^2 + 4*c*d*x + 2
*c^2 - d^2)*cos(x)*sin(x) + (c^2 - d^2)*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(70) = 140$.
time = 4.07, size = 155, normalized size = 2.12

$$c^2x+c^2\sin(2x)-2cdx^2\sin^2(x)-2cdx^2\cos^2(x)+3cdx^2+4cdx\sin(x)\cos(x)+2cd\cos^2(x)-\frac{2d^2x^3\sin^2(x)}{3}-\frac{2d^2x^3\cos^2(x)}{3}+d^2x^3+2d^2x^2\sin(x)\cos(x)-d^2x\sin^2(x)+d^2x\cos^2(x)-d^2\sin(x)\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(x)*sin(3*x),x)
```

```
[Out] c**2*x + c**2*sin(2*x) - 2*c*d*x**2*sin(x)**2 - 2*c*d*x**2*cos(x)**2 + 3*c*
d*x**2 + 4*c*d*x*sin(x)*cos(x) + 2*c*d*cos(x)**2 - 2*d**2*x**3*sin(x)**2/3
- 2*d**2*x**3*cos(x)**2/3 + d**2*x**3 + 2*d**2*x**2*sin(x)*cos(x) - d**2*x*
sin(x)**2 + d**2*x*cos(x)**2 - d**2*sin(x)*cos(x)
```

Giac [A]
time = 0.39, size = 64, normalized size = 0.88

$$\frac{1}{3}\,d^2x^3 + cdx^2 + c^2x + \left(d^2x + cd\right)\cos\left(2\,x\right) + \frac{1}{2}\,\left(2\,d^2x^2 + 4\,cdx + 2\,c^2 - d^2\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 1/3*d^2*x^3 + c*d*x^2 + c^2*x + (d^2*x + c*d)*cos(2*x) + 1/2*(2*d^2*x^2 + 4
*c*d*x + 2*c^2 - d^2)*sin(2*x)
```

Mupad [B]
time = 1.82, size = 73, normalized size = 1.00

$$c^2\,\sin\left(2\,x\right)-\frac{d^2\,\sin\left(2\,x\right)}{2}+c^2\,x+\frac{d^2\,x^3}{3}+d^2\,x^2\,\sin\left(2\,x\right)+c\,d\,\cos\left(2\,x\right)+d^2\,x\,\cos\left(2\,x\right)+c\,d\,x^2+2\,c\,d\,x\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*x)*(c + d*x)^2)/sin(x),x)
```

```
[Out] c^2*sin(2*x) - (d^2*sin(2*x))/2 + c^2*x + (d^2*x^3)/3 + d^2*x^2*sin(2*x) +
c*d*cos(2*x) + d^2*x*cos(2*x) + c*d*x^2 + 2*c*d*x*sin(2*x)
```

3.364 $\int (c+dx)\csc(x)\sin(3x)\,dx$

Optimal. Leaf size=41

$$cx+\frac{dx^2}{2}+\frac{3}{4}d\cos^2(x)+2(c+dx)\cos(x)\sin(x)-\frac{1}{4}d\sin^2(x)$$

[Out] c*x+1/2*d*x^2+3/4*d*cos(x)^2+2*(d*x+c)*cos(x)*sin(x)-1/4*d*sin(x)^2

Rubi [A]
time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4516, 3391}

$$2\sin(x)\cos(x)(c+dx)+cx+\frac{dx^2}{2}-\frac{1}{4}d\sin^2(x)+\frac{3}{4}d\cos^2(x)$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Csc[x]*Sin[3*x],x]

[Out] c*x + (d*x^2)/2 + (3*d*Cos[x]^2)/4 + 2*(c + d*x)*Cos[x]*Sin[x] - (d*Sin[x]^2)/4

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\csc(x)\sin(3x)\,dx &= \int \left(3(c+dx)\cos^2(x)-(c+dx)\sin^2(x)\right)\,dx \\ &= 3\int (c+dx)\cos^2(x)\,dx - \int (c+dx)\sin^2(x)\,dx \\ &= \frac{3}{4}d\cos^2(x)+2(c+dx)\cos(x)\sin(x)-\frac{1}{4}d\sin^2(x)-\frac{1}{2}\int (c+dx)\,dx+\frac{3}{2}\int (c \\ &= cx+\frac{dx^2}{2}+\frac{3}{4}d\cos^2(x)+2(c+dx)\cos(x)\sin(x)-\frac{1}{4}d\sin^2(x)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 34, normalized size = 0.83

$$cx+\frac{dx^2}{2}+\frac{1}{2}d\cos(2x)+c\sin(2x)+dx\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[x]*Sin[3*x],x]
```

```
[Out] c*x + (d*x^2)/2 + (d*Cos[2*x])/2 + c*Sin[2*x] + d*x*Sin[2*x]
```

Maple [A]
time = 0.07, size = 52, normalized size = 1.27

method	result	size
risch	$\frac{d\,x^2}{2}+cx+\frac{d\cos(2x)}{2}+(dx+c)\sin(2x)$	28
default	$4d\left(x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{x^2}{4}-\frac{(\sin^2(x))}{4}\right)+4c\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{d\,x^2}{2}-cx$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 4*d*(x*(1/2*cos(x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)+4*c*(1/2*cos(x)*sin(
x)+1/2*x)-1/2*d*x^2-c*x
```

Maxima [A]
time = 0.27, size = 27, normalized size = 0.66

$$\frac{1}{2}\left(x^2+2\,x\sin\left(2\,x\right)+\cos\left(2\,x\right)\right)d+c(x+\sin\left(2\,x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 1/2*(x^2 + 2*x*sin(2*x) + cos(2*x))*d + c*(x + sin(2*x))
```

Fricas [A]
time = 2.17, size = 27, normalized size = 0.66

$$\frac{1}{2}dx^2 + d\cos(x)^2 + 2(dx + c)\cos(x)\sin(x) + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="fricas")
[Out] 1/2*d*x^2 + d*cos(x)^2 + 2*(d*x + c)*cos(x)*sin(x) + c*x
```

Sympy [A]
time = 2.19, size = 56, normalized size = 1.37

$$cx + c\sin(2x) - dx^2\sin^2(x) - dx^2\cos^2(x) + \frac{3dx^2}{2} + 2dx\sin(x)\cos(x) + d\cos^2(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(x)*sin(3*x),x)
[Out] c*x + c*sin(2*x) - d*x**2*sin(x)**2 - d*x**2*cos(x)**2 + 3*d*x**2/2 + 2*d*x
*sin(x)*cos(x) + d*cos(x)**2
```

Giac [A]
time = 0.39, size = 27, normalized size = 0.66

$$\frac{1}{2}dx^2 + cx + \frac{1}{2}d\cos(2x) + (dx + c)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="giac")
[Out] 1/2*d*x^2 + c*x + 1/2*d*cos(2*x) + (d*x + c)*sin(2*x)
```

Mupad [B]
time = 1.72, size = 30, normalized size = 0.73

$$c\sin(2x) + cx + \frac{dx^2}{2} + \frac{d\cos(2x)}{2} + dx\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*x)*(c + d*x))/sin(x),x)
[Out] c*sin(2*x) + c*x + (d*x^2)/2 + (d*cos(2*x))/2 + d*x*sin(2*x)
```

3.365 $\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx$

Optimal. Leaf size=57

$$\frac{2\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d}+\frac{\log(c+dx)}{d}+\frac{2\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d}$$

```
[Out] 2*Ci(2*c/d+2*x)*cos(2*c/d)/d+ln(d*x+c)/d+2*Si(2*c/d+2*x)*sin(2*c/d)/d
```

Rubi [A]
time = 0.19, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.357$, Rules used = {4516, 3393, 3384, 3380, 3383}

$$\frac{2\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d}+\frac{2\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d}+\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]*Sin[3*x])/(c + d*x),x]
```

```
[Out] (2*Cos[(2*c)/d]*CosIntegral[(2*c)/d + 2*x])/d + Log[c + d*x]/d + (2*Sin[(2*
c)/d]*SinIntegral[(2*c)/d + 2*x])/d
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx &= \int \left(\frac{3\cos^2(x)}{c+dx}-\frac{\sin^2(x)}{c+dx}\right)dx \\
&= 3\int \frac{\cos^2(x)}{c+dx}\,dx-\int \frac{\sin^2(x)}{c+dx}\,dx \\
&= 3\int \left(\frac{1}{2(c+dx)}+\frac{\cos(2x)}{2(c+dx)}\right)dx-\int \left(\frac{1}{2(c+dx)}-\frac{\cos(2x)}{2(c+dx)}\right)dx \\
&= \frac{\log(c+dx)}{d}+\frac{1}{2}\int \frac{\cos(2x)}{c+dx}\,dx+\frac{3}{2}\int \frac{\cos(2x)}{c+dx}\,dx \\
&= \frac{\log(c+dx)}{d}+\frac{1}{2}\cos\left(\frac{2c}{d}\right)\int \frac{\cos\left(\frac{2c}{d}+2x\right)}{c+dx}\,dx+\frac{1}{2}\left(3\cos\left(\frac{2c}{d}\right)\right)\int \frac{\cos\left(\frac{2c}{d}+2x\right)}{c+dx}\,d \\
&= \frac{2\cos\left(\frac{2c}{d}\right)\text{Ci}\left(\frac{2c}{d}+2x\right)}{d}+\frac{\log(c+dx)}{d}+\frac{2\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 0.86

$$\frac{2\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(2\left(\frac{c}{d}+x\right)\right)+\log(c+dx)+2\sin\left(\frac{2c}{d}\right)\text{Si}\left(2\left(\frac{c}{d}+x\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]*Sin[3*x])/(c + d*x),x]
```

```
[Out] (2*Cos[(2*c)/d]*CosIntegral[2*(c/d + x)] + Log[c + d*x] + 2*Sin[(2*c)/d]*Si
nIntegral[2*(c/d + x)])/d
```

Maple [A]
time = 0.10, size = 58, normalized size = 1.02

method	result	size
default	$\frac{2\,\text{cosineIntegral}\left(\frac{2c}{d}+2x\right)\cos\left(\frac{2c}{d}\right)}{d}+\frac{\ln(dx+c)}{d}+\frac{2\,\text{sinIntegral}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d}$	58

risch	$\frac{\ln(dx+c)}{d}-\frac{e^{\frac{2ic}{d}}\,\mathrm{expIntegral}\left(1,2ix+\frac{2ic}{d}\right)}{d}-\frac{e^{-\frac{2ic}{d}}\,\mathrm{expIntegral}\left(1,-2ix-\frac{2ic}{d}\right)}{d}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*sin(3*x)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2*Ci(2*c/d+2*x)*cos(2*c/d)/d+ln(d*x+c)/d+2*Si(2*c/d+2*x)*sin(2*c/d)/d
```

Maxima [C] Result contains complex when optimal does not.
time = 0.30, size = 97, normalized size = 1.70

$$-\frac{\left(E_1\left(\frac{2(-idx-ic)}{d}\right)+E_1\left(-\frac{2(-idx-ic)}{d}\right)\right)\cos\left(\frac{2c}{d}\right)-\left(i\,E_1\left(\frac{2(-idx-ic)}{d}\right)-i\,E_1\left(-\frac{2(-idx-ic)}{d}\right)\right)\sin\left(\frac{2c}{d}\right)-\log\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -((exp_integral_e(1, 2*(-I*d*x - I*c)/d) + exp_integral_e(1, -2*(-I*d*x - I
*c)/d))*cos(2*c/d) - (I*exp_integral_e(1, 2*(-I*d*x - I*c)/d) - I*exp_integ
ral_e(1, -2*(-I*d*x - I*c)/d))*sin(2*c/d) - log(d*x + c))/d
```

Fricas [A]
time = 2.49, size = 62, normalized size = 1.09

$$\frac{\left(\mathrm{Ci}\left(\frac{2(dx+c)}{d}\right)+\mathrm{Ci}\left(-\frac{2(dx+c)}{d}\right)\right)\cos\left(\frac{2c}{d}\right)+2\sin\left(\frac{2c}{d}\right)\mathrm{Si}\left(\frac{2(dx+c)}{d}\right)+\log\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="fricas")
```

```
[Out] ((cos_integral(2*(d*x + c)/d) + cos_integral(-2*(d*x + c)/d))*cos(2*c/d) +
2*sin(2*c/d)*sin_integral(2*(d*x + c)/d) + log(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(3x\right)\csc\left(x\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c),x)
```

```
[Out] Integral(sin(3*x)*csc(x)/(c + d*x), x)
```

Giac [A]
time = 0.39, size = 51, normalized size = 0.89

$$\frac{2\cos\left(\frac{2c}{d}\right)\mathrm{Ci}\left(\frac{2(dx+c)}{d}\right)+2\sin\left(\frac{2c}{d}\right)\mathrm{Si}\left(\frac{2(dx+c)}{d}\right)+\log\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="giac")
```

```
[Out] (2*cos(2*c/d)*cos_integral(2*(d*x + c)/d) + 2*sin(2*c/d)*sin_integral(2*(d*
x + c)/d) + log(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin(3\,x)}{\sin(x)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)/(sin(x)*(c + d*x)),x)
```

```
[Out] int(sin(3*x)/(sin(x)*(c + d*x)), x)
```

3.366 $\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx$

Optimal. Leaf size=78

$$-\frac{3\cos^2(x)}{d(c+dx)}+\frac{4\text{CosIntegral}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d^2}+\frac{\sin^2(x)}{d(c+dx)}-\frac{4\cos\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^2}$$

```
[Out] -3*cos(x)^2/d/(d*x+c)-4*cos(2*c/d)*Si(2*c/d+2*x)/d^2+4*Ci(2*c/d+2*x)*sin(2*
c/d)/d^2+sin(x)^2/d/(d*x+c)
```

Rubi [A]
time = 0.18, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4516, 3394, 12, 3384, 3380, 3383}

$$\frac{4\sin\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d^2}-\frac{4\cos\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^2}+\frac{\sin^2(x)}{d(c+dx)}-\frac{3\cos^2(x)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]*Sin[3*x])/(c + d*x)^2,x]
```

```
[Out] (-3*Cos[x]^2)/(d*(c + d*x)) + (4*CosIntegral[(2*c)/d + 2*x]*Sin[(2*c)/d])/d
^2 + Sin[x]^2/(d*(c + d*x)) - (4*Cos[(2*c)/d]*SinIntegral[(2*c)/d + 2*x])/d
^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx &= \int \left(\frac{3\cos^2(x)}{(c+dx)^2}-\frac{\sin^2(x)}{(c+dx)^2}\right)dx\\
&= 3\int \frac{\cos^2(x)}{(c+dx)^2}\,dx-\int \frac{\sin^2(x)}{(c+dx)^2}\,dx\\
&= -\frac{3\cos^2(x)}{d(c+dx)}+\frac{\sin^2(x)}{d(c+dx)}-\frac{2\int \frac{\sin(2x)}{2(c+dx)}\,dx}{d}+\frac{6\int -\frac{\sin(2x)}{2(c+dx)}\,dx}{d}\\
&= -\frac{3\cos^2(x)}{d(c+dx)}+\frac{\sin^2(x)}{d(c+dx)}-\frac{\int \frac{\sin(2x)}{c+dx}\,dx}{d}-\frac{3\int \frac{\sin(2x)}{c+dx}\,dx}{d}\\
&= -\frac{3\cos^2(x)}{d(c+dx)}+\frac{\sin^2(x)}{d(c+dx)}-\frac{\cos\left(\frac{2c}{d}\right)\int \frac{\sin\left(\frac{2c}{d}+2x\right)}{c+dx}\,dx}{d}-\frac{\left(3\cos\left(\frac{2c}{d}\right)\right)\int \frac{\sin\left(\frac{2c}{d}+2x\right)}{c+dx}\,dx}{d}+\\
&= -\frac{3\cos^2(x)}{d(c+dx)}+\frac{4\operatorname{Ci}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d^2}+\frac{\sin^2(x)}{d(c+dx)}-\frac{4\cos\left(\frac{2c}{d}\right)\operatorname{Si}\left(\frac{2c}{d}+2x\right)}{d^2}
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 61, normalized size = 0.78

$$
\frac{-\frac{d(1+2\cos(2x))}{c+dx}+4\operatorname{CosIntegral}\left(2\left(\frac{c}{d}+x\right)\right)\sin\left(\frac{2c}{d}\right)-4\cos\left(\frac{2c}{d}\right)\operatorname{Si}\left(2\left(\frac{c}{d}+x\right)\right)}{d^2}
$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x]*Sin[3*x])/(c + d*x)^2,x]

[Out] (-((d*(1 + 2*Cos[2*x]))/(c + d*x)) + 4*CosIntegral[2*(c/d + x)]*Sin[(2*c)/d
] - 4*Cos[(2*c)/d]*SinIntegral[2*(c/d + x)])/d^2

Maple [A]
time = 0.12, size = 82, normalized size = 1.05

method	result	size
default	$-\frac{2\cos(2x)}{(dx+c)d}-\frac{2\left(\frac{2\,\mathrm{sinIntegral}\left(\frac{2c}{d}+2x\right)\cos\left(\frac{2c}{d}\right)}{d}-\frac{2\,\mathrm{cosineIntegral}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d}\right)}{d}-\frac{1}{d(dx+c)}$	82
risch	$-\frac{1}{d(dx+c)}+\frac{2i\,\mathrm{e}^{\frac{2ic}{d}}\,\mathrm{expIntegral}\left(1,2ix+\frac{2ic}{d}\right)}{d^2}-\frac{2i\,\mathrm{e}^{-\frac{2ic}{d}}\,\mathrm{expIntegral}\left(1,-2ix-\frac{2ic}{d}\right)}{d^2}-\frac{2i\cos(2x)}{d^2\left(ix+\frac{ic}{d}\right)}$	94

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)*sin(3*x)/(d*x+c)^2,x,method=_RETURNVERBOSE)

[Out] -2*cos(2*x)/(d*x+c)/d-2*(2*Si(2*c/d+2*x)*cos(2*c/d)/d-2*Ci(2*c/d+2*x)*sin(2
*c/d)/d)/d-1/d/(d*x+c)

Maxima [C] Result contains complex when optimal does not.
time = 0.31, size = 330, normalized size = 4.23

$$-\frac{\left(E_2\left(\frac{2(-idx-ic)}{d}\right)+E_2\left(-\frac{2(-idx-ic)}{d}\right)\right)\cos\left(\frac{2c}{d}\right)^3+\left(-iE_2\left(\frac{2(-idx-ic)}{d}\right)+iE_2\left(-\frac{2(-idx-ic)}{d}\right)\right)\sin\left(\frac{2c}{d}\right)^3+\left(\left(E_2\left(\frac{2(-idx-ic)}{d}\right)+E_2\left(-\frac{2(-idx-ic)}{d}\right)\right)\cos\left(\frac{2c}{d}\right)+2\right)\sin\left(\frac{2c}{d}\right)^2+\left(E_2\left(\frac{2(-idx-ic)}{d}\right)+E_2\left(-\frac{2(-idx-ic)}{d}\right)\right)\cos\left(\frac{2c}{d}\right)+2\cos\left(\frac{2c}{d}\right)^2+\left(\left(-iE_2\left(\frac{2(-idx-ic)}{d}\right)+iE_2\left(-\frac{2(-idx-ic)}{d}\right)\right)\cos\left(\frac{2c}{d}\right)^2-iE_2\left(\frac{2(-idx-ic)}{d}\right)+iE_2\left(-\frac{2(-idx-ic)}{d}\right)\right)\sin\left(\frac{2c}{d}\right)}{2\left(\left(\cos\left(\frac{2c}{d}\right)^2+\sin\left(\frac{2c}{d}\right)^2\right)d^2x+\left(c\cos\left(\frac{2c}{d}\right)^2+c\sin\left(\frac{2c}{d}\right)^2\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="maxima")

[Out] -1/2*((exp_integral_e(2, 2*(-I*d*x - I*c)/d) + exp_integral_e(2, -2*(-I*d*x
 - I*c)/d))*cos(2*c/d)^3 + (-I*exp_integral_e(2, 2*(-I*d*x - I*c)/d) + I*ex
p_integral_e(2, -2*(-I*d*x - I*c)/d))*sin(2*c/d)^3 + ((exp_integral_e(2, 2*
(-I*d*x - I*c)/d) + exp_integral_e(2, -2*(-I*d*x - I*c)/d))*cos(2*c/d) + 2)
*sin(2*c/d)^2 + (exp_integral_e(2, 2*(-I*d*x - I*c)/d) + exp_integral_e(2,
-2*(-I*d*x - I*c)/d))*cos(2*c/d) + 2*cos(2*c/d)^2 + ((-I*exp_integral_e(2,
2*(-I*d*x - I*c)/d) + I*exp_integral_e(2, -2*(-I*d*x - I*c)/d))*cos(2*c/d)^
2 - I*exp_integral_e(2, 2*(-I*d*x - I*c)/d) + I*exp_integral_e(2, -2*(-I*d*
x - I*c)/d))*sin(2*c/d))/((cos(2*c/d)^2 + sin(2*c/d)^2)*d^2*x + (c*cos(2*c/
d)^2 + c*sin(2*c/d)^2)*d)

Fricas [A]
time = 2.69, size = 95, normalized size = 1.22

$$-\frac{4\,d\cos(x)^2+4\,(dx+c)\cos\left(\frac{2\,c}{d}\right)\mathrm{Si}\left(\frac{2\,(dx+c)}{d}\right)-2\left((dx+c)\,\mathrm{Ci}\left(\frac{2\,(dx+c)}{d}\right)+(dx+c)\,\mathrm{Ci}\left(-\frac{2\,(dx+c)}{d}\right)\right)\sin\left(\frac{2\,c}{d}\right)-d}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(4*d*cos(x)^2 + 4*(d*x + c)*cos(2*c/d)*sin_integral(2*(d*x + c)/d) - 2*((d
*x + c)*cos_integral(2*(d*x + c)/d) + (d*x + c)*cos_integral(-2*(d*x + c)/d
))*sin(2*c/d) - d)/(d^3*x + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(3x)\csc(x)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(3*x)*csc(x)/(c + d*x)**2, x)
```

Giac [A]
time = 0.40, size = 111, normalized size = 1.42

$$\frac{4\,dx\,\mathrm{Ci}\left(\frac{2\,(dx+c)}{d}\right)\sin\left(\frac{2\,c}{d}\right)-4\,dx\cos\left(\frac{2\,c}{d}\right)\mathrm{Si}\left(\frac{2\,(dx+c)}{d}\right)+4\,c\,\mathrm{Ci}\left(\frac{2\,(dx+c)}{d}\right)\sin\left(\frac{2\,c}{d}\right)-4\,c\cos\left(\frac{2\,c}{d}\right)\mathrm{Si}\left(\frac{2\,(dx+c)}{d}\right)-2\,d\cos\left(2\,x\right)-d}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (4*d*x*cos_integral(2*(d*x + c)/d)*sin(2*c/d) - 4*d*x*cos(2*c/d)*sin_integr
al(2*(d*x + c)/d) + 4*c*cos_integral(2*(d*x + c)/d)*sin(2*c/d) - 4*c*cos(2*
c/d)*sin_integral(2*(d*x + c)/d) - 2*d*cos(2*x) - d)/(d^3*x + c*d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3\,x)}{\sin(x)\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)/(sin(x)*(c + d*x)^2),x)
```

```
[Out] int(sin(3*x)/(sin(x)*(c + d*x)^2), x)
```

3.367 $\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx$

Optimal. Leaf size=99

$$-\frac{3\cos^2(x)}{2d(c+dx)^2}-\frac{4\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d^3}+\frac{4\cos(x)\sin(x)}{d^2(c+dx)}+\frac{\sin^2(x)}{2d(c+dx)^2}-\frac{4\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^3}$$

```
[Out] -4*Ci(2*c/d+2*x)*cos(2*c/d)/d^3-3/2*cos(x)^2/d/(d*x+c)^2-4*Si(2*c/d+2*x)*si
n(2*c/d)/d^3+4*cos(x)*sin(x)/d^2/(d*x+c)+1/2*sin(x)^2/d/(d*x+c)^2
```

Rubi [A]
time = 0.23, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4516, 3395, 31, 3393, 3384, 3380, 3383}

$$-\frac{4\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d^3}-\frac{4\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^3}+\frac{4\sin(x)\cos(x)}{d^2(c+dx)}+\frac{\sin^2(x)}{2d(c+dx)^2}-\frac{3\cos^2(x)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]*Sin[3*x])/(c + d*x)^3,x]
```

```
[Out] (-3*Cos[x]^2)/(2*d*(c + d*x)^2) - (4*Cos[(2*c)/d]*CosIntegral[(2*c)/d + 2*x
])/d^3 + (4*Cos[x]*Sin[x])/(d^2*(c + d*x)) + Sin[x]^2/(2*d*(c + d*x)^2) - (
4*Sin[(2*c)/d]*SinIntegral[(2*c)/d + 2*x])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx = \int \left(\frac{3\cos^2(x)}{(c+dx)^3} - \frac{\sin^2(x)}{(c+dx)^3}\right)dx$$

$$= 3\int \frac{\cos^2(x)}{(c+dx)^3}\,dx - \int \frac{\sin^2(x)}{(c+dx)^3}\,dx$$

$$= -\frac{3\cos^2(x)}{2d(c+dx)^2} + \frac{4\cos(x)\sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{\int \frac{1}{c+dx}\,dx}{d^2} + \frac{2\int \frac{\sin^2(x)}{c+dx}\,dx}{d^2} + \frac{3\int \frac{1}{c+d}}{d^2}$$

$$= -\frac{3\cos^2(x)}{2d(c+dx)^2} + \frac{2\log(c+dx)}{d^3} + \frac{4\cos(x)\sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} + \frac{2\int \left(\frac{1}{2(c+dx)} - \frac{\cos(2}{2(c+d}\right.}{d^2}$$

$$= -\frac{3\cos^2(x)}{2d(c+dx)^2} + \frac{4\cos(x)\sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{\int \frac{\cos(2x)}{c+dx}\,dx}{d^2} - \frac{3\int \frac{\cos(2x)}{c+dx}\,dx}{d^2}$$

$$= -\frac{3\cos^2(x)}{2d(c+dx)^2} + \frac{4\cos(x)\sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{\cos\left(\frac{2c}{d}\right)\int \frac{\cos\left(\frac{2c}{d}+2x\right)}{c+dx}\,dx}{d^2} - \frac{\left(3\cos\left(\frac{2c}{d}\right.\right.}{}$$

$$= -\frac{3\cos^2(x)}{2d(c+dx)^2} - \frac{4\cos\left(\frac{2c}{d}\right)\operatorname{Ci}\left(\frac{2c}{d}+2x\right)}{d^3} + \frac{4\cos(x)\sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{4\sin\left(\frac{2c}{d}\right)\operatorname{S}}{d}$$

Mathematica [A]
time = 0.27, size = 77, normalized size = 0.78

$$\frac{-8\cos\left(\frac{2c}{d}\right)\operatorname{CosIntegral}\left(2\left(\frac{c}{d}+x\right)\right) + \frac{d(-d-2d\cos(2x)+4(c+dx)\sin(2x))}{(c+dx)^2} - 8\sin\left(\frac{2c}{d}\right)\operatorname{Si}\left(2\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]*Sin[3*x])/(c + d*x)^3,x]
```

```
[Out] (-8*Cos[(2*c)/d]*CosIntegral[2*(c/d + x)] + (d*(-d - 2*d*Cos[2*x] + 4*(c +
d*x)*Sin[2*x]))/(c + d*x)^2 - 8*Sin[(2*c)/d]*SinIntegral[2*(c/d + x)])/(2*d
^3)
```

Maple [A]
time = 0.13, size = 104, normalized size = 1.05

method	result	si
default	$-\frac{\cos(2x)}{(dx+c)^2 d} - \frac{-\frac{2\sin(2x)}{(dx+c)d} + \frac{\frac{4\operatorname{sinIntegral}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d} + \frac{4\operatorname{cosineIntegral}\left(\frac{2c}{d}+2x\right)\cos\left(\frac{2c}{d}\right)}{d}}{d}}{d} - \frac{1}{2d(dx+c)^2}$	10
risch	$-\frac{1}{2d(dx+c)^2} + \frac{2\,e^{\frac{2ic}{d}}\operatorname{expIntegral}\left(1, 2ix+\frac{2ic}{d}\right)}{d^3} + \frac{2\,e^{-\frac{2ic}{d}}\operatorname{expIntegral}\left(1, -2ix-\frac{2ic}{d}\right)}{d^3} - \frac{\cos(2x)}{(dx+c)^2 d} + \frac{i(-4idx-4ic)\sin(2x)}{2(dx+c)^2 d^2}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*sin(3*x)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -cos(2*x)/(d*x+c)^2/d-(-2*sin(2*x)/(d*x+c)/d+2*(2*Si(2*c/d+2*x)*sin(2*c/d)/
d+2*Ci(2*c/d+2*x)*cos(2*c/d)/d)/d)/d-1/2/d/(d*x+c)^2
```

Maxima [C] Result contains complex when optimal does not.
time = 0.33, size = 365, normalized size = 3.69

$$-\frac{\left(E_3\left(\frac{2\,(-i\,dx-i\,c)}{d}\right)+E_3\left(-\frac{2\,(-i\,dx-i\,c)}{d}\right)\right)\cos\left(\frac{2\,c}{d}\right)^3-\left(i\,E_3\left(\frac{2\,(-i\,dx-i\,c)}{d}\right)-i\,E_3\left(-\frac{2\,(-i\,dx-i\,c)}{d}\right)\right)\sin\left(\frac{2\,c}{d}\right)^3+\left(\left(E_3\left(\frac{2\,(-i\,dx-i\,c)}{d}\right)+E_3\left(-\frac{2\,(-i\,dx-i\,c)}{d}\right)\right)\cos\left(\frac{2\,c}{d}\right)+1\right)\sin\left(\frac{2\,c}{d}\right)^2+\left(E_3\left(\frac{2\,(-i\,dx-i\,c)}{d}\right)+E_3\left(-\frac{2\,(-i\,dx-i\,c)}{d}\right)\right)\cos\left(\frac{2\,c}{d}\right)+\cos\left(\frac{2\,c}{d}\right)^2-\left(\left(i\,E_3\left(\frac{2\,(-i\,dx-i\,c)}{d}\right)-i\,E_3\left(-\frac{2\,(-i\,dx-i\,c)}{d}\right)\right)\cos\left(\frac{2\,c}{d}\right)^2+i\,E_3\left(\frac{2\,(-i\,dx-i\,c)}{d}\right)-i\,E_3\left(-\frac{2\,(-i\,dx-i\,c)}{d}\right)\right)\sin\left(\frac{2\,c}{d}\right)}{2\left(\left(\cos\left(\frac{2\,c}{d}\right)^2+\sin\left(\frac{2\,c}{d}\right)^2\right)d^3x^2+2\left(c\cos\left(\frac{2\,c}{d}\right)^2+c\sin\left(\frac{2\,c}{d}\right)^2\right)d^2x+\left(c^2\cos\left(\frac{2\,c}{d}\right)^2+c^2\sin\left(\frac{2\,c}{d}\right)^2\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((exp_integral_e(3, 2*(-I*d*x - I*c)/d) + exp_integral_e(3, -2*(-I*d*x
 - I*c)/d))*cos(2*c/d)^3 - (I*exp_integral_e(3, 2*(-I*d*x - I*c)/d) - I*exp
_integral_e(3, -2*(-I*d*x - I*c)/d))*sin(2*c/d)^3 + ((exp_integral_e(3, 2*(
-I*d*x - I*c)/d) + exp_integral_e(3, -2*(-I*d*x - I*c)/d))*cos(2*c/d) + 1)*
sin(2*c/d)^2 + (exp_integral_e(3, 2*(-I*d*x - I*c)/d) + exp_integral_e(3, -
2*(-I*d*x - I*c)/d))*cos(2*c/d) + cos(2*c/d)^2 - ((I*exp_integral_e(3, 2*(-
I*d*x - I*c)/d) - I*exp_integral_e(3, -2*(-I*d*x - I*c)/d))*cos(2*c/d)^2 +
I*exp_integral_e(3, 2*(-I*d*x - I*c)/d) - I*exp_integral_e(3, -2*(-I*d*x -
I*c)/d))*sin(2*c/d))/((cos(2*c/d)^2 + sin(2*c/d)^2)*d^3*x^2 + 2*(c*cos(2*c/
d)^2 + c*sin(2*c/d)^2)*d^2*x + (c^2*cos(2*c/d)^2 + c^2*sin(2*c/d)^2)*d)
```

Fricas [A]
time = 2.07, size = 158, normalized size = 1.60

$$-\frac{4\,d^2\cos\left(x\right)^2-8\left(d^2x+cd\right)\cos\left(x\right)\sin\left(x\right)+8\left(d^2x^2+2\,cdx+c^2\right)\sin\left(\frac{2\,c}{d}\right)\operatorname{Si}\left(\frac{2\,(dx+c)}{d}\right)-d^2+4\left(\left(d^2x^2+2\,cdx+c^2\right)\operatorname{Ci}\left(\frac{2\,(dx+c)}{d}\right)+\left(d^2x^2+2\,cdx+c^2\right)\operatorname{Ci}\left(-\frac{2\,(dx+c)}{d}\right)\right)\cos\left(\frac{2\,c}{d}\right)}{2\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*d^2*cos(x)^2 - 8*(d^2*x + c*d)*cos(x)*sin(x) + 8*(d^2*x^2 + 2*c*d*x
 + c^2)*sin(2*c/d)*sin_integral(2*(d*x + c)/d) - d^2 + 4*((d^2*x^2 + 2*c*d*
x + c^2)*cos_integral(2*(d*x + c)/d) + (d^2*x^2 + 2*c*d*x + c^2)*cos_integr
al(-2*(d*x + c)/d))*cos(2*c/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(3x\right)\csc\left(x\right)}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)**3,x)
```

```
[Out] Integral(sin(3*x)*csc(x)/(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(95) = 190.
time = 0.40, size = 201, normalized size = 2.03

$$-\frac{8\,d^2x^2\cos\left(\frac{2\,c}{d}\right)\operatorname{Ci}\left(\frac{2\,(dx+c)}{d}\right)+8\,d^2x^2\sin\left(\frac{2\,c}{d}\right)\operatorname{Si}\left(\frac{2\,(dx+c)}{d}\right)+16\,cdx\cos\left(\frac{2\,c}{d}\right)\operatorname{Ci}\left(\frac{2\,(dx+c)}{d}\right)+16\,cdx\sin\left(\frac{2\,c}{d}\right)\operatorname{Si}\left(\frac{2\,(dx+c)}{d}\right)+8\,c^2\cos\left(\frac{2\,c}{d}\right)\operatorname{Ci}\left(\frac{2\,(dx+c)}{d}\right)-4\,d^2x\sin\left(2\,x\right)+8\,c^2\sin\left(\frac{2\,c}{d}\right)\operatorname{Si}\left(\frac{2\,(dx+c)}{d}\right)+2\,d^2\cos\left(2\,x\right)-4\,cd\sin\left(2\,x\right)+d^2}{2\,\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(8*d^2*x^2*cos(2*c/d)*cos_integral(2*(d*x + c)/d) + 8*d^2*x^2*sin(2*c/
d)*sin_integral(2*(d*x + c)/d) + 16*c*d*x*cos(2*c/d)*cos_integral(2*(d*x +
c)/d) + 16*c*d*x*sin(2*c/d)*sin_integral(2*(d*x + c)/d) + 8*c^2*cos(2*c/d)*
cos_integral(2*(d*x + c)/d) - 4*d^2*x*sin(2*x) + 8*c^2*sin(2*c/d)*sin_integ
ral(2*(d*x + c)/d) + 2*d^2*cos(2*x) - 4*c*d*sin(2*x) + d^2)/(d^5*x^2 + 2*c*
d^4*x + c^2*d^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,x\right)}{\sin\left(x\right)\,{\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)/(sin(x)*(c + d*x)^3),x)
```

```
[Out] int(sin(3*x)/(sin(x)*(c + d*x)^3), x)
```

3.368 $\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=198

$$\frac{3d^4x}{2b^4}-\frac{d(c+dx)^3}{b^2}+\frac{(c+dx)^5}{5d}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}+\frac{3d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{3d^4\cos(a+bx)\sin(a-}{b^5}$$

```
[Out] 3/2*d^4*x/b^4-d*(d*x+c)^3/b^2+1/5*(d*x+c)^5/d-9/2*d^3*(d*x+c)*cos(b*x+a)^2/
b^4+3*d*(d*x+c)^3*cos(b*x+a)^2/b^2+3*d^4*cos(b*x+a)*sin(b*x+a)/b^5-6*d^2*(d
*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^3+2*(d*x+c)^4*cos(b*x+a)*sin(b*x+a)/b+3/2*d
^3*(d*x+c)*sin(b*x+a)^2/b^4-d*(d*x+c)^3*sin(b*x+a)^2/b^2
```

Rubi [A]
time = 0.18, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4516, 3392, 32, 2715, 8}

$$\frac{3d^4\sin(a+bx)\cos(a+bx)}{b^5}+\frac{3d^3(c+dx)\sin^2(a+bx)}{2b^4}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}-\frac{6d^2(c+dx)^2\sin(a+bx)\cos(a+bx)}{b^3}-\frac{d(c+dx)^3\sin^2(a+bx)}{b^2}+\frac{3d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{2(c+dx)^4\sin(a+bx)\cos(a+bx)}{b}+\frac{3d^4x}{2b^4}-\frac{d(c+dx)^3}{b^2}+\frac{(c+dx)^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (3*d^4*x)/(2*b^4) - (d*(c + d*x)^3)/b^2 + (c + d*x)^5/(5*d) - (9*d^3*(c + d
*x)*Cos[a + b*x]^2)/(2*b^4) + (3*d*(c + d*x)^3*Cos[a + b*x]^2)/b^2 + (3*d^4
*Cos[a + b*x]*Sin[a + b*x])/b^5 - (6*d^2*(c + d*x)^2*Cos[a + b*x]*Sin[a + b
*x])/b^3 + (2*(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x])/b + (3*d^3*(c + d*x)*S
in[a + b*x]^2)/(2*b^4) - (d*(c + d*x)^3*Sin[a + b*x]^2)/b^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
```

```
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^4\csc(a+bx)\sin(3a+3bx)\,dx &= \int \left(3(c+dx)^4\cos^2(a+bx)-(c+dx)^4\sin^2(a+bx)\right)\,dx\\ &= 3\int (c+dx)^4\cos^2(a+bx)\,dx-\int (c+dx)^4\sin^2(a+bx)\,dx\\ &= \frac{3d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{2(c+dx)^4\cos(a+bx)\sin(a+bx)}{b}-\\ &= \frac{(c+dx)^5}{5d}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}+\frac{3d(c+dx)^3\cos^2(a+bx)}{b^2}\\ &= -\frac{d(c+dx)^3}{b^2}+\frac{(c+dx)^5}{5d}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}+\frac{3d(c+d}{}\\ &= \frac{3d^4x}{2b^4}-\frac{d(c+dx)^3}{b^2}+\frac{(c+dx)^5}{5d}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}+\frac{3d}{}\end{aligned}$$

Mathematica [**A**]
time = 0.64, size = 128, normalized size = 0.65

$$c^4x+2c^3dx^2+2c^2d^2x^3+cd^3x^4+\frac{d^4x^5}{5}+\frac{d(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\cos(2(a+bx))}{b^4}+\frac{\left(3d^4-6b^2d^2(c+dx)^2+2b^4(c+dx)^4\right)\sin(2(a+bx))}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] c^4*x + 2*c^3*d*x^2 + 2*c^2*d^2*x^3 + c*d^3*x^4 + (d^4*x^5)/5 + (d*(c + d*x
)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])/b^4 + ((3*d^4 - 6*b^2*d^2*
(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sin[2*(a + b*x)])/(2*b^5)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 999 vs. 2(190) = 380.
time = 0.13, size = 1000, normalized size = 5.05

method	result
risch	$\frac{d^4x^5}{5}+d^3c\,x^4+2d^2c^2x^3+2d\,c^3x^2+c^4x+\frac{c^5}{5d}+\frac{d(2b^2d^3x^3+6b^2c\,d^2x^2+6b^2c^2dx+2b^2c^3-3d^3x-3c\,d^2)\cos(2bx+2a)}{b^4}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

```
[Out] -c^4*x-1/5*d^4*x^5+4*c^4/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-d^3*c*
x^4-2*d^2*c^2*x^3-2*d*c^3*x^2+4*d^4/b^5*((b*x+a)^4*(1/2*cos(b*x+a)*sin(b*x+
a)+1/2*b*x+1/2*a)+(b*x+a)^3*cos(b*x+a)^2-3*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*
x+a)+1/2*b*x+1/2*a)-3/2*(b*x+a)*cos(b*x+a)^2+3/4*cos(b*x+a)*sin(b*x+a)+3/4*
b*x+3/4*a+(b*x+a)^3-2/5*(b*x+a)^5-4*a*((b*x+a)^3*(1/2*cos(b*x+a)*sin(b*x+a)
+1/2*b*x+1/2*a)+3/4*(b*x+a)^2*cos(b*x+a)^2-3/2*(b*x+a)*(1/2*cos(b*x+a)*sin(
b*x+a)+1/2*b*x+1/2*a)+3/8*(b*x+a)^2+3/8*sin(b*x+a)^2-3/8*(b*x+a)^4)+6*a^2*(
(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^
2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)-4*a^3*((b*x+a)*(1/
2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+a^4*
(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))+16*d^3*c/b^4*((b*x+a)^3*(1/2*cos
(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)^2*cos(b*x+a)^2-3/2*(b*x+a)*(1
/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/8*(b*x+a)^2+3/8*sin(b*x+a)^2-3/8*
(b*x+a)^4-3*a*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x
+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)+3*a
^2*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin
(b*x+a)^2)-a^3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))+24*c^2*d^2/b^3*((
b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2
-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3-2*a*((b*x+a)*(1/2*co
s(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+a^2*(1/2
*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))+16*c^3*d/b^2*((b*x+a)*(1/2*cos(b*x+a
)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2-a*(1/2*cos(b*x+a
)*sin(b*x+a)+1/2*b*x+1/2*a))
```

Maxima [A]

time = 0.31, size = 244, normalized size = 1.23

$$\frac{(bx+\sin(2\,bx+2\,a))c^4}{b}+\frac{2\,(b^2x^2+2\,bx\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a))c^3d}{b^2}+\frac{(2\,b^3x^3+6\,bx\cos(2\,bx+2\,a)+3\,(2\,b^2x^2-1)\sin(2\,bx+2\,a))c^2d^2}{b^3}+\frac{(b^4x^4+3\,(2\,b^2x^2-1)\cos(2\,bx+2\,a)+2\,(2\,b^3x^3-3\,bx)\sin(2\,bx+2\,a))cd^3}{b^4}+\frac{(2\,b^5x^5+10\,(2\,b^3x^3-3\,bx)\cos(2\,bx+2\,a)+5\,(2\,b^4x^4-6\,b^2x^2+3)\sin(2\,bx+2\,a))d^4}{10\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] (b*x + sin(2*b*x + 2*a))*c^4/b + 2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos(
2*b*x + 2*a))*c^3*d/b^2 + (2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(2*b^2*x^
2 - 1)*sin(2*b*x + 2*a))*c^2*d^2/b^3 + (b^4*x^4 + 3*(2*b^2*x^2 - 1)*cos(2*b
*x + 2*a) + 2*(2*b^3*x^3 - 3*b*x)*sin(2*b*x + 2*a))*c*d^3/b^4 + 1/10*(2*b^5
```

```
*x^5 + 10*(2*b^3*x^3 - 3*b*x)*cos(2*b*x + 2*a) + 5*(2*b^4*x^4 - 6*b^2*x^2 +
 3)*sin(2*b*x + 2*a))*d^4/b^5
```

Fricas [A]
time = 1.97, size = 283, normalized size = 1.43

$$\frac{b^5d^4x^5 + 5\,b^5cd^3x^4 + 10\,(b^5c^2d^2 - b^3d^4)x^3 + 10\,(b^5c^3d - 3\,b^3cd^3)x^2 + 10\,(2\,b^3d^4x^3 + 6\,b^3cd^3x^2 + 2\,b^3c^3d - 3\,bcd^3 + 3\,(2\,b^3c^2d^2 - bd^4)x)\cos(bx+a)^2 + 5\,(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 2\,b^4c^4 - 6\,b^2c^2d^2 + 3\,d^4 + 6\,(2\,b^4c^2d^2 - b^2d^4)x^2 + 4\,(2\,b^4c^3d - 3\,b^2cd^3)x)\cos(bx+a)\sin(bx+a) + 5\,(b^5c^4 - 6\,b^3c^2d^2 + 3\,bd^4)x}{5\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/5*(b^5*d^4*x^5 + 5*b^5*c*d^3*x^4 + 10*(b^5*c^2*d^2 - b^3*d^4)*x^3 + 10*(b
^5*c^3*d - 3*b^3*c*d^3)*x^2 + 10*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c
^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 + 5*(2*b^4*d
^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2
*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)*sin(b*x
 + a) + 5*(b^5*c^4 - 6*b^3*c^2*d^2 + 3*b*d^4)*x)/b^5
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1255 vs. 2(190) = 380.
time = 0.54, size = 1255, normalized size = 6.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] 1/5*(10*b^4*c^4*cos(b*x + a)*sin(b*x + a) + 40*(b*x + a)*b^3*c^3*d*cos(b*x
+ a)*sin(b*x + a) - 40*a*b^3*c^3*d*cos(b*x + a)*sin(b*x + a) + 60*(b*x + a)
^2*b^2*c^2*d^2*cos(b*x + a)*sin(b*x + a) - 120*(b*x + a)*a*b^2*c^2*d^2*cos(
b*x + a)*sin(b*x + a) + 60*a^2*b^2*c^2*d^2*cos(b*x + a)*sin(b*x + a) + 40*(
b*x + a)^3*b*c*d^3*cos(b*x + a)*sin(b*x + a) - 120*(b*x + a)^2*a*b*c*d^3*co
s(b*x + a)*sin(b*x + a) + 120*(b*x + a)*a^2*b*c*d^3*cos(b*x + a)*sin(b*x +
a) - 40*a^3*b*c*d^3*cos(b*x + a)*sin(b*x + a) + 10*(b*x + a)^4*d^4*cos(b*x
+ a)*sin(b*x + a) - 40*(b*x + a)^3*a*d^4*cos(b*x + a)*sin(b*x + a) + 60*(b*
x + a)^2*a^2*d^4*cos(b*x + a)*sin(b*x + a) - 40*(b*x + a)*a^3*d^4*cos(b*x +
```

```
 a)*sin(b*x + a) + 10*a^4*d^4*cos(b*x + a)*sin(b*x + a) + 5*(b*x + a)*b^4*c
^4 + 10*(b*x + a)^2*b^3*c^3*d - 20*(b*x + a)*a*b^3*c^3*d + 10*(b*x + a)^3*b
^2*c^2*d^2 - 30*(b*x + a)^2*a*b^2*c^2*d^2 + 30*(b*x + a)*a^2*b^2*c^2*d^2 +
5*(b*x + a)^4*b*c*d^3 - 20*(b*x + a)^3*a*b*c*d^3 + 30*(b*x + a)^2*a^2*b*c*d
^3 - 20*(b*x + a)*a^3*b*c*d^3 + (b*x + a)^5*d^4 - 5*(b*x + a)^4*a*d^4 + 10*
(b*x + a)^3*a^2*d^4 - 10*(b*x + a)^2*a^3*d^4 + 5*(b*x + a)*a^4*d^4 + 10*b^3
*c^3*d*cos(b*x + a)^2 + 30*(b*x + a)*b^2*c^2*d^2*cos(b*x + a)^2 - 30*a*b^2*
c^2*d^2*cos(b*x + a)^2 + 30*(b*x + a)^2*b*c*d^3*cos(b*x + a)^2 - 60*(b*x +
a)*a*b*c*d^3*cos(b*x + a)^2 + 30*a^2*b*c*d^3*cos(b*x + a)^2 + 10*(b*x + a)^
3*d^4*cos(b*x + a)^2 - 30*(b*x + a)^2*a*d^4*cos(b*x + a)^2 + 30*(b*x + a)*a
^2*d^4*cos(b*x + a)^2 - 10*a^3*d^4*cos(b*x + a)^2 - 10*b^3*c^3*d*sin(b*x +
a)^2 - 30*(b*x + a)*b^2*c^2*d^2*sin(b*x + a)^2 + 30*a*b^2*c^2*d^2*sin(b*x +
 a)^2 - 30*(b*x + a)^2*b*c*d^3*sin(b*x + a)^2 + 60*(b*x + a)*a*b*c*d^3*sin(
b*x + a)^2 - 30*a^2*b*c*d^3*sin(b*x + a)^2 - 10*(b*x + a)^3*d^4*sin(b*x + a
)^2 + 30*(b*x + a)^2*a*d^4*sin(b*x + a)^2 - 30*(b*x + a)*a^2*d^4*sin(b*x +
a)^2 + 10*a^3*d^4*sin(b*x + a)^2 - 30*b^2*c^2*d^2*cos(b*x + a)*sin(b*x + a)
 - 60*(b*x + a)*b*c*d^3*cos(b*x + a)*sin(b*x + a) + 60*a*b*c*d^3*cos(b*x +
a)*sin(b*x + a) - 30*(b*x + a)^2*d^4*cos(b*x + a)*sin(b*x + a) + 60*(b*x +
a)*a*d^4*cos(b*x + a)*sin(b*x + a) - 30*a^2*d^4*cos(b*x + a)*sin(b*x + a) -
 15*b*c*d^3*cos(b*x + a)^2 - 15*(b*x + a)*d^4*cos(b*x + a)^2 + 15*a*d^4*cos
(b*x + a)^2 + 15*b*c*d^3*sin(b*x + a)^2 + 15*(b*x + a)*d^4*sin(b*x + a)^2 -
 15*a*d^4*sin(b*x + a)^2 + 15*d^4*cos(b*x + a)*sin(b*x + a))/b^5
```

Mupad [B]
time = 0.65, size = 344, normalized size = 1.74

$$\frac{\frac{3\,d^4\,\sin\left(2\,a+2\,b\,x\right)}{2}+b^5\,c^4\,x+b^4\,c^4\,\sin\left(2\,a+2\,b\,x\right)+\frac{b^5\,d^4\,x^5}{5}+2\,b^3\,c^3\,d\,\cos\left(2\,a+2\,b\,x\right)+2\,b^5\,c^3\,d\,x^2+b^5\,c\,d^3\,x^4-3\,b^2\,c^2\,d^2\,\sin\left(2\,a+2\,b\,x\right)+2\,b^3\,d^4\,x^3\,\cos\left(2\,a+2\,b\,x\right)+2\,b^5\,c^2\,d^2\,x^3-3\,b^2\,d^4\,x^2\,\sin\left(2\,a+2\,b\,x\right)+b^4\,d^4\,x^4\,\sin\left(2\,a+2\,b\,x\right)-3\,b\,c\,d^3\,\cos\left(2\,a+2\,b\,x\right)-3\,b\,d^4\,x\,\cos\left(2\,a+2\,b\,x\right)+6\,b^4\,c^2\,d^2\,x^2\,\sin\left(2\,a+2\,b\,x\right)-6\,b^2\,c\,d^3\,x\,\sin\left(2\,a+2\,b\,x\right)+4\,b^4\,c^3\,d\,x\,\sin\left(2\,a+2\,b\,x\right)+6\,b^3\,c^2\,d^2\,x\,\cos\left(2\,a+2\,b\,x\right)+6\,b^3\,c\,d^3\,x^2\,\cos\left(2\,a+2\,b\,x\right)+4\,b^4\,c\,d^3\,x^3\,\sin\left(2\,a+2\,b\,x\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^4)/sin(a + b*x),x)
```

```
[Out] ((3*d^4*sin(2*a + 2*b*x))/2 + b^5*c^4*x + b^4*c^4*sin(2*a + 2*b*x) + (b^5*d
^4*x^5)/5 + 2*b^3*c^3*d*cos(2*a + 2*b*x) + 2*b^5*c^3*d*x^2 + b^5*c*d^3*x^4
- 3*b^2*c^2*d^2*sin(2*a + 2*b*x) + 2*b^3*d^4*x^3*cos(2*a + 2*b*x) + 2*b^5*c
^2*d^2*x^3 - 3*b^2*d^4*x^2*sin(2*a + 2*b*x) + b^4*d^4*x^4*sin(2*a + 2*b*x)
- 3*b*c*d^3*cos(2*a + 2*b*x) - 3*b*d^4*x*cos(2*a + 2*b*x) + 6*b^4*c^2*d^2*x
^2*sin(2*a + 2*b*x) - 6*b^2*c*d^3*x*sin(2*a + 2*b*x) + 4*b^4*c^3*d*x*sin(2*
a + 2*b*x) + 6*b^3*c^2*d^2*x*cos(2*a + 2*b*x) + 6*b^3*c*d^3*x^2*cos(2*a + 2
*b*x) + 4*b^4*c*d^3*x^3*sin(2*a + 2*b*x))/b^5
```

3.369 $\int (c+dx)^3 \csc(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=171

$$-\frac{3cd^2x}{2b^2}-\frac{3d^3x^2}{4b^2}+\frac{(c+dx)^4}{4d}-\frac{9d^3\cos^2(a+bx)}{8b^4}+\frac{9d(c+dx)^2\cos^2(a+bx)}{4b^2}-\frac{3d^2(c+dx)\cos(a+bx)\sin(a+bx)}{b^3}$$

```
[Out] -3/2*c*d^2*x/b^2-3/4*d^3*x^2/b^2+1/4*(d*x+c)^4/d-9/8*d^3*cos(b*x+a)^2/b^4+9
/4*d*(d*x+c)^2*cos(b*x+a)^2/b^2-3*d^2*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^3+2*(
d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b+3/8*d^3*sin(b*x+a)^2/b^4-3/4*d*(d*x+c)^2*s
in(b*x+a)^2/b^2
```

Rubi [A]
time = 0.13, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4516, 3392, 32, 3391}

$$\frac{3d^3\sin^2(a+bx)}{8b^4}-\frac{9d^3\cos^2(a+bx)}{8b^4}-\frac{3d^2(c+dx)\sin(a+bx)\cos(a+bx)}{b^3}-\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}+\frac{9d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{2(c+dx)^3\sin(a+bx)\cos(a+bx)}{b}-\frac{3cd^2x}{2b^2}-\frac{3d^3x^2}{4b^2}+\frac{(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (-3*c*d^2*x)/(2*b^2) - (3*d^3*x^2)/(4*b^2) + (c + d*x)^4/(4*d) - (9*d^3*Cos
[a + b*x]^2)/(8*b^4) + (9*d*(c + d*x)^2*Cos[a + b*x]^2)/(4*b^2) - (3*d^2*(c
 + d*x)*Cos[a + b*x]*Sin[a + b*x])/b^3 + (2*(c + d*x)^3*Cos[a + b*x]*Sin[a
+ b*x])/b + (3*d^3*Sin[a + b*x]^2)/(8*b^4) - (3*d*(c + d*x)^2*Sin[a + b*x]^
2)/(4*b^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
```

```
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\csc(a+bx)\sin(3a+3bx)\,dx &= \int \left(3(c+dx)^3\cos^2(a+bx)-(c+dx)^3\sin^2(a+bx)\right)\,dx\\ &= 3\int (c+dx)^3\cos^2(a+bx)\,dx-\int (c+dx)^3\sin^2(a+bx)\,dx\\ &= \frac{9d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{2(c+dx)^3\cos(a+bx)\sin(a+bx)}{b}\\ &= \frac{(c+dx)^4}{4d}-\frac{9d^3\cos^2(a+bx)}{8b^4}+\frac{9d(c+dx)^2\cos^2(a+bx)}{4b^2}-\frac{3d^2\\ &= -\frac{3cd^2x}{2b^2}-\frac{3d^3x^2}{4b^2}+\frac{(c+dx)^4}{4d}-\frac{9d^3\cos^2(a+bx)}{8b^4}+\frac{9d(c+dx)^2}{4}\end{aligned}$$

Mathematica [A]
time = 0.40, size = 105, normalized size = 0.61

$$\frac{b^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)+3d(-d^2+2b^2(c+dx)^2)\cos(2(a+bx))+2b(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\sin(2(a+bx))}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 3*d*(-d^2 + 2*b^2*(c +
 d*x)^2)*Cos[2*(a + b*x)] + 2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Sin[
2*(a + b*x)])/(4*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(157) = 314$.
time = 0.09, size = 580, normalized size = 3.39

method	result

risch	$\frac{d^3x^4}{4}+d^2c\,x^3+\frac{3d\,c^2x^2}{2}+c^3x+\frac{c^4}{4d}+\frac{3d\left(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2\right)\cos(2bx+2a)}{4b^4}+$ $(2b^2d^3x^3+6b^2c\,d^2x^2+6b^2c^2dx+2$ … / $2b^3$ …
default	$-c^3x-\frac{d^3x^4}{4}+\frac{4c^3\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}-d^2c\,x^3-\frac{3d\,c^2x^2}{2}+$ $4d^3\left((bx+a)^3\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\right.$ $3(bx+$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

```
[Out] -c^3*x-1/4*d^3*x^4+4*c^3/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-d^2*c*
x^3-3/2*d*c^2*x^2+4*d^3/b^4*((b*x+a)^3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1
/2*a)+3/4*(b*x+a)^2*cos(b*x+a)^2-3/2*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2
*b*x+1/2*a)+3/8*(b*x+a)^2+3/8*sin(b*x+a)^2-3/8*(b*x+a)^4-3*a*((b*x+a)^2*(1/
2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x
+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)+3*a^2*((b*x+a)*(1/2*cos(b*x+a)*
sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-a^3*(1/2*cos(b*x+
a)*sin(b*x+a)+1/2*b*x+1/2*a))+12*c*d^2/b^3*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b
*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4
*b*x-1/4*a-1/3*(b*x+a)^3-2*a*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/
2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+a^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x
+1/2*a))+12*c^2*d/b^2*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/
4*(b*x+a)^2-1/4*sin(b*x+a)^2-a*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))
```

Maxima [A]
time = 0.29, size = 173, normalized size = 1.01

$$\frac{(bx+\sin(2\,bx+2\,a))c^3}{b}+\frac{3\,(b^2x^2+2\,bx\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a))c^2d}{2\,b^2}+\frac{(2\,b^3x^3+6\,bx\cos(2\,bx+2\,a)+3\,(2\,b^2x^2-1)\sin(2\,bx+2\,a))cd^2}{2\,b^3}+\frac{(b^4x^4+3\,(2\,b^2x^2-1)\cos(2\,bx+2\,a)+2\,(2\,b^3x^3-3\,bx)\sin(2\,bx+2\,a))d^3}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] (b*x + sin(2*b*x + 2*a))*c^3/b + 3/2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + co
s(2*b*x + 2*a))*c^2*d/b^2 + 1/2*(2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(2*
b^2*x^2 - 1)*sin(2*b*x + 2*a))*c*d^2/b^3 + 1/4*(b^4*x^4 + 3*(2*b^2*x^2 - 1)
*cos(2*b*x + 2*a) + 2*(2*b^3*x^3 - 3*b*x)*sin(2*b*x + 2*a))*d^3/b^4
```

Fricas [A]
time = 1.65, size = 188, normalized size = 1.10

$$\frac{b^4d^3x^4+4\,b^4cd^2x^3+6\,(b^4c^2d-b^2d^3)x^2+6\,(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3)\cos(bx+a)^2+4\,(2\,b^3d^3x^3+6\,b^3cd^2x^2+2\,b^3c^3-3\,bcd^2+3\,(2\,b^3c^2d-bd^3)x)\cos(bx+a)\sin(bx+a)+4\,(b^4c^3-3\,b^2cd^2)x}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 6*(b^4*c^2*d - b^2*d^3)*x^2 + 6*(2*b^2
*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 4*(2*b^3*d^3
*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)
*cos(b*x + a)*sin(b*x + a) + 4*(b^4*c^3 - 3*b^2*c*d^2)*x)/b^4
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 682 vs. 2(157) = 314.
time = 0.48, size = 682, normalized size = 3.99

$$\frac{8 b^3 c^3 \cos(bx+a)\sin(bx+a) + 24(bx+a)b^2c^2d\cos(bx+a)\sin(bx+a) - 24ab^2c^2d\cos(bx+a)\sin(bx+a) + 24(bx+a)^2bcd^2\cos(bx+a)\sin(bx+a) - 48(bx+a)abcd^2\cos(bx+a)\sin(bx+a) + 24a^2bcd^2\cos(bx+a)\sin(bx+a) + 8(bx+a)^3d^3\cos(bx+a)\sin(bx+a) - 24(bx+a)^2ad^3\cos(bx+a)\sin(bx+a) + 24(bx+a)a^2d^3\cos(bx+a)\sin(bx+a) - 8a^3d^3\cos(bx+a)\sin(bx+a) + 4(bx+a)b^3c^3 + 6(bx+a)^2b^2c^2d - 12(bx+a)ab^2c^2d + 4(bx+a)^3bcd^2 - 12(bx+a)^2abcd^2 + 12(bx+a)a^2bcd^2 + (bx+a)^4d^3 - 4(bx+a)^3ad^3 + 6(bx+a)^2a^2d^3 - 4(bx+a)a^3d^3 + 6b^2c^2d\cos(bx+a)^2 + 12(bx+a)bcd^2\cos(bx+a)^2 - 12abcd^2\cos(bx+a)^2 + 6(bx+a)^2d^3\cos(bx+a)^2 - 12(bx+a)ad^3\cos(bx+a)^2 + 6a^2d^3\cos(bx+a)^2 - 6b^2c^2d\sin(bx+a)^2 - 12(bx+a)bcd^2\sin(bx+a)^2 + 12abcd^2\sin(bx+a)^2 - 6(bx+a)^2d^3\sin(bx+a)^2 + 12(bx+a)ad^3\sin(bx+a)^2 - 6a^2d^3\sin(bx+a)^2 - 12bcd^2\cos(bx+a)\sin(bx+a) - 12(bx+a)d^3\cos(bx+a)\sin(bx+a) + 12ad^3\cos(bx+a)\sin(bx+a) - 3d^3\cos(bx+a)^2 + 3d^3\sin(bx+a)^2}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] 1/4*(8*b^3*c^3*cos(b*x + a)*sin(b*x + a) + 24*(b*x + a)*b^2*c^2*d*cos(b*x +
 a)*sin(b*x + a) - 24*a*b^2*c^2*d*cos(b*x + a)*sin(b*x + a) + 24*(b*x + a)^
2*b*c*d^2*cos(b*x + a)*sin(b*x + a) - 48*(b*x + a)*a*b*c*d^2*cos(b*x + a)*s
in(b*x + a) + 24*a^2*b*c*d^2*cos(b*x + a)*sin(b*x + a) + 8*(b*x + a)^3*d^3*
cos(b*x + a)*sin(b*x + a) - 24*(b*x + a)^2*a*d^3*cos(b*x + a)*sin(b*x + a)
+ 24*(b*x + a)*a^2*d^3*cos(b*x + a)*sin(b*x + a) - 8*a^3*d^3*cos(b*x + a)*s
in(b*x + a) + 4*(b*x + a)*b^3*c^3 + 6*(b*x + a)^2*b^2*c^2*d - 12*(b*x + a)*
a*b^2*c^2*d + 4*(b*x + a)^3*b*c*d^2 - 12*(b*x + a)^2*a*b*c*d^2 + 12*(b*x +
a)*a^2*b*c*d^2 + (b*x + a)^4*d^3 - 4*(b*x + a)^3*a*d^3 + 6*(b*x + a)^2*a^2*
d^3 - 4*(b*x + a)*a^3*d^3 + 6*b^2*c^2*d*cos(b*x + a)^2 + 12*(b*x + a)*b*c*d
^2*cos(b*x + a)^2 - 12*a*b*c*d^2*cos(b*x + a)^2 + 6*(b*x + a)^2*d^3*cos(b*x
 + a)^2 - 12*(b*x + a)*a*d^3*cos(b*x + a)^2 + 6*a^2*d^3*cos(b*x + a)^2 - 6*
b^2*c^2*d*sin(b*x + a)^2 - 12*(b*x + a)*b*c*d^2*sin(b*x + a)^2 + 12*a*b*c*d
^2*sin(b*x + a)^2 - 6*(b*x + a)^2*d^3*sin(b*x + a)^2 + 12*(b*x + a)*a*d^3*s
in(b*x + a)^2 - 6*a^2*d^3*sin(b*x + a)^2 - 12*b*c*d^2*cos(b*x + a)*sin(b*x
+ a) - 12*(b*x + a)*d^3*cos(b*x + a)*sin(b*x + a) + 12*a*d^3*cos(b*x + a)*s
in(b*x + a) - 3*d^3*cos(b*x + a)^2 + 3*d^3*sin(b*x + a)^2)/b^4
```

Mupad [**B**]
time = 2.18, size = 216, normalized size = 1.26

$$c^3 x + \frac{d^3 x^4}{4} + \frac{3 c^2 d x^2}{2} + c d^2 x^3 - \frac{3 d^3 \cos(2a+2bx)}{4 b^4} + \frac{c^3 \sin(2a+2bx)}{b} + \frac{3 c^2 d \cos(2a+2bx)}{2 b^2} - \frac{3 c d^2 \sin(2a+2bx)}{2 b^3} - \frac{3 d^3 x \sin(2a+2bx)}{2 b^3} + \frac{3 d^3 x^2 \cos(2a+2bx)}{2 b^2} + \frac{d^3 x^3 \sin(2a+2bx)}{b} + \frac{3 c d^2 x \cos(2a+2bx)}{b^2} + \frac{3 c^2 d x \sin(2a+2bx)}{b} + \frac{3 c d^2 x^2 \sin(2a+2bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^3)/sin(a + b*x),x)
```

```
[Out] c^3*x + (d^3*x^4)/4 + (3*c^2*d*x^2)/2 + c*d^2*x^3 - (3*d^3*cos(2*a + 2*b*x)
)/(4*b^4) + (c^3*sin(2*a + 2*b*x))/b + (3*c^2*d*cos(2*a + 2*b*x))/(2*b^2) -
 (3*c*d^2*sin(2*a + 2*b*x))/(2*b^3) - (3*d^3*x*sin(2*a + 2*b*x))/(2*b^3) +
(3*d^3*x^2*cos(2*a + 2*b*x))/(2*b^2) + (d^3*x^3*sin(2*a + 2*b*x))/b + (3*c*
d^2*x*cos(2*a + 2*b*x))/b^2 + (3*c^2*d*x*sin(2*a + 2*b*x))/b + (3*c*d^2*x^2
*sin(2*a + 2*b*x))/b
```

3.370 $\int (c+dx)^2 \csc(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=112

$$-\frac{d^2x}{2b^2}+\frac{(c+dx)^3}{3d}+\frac{3d(c+dx)\cos^2(a+bx)}{2b^2}-\frac{d^2\cos(a+bx)\sin(a+bx)}{b^3}+\frac{2(c+dx)^2\cos(a+bx)\sin(a+bx)}{b}$$

```
[Out] -1/2*d^2*x/b^2+1/3*(d*x+c)^3/d+3/2*d*(d*x+c)*cos(b*x+a)^2/b^2-d^2*cos(b*x+a
)*sin(b*x+a)/b^3+2*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b-1/2*d*(d*x+c)*sin(b*x+
a)^2/b^2
```

Rubi [A]

time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.217$, Rules used = {4516, 3392, 32, 2715, 8}

$$-\frac{d^2\sin(a+bx)\cos(a+bx)}{b^3}-\frac{d(c+dx)\sin^2(a+bx)}{2b^2}+\frac{3d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{2(c+dx)^2\sin(a+bx)\cos(a+bx)}{b}-\frac{d^2x}{2b^2}+\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] -1/2*(d^2*x)/b^2 + (c + d*x)^3/(3*d) + (3*d*(c + d*x)*Cos[a + b*x]^2)/(2*b^
2) - (d^2*Cos[a + b*x]*Sin[a + b*x])/b^3 + (2*(c + d*x)^2*Cos[a + b*x]*Sin[
a + b*x])/b - (d*(c + d*x)*Sin[a + b*x]^2)/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
```

```
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int (c+dx)^2 \csc(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^2\cos^2(a+bx) - (c+dx)^2\sin^2(a+bx)\right)\,dx$$
$$= 3\int (c+dx)^2\cos^2(a+bx)\,dx - \int (c+dx)^2\sin^2(a+bx)\,dx$$
$$= \frac{3d(c+dx)\cos^2(a+bx)}{2b^2} + \frac{2(c+dx)^2\cos(a+bx)\sin(a+bx)}{b} - \cdots$$
$$= \frac{(c+dx)^3}{3d} + \frac{3d(c+dx)\cos^2(a+bx)}{2b^2} - \frac{d^2\cos(a+bx)\sin(a+bx)}{b^3} \cdots$$
$$= -\frac{d^2x}{2b^2} + \frac{(c+dx)^3}{3d} + \frac{3d(c+dx)\cos^2(a+bx)}{2b^2} - \frac{d^2\cos(a+bx)\sin \cdots}{b^3}$$

Mathematica [A]
time = 0.40, size = 73, normalized size = 0.65

$$c^2x + cdx^2 + \frac{d^2x^3}{3} + \frac{d(c+dx)\cos(2(a+bx))}{b^2} + \frac{(-d^2+2b^2(c+dx)^2)\sin(2(a+bx))}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] c^2*x + c*d*x^2 + (d^2*x^3)/3 + (d*(c + d*x)*Cos[2*(a + b*x)])/b^2 + ((-d^2
 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(2*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(104) = 208.
time = 0.08, size = 294, normalized size = 2.62

method	result

risch	$\frac{d^2x^3}{3}+cd\,x^2+c^2x+\frac{c^3}{3d}+\frac{d(dx+c)\cos(2bx+2a)}{b^2}+\frac{(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2)\sin(2bx+2a)}{2b^3}$
default	$-c^2x-\frac{d^2x^3}{3}+\frac{4c^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}-cd\,x^2+\frac{4d^2\left((bx+a)^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\frac{(bx+a)\left(\cos^2(bx+\right.}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)

[Out] -c^2*x-1/3*d^2*x^3+4*c^2/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-c*d*x^2+4*d^2/b^3*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3-2*a*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+a^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))+8*c*d/b^2*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2-a*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))

Maxima [A]
time = 0.28, size = 108, normalized size = 0.96

$$\frac{(bx+\sin(2\,bx+2\,a))c^2}{b}+\frac{(b^2x^2+2\,bx\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a))cd}{b^2}+\frac{(2\,b^3x^3+6\,bx\cos(2\,bx+2\,a)+3\,(2\,b^2x^2-1)\sin(2\,bx+2\,a))d^2}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")

[Out] (b*x + sin(2*b*x + 2*a))*c^2/b + (b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*c*d/b^2 + 1/6*(2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(2*b^2*x^2 - 1)*sin(2*b*x + 2*a))*d^2/b^3

Fricas [A]
time = 2.37, size = 111, normalized size = 0.99

$$\frac{b^3d^2x^3+3\,b^3cdx^2+6\,(bd^2x+bcd)\cos(bx+a)^2+3\,(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\cos(bx+a)\sin(bx+a)+3\,(b^3c^2-bd^2)x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")

[Out] 1/3*(b^3*d^2*x^3 + 3*b^3*c*d*x^2 + 6*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(b*x + a)*sin(b*x + a) + 3*(b^3*c^2 - b*d^2)*x)/b^3

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(c+dx)^2\sin(3a+3bx)\csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Integral((c + d*x)**2*sin(3*a + 3*b*x)*csc(a + b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(104) = 208.
time = 0.46, size = 313, normalized size = 2.79

$$\frac{6\,b^2c^2\cos\left(bx+a\right)\sin\left(bx+a\right)+12\,(bx+a)bcd\cos\left(bx+a\right)\sin\left(bx+a\right)-12\,abcd\cos\left(bx+a\right)\sin\left(bx+a\right)+6\,(bx+a)^2d^2\cos\left(bx+a\right)\sin\left(bx+a\right)-12\,(bx+a)ad^2\cos\left(bx+a\right)\sin\left(bx+a\right)+6\,a^2d^2\cos\left(bx+a\right)\sin\left(bx+a\right)+3\,(bx+a)b^2c^2+3\,(bx+a)^2bcd-6\,(bx+a)abcd+(bx+a)^3d^2-3\,(bx+a)^2ad^2+3\,(bx+a)a^2d^2+3\,bcd\cos\left(bx+a\right)^2+3\,(bx+a)d^2\cos\left(bx+a\right)^2-3\,ad^2\cos\left(bx+a\right)^2-3\,bcd\sin\left(bx+a\right)^2-3\,(bx+a)d^2\sin\left(bx+a\right)^2+3\,ad^2\sin\left(bx+a\right)^2-3\,d^2\cos\left(bx+a\right)\sin\left(bx+a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] 1/3*(6*b^2*c^2*cos(b*x + a)*sin(b*x + a) + 12*(b*x + a)*b*c*d*cos(b*x + a)*
sin(b*x + a) - 12*a*b*c*d*cos(b*x + a)*sin(b*x + a) + 6*(b*x + a)^2*d^2*cos
(b*x + a)*sin(b*x + a) - 12*(b*x + a)*a*d^2*cos(b*x + a)*sin(b*x + a) + 6*a
^2*d^2*cos(b*x + a)*sin(b*x + a) + 3*(b*x + a)*b^2*c^2 + 3*(b*x + a)^2*b*c*
d - 6*(b*x + a)*a*b*c*d + (b*x + a)^3*d^2 - 3*(b*x + a)^2*a*d^2 + 3*(b*x +
a)*a^2*d^2 + 3*b*c*d*cos(b*x + a)^2 + 3*(b*x + a)*d^2*cos(b*x + a)^2 - 3*a*
d^2*cos(b*x + a)^2 - 3*b*c*d*sin(b*x + a)^2 - 3*(b*x + a)*d^2*sin(b*x + a)^
2 + 3*a*d^2*sin(b*x + a)^2 - 3*d^2*cos(b*x + a)*sin(b*x + a))/b^3
```

Mupad [B]
time = 0.31, size = 121, normalized size = 1.08

$$c^2\,x+\frac{d^2\,x^3}{3}+\frac{c^2\,\sin\left(2\,a+2\,b\,x\right)}{b}-\frac{d^2\,\sin\left(2\,a+2\,b\,x\right)}{2\,b^3}+c\,d\,x^2+\frac{d^2\,x\,\cos\left(2\,a+2\,b\,x\right)}{b^2}+\frac{d^2\,x^2\,\sin\left(2\,a+2\,b\,x\right)}{b}+\frac{c\,d\,\cos\left(2\,a+2\,b\,x\right)}{b^2}+\frac{2\,c\,d\,x\,\sin\left(2\,a+2\,b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^2)/sin(a + b*x),x)
```

```
[Out] c^2*x + (d^2*x^3)/3 + (c^2*sin(2*a + 2*b*x))/b - (d^2*sin(2*a + 2*b*x))/(2*
b^3) + c*d*x^2 + (d^2*x*cos(2*a + 2*b*x))/b^2 + (d^2*x^2*sin(2*a + 2*b*x))/
b + (c*d*cos(2*a + 2*b*x))/b^2 + (2*c*d*x*sin(2*a + 2*b*x))/b
```

3.371 $\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=66

$$cx+\frac{dx^2}{2}+\frac{3d\cos^2(a+bx)}{4b^2}+\frac{2(c+dx)\cos(a+bx)\sin(a+bx)}{b}-\frac{d\sin^2(a+bx)}{4b^2}$$

```
[Out] c*x+1/2*d*x^2+3/4*d*cos(b*x+a)^2/b^2+2*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b-1/4*
d*sin(b*x+a)^2/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4516, 3391}

$$-\frac{d\sin^2(a+bx)}{4b^2}+\frac{3d\cos^2(a+bx)}{4b^2}+\frac{2(c+dx)\sin(a+bx)\cos(a+bx)}{b}+cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] c*x + (d*x^2)/2 + (3*d*Cos[a + b*x]^2)/(4*b^2) + (2*(c + d*x)*Cos[a + b*x]*
Sin[a + b*x])/b - (d*Sin[a + b*x]^2)/(4*b^2)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)\cos^2(a+bx) - (c+dx)\sin^2(a+bx)\right)\,dx$$
$$= 3\int (c+dx)\cos^2(a+bx)\,dx - \int (c+dx)\sin^2(a+bx)\,dx$$
$$= \frac{3d\cos^2(a+bx)}{4b^2} + \frac{2(c+dx)\cos(a+bx)\sin(a+bx)}{b} - \frac{d\sin^2(a+}{4b^2}$$
$$= cx + \frac{dx^2}{2} + \frac{3d\cos^2(a+bx)}{4b^2} + \frac{2(c+dx)\cos(a+bx)\sin(a+bx)}{b} -$$

Mathematica [A]
time = 0.16, size = 46, normalized size = 0.70

$$\frac{d\cos(2(a+bx)) + b(bx(2c+dx) + 2(c+dx)\sin(2(a+bx)))}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
[Out] (d*Cos[2*(a + b*x)] + b*(b*x*(2*c + d*x) + 2*(c + d*x)*Sin[2*(a + b*x)]))/(
2*b^2)
```

Maple [A]
time = 0.06, size = 119, normalized size = 1.80

method	result
risch	$\frac{d\,x^2}{2} + cx + \frac{d\cos(2bx+2a)}{2b^2} + \frac{(dx+c)\sin(2bx+2a)}{b}$
default	$-\frac{d\,x^2}{2} - cx + \frac{4c\left(\frac{\cos(bx+a)\sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2}\right)}{b} + \frac{4d\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2}\right) - \frac{(bx+a)^2}{4} - \frac{\left(\sin^2(bx+a)\right)}{4} - a\left(\frac{\cos(bx+}{}\right.\right.}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
[Out] -1/2*d*x^2-c*x+4*c/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+4*d/b^2*((b*
x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)
^2-a*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))
```

Maxima [A]
time = 0.29, size = 55, normalized size = 0.83

$$\frac{(bx + \sin(2\,bx + 2\,a))c}{b} + \frac{(b^2x^2 + 2\,bx\sin(2\,bx + 2\,a) + \cos(2\,bx + 2\,a))d}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] (b*x + sin(2*b*x + 2*a))*c/b + 1/2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos(
2*b*x + 2*a))*d/b^2
```

Fricas [A]
time = 1.76, size = 54, normalized size = 0.82

$$\frac{b^2dx^2 + 2\,b^2cx + 2\,d\cos\left(bx+a\right)^2 + 4\left(bdx + bc\right)\cos\left(bx+a\right)\sin\left(bx+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d*x^2 + 2*b^2*c*x + 2*d*cos(b*x + a)^2 + 4*(b*d*x + b*c)*cos(b*x +
 a)*sin(b*x + a))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c + dx\right)\sin\left(3a + 3bx\right)\csc\left(a + bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Integral((c + d*x)*sin(3*a + 3*b*x)*csc(a + b*x), x)
```

Giac [A]
time = 0.41, size = 106, normalized size = 1.61

$$\frac{4\,bc\cos\left(bx+a\right)\sin\left(bx+a\right) + 4\left(bx+a\right)d\cos\left(bx+a\right)\sin\left(bx+a\right) - 4\,ad\cos\left(bx+a\right)\sin\left(bx+a\right) + 2\left(bx+a\right)bc + \left(bx+a\right)^2d - 2\left(bx+a\right)ad + d\cos\left(bx+a\right)^2 - d\sin\left(bx+a\right)^2}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] 1/2*(4*b*c*cos(b*x + a)*sin(b*x + a) + 4*(b*x + a)*d*cos(b*x + a)*sin(b*x +
 a) - 4*a*d*cos(b*x + a)*sin(b*x + a) + 2*(b*x + a)*b*c + (b*x + a)^2*d - 2
*(b*x + a)*a*d + d*cos(b*x + a)^2 - d*sin(b*x + a)^2)/b^2
```

Mupad [B]
time = 0.21, size = 53, normalized size = 0.80

$$c\,x + \frac{d\,x^2}{2} + \frac{\frac{d\,\cos\left(2\,a+2\,b\,x\right)}{2} + b\left(c\,\sin\left(2\,a+2\,b\,x\right) + d\,x\,\sin\left(2\,a+2\,b\,x\right)\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x))/sin(a + b*x),x)
```

```
[Out] c*x + (d*x^2)/2 + ((d*cos(2*a + 2*b*x))/2 + b*(c*sin(2*a + 2*b*x) + d*x*sin
(2*a + 2*b*x)))/b^2
```

3.372 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

Optimal. Leaf size=71

$$\frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{\log(c+dx)}{d}-\frac{2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}$$

```
[Out] 2*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d+ln(d*x+c)/d-2*Si(2*b*c/d+2*b*x)*sin(
2*a-2*b*c/d)/d
```

Rubi [A]
time = 0.20, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4516, 3393, 3384, 3380, 3383}

$$\frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}-\frac{2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] (2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d + Log[c + d*x]/d
- (2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx &= \int \left(\frac{3\cos^2(a+bx)}{c+dx}-\frac{\sin^2(a+bx)}{c+dx}\right)dx \\
&= 3\int \frac{\cos^2(a+bx)}{c+dx}\,dx-\int \frac{\sin^2(a+bx)}{c+dx}\,dx \\
&= 3\int \left(\frac{1}{2(c+dx)}+\frac{\cos(2a+2bx)}{2(c+dx)}\right)dx-\int \left(\frac{1}{2(c+dx)}-\frac{\cos(2a+2bx)}{2(c+dx)}\right. \\
&= \frac{\log(c+dx)}{d}+\frac{1}{2}\int \frac{\cos(2a+2bx)}{c+dx}\,dx+\frac{3}{2}\int \frac{\cos(2a+2bx)}{c+dx}\,dx \\
&= \frac{\log(c+dx)}{d}+\frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx+\frac{1}{2}\left(3\cos\left(2a-\right.\right. \\
&= \frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{\log(c+dx)}{d}-\frac{2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+\right.}{d}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 63, normalized size = 0.89

$$\frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)+\log(c+dx)-2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
[Out] (2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + Log[c + d*x] - 2*S
in[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d
```

Maple [A]
time = 0.10, size = 116, normalized size = 1.63

method	result
risch	$\frac{\ln(dx+c)}{d}-\frac{e^{-\frac{2i(ad-cb)}{d}}\operatorname{expIntegral}\left(1,2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{d}-\frac{e^{\frac{2i(ad-cb)}{d}}\operatorname{expIntegral}\left(1,-2ibx-2ia-\frac{2(-iad+ibc)}{d}\right)}{d}$
default	$-\frac{\ln(dx+c)}{d}+\frac{2\operatorname{sinIntegral}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}+\frac{2\operatorname{cosineIntegral}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}+\frac{2\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(d*x+c)/d+2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*
x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d+2*ln(-a*d+c*b+d*(b*x+a))/d
```

Maxima [C] Result contains complex when optimal does not.
time = 0.33, size = 119, normalized size = 1.68

$$-\frac{\left(E_1\left(\frac{2(-ibdx-ibc)}{d}\right)+E_1\left(-\frac{2(-ibdx-ibc)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)-\left(-iE_1\left(\frac{2(-ibdx-ibc)}{d}\right)+iE_1\left(-\frac{2(-ibdx-ibc)}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -((exp_integral_e(1, 2*(-I*b*d*x - I*b*c)/d) + exp_integral_e(1, -2*(-I*b*d
*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (-I*exp_integral_e(1, 2*(-I*b*d*x -
 I*b*c)/d) + I*exp_integral_e(1, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c - a*
d)/d) - log(d*x + c))/d
```

Fricas [A]
time = 1.87, size = 85, normalized size = 1.20

$$\frac{\left(\operatorname{Ci}\left(\frac{2(bdx+bc)}{d}\right)+\operatorname{Ci}\left(-\frac{2(bdx+bc)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)-2\sin\left(-\frac{2(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2(bdx+bc)}{d}\right)+\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] ((cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d))*cos(-
2*(b*c - a*d)/d) - 2*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d)
+ log(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin(3a+3bx)\csc(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] Integral(sin(3*a + 3*b*x)*csc(a + b*x)/(c + d*x), x)
```

Giac [**A**]
time = 0.43, size = 102, normalized size = 1.44

$$\frac{2\,b\cos\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{2\,(bc+(bx+a)d-ad)}{d}\right)+2\,b\sin\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Si}\left(-\frac{2\,(bc+(bx+a)d-ad)}{d}\right)+b\log\left(bc+(bx+a)d-ad\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

```
[Out] (2*b*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d) + 2*
b*sin(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d) + b*lo
g(b*c + (b*x + a)*d - a*d))/(b*d)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)}{\sin\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)), x)
```

3.373 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=102

$$-\frac{3\cos^2(a+bx)}{d(c+dx)}-\frac{4b\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^2}+\frac{\sin^2(a+bx)}{d(c+dx)}-\frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}$$

```
[Out] -3*cos(b*x+a)^2/d/(d*x+c)-4*b*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^2-4*b*Ci
(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^2+sin(b*x+a)^2/d/(d*x+c)
```

Rubi [A]
time = 0.21, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4516, 3394, 12, 3384, 3380, 3383}

$$-\frac{4b\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin^2(a+bx)}{d(c+dx)}-\frac{3\cos^2(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] (-3*Cos[a + b*x]^2)/(d*(c + d*x)) - (4*b*CosIntegral[(2*b*c)/d + 2*b*x]*Sin
[2*a - (2*b*c)/d])/d^2 + Sin[a + b*x]^2/(d*(c + d*x)) - (4*b*Cos[2*a - (2*b
*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx &= \int \left(\frac{3\cos^2(a+bx)}{(c+dx)^2}-\frac{\sin^2(a+bx)}{(c+dx)^2}\right)dx \\
&= 3\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx-\int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx \\
&= -\frac{3\cos^2(a+bx)}{d(c+dx)}+\frac{\sin^2(a+bx)}{d(c+dx)}-\frac{(2b)\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx}{d}+\frac{(6b)\int -\frac{\sin(2a+2}{2(c+dx}}{d} \\
&= -\frac{3\cos^2(a+bx)}{d(c+dx)}+\frac{\sin^2(a+bx)}{d(c+dx)}-\frac{b\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d}-\frac{(3b)\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d} \\
&= -\frac{3\cos^2(a+bx)}{d(c+dx)}+\frac{\sin^2(a+bx)}{d(c+dx)}-\frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d}- \\
&= -\frac{3\cos^2(a+bx)}{d(c+dx)}-\frac{4b\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^2}+\frac{\sin^2(a+bx)}{d(c+dx)}-\frac{4b\,\mathrm{c}}{}
\end{aligned}$$

Mathematica [A]
time = 0.50, size = 81, normalized size = 0.79

$$-\frac{\frac{d(1+2\cos(2(a+bx)))}{c+dx}+4b\,\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right)+4b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]

[Out] -(((d*(1 + 2*Cos[2*(a + b*x)]))/(c + d*x) + 4*b*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + 4*b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^2)

Maple [A]
time = 0.13, size = 169, normalized size = 1.66

method	result
risch	$-\frac{1}{d(dx+c)}+\frac{2ib\,e^{-\frac{2i(ad-cb)}{d}}\,\text{expIntegral}\left(1,2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{d^2}-\frac{2ib\,e^{\frac{2i(ad-cb)}{d}}\,\text{expIntegral}\left(1,-2ibx-2ia-\frac{2(-iad+ibc)}{d}\right)}{d^2}-$
default	$\frac{1}{d(dx+c)}+\frac{b^2\left(-\frac{2\cos(2bx+2a)}{(-ad+cb+d(bx+a))d}-\frac{2\left(\frac{2\,\text{sinIntegral}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\text{cosineIntegral}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2}{}\right.}{d}\right.}{d}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x,method=_RETURNVERBOSE)

[Out] 1/d/(d*x+c)+4/b*(1/4*b^2*(-2*cos(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)-1/2*b^2/(-a*d+c*b+d*(b*x+a))/d)

Maxima [C] Result contains complex when optimal does not.
time = 0.35, size = 120, normalized size = 1.18

$$-\frac{\left(E_2\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)+E_2\left(-\frac{2(-i\,bdx-i\,bc)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)-\left(-i\,E_2\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)+i\,E_2\left(-\frac{2(-i\,bdx-i\,bc)}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+1}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")

[Out] -((exp_integral_e(2, 2*(-I*b*d*x - I*b*c)/d) + exp_integral_e(2, -2*(-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (-I*exp_integral_e(2, 2*(-I*b*d*x - I*b*c)/d) + I*exp_integral_e(2, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c - a*d)/d) + 1)/(d^2*x + c*d)

Fricas [A]
time = 1.95, size = 131, normalized size = 1.28

$$-\frac{4\,d\cos(bx+a)^2+4\,(bdx+bc)\cos\left(-\frac{2(bc-ad)}{d}\right)\text{Si}\left(\frac{2(bdx+bc)}{d}\right)+2\left((bdx+bc)\,\text{Ci}\left(\frac{2(bdx+bc)}{d}\right)+(bdx+bc)\,\text{Ci}\left(-\frac{2(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-d}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")

```
[Out] -(4*d*cos(b*x + a)^2 + 4*(b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*sin_integral(2
*(b*d*x + b*c)/d) + 2*((b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d) + (b*d
*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d) - d)/(d^3
*x + c*d^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(102) = 204.
time = 0.44, size = 308, normalized size = 3.02

$$-\frac{4\,b^{3}c\,\mathrm{Ci}\left(\frac{2\,(bc+(bx+a)d-ad)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+4\,(bx+a)b^{2}d\,\mathrm{Ci}\left(\frac{2\,(bc+(bx+a)d-ad)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\,ab^{2}d\,\mathrm{Ci}\left(\frac{2\,(bc+(bx+a)d-ad)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\,b^{3}c\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(-\frac{2\,(bc+(bx+a)d-ad)}{d}\right)-4\,(bx+a)b^{2}d\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(-\frac{2\,(bc+(bx+a)d-ad)}{d}\right)+4\,ab^{2}d\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(-\frac{2\,(bc+(bx+a)d-ad)}{d}\right)+2\,b^{2}d\cos\left(2\,bx+2\,a\right)+b^{2}d}{(bcd^{2}+(bx+a)d^{3}-ad^{3})b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -(4*b^3*c*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c - a*d)/d)
 + 4*(b*x + a)*b^2*d*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*
c - a*d)/d) - 4*a*b^2*d*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*
(b*c - a*d)/d) - 4*b^3*c*cos(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x
+ a)*d - a*d)/d) - 4*(b*x + a)*b^2*d*cos(-2*(b*c - a*d)/d)*sin_integral(-2*
(b*c + (b*x + a)*d - a*d)/d) + 4*a*b^2*d*cos(-2*(b*c - a*d)/d)*sin_integral
(-2*(b*c + (b*x + a)*d - a*d)/d) + 2*b^2*d*cos(2*b*x + 2*a) + b^2*d)/((b*c*
d^2 + (b*x + a)*d^3 - a*d^3)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)}{\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^2), x)
```

3.374 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=136

$$-\frac{3\cos^2(a+bx)}{2d(c+dx)^2}-\frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)}+\frac{\sin^2(a+bx)}{2d(c+dx)^2}+\frac{4b^2\,\mathrm{s}}{}$$

```
[Out] -4*b^2*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^3-3/2*cos(b*x+a)^2/d/(d*x+c)^2+
4*b^2*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^3+4*b*cos(b*x+a)*sin(b*x+a)/d^2/
(d*x+c)+1/2*sin(b*x+a)^2/d/(d*x+c)^2
```

Rubi [A]
time = 0.27, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4516, 3395, 31, 3393, 3384, 3380, 3383}

$$-\frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{4b^2\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{4b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}+\frac{\sin^2(a+bx)}{2d(c+dx)^2}-\frac{3\cos^2(a+bx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] (-3*Cos[a + b*x]^2)/(2*d*(c + d*x)^2) - (4*b^2*Cos[2*a - (2*b*c)/d]*CosInte
gral[(2*b*c)/d + 2*b*x])/d^3 + (4*b*Cos[a + b*x]*Sin[a + b*x])/(d^2*(c + d*
x)) + Sin[a + b*x]^2/(2*d*(c + d*x)^2) + (4*b^2*Sin[2*a - (2*b*c)/d]*SinInt
egral[(2*b*c)/d + 2*b*x])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \left(\frac{3\cos^2(a+bx)}{(c+dx)^3} - \frac{\sin^2(a+bx)}{(c+dx)^3}\right)dx$$
$$= 3\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx - \int \frac{\sin^2(a+bx)}{(c+dx)^3}\,dx$$
$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} + \frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{b^2\int \frac{1}{c+dx}\,d}{d^2}$$
$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} + \frac{2b^2\log(c+dx)}{d^3} + \frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a-}{2d(c+}$$
$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} + \frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{b^2\int \frac{\cos(2a-}{c+d}}{d^2}$$
$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} + \frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{(b^2\cos(2a}{}$$
$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} - \frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^3} + \frac{4b\cos(a+bx)\sin(a}{d^2(c+dx)}$$

Mathematica [A]
time = 0.92, size = 104, normalized size = 0.76

$$-\frac{8b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) + \frac{d(d+2d\cos(2(a+bx))-4b(c+dx)\sin(2(a+bx)))}{(c+dx)^2} - 8b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] -1/2*(8*b^2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + (d*(d + 2
*d*Cos[2*(a + b*x)] - 4*b*(c + d*x)*Sin[2*(a + b*x)]))/(c + d*x)^2 - 8*b^2*
Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^3
```

Maple [A]
time = 0.16, size = 207, normalized size = 1.52

method	result
default	$\frac{1}{2d(dx+c)^2} + \frac{b^3\left(-\frac{\cos(2bx+2a)}{(-ad+cb+d(bx+a))^2 d} - \frac{-\frac{2\sin(2bx+2a)}{(-ad+cb+d(bx+a))d} + \frac{\frac{4\,\text{sinIntegral}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d} + \frac{4\,\text{cosineIntegral}\left(2bx\right.}{d}}{d}}{}\right.}{b}$

risch	$-\frac{1}{2d(dx+c)^2}+\frac{2b^2 \mathrm{e}^{-\frac{2i(ad-cb)}{d}} \operatorname{expIntegral}\left(1,2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{d^3}+\frac{2b^2 \mathrm{e}^{\frac{2i(ad-cb)}{d}} \operatorname{expIntegral}\left(1,-2ibx-2ia-\frac{2(-iad+ibc)}{d}\right)}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/(d*x+c)^2+4/b*(1/4*b^3*(-cos(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d-(-2*
sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin
(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)
/d)-1/4*b^3/(-a*d+c*b+d*(b*x+a))^2/d)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.33, size = 132, normalized size = 0.97

$$-\frac{2\left(E_3\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)+E_3\left(-\frac{2(-i\,bdx-i\,bc)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+2\left(i\,E_3\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)-i\,E_3\left(-\frac{2(-i\,bdx-i\,bc)}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+1}{2\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(exp_integral_e(3, 2*(-I*b*d*x - I*b*c)/d) + exp_integral_e(3, -2*(
-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*(I*exp_integral_e(3, 2*(-I*
b*d*x - I*b*c)/d) - I*exp_integral_e(3, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b
*c - a*d)/d) + 1)/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [A]
time = 1.41, size = 225, normalized size = 1.65

$$-\frac{4\,d^2\cos\left(bx+a\right)^2-8\left(bd^2x+bcd\right)\cos\left(bx+a\right)\sin\left(bx+a\right)-8\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{2(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2(bdx+bc)}{d}\right)-d^2+4\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(\frac{2(bdx+bc)}{d}\right)+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(-\frac{2(bdx+bc)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)}{2\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*d^2*cos(b*x + a)^2 - 8*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a)
- 8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-2*(b*c - a*d)/d)*sin_integra
l(2*(b*d*x + b*c)/d) - d^2 + 4*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_i
ntegral(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_inte
gral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2
*d^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**3,x)

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 704 vs. 2(132) = 264.
time = 0.49, size = 704, normalized size = 5.18

$$-\frac{8 b^5 c^2 \cos\left(-\frac{2(bc-ad)}{d}\right) \operatorname{Ci}\left(\frac{2(bc+(bx+a)d-ad)}{d}\right) + 16 (bx+a) b^4 cd \cos\left(-\frac{2(bc-ad)}{d}\right) \operatorname{Ci}\left(\frac{2(bc+(bx+a)d-ad)}{d}\right) - 16 ab^4 cd \cos\left(-\frac{2(bc-ad)}{d}\right) \operatorname{Ci}\left(\frac{2(bc+(bx+a)d-ad)}{d}\right) + 8 (bx+a)^2 b^3 d^2 \cos\left(-\frac{2(bc-ad)}{d}\right) \operatorname{Ci}\left(\frac{2(bc+(bx+a)d-ad)}{d}\right) - 16 (bx+a) ab^3 d^2 \cos\left(-\frac{2(bc-ad)}{d}\right) \operatorname{Ci}\left(\frac{2(bc+(bx+a)d-ad)}{d}\right) + 8 a^2 b^3 d^2 \cos\left(-\frac{2(bc-ad)}{d}\right) \operatorname{Ci}\left(\frac{2(bc+(bx+a)d-ad)}{d}\right) + 8 b^5 c^2 \sin\left(-\frac{2(bc-ad)}{d}\right) \operatorname{Si}\left(-\frac{2(bc+(bx+a)d-ad)}{d}\right) + 16 (bx+a) b^4 cd \sin\left(-\frac{2(bc-ad)}{d}\right) \operatorname{Si}\left(-\frac{2(bc+(bx+a)d-ad)}{d}\right) - 16 ab^4 cd \sin\left(-\frac{2(bc-ad)}{d}\right) \operatorname{Si}\left(-\frac{2(bc+(bx+a)d-ad)}{d}\right) + 8 (bx+a)^2 b^3 d^2 \sin\left(-\frac{2(bc-ad)}{d}\right) \operatorname{Si}\left(-\frac{2(bc+(bx+a)d-ad)}{d}\right) - 16 (bx+a) ab^3 d^2 \sin\left(-\frac{2(bc-ad)}{d}\right) \operatorname{Si}\left(-\frac{2(bc+(bx+a)d-ad)}{d}\right) + 8 a^2 b^3 d^2 \sin\left(-\frac{2(bc-ad)}{d}\right) \operatorname{Si}\left(-\frac{2(bc+(bx+a)d-ad)}{d}\right) - 4 b^4 cd \sin(2bx+2a) - 4 (bx+a) b^3 d^2 \sin(2bx+2a) + 4 ab^3 d^2 \sin(2bx+2a) + 2 b^3 d^2 \cos(2bx+2a) + b^3 d^2}{2 \left(b^2 c^2 d^3 + 2 (bx+a) bcd^4 - 2 abcd^4 + (bx+a)^2 d^5 - 2 (bx+a) ad^5 + a^2 d^5\right) b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")

```
[Out] -1/2*(8*b^5*c^2*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*c + (b*x + a)*d - a
*d)/d) + 16*(b*x + a)*b^4*c*d*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*c + (
b*x + a)*d - a*d)/d) - 16*a*b^4*c*d*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b
*c + (b*x + a)*d - a*d)/d) + 8*(b*x + a)^2*b^3*d^2*cos(-2*(b*c - a*d)/d)*co
s_integral(2*(b*c + (b*x + a)*d - a*d)/d) - 16*(b*x + a)*a*b^3*d^2*cos(-2*(
b*c - a*d)/d)*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d) + 8*a^2*b^3*d^2*c
os(-2*(b*c - a*d)/d)*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d) + 8*b^5*c^
2*sin(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d) + 16*(
b*x + a)*b^4*c*d*sin(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d -
 a*d)/d) - 16*a*b^4*c*d*sin(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x +
 a)*d - a*d)/d) + 8*(b*x + a)^2*b^3*d^2*sin(-2*(b*c - a*d)/d)*sin_integral(
-2*(b*c + (b*x + a)*d - a*d)/d) - 16*(b*x + a)*a*b^3*d^2*sin(-2*(b*c - a*d)
/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d) + 8*a^2*b^3*d^2*sin(-2*(b*
c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d) - 4*b^4*c*d*sin(2*
b*x + 2*a) - 4*(b*x + a)*b^3*d^2*sin(2*b*x + 2*a) + 4*a*b^3*d^2*sin(2*b*x +
 2*a) + 2*b^3*d^2*cos(2*b*x + 2*a) + b^3*d^2)/((b^2*c^2*d^3 + 2*(b*x + a)*b
*c*d^4 - 2*a*b*c*d^4 + (b*x + a)^2*d^5 - 2*(b*x + a)*a*d^5 + a^2*d^5)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3a+3bx)}{\sin(a+bx)\,(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^3),x)

[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^3), x)

3.375 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=205

$$-\frac{2b^2}{3d^3(c+dx)}-\frac{\cos^2(a+bx)}{d(c+dx)^3}+\frac{2b^2\cos^2(a+bx)}{d^3(c+dx)}+\frac{8b^3\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{3d^4}+\frac{4b\cos(a+bx)\,\mathrm{s}}{3d^2(c+d}$$

```
[Out] -2/3*b^2/d^3/(d*x+c)-cos(b*x+a)^2/d/(d*x+c)^3+2*b^2*cos(b*x+a)^2/d^3/(d*x+c
)+8/3*b^3*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^4+8/3*b^3*Ci(2*b*c/d+2*b*x)*
sin(2*a-2*b*c/d)/d^4+4/3*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)^2+1/3*sin(b*x+
a)^2/d/(d*x+c)^3-2/3*b^2*sin(b*x+a)^2/d^3/(d*x+c)
```

Rubi [A]
time = 0.26, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4516, 3395, 32, 3394, 12, 3384, 3380, 3383}

$$\frac{8b^3\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{8b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{2b^2\sin^2(a+bx)}{3d^3(c+dx)}+\frac{2b^2\cos^2(a+bx)}{d^3(c+dx)}+\frac{4b\sin(a+bx)\cos(a+bx)}{3d^2(c+dx)^2}+\frac{\sin^2(a+bx)}{3d(c+dx)^3}-\frac{\cos^2(a+bx)}{d(c+dx)^3}-\frac{2b^2}{3d^3(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^4,x]
```

```
[Out] (-2*b^2)/(3*d^3*(c + d*x)) - Cos[a + b*x]^2/(d*(c + d*x)^3) + (2*b^2*Cos[a
+ b*x]^2)/(d^3*(c + d*x)) + (8*b^3*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a -
 (2*b*c)/d])/(3*d^4) + (4*b*Cos[a + b*x]*Sin[a + b*x])/(3*d^2*(c + d*x)^2)
+ Sin[a + b*x]^2/(3*d*(c + d*x)^3) - (2*b^2*Sin[a + b*x]^2)/(3*d^3*(c + d*x
)) + (8*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(3*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx &= \int \left(\frac{3\cos^2(a+bx)}{(c+dx)^4} - \frac{\sin^2(a+bx)}{(c+dx)^4}\right) dx \\
&= 3\int \frac{\cos^2(a+bx)}{(c+dx)^4}\,dx - \int \frac{\sin^2(a+bx)}{(c+dx)^4}\,dx \\
&= -\frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{4b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} + \frac{\sin^2(a+bx)}{3d(c+dx)^3} - \frac{b^2\int \frac{1}{(c+dx)}}{3d^2} \\
&= -\frac{2b^2}{3d^3(c+dx)} - \frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{2b^2\cos^2(a+bx)}{d^3(c+dx)} + \frac{4b\cos(a+bx)\sin(a}{3d^2(c+dx)^2} \\
&= -\frac{2b^2}{3d^3(c+dx)} - \frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{2b^2\cos^2(a+bx)}{d^3(c+dx)} + \frac{4b\cos(a+bx)\sin(a}{3d^2(c+dx)^2} \\
&= -\frac{2b^2}{3d^3(c+dx)} - \frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{2b^2\cos^2(a+bx)}{d^3(c+dx)} + \frac{4b\cos(a+bx)\sin(a}{3d^2(c+dx)^2} \\
&= -\frac{2b^2}{3d^3(c+dx)} - \frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{2b^2\cos^2(a+bx)}{d^3(c+dx)} + \frac{8b^3\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)\sin}{3d^4}
\end{aligned}
$$

Mathematica [A]
time = 0.92, size = 125, normalized size = 0.61

$$
\frac{8b^3\operatorname{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right) + \frac{d\left((-2d^2+4b^2(c+dx)^2)\cos(2(a+bx))+d(-d+2b(c+dx)\sin(2(a+bx)))\right)}{(c+dx)^3} + 8b^3\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right)}{3d^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^4,x]
```

```
[Out] (8*b^3*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*((-2*d^2 +
4*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + d*(-d + 2*b*(c + d*x)*Sin[2*(a + b*x)
])))/(c + d*x)^3 + 8*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d
])/(3*d^4)
```

Maple [A]
time = 0.19, size = 243, normalized size = 1.19

method	result

default	$\frac{1}{3d(dx+c)^3}+\frac{b^4\left(-\frac{2\cos(2bx+2a)}{3(-ad+cb+d(bx+a))^3 d}-\frac{2\left(-\frac{\sin(2bx+2a)}{(-ad+cb+d(bx+a))^2 d}+\frac{-\frac{2\cos(2bx+2a)}{(-ad+cb+d(bx+a))d}-\frac{2\left(\frac{2\,\mathrm{sinIntegral}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\cos(\ldots}{d}\ldots}{\ldots}}{\ldots}\right)}{3d}\ldots\right)}{b}\ldots$
risch	$-\frac{1}{3d(dx+c)^3}-\frac{4ib^3 e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{3d^4}+\frac{4ib^3 e^{\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}\left(1,-2ibx-2ia-\frac{2(-iad+ibc)}{d}\right)}{3d^4}\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d/(d*x+c)^3+4/b*(1/4*b^4*(-2/3*cos(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^3/d-
2/3*(-sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d+(-2*cos(2*b*x+2*a)/(-a*d+c*b+
d*(b*x+a))/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2
*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d)/d)-1/6*b^4/(-a*d+c*b+
d*(b*x+a))^3/d)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.36, size = 143, normalized size = 0.70

$$-\frac{3\left(E_4\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)+E_4\left(-\frac{2(-i\,bdx-i\,bc)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+3\left(i\,E_4\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)-i\,E_4\left(-\frac{2(-i\,bdx-i\,bc)}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+1}{3\left(d^4x^3+3\,cd^3x^2+3\,c^2d^2x+c^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*(exp_integral_e(4, 2*(-I*b*d*x - I*b*c)/d) + exp_integral_e(4, -2*(
-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 3*(I*exp_integral_e(4, 2*(-I*
b*d*x - I*b*c)/d) - I*exp_integral_e(4, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b
*c - a*d)/d) + 1)/(d^4*x^3 + 3*c*d^3*x^2 + 3*c^2*d^2*x + c^3*d)
```

Fricas [A]
time = 2.69, size = 343, normalized size = 1.67

$$-\frac{4\,b^2d^3x^2+8\,b^2cd^2x+4\,b^2c^2d-d^3-4\left(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3\right)\cos\left(bx+a\right)^2-4\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)\sin\left(bx+a\right)-8\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2(bdx+bc)}{d}\right)-4\left(\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\mathrm{Ci}\left(\frac{2(bdx+bc)}{d}\right)+\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\mathrm{Ci}\left(-\frac{2(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)}{3\left(d^7x^3+3\,cd^6x^2+3\,c^2d^5x+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(4*b^2*d^3*x^2 + 8*b^2*c*d^2*x + 4*b^2*c^2*d - d^3 - 4*(2*b^2*d^3*x^2
+ 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 - 4*(b*d^3*x + b*c*d^2)
```

```
*cos(b*x + a)*sin(b*x + a) - 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d
*x + b^3*c^3)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - 4*((b
^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(2*(b*d
*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*co
s_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x
^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1305 vs. 2(193) = 386.
time = 0.50, size = 1305, normalized size = 6.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/3*(8*b^7*c^3*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c - a*
d)/d) + 24*(b*x + a)*b^6*c^2*d*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*
sin(-2*(b*c - a*d)/d) - 24*a*b^6*c^2*d*cos_integral(2*(b*c + (b*x + a)*d -
a*d)/d)*sin(-2*(b*c - a*d)/d) + 24*(b*x + a)^2*b^5*c*d^2*cos_integral(2*(b*
c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c - a*d)/d) - 48*(b*x + a)*a*b^5*c*d^2*
cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c - a*d)/d) + 24*a^2*
b^5*c*d^2*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c - a*d)/d)
 + 8*(b*x + a)^3*b^4*d^3*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2
*(b*c - a*d)/d) - 24*(b*x + a)^2*a*b^4*d^3*cos_integral(2*(b*c + (b*x + a)*
d - a*d)/d)*sin(-2*(b*c - a*d)/d) + 24*(b*x + a)*a^2*b^4*d^3*cos_integral(2
*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c - a*d)/d) - 8*a^3*b^4*d^3*cos_int
egral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c - a*d)/d) - 8*b^7*c^3*cos(
-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d) - 24*(b*x +
a)*b^6*c^2*d*cos(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d
)/d) + 24*a*b^6*c^2*d*cos(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a
)*d - a*d)/d) - 24*(b*x + a)^2*b^5*c*d^2*cos(-2*(b*c - a*d)/d)*sin_integral
(-2*(b*c + (b*x + a)*d - a*d)/d) + 48*(b*x + a)*a*b^5*c*d^2*cos(-2*(b*c - a
*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d) - 24*a^2*b^5*c*d^2*cos(
-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d) - 8*(b*x + a
)^3*b^4*d^3*cos(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)
```

```
/d) + 24*(b*x + a)^2*a*b^4*d^3*cos(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c +
 (b*x + a)*d - a*d)/d) - 24*(b*x + a)*a^2*b^4*d^3*cos(-2*(b*c - a*d)/d)*sin
_integral(-2*(b*c + (b*x + a)*d - a*d)/d) + 8*a^3*b^4*d^3*cos(-2*(b*c - a*d
)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d) + 4*b^6*c^2*d*cos(2*b*x +
 2*a) + 8*(b*x + a)*b^5*c*d^2*cos(2*b*x + 2*a) - 8*a*b^5*c*d^2*cos(2*b*x +
2*a) + 4*(b*x + a)^2*b^4*d^3*cos(2*b*x + 2*a) - 8*(b*x + a)*a*b^4*d^3*cos(2
*b*x + 2*a) + 4*a^2*b^4*d^3*cos(2*b*x + 2*a) + 2*b^5*c*d^2*sin(2*b*x + 2*a)
 + 2*(b*x + a)*b^4*d^3*sin(2*b*x + 2*a) - 2*a*b^4*d^3*sin(2*b*x + 2*a) - 2*
b^4*d^3*cos(2*b*x + 2*a) - b^4*d^3)/((b^3*c^3*d^4 + 3*(b*x + a)*b^2*c^2*d^5
 - 3*a*b^2*c^2*d^5 + 3*(b*x + a)^2*b*c*d^6 - 6*(b*x + a)*a*b*c*d^6 + 3*a^2*
b*c*d^6 + (b*x + a)^3*d^7 - 3*(b*x + a)^2*a*d^7 + 3*(b*x + a)*a^2*d^7 - a^3
*d^7)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(3a+3bx)}{\sin(a+bx)\ (c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^4),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^4), x)
```

3.376 $\int (c+dx)^3 \csc^2(a+bx) \sin(3a+3bx) \, dx$

Optimal. Leaf size=255

$$-\frac{6(c+dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{24d^2(c+dx)\cos(a+bx)}{b^3}+\frac{4(c+dx)^3\cos(a+bx)}{b}+\frac{9id(c+dx)^2\text{PolyLog}(2,$$

```
[Out] -6*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b-24*d^2*(d*x+c)*cos(b*x+a)/b^3+4*(d*x
+c)^3*cos(b*x+a)/b+9*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^2-9*I*d*(d*
x+c)^2*polylog(2,exp(I*(b*x+a)))/b^2-18*d^2*(d*x+c)*polylog(3,-exp(I*(b*x+a
)))/b^3+18*d^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-18*I*d^3*polylog(4,-ex
p(I*(b*x+a)))/b^4+18*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+24*d^3*sin(b*x+a)/
b^4-12*d*(d*x+c)^2*sin(b*x+a)/b^2
```

Rubi [A]

time = 0.24, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4516, 4493, 3377, 2717, 4268, 2611, 6744, 2320, 6724}

$$-\frac{18id^3\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}+\frac{18id^3\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}+\frac{24d^3\sin(a+bx)}{b^4}-\frac{18d^2(c+dx)\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{18d^2(c+dx)\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{24d^2(c+dx)\cos(a+bx)}{b^3}+\frac{9id(c+dx)^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{9id(c+dx)^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{12d(c+dx)^2\sin(a+bx)}{b^2}+\frac{4(c+dx)^3\cos(a+bx)}{b}-\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (-6*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (24*d^2*(c + d*x)*Cos[a + b*x
])/b^3 + (4*(c + d*x)^3*Cos[a + b*x])/b + ((9*I)*d*(c + d*x)^2*PolyLog[2, -
E^(I*(a + b*x))])/b^2 - ((9*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b
^2 - (18*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (18*d^2*(c + d*x
)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((18*I)*d^3*PolyLog[4, -E^(I*(a + b*x)
)])/b^4 + ((18*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + (24*d^3*Sin[a + b*
x])/b^4 - (12*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \csc^2(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^3\cos(a+bx)\cot(a+bx)-(c+dx)^3\sin(a+bx)\right)$$

$$= 3\int (c+dx)^3\cos(a+bx)\cot(a+bx)\,dx - \int (c+dx)^3\sin(a+$$

$$= \frac{(c+dx)^3\cos(a+bx)}{b} + 3\int (c+dx)^3\csc(a+bx)\,dx - 3\int (c+$$

$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4(c+dx)^3\cos(a+bx)}{b} - \frac{3d(c}{}$$

$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{4(c}{}$$

$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{24d^2(c+dx)\cos(a+bx)}{b^3} + \frac{4(}{}$$

$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{24d^2(c+dx)\cos(a+bx)}{b^3} + \frac{4(}{}$$

$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{24d^2(c+dx)\cos(a+bx)}{b^3} + \frac{4(}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 541 vs. $2(255) = 510$.
time = 1.04, size = 541, normalized size = 2.12

$$\frac{4b^3c^3\cos(a+bx)-24bcd^2\cos(a+bx)+12b^3c^2dx\cos(a+bx)-24bd^3x\cos(a+bx)+12b^3cd^2x^2\cos(a+bx)+4b^3d^3x^3\cos(a+bx)+3b^3c^3\log\left(1-e^{i(a+bx)}\right)+9b^3c^2dx\log\left(1-e^{i(a+bx)}\right)+9b^3cd^2x^2\log\left(1-e^{i(a+bx)}\right)+3b^3d^3x^3\log\left(1-e^{i(a+bx)}\right)-3b^3c^3\log\left(1+e^{i(a+bx)}\right)-9b^3c^2dx\log\left(1+e^{i(a+bx)}\right)-9b^3cd^2x^2\log\left(1+e^{i(a+bx)}\right)-3b^3d^3x^3\log\left(1+e^{i(a+bx)}\right)+9ib^2d(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)-9ib^2d(c+dx)^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)-18bcd^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)-18bd^3x\text{PolyLog}\left(3,-e^{i(a+bx)}\right)+18bcd^2\text{PolyLog}\left(3,e^{i(a+bx)}\right)+18bd^3x\text{PolyLog}\left(3,e^{i(a+bx)}\right)-18id^3\text{PolyLog}\left(4,-e^{i(a+bx)}\right)+18id^3\text{PolyLog}\left(4,e^{i(a+bx)}\right)-12b^2c^2d\sin(a+bx)+24d^3\sin(a+bx)-24b^2cd^2x\sin(a+bx)-12b^2d^3x^2\sin(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (4*b^3*c^3*Cos[a + b*x] - 24*b*c*d^2*Cos[a + b*x] + 12*b^3*c^2*d*x*Cos[a +
b*x] - 24*b*d^3*x*Cos[a + b*x] + 12*b^3*c*d^2*x^2*Cos[a + b*x] + 4*b^3*d^3*
x^3*Cos[a + b*x] + 3*b^3*c^3*Log[1 - E^(I*(a + b*x))] + 9*b^3*c^2*d*x*Log[1
 - E^(I*(a + b*x))] + 9*b^3*c*d^2*x^2*Log[1 - E^(I*(a + b*x))] + 3*b^3*d^3*
x^3*Log[1 - E^(I*(a + b*x))] - 3*b^3*c^3*Log[1 + E^(I*(a + b*x))] - 9*b^3*c
^2*d*x*Log[1 + E^(I*(a + b*x))] - 9*b^3*c*d^2*x^2*Log[1 + E^(I*(a + b*x))]
- 3*b^3*d^3*x^3*Log[1 + E^(I*(a + b*x))] + (9*I)*b^2*d*(c + d*x)^2*PolyLog[
2, -E^(I*(a + b*x))] - (9*I)*b^2*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))]
```

```
- 18*b*c*d^2*PolyLog[3, -E^(I*(a + b*x))] - 18*b*d^3*x*PolyLog[3, -E^(I*(a
+ b*x))] + 18*b*c*d^2*PolyLog[3, E^(I*(a + b*x))] + 18*b*d^3*x*PolyLog[3, E
^(I*(a + b*x))] - (18*I)*d^3*PolyLog[4, -E^(I*(a + b*x))] + (18*I)*d^3*Poly
Log[4, E^(I*(a + b*x))] - 12*b^2*c^2*d*Sin[a + b*x] + 24*d^3*Sin[a + b*x] -
 24*b^2*c*d^2*x*Sin[a + b*x] - 12*b^2*d^3*x^2*Sin[a + b*x])/b^4
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(237) = 474$.
time = 0.12, size = 849, normalized size = 3.33

method	result
risch	$-\frac{9c^2 d \ln(e^{i(bx+a)}+1)a}{b^2} - \frac{6c^3 \operatorname{arctanh}(e^{i(bx+a)})}{b} - \frac{18icd^2 \operatorname{polylog}(2,e^{i(bx+a)})x}{b^2} + \frac{18icd^2 \operatorname{polylog}(2,-e^{i(bx+a)})x}{b^2} - \frac{3d^3 \ln(e^{i(b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

```
[Out] 18/b^2*c^2*d*a*arctanh(exp(I*(b*x+a)))-18/b^3*c*d^2*a^2*arctanh(exp(I*(b*x+
a)))+9/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))+1)-3/b^4*d^3*ln(exp(I*(b*x+a))+1)*a^
3+18*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+18/b^3*c*d^2*polylog(3,exp(I*(b*x+
a)))-18/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))-18/b^3*d^3*polylog(3,-exp(I*(b
*x+a)))*x+18/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-18*I*d^3*polylog(4,-exp(I*
(b*x+a)))/b^4+6/b^4*d^3*a^3*arctanh(exp(I*(b*x+a)))+2*(d^3*x^3*b^3+3*b^3*c*
d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c
*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))-3/b*d^3*ln(exp(I*(b*x+a))+
1)*x^3+3/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+
2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^
3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))-6/
b*c^3*arctanh(exp(I*(b*x+a)))-9*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-9*I/b
^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-9/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+9/b*c
^2*d*ln(1-exp(I*(b*x+a)))*x+9/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-9/b*c*d^2*ln
(exp(I*(b*x+a))+1)*x^2+9/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-9/b^3*c*d^2*ln(1-
exp(I*(b*x+a)))*a^2-9/b^2*c^2*d*ln(exp(I*(b*x+a))+1)*a+9*I/b^2*d^3*polylog(
2,-exp(I*(b*x+a)))*x^2+9*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))+18*I/b^2*c*
d^2*polylog(2,-exp(I*(b*x+a)))*x-18*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(231) = 462$.
time = 0.42, size = 606, normalized size = 2.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 +
 sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*x)
```

```
*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b - 1/2*(3
6*I*d^3*polylog(4, -e^(I*b*x + I*a)) - 36*I*d^3*polylog(4, e^(I*b*x + I*a))
 - 6*(-I*b^3*d^3*x^3 - 3*I*b^3*c*d^2*x^2 - 3*I*b^3*c^2*d*x)*arctan2(sin(b*x
 + a), cos(b*x + a) + 1) - 6*(-I*b^3*d^3*x^3 - 3*I*b^3*c*d^2*x^2 - 3*I*b^3*
c^2*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 8*(b^3*d^3*x^3 + 3*b^3*
c*d^2*x^2 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a) - 18*(I*b^2
*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(-e^(I*b*x + I*a)) - 18*(-I*
b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(e^(I*b*x + I*a)) + 3*(b^
3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x)*log(cos(b*x + a)^2 + sin(b*x +
 a)^2 + 2*cos(b*x + a) + 1) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*
d*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 36*(b*d^3*
x + b*c*d^2)*polylog(3, -e^(I*b*x + I*a)) - 36*(b*d^3*x + b*c*d^2)*polylog(
3, e^(I*b*x + I*a)) + 24*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*
sin(b*x + a))/b^4
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(231) = 462$.
time = 1.85, size = 929, normalized size = 3.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(18*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 18*I*d^3*polylog(
4, cos(b*x + a) - I*sin(b*x + a)) + 18*I*d^3*polylog(4, -cos(b*x + a) + I*s
in(b*x + a)) - 18*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 8*(b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*
x)*cos(b*x + a) - 9*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(c
os(b*x + a) + I*sin(b*x + a)) - 9*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2
*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 9*(I*b^2*d^3*x^2 + 2*I*b^2*c
*d^2*x + I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 9*(-I*b^2*d^3
*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x + a))
 - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x
+ a) + I*sin(b*x + a) + 1) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d
*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 3*(b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a
) + 1/2) + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*c
os(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
+ 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a
) + I*sin(b*x + a) + 1) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x
+ 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x +
a) + 1) + 18*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a))
+ 18*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 18*(b*
d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 18*(b*d^3*x +
```

```
b*c*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) - 24*(b^2*d^3*x^2 + 2*
b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a))/b^4
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**2*sin(3*b*x+3*a),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
[Out] integrate((d*x + c)^3*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^3)/sin(a + b*x)^2,x)
[Out] \text{Hanged}
```

3.377 $\int (c+dx)^2 \csc^2(a+bx) \sin(3a+3bx) \, dx$

Optimal. Leaf size=172

$$-\frac{6(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{8d^2 \cos(a+bx)}{b^3}+\frac{4(c+dx)^2 \cos(a+bx)}{b}+\frac{6id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}$$

```
[Out] -6*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-8*d^2*cos(b*x+a)/b^3+4*(d*x+c)^2*cos
(b*x+a)/b+6*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-6*I*d*(d*x+c)*polylo
g(2,exp(I*(b*x+a)))/b^2-6*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+6*d^2*polylog(
3,exp(I*(b*x+a)))/b^3-8*d*(d*x+c)*sin(b*x+a)/b^2
```

Rubi [A]
time = 0.17, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4516, 4493, 3377, 2718, 4268, 2611, 2320, 6724}

$$-\frac{6d^2\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{8d^2\cos(a+bx)}{b^3}+\frac{6id(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{6id(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{8d(c+dx)\sin(a+bx)}{b^2}+\frac{4(c+dx)^2\cos(a+bx)}{b}-\frac{6(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (-6*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (8*d^2*Cos[a + b*x])/b^3 + (4
*(c + d*x)^2*Cos[a + b*x])/b + ((6*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x
))])/b^2 - ((6*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (6*d^2*Pol
yLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3 -
(8*d*(c + d*x)*Sin[a + b*x])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2 \csc^2(a+bx)\sin(3a+3bx)\,dx &= \int \left(3(c+dx)^2\cos(a+bx)\cot(a+bx) - (c+dx)^2\sin(a+bx)\right)\\ &= 3\int (c+dx)^2\cos(a+bx)\cot(a+bx)\,dx - \int (c+dx)^2\sin(a+\\ &= \frac{(c+dx)^2\cos(a+bx)}{b} + 3\int (c+dx)^2\csc(a+bx)\,dx - 3\int (c+\\ &= -\frac{6(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4(c+dx)^2\cos(a+bx)}{b} - \frac{2d(c\\ &= -\frac{6(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{4(c+dx)^2 c}{b}\\ &= -\frac{6(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{8d^2\cos(a+bx)}{b^3} + \frac{4(c+dx)^2 c}{b}\\ &= -\frac{6(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{8d^2\cos(a+bx)}{b^3} + \frac{4(c+dx)^2 c}{b}\end{aligned}$$

Mathematica [**A**]
time = 0.69, size = 317, normalized size = 1.84

$$\frac{4b^2c^2\cos(a+bx) - 8d^2\cos(a+bx) + 8b^2cdx\cos(a+bx) + 4b^2d^2x^2\cos(a+bx) + 3b^2c^2\log\left(1-e^{i(a+bx)}\right) + 6b^2cdx\log\left(1-e^{i(a+bx)}\right) + 3b^2d^2x^2\log\left(1-e^{i(a+bx)}\right) - 3b^2c^2\log\left(1+e^{i(a+bx)}\right) - 6b^2cdx\log\left(1+e^{i(a+bx)}\right) - 3b^2d^2x^2\log\left(1+e^{i(a+bx)}\right) + 6ibd(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right) - 6ibd(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right) - 6d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right) + 6d^2\text{PolyLog}\left(3,e^{i(a+bx)}\right) - 8bcd\sin(a+bx) - 8bd^2x\sin(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (4*b^2*c^2*Cos[a + b*x] - 8*d^2*Cos[a + b*x] + 8*b^2*c*d*x*Cos[a + b*x] + 4
*b^2*d^2*x^2*Cos[a + b*x] + 3*b^2*c^2*Log[1 - E^(I*(a + b*x))] + 6*b^2*c*d*
x*Log[1 - E^(I*(a + b*x))] + 3*b^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] - 3*b^2
*c^2*Log[1 + E^(I*(a + b*x))] - 6*b^2*c*d*x*Log[1 + E^(I*(a + b*x))] - 3*b^
2*d^2*x^2*Log[1 + E^(I*(a + b*x))] + (6*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(
a + b*x))] - (6*I)*b*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] - 6*d^2*PolyLo
g[3, -E^(I*(a + b*x))] + 6*d^2*PolyLog[3, E^(I*(a + b*x))] - 8*b*c*d*Sin[a
+ b*x] - 8*b*d^2*x*Sin[a + b*x])/b^3
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(160) = 320$.
time = 0.26, size = 481, normalized size = 2.80

method	result
risch	$\frac{2\left(x^2d^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2\right)\mathrm{e}^{i(bx+a)}}{b^3} + \frac{2\left(x^2d^2b^2+2b^2cdx-2ib\,d^2x+b^2c^2-2ibcd-2d^2\right)\mathrm{e}^{-i(bx+a)}}{b^3} - \frac{6cd\ln\left(\mathrm{e}^{i(bx}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(I*(
b*x+a))+2*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)/b^3
*exp(-I*(b*x+a))-6/b*c*d*ln(exp(I*(b*x+a))+1)*x-6/b^2*c*d*ln(exp(I*(b*x+a))
+1)*a+6/b*c*d*ln(1-exp(I*(b*x+a)))*x+6/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+12/b^
2*c*d*a*arctanh(exp(I*(b*x+a)))-6*I/b^2*polylog(2,exp(I*(b*x+a)))*d^2*x-6/b
*c^2*arctanh(exp(I*(b*x+a)))+6*d^2*polylog(3,exp(I*(b*x+a)))/b^3-6*d^2*poly
log(3,-exp(I*(b*x+a)))/b^3+3/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/b^3*d^2*ln(1-
exp(I*(b*x+a)))*a^2+6*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-3/b*d^2*ln(exp
(I*(b*x+a))+1)*x^2+3/b^3*d^2*ln(exp(I*(b*x+a))+1)*a^2-6*I/b^2*c*d*polylog(2
,exp(I*(b*x+a)))+6*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-6/b^3*d^2*a^2*arcta
nh(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(156) = 312$.
time = 0.39, size = 413, normalized size = 2.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 +
 sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*x)
*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b - 1/2*(1
2*d^2*polylog(3, -e^(I*b*x + I*a)) - 12*d^2*polylog(3, e^(I*b*x + I*a)) - 6
*(-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x)*arctan2(sin(b*x + a), cos(b*x + a) + 1) -
 6*(-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1
) - 8*(b^2*d^2*x^2 + 2*b^2*c*d*x - 2*d^2)*cos(b*x + a) - 12*(I*b*d^2*x + I*
b*c*d)*dilog(-e^(I*b*x + I*a)) - 12*(-I*b*d^2*x - I*b*c*d)*dilog(e^(I*b*x +
 I*a)) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2
+ 2*cos(b*x + a) + 1) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x)*log(cos(b*x + a)^2 +
sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 16*(b*d^2*x + b*c*d)*sin(b*x + a))/b
^3
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(156) = 312$.
time = 1.33, size = 566, normalized size = 3.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*d^2*polylog(3, cos
(b*x + a) - I*sin(b*x + a)) - 6*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x +
```

```
a)) - 6*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 8*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) - 6*(I*b*d^2*x + I*b*c*d)*dilog(
cos(b*x + a) + I*sin(b*x + a)) - 6*(-I*b*d^2*x - I*b*c*d)*dilog(cos(b*x + a
) - I*sin(b*x + a)) - 6*(I*b*d^2*x + I*b*c*d)*dilog(-cos(b*x + a) + I*sin(b
*x + a)) - 6*(-I*b*d^2*x - I*b*c*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) -
 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*sin(b*x + a)
+ 1) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) - I*sin(b*x
 + a) + 1) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a) + 1/2*
I*sin(b*x + a) + 1/2) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x
+ a) - 1/2*I*sin(b*x + a) + 1/2) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d
 - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*(b^2*d^2*x^2 + 2*b^
2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 16
*(b*d^2*x + b*c*d)*sin(b*x + a))/b^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2*sin(3*b*x+3*a),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
[Out] integrate((d*x + c)^2*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^2)/sin(a + b*x)^2,x)
[Out] \text{Hanged}
```

3.378 $\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=95

$$-\frac{6(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{4(c+dx)\cos(a+bx)}{b}+\frac{3id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{4}{}$$

```
[Out] -6*(d*x+c)*arctanh(exp(I*(b*x+a)))/b+4*(d*x+c)*cos(b*x+a)/b+3*I*d*polylog(2
,-exp(I*(b*x+a)))/b^2-3*I*d*polylog(2,exp(I*(b*x+a)))/b^2-4*d*sin(b*x+a)/b^
2
```

Rubi [A]
time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4516, 4493, 3377, 2717, 4268, 2317, 2438}

$$\frac{3id\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{3id\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{4d\sin(a+bx)}{b^2}+\frac{4(c+dx)\cos(a+bx)}{b}-\frac{6(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (-6*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + (4*(c + d*x)*Cos[a + b*x])/b +
((3*I)*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d*PolyLog[2, E^(I*(a +
b*x))])/b^2 - (4*d*Sin[a + b*x])/b^2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx &= \int (3(c+dx)\cos(a+bx)\cot(a+bx)-(c+dx)\sin(a+bx))\,dx \\
&= 3\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx-\int (c+dx)\sin(a+bx \\
&= \frac{(c+dx)\cos(a+bx)}{b}+3\int (c+dx)\csc(a+bx)\,dx-3\int (c+d \\
&= -\frac{6(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{4(c+dx)\cos(a+bx)}{b}-\frac{d\sin(a}{b^2} \\
&= -\frac{6(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{4(c+dx)\cos(a+bx)}{b}-\frac{4d\sin(}{b} \\
&= -\frac{6(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{4(c+dx)\cos(a+bx)}{b}+\frac{3id\mathrm{Li}_2(}{}
\end{aligned}$$

Mathematica [A]
time = 0.33, size = 171, normalized size = 1.80

$$\frac{4c\cos(a+bx)}{b}+\frac{4dx\cos(a+bx)}{b}-\frac{3c\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{3c\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}-\frac{3ad\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)}{b^2}+\frac{3d\left((a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)+i\left(\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)-\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)\right)\right)}{b^2}-\frac{4d\sin(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (4*c*Cos[a + b*x])/b + (4*d*x*Cos[a + b*x])/b - (3*c*Log[Cos[(a + b*x)/2]])
/b + (3*c*Log[Sin[(a + b*x)/2]])/b - (3*a*d*Log[Tan[(a + b*x)/2]])/b^2 + (3
*d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(Po
lyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x))])))/b^2 - (4*d*Sin[
a + b*x])/b^2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(87) = 174$.
time = 0.12, size = 204, normalized size = 2.15

method	result
default	$\frac{3c\ln(\csc(bx+a)-\cot(bx+a))}{b}+\frac{3d\ln(1-e^{i(bx+a)})x}{b}-\frac{3d\ln(e^{i(bx+a)}+1)x}{b}-\frac{3da\ln(\csc(bx+a)-\cot(bx+a))}{b^2}-\frac{3id\,\mathrm{dilog}(1-e^{i(bx}}{b^2}$
risch	$\frac{2(dxb+cb+id)e^{i(bx+a)}}{b^2}+\frac{2(dxb+cb-id)e^{-i(bx+a)}}{b^2}-\frac{6c\,\mathrm{arctanh}(e^{i(bx+a)})}{b}+\frac{3d\ln(1-e^{i(bx+a)})x}{b}+\frac{3d\ln(1-e^{i(bx+a)})a}{b^2}-\frac{3id}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

```
[Out] 3*c/b*ln(csc(b*x+a)-cot(b*x+a))+3/b*d*ln(1-exp(I*(b*x+a)))*x-3/b*d*ln(exp(I
*(b*x+a))+1)*x-3*d/b^2*a*ln(csc(b*x+a)-cot(b*x+a))-3*I*d/b^2*dilog(1-exp(I*
(b*x+a)))+3*I*d/b^2*dilog(exp(I*(b*x+a))+1)+3/b^2*d*ln(1-exp(I*(b*x+a)))*a-
3/b^2*d*ln(exp(I*(b*x+a))+1)*a+4*c/b*cos(b*x+a)+4*d/b*cos(b*x+a)*x-4*d*sin(
b*x+a)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] 1/2*c*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + s
in(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*x)*c
os(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b + (4*b*x*c
os(b*x + a) + 3*b^2*integrate(x*sin(b*x + a)/(cos(b*x + a)^2 + sin(b*x + a)
^2 + 2*cos(b*x + a) + 1), x) + 3*b^2*integrate(x*sin(b*x + a)/(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1), x) - 4*sin(b*x + a))*d/b^2
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(83) = 166$.
time = 1.87, size = 281, normalized size = 2.96

$$\frac{8(bdx+bc)\cos(bx+a)-3i\,d\mathrm{Li}_2(\cos(bx+a)+i\sin(bx+a))+3i\,d\mathrm{Li}_2(\cos(bx+a)-i\sin(bx+a))-3i\,d\mathrm{Li}_2(-\cos(bx+a)+i\sin(bx+a))+3i\,d\mathrm{Li}_2(-\cos(bx+a)-i\sin(bx+a))-3(bdx+bc)\log(\cos(bx+a)+i\sin(bx+a)+1)-3(bdx+bc)\log(\cos(bx+a)-i\sin(bx+a)+1)+3(bc-ad)\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}i\sin(bx+a)+\frac{1}{2}\right)+3(bc-ad)\log\left(-\frac{1}{2}\cos(bx+a)-\frac{1}{2}i\sin(bx+a)+\frac{1}{2}\right)+3(bdx+ad)\log(-\cos(bx+a)+i\sin(bx+a)+1)+3(bdx+ad)\log(-\cos(bx+a)-i\sin(bx+a)+1)-8d\sin(bx+a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(8*(b*d*x + b*c)*cos(b*x + a) - 3*I*d*dilog(cos(b*x + a) + I*sin(b*x +
a)) + 3*I*d*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*I*d*dilog(-cos(b*x + a
) + I*sin(b*x + a)) + 3*I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b*d*
x + b*c)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b*d*x + b*c)*log(cos(b
*x + a) - I*sin(b*x + a) + 1) + 3*(b*c - a*d)*log(-1/2*cos(b*x + a) + 1/2*I
*sin(b*x + a) + 1/2) + 3*(b*c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x
+ a) + 1/2) + 3*(b*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*(
b*d*x + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 8*d*sin(b*x + a))/b^
2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x))/sin(a + b*x)^2,x)
```

```
[Out] \text{Hanged}
```

3.379 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

Optimal. Leaf size=72

$$-\frac{4\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}-\frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+3\text{Int}\left(\frac{\csc(a+bx)}{c+dx},x\right)$$

```
[Out] -4*cos(a-b*c/d)*Si(b*c/d+b*x)/d-4*Ci(b*c/d+b*x)*sin(a-b*c/d)/d+3*Unintegrable(csc(b*x+a)/(d*x+c),x)
```

Rubi [A]
time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] (-4*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d - (4*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d + 3*Defer[Int][Csc[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx &= \int \left(\frac{3\cos(a+bx)\cot(a+bx)}{c+dx}-\frac{\sin(a+bx)}{c+dx}\right)dx\\
&= 3\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx-\int \frac{\sin(a+bx)}{c+dx}\,dx\\
&= 3\int \frac{\csc(a+bx)}{c+dx}\,dx-3\int \frac{\sin(a+bx)}{c+dx}\,dx-\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right.}{c+dx}\\
&= -\frac{\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+3\int \frac{\csc(a+bx)}{c+dx}\\
&= -\frac{4\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}-\frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+3\int \frac{\csc(a+b}{c+dx}
\end{aligned}$$

Mathematica [A]
time = 6.01, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x), x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^{2}\left(bx+a\right)\right)\sin\left(3bx+3a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -(2*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(1, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - 3*d*integrate(sin(b*x + a)/((d*x + c
)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*cos(b*x + a
) + c), x) - 3*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x +
c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) - 2*(exp_integr
al_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin
(-(b*c - a*d)/d))/d
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)}{{\sin\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)), x)
```

3.380 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=92

$$-\frac{4b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{4\sin(a+bx)}{d(c+dx)}+\frac{4b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+3\text{Int}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] -4*b*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^2+4*b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2+4*si
n(b*x+a)/d/(d*x+c)+3*Unintegrable(csc(b*x+a)/(d*x+c)^2,x)
```

Rubi [A]
time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] (-4*b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2 + (4*Sin[a + b*x])/(
d*(c + d*x)) + (4*b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + 3*De
fer[Int][Csc[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx &= \int \left(\frac{3\cos(a+bx)\cot(a+bx)}{(c+dx)^2}-\frac{\sin(a+bx)}{(c+dx)^2}\right)dx\\
&= 3\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx-\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(a+bx)}{d(c+dx)}+3\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx-3\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx-\frac{b\int \frac{\cos(a+bx)}{c+dx}\,d}{d}\\
&= \frac{4\sin(a+bx)}{d(c+dx)}+3\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx-\frac{(3b)\int \frac{\cos(a+bx)}{c+dx}\,dx}{d}-\frac{\left(b\cos\left(a-\frac{b}{c}\right.\right.}{}\\
&= -\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{4\sin(a+bx)}{d(c+dx)}+\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}\\
&= -\frac{4b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{4\sin(a+bx)}{d(c+dx)}+\frac{4b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+b\right.}{d^2}
\end{aligned}$$

Mathematica [A]
time = 6.60, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2, x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(\csc^2(bx+a))\sin(3bx+3a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -(2*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(2, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - 3*(d^2*x + c*d)*integrate(sin(b*x +
a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2
 + 2*c*d*x + c^2)*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*
x + a)), x) - 3*(d^2*x + c*d)*integrate(sin(b*x + a)/(d^2*x^2 + 2*c*d*x + (
d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(b*x
 + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x) - 2*(exp_inte
gral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*s
in(-(b*c - a*d)/d))/(d^2*x + c*d)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)}{{\sin\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^2),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^2), x)
```

3.381 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=115

$$\frac{2b\cos(a+bx)}{d^2(c+dx)}+\frac{2b^2\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^3}+\frac{2\sin(a+bx)}{d(c+dx)^2}+\frac{2b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^3}+3\text{Int}\left(\frac{\text{csc}}{(c}\right.$$

```
[Out] 2*b*cos(b*x+a)/d^2/(d*x+c)+2*b^2*cos(a-b*c/d)*Si(b*c/d+b*x)/d^3+2*b^2*Ci(b*
c/d+b*x)*sin(a-b*c/d)/d^3+2*sin(b*x+a)/d/(d*x+c)^2+3*Unintegrable(csc(b*x+a
)/(d*x+c)^3,x)
```

Rubi [A]
time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] (2*b*Cos[a + b*x])/(d^2*(c + d*x)) + (2*b^2*CosIntegral[(b*c)/d + b*x]*Sin[
a - (b*c)/d])/d^3 + (2*Sin[a + b*x])/(d*(c + d*x)^2) + (2*b^2*Cos[a - (b*c)
/d]*SinIntegral[(b*c)/d + b*x])/d^3 + 3*Defer[Int][Csc[a + b*x]/(c + d*x)^3
, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \left(\frac{3\cos(a+bx)\cot(a+bx)}{(c+dx)^3} - \frac{\sin(a+bx)}{(c+dx)^3}\right) dx$$
$$= 3\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^3}\,dx - \int \frac{\sin(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{\sin(a+bx)}{2d(c+dx)^2} + 3\int \frac{\csc(a+bx)}{(c+dx)^3}\,dx - 3\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx - \frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}}{2d}$$
$$= \frac{b\cos(a+bx)}{2d^2(c+dx)} + \frac{2\sin(a+bx)}{d(c+dx)^2} + 3\int \frac{\csc(a+bx)}{(c+dx)^3}\,dx + \frac{b^2\int \frac{\sin(a+bx)}{c+dx}\,dx}{2d^2}$$
$$= \frac{2b\cos(a+bx)}{d^2(c+dx)} + \frac{2\sin(a+bx)}{d(c+dx)^2} + 3\int \frac{\csc(a+bx)}{(c+dx)^3}\,dx + \frac{(3b^2)\int \frac{\sin(a+bx)}{c+dx}}{2d^2}$$
$$= \frac{2b\cos(a+bx)}{d^2(c+dx)} + \frac{b^2\operatorname{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{2d^3} + \frac{2\sin(a+bx)}{d(c+dx)^2} + \frac{b^2\cos(a}{}$$
$$= \frac{2b\cos(a+bx)}{d^2(c+dx)} + \frac{2b^2\operatorname{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^3} + \frac{2\sin(a+bx)}{d(c+dx)^2} + \frac{2b^2\cos}{}$$

Mathematica [A]
time = 7.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
[Out] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3, x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^2\left(bx+a\right)\right)\sin\left(3bx+3a\right)}{\left(dx+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
[Out] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -(2*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(3, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - 3*(d^3*x^2 + 2*c*d^2*x + c^2*d)*inte
grate(sin(b*x + a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3 + (d^3*x^3 + 3*
c*d^2*x^2 + 3*c^2*d*x + c^3)*cos(b*x + a)^2 + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^
2*d*x + c^3)*sin(b*x + a)^2 + 2*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*c
os(b*x + a)), x) - 3*(d^3*x^2 + 2*c*d^2*x + c^2*d)*integrate(sin(b*x + a)/(
d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3 + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*
x + c^3)*cos(b*x + a)^2 + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sin(b*x
 + a)^2 - 2*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos(b*x + a)), x) - 2
*(exp_integral_e(3, (I*b*d*x + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x + I*
b*c)/d))*sin(-(b*c - a*d)/d))/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3a+3bx)}{\sin(a+bx)^2 (c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^3),x)
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^3), x)
```

3.382 $\int (c+dx)^4 \sec(a+bx) \sin(3a+3bx)\,dx$

Optimal. Leaf size=299

$$\frac{6cd^3x}{b^3}+\frac{3d^4x^2}{b^3}-\frac{(c+dx)^4}{b}-\frac{i(c+dx)^5}{5d}+\frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{2id(c+dx)^3\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}+\frac{3c}{}$$

```
[Out] 6*c*d^3*x/b^3+3*d^4*x^2/b^3-(d*x+c)^4/b-1/5*I*(d*x+c)^5/d+(d*x+c)^4*ln(1+ex
p(2*I*(b*x+a)))/b-2*I*d*(d*x+c)^3*polylog(2,-exp(2*I*(b*x+a)))/b^2+3*d^2*(d
*x+c)^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+3*I*d^3*(d*x+c)*polylog(4,-exp(2*I
*(b*x+a)))/b^4-3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5-6*d^3*(d*x+c)*cos(b
*x+a)*sin(b*x+a)/b^4+4*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2+3*d^4*sin(b*x+
a)^2/b^5-6*d^2*(d*x+c)^2*sin(b*x+a)^2/b^3+2*(d*x+c)^4*sin(b*x+a)^2/b
```

Rubi [A]
time = 0.35, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {4516, 4489, 3392, 32, 3391, 4492, 3800, 2221, 2611, 6744, 2320, 6724}

$$-\frac{3d^4\mathrm{Li}_5\left(-e^{2i(a+bx)}\right)}{2b^5}+\frac{3d^4\sin^2(a+bx)}{b^5}+\frac{3id^3(c+dx)\mathrm{Li}_4\left(-e^{2i(a+bx)}\right)}{b^4}-\frac{6d^3(c+dx)\sin(a+bx)\cos(a+bx)}{b^4}+\frac{3d^2(c+dx)^2\mathrm{Li}_3\left(-e^{2i(a+bx)}\right)}{b^3}-\frac{6d^2(c+dx)^2\sin^2(a+bx)}{b^3}-\frac{2id(c+dx)^3\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}+\frac{4d(c+dx)^3\sin(a+bx)\cos(a+bx)}{b^2}+\frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{2(c+dx)^4\sin^2(a+bx)}{b}+\frac{6cd^3x}{b^3}+\frac{3d^4x^2}{b^3}-\frac{(c+dx)^4}{b}-\frac{i(c+dx)^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (6*c*d^3*x)/b^3 + (3*d^4*x^2)/b^3 - (c + d*x)^4/b - ((I/5)*(c + d*x)^5)/d +
 ((c + d*x)^4*Log[1 + E^((2*I)*(a + b*x))])/b - ((2*I)*d*(c + d*x)^3*PolyLo
g[2, -E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(
a + b*x))])/b^3 + ((3*I)*d^3*(c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^
4 - (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5) - (6*d^3*(c + d*x)*Cos
[a + b*x]*Sin[a + b*x])/b^4 + (4*d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/b
^2 + (3*d^4*Sin[a + b*x]^2)/b^5 - (6*d^2*(c + d*x)^2*Sin[a + b*x]^2)/b^3 +
(2*(c + d*x)^4*Sin[a + b*x]^2)/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4492

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \sec(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^4\cos(a+bx)\sin(a+bx) - (c+dx)^4\sin^2(a+bx)\,\text{t}\right.$$

$$= 3\int (c+dx)^4\cos(a+bx)\sin(a+bx)\,dx - \int (c+dx)^4\sin^2(a+$$

$$= \frac{3(c+dx)^4\sin^2(a+bx)}{2b} - \frac{(6d)\int (c+dx)^3\sin^2(a+bx)\,dx}{b} + \int ($$

$$= -\frac{i(c+dx)^5}{5d} + \frac{3d(c+dx)^3\cos(a+bx)\sin(a+bx)}{b^2} - \frac{9d^2(c+d}{}$$

$$= -\frac{3(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{9d^3}{}$$

$$= \frac{9cd^3x}{2b^3} + \frac{9d^4x^2}{4b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e\right.}{b}$$

$$= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e\right.}{b}$$

$$= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e\right.}{b}$$

$$= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e\right.}{b}$$

$$= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e\right.}{b}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2517 vs. 2(299) = 598.
time = 6.47, size = 2517, normalized size = 8.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] -1/2*(c^2*d^2*((2*I)*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(1 + E^((2*I)*a))*L
og[1 + E^((2*I)*(a + b*x))]) + (6*I)*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((
2*I)*(a + b*x))] - 3*(1 + E^((2*I)*a))*PolyLog[3, -E^((2*I)*(a + b*x))])*Se
c[a])/(b^3*E^(I*a)) + I*c*d^3*E^(I*a)*(-x^4 + (1 + E^((-2*I)*a))*x^4 - ((1
+ E^((2*I)*a))*(2*b^4*x^4 + (4*I)*b^3*x^3*Log[1 + E^((2*I)*(a + b*x))] + 6*
b^2*x^2*PolyLog[2, -E^((2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[3, -E^((2*I)*(
a + b*x))] - 3*PolyLog[4, -E^((2*I)*(a + b*x))]))/(2*b^4*E^((2*I)*a)))*Sec[
a] + (I/5)*d^4*E^(I*a)*(-x^5 + (1 + E^((-2*I)*a))*x^5 - ((1 + E^((2*I)*a))*
(4*b^5*x^5 + (10*I)*b^4*x^4*Log[1 + E^((2*I)*(a + b*x))] + 20*b^3*x^3*PolyL
og[2, -E^((2*I)*(a + b*x))] + (30*I)*b^2*x^2*PolyLog[3, -E^((2*I)*(a + b*x)
```

)] - 30*b*x*PolyLog[4, -E^((2*I)*(a + b*x))] - (15*I)*PolyLog[5, -E^((2*I)*(a + b*x))]))/(4*b^5*E^((2*I)*a)))*Sec[a] + (c^4*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (2*c^3*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + Sec[a]*(Cos[2*a + 2*b*x]/(40*b^5) - ((I/40)*Sin[2*a + 2*b*x])/b^5)*(-20*b^4*c^4*Cos[a] + (40*I)*b^3*c^3*d*Cos[a] + 60*b^2*c^2*d^2*Cos[a] - (60*I)*b*c*d^3*Cos[a] - 30*d^4*Cos[a] - 80*b^4*c^3*d*x*Cos[a] + (120*I)*b^3*c^2*d^2*x*Cos[a] + 120*b^2*c*d^3*x*Cos[a] - (60*I)*b*d^4*x*Cos[a] - 120*b^4*c^2*d^2*x^2*Cos[a] + (120*I)*b^3*c*d^3*x^2*Cos[a] + 60*b^2*d^4*x^2*Cos[a] - 80*b^4*c*d^3*x^3*Cos[a] + (40*I)*b^3*d^4*x^3*Cos[a] - 20*b^4*d^4*x^4*Cos[a] - (20*I)*b^5*c^4*x*Cos[a + 2*b*x] - (40*I)*b^5*c^3*d*x^2*Cos[a + 2*b*x] - (40*I)*b^5*c^2*d^2*x^3*Cos[a + 2*b*x] - (20*I)*b^5*c*d^3*x^4*Cos[a + 2*b*x] - (4*I)*b^5*d^4*x^5*Cos[a + 2*b*x] + (20*I)*b^5*c^4*x*Cos[3*a + 2*b*x] + (40*I)*b^5*c^3*d*x^2*Cos[3*a + 2*b*x] + (40*I)*b^5*c^2*d^2*x^3*Cos[3*a + 2*b*x] + (20*I)*b^5*c*d^3*x^4*Cos[3*a + 2*b*x] + (4*I)*b^5*d^4*x^5*Cos[3*a + 2*b*x] - 10*b^4*c^4*Cos[3*a + 4*b*x] - (20*I)*b^3*c^3*d*Cos[3*a + 4*b*x] + 30*b^2*c^2*d^2*Cos[3*a + 4*b*x] + (30*I)*b*c*d^3*Cos[3*a + 4*b*x] - 15*d^4*Cos[3*a + 4*b*x] - 40*b^4*c^3*d*x*Cos[3*a + 4*b*x] - (60*I)*b^3*c^2*d^2*x*Cos[3*a + 4*b*x] + 60*b^2*c*d^3*x*Cos[3*a + 4*b*x] + (30*I)*b*d^4*x*Cos[3*a + 4*b*x] - 60*b^4*c^2*d^2*x^2*Cos[3*a + 4*b*x] - (60*I)*b^3*c*d^3*x^2*Cos[3*a + 4*b*x] + 30*b^2*d^4*x^2*Cos[3*a + 4*b*x] - 40*b^4*c*d^3*x^3*Cos[3*a + 4*b*x] - (20*I)*b^3*d^4*x^3*Cos[3*a + 4*b*x] - 10*b^4*d^4*x^4*Cos[3*a + 4*b*x] - 10*b^4*c^4*Cos[5*a + 4*b*x] - (20*I)*b^3*c^3*d*Cos[5*a + 4*b*x] + 30*b^2*c^2*d^2*Cos[5*a + 4*b*x] + (30*I)*b*c*d^3*Cos[5*a + 4*b*x] - 15*d^4*Cos[5*a + 4*b*x] - 40*b^4*c^3*d*x*Cos[5*a + 4*b*x] - (60*I)*b^3*c^2*d^2*x*Cos[5*a + 4*b*x] + 60*b^2*c*d^3*x*Cos[5*a + 4*b*x] + (30*I)*b*d^4*x*Cos[5*a + 4*b*x] - 60*b^4*c^2*d^2*x^2*Cos[5*a + 4*b*x] - (60*I)*b^3*c*d^3*x^2*Cos[5*a + 4*b*x] + 30*b^2*d^4*x^2*Cos[5*a + 4*b*x] - 40*b^4*c*d^3*x^3*Cos[5*a + 4*b*x] - (20*I)*b^3*d^4*x^3*Cos[5*a + 4*b*x] - 10*b^4*d^4*x^4*Cos[5*a + 4*b*x] + 20*b^5*c^4*x*Sin[a + 2*b*x] + 40*b^5*c^3*d*x^2*Sin[a + 2*b*x] + 40*b^5*c^2*d^2*x^3*Sin[a + 2*b*x] + 20*b^5*c*d^3*x^4*Sin[a + 2*b*x] + 4*b^5*d^4*x^5*Sin[a + 2*b*x] - 20*b^5*c^4*x*Sin[3*a + 2*b*x] - 40*b^5*c^3*d*x^2*Sin[3*a + 2*b*x] - 40*b^5*c^2*d^2*x^3*Sin[3*a + 2*b*x] - 20*b^5*c*d^3*x^4*Sin[3*a + 2*b*x] - 4*b^5*d^4*x^5*Sin[3*a + 2*b*x] - (10*I)*b^4*c^4*Sin[3*a + 4*b*x] + 20*b^3*c^3*d*Sin[3*a + 4*b*x] + (30*I)*b^2*c^2*d^2*Sin[3*a + 4*b*x] - 30*b*c*d^3*Sin[3*a + 4*b*x] - (15*I)*d^4*Sin[3*a + 4*b*x] - (40*I)*b^4*c^3*d*x*Sin[3*a + 4*b*x] + 60*b^3*c^2*d^2*x*Sin[3*a + 4*b*x] + (60*I)*b^2*c*d^3*x*Sin[3*a + 4*b*x] - 30*b*d^4*x*Sin[3*a + 4*b*x] - (60*I)*b^4*c^2*d^2*x^2*Sin[3*a + 4*b*x] + 60*b^3*c*d^3*x^2*Sin[3*a + 4*b*x] + (30*I)*b^2*d^4*x^2*Sin[3*a + 4*b*x] - (40*I)*b^4*c*d^3*x^3*Sin[3*a + 4*b*x] + 20*b^3*d^4*x^3*Sin[3*a + 4*b*x] - (10*I)*b^4*d^4*x^4*Sin[3*a + 4*b*x] - (10*I)*b^4*c^4*Sin[5*a + 4*b*x

```
] + 20*b^3*c^3*d*Sin[5*a + 4*b*x] + (30*I)*b^2*c^2*d^2*Sin[5*a + 4*b*x] - 3
0*b*c*d^3*Sin[5*a + 4*b*x] - (15*I)*d^4*Sin[5*a + 4*b*x] - (40*I)*b^4*c^3*d
*x*Sin[5*a + 4*b*x] + 60*b^3*c^2*d^2*x*Sin[5*a + 4*b*x] + (60*I)*b^2*c*d^3*
x*Sin[5*a + 4*b*x] - 30*b*d^4*x*Sin[5*a + 4*b*x] - (60*I)*b^4*c^2*d^2*x^2*S
in[5*a + 4*b*x] + 60*b^3*c*d^3*x^2*Sin[5*a + 4*b*x] + (30*I)*b^2*d^4*x^2*Si
n[5*a + 4*b*x] - (40*I)*b^4*c*d^3*x^3*Sin[5*a + 4*b*x] + 20*b^3*d^4*x^3*Sin
[5*a + 4*b*x] - (10*I)*b^4*d^4*x^4*Sin[5*a + 4*b*x])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 964 vs. $2(282) = 564$.
time = 0.14, size = 965, normalized size = 3.23

method	result
risch	$\frac{8c\,d^3a^3\ln(\mathrm{e}^{i(bx+a)})}{b^4}-\frac{8ic\,d^3a^3x}{b^3}+\frac{12id^2c^2a^2x}{b^2}-\frac{8ic^3dax}{b}-\frac{12c^2d^2a^2\ln(\mathrm{e}^{i(bx+a)})}{b^3}+\frac{8c^3da\ln(\mathrm{e}^{i(bx+a)})}{b^2}+\frac{2id^4a^4x}{b^4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

```
[Out] -6*I/b^2*c^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-6*I/b^2*c*d^3*polylog(2,-ex
p(2*I*(b*x+a)))*x^2-8*I/b^3*c*d^3*a^3*x+12*I/b^2*d^2*c^2*a^2*x-8*I/b*c^3*d*
a*x-2*I/b^2*c^3*d*polylog(2,-exp(2*I*(b*x+a)))+3*I/b^4*c*d^3*polylog(4,-exp
(2*I*(b*x+a)))-2*I/b^2*d^4*polylog(2,-exp(2*I*(b*x+a)))*x^3+3*I/b^4*d^4*pol
ylog(4,-exp(2*I*(b*x+a)))*x+4/b*c*d^3*ln(1+exp(2*I*(b*x+a)))*x^3-3/2*d^4*po
lylog(5,-exp(2*I*(b*x+a)))/b^5-12/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)))+8/b^2*
c^3*d*a*ln(exp(I*(b*x+a)))+2*I/b^4*d^4*a^4*x-4*I/b^2*c^3*d*a^2-6*I/b^4*c*d^
3*a^4+8*I/b^3*d^2*c^2*a^3-1/4*(2*d^4*x^4*b^4+4*I*b^3*d^4*x^3+8*b^4*c*d^3*x^
3+12*I*b^3*c*d^3*x^2+12*b^4*c^2*d^2*x^2+12*I*b^3*c^2*d^2*x+8*b^4*c^3*d*x+4*
I*b^3*c^3*d+2*b^4*c^4-6*b^2*d^4*x^2-6*I*b*d^4*x-12*b^2*c*d^3*x-6*I*b*c*d^3-
6*b^2*c^2*d^2+3*d^4)/b^5*exp(2*I*(b*x+a))-1/4*(2*d^4*x^4*b^4-4*I*b^3*d^4*x^
3+8*b^4*c*d^3*x^3-12*I*b^3*c*d^3*x^2+12*b^4*c^2*d^2*x^2-12*I*b^3*c^2*d^2*x+
8*b^4*c^3*d*x-4*I*b^3*c^3*d+2*b^4*c^4-6*b^2*d^4*x^2+6*I*b*d^4*x-12*b^2*c*d^
3*x+6*I*b*c*d^3-6*b^2*c^2*d^2+3*d^4)/b^5*exp(-2*I*(b*x+a))+I*c^4*x+1/5*I/d*
c^5-2/b*c^4*ln(exp(I*(b*x+a)))-1/5*I*d^4*x^5-2/b^5*d^4*a^4*ln(exp(I*(b*x+a)
))+1/b*c^4*ln(1+exp(2*I*(b*x+a)))+8/5*I/b^5*a^5*d^4-I*d^3*c*x^4-2*I*d^2*c^2
*x^3-2*I*d*c^3*x^2+3/b^3*c^2*d^2*polylog(3,-exp(2*I*(b*x+a)))+3/b^3*d^4*pol
ylog(3,-exp(2*I*(b*x+a)))*x^2+4/b*c^3*d*ln(1+exp(2*I*(b*x+a)))*x+6/b*c^2*d^
2*ln(1+exp(2*I*(b*x+a)))*x^2+1/b*d^4*ln(1+exp(2*I*(b*x+a)))*x^4+6/b^3*c*d^3
*polylog(3,-exp(2*I*(b*x+a)))*x+8/b^4*c*d^3*a^3*ln(exp(I*(b*x+a)))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(278) = 556$.
time = 0.42, size = 610, normalized size = 2.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -1/2*c^4*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + c
os(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/30*(-
6*I*b^5*d^4*x^5 - 30*I*b^5*c*d^3*x^4 - 60*I*b^5*c^2*d^2*x^3 - 60*I*b^5*c^3*
d*x^2 - 90*d^4*polylog(5, -e^(2*I*b*x + 2*I*a)) - 20*(-3*I*b^4*d^4*x^4 - 8*
I*b^4*c*d^3*x^3 - 9*I*b^4*c^2*d^2*x^2 - 6*I*b^4*c^3*d*x)*arctan2(sin(2*b*x
+ 2*a), cos(2*b*x + 2*a) + 1) - 15*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 - 6*b^2
*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2
*c*d^3)*x)*cos(2*b*x + 2*a) - 60*(2*I*b^3*d^4*x^3 + 4*I*b^3*c*d^3*x^2 + 3*I
*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(-e^(2*I*b*x + 2*I*a)) + 10*(3*b^4*d^4*x
^4 + 8*b^4*c*d^3*x^3 + 9*b^4*c^2*d^2*x^2 + 6*b^4*c^3*d*x)*log(cos(2*b*x + 2
*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 60*(-3*I*b*d^4*x - 2
*I*b*c*d^3)*polylog(4, -e^(2*I*b*x + 2*I*a)) + 30*(6*b^2*d^4*x^2 + 8*b^2*c*
d^3*x + 3*b^2*c^2*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) + 30*(2*b^3*d^4*x^3
 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)
*sin(2*b*x + 2*a))/b^5
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1652 vs. $2(278) = 556$.
time = 4.15, size = 1652, normalized size = 5.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 - 24*d^4*polylog(5, I*cos(b*x + a) + s
in(b*x + a)) - 24*d^4*polylog(5, I*cos(b*x + a) - sin(b*x + a)) - 24*d^4*po
lylog(5, -I*cos(b*x + a) + sin(b*x + a)) - 24*d^4*polylog(5, -I*cos(b*x + a
) - sin(b*x + a)) + 6*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - 2*(2*b^4*d^4*x^4 + 8*
b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^2*
d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4*(2*b^3*d^4*x
^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*
x)*cos(b*x + a)*sin(b*x + a) + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x - 4*(-I*b^3*
d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog(I*cos(
b*x + a) + sin(b*x + a)) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c
^2*d^2*x + I*b^3*c^3*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 4*(I*b^3*d^4
*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(-I*cos(b*
x + a) + sin(b*x + a)) - 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^
2*d^2*x - I*b^3*c^3*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a
) + I*sin(b*x + a) + I) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*
a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b^4*d^4*x^
4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6
*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(b*x + a) + sin(b*x +
```

```
a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*
x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(
b*x + a) - sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d
^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3
- a^4*d^4)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c
*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^
2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) +
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log
(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2
*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-cos(b*x + a) - I*sin(b*x + a) + I)
 - 24*(I*b*d^4*x + I*b*c*d^3)*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 2
4*(-I*b*d^4*x - I*b*c*d^3)*polylog(4, I*cos(b*x + a) - sin(b*x + a)) - 24*(
-I*b*d^4*x - I*b*c*d^3)*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) - 24*(I*
b*d^4*x + I*b*c*d^3)*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) + 12*(b^2*d
^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, I*cos(b*x + a) + sin(b*x +
 a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, I*cos(b*x
+ a) - sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polyl
og(3, -I*cos(b*x + a) + sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b
^2*c^2*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)))/b^5
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sec(b*x+a)*sin(3*b*x+3*a),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
[Out] integrate((d*x + c)^4*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)\,{\left(c+d\,x\right)}^4}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^4)/cos(a + b*x),x)
[Out] int((sin(3*a + 3*b*x)*(c + d*x)^4)/cos(a + b*x), x)
```

3.383 $\int (c+dx)^3 \sec(a+bx) \sin(3a+3bx) \, dx$

Optimal. Leaf size=242

$$\frac{3d^3x}{2b^3}-\frac{(c+dx)^3}{b}-\frac{i(c+dx)^4}{4d}+\frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{3d^2(c+dx}{}$$

```
[Out] 3/2*d^3*x/b^3-(d*x+c)^3/b-1/4*I*(d*x+c)^4/d+(d*x+c)^3*ln(1+exp(2*I*(b*x+a))
)/b-3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/b^2+3/2*d^2*(d*x+c)*poly
log(3,-exp(2*I*(b*x+a)))/b^3+3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4-3/2
*d^3*cos(b*x+a)*sin(b*x+a)/b^4+3*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^2-3*d^
2*(d*x+c)*sin(b*x+a)^2/b^3+2*(d*x+c)^3*sin(b*x+a)^2/b
```

Rubi [A]
time = 0.30, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {4516, 4489, 3392, 32, 2715, 8, 4492, 3800, 2221, 2611, 6744, 2320, 6724}

$$\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^3\sin(a+bx)\cos(a+bx)}{2b^4}+\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}-\frac{3d^2(c+dx)\sin^2(a+bx)}{b^3}-\frac{3id(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}+\frac{3d(c+dx)^2\sin(a+bx)\cos(a+bx)}{b^2}+\frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{2(c+dx)^3\sin^2(a+bx)}{b}+\frac{3d^3x}{2b^3}-\frac{(c+dx)^3}{b}-\frac{i(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (3*d^3*x)/(2*b^3) - (c + d*x)^3/b - ((I/4)*(c + d*x)^4)/d + ((c + d*x)^3*Lo
g[1 + E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*
I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/(2
*b^3) + (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 - (3*d^3*Cos[a
 + b*x]*Sin[a + b*x])/(2*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])
/b^2 - (3*d^2*(c + d*x)*Sin[a + b*x]^2)/b^3 + (2*(c + d*x)^3*Sin[a + b*x]^2
)/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
```

```
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4492

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \sec(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^3\cos(a+bx)\sin(a+bx) - (c+dx)^3\sin^2(a+bx)\,\mathrm{t}\right.$$

$$= 3\int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx - \int (c+dx)^3\sin^2(a+$$

$$= \frac{3(c+dx)^3\sin^2(a+bx)}{2b} - \frac{(9d)\int (c+dx)^2\sin^2(a+bx)\,dx}{2b} + \int ($$

$$= -\frac{i(c+dx)^4}{4d} + \frac{9d(c+dx)^2\cos(a+bx)\sin(a+bx)}{4b^2} - \frac{9d^2(c+d}{}$$

$$= -\frac{3(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{9d^3}{}$$

$$= \frac{9d^3x}{8b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} -$$

$$= \frac{3d^3x}{2b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} -$$

$$= \frac{3d^3x}{2b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} -$$

$$= \frac{3d^3x}{2b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} -$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1733 vs. $2(242) = 484$.
time = 6.41, size = 1733, normalized size = 7.16

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] -1/4*(c*d^2*((2*I)*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(1 + E^((2*I)*a))*Log
[1 + E^((2*I)*(a + b*x))]) + (6*I)*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((2*
I)*(a + b*x))] - 3*(1 + E^((2*I)*a))*PolyLog[3, -E^((2*I)*(a + b*x))])*Sec[
a])/(b^3*E^(I*a)) + (I/4)*d^3*E^(I*a)*(-x^4 + (1 + E^((-2*I)*a))*x^4 - ((1
+ E^((2*I)*a))*(2*b^4*x^4 + (4*I)*b^3*x^3*Log[1 + E^((2*I)*(a + b*x))] + 6*
b^2*x^2*PolyLog[2, -E^((2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[3, -E^((2*I)*(
a + b*x))] - 3*PolyLog[4, -E^((2*I)*(a + b*x))]))/(2*b^4*E^((2*I)*a)))*Sec[
a] + (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a
]))/(b*(Cos[a]^2 + Sin[a]^2)) + (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[
a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)]
 - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*
Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog
```

```
[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*
Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + Sec[a]*(Cos[2*a + 2*b*x]/(16*b^4) -
 ((I/16)*Sin[2*a + 2*b*x])/b^4)*(-8*b^3*c^3*Cos[a] + (12*I)*b^2*c^2*d*Cos[a
] + 12*b*c*d^2*Cos[a] - (6*I)*d^3*Cos[a] - 24*b^3*c^2*d*x*Cos[a] + (24*I)*b
^2*c*d^2*x*Cos[a] + 12*b*d^3*x*Cos[a] - 24*b^3*c*d^2*x^2*Cos[a] + (12*I)*b^
2*d^3*x^2*Cos[a] - 8*b^3*d^3*x^3*Cos[a] - (8*I)*b^4*c^3*x*Cos[a + 2*b*x] -
(12*I)*b^4*c^2*d*x^2*Cos[a + 2*b*x] - (8*I)*b^4*c*d^2*x^3*Cos[a + 2*b*x] -
(2*I)*b^4*d^3*x^4*Cos[a + 2*b*x] + (8*I)*b^4*c^3*x*Cos[3*a + 2*b*x] + (12*I
)*b^4*c^2*d*x^2*Cos[3*a + 2*b*x] + (8*I)*b^4*c*d^2*x^3*Cos[3*a + 2*b*x] + (
2*I)*b^4*d^3*x^4*Cos[3*a + 2*b*x] - 4*b^3*c^3*Cos[3*a + 4*b*x] - (6*I)*b^2*
c^2*d*Cos[3*a + 4*b*x] + 6*b*c*d^2*Cos[3*a + 4*b*x] + (3*I)*d^3*Cos[3*a + 4
*b*x] - 12*b^3*c^2*d*x*Cos[3*a + 4*b*x] - (12*I)*b^2*c*d^2*x*Cos[3*a + 4*b*
x] + 6*b*d^3*x*Cos[3*a + 4*b*x] - 12*b^3*c*d^2*x^2*Cos[3*a + 4*b*x] - (6*I)
*b^2*d^3*x^2*Cos[3*a + 4*b*x] - 4*b^3*d^3*x^3*Cos[3*a + 4*b*x] - 4*b^3*c^3*
Cos[5*a + 4*b*x] - (6*I)*b^2*c^2*d*Cos[5*a + 4*b*x] + 6*b*c*d^2*Cos[5*a + 4
*b*x] + (3*I)*d^3*Cos[5*a + 4*b*x] - 12*b^3*c^2*d*x*Cos[5*a + 4*b*x] - (12*
I)*b^2*c*d^2*x*Cos[5*a + 4*b*x] + 6*b*d^3*x*Cos[5*a + 4*b*x] - 12*b^3*c*d^2
*x^2*Cos[5*a + 4*b*x] - (6*I)*b^2*d^3*x^2*Cos[5*a + 4*b*x] - 4*b^3*d^3*x^3*
Cos[5*a + 4*b*x] + 8*b^4*c^3*x*Sin[a + 2*b*x] + 12*b^4*c^2*d*x^2*Sin[a + 2*
b*x] + 8*b^4*c*d^2*x^3*Sin[a + 2*b*x] + 2*b^4*d^3*x^4*Sin[a + 2*b*x] - 8*b^
4*c^3*x*Sin[3*a + 2*b*x] - 12*b^4*c^2*d*x^2*Sin[3*a + 2*b*x] - 8*b^4*c*d^2*
x^3*Sin[3*a + 2*b*x] - 2*b^4*d^3*x^4*Sin[3*a + 2*b*x] - (4*I)*b^3*c^3*Sin[3
*a + 4*b*x] + 6*b^2*c^2*d*Sin[3*a + 4*b*x] + (6*I)*b*c*d^2*Sin[3*a + 4*b*x]
 - 3*d^3*Sin[3*a + 4*b*x] - (12*I)*b^3*c^2*d*x*Sin[3*a + 4*b*x] + 12*b^2*c*
d^2*x*Sin[3*a + 4*b*x] + (6*I)*b*d^3*x*Sin[3*a + 4*b*x] - (12*I)*b^3*c*d^2*
x^2*Sin[3*a + 4*b*x] + 6*b^2*d^3*x^2*Sin[3*a + 4*b*x] - (4*I)*b^3*d^3*x^3*S
in[3*a + 4*b*x] - (4*I)*b^3*c^3*Sin[5*a + 4*b*x] + 6*b^2*c^2*d*Sin[5*a + 4*
b*x] + (6*I)*b*c*d^2*Sin[5*a + 4*b*x] - 3*d^3*Sin[5*a + 4*b*x] - (12*I)*b^3
*c^2*d*x*Sin[5*a + 4*b*x] + 12*b^2*c*d^2*x*Sin[5*a + 4*b*x] + (6*I)*b*d^3*x
*Sin[5*a + 4*b*x] - (12*I)*b^3*c*d^2*x^2*Sin[5*a + 4*b*x] + 6*b^2*d^3*x^2*S
in[5*a + 4*b*x] - (4*I)*b^3*d^3*x^3*Sin[5*a + 4*b*x])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 647 vs. $2(219) = 438$.
time = 0.11, size = 648, normalized size = 2.68

method	result
risch	$-\frac{i d^3 x^4}{4}-\frac{2 c^3 \ln\left(e^{i(bx+a)}\right)}{b}+i c^3 x+\frac{i c^4}{4d}+\frac{6 i c d^2 a^2 x}{b^2}-\frac{6 i c^2 d a x}{b}-\frac{\left(4 d^3 x^3 b^3+12 b^3 c d^2 x^2+6 i b^2 d^3 x^2+12 b^3 c^2 d x+12 i b^2 c d^2 x\right.}{8 b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*I/b^4*d^3*a^4-I*d^2*c*x^3-3/2*I*d*c^2*x^2+2/b^4*d^3*a^3*ln(exp(I*(b*x+
a)))-1/4*I*d^3*x^4-2/b*c^3*ln(exp(I*(b*x+a)))+I*c^3*x+1/4*I/d*c^4-3/2*I/b^2
```

```
*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2-3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b
*x+a)))+6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))+1/b*c^3*ln(1+exp(2*I*(b*x+a)))+3/2
/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))+3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x
+a)))*x-1/8*(4*d^3*x^3*b^3+6*I*b^2*d^3*x^2+12*b^3*c*d^2*x^2+12*I*b^2*c*d^2*
x+12*b^3*c^2*d*x+6*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x-3*I*d^3-6*c*d^2*b)/b^4*e
xp(2*I*(b*x+a))-1/8*(4*d^3*x^3*b^3-6*I*b^2*d^3*x^2+12*b^3*c*d^2*x^2-12*I*b^
2*c*d^2*x+12*b^3*c^2*d*x-6*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x+3*I*d^3-6*c*d^2*
b)/b^4*exp(-2*I*(b*x+a))-3*I/b^2*c*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-6/b^3
*c*d^2*a^2*ln(exp(I*(b*x+a)))+4*I/b^3*c*d^2*a^3-3*I/b^2*c^2*d*a^2-2*I/b^3*d
^3*a^3*x+3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/b*c^2*d*ln(1+exp(2*I*
(b*x+a)))*x+3/b*c*d^2*ln(1+exp(2*I*(b*x+a)))*x^2+1/b*d^3*ln(1+exp(2*I*(b*x+
a)))*x^3+6*I/b^2*c*d^2*a^2*x-6*I/b*c^2*d*a*x
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(215) = 430$.
time = 0.38, size = 444, normalized size = 1.83

$$-\frac{c^3\left(2\cos\left(2bx+2a\right)-\log\left(\cos\left(2bx\right)^2+2\cos\left(2bx\right)\cos\left(2a\right)+\cos\left(2a\right)^2+\sin\left(2bx\right)^2-2\sin\left(2bx\right)\sin\left(2a\right)+\sin\left(2a\right)^2\right)\right)}{2b}+\frac{-3ib^4d^3x^4-12ib^4cd^2x^3-18ib^4c^2dx^2+12id^3\mathrm{Li}_4\left(-e^{(2ibx+2ia)}\right)-4\left(-4ib^3d^3x^3-9ib^3cd^2x^2-9ib^3c^2dx\right)\arctan\left(\sin\left(2bx+2a\right),\cos\left(2bx+2a\right)+1\right)-6\left(2b^3d^3x^3+6b^3cd^2x^2-3bcd^2+3\left(2b^3c^2d-bd^3\right)x\right)\cos\left(2bx+2a\right)-6\left(4ib^2d^3x^2+6ib^2cd^2x+3ib^2c^2d\right)\mathrm{Li}_2\left(-e^{(2ibx+2ia)}\right)+2\left(4b^3d^3x^3+9b^3cd^2x^2+9b^3c^2dx\right)\log\left(\cos\left(2bx+2a\right)^2+\sin\left(2bx+2a\right)^2+2\cos\left(2bx+2a\right)+1\right)+6\left(4bd^3x+3bcd^2\right)\mathrm{Li}_3\left(-e^{(2ibx+2ia)}\right)+9\left(2b^2d^3x^2+4b^2cd^2x+2b^2c^2d-d^3\right)\sin\left(2bx+2a\right)}{12b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -1/2*c^3*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + c
os(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/12*(-
3*I*b^4*d^3*x^4 - 12*I*b^4*c*d^2*x^3 - 18*I*b^4*c^2*d*x^2 + 12*I*d^3*polylo
g(4, -e^(2*I*b*x + 2*I*a)) - 4*(-4*I*b^3*d^3*x^3 - 9*I*b^3*c*d^2*x^2 - 9*I*
b^3*c^2*d*x)*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(2*b^3*d^3
*x^3 + 6*b^3*c*d^2*x^2 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(2*b*x +
 2*a) - 6*(4*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-e^(2*I
*b*x + 2*I*a)) + 2*(4*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x)*log(co
s(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 6*(4*b*d^
3*x + 3*b*c*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) + 9*(2*b^2*d^3*x^2 + 4*b^
2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin(2*b*x + 2*a))/b^4
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1126 vs. $2(215) = 430$.
time = 4.66, size = 1126, normalized size = 4.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 6*I*d^3*polylog(4, I*cos(b*x + a) +
sin(b*x + a)) + 6*I*d^3*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 6*I*d^3
*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x
```

```
 + a) - sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*
b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b^2*d^3*x^2 + 4*
b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 3*(2*b^3*c^2*d
 - b*d^3)*x - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(I*co
s(b*x + a) + sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2
*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2
*x + I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2
 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin
(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(co
s(b*x + a) - I*sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c
^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(
b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^
3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
 + a^3*d^3)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*
c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I
*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^
2 - a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 6
*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x
 + b*c*d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*(b*d^3*x + b*c*d^
2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polyl
og(3, -I*cos(b*x + a) - sin(b*x + a)))/b^4
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin (3\,a+3\,b\,x)\,\left(c+d\,x\right)^3}{\cos (a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^3)/cos(a + b*x),x)
[Out] int((sin(3*a + 3*b*x)*(c + d*x)^3)/cos(a + b*x), x)
```

3.384 $\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=173

$$-\frac{2cdx}{b}-\frac{d^2x^2}{b}-\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}(3,-}{2b^3}$$

```
[Out] -2*c*d*x/b-d^2*x^2/b-1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b-I
*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/2*d^2*polylog(3,-exp(2*I*(b*x
+a)))/b^3+2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2-d^2*sin(b*x+a)^2/b^3+2*(d*x
+c)^2*sin(b*x+a)^2/b
```

Rubi [A]
time = 0.23, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {4516, 4489, 3391, 4492, 3800, 2221, 2611, 2320, 6724}

$$\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}-\frac{d^2\sin^2(a+bx)}{b^3}-\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}+\frac{2d(c+dx)\sin(a+bx)\cos(a+bx)}{b^2}+\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{2(c+dx)^2\sin^2(a+bx)}{b}-\frac{2cdx}{b}-\frac{d^2x^2}{b}-\frac{i(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (-2*c*d*x)/b - (d^2*x^2)/b - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[1 + E
^((2*I)*(a + b*x))])/b - (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b
^2 + (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (2*d*(c + d*x)*Cos[a
+ b*x]*Sin[a + b*x])/b^2 - (d^2*Sin[a + b*x]^2)/b^3 + (2*(c + d*x)^2*Sin[a
+ b*x]^2)/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

<u>Rule 3391</u>

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

<u>Rule 3800</u>

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

<u>Rule 4489</u>

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

<u>Rule 4492</u>

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

<u>Rule 4516</u>

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

<u>Rule 6724</u>

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^2\cos(a+bx)\sin(a+bx) - (c+dx)^2\sin^2(a+bx)\,\mathrm{ta}\right.$$

$$= 3\int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx - \int (c+dx)^2\sin^2(a+b$$

$$= \frac{3(c+dx)^2\sin^2(a+bx)}{2b} - \frac{(3d)\int (c+dx)\sin^2(a+bx)\,dx}{b} + \int (c$$

$$= -\frac{i(c+dx)^3}{3d} + \frac{3d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2} - \frac{3d^2\sin^2(a+}{4b^3}$$

$$= -\frac{3cdx}{2b} - \frac{3d^2x^2}{4b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2c}{}$$

$$= -\frac{2cdx}{b} - \frac{d^2x^2}{b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{id(}{}$$

$$= -\frac{2cdx}{b} - \frac{d^2x^2}{b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{id(}{}$$

$$= -\frac{2cdx}{b} - \frac{d^2x^2}{b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{id(}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 523 vs. 2(173) = 346.
time = 6.40, size = 523, normalized size = 3.02

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] -1/12*(d^2*((2*I)*b^2*x^2*(2*b*E^((2*I)*a)*x + (3*I)*(1 + E^((2*I)*a))*Log[
1 + E^((2*I)*(a + b*x))]) + (6*I)*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((2*I
)*(a + b*x))] - 3*(1 + E^((2*I)*a))*PolyLog[3, -E^((2*I)*(a + b*x))])*Sec[a
])/(b^3*E^(I*a)) + (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x
]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (c*d*Csc[a]*((b^2*x^2)/E^(I*A
rcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-
2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]
]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] +
 I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a
])/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (Cos[2*b*x]*(2*b^2*c^2*Cos[
```

```
2*a] - d^2*Cos[2*a] + 4*b^2*c*d*x*Cos[2*a] + 2*b^2*d^2*x^2*Cos[2*a] - 2*b*c
*d*Sin[2*a] - 2*b*d^2*x*Sin[2*a]))/(2*b^3) + ((2*b*c*d*Cos[2*a] + 2*b*d^2*x
*Cos[2*a] + 2*b^2*c^2*Sin[2*a] - d^2*Sin[2*a] + 4*b^2*c*d*x*Sin[2*a] + 2*b^
2*d^2*x^2*Sin[2*a])*Sin[2*b*x])/(2*b^3) - (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Ta
n[a])/3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(161) = 322$.
time = 0.15, size = 386, normalized size = 2.23

method	result
risch	$-\frac{2icd\,a^2}{b^2} - \frac{id^2x^3}{3} + \frac{ic^3}{3d} + \frac{2id^2a^2x}{b^2} - \frac{(2x^2d^2b^2+4b^2cdx+2ib\,d^2x+2b^2c^2+2ibcd-d^2)\,\mathrm{e}^{2i(bx+a)}}{4b^3} - \frac{(2x^2d^2b^2+4b^2cdx-2ib\,d^2x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/b^2*c*d*a^2-1/3*I*d^2*x^3-I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))+1/3*I
/d*c^3-1/4*(2*x^2*d^2*b^2+2*I*b*d^2*x+4*b^2*c*d*x+2*I*b*c*d+2*b^2*c^2-d^2)/
b^3*exp(2*I*(b*x+a))-1/4*(2*x^2*d^2*b^2-2*I*b*d^2*x+4*b^2*c*d*x-2*I*b*c*d+2
*b^2*c^2-d^2)/b^3*exp(-2*I*(b*x+a))+1/b*c^2*ln(1+exp(2*I*(b*x+a)))-2/b*c^2*
ln(exp(I*(b*x+a)))-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))+2*I/b^2*d^2*a^2*x-I/b^2
*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-I*d*c*x^2-4*I/b*c*d*a*x+1/b*d^2*ln(1+ex
p(2*I*(b*x+a)))*x^2+4/3*I/b^3*d^2*a^3+1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/
b^3+4/b^2*c*d*a*ln(exp(I*(b*x+a)))+I*c^2*x+2/b*c*d*ln(1+exp(2*I*(b*x+a)))*x
```

Maxima [A]
time = 0.36, size = 303, normalized size = 1.75

$$-\frac{c^2\left(2\cos\left(2bx+2a\right)-\log\left(\cos\left(2bx\right)^2+2\cos\left(2bx\right)\cos\left(2a\right)+\cos\left(2a\right)^2+\sin\left(2bx\right)^2-2\sin\left(2bx\right)\sin\left(2a\right)+\sin\left(2a\right)^2\right)\right)}{2b}+\frac{-2ib^3d^2x^3-6ib^3cdx^2+3d^2\mathrm{Li}_3(-e^{(2ibx+2ia)})-6\left(-ib^2d^2x^2-2ib^2cdx\right)\arctan\left(\sin\left(2bx+2a\right),\cos\left(2bx+2a\right)+1\right)-3\left(2b^2d^2x^2+4b^2cdx-d^2\right)\cos\left(2bx+2a\right)-6\left(ibd^2x+ibcd\right)\mathrm{Li}_2(-e^{(2ibx+2ia)})+3\left(b^2d^2x^2+2b^2cdx\right)\log\left(\cos\left(2bx+2a\right)^2+\sin\left(2bx+2a\right)^2+2\cos\left(2bx+2a\right)+1\right)+6\left(bd^2x+bcd\right)\sin\left(2bx+2a\right)}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -1/2*c^2*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + c
os(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/6*(-2
*I*b^3*d^2*x^3 - 6*I*b^3*c*d*x^2 + 3*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) -
 6*(-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x)*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2
*a) + 1) - 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x - d^2)*cos(2*b*x + 2*a) - 6*(I*b*
d^2*x + I*b*c*d)*dilog(-e^(2*I*b*x + 2*I*a)) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x
)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 6
*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a))/b^3
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 681 vs. $2(158) = 316$.
time = 4.48, size = 681, normalized size = 3.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x - 2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c
^2 - d^2)*cos(b*x + a)^2 + 2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a))
+ 2*d^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos
(b*x + a) + sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)
) + 4*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) - 2*(-I*b*d^2*x - I*b*c*d
)*dilog(I*cos(b*x + a) + sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(I*co
s(b*x + a) - sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(-I*cos(b*x + a)
+ sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x + a) - sin(b*x
+ a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*x + a) +
 I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*sin(b*x + a) + I
) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) +
sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I
*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d
- a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c
*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/b^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)\,{\left(c+d\,x\right)}^2}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^2)/cos(a + b*x),x)
[Out] int((sin(3*a + 3*b*x)*(c + d*x)^2)/cos(a + b*x), x)
```

3.385 $\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=107

$$-\frac{dx}{b}-\frac{i(c+dx)^2}{2d}+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{d\cos(a+bx)\sin(a+bx)}{b^2}+\frac{2(c+d$$

```
[Out] -d*x/b-1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b-1/2*I*d*polylog(2
,-exp(2*I*(b*x+a)))/b^2+d*cos(b*x+a)*sin(b*x+a)/b^2+2*(d*x+c)*sin(b*x+a)^2/
b
```

Rubi [A]
time = 0.13, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4516, 4489, 2715, 8, 4492, 3800, 2221, 2317, 2438}

$$-\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}+\frac{d\sin(a+bx)\cos(a+bx)}{b^2}+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{2(c+dx)\sin^2(a+bx)}{b}-\frac{dx}{b}-\frac{i(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] -((d*x)/b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 + E^((2*I)*(a + b*x))
])/b - ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + (d*Cos[a + b*x]*Sin
[a + b*x])/b^2 + (2*(c + d*x)*Sin[a + b*x]^2)/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4492

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)\cos(a+bx)\sin(a+bx) - (c+dx)\sin^2(a+bx)\tan(a\right.$$

$$= 3\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx - \int (c+dx)\sin^2(a+bx)\,\mathrm{t}$$

$$= \frac{3(c+dx)\sin^2(a+bx)}{2b} - \frac{(3d)\int \sin^2(a+bx)\,dx}{2b} + \int (c+dx)\cos($$

$$= -\frac{i(c+dx)^2}{2d} + \frac{3d\cos(a+bx)\sin(a+bx)}{4b^2} + \frac{2(c+dx)\sin^2(a+bx}{b}$$

$$= -\frac{3dx}{4b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{d\cos(a+bx)\,\mathrm{s}}{b^2}$$

$$= -\frac{dx}{b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{d\cos(a+bx)\,\mathrm{si}}{b^2}$$

$$= -\frac{dx}{b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{id\mathrm{Li}_2\left(-e^{2i(a+bx}\right.}{2b^2}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 254 vs. $2(107) = 214$.
time = 6.13, size = 254, normalized size = 2.37

$$\frac{d\csc(a)\left(b^2e^{-i\mathrm{ArcTan}(\cot(a))}x^2 - \frac{\cot(a)\left(ibx(-\pi-2\mathrm{ArcTan}(\cot(a)))-\pi\log(1+e^{-2ibx})-2(bx-\mathrm{ArcTan}(\cot(a)))\log\left(1-e^{2i(bx-\mathrm{ArcTan}(\cot(a)))}\right)+\pi\log(\cos(bx))-2\mathrm{ArcTan}(\cot(a))\log(\sin(bx-\mathrm{ArcTan}(\cot(a))))+i\mathrm{PolyLog}\left(2,e^{2i(bx-\mathrm{ArcTan}(\cot(a)))}\right)\right)}{\sqrt{1+\cot^2(a)}}\right)\sec(a)}{2b^2\sqrt{\csc^2(a)\left(\cos^2(a)+\sin^2(a)\right)}} - \frac{d\cos(2bx)(2bx\cos(2a)-\sin(2a))}{2b^2} + \frac{d(\cos(2a)+2bx\sin(2a))\sin(2bx)}{2b^2} + \frac{c(\log(\cos(a+bx))+2\sin^2(a+bx))}{b} - \frac{1}{2}dx^2\tan(a)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[
Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^
((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[S
in[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))])
)/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)])
- (d*Cos[2*b*x]*(2*b*x*Cos[2*a] - Sin[2*a]))/(2*b^2) + (d*(Cos[2*a] + 2*b*x
*Sin[2*a])*Sin[2*b*x])/(2*b^2) + (c*(Log[Cos[a + b*x]] + 2*Sin[a + b*x]^2))
/b - (d*x^2*Tan[a])/2
```

Maple [A]
time = 0.32, size = 177, normalized size = 1.65

method	result
risch	$-\frac{idx^2}{2} + icx - \frac{(2dxb+2cb+id)\mathrm{e}^{2i(bx+a)}}{4b^2} - \frac{(2dxb+2cb-id)\mathrm{e}^{-2i(bx+a)}}{4b^2} + \frac{c\ln\left(1+\mathrm{e}^{2i(bx+a)}\right)}{b} - \frac{2c\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b} - \frac{2idax}{b} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*d*x^2+I*c*x-1/4*(2*d*x*b+I*d+2*c*b)/b^2*exp(2*I*(b*x+a))-1/4*(2*d*x*
b-I*d+2*c*b)/b^2*exp(-2*I*(b*x+a))+1/b*c*ln(1+exp(2*I*(b*x+a)))-2/b*c*ln(ex
p(I*(b*x+a)))-2*I/b*d*a*x-I/b^2*d*a^2+1/b*d*ln(1+exp(2*I*(b*x+a)))*x-1/2*I*
d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b^2*d*a*ln(exp(I*(b*x+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -1/2*c*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos
(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b - 1/2*(2*b*
x*cos(2*b*x + 2*a) + 4*b^2*integrate(x*sin(2*b*x + 2*a)/(cos(2*b*x + 2*a)^2
 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1), x) - sin(2*b*x + 2*a))*d/b
^2
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(94) = 188.
time = 3.44, size = 340, normalized size = 3.18

$$\frac{2\,bdx - 4\,(bdx + bc)\cos\left(bx + a\right)^{2} + 2\,d\cos\left(bx + a\right)\sin\left(bx + a\right) + i\,d\mathrm{Li}_2(i\,\cos\left(bx + a\right) + \sin\left(bx + a\right)) - i\,d\mathrm{Li}_2(i\,\cos\left(bx + a\right) - \sin\left(bx + a\right)) - i\,d\mathrm{Li}_2(-i\,\cos\left(bx + a\right) + \sin\left(bx + a\right)) + i\,d\mathrm{Li}_2(-i\,\cos\left(bx + a\right) - \sin\left(bx + a\right)) + (bc - ad)\log\left(\cos\left(bx + a\right) + i\,\sin\left(bx + a\right) + i\right) + (bc - ad)\log\left(\cos\left(bx + a\right) - i\,\sin\left(bx + a\right) + i\right) + (bdx + ad)\log\left(i\,\cos\left(bx + a\right) + \sin\left(bx + a\right) + 1\right) + (bdx + ad)\log\left(i\,\cos\left(bx + a\right) - \sin\left(bx + a\right) + 1\right) + (bdx + ad)\log\left(-i\,\cos\left(bx + a\right) + \sin\left(bx + a\right) + 1\right) + (bdx + ad)\log\left(-i\,\cos\left(bx + a\right) - \sin\left(bx + a\right) + 1\right) + (bc - ad)\log\left(-\cos\left(bx + a\right) + i\,\sin\left(bx + a\right) + i\right) + (bc - ad)\log\left(-\cos\left(bx + a\right) - i\,\sin\left(bx + a\right) + i\right)}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x - 4*(b*d*x + b*c)*cos(b*x + a)^2 + 2*d*cos(b*x + a)*sin(b*x +
a) + I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) - I*d*dilog(I*cos(b*x + a) -
sin(b*x + a)) - I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(-I*co
s(b*x + a) - sin(b*x + a)) + (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a)
+ I) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b*d*x + a*d)*l
og(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a*d)*log(I*cos(b*x + a) -
sin(b*x + a) + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) +
 (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*log(-c
os(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(-cos(b*x + a) - I*sin(b
*x + a) + I))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)\sin\left(3a + 3bx\right)\sec\left(a + bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Integral((c + d*x)*sin(3*a + 3*b*x)*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)\,\left(c+d\,x\right)}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x))/cos(a + b*x),x)
```

```
[Out] int((sin(3*a + 3*b*x)*(c + d*x))/cos(a + b*x), x)
```

3.386 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

Optimal. Leaf size=80

$$\frac{2\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d}+\frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}-\text{Int}\left(\frac{\tan(a+bx)}{c+dx},x\right)$$

```
[Out] 2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d+2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)
/d-Unintegrable(tan(b*x+a)/(d*x+c),x)
```

Rubi [A]

time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] (2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/d + (2*Cos[2*a - (2
*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d - Defer[Int][Tan[a + b*x]/(c + d
*x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx &= \int \left(\frac{3\cos(a+bx)\sin(a+bx)}{c+dx}-\frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\right)dx\\
&= 3\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx-\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx\\
&= 3\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx+\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx-\int \frac{\tan(a+bx)}{c+dx}\,d\\
&= \frac{3}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx+\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx-\int \frac{\tan(a+bx)}{c+dx}\,dx\\
&= \frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx+\frac{1}{2}\left(3\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx+\\
&= \frac{3\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d}+\frac{3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{1}{2}\cos\left(\right.\\
&= \frac{2\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d}+\frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}-\int \frac{\tan(}{c}
\end{aligned}$$

Mathematica [A]
time = 3.25, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x), x]
```

Maple [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(3bx+3a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -((-I*exp_integral_e(1, 2*(-I*b*d*x - I*b*c)/d) + I*exp_integral_e(1, -2*(-
I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*d*integrate(sin(2*b*x + 2*a)
/((d*x + c)*cos(2*b*x + 2*a)^2 + (d*x + c)*sin(2*b*x + 2*a)^2 + d*x + 2*(d*
x + c)*cos(2*b*x + 2*a) + c), x) + (exp_integral_e(1, 2*(-I*b*d*x - I*b*c)/
d) + exp_integral_e(1, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c - a*d)/d))/d
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
[Out] integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)}{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)),x)
[Out] int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)), x)
```

3.387 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=103

$$\frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{2\sin(2a+2bx)}{d(c+dx)}-\frac{4b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\text{Int}\left(\frac{\tan(a+bx)}{(c+dx)^2}$$

```
[Out] 4*b*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^2-4*b*Si(2*b*c/d+2*b*x)*sin(2*a-2*
b*c/d)/d^2-2*sin(2*b*x+2*a)/d/(d*x+c)-Unintegrable(tan(b*x+a)/(d*x+c)^2,x)
```

Rubi [A]
time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] (4*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2 - (2*Sin[2*a
+ 2*b*x])/(d*(c + d*x)) - (4*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d +
 2*b*x])/d^2 - Defer[Int][Tan[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \left(\frac{3\cos(a+bx)\sin(a+bx)}{(c+dx)^2} - \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\right)dx$$

$$= 3\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx - \int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

$$= 3\int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx + \int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx - \int \frac{\tan(a+bx)}{(c+dx)^2}$$

$$= \frac{3}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx + \int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx - \int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$$

$$= -\frac{3\sin(2a+2bx)}{2d(c+dx)} + \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx + \frac{(3b)\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d} - \int$$

$$= -\frac{2\sin(2a+2bx)}{d(c+dx)} + \frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d} + \frac{\left(3b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}}{d}$$

$$= \frac{3b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2} - \frac{2\sin(2a+2bx)}{d(c+dx)} - \frac{3b\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}(}{d^2}$$

$$= \frac{4b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2} - \frac{2\sin(2a+2bx)}{d(c+dx)} - \frac{4b\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}(}{d^2}$$

Mathematica [A]
time = 4.42, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
[Out] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2, x]
```

Maple [A]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(3bx+3a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)
[Out] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((-I*exp_integral_e(2, 2*(-I*b*d*x - I*b*c)/d) + I*exp_integral_e(2, -2*(-
I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*(d^2*x + c*d)*integrate(sin(
2*b*x + 2*a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x + 2*a
)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(2*b*x + 2*a)^2 + c^2 + 2*(d^2*x^2 + 2*c
*d*x + c^2)*cos(2*b*x + 2*a)), x) + (exp_integral_e(2, 2*(-I*b*d*x - I*b*c)
/d) + exp_integral_e(2, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c - a*d)/d))/(d
^2*x + c*d)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (3\,a+3\,b\,x)}{\cos (a+b\,x)\,\,{\left (c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^2), x)
```

3.388 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=129

$$-\frac{2b\cos(2a+2bx)}{d^2(c+dx)}-\frac{4b^2\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^3}-\frac{\sin(2a+2bx)}{d(c+dx)^2}-\frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}$$

```
[Out] -2*b*cos(2*b*x+2*a)/d^2/(d*x+c)-4*b^2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^
3-4*b^2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^3-sin(2*b*x+2*a)/d/(d*x+c)^2-U
nintegrable(tan(b*x+a)/(d*x+c)^3,x)
```

Rubi [A]
time = 0.27, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] (-2*b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)) - (4*b^2*CosIntegral[(2*b*c)/d + 2*
b*x]*Sin[2*a - (2*b*c)/d])/d^3 - Sin[2*a + 2*b*x]/(d*(c + d*x)^2) - (4*b^2*
Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^3 - Defer[Int][Tan[a
 + b*x]/(c + d*x)^3, x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \left(\frac{3\cos(a+bx)\sin(a+bx)}{(c+dx)^3} - \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^3}\right) dx$$

$$= 3\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx - \int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^3}\,dx$$

$$= 3\int \frac{\sin(2a+2bx)}{2(c+dx)^3}\,dx + \int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx - \int \frac{\tan(a+bx)}{(c+dx)^3}\,c$$

$$= \frac{3}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx + \int \frac{\sin(2a+2bx)}{2(c+dx)^3}\,dx - \int \frac{\tan(a+bx)}{(c+dx)^3}\,dx$$

$$= -\frac{3\sin(2a+2bx)}{4d(c+dx)^2} + \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx + \frac{(3b)\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{2d} - \int \frac{\text{ta}}{(}$$

$$= -\frac{3b\cos(2a+2bx)}{2d^2(c+dx)} - \frac{\sin(2a+2bx)}{d(c+dx)^2} - \frac{(3b^2)\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d^2} + \frac{b\int \frac{\cos(2a+2}{(c+dx)}}{2d}$$

$$= -\frac{2b\cos(2a+2bx)}{d^2(c+dx)} - \frac{\sin(2a+2bx)}{d(c+dx)^2} - \frac{b^2\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d^2} - \frac{\left(3b^2\cos\left(2a-\right.\right.}{}$$

$$= -\frac{2b\cos(2a+2bx)}{d^2(c+dx)} - \frac{3b^2\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^3} - \frac{\sin(2a+2bx)}{d(c+dx)^2} -$$

$$= -\frac{2b\cos(2a+2bx)}{d^2(c+dx)} - \frac{4b^2\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^3} - \frac{\sin(2a+2bx)}{d(c+dx)^2} -$$

Mathematica [A]
time = 6.13, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
[Out] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3, x]
```

Maple [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(3bx+3a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

```
[Out] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -((-I*exp_integral_e(3, 2*(-I*b*d*x - I*b*c)/d) + I*exp_integral_e(3, -2*(-
I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*(d^3*x^2 + 2*c*d^2*x + c^2*d
)*integrate(sin(2*b*x + 2*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3 + (d^
3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos(2*b*x + 2*a)^2 + (d^3*x^3 + 3*c*
d^2*x^2 + 3*c^2*d*x + c^3)*sin(2*b*x + 2*a)^2 + 2*(d^3*x^3 + 3*c*d^2*x^2 +
3*c^2*d*x + c^3)*cos(2*b*x + 2*a)), x) + (exp_integral_e(3, 2*(-I*b*d*x - I
*b*c)/d) + exp_integral_e(3, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c - a*d)/d
))/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
 c^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)}{\cos\left(a+b\,x\right)\,{\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^3),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^3), x)
```

3.389 $\int (c+dx)^3 \sec^2(a+bx) \sin(3a+3bx)\,dx$

Optimal. Leaf size=230

$$-\frac{6id(c+dx)^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}+\frac{24d^2(c+dx)\cos(a+bx)}{b^3}-\frac{4(c+dx)^3\cos(a+bx)}{b}+\frac{6id^2(c+dx)\text{PolyLog}(2$$

```
[Out] -6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2+24*d^2*(d*x+c)*cos(b*x+a)/b^3-4
*(d*x+c)^3*cos(b*x+a)/b+6*I*d^2*(d*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^3-6*
I*d^2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^3-6*d^3*polylog(3,-I*exp(I*(b*x
+a)))/b^4+6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4-(d*x+c)^3*sec(b*x+a)/b-24*d
^3*sin(b*x+a)/b^4+12*d*(d*x+c)^2*sin(b*x+a)/b^2
```

Rubi [A]
time = 0.23, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4516, 3377, 2717, 4492, 4494, 4266, 2611, 2320, 6724}

$$-\frac{6id(c+dx)^2\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}-\frac{6d^3\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}-\frac{24d^3\sin(a+bx)}{b^4}+\frac{6id^2(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}+\frac{24d^2(c+dx)\cos(a+bx)}{b^3}+\frac{12d(c+dx)^2\sin(a+bx)}{b^2}-\frac{4(c+dx)^3\cos(a+bx)}{b}-\frac{(c+dx)^3\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] ((-6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 + (24*d^2*(c + d*x)*Cos[
a + b*x])/b^3 - (4*(c + d*x)^3*Cos[a + b*x])/b + ((6*I)*d^2*(c + d*x)*PolyL
og[2, (-I)*E^(I*(a + b*x))])/b^3 - ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(I*(
a + b*x))])/b^3 - (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 + (6*d^3*Pol
yLog[3, I*E^(I*(a + b*x))])/b^4 - ((c + d*x)^3*Sec[a + b*x])/b - (24*d^3*Si
n[a + b*x])/b^4 + (12*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4492

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \sec^2(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^3\sin(a+bx) - (c+dx)^3\sin(a+bx)\tan^2(a+bx)\right)$$

$$= 3\int (c+dx)^3\sin(a+bx)\,dx - \int (c+dx)^3\sin(a+bx)\tan^2(a+$$

$$= -\frac{3(c+dx)^3\cos(a+bx)}{b} + \frac{(9d)\int (c+dx)^2\cos(a+bx)\,dx}{b} + \int$$

$$= -\frac{4(c+dx)^3\cos(a+bx)}{b} - \frac{(c+dx)^3\sec(a+bx)}{b} + \frac{9d(c+dx)^2}{b^2}$$

$$= -\frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{18d^2(c+dx)\cos(a+bx)}{b^3} - 4($$

$$= -\frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^2(c+dx)\cos(a+bx)}{b^3} - 4($$

$$= -\frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^2(c+dx)\cos(a+bx)}{b^3} - 4($$

$$= -\frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^2(c+dx)\cos(a+bx)}{b^3} - 4($$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 532 vs. $2(230) = 460$.
time = 2.29, size = 532, normalized size = 2.31

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] -((Sec[a + b*x]*(3*b^3*c^3 - 12*b*c*d^2 + 9*b^3*c^2*d*x - 12*b*d^3*x + 9*b^
3*c*d^2*x^2 + 3*b^3*d^3*x^3 + (6*I)*b^2*c^2*d*ArcTan[E^(I*(a + b*x))]*Cos[a
 + b*x] + 2*b^3*c^3*Cos[2*(a + b*x)] - 12*b*c*d^2*Cos[2*(a + b*x)] + 6*b^3*
c^2*d*x*Cos[2*(a + b*x)] - 12*b*d^3*x*Cos[2*(a + b*x)] + 6*b^3*c*d^2*x^2*Co
s[2*(a + b*x)] + 2*b^3*d^3*x^3*Cos[2*(a + b*x)] - 6*b^2*c*d^2*x*Cos[a + b*x
]*Log[1 - I*E^(I*(a + b*x))] - 3*b^2*d^3*x^2*Cos[a + b*x]*Log[1 - I*E^(I*(a
 + b*x))] + 6*b^2*c*d^2*x*Cos[a + b*x]*Log[1 + I*E^(I*(a + b*x))] + 3*b^2*d
^3*x^2*Cos[a + b*x]*Log[1 + I*E^(I*(a + b*x))] - (6*I)*b*d^2*(c + d*x)*Cos[
a + b*x]*PolyLog[2, (-I)*E^(I*(a + b*x))] + (6*I)*b*d^2*(c + d*x)*Cos[a + b
*x]*PolyLog[2, I*E^(I*(a + b*x))] + 6*d^3*Cos[a + b*x]*PolyLog[3, (-I)*E^(I
*(a + b*x))] - 6*d^3*Cos[a + b*x]*PolyLog[3, I*E^(I*(a + b*x))] - 6*b^2*c^2
*d*Sin[2*(a + b*x)] + 12*d^3*Sin[2*(a + b*x)] - 12*b^2*c*d^2*x*Sin[2*(a + b
*x)] - 6*b^2*d^3*x^2*Sin[2*(a + b*x)]))/b^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(213) = 426$.
time = 0.16, size = 677, normalized size = 2.94

method	result
risch	$-\frac{2\left(d^{3} x^{3} b^{3}+3 b^{3} c\, d^{2} x^{2}+3 i b^{2} d^{3} x^{2}+3 b^{3} c^{2} d x+6 i b^{2} c\, d^{2} x+b^{3} c^{3}+3 i b^{2} c^{2} d-6 b\, d^{3} x-6 c\, d^{2} b-6 i d^{3}\right) {\mathrm e}^{i \left(b x+a\right)}}{b^{4}}-\frac{2\left(d^{3} x^{3} b^{3}+3 b^{3} c\, d^{2} x^{2}-3 i b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*d
^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))-2*
(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^3*
x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))-2*ex
p(I*(b*x+a))*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(1+exp(2*I*(b*x+a)))-6*I
/b^3*d^3*polylog(2,I*exp(I*(b*x+a)))*x+6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^
4+6/b^2*d^2*c*ln(1-I*exp(I*(b*x+a)))*x+6*I/b^3*d^3*polylog(2,-I*exp(I*(b*x+
a)))*x-3/b^2*d^3*ln(1+I*exp(I*(b*x+a)))*x^2+6/b^3*d^2*c*ln(1-I*exp(I*(b*x+a
)))*a+3/b^2*d^3*ln(1-I*exp(I*(b*x+a)))*x^2-6/b^3*d^2*c*ln(1+I*exp(I*(b*x+a)
))*a-6*I/b^2*d*c^2*arctan(exp(I*(b*x+a)))-6*I/b^4*d^3*a^2*arctan(exp(I*(b*x
+a)))+6*I/b^3*d^2*c*polylog(2,-I*exp(I*(b*x+a)))-6*I/b^3*d^2*c*polylog(2,I*
exp(I*(b*x+a)))+3/b^4*d^3*a^2*ln(1+I*exp(I*(b*x+a)))-6*d^3*polylog(3,-I*exp
(I*(b*x+a)))/b^4+12*I/b^3*d^2*c*a*arctan(exp(I*(b*x+a)))-3/b^4*d^3*a^2*ln(1
-I*exp(I*(b*x+a)))-6/b^2*d^2*c*ln(1+I*exp(I*(b*x+a)))*x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -2*((cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) + (3*cos(2*b*x + 2*a
) + 1)*cos(3*b*x + 3*a) + 3*cos(2*b*x + 2*a)*cos(b*x + a) + (sin(3*b*x + 3*
a) + sin(b*x + a))*sin(4*b*x + 4*a) + 3*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) +
 3*sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))*c^3/(b*cos(3*b*x + 3*a)^2
+ 2*b*cos(3*b*x + 3*a)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x + 3*a)
^2 + 2*b*sin(3*b*x + 3*a)*sin(b*x + a) + b*sin(b*x + a)^2) - 3/2*(4*(cos(a)
^2 + sin(a)^2)*b*x*cos(b*x + a) + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) +
 b*x*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(
b*x + 2*a)*sin(a))*cos(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))
*cos(2*b*x + 3*a)^2 + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b*x*cos(b*x
 + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*x + 2*a)*s
```

in(a))*sin(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 3*a)^2 + 4*((b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*cos(3*b*x + 3*a)^2 + (b*x*cos(a) + sin(a))*cos(b*x + a)^2 + (b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*sin(3*b*x + 3*a)^2 + (b*x*cos(a) + sin(a))*sin(b*x + a)^2 + 2*(b*x*cos(2*b*x + 3*a)*cos(b*x + a) + (b*x*cos(a) + sin(a))*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(b*x*cos(2*b*x + 3*a)*sin(b*x + a) + (b*x*cos(a) + sin(a))*sin(b*x + a) + sin(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a) + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 3*a))*cos(3*b*x + 4*a) + 4*(6*b*x*cos(b*x + 2*a)*cos(b*x + a)*cos(a) + 6*b*x*cos(b*x + a)*sin(b*x + 2*a)*sin(a) + b*x*cos(2*b*x + 3*a)^2 + b*x*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*b*x + 2*(3*b*x*cos(b*x + 2*a)*cos(b*x + a) + b*x*cos(a))*cos(2*b*x + 3*a) + 2*(3*b*x*cos(b*x + a)*sin(b*x + 2*a) + b*x*sin(a))*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + 4*(2*b*x*cos(b*x + a)*cos(a) + 3*(b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(b*x + 2*a) - 2*cos(a)*sin(b*x + a))*cos(2*b*x + 3*a) + 12*(b*x*cos(b*x + a)^2*cos(a) + b*x*cos(a)*sin(b*x + a)^2)*cos(b*x + 2*a) - ((cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*cos(b*x + a)^2 + (cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*sin(b*x + a)^2 + 2*(cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a) + cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2*cos(a) + cos(a)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(cos(2*b*x + 3*a)^2*sin(b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + sin(2*b*x + 3*a)^2*sin(b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (cos(a)^2 + sin(a)^2)*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(cos(b*x + a)^2*sin(a) + sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + ((cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*cos(b*x + a)^2 + (cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*sin(b*x + a)^2 + 2*(cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a) + cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2*cos(a) + cos(a)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(cos(2*b*x + 3*a)^2*sin(b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + sin(2*b*x + 3*a)^2*sin(b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (cos(a)^2 + sin(a)^2)*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(cos(b*x + a)^2*sin(a) + sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a))*log(cos(b*x

```
+ a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + 4*((b*x*sin(2*b*x + 3*a) +
b*x*sin(a) - cos(2*b*x + 3*a) - cos(a))*cos(3*b*x + 3*a)^2 + (b*x*sin(a) -
cos(a))*cos(b*x + a)^2 + (b*x*sin(2*b*x + 3*a) + b*x*sin(a) - cos(2*b*x + 3
*a) - cos(a))*sin(3*b*x + 3*a)^2 + (b*x*sin(a) - cos(a))*sin(b*x + a)^2 + 2
*(b*x*cos(b*x + a)*sin(2*b*x + 3*a) + (b*x*sin(a) - cos(a))*cos(b*x + a) -
cos(2*b*x + 3*a)*cos(b*x + a))*cos(3*b*x + 3*a) - (cos(b*x + a)^2 + sin(b*x
 + a)^2)*cos(2*b*x + 3*a) + 2*(b*x*sin(2*b*x + 3*a)*sin(b*x + a) + (b*x*sin
(a) - cos(a))*sin(b*x + a) - cos(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a
) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*sin(2*b*x + 3*a))*sin(3*b*x +
 4*a) + 4*(6*b*x*cos(b*x + 2*a)*cos(a)*sin(b*x + a) + 6*b*x*sin(b*x + 2*a)*
sin(b*x + a)*sin(a) + 2*(3*b*x*cos(b*x + 2*a)*s...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(204) = 408$.
time = 5.24, size = 896, normalized size = 3.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*c
os(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*
polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, -
I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x +
 a) - sin(b*x + a)) + 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^
2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 6*(I*b*d^3*x + I*b*c*d^2)*c
os(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) + 6*(I*b*d^3*x + I*b*c*d^2
)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) + 6*(-I*b*d^3*x - I*b*c
*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 6*(-I*b*d^3*x -
I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) +
 I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) -
 I*sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d
^3)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 +
2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - si
n(b*x + a) + 1) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*c
os(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^
2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b
*x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(
b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(
b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 24*(b^2*d^3*x^2 + 2*b^2*
c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)*sin(b*x + a))/(b^4*cos(b*x + a))
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)**2*sin(3*b*x+3*a),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
[Out] integrate((d*x + c)^3*sec(b*x + a)^2*sin(3*b*x + 3*a), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^3)/cos(a + b*x)^2,x)
[Out] \text{Hanged}
```

3.390 $\int (c+dx)^2 \sec^2(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=147

$$-\frac{4id(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}+\frac{8d^2\cos(a+bx)}{b^3}-\frac{4(c+dx)^2\cos(a+bx)}{b}+\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}-\frac{2id}{}$$

[Out] -4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2+8*d^2*cos(b*x+a)/b^3-4*(d*x+c)^2*cos(b*x+a)/b+2*I*d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3-2*I*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3-(d*x+c)^2*sec(b*x+a)/b+8*d*(d*x+c)*sin(b*x+a)/b^2

Rubi [A]
time = 0.15, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4516, 3377, 2718, 4492, 4494, 4266, 2317, 2438}

$$-\frac{4id(c+dx)\text{ArcTan}\left(e^{i(a+bx)}\right)}{b^2}+\frac{2id^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}+\frac{8d^2\cos(a+bx)}{b^3}+\frac{8d(c+dx)\sin(a+bx)}{b^2}-\frac{4(c+dx)^2\cos(a+bx)}{b}-\frac{(c+dx)^2\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]

[Out] ((-4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 + (8*d^2*Cos[a + b*x])/b^3 - (4*(c + d*x)^2*Cos[a + b*x])/b + ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 - ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((c + d*x)^2*Sec[a + b*x])/b + (8*d*(c + d*x)*Sin[a + b*x])/b^2

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4492

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int (c+dx)^2\sec^2(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^2\sin(a+bx) - (c+dx)^2\sin(a+bx)\tan^2(a+bx)\right.$$

$$= 3\int (c+dx)^2\sin(a+bx)\,dx - \int (c+dx)^2\sin(a+bx)\tan^2(a+$$

$$= -\frac{3(c+dx)^2\cos(a+bx)}{b} + \frac{(6d)\int (c+dx)\cos(a+bx)\,dx}{b} + \int$$

$$= -\frac{4(c+dx)^2\cos(a+bx)}{b} - \frac{(c+dx)^2\sec(a+bx)}{b} + \frac{6d(c+dx)}{b}$$

$$= -\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6d^2\cos(a+bx)}{b^3} - \frac{4(c+dx)^2 c}{b}$$

$$= -\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{8d^2\cos(a+bx)}{b^3} - \frac{4(c+dx)^2 c}{b}$$

$$= -\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{8d^2\cos(a+bx)}{b^3} - \frac{4(c+dx)^2 c}{b}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 364 vs. 2(147) = 294.
time = 3.82, size = 364, normalized size = 2.48

$$\frac{4bcd\tanh^{-1}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right) + 2d^2\left(2\text{ArcTan}(\cot(a))\tanh^{-1}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right) - \frac{\csc(a)\left((bx-\text{ArcTan}(\cot(a)))\left(\log\left(1-e^{i(bx-\text{ArcTan}(\cot(a)))}\right)-\log\left(1+e^{i(bx-\text{ArcTan}(\cot(a)))}\right)\right)+i\text{PolyLog}\left(2,-e^{i(bx-\text{ArcTan}(\cot(a)))}\right)-i\text{PolyLog}\left(2,e^{i(bx-\text{ArcTan}(\cot(a)))}\right)\right)}{\sqrt{\csc^2(a)}}\right) - b^2(c+dx)^2\sec(a) - 4\cos(bx)\left((-2d^2+b^2(c+dx)^2)\cos(a)-2bd(c+dx)\sin(a)\right) + 4\left(2bd(c+dx)\cos(a)+(-2d^2+b^2(c+dx)^2)\sin(a)\right)\sin(bx) - \frac{b^2(c+dx)^2\sin\left(\frac{bx}{2}\right)}{\left(\cos\left(\frac{a}{2}\right)-\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)} + \frac{b^2(c+dx)^2\sin\left(\frac{bx}{2}\right)}{\left(\cos\left(\frac{a}{2}\right)+\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)}}{b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + 2*d^2*(2*ArcTan[Cot[a]]*Ar
cTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - (Csc[a]*((b*x - ArcTan[Cot[a]])*(Log[
1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a]]))])
+ I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2, E^(I*(b*x - Ar
cTan[Cot[a]]))]))/Sqrt[Csc[a]^2]) - b^2*(c + d*x)^2*Sec[a] - 4*Cos[b*x]*((-
2*d^2 + b^2*(c + d*x)^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) + 4*(2*b*d*(c + d
*x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] - (b^2*(c + d*x)^2
*Sin[(b*x)/2])/((Cos[a/2] - Sin[a/2])*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])
) + (b^2*(c + d*x)^2*Sin[(b*x)/2])/((Cos[a/2] + Sin[a/2])*(Cos[(a + b*x)/2]
 + Sin[(a + b*x)/2])))/b^3
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(136) = 272.
time = 0.12, size = 369, normalized size = 2.51

method	result

risch	$-\frac{2\left(x^2d^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2\right)e^{i(bx+a)}}{b^3}-\frac{2\left(x^2d^2b^2+2b^2cdx-2ib\,d^2x+b^2c^2-2ibcd-2d^2\right)e^{-i(bx+a)}}{b^3}-\frac{2\,e^{i(bx+a)}(}{b(1+}$
default	$-\frac{4c^2\cos(bx+a)}{b}-\frac{c^2}{\cos(bx+a)b}+\frac{4d^2\left(-(bx+a)^2\cos(bx+a)+2\cos(bx+a)+2(bx+a)\sin(bx+a)-2a(\sin(bx+a)-(bx+a)\cos(bx+a))\right)}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

```
[Out] -4*c^2*cos(b*x+a)/b-c^2/cos(b*x+a)/b+4*d^2/b^3*(-(b*x+a)^2*cos(b*x+a)+2*cos
(b*x+a)+2*(b*x+a)*sin(b*x+a)-2*a*(sin(b*x+a)-(b*x+a)*cos(b*x+a))-a^2*cos(b*
x+a))-d^2/b/cos(b*x+a)*x^2-2*d^2/b^2*ln(1+I*exp(I*(b*x+a)))*x-2*d^2/b^3*ln(
1+I*exp(I*(b*x+a)))*a+2*d^2/b^2*ln(1-I*exp(I*(b*x+a)))*x+2*d^2/b^3*ln(1-I*e
xp(I*(b*x+a)))*a+2*I*d^2/b^3*dilog(1+I*exp(I*(b*x+a)))-2*I*d^2/b^3*dilog(1-
I*exp(I*(b*x+a)))-2*d^2/b^3*a*ln(sec(b*x+a)+tan(b*x+a))+8*c*d/b^2*(sin(b*x+
a)-(b*x+a)*cos(b*x+a)+a*cos(b*x+a))-2*c*d/b/cos(b*x+a)*x+2*c*d/b^2*ln(sec(b
*x+a)+tan(b*x+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -2*((cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) + (3*cos(2*b*x + 2*a
) + 1)*cos(3*b*x + 3*a) + 3*cos(2*b*x + 2*a)*cos(b*x + a) + (sin(3*b*x + 3*
a) + sin(b*x + a))*sin(4*b*x + 4*a) + 3*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) +
 3*sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))*c^2/(b*cos(3*b*x + 3*a)^2
+ 2*b*cos(3*b*x + 3*a)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x + 3*a)
^2 + 2*b*sin(3*b*x + 3*a)*sin(b*x + a) + b*sin(b*x + a)^2) - (4*(cos(a)^2 +
 sin(a)^2)*b*x*cos(b*x + a) + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b*x
*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*x
+ 2*a)*sin(a))*cos(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*cos
(2*b*x + 3*a)^2 + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b*x*cos(b*x + 2
*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*x + 2*a)*sin(a
))*sin(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 3*a
)^2 + 4*((b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*co
s(3*b*x + 3*a)^2 + (b*x*cos(a) + sin(a))*cos(b*x + a)^2 + (b*x*cos(2*b*x +
3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*sin(3*b*x + 3*a)^2 + (b*x*co
s(a) + sin(a))*sin(b*x + a)^2 + 2*(b*x*cos(2*b*x + 3*a)*cos(b*x + a) + (b*x
*cos(a) + sin(a))*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 3*a))*cos(3*b*x +
 3*a) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(b*x
*cos(2*b*x + 3*a)*sin(b*x + a) + (b*x*cos(a) + sin(a))*sin(b*x + a) + sin(2
```

*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a) + (cos(b*x + a)^2 + sin(b*x + a)
^2)*sin(2*b*x + 3*a))*cos(3*b*x + 4*a) + 4*(6*b*x*cos(b*x + 2*a)*cos(b*x +
a)*cos(a) + 6*b*x*cos(b*x + a)*sin(b*x + 2*a)*sin(a) + b*x*cos(2*b*x + 3*a)
^2 + b*x*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*b*x + 2*(3*b*x*cos(b*x
+ 2*a)*cos(b*x + a) + b*x*cos(a))*cos(2*b*x + 3*a) + 2*(3*b*x*cos(b*x + a)*
sin(b*x + 2*a) + b*x*sin(a))*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + 4*(2*b*x*
cos(b*x + a)*cos(a) + 3*(b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(b*x +
 2*a) - 2*cos(a)*sin(b*x + a))*cos(2*b*x + 3*a) + 12*(b*x*cos(b*x + a)^2*co
s(a) + b*x*cos(a)*sin(b*x + a)^2)*cos(b*x + 2*a) - ((cos(2*b*x + 3*a)^2 + 2
*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*
a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2
)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*cos(b*x + a)^2 + (cos(2*b*x +
3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(
2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(
b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*sin(b*x + a)^2 + 2*(
cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a) +
cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a) +
(cos(a)^2 + sin(a)^2)*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2*co
s(a) + cos(a)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(cos(2*b*x + 3*a)^2*sin(
b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + sin(2*b*x + 3*a)^2*sin(
b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (cos(a)^2 + sin(a)^2)*s
in(b*x + a))*sin(3*b*x + 3*a) + 2*(cos(b*x + a)^2*sin(a) + sin(b*x + a)^2*s
in(a))*sin(2*b*x + 3*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x +
a) + 1) + ((cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin
(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a)^2
+ (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)
^2)*cos(b*x + a)^2 + (cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(
a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3*b*x
 + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos(a)^
2 + sin(a)^2)*sin(b*x + a)^2 + 2*(cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*cos(2
*b*x + 3*a)*cos(b*x + a)*cos(a) + cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*cos(b
*x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*cos(b*x + a))*cos(3
*b*x + 3*a) + 2*(cos(b*x + a)^2*cos(a) + cos(a)*sin(b*x + a)^2)*cos(2*b*x +
 3*a) + 2*(cos(2*b*x + 3*a)^2*sin(b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*sin(
b*x + a) + sin(2*b*x + 3*a)^2*sin(b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x + a
)*sin(a) + (cos(a)^2 + sin(a)^2)*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(cos(b*
x + a)^2*sin(a) + sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + 4*((b*x*sin(2*b*x + 3*a) + b*x*
sin(a) - cos(2*b*x + 3*a) - cos(a))*cos(3*b*x + 3*a)^2 + (b*x*sin(a) - cos(
a))*cos(b*x + a)^2 + (b*x*sin(2*b*x + 3*a) + b*x*sin(a) - cos(2*b*x + 3*a)
- cos(a))*sin(3*b*x + 3*a)^2 + (b*x*sin(a) - cos(a))*sin(b*x + a)^2 + 2*(b*
x*cos(b*x + a)*sin(2*b*x + 3*a) + (b*x*sin(a) - cos(a))*cos(b*x + a) - cos(
2*b*x + 3*a)*cos(b*x + a))*cos(3*b*x + 3*a) - (cos(b*x + a)^2 + sin(b*x + a
)^2)*cos(2*b*x + 3*a) + 2*(b*x*sin(2*b*x + 3*a)*sin(b*x + a) + (b*x*sin(a)
- cos(a))*sin(b*x + a) - cos(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a) +

```
(b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*sin(2*b*x + 3*a))*sin(3*b*x + 4*a
) + 4*(6*b*x*cos(b*x + 2*a)*cos(a)*sin(b*x + a) + 6*b*x*sin(b*x + 2*a)*sin(
b*x + a)*sin(a) + 2*(3*b*x*cos(b*x + 2*a)*sin(b...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(129) = 258$.
time = 2.99, size = 513, normalized size = 3.49

$$-\frac{b^2d^2x^2+2\,b^2cdx+b^2c^2+i\,d^2\cos(bx+a)\,\mathrm{Li}_2(i\cos(bx+a)+\sin(bx+a))+i\,d^2\cos(bx+a)\,\mathrm{Li}_2(i\cos(bx+a)-\sin(bx+a))-i\,d^2\cos(bx+a)\,\mathrm{Li}_2(-i\cos(bx+a)+\sin(bx+a))-i\,d^2\cos(bx+a)\,\mathrm{Li}_2(-i\cos(bx+a)-\sin(bx+a))+4\,(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2)\cos(bx+a)^2-(bcd-ad^2)\cos(bx+a)\log(\cos(bx+a)+i\sin(bx+a)+i)+(bcd-ad^2)\cos(bx+a)\log(\cos(bx+a)-i\sin(bx+a)+i)-(bd^2x+ad^2)\cos(bx+a)\log(i\cos(bx+a)+\sin(bx+a)+1)+(bd^2x+ad^2)\cos(bx+a)\log(i\cos(bx+a)-\sin(bx+a)+1)-(bd^2x+ad^2)\cos(bx+a)\log(-i\cos(bx+a)+\sin(bx+a)+1)+(bd^2x+ad^2)\cos(bx+a)\log(-i\cos(bx+a)-\sin(bx+a)+1)-(bcd-ad^2)\cos(bx+a)\log(-\cos(bx+a)+i\sin(bx+a)+i)+(bcd-ad^2)\cos(bx+a)\log(-\cos(bx+a)-i\sin(bx+a)+i)-8\,(bd^2x+bcd)\cos(bx+a)\sin(bx+a)}{b^3\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] -(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*cos(b*x + a)*dilog(I*cos(b*x
+ a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x +
a)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d^2*cos(
b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 4*(b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - (b*c*d - a*d^2)*cos(b*x + a)*log(cos
(b*x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x
+ a) - I*sin(b*x + a) + I) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x +
 a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a)
 - sin(b*x + a) + 1) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) +
 sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - s
in(b*x + a) + 1) - (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b
*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x +
 a) + I) - 8*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a)
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)^2*sin(3*b*x + 3*a), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^2)/cos(a + b*x)^2,x)
```

```
[Out] \text{Hanged}
```

3.391 $\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=57

$$\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{4(c+dx)\cos(a+bx)}{b}-\frac{(c+dx)\sec(a+bx)}{b}+\frac{4d\sin(a+bx)}{b^2}$$

```
[Out] d*arctanh(sin(b*x+a))/b^2-4*(d*x+c)*cos(b*x+a)/b-(d*x+c)*sec(b*x+a)/b+4*d*s
in(b*x+a)/b^2
```

Rubi [A]
time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4516, 3377, 2717, 4492, 4494, 3855}

$$\frac{4d\sin(a+bx)}{b^2}+\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{4(c+dx)\cos(a+bx)}{b}-\frac{(c+dx)\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (d*ArcTanh[Sin[a + b*x]])/b^2 - (4*(c + d*x)*Cos[a + b*x])/b - ((c + d*x)*S
ec[a + b*x])/b + (4*d*Sin[a + b*x])/b^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4492

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)\sin(a+bx)-(c+dx)\sin(a+bx)\tan^2(a+bx)\right)\,d$$

$$= 3\int (c+dx)\sin(a+bx)\,dx - \int (c+dx)\sin(a+bx)\tan^2(a+bx$$

$$= -\frac{3(c+dx)\cos(a+bx)}{b} + \frac{(3d)\int \cos(a+bx)\,dx}{b} + \int (c+dx)\,\mathrm{si}$$

$$= -\frac{4(c+dx)\cos(a+bx)}{b} - \frac{(c+dx)\sec(a+bx)}{b} + \frac{3d\sin(a+bx)}{b^2}$$

$$= \frac{d\tanh^{-1}(\sin(a+bx))}{b^2} - \frac{4(c+dx)\cos(a+bx)}{b} - \frac{(c+dx)\sec(a}{b}$$

Mathematica [A]
time = 0.45, size = 105, normalized size = 1.84

$$-\frac{\sec(a+bx)\left(3bc+3bdx+2b(c+dx)\cos(2(a+bx))+d\cos(a+bx)\left(\log\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)\right)-2d\sin(2(a+bx))\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] -((Sec[a + b*x]*(3*b*c + 3*b*d*x + 2*b*(c + d*x)*Cos[2*(a + b*x)] + d*Cos[a
 + b*x]*(Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] - Log[Cos[(a + b*x)/2] +
Sin[(a + b*x)/2]]) - 2*d*Sin[2*(a + b*x)]))/b^2)
```

Maple [A]
time = 0.11, size = 96, normalized size = 1.68

method	result
default	$-\frac{4c\cos(bx+a)}{b}-\frac{c}{\cos(bx+a)b}+\frac{4d(\sin(bx+a)-(bx+a)\cos(bx+a)+a\cos(bx+a))}{b^2}-\frac{dx}{b\cos(bx+a)}+\frac{d\ln(\sec(bx+a)+\tan(bx+a))}{b^2}$
risch	$-\frac{2(dxb+cb+id)e^{i(bx+a)}}{b^2}-\frac{2(dxb+cb-id)e^{-i(bx+a)}}{b^2}-\frac{2\,e^{i(bx+a)}(dx+c)}{b(1+e^{2i(bx+a)})}+\frac{d\ln(e^{i(bx+a)}+i)}{b^2}-\frac{d\ln(e^{i(bx+a)}-i)}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

```
[Out] -4*c/b*cos(b*x+a)-c/cos(b*x+a)/b+4*d/b^2*(sin(b*x+a)-(b*x+a)*cos(b*x+a)+a*c
os(b*x+a))-d/b/cos(b*x+a)*x+d/b^2*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3330 vs. $2(57) = 114$.
time = 0.56, size = 3330, normalized size = 58.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -2*((cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) + (3*cos(2*b*x + 2*a
) + 1)*cos(3*b*x + 3*a) + 3*cos(2*b*x + 2*a)*cos(b*x + a) + (sin(3*b*x + 3*
a) + sin(b*x + a))*sin(4*b*x + 4*a) + 3*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) +
 3*sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))*c/(b*cos(3*b*x + 3*a)^2 +
2*b*cos(3*b*x + 3*a)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x + 3*a)^2
 + 2*b*sin(3*b*x + 3*a)*sin(b*x + a) + b*sin(b*x + a)^2) - 1/2*(4*(cos(a)^2
 + sin(a)^2)*b*x*cos(b*x + a) + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b
*x*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*
x + 2*a)*sin(a))*cos(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*c
os(2*b*x + 3*a)^2 + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b*x*cos(b*x +
 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*x + 2*a)*sin
(a))*sin(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 3
*a)^2 + 4*((b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*
cos(3*b*x + 3*a)^2 + (b*x*cos(a) + sin(a))*cos(b*x + a)^2 + (b*x*cos(2*b*x
+ 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*sin(3*b*x + 3*a)^2 + (b*x*
cos(a) + sin(a))*sin(b*x + a)^2 + 2*(b*x*cos(2*b*x + 3*a)*cos(b*x + a) + (b
*x*cos(a) + sin(a))*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 3*a))*cos(3*b*x
 + 3*a) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(b
*x*cos(2*b*x + 3*a)*sin(b*x + a) + (b*x*cos(a) + sin(a))*sin(b*x + a) + sin
(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a) + (cos(b*x + a)^2 + sin(b*x +
a)^2)*sin(2*b*x + 3*a))*cos(3*b*x + 4*a) + 4*(6*b*x*cos(b*x + 2*a)*cos(b*x
+ a)*cos(a) + 6*b*x*cos(b*x + a)*sin(b*x + 2*a)*sin(a) + b*x*cos(2*b*x + 3*
a)^2 + b*x*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*b*x + 2*(3*b*x*cos(b*
```

x + 2*a)*cos(b*x + a) + b*x*cos(a))*cos(2*b*x + 3*a) + 2*(3*b*x*cos(b*x + a
)*sin(b*x + 2*a) + b*x*sin(a))*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + 4*(2*b*
x*cos(b*x + a)*cos(a) + 3*(b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(b*x
 + 2*a) - 2*cos(a)*sin(b*x + a))*cos(2*b*x + 3*a) + 12*(b*x*cos(b*x + a)^2*
cos(a) + b*x*cos(a)*sin(b*x + a)^2)*cos(b*x + 2*a) - ((cos(2*b*x + 3*a)^2 +
 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x +
3*a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)
^2)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*cos(b*x + a)^2 + (cos(2*b*x
+ 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*si
n(2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + si
n(b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*sin(b*x + a)^2 + 2
*(cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a)
+ cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a)
+ (cos(a)^2 + sin(a)^2)*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2*
cos(a) + cos(a)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(cos(2*b*x + 3*a)^2*si
n(b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + sin(2*b*x + 3*a)^2*si
n(b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (cos(a)^2 + sin(a)^2)
*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(cos(b*x + a)^2*sin(a) + sin(b*x + a)^2
*sin(a))*sin(2*b*x + 3*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x
+ a) + 1) + ((cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + s
in(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a)^
2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + sin(
a)^2)*cos(b*x + a)^2 + (cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + co
s(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3*b
*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos(a
)^2 + sin(a)^2)*sin(b*x + a)^2 + 2*(cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*cos
(2*b*x + 3*a)*cos(b*x + a)*cos(a) + cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*cos
(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*cos(b*x + a))*cos
(3*b*x + 3*a) + 2*(cos(b*x + a)^2*cos(a) + cos(a)*sin(b*x + a)^2)*cos(2*b*x
 + 3*a) + 2*(cos(2*b*x + 3*a)^2*sin(b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*si
n(b*x + a) + sin(2*b*x + 3*a)^2*sin(b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x +
 a)*sin(a) + (cos(a)^2 + sin(a)^2)*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(cos(
b*x + a)^2*sin(a) + sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a))*log(cos(b*x +
a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + 4*((b*x*sin(2*b*x + 3*a) + b*
x*sin(a) - cos(2*b*x + 3*a) - cos(a))*cos(3*b*x + 3*a)^2 + (b*x*sin(a) - co
s(a))*cos(b*x + a)^2 + (b*x*sin(2*b*x + 3*a) + b*x*sin(a) - cos(2*b*x + 3*a
) - cos(a))*sin(3*b*x + 3*a)^2 + (b*x*sin(a) - cos(a))*sin(b*x + a)^2 + 2*(
b*x*cos(b*x + a)*sin(2*b*x + 3*a) + (b*x*sin(a) - cos(a))*cos(b*x + a) - co
s(2*b*x + 3*a)*cos(b*x + a))*cos(3*b*x + 3*a) - (cos(b*x + a)^2 + sin(b*x +
 a)^2)*cos(2*b*x + 3*a) + 2*(b*x*sin(2*b*x + 3*a)*sin(b*x + a) + (b*x*sin(a
) - cos(a))*sin(b*x + a) - cos(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a)
+ (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*sin(2*b*x + 3*a))*sin(3*b*x + 4
a) + 4(6*b*x*cos(b*x + 2*a)*cos(a)*sin(b*x + a) + 6*b*x*sin(b*x + 2*a)*si
n(b*x + a)*sin(a) + 2*(3*b*x*cos(b*x + 2*a)*sin...

Fricas [A]
time = 3.54, size = 93, normalized size = 1.63

$$-\frac{2\,bdx+8\,(bdx+bc)\cos\left(bx+a\right)^{2}-d\cos\left(bx+a\right)\log\left(\sin\left(bx+a\right)+1\right)+d\cos\left(bx+a\right)\log\left(-\sin\left(bx+a\right)+1\right)-8\,d\cos\left(bx+a\right)\sin\left(bx+a\right)+2\,bc}{2\,b^{2}\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + 8*(b*d*x + b*c)*cos(b*x + a)^2 - d*cos(b*x + a)*log(sin(b*x
 + a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a) + 1) - 8*d*cos(b*x + a)*sin(b
*x + a) + 2*b*c)/(b^2*cos(b*x + a))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(57) = 114.
time = 0.50, size = 365, normalized size = 6.40

$$\frac{10\,bc\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{4}+10\,(bx+a)d\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{4}-10\,ad\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{4}+d\log\left(\frac{2\,\left(\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}+2\,\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)+1\right)}{\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}+1}\right)\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{4}-d\log\left(\frac{2\,\left(\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}-2\,\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)+1\right)}{\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}+1}\right)\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{4}-12\,bc\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}-12\,(bx+a)d\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}+12\,ad\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}+16\,d\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{3}+10\,bc+10\,(bx+a)d-10\,ad-d\log\left(\frac{2\,\left(\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}+2\,\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)+1\right)}{\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}+1}\right)+d\log\left(\frac{2\,\left(\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}-2\,\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)+1\right)}{\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}+1}\right)-16\,d\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}{2\,\left(b\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{4}-b\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] 1/2*(10*b*c*tan(1/2*b*x + 1/2*a)^4 + 10*(b*x + a)*d*tan(1/2*b*x + 1/2*a)^4
- 10*a*d*tan(1/2*b*x + 1/2*a)^4 + d*log(2*(tan(1/2*b*x + 1/2*a)^2 + 2*tan(1
/2*b*x + 1/2*a) + 1)/(tan(1/2*b*x + 1/2*a)^2 + 1))*tan(1/2*b*x + 1/2*a)^4 -
 d*log(2*(tan(1/2*b*x + 1/2*a)^2 - 2*tan(1/2*b*x + 1/2*a) + 1)/(tan(1/2*b*x
 + 1/2*a)^2 + 1))*tan(1/2*b*x + 1/2*a)^4 - 12*b*c*tan(1/2*b*x + 1/2*a)^2 -
12*(b*x + a)*d*tan(1/2*b*x + 1/2*a)^2 + 12*a*d*tan(1/2*b*x + 1/2*a)^2 + 16*
d*tan(1/2*b*x + 1/2*a)^3 + 10*b*c + 10*(b*x + a)*d - 10*a*d - d*log(2*(tan(
1/2*b*x + 1/2*a)^2 + 2*tan(1/2*b*x + 1/2*a) + 1)/(tan(1/2*b*x + 1/2*a)^2 +
1)) + d*log(2*(tan(1/2*b*x + 1/2*a)^2 - 2*tan(1/2*b*x + 1/2*a) + 1)/(tan(1/
2*b*x + 1/2*a)^2 + 1)) - 16*d*tan(1/2*b*x + 1/2*a))/((b*tan(1/2*b*x + 1/2*a
)^4 - b)*b)
```

Mupad [B]
time = 1.31, size = 150, normalized size = 2.63

$$\mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}\,\left(\frac{-2\,b\,c+d\,2\mathrm{i}}{b^2}-\frac{2\,d\,x}{b}\right)-\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\frac{2\,b\,c+d\,2\mathrm{i}}{b^2}+\frac{2\,d\,x}{b}\right)-\frac{d\,\ln\left({\mathrm{e}}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}-\mathrm{i}\right)}{b^2}+\frac{d\,\ln\left({\mathrm{e}}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}+1\mathrm{i}\right)}{b^2}-\frac{{\mathrm{e}}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(c+d\,x\right)\,2\mathrm{i}}{b\,\left({\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x))/cos(a + b*x)^2,x)
```

```
[Out] exp(- a*1i - b*x*1i)*((d*2i - 2*b*c)/b^2 - (2*d*x)/b) - exp(a*1i + b*x*1i)*
((d*2i + 2*b*c)/b^2 + (2*d*x)/b) - (d*log(exp(a*1i + b*x*1i) - 1i))/b^2 + (
d*log(exp(a*1i + b*x*1i) + 1i))/b^2 - (exp(a*1i + b*x*1i)*(c + d*x)*2i)/(b*
(exp(a*2i + b*x*2i)*1i + 1i))
```

3.392 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

Optimal. Leaf size=78

$$\frac{4\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}+\frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}-\text{Int}\left(\frac{\sec(a+bx)\tan(a+bx)}{c+dx},x\right)$$

```
[Out] -CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)+4*cos(a-b*c/d)*Si(b*c/d+b
*x)/d+4*Ci(b*c/d+b*x)*sin(a-b*c/d)/d
```

Rubi [A]
time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] (4*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d + (4*Cos[a - (b*c)/d]*Sin
Integral[(b*c)/d + b*x])/d - Defer[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*
x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx &= \int \left(\frac{3\sin(a+bx)}{c+dx}-\frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\right)dx\\
&= 3\int \frac{\sin(a+bx)}{c+dx}\,dx-\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx\\
&= \left(3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\left(3\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+\right.}{c+dx}\\
&= \frac{3\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}+\frac{3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\cos\left(a-\frac{bc}{d}\right)\int\\
&= \frac{4\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}+\frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}-\int \frac{\sec(a+bx)\,\text{ta}}{c+}
\end{aligned}$$

Mathematica [A]
time = 13.42, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x), x]
```

Maple [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(\sec^2(bx+a))\sin(3bx+3a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] 2*(b*c*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(1, -(I
*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(1, (I*b*d*x +
 I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) +
 (b*c*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(1, -(I*
b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(1, (I*b*d*x +
I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) +
(b*d*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(1, -(I*b
*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integral_e(1, (I*b*d*x + I
*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x)
*cos(2*b*x + 2*a)^2 + (b*c*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*e
xp_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_inte
gral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*s
in(-(b*c - a*d)/d) + (b*d*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*ex
p_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integ
ral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*si
n(-(b*c - a*d)/d))*x)*sin(2*b*x + 2*a)^2 - d*sin(2*b*x + 2*a)*sin(b*x + a)
+ (b*d*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(1, -(I
*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integral_e(1, (I*b*d*x +
 I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*
x + (2*b*c*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(1,
 -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - 2*b*c*(exp_integral_e(1, (I*b
```

```
*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d
)/d) + 2*(b*d*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integral_e
(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integral_e(1, (I*
b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*
d)/d))*x - d*cos(b*x + a))*cos(2*b*x + 2*a) - d*cos(b*x + a) - (b*d^3*x + b
*c*d^2 + (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x + b*c*d^2)*sin(2
*b*x + 2*a)^2 + 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*integrate((cos(2*b*
x + 2*a)*cos(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d^
2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a
)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x^2 + 2
*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)), x))/(b*d^2*x + b*c*d + (b*d^2*x + b*c*
d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x +
 b*c*d)*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
[Out] integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
[Out] integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)}{{\cos\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)),x)
[Out] int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)), x)
```

3.393 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=98

$$\frac{4b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{4\sin(a+bx)}{d(c+dx)}-\frac{4b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}-\text{Int}\left(\frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\right.$$

```
[Out] -CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)+4*b*Ci(b*c/d+b*x)*cos(a
-b*c/d)/d^2-4*b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2-4*sin(b*x+a)/d/(d*x+c)
```

Rubi [A]
time = 0.27, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] (4*b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2 - (4*Sin[a + b*x])/(d
*(c + d*x)) - (4*b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 - Defer
[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx &= \int \left(\frac{3\sin(a+bx)}{(c+dx)^2}-\frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\right)dx\\
&= 3\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx-\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx\\
&= -\frac{3\sin(a+bx)}{d(c+dx)}+\frac{(3b)\int \frac{\cos(a+bx)}{c+dx}\,dx}{d}+\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx-\int \frac{\sec(a+bx)}{(c+\cdots}\\
&= -\frac{4\sin(a+bx)}{d(c+dx)}+\frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{d}+\frac{\left(3b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}-\frac{(3\cdots}{}\\
&= \frac{3b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{4\sin(a+bx)}{d(c+dx)}-\frac{3b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}\\
&= \frac{4b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{4\sin(a+bx)}{d(c+dx)}-\frac{4b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}
\end{aligned}$$

Mathematica [A]
time = 16.83, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
[Out] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2, x]
```

Maple [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(\sec^2(bx+a))\sin(3bx+3a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
[Out] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
[Out] 2*(b*c*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(2, -(I
*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(2, (I*b*d*x +
 I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) +
 (b*c*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(2, -(I*
b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(2, (I*b*d*x +
I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) +
(b*d*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(2, -(I*b
*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integral_e(2, (I*b*d*x + I
*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x)
*cos(2*b*x + 2*a)^2 + (b*c*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*e
xp_integral_e(2, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_inte
gral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*s
in(-(b*c - a*d)/d) + (b*d*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*ex
p_integral_e(2, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integ
ral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*si
n(-(b*c - a*d)/d))*x)*sin(2*b*x + 2*a)^2 - d*sin(2*b*x + 2*a)*sin(b*x + a)
```

```
+ (b*d*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(2, -(I
*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integral_e(2, (I*b*d*x +
 I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*
x + (2*b*c*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(2,
 -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - 2*b*c*(exp_integral_e(2, (I*b
*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d
)/d) + 2*(b*d*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integral_e
(2, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integral_e(2, (I*
b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*
d)/d))*x - d*cos(b*x + a))*cos(2*b*x + 2*a) - d*cos(b*x + a) - 2*(b*d^4*x^2
 + 2*b*c*d^3*x + b*c^2*d^2 + (b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*cos(2*b*
x + 2*a)^2 + (b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*sin(2*b*x + 2*a)^2 + 2*(
b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*cos(2*b*x + 2*a))*integrate((cos(2*b*x
 + 2*a)*cos(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d^3
*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3
*b*c^2*d*x + b*c^3)*cos(2*b*x + 2*a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c
^2*d*x + b*c^3)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2
*d*x + b*c^3)*cos(2*b*x + 2*a)), x))/(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (
b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*c*
d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)
*cos(2*b*x + 2*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)}{{\cos\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^2),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^2), x)
```

3.394 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=121

$$-\frac{2b\cos(a+bx)}{d^2(c+dx)}-\frac{2b^2\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^3}-\frac{2\sin(a+bx)}{d(c+dx)^2}-\frac{2b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^3}-\text{Int}\left(\frac{\text{se}}{}\right.$$

```
[Out] -CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^3,x)-2*b*cos(b*x+a)/d^2/(d*x
+c)-2*b^2*cos(a-b*c/d)*Si(b*c/d+b*x)/d^3-2*b^2*Ci(b*c/d+b*x)*sin(a-b*c/d)/d
^3-2*sin(b*x+a)/d/(d*x+c)^2
```

Rubi [A]
time = 0.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] (-2*b*Cos[a + b*x])/(d^2*(c + d*x)) - (2*b^2*CosIntegral[(b*c)/d + b*x]*Sin
[a - (b*c)/d])/d^3 - (2*Sin[a + b*x])/(d*(c + d*x)^2) - (2*b^2*Cos[a - (b*c
)/d]*SinIntegral[(b*c)/d + b*x])/d^3 - Defer[Int][(Sec[a + b*x]*Tan[a + b*x
])/(c + d*x)^3, x]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \left(\frac{3\sin(a+bx)}{(c+dx)^3} - \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^3}\right)dx$$
$$= 3\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx - \int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^3}\,dx$$
$$= -\frac{3\sin(a+bx)}{2d(c+dx)^2} + \frac{(3b)\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{2d} + \int \frac{\sin(a+bx)}{(c+dx)^3}\,dx - \int \frac{\sec(a+b}{(c}$$
$$= -\frac{3b\cos(a+bx)}{2d^2(c+dx)} - \frac{2\sin(a+bx)}{d(c+dx)^2} - \frac{(3b^2)\int \frac{\sin(a+bx)}{c+dx}\,dx}{2d^2} + \frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{2d}$$
$$= -\frac{2b\cos(a+bx)}{d^2(c+dx)} - \frac{2\sin(a+bx)}{d(c+dx)^2} - \frac{b^2\int \frac{\sin(a+bx)}{c+dx}\,dx}{2d^2} - \frac{\left(3b^2\cos\left(a-\frac{bc}{d}\right)\right)}{2d}$$
$$= -\frac{2b\cos(a+bx)}{d^2(c+dx)} - \frac{3b^2\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{2d^3} - \frac{2\sin(a+bx)}{d(c+dx)^2} - \frac{3b^2\text{co}}{}$$
$$= -\frac{2b\cos(a+bx)}{d^2(c+dx)} - \frac{2b^2\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^3} - \frac{2\sin(a+bx)}{d(c+dx)^2} - \frac{2b^2\text{co}}{}$$

<u>**Mathematica**</u> [A]
time = 19.80, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
[Out] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3, x]
```

<u>**Maple**</u> [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(\sec^2(bx+a))\sin(3bx+3a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
[Out] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

<u>**Maxima**</u> [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")

[Out] 2*(b*c*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(3, (I*b*d*x + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) + (b*c*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(3, (I*b*d*x + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integral_e(3, (I*b*d*x + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*cos(2*b*x + 2*a)^2 + (b*c*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(3, (I*b*d*x + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integral_e(3, (I*b*d*x + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*sin(2*b*x + 2*a)^2 - d*sin(2*b*x + 2*a)*sin(b*x + a) + (b*d*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integral_e(3, (I*b*d*x + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x + (2*b*c*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - 2*b*c*(exp_integral_e(3, (I*b*d*x + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) + 2*(b*d*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integral_e(3, (I*b*d*x + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x - d*cos(b*x + a))*cos(2*b*x + 2*a) - d*cos(b*x + a) - 3*(b*d^5*x^3 + 3*b*c*d^4*x^2 + 3*b*c^2*d^3*x + b*c^3*d^2 + (b*d^5*x^3 + 3*b*c*d^4*x^2 + 3*b*c^2*d^3*x + b*c^3*d^2)*cos(2*b*x + 2*a)^2 + (b*d^5*x^3 + 3*b*c*d^4*x^2 + 3*b*c^2*d^3*x + b*c^3*d^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^5*x^3 + 3*b*c*d^4*x^2 + 3*b*c^2*d^3*x + b*c^3*d^2)*cos(2*b*x + 2*a))*integrate((cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + (b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4)*cos(2*b*x + 2*a)^2 + (b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4)*sin(2*b*x + 2*a)^2 + 2*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4)*cos(2*b*x + 2*a)), x))/(b*d^4*x^3 + 3*b*c*d^3*x^2 + 3*b*c^2*d^2*x + b*c^3*d + (b*d^4*x^3 + 3*b*c*d^3*x^2 + 3*b*c^2*d^2*x + b*c^3*d)*cos(2*b*x + 2*a)^2 + (b*d^4*x^3 + 3*b*c*d^3*x^2 + 3*b*c^2*d^2*x + b*c^3*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^4*x^3 + 3*b*c*d^3*x^2 + 3*b*c^2*d^2*x + b*c^3*d)*cos(2*b*x + 2*a))

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)}{{\cos\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^3),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^3), x)
```

3.395 $\int x \cos(2x) \sec(x) \, dx$

Optimal. Leaf size=57

$$2ix\text{ArcTan}\left(e^{ix}\right) + 2\cos(x) - i\text{PolyLog}\left(2, -ie^{ix}\right) + i\text{PolyLog}\left(2, ie^{ix}\right) + 2x\sin(x)$$

```
[Out] 2*I*x*arctan(exp(I*x))+2*cos(x)-I*polylog(2,-I*exp(I*x))+I*polylog(2,I*exp(
I*x))+2*x*sin(x)
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {4516, 3377, 2718, 4492, 4266, 2317, 2438}

$$2ix\text{ArcTan}\left(e^{ix}\right) - i\text{Li}_2\left(-ie^{ix}\right) + i\text{Li}_2\left(ie^{ix}\right) + 2x\sin(x) + 2\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[2*x]*Sec[x],x]
```

```
[Out] (2*I)*x*ArcTan[E^(I*x)] + 2*Cos[x] - I*PolyLog[2, (-I)*E^(I*x)] + I*PolyLog
[2, I*E^(I*x)] + 2*x*Sin[x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
```

```
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4492

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}
\int x\cos(2x)\sec(x)\,dx &= \int (x\cos(x)-x\sin(x)\tan(x))\,dx\\
&= \int x\cos(x)\,dx-\int x\sin(x)\tan(x)\,dx\\
&= x\sin(x)+\int x\cos(x)\,dx-\int x\sec(x)\,dx-\int \sin(x)\,dx\\
&= 2ix\tan^{-1}\left(e^{ix}\right)+\cos(x)+2x\sin(x)+\int \log\left(1-ie^{ix}\right)\,dx-\int \log\left(1+ie^{ix}\right)\,dx -\\
&= 2ix\tan^{-1}\left(e^{ix}\right)+2\cos(x)+2x\sin(x)-i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{ix}\right)+i\text{Subs}\\
&= 2ix\tan^{-1}\left(e^{ix}\right)+2\cos(x)-i\text{Li}_2\left(-ie^{ix}\right)+i\text{Li}_2\left(ie^{ix}\right)+2x\sin(x)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 77, normalized size = 1.35

$$2\cos(x)-x\left(\log\left(1-ie^{ix}\right)-\log\left(1+ie^{ix}\right)\right)-i\left(\text{PolyLog}\left(2,-ie^{ix}\right)-\text{PolyLog}\left(2,ie^{ix}\right)\right)+2x\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[2*x]*Sec[x],x]
```

```
[Out] 2*Cos[x] - x*(Log[1 - I*E^(I*x)] - Log[1 + I*E^(I*x)]) - I*(PolyLog[2, (-I)
*E^(I*x)] - PolyLog[2, I*E^(I*x)]) + 2*x*Sin[x]
```

Maple [A]
time = 0.11, size = 66, normalized size = 1.16

method	result
default	$x \ln \left(1+i \mathrm{e}^{i x}\right)-x \ln \left(1-i \mathrm{e}^{i x}\right)-i \operatorname{dilog}\left(1+i \mathrm{e}^{i x}\right)+i \operatorname{dilog}\left(1-i \mathrm{e}^{i x}\right)+2 \cos (x)+2 x \sin (x)$
risch	$-i(x+i) \mathrm{e}^{i x}+i(x-i) \mathrm{e}^{-i x}+x \ln \left(1+i \mathrm{e}^{i x}\right)-x \ln \left(1-i \mathrm{e}^{i x}\right)-i \operatorname{dilog}\left(1+i \mathrm{e}^{i x}\right)+i \operatorname{dilog}\left(1-i \mathrm{e}^{i x}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x)*sec(x),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(1+I*exp(I*x))-x*ln(1-I*exp(I*x))-I*dilog(1+I*exp(I*x))+I*dilog(1-I*exp
(I*x))+2*cos(x)+2*x*sin(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x),x, algorithm="maxima")
```

```
[Out] 2*x*sin(x) + 2*cos(x) - 2*integrate((x*cos(2*x)*cos(x) + x*sin(2*x)*sin(x)
+ x*cos(x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(36) = 72$.
time = 3.03, size = 106, normalized size = 1.86

$$-\frac{1}{2} x \log \left(i \cos (x)+\sin (x)+1\right)+\frac{1}{2} x \log \left(i \cos (x)-\sin (x)+1\right)-\frac{1}{2} x \log \left(-i \cos (x)+\sin (x)+1\right)+\frac{1}{2} x \log \left(-i \cos (x)-\sin (x)+1\right)+2 x \sin (x)+2 \cos (x)+\frac{1}{2} i \operatorname{Li}_2(i \cos (x)+\sin (x))+\frac{1}{2} i \operatorname{Li}_2(i \cos (x)-\sin (x))-\frac{1}{2} i \operatorname{Li}_2(-i \cos (x)+\sin (x))-\frac{1}{2} i \operatorname{Li}_2(-i \cos (x)-\sin (x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x),x, algorithm="fricas")
```

```
[Out] -1/2*x*log(I*cos(x) + sin(x) + 1) + 1/2*x*log(I*cos(x) - sin(x) + 1) - 1/2*
x*log(-I*cos(x) + sin(x) + 1) + 1/2*x*log(-I*cos(x) - sin(x) + 1) + 2*x*sin
(x) + 2*cos(x) + 1/2*I*dilog(I*cos(x) + sin(x)) + 1/2*I*dilog(I*cos(x) - si
n(x)) - 1/2*I*dilog(-I*cos(x) + sin(x)) - 1/2*I*dilog(-I*cos(x) - sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \cos (2x) \sec (x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x),x)
```

```
[Out] Integral(x*cos(2*x)*sec(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x),x, algorithm="giac")
```

```
[Out] integrate(x*cos(2*x)*sec(x), x)
```

Mupad [B]
time = 2.21, size = 46, normalized size = 0.81

$$2\cos(x) + 2x\sin(x) - \mathrm{polylog}\left(2, -\mathrm{e}^{x\,1\mathrm{i}}\,1\mathrm{i}\right)\,1\mathrm{i} + \mathrm{polylog}\left(2, \mathrm{e}^{x\,1\mathrm{i}}\,1\mathrm{i}\right)\,1\mathrm{i} + x\,\mathrm{atan}\left(\mathrm{e}^{x\,1\mathrm{i}}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(2*x))/cos(x),x)
```

```
[Out] 2*cos(x) - polylog(2, -exp(x*1i)*1i)*1i + polylog(2, exp(x*1i)*1i)*1i + x*a
tan(exp(x*1i))*2i + 2*x*sin(x)
```

3.396 $\int x \cos(2x) \sec^2(x) \, dx$

Optimal. Leaf size=14

$$x^2 - \log(\cos(x)) - x \tan(x)$$

```
[Out] x^2-ln(cos(x))-x*tan(x)
```

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4516, 3801, 3556, 30}

$$x^2 - x \tan(x) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[2*x]*Sec[x]^2,x]
```

```
[Out] x^2 - Log[Cos[x]] - x*Tan[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$
\begin{aligned}
\int x\cos(2x)\sec^2(x)\,dx &= \int \left(x - x\tan^2(x)\right)\,dx \\
&= \frac{x^2}{2} - \int x\tan^2(x)\,dx \\
&= \frac{x^2}{2} - x\tan(x) + \int x\,dx + \int \tan(x)\,dx \\
&= x^2 - \log(\cos(x)) - x\tan(x)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 14, normalized size = 1.00

$$x^2 - \log(\cos(x)) - x\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[2*x]*Sec[x]^2,x]
```

```
[Out] x^2 - Log[Cos[x]] - x*Tan[x]
```

Maple [A]
time = 0.08, size = 15, normalized size = 1.07

method	result	size
default	$x^2 - \ln(\cos(x)) - x\tan(x)$	15
risch	$x^2 + 2ix - \frac{2ix}{e^{2ix}+1} - \ln(e^{2ix}+1)$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x)*sec(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^2-ln(cos(x))-x*tan(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(14) = 28.
time = 0.49, size = 111, normalized size = 7.93

$$\frac{2\,x^2\cos(2\,x)^2 + 2\,x^2\sin(2\,x)^2 + 4\,x^2\cos(2\,x) + 2\,x^2 - \left(\cos(2\,x)^2 + \sin(2\,x)^2 + 2\,\cos(2\,x) + 1\right)\log\left(\cos(2\,x)^2 + \sin(2\,x)^2 + 2\,\cos(2\,x) + 1\right) - 4\,x\sin(2\,x)}{2\left(\cos(2\,x)^2 + \sin(2\,x)^2 + 2\,\cos(2\,x) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*x^2*cos(2*x)^2 + 2*x^2*sin(2*x)^2 + 4*x^2*cos(2*x) + 2*x^2 - (cos(2*
x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x
) + 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [A]
time = 3.15, size = 26, normalized size = 1.86

$$\frac{x^2\cos(x) - \cos(x)\log(-\cos(x)) - x\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^2,x, algorithm="fricas")
```

```
[Out] (x^2*cos(x) - cos(x)*log(-cos(x)) - x*sin(x))/cos(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(12) = 24.
time = 4.08, size = 144, normalized size = 10.29

$$x^2 + \frac{2x\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} - \frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} + \frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{\tan^2\left(\frac{x}{2}\right)-1} - \frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} + \frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{\tan^2\left(\frac{x}{2}\right)-1} + \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} - \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{\tan^2\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)**2,x)
```

```
[Out] x**2 + 2*x*tan(x/2)/(tan(x/2)**2 - 1) - log(tan(x/2) - 1)*tan(x/2)**2/(tan(
x/2)**2 - 1) + log(tan(x/2) - 1)/(tan(x/2)**2 - 1) - log(tan(x/2) + 1)*tan(
x/2)**2/(tan(x/2)**2 - 1) + log(tan(x/2) + 1)/(tan(x/2)**2 - 1) + log(tan(x
/2)**2 + 1)*tan(x/2)**2/(tan(x/2)**2 - 1) - log(tan(x/2)**2 + 1)/(tan(x/2)*
*2 - 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(14) = 28.
time = 0.43, size = 118, normalized size = 8.43

$$\frac{2\,x^2\tan\left(\frac{1}{2}\,x\right)^2 - \log\left(\frac{4\left(\tan\left(\frac{1}{2}\,x\right)^4-2\,\tan\left(\frac{1}{2}\,x\right)^2+1\right)}{\tan\left(\frac{1}{2}\,x\right)^4+2\,\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^2 - 2\,x^2 + 4\,x\tan\left(\frac{1}{2}\,x\right) + \log\left(\frac{4\left(\tan\left(\frac{1}{2}\,x\right)^4-2\,\tan\left(\frac{1}{2}\,x\right)^2+1\right)}{\tan\left(\frac{1}{2}\,x\right)^4+2\,\tan\left(\frac{1}{2}\,x\right)^2+1}\right)}{2\left(\tan\left(\frac{1}{2}\,x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*x^2*tan(1/2*x)^2 - log(4*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1/
2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - 2*x^2 + 4*x*tan(1/2*x) + log(4
*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/
(tan(1/2*x)^2 - 1)
```

Mupad [B]
time = 2.35, size = 31, normalized size = 2.21

$$x^2 - \ln\left(\mathrm{e}^{x\,2\mathrm{i}}+1\right) + x\,2\mathrm{i} - \frac{x\,2\mathrm{i}}{\mathrm{e}^{x\,2\mathrm{i}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(2*x))/cos(x)^2,x)
[Out] x*2i - log(exp(x*2i) + 1) - (x*2i)/(exp(x*2i) + 1) + x^2
```

3.397 $\int x\cos(2x)\sec^3(x)\,dx$

Optimal. Leaf size=67

$$-3ix\text{ArcTan}\left(e^{ix}\right)+\frac{3}{2}i\text{PolyLog}\left(2,-ie^{ix}\right)-\frac{3}{2}i\text{PolyLog}\left(2,ie^{ix}\right)+\frac{\sec(x)}{2}-\frac{1}{2}x\sec(x)\tan(x)$$

```
[Out] -3*I*x*arctan(exp(I*x))+3/2*I*polylog(2,-I*exp(I*x))-3/2*I*polylog(2,I*exp(
I*x))+1/2*sec(x)-1/2*x*sec(x)*tan(x)
```

Rubi [A]
time = 0.09, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.600$, Rules used = {4516, 4266, 2317, 2438, 4498, 4270}

$$-3ix\text{ArcTan}\left(e^{ix}\right)+\frac{3}{2}i\text{Li}_2\left(-ie^{ix}\right)-\frac{3}{2}i\text{Li}_2\left(ie^{ix}\right)+\frac{\sec(x)}{2}-\frac{1}{2}x\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[2*x]*Sec[x]^3,x]
```

```
[Out] (-3*I)*x*ArcTan[E^(I*x)] + ((3*I)/2)*PolyLog[2, (-I)*E^(I*x)] - ((3*I)/2)*P
olyLog[2, I*E^(I*x)] + Sec[x]/2 - (x*Sec[x]*Tan[x])/2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
```

```
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4498

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4516

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}
\int x\cos(2x)\sec^3(x)\,dx &= \int \left(x\sec(x)-x\sec(x)\tan^2(x)\right)\,dx\\
&= \int x\sec(x)\,dx-\int x\sec(x)\tan^2(x)\,dx\\
&= -2ix\tan^{-1}\left(e^{ix}\right)-\int \log\left(1-ie^{ix}\right)\,dx+\int \log\left(1+ie^{ix}\right)\,dx+\int x\sec(x)\,dx-\\
&= -4ix\tan^{-1}\left(e^{ix}\right)+\frac{\sec(x)}{2}-\frac{1}{2}x\sec(x)\tan(x)+i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{ix}\right)\\
&= -3ix\tan^{-1}\left(e^{ix}\right)+i\text{Li}_2\left(-ie^{ix}\right)-i\text{Li}_2\left(ie^{ix}\right)+\frac{\sec(x)}{2}-\frac{1}{2}x\sec(x)\tan(x)+i\text{Subs}\\
&= -3ix\tan^{-1}\left(e^{ix}\right)+2i\text{Li}_2\left(-ie^{ix}\right)-2i\text{Li}_2\left(ie^{ix}\right)+\frac{\sec(x)}{2}-\frac{1}{2}x\sec(x)\tan(x)-\frac{1}{2}i\text{S}\\
&= -3ix\tan^{-1}\left(e^{ix}\right)+\frac{3}{2}i\text{Li}_2\left(-ie^{ix}\right)-\frac{3}{2}i\text{Li}_2\left(ie^{ix}\right)+\frac{\sec(x)}{2}-\frac{1}{2}x\sec(x)\tan(x)
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 146 vs. 2(67) = 134.
time = 0.33, size = 146, normalized size = 2.18

$$\frac{1}{4}\left(6x\log\left(1-ie^{ix}\right)-6x\log\left(1+ie^{ix}\right)+6i\text{PolyLog}\left(2,-ie^{ix}\right)-6i\text{PolyLog}\left(2,ie^{ix}\right)+\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}+\frac{x}{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2}-\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}+\frac{x}{-1+\sin(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[2*x]*Sec[x]^3,x]
```

```
[Out] (6*x*Log[1 - I*E^(I*x)] - 6*x*Log[1 + I*E^(I*x)] + (6*I)*PolyLog[2, (-I)*E^
(I*x)] - (6*I)*PolyLog[2, I*E^(I*x)] + (2*Sin[x/2])/(Cos[x/2] - Sin[x/2]) +
 x/(Cos[x/2] + Sin[x/2])^2 - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2]) + x/(-1 + S
in[x]))/4
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(48) = 96$.
time = 0.16, size = 102, normalized size = 1.52

method	result	size
risch	$\frac{i\left(\mathrm{e}^{3ix}x-x\,\mathrm{e}^{ix}-i\mathrm{e}^{3ix}-i\mathrm{e}^{ix}\right)}{\left(\mathrm{e}^{2ix}+1\right)^{2}}-\frac{3x\ln\left(1+i\mathrm{e}^{ix}\right)}{2}+\frac{3x\ln\left(1-i\mathrm{e}^{ix}\right)}{2}+\frac{3i\,\mathrm{dilog}\left(1+i\mathrm{e}^{ix}\right)}{2}-\frac{3i\,\mathrm{dilog}\left(1-i\mathrm{e}^{ix}\right)}{2}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x)*sec(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] I/(exp(2*I*x)+1)^2*(exp(3*I*x)*x-x*exp(I*x)-I*exp(3*I*x)-I*exp(I*x))-3/2*x*
ln(1+I*exp(I*x))+3/2*x*ln(1-I*exp(I*x))+3/2*I*dilog(1+I*exp(I*x))-3/2*I*dil
og(1-I*exp(I*x))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^3,x, algorithm="maxima")
```

```
[Out] -((x*sin(3*x) - x*sin(x) - cos(3*x) - cos(x))*cos(4*x) - (2*x*sin(2*x) + 2*
cos(2*x) + 1)*cos(3*x) - 2*(x*sin(x) + cos(x))*cos(2*x) - 3*(2*(2*cos(2*x)
+ 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x
) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)*integrate((x*cos(2*x)*cos(x) + x*sin(2*x
)*sin(x) + x*cos(x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1), x) - (x*co
s(3*x) - x*cos(x) + sin(3*x) + sin(x))*sin(4*x) + (2*x*cos(2*x) + x - 2*sin
(2*x))*sin(3*x) + 2*(x*cos(x) - sin(x))*sin(2*x) - x*sin(x) - cos(x))/(2*(2
*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*
x)*sin(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(38) = 76$.
time = 2.99, size = 144, normalized size = 2.15

$$\frac{3\,x\cos\left(x\right)^{2}\log\left(i\,\cos\left(x\right)+\sin\left(x\right)+1\right)-3\,x\cos\left(x\right)^{2}\log\left(i\,\cos\left(x\right)-\sin\left(x\right)+1\right)+3\,x\cos\left(x\right)^{2}\log\left(-i\,\cos\left(x\right)+\sin\left(x\right)+1\right)-3\,x\cos\left(x\right)^{2}\log\left(-i\,\cos\left(x\right)-\sin\left(x\right)+1\right)-3i\,\cos\left(x\right)^{2}\mathrm{Li}_2(i\,\cos\left(x\right)+\sin\left(x\right))-3i\,\cos\left(x\right)^{2}\mathrm{Li}_2(i\,\cos\left(x\right)-\sin\left(x\right))+3i\,\cos\left(x\right)^{2}\mathrm{Li}_2(-i\,\cos\left(x\right)+\sin\left(x\right))+3i\,\cos\left(x\right)^{2}\mathrm{Li}_2(-i\,\cos\left(x\right)-\sin\left(x\right))-2\,x\sin\left(x\right)+2\,\cos\left(x\right)}{4\,\cos\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*x*cos(x)^2*log(I*cos(x) + sin(x) + 1) - 3*x*cos(x)^2*log(I*cos(x) -
sin(x) + 1) + 3*x*cos(x)^2*log(-I*cos(x) + sin(x) + 1) - 3*x*cos(x)^2*log(-
I*cos(x) - sin(x) + 1) - 3*I*cos(x)^2*dilog(I*cos(x) + sin(x)) - 3*I*cos(x)
^2*dilog(I*cos(x) - sin(x)) + 3*I*cos(x)^2*dilog(-I*cos(x) + sin(x)) + 3*I*
cos(x)^2*dilog(-I*cos(x) - sin(x)) - 2*x*sin(x) + 2*cos(x))/cos(x)^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(2x)\sec^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)**3,x)
```

```
[Out] Integral(x*cos(2*x)*sec(x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cos(2*x)*sec(x)^3, x)
```

Mupad [B]
time = 2.32, size = 63, normalized size = 0.94

$$\frac{1}{2\,\cos(x)}+x\,\mathrm{atanh}\left(\mathrm{e}^{x\,1\mathrm{i}}\,1\mathrm{i}\right)-\frac{x\,\sin(x)}{2\cos(x)^2}+\frac{\mathrm{polylog}(2,-\mathrm{e}^{x\,1\mathrm{i}}\,1\mathrm{i})\;3\mathrm{i}}{2}-\frac{\mathrm{polylog}(2,\mathrm{e}^{x\,1\mathrm{i}}\,1\mathrm{i})\;3\mathrm{i}}{2}-x\,\mathrm{atan}\left(\mathrm{e}^{x\,1\mathrm{i}}\right)\;4\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(2*x))/cos(x)^3,x)
```

```
[Out] (polylog(2, -exp(x*1i)*1i)*3i)/2 - (polylog(2, exp(x*1i)*1i)*3i)/2 + 1/(2*c
os(x)) - x*atan(exp(x*1i))*4i + x*atanh(exp(x*1i)*1i) - (x*sin(x))/(2*cos(x
)^2)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:  #isinstance(expn,list):
      return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:  #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])  #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstar
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)    #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(5,m1)    #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(6,m1)    #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```